Thermalisation of pure states

(à la recherche de l’information perdue)

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Many Body Quantum Chaos, Bad Metals & Holography
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work with

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The Mathematics of Physics
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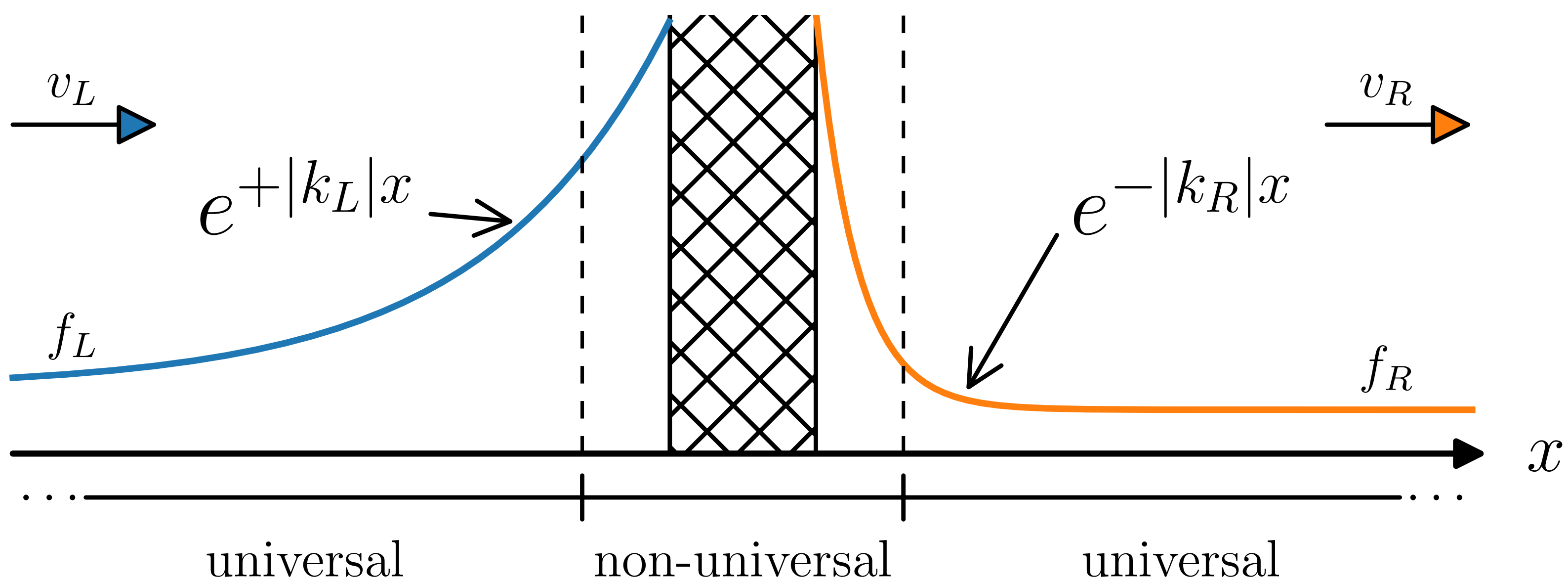
Universality of NESS

What I don't have time to talk about today, but wish I had



Universal features of spatial structure of NESS are determined cleanly by

$$\eta/s$$

Can(?) measure e.g. in thermoelectric probes of graphene
[Benjamin Withers & Julian Sonner, PRL 2017]

back to the topic of my talk

Setting the stage

AdS/CFT relates gravity (often in AdS) to unitary field theory (often CFT)
Familiar notions of quantum field theory are geometrized

Want to explore CFT ➛ (quantum) gravity
recent revival of interest in low-D toy models (AdS_3/CFT_2, SYK,…)

➙ relevant developments in CFT, many-body physics:

- time evolution and spread of entanglement
- thermalization of closed quantum systems (e.g. via eigenstates)
- non-perturbative methods (e.g. bootstrap)

Thermalization ➛ BH formation (& evaporation)

Unitarity at stake

[Hawking, Maldacena]

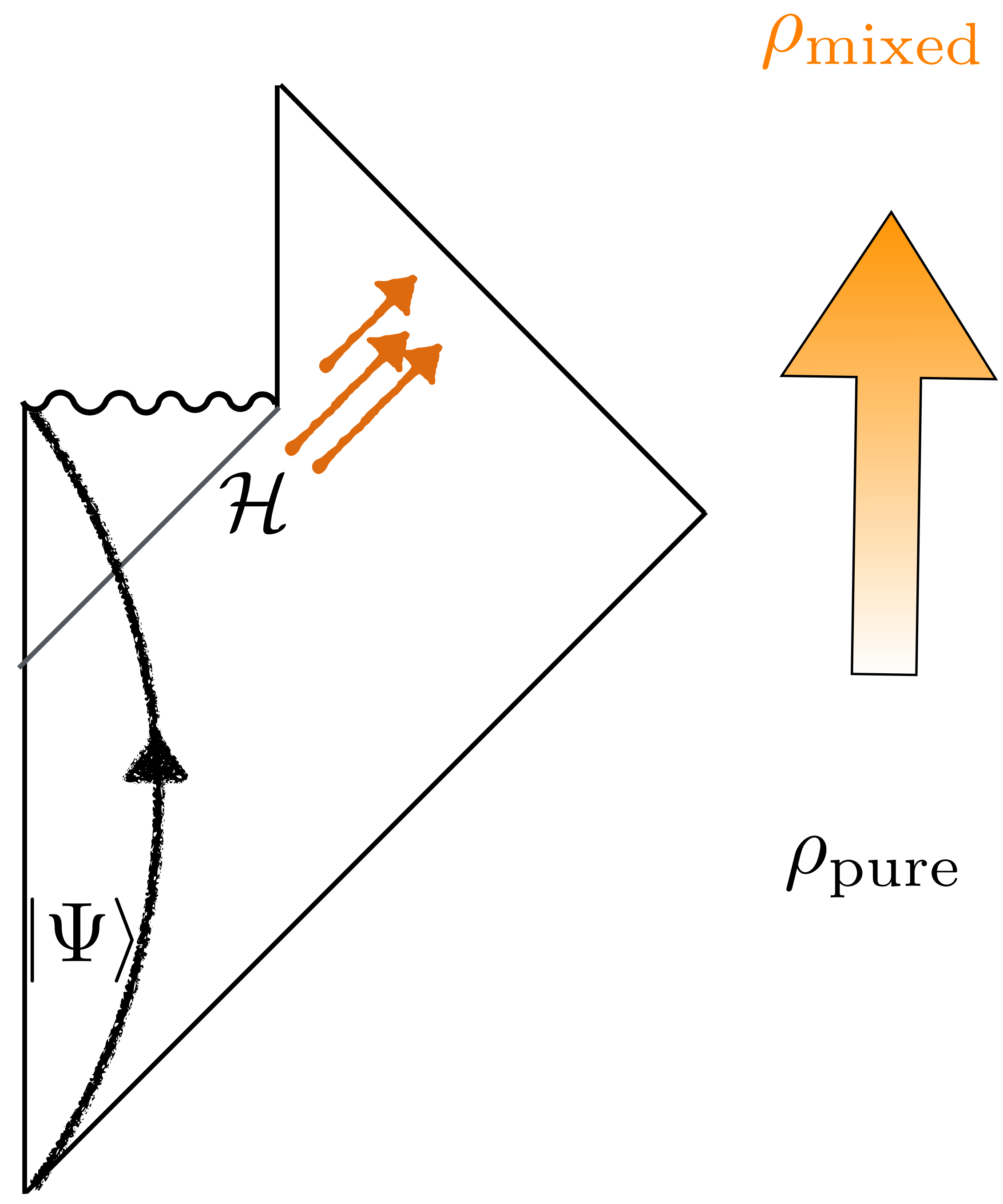
- gravity as an EFT implies pure to mixed evolution

