AdS/CMT and (*pseudo*-)symmetry breaking

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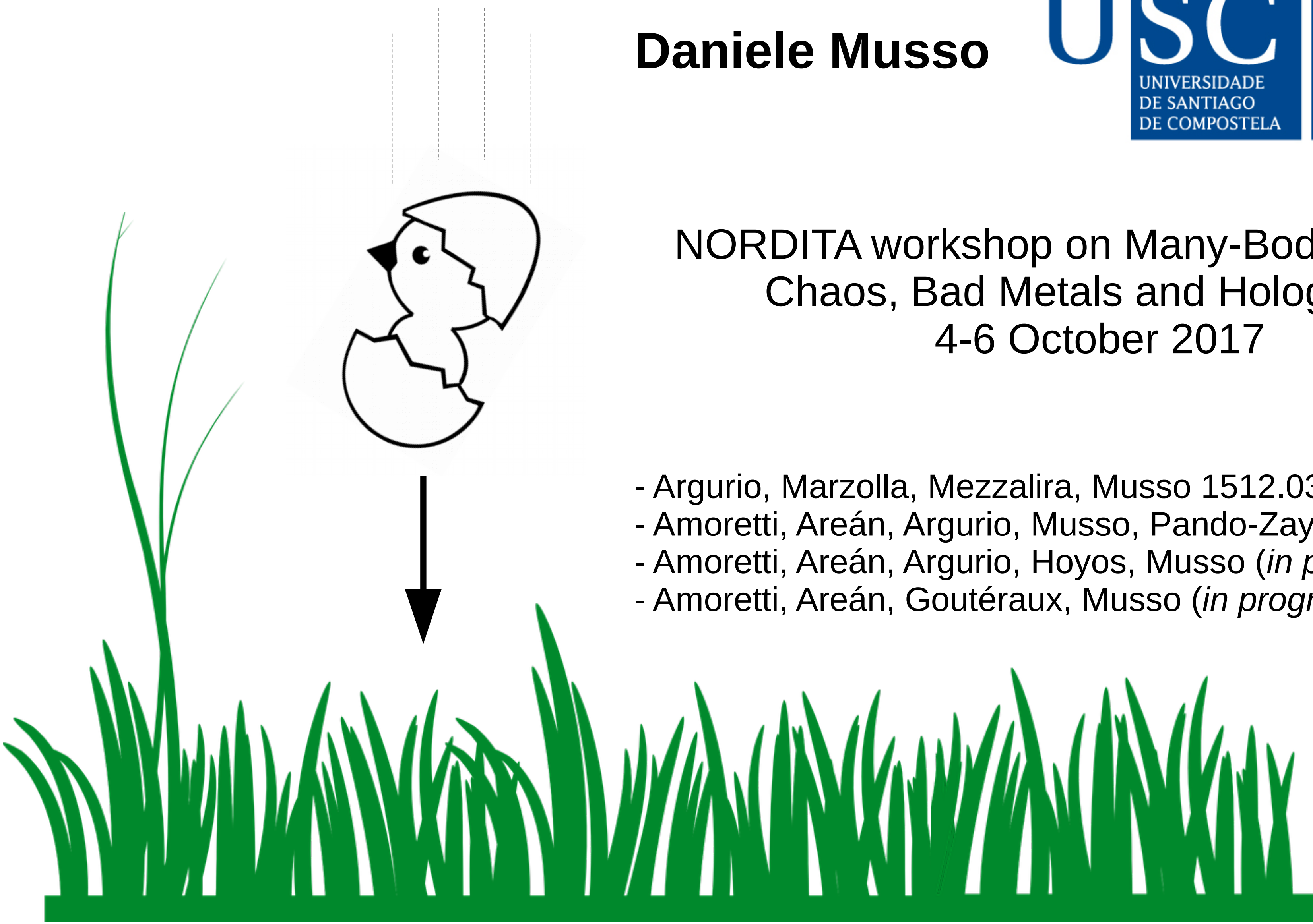
