Compressible quark matter in $\mathcal{N} = 4$ SYM

Christiana Pantelidou

Durham University
(before: University of Barcelona)

ArXiv:1707.06989, 1611.05808
with Anton Faedo, David Mateos and Javier Tarrio

Applied holography

Idea: Use holography to study strongly coupled systems that are not amenable to analysis using conventional techniques. So far, applications to:

- Condensed matter systems, e.g. high-T_c superconductors, heavy fermions etc.
- Non-perturbative regime of Quantum Chromodynamics.

Applied holography

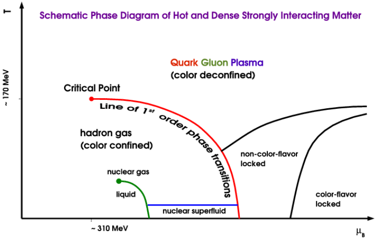
Idea: Use holography to study strongly coupled systems that are not amenable to analysis using conventional techniques. So far, applications to:

- Condensed matter systems, e.g. high-T_c superconductors, heavy fermions etc.
  Current work: e.g. conductivity curvature corrections, Abrikosov lattices, large D holography etc
- Non-perturbative regime of Quantum Chromodynamics.
  Today: status report for David's ERC that just finished.

Introduction: AdS/QCD

The strong nuclear interactions between quarks and gluons are described by **Quantum Chromodynamics (QCD)**.

- Asymptocally free!
- Some progress achieved using lattice simulations: restrictions due to the *sign problem*.
- Non-perturbative aspects remain challeging: confinement, chiral symmetry breaking, phase transitions etc.



QCD string dual

Study using the **gauge/string duality**?

$\rightarrow$ String duals of many gauge theories are known, but QCD itself is difficult to construct.

$\rightarrow$ Try to extract 'universal' behaviour, predictions that are robust enough to apply to QCD as well.

In this talk

Model of interest:

- Study **d=4 SYM** coupled to massless quarks at finite charge density and finite temperature.
- String dual described by the **D3-D7 system** with an **electric flux** on the D7's.

Pros: right dimensionality, natural generalisation of YM.
Cons: not asymptotically free, exhibits a Landau Pole [Benini at al].

- restrict the energy range to that sufficiently below the LP to extract results insensitive to the UV.
- shed light on holography for UV-incomplete theories in general?

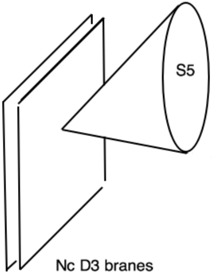
Outline

1. Introduce the various ingredients step-by-step:
   - D3 branes
   - Add D7 flavour branes
   - Add charge density

2. Discuss thermodynamics: towards the phase diagram.

Step 1: d=4 SYM

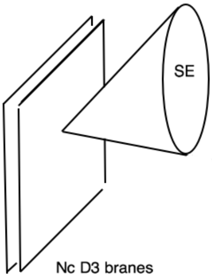
d=4 $\mathcal{N}$=4 $SU(N_c)$ SYM theory $\Leftrightarrow$ N_c D3-branes in flat space



- This corresponds to AdS_5XS^5, supported by F_5 flux.

Step 1: d=4 SYM

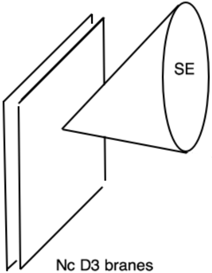
d=4 $\mathcal{N}$=4 $SU(N_c)$ SYM theory $\Leftrightarrow$ N_c D3-branes in flat space



- This corresponds to $AdS_5 X S^5$, supported by F_5 flux.
- Susy can be reduced to $\mathcal{N}$=1 by generalising the internal manifold to a SE: quiver gauge theories.
- Dimensionless coupling constant: $g^2_{YM} \sim (length)^{d-4}$, the gauge theory is conformal ($\beta = 0$).