- fundamentally incompatible with a unitary S-matrix

Use simplified laboratory of AdS_3/CFT_2

1. Signatures of information loss in CFT correlations @ large c

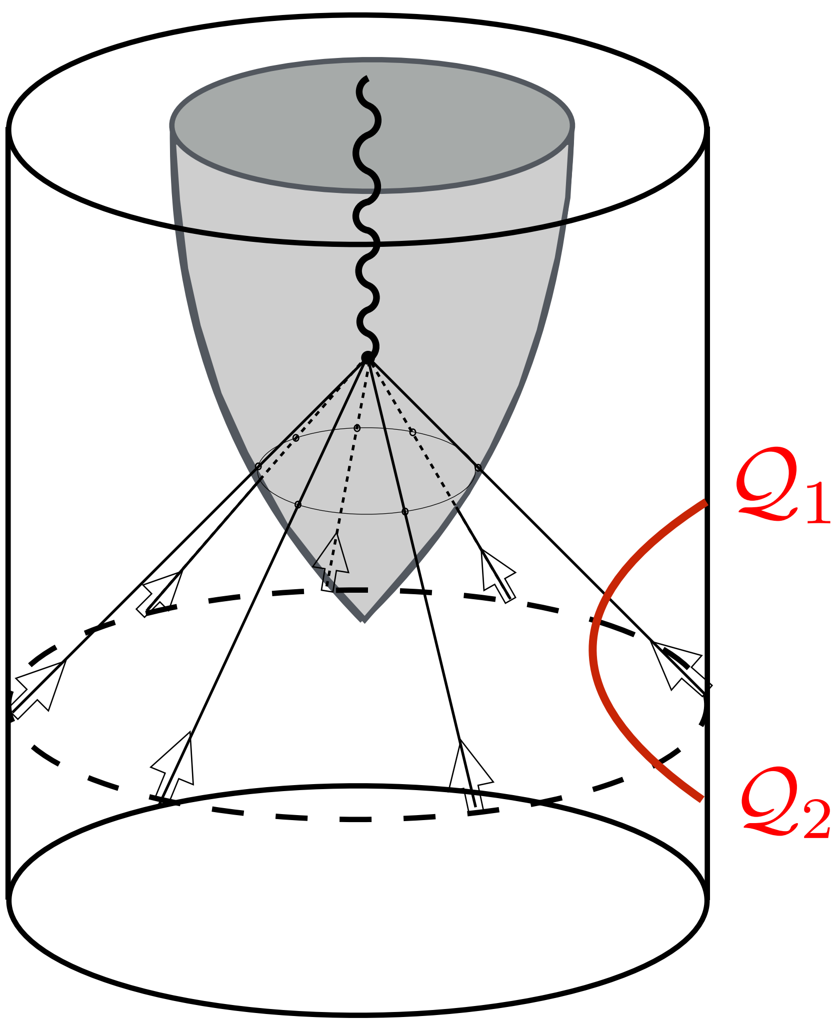
2. New results on bulk-boundary relation in semiclassical limit



Approach

Tension with unitarity is sharpest for collapsing black hole

→ how do we describe black-hole collapse in CFT?



$$\langle \mathcal{V} | \mathcal{Q}_1(t,0)\, \mathcal{Q}_2(0) | \mathcal{V} \rangle$$

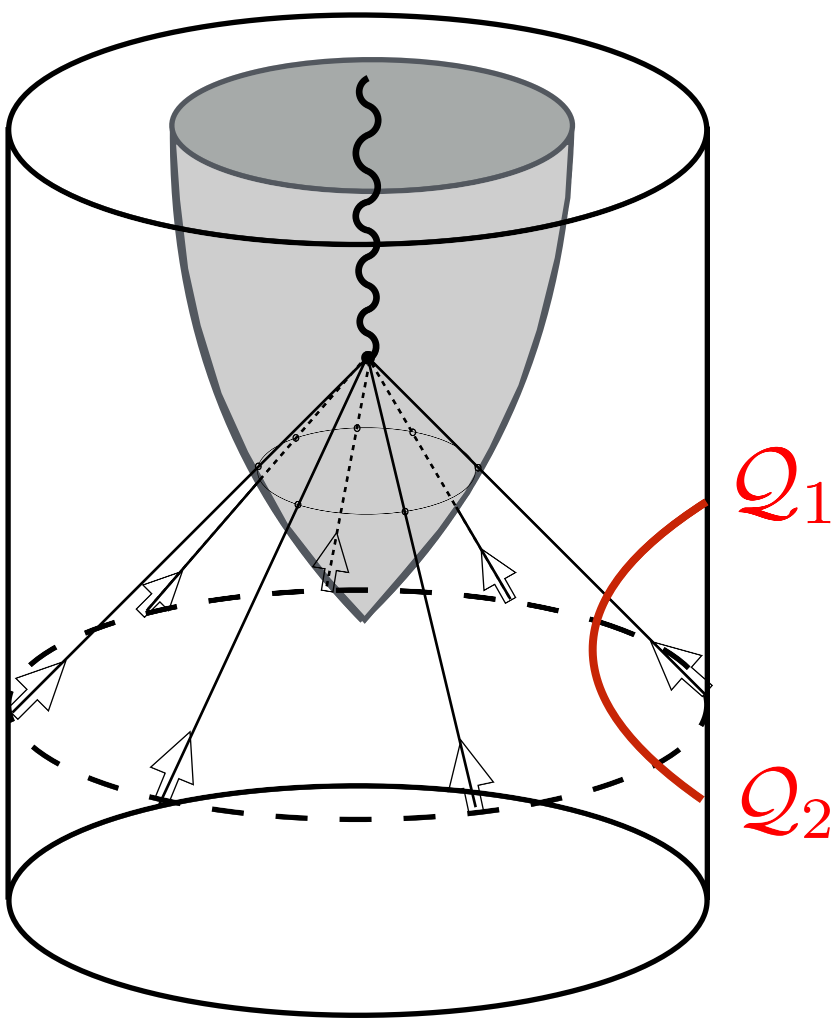
$|\mathcal{V}\rangle$ heavy pure state → BH collapse

measure correlations of light probe operators $\mathcal{Q}$

Results

Follow CFT from quench to thermalisation at large c
[also: Calabrese, Cardy; Hartman, Maldacena]

Calculate Lorentzian physics via continuum monodromy method: entanglement, autocorrelation,…



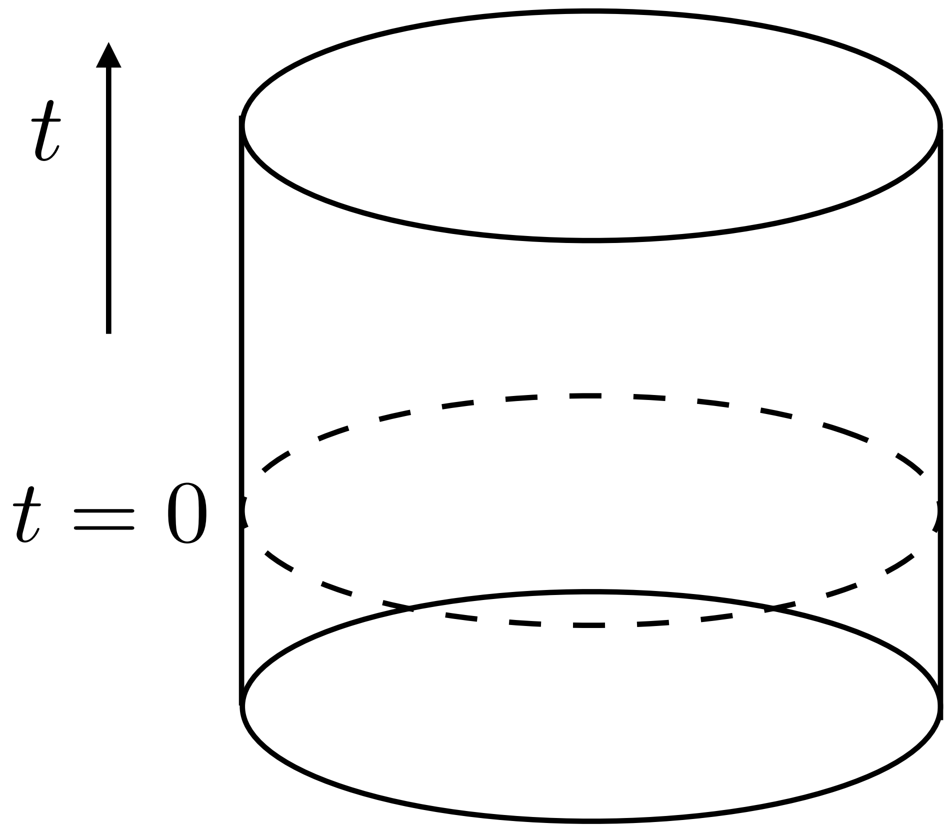
Results at large c: match gravity calculations in Vaidya