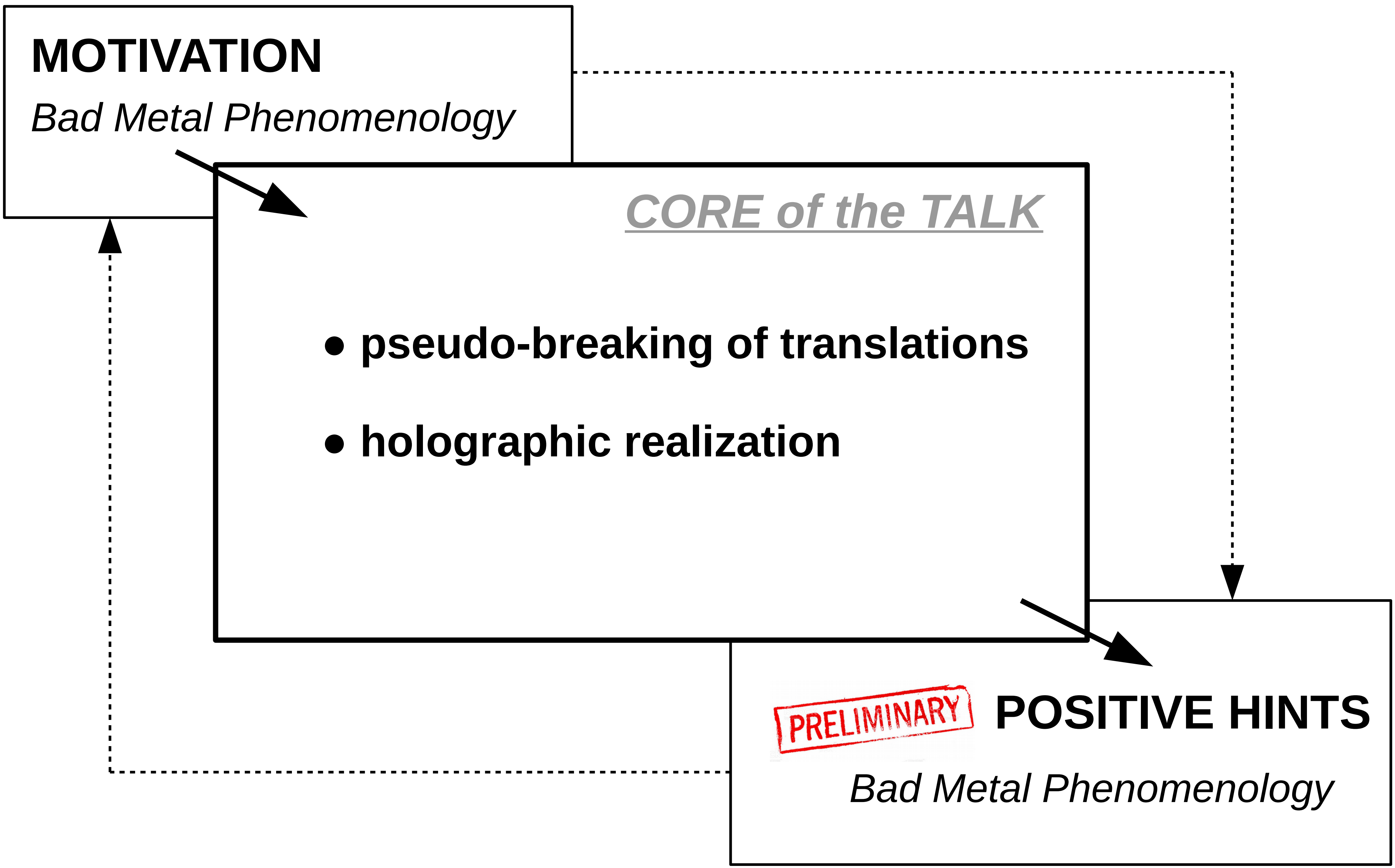
NORDITA workshop on Many-Body Quantum Chaos, Bad Metals and Holography
4-6 October 2017