Step 1: d=4 SYM

d=4 $\mathcal{N}$=4 $SU(N_c)$ SYM theory $\Leftrightarrow$ N_c D3-branes in flat space



- This corresponds to AdS_5XS^5, supported by F_5 flux.
- Susy can be reduced to $\mathcal{N}$=1 by generalising the internal manifold to a SE: quiver gauge theories.
- Dimensionless coupling constant: $g^2_{YM} \sim (length)^{d-4}$, the gauge theory is conformal ($\beta = 0$).

- Degrees of freedom in the adjoint... quarks? Need to add D7 "flavour" branes. [Karch,Katz]

Step 2: Add flavour I

Add N_f **D7 branes** corresponding to massless quarks.

	x^1	x^2	x^3	r			SE		
D3	$\times$	$\times$	$\times$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\cdot$	$\cdot$

$\rightarrow$ Solving for metric and fluxes is a difficult: solve PDEs.
$\rightarrow$ Simplify by smearing homogeneously the D7's in the internal directions: introducing quarks with all possible quantum numbers. [Bigazzi et al.]
$\rightarrow$ Solve (analytically) the eom, including backreaction, to construct the geometry and the fluxes.

Step 2: Add flavour II

One finds: [Benini et al.]

→ Gravity side:

- Flavour is irrelevant in the IR: (log) AdS_5 *at low energies.*
- The UV is altered significantly. The dilaton blows up: the theory develops a LP.

→ FT side: This generates an RG flow with $\beta \sim \frac{N_f}{N_c} > 0$.

Interesting: LP is described by a hyperscaling violating metric (with $\theta = 7/2$). [CP et al.]

Landau Pole physics: UV cut-off

Questions:

1. How is the UV **cut-off** manifested in the geometry? the $D = 5$ metric, g, is non-monotonic. [CP et al.]

- maximum number of degrees of freedom $n \sim g_{xx}^{3/2}$.
- the radial proper distance is finite: $\int \sqrt{g_{\rho\rho}} d\rho$ converges.

2. Are the solutions valid? Some of the effects of the Landau Pole are within the region where supergravity **can be trusted**.

Thermodynamics I

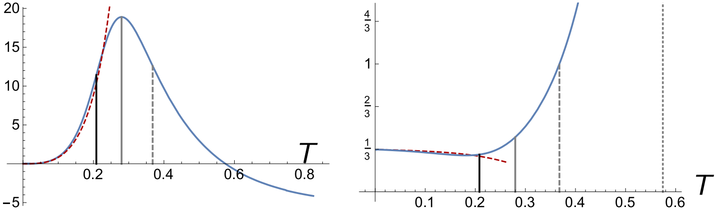
Upto now everything was at $T = 0$. Add temperature.

- The RG flows (numerical) are essentially like before, just cut by the horizon.
- Study thermodynamics: regularise action.
  $\rightarrow$ Usual counter-terms are enough! Why?
  $\rightarrow$ Relate the HV metric to AdS by analytic continuation in the number of dimension.

Thermodynamics II

- Define all the thermodynamic quantities in the usual way:

$$C_v = TdS/dT \qquad c_s^2 = dp/d\epsilon = S/C_v$$



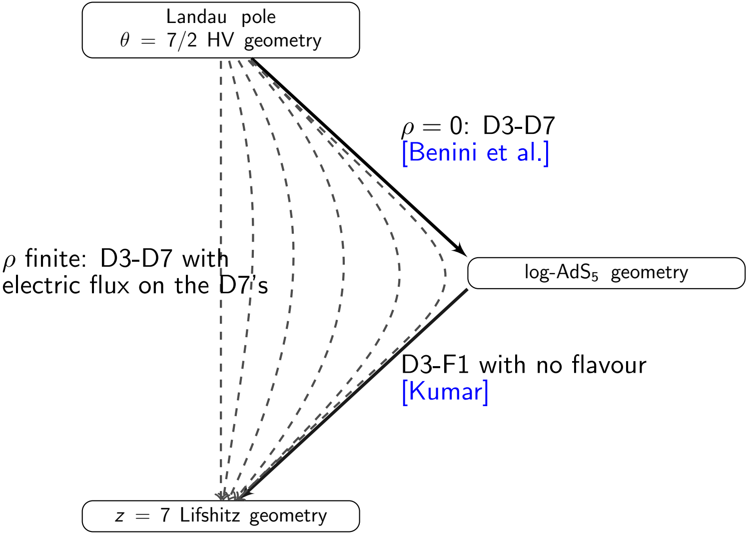
- Specific heat: becomes negative, signaling an instability.
- Speed of sound: grows above 1/3 (conformal value) and, in fact, diverges at some finite temperature.