Autocorrelation: signs of information loss and retrieval

General correlation function: from conformal blocks to path integral

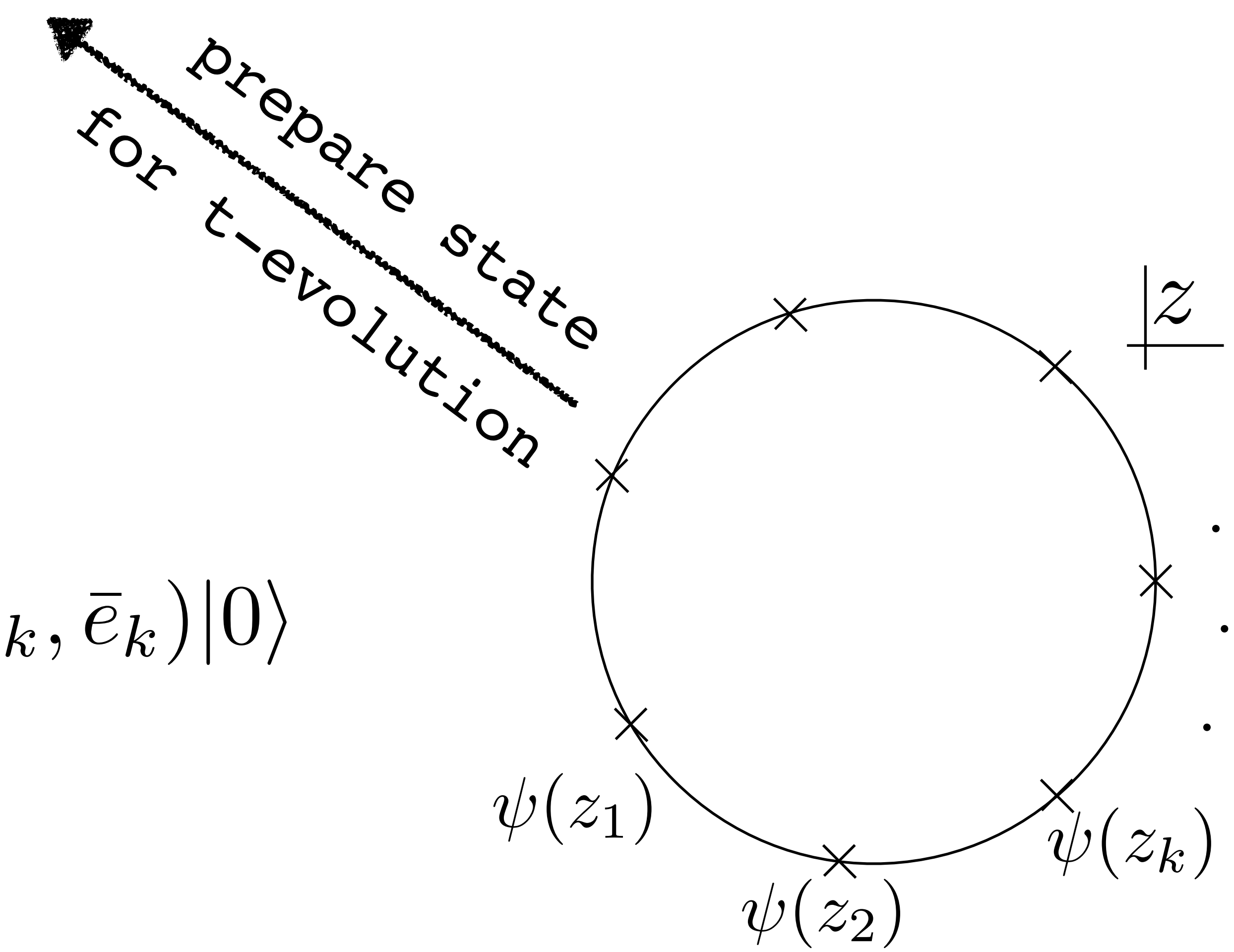
information loss in CFT

BH collapse in CFT



start in excited state at t=0:

prepare by Euclidean path integral
➛ regulator σ



$$|\mathcal{V}\rangle = \frac{1}{\mathcal{N}} \prod_{k=1}^{n} \psi(e_k, \bar{e}_k)|0\rangle$$

ψ is primary

Vacuum dominance

in the semi-classical limit (large c), get sum of exponentials

$$\langle \mathcal{V} | \mathcal{Q}_1(x_1) \mathcal{Q}_1(x_2) | \mathcal{V} \rangle = \sum_{\text{blocks}} a_k e^{-\frac{c}{6} f_k^{(n)}(x_1, x_2)}$$

correlator approximated by largest term, the identity block

"it from **id**"

the dominant contribution comes from the identity Virasoro block, that is the unit operator **id** *and all its descendants $T, \partial T, T^2\, T\partial T \ldots$, (multi-graviton exchange in bulk)*

subleading corrections exponentially suppressed in $e^{-c} \sim e^{-1/G}$

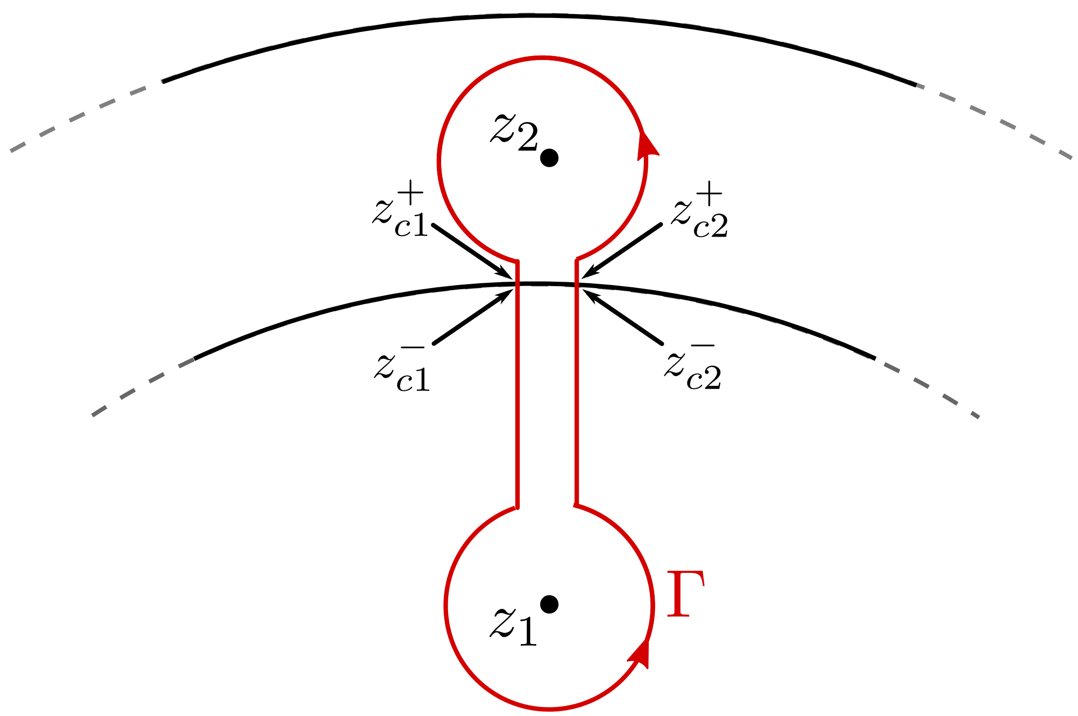
Autocorrelation

let us now return to the black hole and compute

$$G(t_1, t_2) = \langle \mathcal{V} | \mathcal{Q}_1(t_1, 0) \mathcal{Q}_2(t_2, 0) | \mathcal{V} \rangle$$