- Argurio, Marzolla, Mezzalira, Musso 1512.03750
- Amoretti, Areán, Argurio, Musso, Pando-Zayas 1611.09344
- Amoretti, Areán, Argurio, Hoyos, Musso (*in progress*)
- Amoretti, Areán, Goutéraux, Musso (*in progress*)

Plan of the talk



Bad metals phenomenology

[Hussey,Takenaka,Takagi cond-mat/0404263]

- Resistivities exceeding the Mott-Ioffe-Regel (MIR) resistivity bound
  (quasi-particles whose mfp is comparable with interatomic spacing)

- Metallic behavior in the absence of a Drude peak (~~$\Gamma_{\rm mom} \ll T$~~)
  (loss of coherent transport)

'Parallel resistor formula'
(empirical, no second conducting channel)

$$\frac{1}{\rho} = \frac{1}{\rho_{\rm ideal}} + \frac{1}{\rho_{\rm sat}}$$

Absence of minimum metallic conductivity compatible with minimum mean free path, ***however***, MIR criterion for loss of coherence is preserved (also for strong electron correlation)

The *<u>effective scattering rate</u>* (from optical spectra) do saturates for both good and bad metals

→ *(need to focus on the optical properties)*

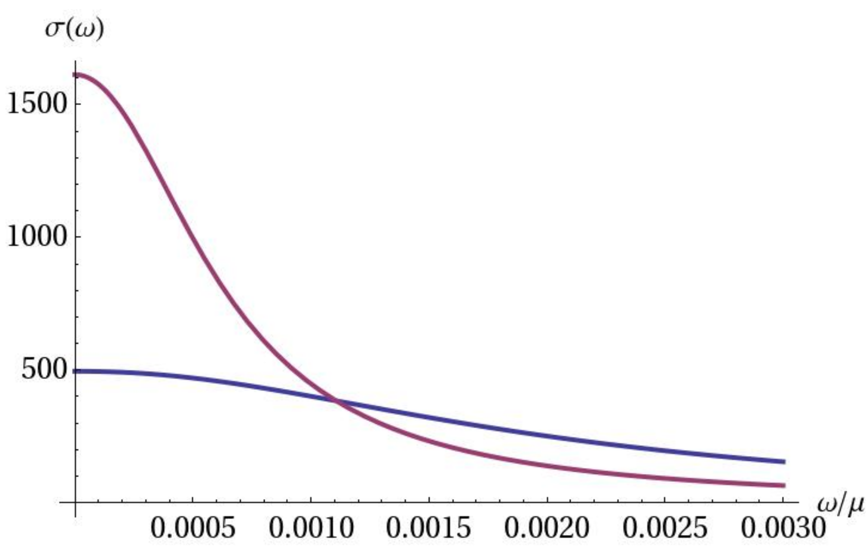
$$\frac{1}{\Gamma_{\rm eff}(T,\omega)} = \frac{1}{\Gamma_{\rm ideal}(T,\omega)} + \frac{1}{\Gamma_{\rm max}}$$

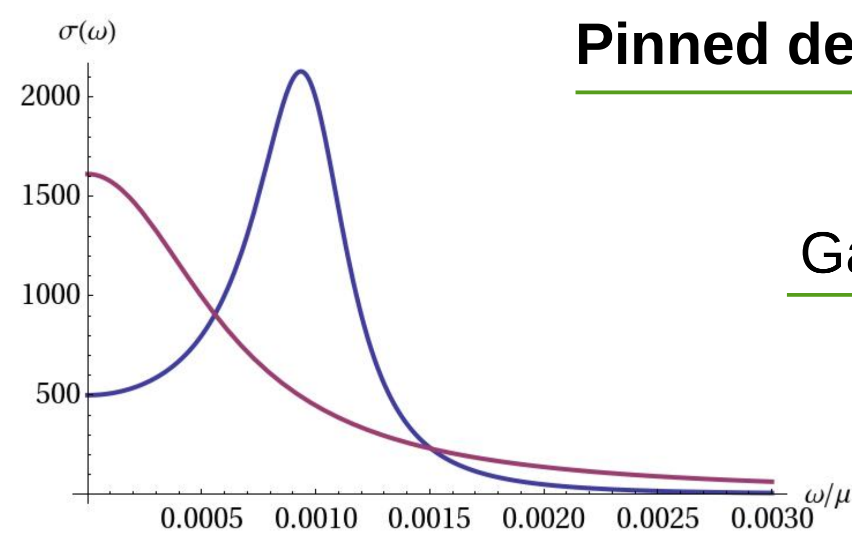
A possible mechanism...

[Basov,Dabrowski,Timusk PRL **81**; Delacrétaz,Goutéraux,Hartnoll,Karlsson 1612.04381]

- No ever increasing scattering rate, no strong disorder. Rather, a mechanism for decoherence
(absence of quasi particle is not sufficient to produce bad metallic behavior)

- *“Bad metallic behavior is associated largely with the absence of a zero frequency collective mode”*. [Hussey,Takenaka,Takagi cond-mat/0404263]
Suppression of low-frequency spectral weight.





Pinned density wave

Gapped peak

PSEUDO-NAMBU-GOLDSTONE MODE

(for holographic pinning see Jokela,Järvinen, Lippert 1708.07837; Andrade,Baggioli,Krikun,Poovuttikul 1708.08306*)*

… supported by a hydro analysis

Pseudo-NG mode due to ***weakly pinned, incommensurate density wave order***

hydro provides *"a unified theoretical description of* AC/DC *transport in terms of short range, quantum critical fluctuations of incommensurate density wave order".* [Delacrétaz,Goutéraux,Hartnoll,Karlsson 1702.05104]

Leads to a formula for the conductivity which encodes the gapped peak mechanism

"Phase relaxation rate" (long-range order disrupted by phase-disordering mobile dislocations)

"Drude weight"

$$\sigma(\omega) = \sigma_{\rm inc} + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

"incoherent conductivity", roughly the conductivity on the right of the peak

"Pinning", position of the gapped peak

"Momentum relaxation rate"

Technical stop…

● Control on the thermodynamic and transport properties of a strongly correlated system *(quantum critical fluctions)*.