Step 3: Add charge

Turn on a chemical potential by adding N_{st} units of **electric flux** on the D7 flavour branes: [Witten]

- Only parameter appearing in eom is $\rho \sim \frac{N_c^{1/4} N_{st}}{4 N_f^{1/2}}$.
- The charge is relevant in the UV: doesn't change the asymptotics.
- Conformality in the IR is broken by the new scale: the theory now *flows to a Lif solution with* $z = 7$: $t \to \lambda^7 t, \quad x \to \lambda x$.

Pictorial representation RG flows



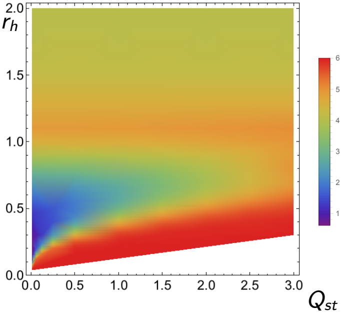
Constructing the flows

→ Solve **numerically** a set of coupled ODEs, for various values of the quark density.
→ We recover the expected behaviour:

Scaling power of the dilaton at the horizon:

$$c = \left. \frac{r\,(e^{\phi})'}{e^{\phi}} \right|_{r=r_h}$$

- LP: $e^{\phi} \sim r^4 \Rightarrow c = 4$
- Lif: $e^{\phi} \sim r^6 \Rightarrow c = 6$
- AdS: $e^{\phi} \sim const \Rightarrow c = 0$



Towards the Phase diagram

$\rightarrow$ Study thermodynamics: renormalise the action, like before.

$\rightarrow$ Stability properties?

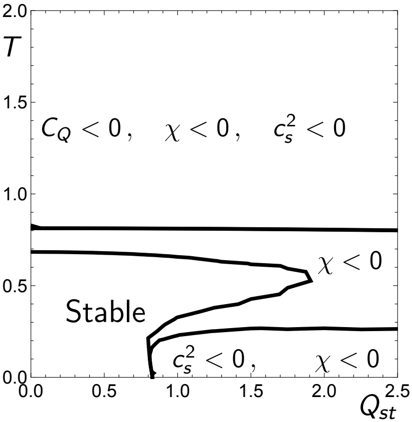
- Thermodynamic stability requirement: Hessian is positive definite.

$$C_Q = T \frac{\partial s}{\partial T}\Big|_{Q_{st}} > 0 \,, \qquad \chi = \frac{\partial Q_{st}}{\partial \mu}\Big|_{T} > 0 \,.$$

- Dynamical stability: How does the speed of sound, c_s^2, behave?

$$c_s^2 > 0 \,.$$

The Phase diagram



- Unstable at high T due to LP, as in chargeless case.
- Unstable at low T and high Q_{st} : inhomogeneous phase.
- $c_s^2 < 0$: the sound mode goes unstable.

Conclusions

We studied d=4 $\mathcal{N}$=4 SYM theory with dynamical quarks at finite density and finite temperature.

- Hints toward spatially modulated phase transitions. Construct them?
- Study Colour Flavour Locking superconductors? consider instantons ($F \wedge F \neq 0$) on this background: desolved D3's in the D7's.
- Study FFLO (spatially modulated superconducting) state? [Donos,Gauntlett,CP]

Thank you!