Dominated by a **single** id channel

$$\mathcal{F}_0^{\Gamma(0)} = \exp\left[-\frac{c}{6} f_0^{\infty}(t_1, t_2) \right]$$



Determine semiclassical block
from monodromy problem
[Zamolochikov]

$$\boxed{G(t_1, t_2) = \left(\frac{1}{\pi T} \cos\left(\frac{t_1}{2} \right) \sinh\left(\pi T t_2 \right) - 2 \sin\left(\frac{t_1}{2} \right) \cosh\left(\pi T t_2 \right) \right)^{-2\Delta^{\mathcal{Q}}}}$$

Late Lorentzian times

Let us return to the original question of information loss

The correlation function decays without bound at large time

$$\boxed{G(t_1,t_2) \sim \exp(-\frac{2\pi\Delta^{\mathcal{Q}} t}{\beta})}$$

Manifestly in conflict with unitarity: **CFT loses information!**

But leading result comes with non-perturbative corrections

$$G(t_1,t_2) = \underbrace{a_0 e^{-\frac{c}{6} f_0^\infty}}_{\text{Vaidya geometry}} + \sum_{k \neq \text{vac}} \underbrace{a_k e^{-\frac{c}{6} f_k^\infty}}_{\text{Other states}}$$

On information loss

This is the anti-information paradox: what happened to unitarity?

$$\overline{|G(t)|} = \overline{\left|\sum_{n,k} e^{i(E_n - E_k)t}\Psi_n^*(\mathcal{V})\langle n|\mathcal{Q}|k\rangle\langle k|\mathcal{Q}|\mathcal{V}\rangle\right|} \neq 0$$

➙ (average) correlations cannot become arbitrarily small
(see also [Barbon & Rabonivici])

Neglected non-perturbative corrections. They contribute

$$\sum_{k\neq \mathrm{vac}} a_k e^{-\frac{c}{6}f_k^{\infty}(1,2,\ldots p)} \sim e^{-S}$$

restore unitary at large time ➙ non-perturbative effects in $1/G_N$

Comments

Boundary story is that of thermalization.
Non-unitary truncation, corresponds to leading bulk answer

Can investigate similar questions for heavy eigenstates

$$\langle \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \mathcal{O}_H \rangle \sim \langle \mathcal{O}_L \mathcal{O}_L \rangle_{T_H}$$

Closely related to study of ETH in CFT [Dymarsky et al.; Datta et al., JS & Vielma]

[Kaplan et al.] looked at contributions from higher blocks:
non-exponential late time behaviour $t^{-3/2}$

Not good enough: need to sum over all heavy blocks
Similar story for spectral form factor [Dyer & Gur-Ari]

from conformal blocks to path integrals

General correlation function

suppose we would like to compute

$$G(t_1, x_1 | t_2, x_2) = \langle \mathcal{V} | \mathcal{Q}_1(t_1, x_1) \mathcal{Q}_2(t_2, x_2) | \mathcal{V} \rangle$$

no longer dominated by a **single** id channel. Prescription:

$$\boxed{G(t_1, x_1 | t_2, x_2) = \int dx_c \left| \mathcal{F}_0^{\Gamma(x_c)} \right|^2}$$

Sum over **id** in all channels (looks odd from CFT perspective)

(remark: **id** in one channel = sum over heavies in another)

Complex saddle points

consider probe with $1 \ll h_{\mathcal{Q}} \ll c$

evaluate correlator via saddle-point

$$G(t_1, x_1 | t_2, x_2) = \int dx_c \underbrace{\mathcal{F}_0^{\Gamma} \overline{\mathcal{F}_0^{\Gamma}}}_{\in \mathbb{C} \text{ (continuation to Lorentzian)}}$$

we find complex saddle points: $x_c \in \mathbb{C}$

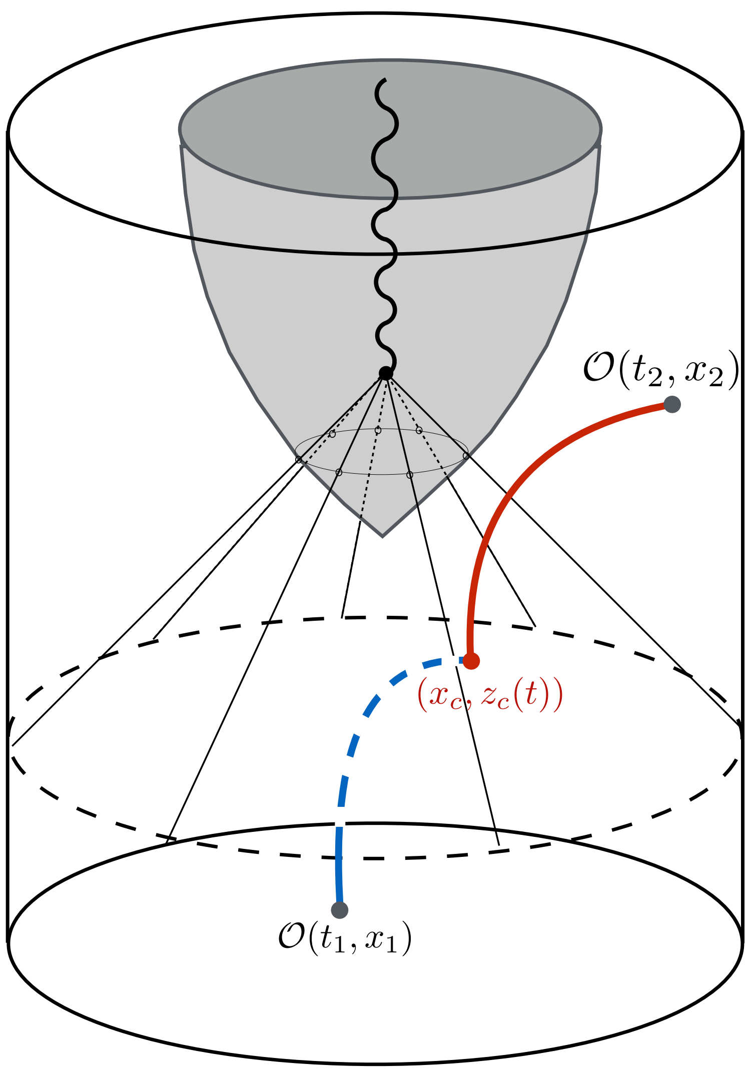
radical change of philosophy of Virasoro id block:

bulk physics is **not** well approximated by **id** in any single channel

Bulk perspective

$$G(t_1, x_1|t_2, x_2) = \int [Dx(\tau)]\, e^{im\int d\tau}$$

$1 \ll h_{\mathcal{Q}} \ll c$ → geodesic approximation



$$\boxed{G(t_1, x_1|t_2, x_2) = \int dx_c e^{i\Delta\mathcal{L}(x_1^\mu, x_c) + i\Delta\mathcal{L}(x_c, x_2^\mu)}}$$

Gravity saddle point = CFT saddle point

for same kinematics, get complex saddle point (analytically continued geodesic)

Comments

$\sum_{\Gamma_0}$ vs. $\max_{\Gamma_0}$ prescription:

All previous cases: real exponential, so $\sum_{\Gamma_0} = \max_{\Gamma_0}$

Lorentzian dynamics allows to distinguish, and $\sum_{\Gamma_0}$ comes out on top

Aren't we overcounting?