● Control on the low-energy dynamic (EFT) of a system featuring coexistence of spontaneous and explicit symmetry breaking.

Let us resort to ***holography***

- As an alternative formulation of QFT we have all the standard techniques and machinery to control the ***"kinetics"*** of pseudo-symmetry breaking

- By means of explicit models and solutions thereof we have also the ***"dynamical"*** information

Ward identities structure

Action defining the generating functional (GF) of the theory

$$S_{\text{tot}} = S_{\text{inv}} + \int d^d x \, \frac{1}{2} m \, O_\phi + c.c.$$

A variation of the GF with respect to the symmetry leads to the ***current conservation*** operator identity
(i.e. 1-point functions at sources on)

$$\partial_\mu J^\mu = m \, \text{Im} O_\phi$$

Further functional derivations and eventual switching off of the sources leads to n-points Ward-Takahashi identities

$$\langle \partial_\mu J^\mu(x) \text{Im} O_\phi(0) \rangle = m \langle \text{Im} O_\phi(x) \text{Im} O_\phi(0) \rangle + i \langle \text{Re} O_\phi \rangle \delta^d(x)$$

$$\langle \text{Im} O_\phi \text{Im} O_\phi \rangle = -i f(\Box)$$

$$\langle \partial_\mu J^\mu \text{Im} O_\phi \rangle = -im f(\Box) + iv$$

$$\langle \partial_\mu J^\mu \partial_\nu J^\nu \rangle = -im^2 f(\Box) + imv$$

$$\langle O_\phi \rangle = v \in \mathbb{R}$$

Pseudo-spontaneous breaking

2-pt WT id in momentum space $$ik_\mu\langle J^\mu \mathrm{Im} O_\phi\rangle = -imf(k^2) + iv$$

Relying on relativistic invariance $$\langle J^\mu \mathrm{Im} O_\phi\rangle = k^\mu g(k^2)$$

$$\langle J^\mu \mathrm{Im} O_\phi\rangle = -\frac{k^\mu}{k^2}(mf(k^2) - v)$$

Consistency requirements (I) :

- no massless pole at source on

$$mf(0) - v = 0$$

- massless pole at source off
(Goldstone theorem)

$$f(k^2) \to \frac{\mu}{k^2}$$

$$f(k^2) \simeq \frac{\mu}{k^2 + M^2} - \frac{\mu}{M^2} + \frac{v}{m}$$

Consistency requirements (II) :

- no divergence for vanishing source

$$M^2 = \frac{\mu}{v} m$$

Gell Mann Oaks Renner relation

Holographic model: superconductor

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F^{MN} F_{MN} - D_M \phi^* D^M \phi + 2\phi^* \phi \right\}$$

$$\phi_b = mz + vz^2$$

Encoding source and VEV

Studying the fluctuations leads to:

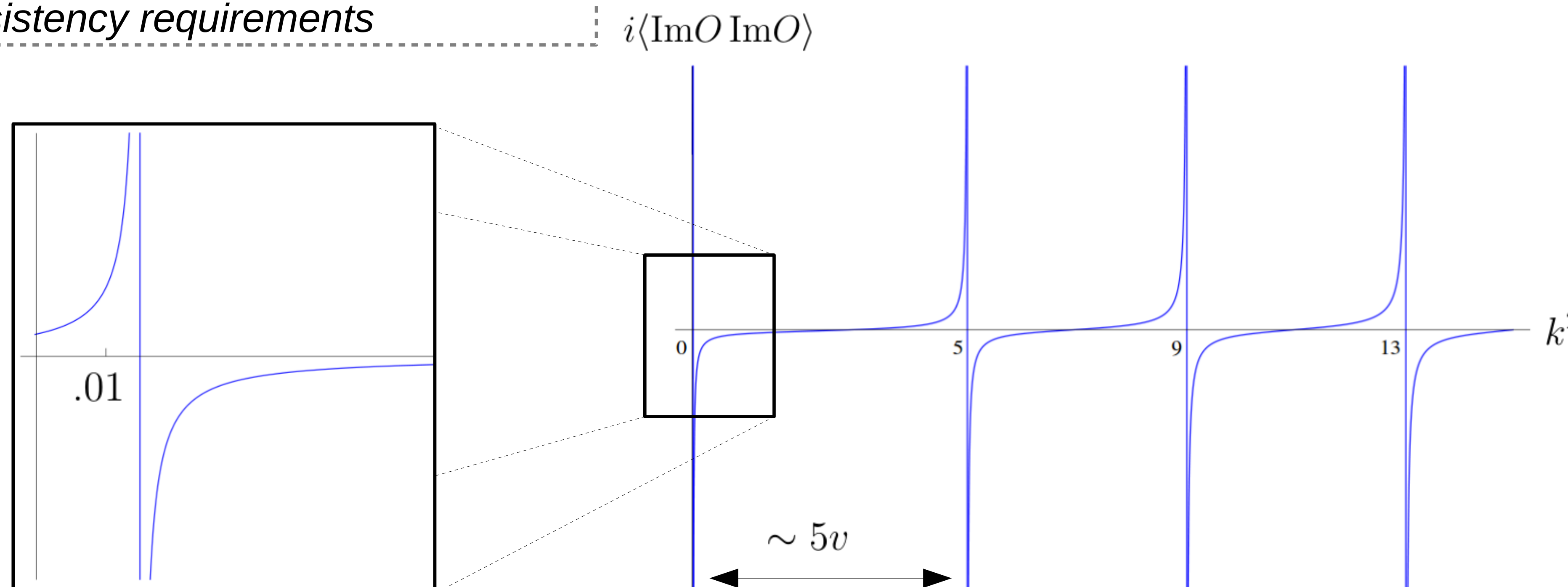
- *Analytic expressions for the probe correlators*
(and insensitivity to the backreaction)

