Ward identities and relations between conductivities and viscosities in holography

David Rodríguez Fernández

Oviedo University

C. Hoyos and D. Rodríguez Fernández

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We shall consider a **strongly coupled QFT** at finite temperature in $2+1$ dimensions with gravity dual and its relations between transport coefficients through the **Ward Identities**

For $\tilde{\Gamma}^{\alpha\beta\mu\nu} = \langle T^{\alpha\beta} T^{\mu\nu} \rangle_R$, the Ward identities are

First kind

$$\frac{\partial}{\partial x^\alpha} \widetilde{\Gamma}^{\alpha\beta\mu\nu}(x, x') \approx 0$$

Well known from "kinematics" (near boundary expansion and EOMS)

Second kind

$$\frac{\partial}{\partial x'^\mu} \widetilde{\Gamma}^{\alpha\beta\mu\nu}(x, x') \approx 0$$

Necessary a further "ingredient"

Motivation

In momentum space and $d = 2 + 1$,

Kubo formulas

$$\text{Momentum conductivity} \;\Rightarrow \kappa_{ij} = -\frac{1}{\omega}\,\mathrm{Im}\,\Gamma_{0i0j}(\omega, k),$$

$$\text{Shear viscosity} \;\Rightarrow \eta = -\frac{1}{\omega}\,\mathrm{Im}\,\Gamma_{xyxy}(\omega, k),$$

$$\text{Bulk viscosity and shear viscosity} \;\Rightarrow \eta + \frac{\zeta}{2} = -\frac{1}{\omega}\,\mathrm{Im}\,\Gamma_{xxxx}(\omega, k).$$

Ward Ids. + Kubo Formulas

$\Downarrow$

Relations between transport coefficients

We will consider the bulk Lagrangian density

$$\mathcal{L} = \mathcal{R} - 2\Lambda - (\partial\phi)^2 - V(\phi)\,, \qquad \Lambda = -3\,.$$

From the bulk EOS $\exists$ **conserved quantity + parity invariant**

$$\partial_r \mathcal{J}_{\text{on-shell}} = 0 \implies \mathcal{J}_{\text{Boun}} = \mathcal{J}_{\text{Hor}}$$

$$\frac{1}{2}\left[\mathcal{J}(k) - \mathcal{J}(-k)\right]_{\text{Boun,Hor}} = \left[\mathcal{J}(k)\right]_{\textsf{odd}\ \text{Boun,Hor}} = 0$$

Knowledge of the 2^{nd} kind W.Ids. $\partial_\mu \tilde{\Gamma}^{\alpha\beta\mu\nu} \approx 0$

$\xi \neq 0 \implies$ non-CFT, $[\mathcal{J}]_{\textsf{odd}} \neq 0 \implies$ but, we will still draw useful information

Conclusions

For $k <<$

$$\kappa_{ij} \simeq \kappa_{ij}^{(0)} + (k^2\delta^{ij} - k^i k^j)\kappa_T^{(2)} + k^i k^j \kappa_L^{(2)} + \cdots,$$
$$\eta \simeq \eta^{(0)} + O(k^2), \quad \zeta = \zeta^{(0)} + O(k^2).$$

$+$

Second kind W.ids.

$$\left[\omega^2\Gamma_{0y0y} + k^2\overline{\Gamma}_{xyxy}\right]_{\text{even}} = 0,$$

$$\left[\omega^2\Gamma_{0x0x} + k^2\overline{\Gamma}_{xxxx}\right]_{\text{even}} = \left(\omega^2 + k^2\right) P + k\omega W_{\text{odd}}$$

$+$

Kubo formulas

$\Downarrow$

relations between $\zeta \Leftrightarrow \eta \Leftrightarrow \kappa$

Conclusions

$W_{\rm odd}$ is a mixture of contact terms and cannot be fully determined without solving the EOMS. Expanding $W_{\rm odd} \simeq kW^{(1)}_{\rm odd} + \cdots$, we get

$$\kappa_T^{(2)} = \frac{1}{\omega^2}\eta^{(0)},$$

$$\kappa_L^{(2)} = \frac{1}{\omega^2}\left(\eta^{(0)} + \frac{\zeta^{(0)}}{2} - \mathrm{Im}\, W^{(1)}_{\rm odd}\right).$$

$\Downarrow$ Agrees with field theory

$\Downarrow$ Has the right structure, but we do not know from general arguments what is the contribution from $W_{\rm odd}$.

Up to contact terms ($W_{\rm odd}$), we have derived the transport relations from the second kind W.Ids.

THANK YOU!