Usually sum over blocks, not channels

Working assumption: no overlap between **id** in different channels, when dualized in to a single channel (at large c)

Creates subtlety when looking at 1/c corrections

Summary

time-dependent 3D quantum gravity with matter in 1/c expansion
'it from id' ➙ ideal arena to think about quantum BHs

CFT correlation functions seemingly violate unitarity (naïve).
non-perturbative corrections in c restore unitarity

on gravity side these correspond to non-perturbative effects in $G_{N.}$
geometric interpretation? bulk interpretation?

monodromy method identifies off-shell contributions on both sides:

General map from conformal block expansion to bulk path int?

The geometry of eigenstates

Philosophy

Take a step back: why do closed quantum systems thermalise?

As alluded to before, eigenstate thermalisation gives an answer

$$\boxed{\langle m|\mathcal{O}|n\rangle = \overline{\mathcal{O}}_{\mathrm{mc}}(\overline{E})\delta_{mn} + e^{-S(\overline{E})/2} f(\overline{E},\omega) R_{mn}}$$

Thermalisation = dephasing the levels of a chaotic quantum system

Individual eigenstates are thermal.

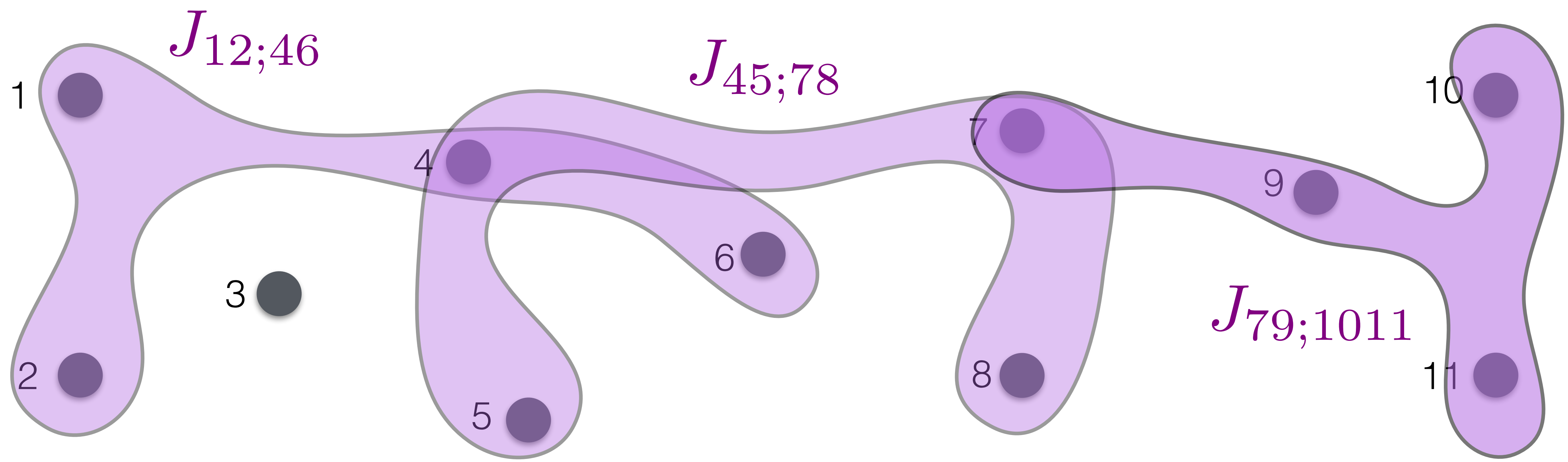
Is there a bulk dual of an individual eigenstate?

Typically, many-body spectrum out of reach, but not in SYK!

[JS, Vielma; Maldacena & Kourkoulou, Maldacena & Stanford, Polchinski & Rosenhaus]

The model

random (quenched) disorder model with all-to-all couplings



$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l$$

$$\left\{ c_i^\dagger, c_j \right\} = \delta_{ij} \qquad \{c_i, c_j\} = 0$$

Couplings $J_{ij;kl}$ are drawn from a Gaussian random distribution with $\overline{J_{ij;kl}} = 0$ and $\overline{|J_{ij;kl}|^2} = J^2$

Comments

I) Origin: construct a controlled spin glass phase [Sachdev & Ye, Parcollet & Georges]

II) Model revived independently by Kitaev: random Majorana fermions, connection to AdS_2 BH

III) Model can be solved in a 1/N expansion: almost conformal at low temperature, finite residual entropy, maximally chaotic [Sachdev & Ye, Parcollet & Georges, Kitaev,…]

IV) Model can be solved in ED for N ~ 20. Spectral properties, dynamics, eigenstate thermalisation [JS, Vielma]

Eigenstates

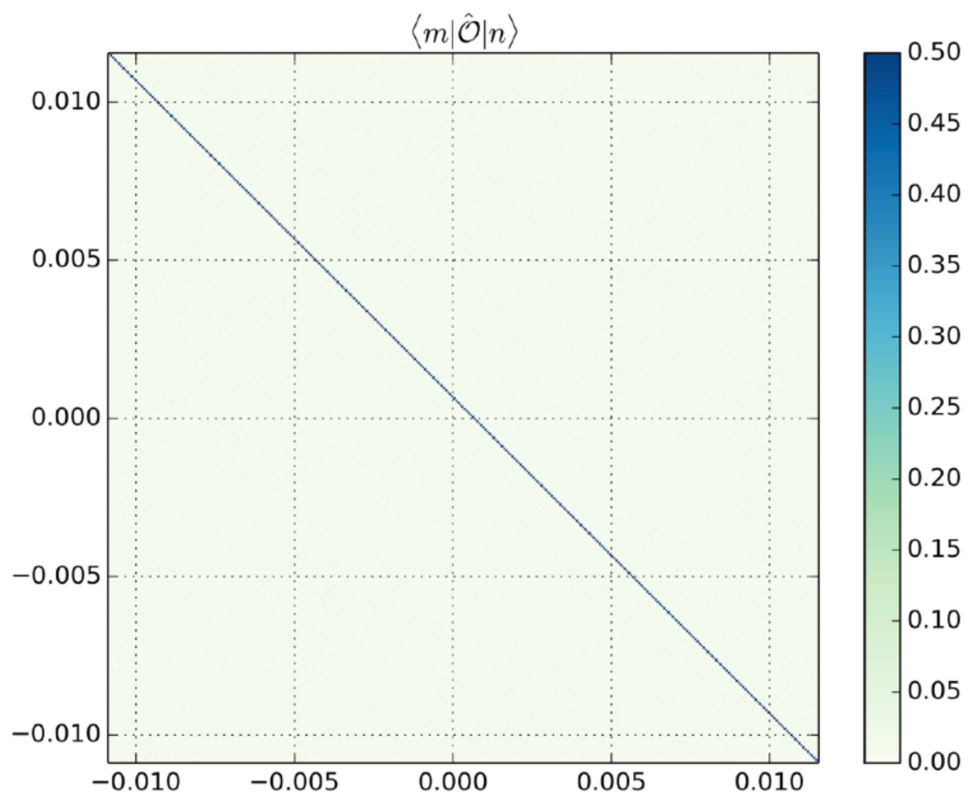
[JS & Vielma]