- *Verification of all the symmetry consistency requirements*

pseudo-breaking:

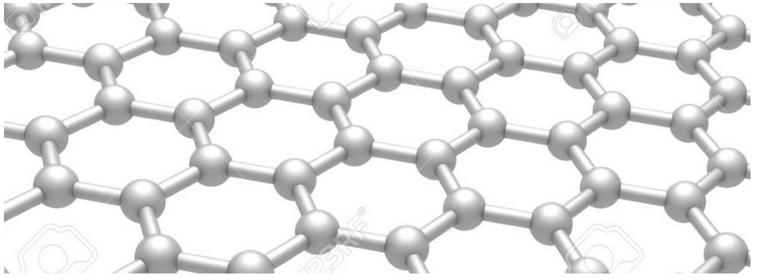
light pole and gapped spectrum

$$\frac{m}{\sqrt{v}} = .01 \ll 1 \qquad v = 1$$



Translational symmetry breaking

(pseudo-)Phonons: *(pseudo-)*Nambu-Goldstone modes of translation symmetry breaking.



The **Ward identities** relate various correlators involving the energy-momentum tensor and the operator breaking the symmetry.

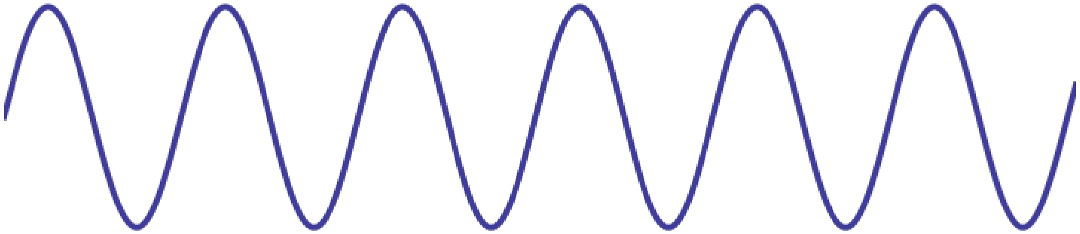
The topic borrows inspiration and applies to high energy and early universe as well *(e.g. QCD crystals, Q-balls, …)*.

[Coleman Nucl.Phys. B262 (1985) 263]

The description of the **RG-flow** of a microscopic translational invariant theory that spontaneously develops a lattice is complicated because, for instance, it entails many-body physics.

“density waves” *(charge-, spin-,...)*

Condensation of a spatially modulated order parameter.



Some **UV** information is ***relevant*** to describe the **IR**.

Example 1: a proper **counting** of Nambu-Goldstone modes comes from the knowledge of the *symmetry breaking pattern* **and** the *representation* the order parameter.

[Brauner hep-ph/1001.5212; Nicolis-Penco-Piazza-Rosen hep-th/1306.1240]

Example 2: *“current algebra forces the phonons to interact”*

[Leutwyler hep-ph/9609466]

Pseudo-Goldstone modes feature a **mass** which is related to the explicit symmetry breaking parameter (*GMOR*)

Simplest toy models

Single complex scalar operator with a **periodic space-dependent VEV**

$$\langle O(x) \rangle = e^{ik\cdot x} v$$

Concomitant breaking of conformal and translational symmetry

Analysis for a generic **Quantum Field Theory**, then realized and dynamically completed through a **holographic** realization.

$$S_0 = \int d^4x \, \sqrt{-g} \left(R - 2\Lambda - \partial_M \Phi \, \partial^M \Phi^* - m^2 \Phi \Phi^* \right)$$

$$\Phi(x, z) = e^{ik\cdot x} \varphi(z)$$

Simplest Q-lattice model, later generalized to phenomenological purposes.