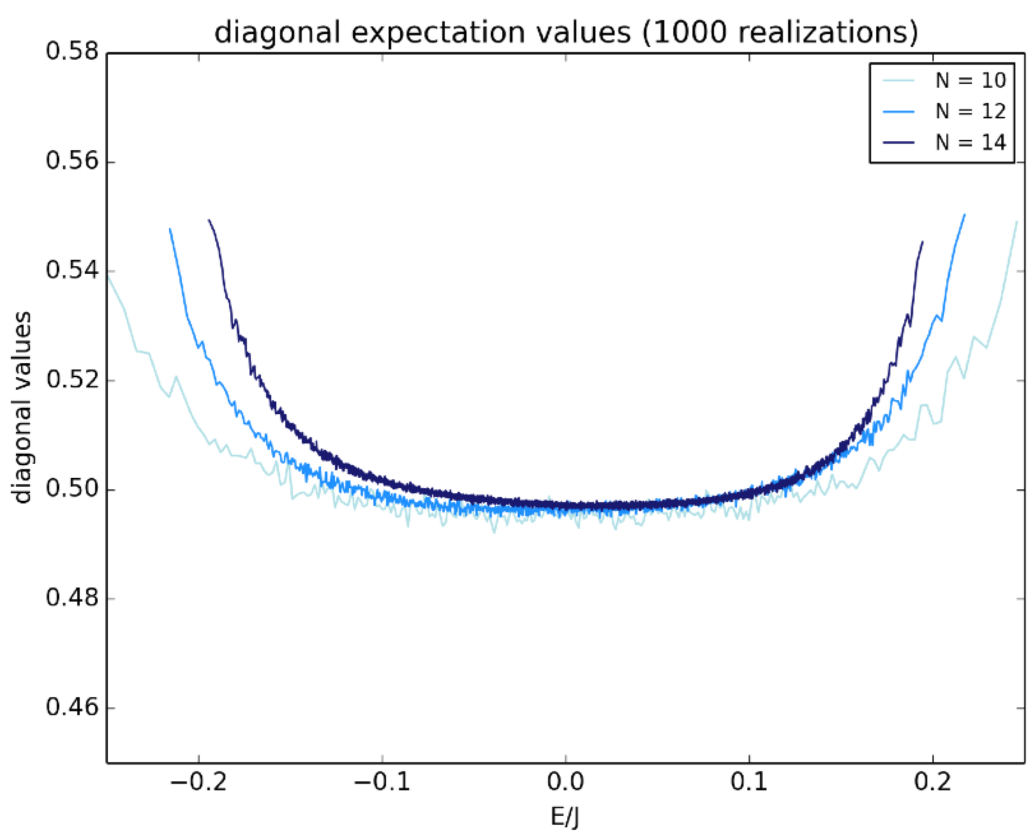
Solve SYK in exact diagonalization

We find (numerically) that indeed ETH is the mechanism in SYK

$$\boxed{\langle m|\mathcal{O}|n\rangle = \overline{\mathcal{O}}_{\rm mc}(\overline{E})\delta_{mn} + e^{-S(\overline{E})/2} f(\overline{E},\omega) R_{mn}}$$

$$\mathcal{O} = \hat{n}_k \quad \text{(for some site} \quad k)$$

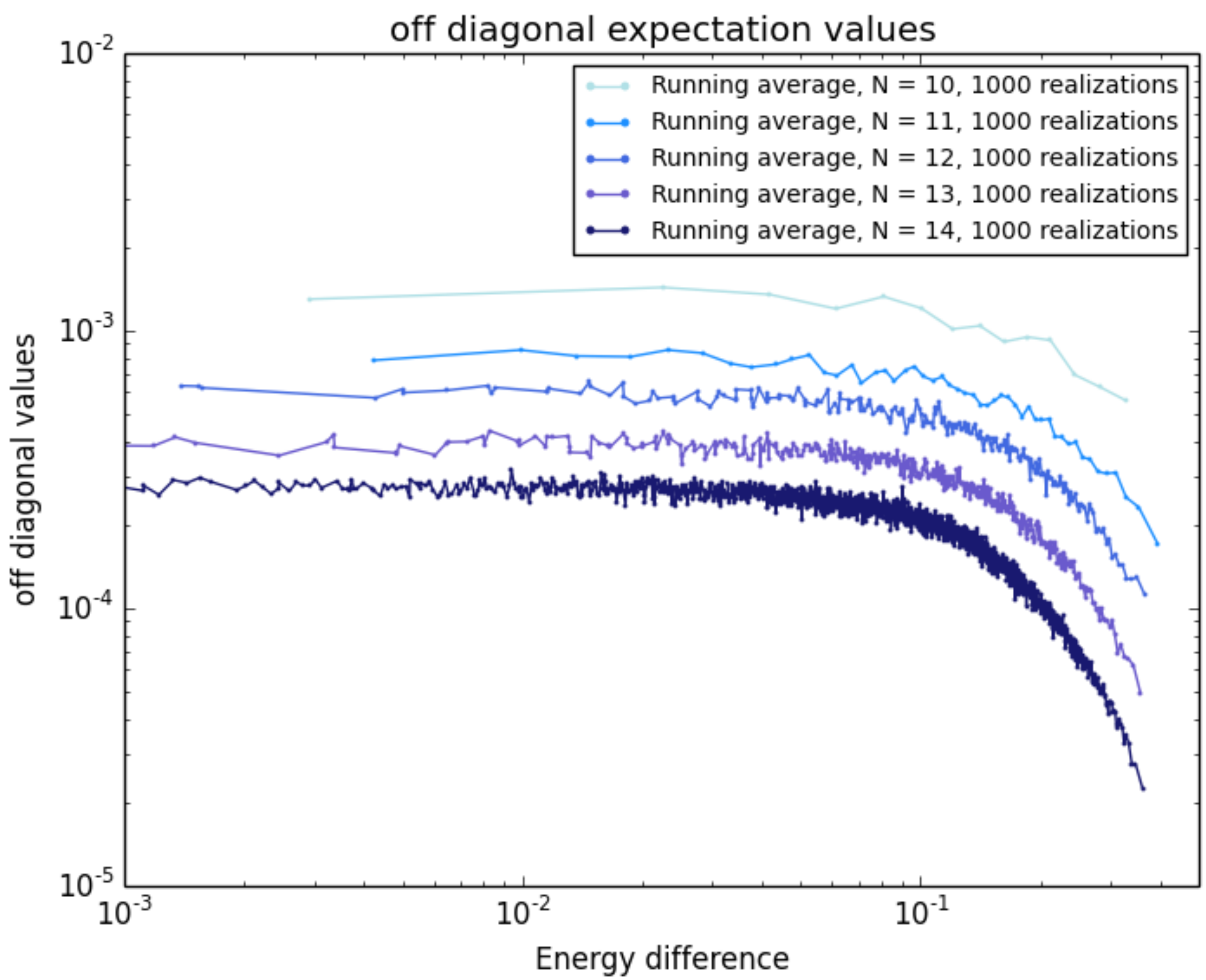




Aside: just random?

Let's look at the off-diagonal matrix elements

$$\boxed{\langle m|\mathcal{O}|n\rangle = \overline{\mathcal{O}}_{\rm mc}(\overline{E})\delta_{mn} + e^{-S(\overline{E})/2} f(\overline{E},\omega)R_{mn}}$$



Crossover from constant (RMT) to non-constant behaviour

Thouless energy

Scrambling in eigenstates

[JS & Vielma]

Consider an out-of-time-order 4-pt function (OTOC)

$$\boxed{\langle A(\tau)B(0)A(\tau)B(0)\rangle \sim e^{-2\lambda_L t} \qquad \lambda_L = \frac{2\pi}{\beta}}$$

= upper bound on Lyapunov exponent [Maldacena, Shenker, Stanford,…]

Matches precisely with result for a black hole. Slight reformulation:

$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

State is highly correlated: $\langle \Psi | V_L V_R | \Psi \rangle \sim \mathcal{O}(1)$

State is non-typical: $\langle \Psi' | V_L V_R | \Psi' \rangle \to 0$

Can trace reason for this to behaviour of OTOC above

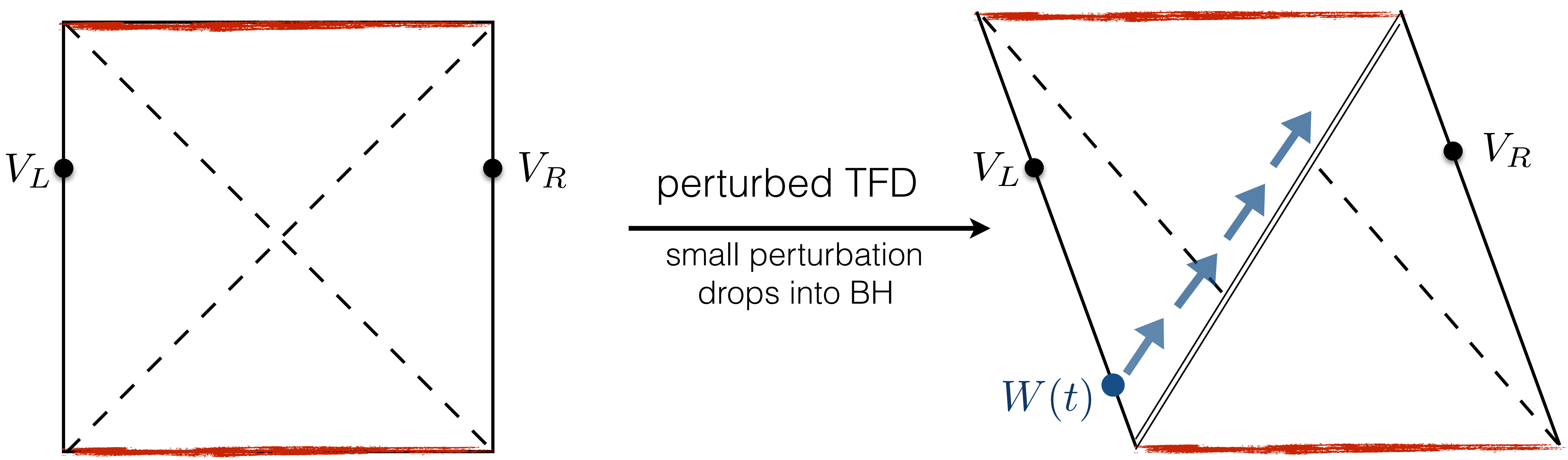
Black holes and chaos

[Shenker & Stanford; … del Campo, Molina-Vilaplana, JS]

'eternal' black hole has two sides (= 'Kruskal extension')