Holographic computations

From the quadratic renormalized action we can compute the **2-pt correlators**.

To obtain the **2-pt Ward identities** we *must* take into account the *(global)* symmetries of the dual **QFT**. These correspond to *(local)* **gauge symmetries** of the bulk model.

To get the **2-pt Ward identity** we have to properly deal with bulk **gauge invariance**.

This is technically involved and mainly relies on two steps:

1) Substituting in the renormalized action the **constraint equations**, *i.e.* the Einstein equations with at least an index along the radial direction.

2) Express the action in terms of **gauge-invariant combinations** of the fluctuation fields.

2pt translation Ward identity

$$\langle \partial_\mu T^{\mu\nu}(x)\, O_\varphi(x')\rangle = -k^\nu \,\langle O_\varphi\rangle\, \delta(x-x') + 2k^\nu\, \varphi\,[f(x-x') - g(x-x')]$$

with the definitions: $O = e^{ik\cdot x} O_\varphi$ $\bar{\Phi} = e^{ik\cdot x}\varphi$

$$\langle O_\varphi(x)\, O^*_\varphi(x')\rangle = \langle O^*_\varphi(x)\, O_\varphi(x')\rangle = 4i f(x-x')$$

$$\langle O_\varphi(x)\, O_\varphi(x')\rangle = 4ig(x-x') \qquad \langle O^*_\varphi(x)\, O^*_\varphi(x')\rangle = 4ig^*(x-x')$$

The **parameter functions f** depend **ONLY** on the difference x'-x.

This constitutes an important **assumption** suggested by the holographic model, indeed the **Q-lattice ansatz** manages to **"factor out"** the spatial features.

In other terms, after having stripped the phase dependent factor off, the system appears formally **translational invariant**.

$\sim$ [Areán,Bertolini,Evslin,Prochazka 1003.5661]

Pseudo-phonon

$$f = \frac{\mu}{q^2 + M^2} + \ldots$$

residue → μ; mass → M^2

$$M^2 = \frac{\mu}{\langle O_\varphi \rangle}\varphi$$

source → φ; VEV → $\langle O_\varphi \rangle$

As in the U(1) case, consistency requirements arising from the presence/absence of the Nambu-Goldstone pole lead to GMOR relations for the pseudo-NG mass.

(in particular we want μ *to neither diverge nor vanish in the spontaneous* $\varphi \to 0$ *limit)*

It is consistent *(though not imposed bv the UV analysis)* to have the phonon residue to vanish in the $k \to 0$ limit.

- so far the result is only ***"kinematic"***

- complemented by a counting study

- insight on the tensorial structure of the correlators

A refined holographic model

PRELIMINARY

$$S=\int d^{d+2}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2-V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{d}\partial\psi_i^2\right]$$

- ***'axion-dilaton' model***, it contains the simplest Q-lattice shown before

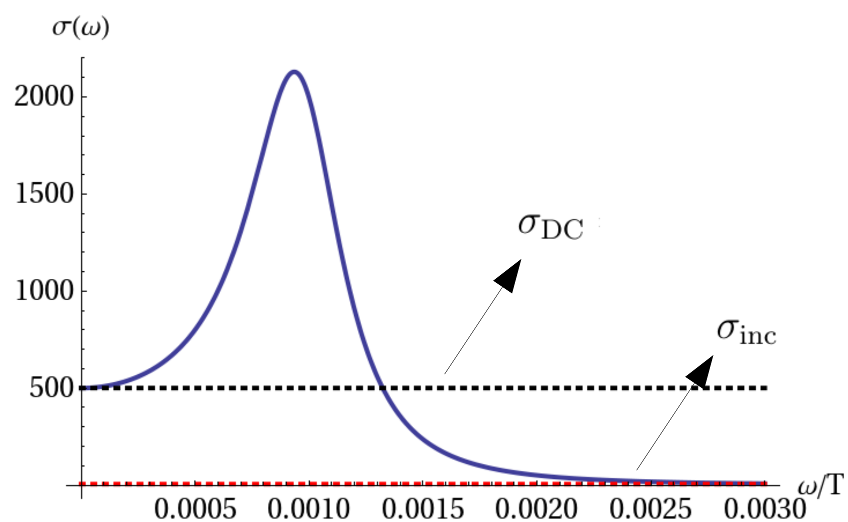
$$\phi=\phi(r)\,,\qquad \psi_i=kx^i$$

- Analytic computations of $\sigma_{\rm DC}$ and $\sigma_{\rm inc}$

- *'Hydrodynamic fit'*

$$\sigma(\omega)=\sigma_{\rm inc}+\frac{\rho^2}{\chi_{\pi\pi}}\frac{\Omega-i\omega}{(\Omega-i\omega)(\Gamma-i\omega)+\omega_o^2}$$