$$\boxed{|\Psi_{\rm BH}\rangle = |\psi(\beta)\rangle}$$

two sides correspond to two sides of thermofield double

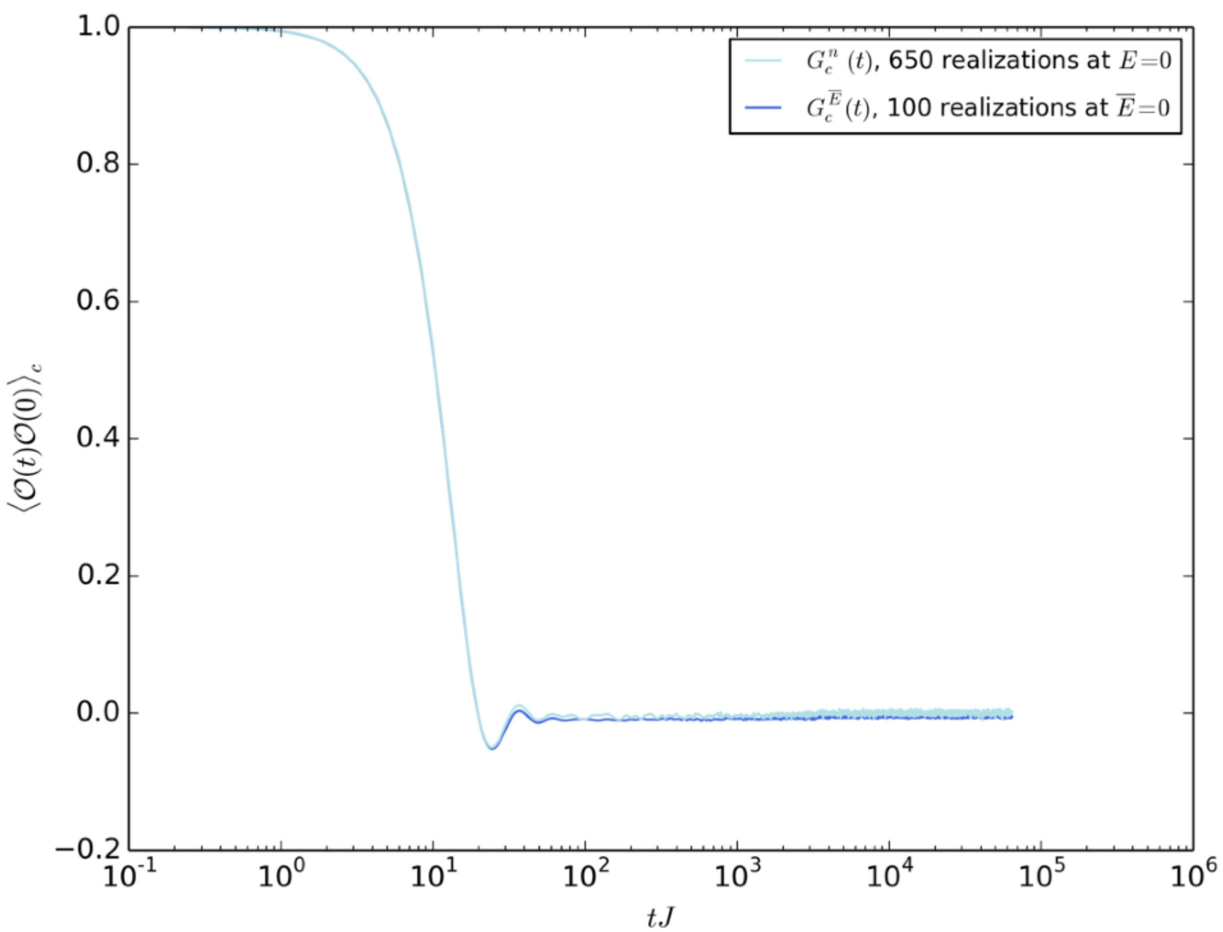


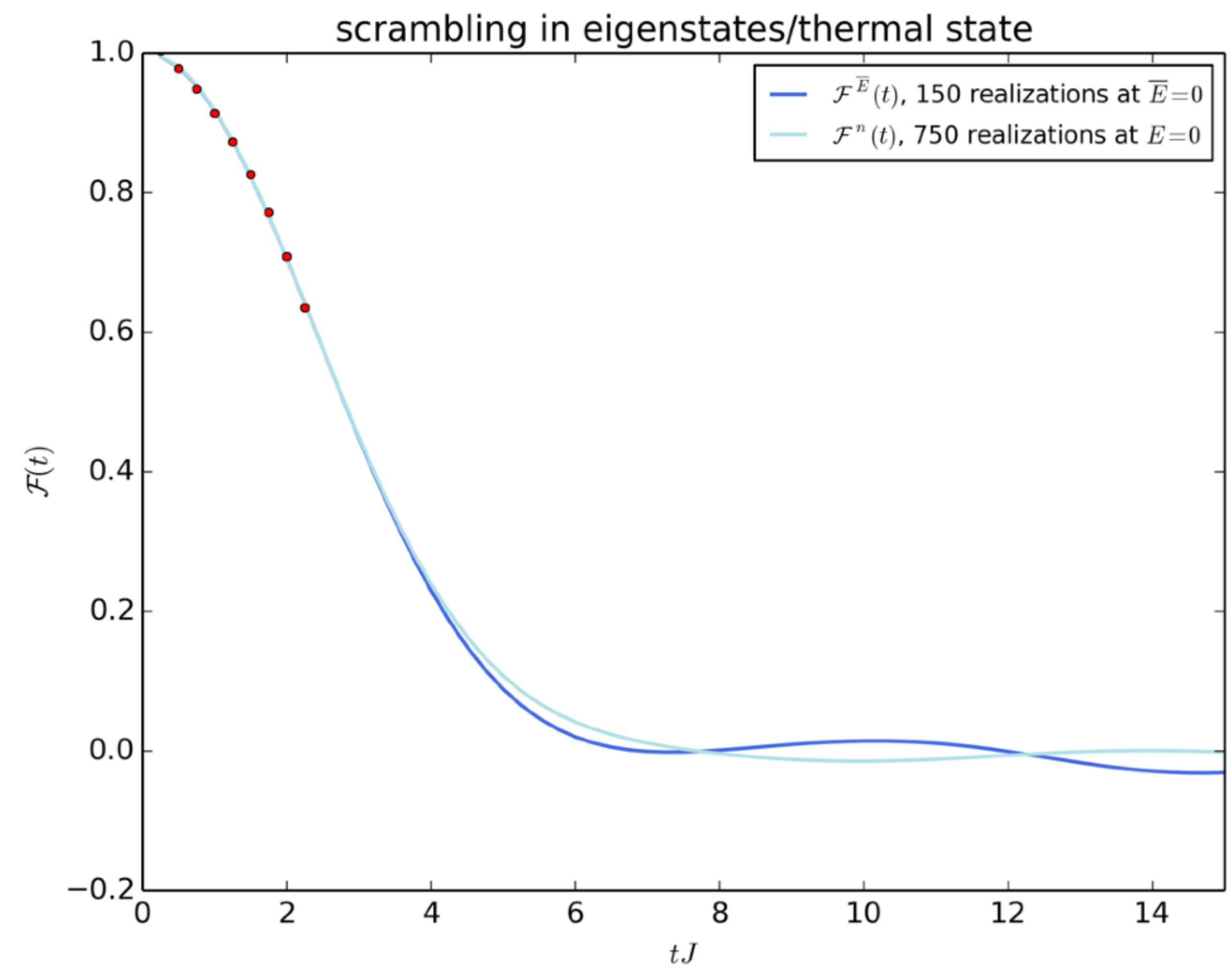
$$\langle\Psi|V_L V_R|\Psi\rangle \sim \mathcal{O}(1) \qquad |\Psi\rangle \to |\Psi'\rangle \sim W(t)|\Psi\rangle \qquad \langle\Psi'|V_L V_R|\Psi'\rangle \to 0$$

Eigenstates and chaos

[JS & Vielma]

Compare OTOC (and 2pt function) in eigenstates to thermal result





Become essentially indistinguishable as system size increases

Conjecture: $\exists\ \lambda_{\mathrm{L}}^{\mathrm{ETH}} = \dfrac{2\pi}{\beta(E)}$

summary and outlook

Summary

Eigenstates in the SYK model are thermal in the sense of ETH

Correlations in individual eigenstates are exponentially close to thermal ones

→ we may operationally treat a single eigenstate as having a dual geometry, up to exponential corrections

We've already seen that these corrections are important to resolve information loss

Comment: [Marolf & Polchinski] used ETH to argue against ER-EPR

Outlook (laundry list)

I) Establish ETH analytically [Nayak, JS & Vielma]

$$\langle m|\mathcal{O}_i|n\rangle = \lim_{m,n\to\infty}\langle \mathcal{O}_m \mathcal{O}_i \mathcal{O}_n\rangle \qquad\qquad \mathcal{O}_k \sim c_i^\dagger \partial^k c_i$$

→ Limit of 6-pt function [Gross & Rosenhaus]

II) Prove conjecture about chaos exponent in eigenstates

III) Attack more generally the problem of bulk reconstruction
Caveat:

- what is the bulk dual of SYK?
- how do we think about random couplings?
- perhaps tensor models are better starting point

Thank you for your attention

Unitarity vs thermalization

(constraints on long-time correlations from unitarity)

Correlations in a closed quantum system, e.g.

$$G(t) = \mathrm{tr} \rho \mathcal{O}(t) \mathcal{O}(0)$$

Time average over a large time T cannot vanish by unitarity

$$\lim_{T\to\infty} \overline{|G(t)|^2} \neq 0$$

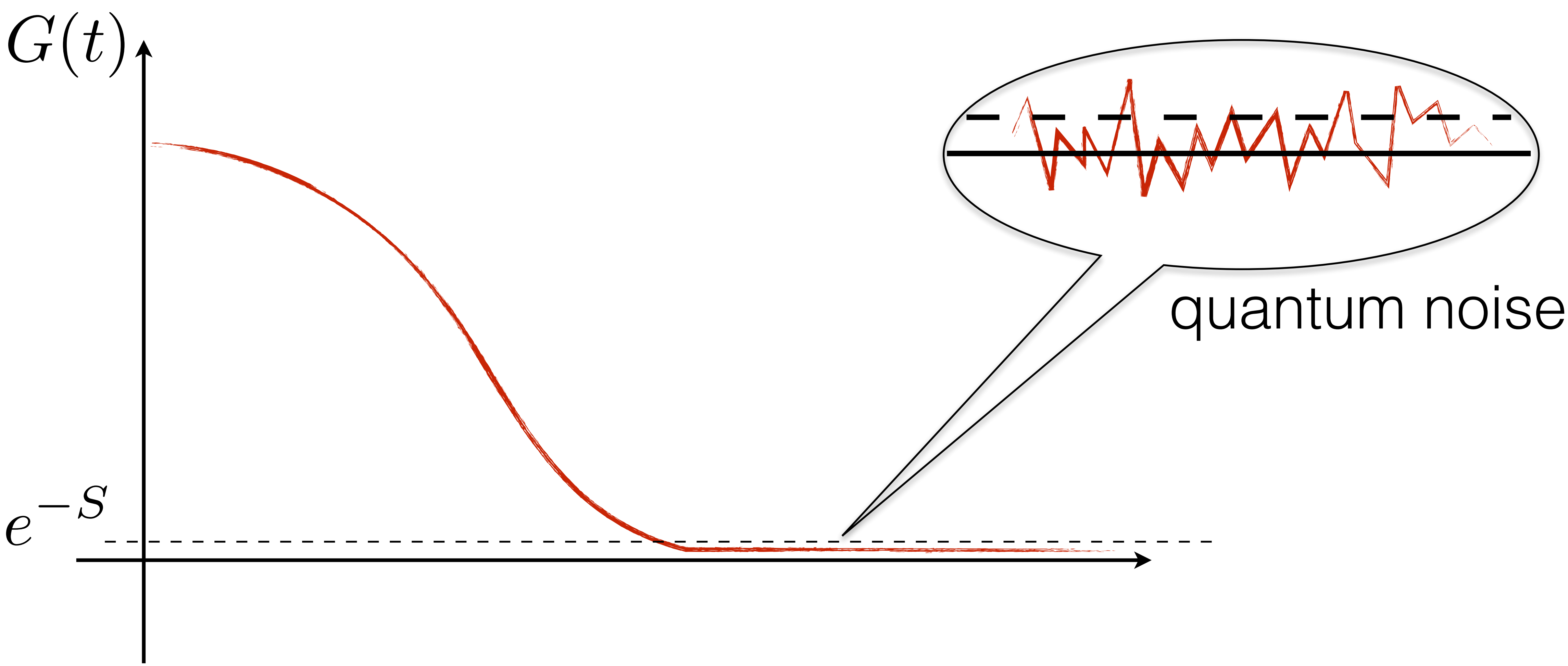
Need to assume spectrum is generic (no specific ordering principle)

➢ connection with ETH

Unitarity vs thermalization

(constraints on long-time correlations from unitarity)

$$\rho = e^{-\beta H}$$



see also [Barbon & Rabonivici]