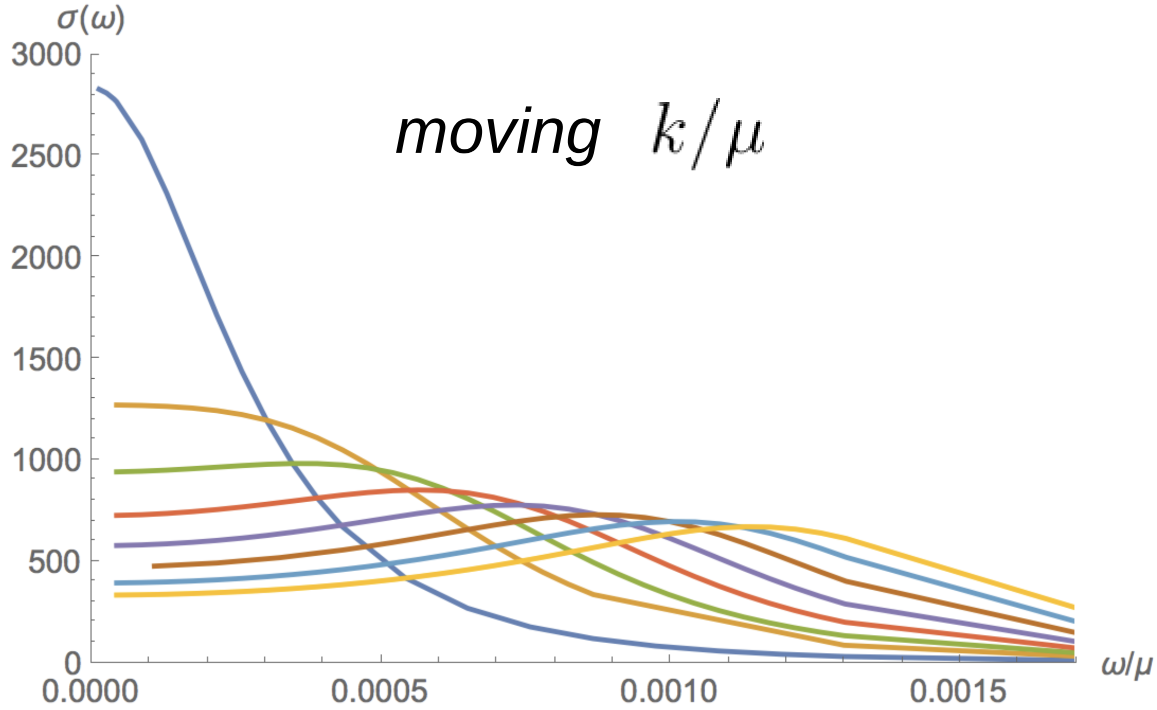
- High frequency saturation to 1 *(not shown)*

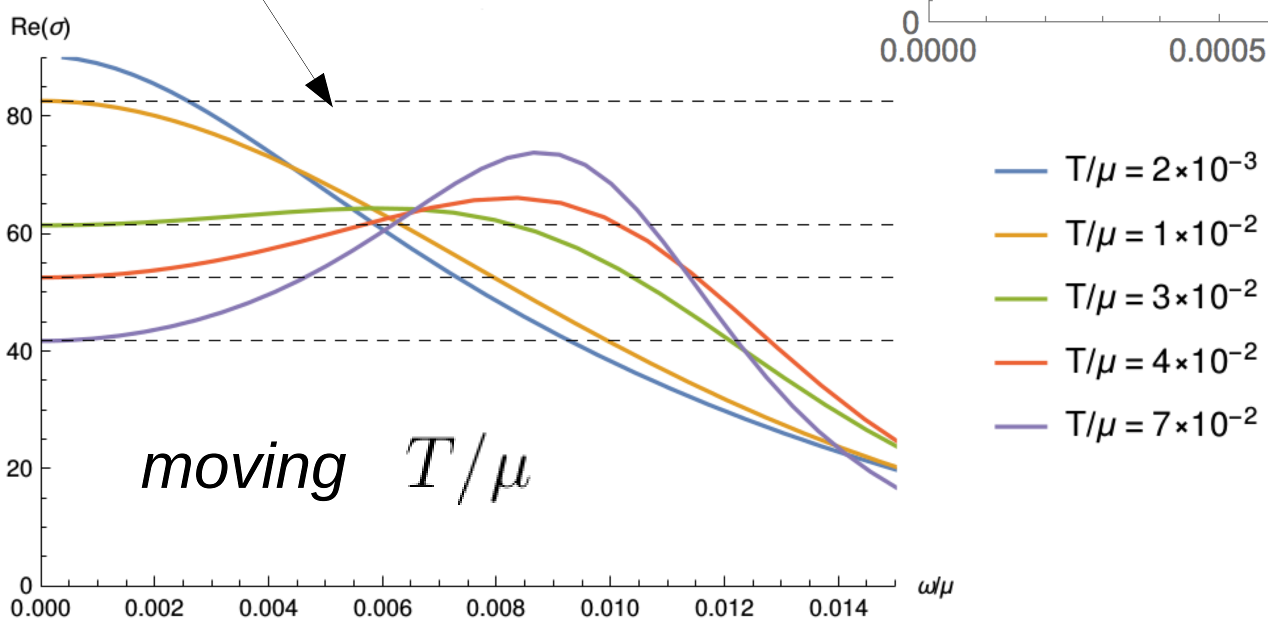


Bad metal phenomenology

PRELIMINARY

- $\omega_{\text{peak}} \sim \Delta\omega$
- decoherence
  (any useful MIR-like hint?)
- a non-hydro peak moving to higher frequencies with T
  (its behavior is non-linear in T)



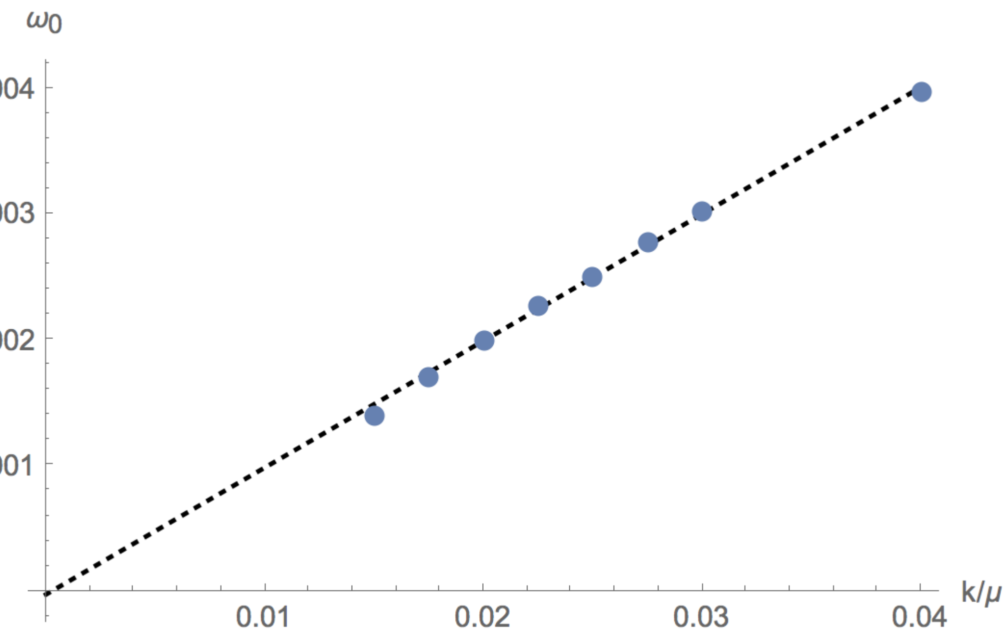


Hydro fit and beyond

PRELIMINARY

$$\sigma(\omega) = \sigma_{\text{inc}} + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

$$\omega_0 \sim k\sqrt{\varphi} + \ldots$$
$$\Omega \sim \varphi + \ldots$$
$$\Gamma \sim k^2\varphi^2 + \ldots$$



- The position of the peak is in line with **GMOR** expectation

(its dependence on k is instead a prediction)

[Amoretti,Areán,Argurio,Musso,Pando-Zayas 1611.09344]

- The '*peak unbalance*' Ω is **unsensitive to** k at leading order

- Γ is in line with ***inhomogeneous lattice*** and ***memory matrix*** results

[Hartnoll-Hofman 1201.3917,Blake,Tong,Vegh 1310.3832]

Conclusions & perspectives

● Bad metals phenomenology admits a description which features ***critical fluctuations*** of ***weakly pinned density waves***.

● Holography captures the pseudo symmetry breaking physics. In particular that of translations.

● Holography allows to extend the hydrodynamic picture as the bad metals experimental data requires.

● Characterizing robustly the bad metal pheno in holography

(parameter space, widening the T range,…)

● Fitting outside hydro

(precise behavior in T of the position of the peak,…)

● Interpreting the parameters

(origin of unbalance,…)