Holographic pinning in probe-brane models

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Many-Body Quantum Chaos, Bad Metals and Holography

Nordita – 6 October 2017

Key references

Mostly about recent work

- Gravity dual of (spontaneous) SDW+CDW [1408.1397]
- Conductivities: sliding stripes [1612.07323]
- Pinning them [1708.07837]

with

- Matti Järvinen (Utrecht)
- Matt Lippert (LIU)

Outline

1. Introduction
2. Phase structure of D3-D7' model
   - Spontaneous striping
3. Sliding SDW+CDW stripes
4. Pinning them!
   - Magnetic lattice
   - Ionic lattice (novel instability)
5. Outlook

Holographic striped phases

Translation symmetry-breaking in holographic models

- Mostly **explicit** breaking
- Modulated sources model ionic lattice and impurities
- Linear scalars (massive gravity, Q-lattice, Bianchi VII etc.) homogeneous
- Symmetry breaking $\rightarrow$ Drude-like conductivity

[Horowitz-Santos-Tong 1204.0519]

[Vegh 1301.0537]

[Arean et al. 1308.1920]

[Blake et al. 1308.4970]

[Goutéraux 1401.5436]

[Donos-Gauntlett 1409.6875]

[. . .]

[Andrade-Baggioli-Krikun-Poovuttikul 1708.08306]

[Alberte-Ammon-Baggioli-Jiménez-Pujolás 1708.08477]

Holographic striped phases

Spontaneous breaking of translation symmetry

- Interesting phenomena
  - Phase transition
  - Goldstone mode
  - Interaction with lattice/impurities
- Modulated instabilities in several models

  [Nakamura-Ooguri-Park 0911.0679]
- Inhomogeneous ground state constructed in some examples

  [Donos 1303.7211]

  [Withers 1304.0129]

  [Rozali et al. 1304.3130]
- Very little work on transport

Our approach

- Top-down: a concrete string theory set-up
- Control over the dual field theory
- Criterion: system with only fermion matter in the fundamental representation
- Specific models with 2+1 d fermions interacting with 3+1 d gauge fields: D3-D7' systems

Holographic cousin models

Brane intersections with $\#\mathrm{ND} = 6$ [Rey]

- Fundamental fermions [Davis,Kraus,Shah]
- Probe *Dq* in *Dp* background [Myers,Wapler]
- No SUSY $\rightarrow$ stability?
- Chern-Simons terms [Alanen,Keski-Vakkuri,Kraus,Suur-Uski]

Familiar example: Sakai-Sugimoto D4-D8-$\overline{\mathrm{D8}}$

QH models:

1. The D3-D7' model [Bergman-NJ-Lifschytz-Lippert]
   - 2+1 d defect, filling fraction ν irrational
2. The D2-D8' model [NJ-Järvinen-Lippert]
   - Fully 2+1 d, $\nu = 1$

Unquenched (massive) ABJM: fully 2+1 d, $\nu = M/2 + func(N_f)$, SUSY even w/ $d \neq 0 \neq B$

[Bea-NJ-Lippert-Ramallo-Zoakos]

D3-D7' model

- D3 background, $AdS_5 \times S^5$ (finite temperature)

$$ds_{10}^2 = r^2 \left(-h(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{r^2}\left(\frac{dr^2}{h(r)} + r^2 d\Omega_5^2\right)$$

$$h(r) = 1 - \left(\frac{r_T}{r}\right)^4$$

- Five-form flux

$$F_5 = dC_4 = (1 + *)dt \wedge dx \wedge dy \wedge dz \wedge d(r^4)\ , \quad \int_{S^5} F_5 = N$$

- Dilaton is a constant
- S^5 fibering: $S^2 \times S^2$ over an interval

$$d\Omega_5^2 = d\psi^2 + \cos^2\psi\ d\Omega_{2(1)}^2 + \sin^2\psi\ d\Omega_{2(2)}^2$$

	0	1	2	z	r	ψ	$S^2_{(1)}$	$S^2_{(2)}$
$D3$	X	X	X	X				
$D7$	X	X	X		X		X	X

Probe D7 brane

- Wraps $S^2 \times S^2 \subset S^5$
- Fermions on 2+1 d defect
- Embedding $\psi(r)$ is tachyonic
- Stabilized by wrapped flux on S^2's

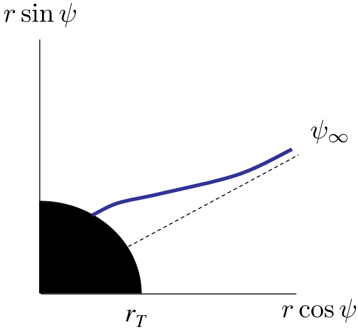
Add magnetic field $F_{12} = B$ and charge density $F_{r0} = A_0'(r)$

D7 probe action

$$S = S_{\rm DBI} + S_{\rm CS} \quad = -T_7 \int d^8x \, e^{-\Phi} \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} - \frac{(2\pi\alpha')^2 T_7}{2} \int P[C_4] \wedge F \wedge F$$

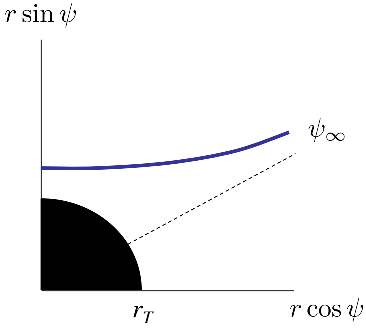
Embeddings

Black Hole



- D7 enters horizon
- Metallic phase
- This talk

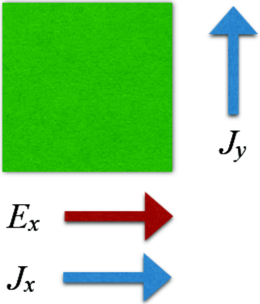
Minkowski



- D7 ends where S^2 shrinks
- QH phase

DC Conductivities

- BH phase: metallic behavior
- Use Karch-O'Bannon method
  [Karch-O'Bannon 0705.3870]
  - Add electric field E_x and charge current J_x and J_y
  - Impose reality below pseudo-horizon
  - Yields nonlinear DC conductivity at finite E_x

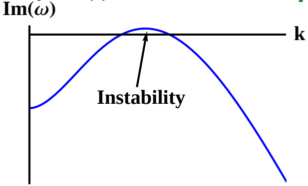


- Finite results
  - N_c D3-branes are momentum sink for probe D7'
  - valid up to $t \sim N_c$ until absorbed $k \sim N_c^2$ s.t. $v \sim \mathcal{O}(1)$
  - act like "smeared impurities"

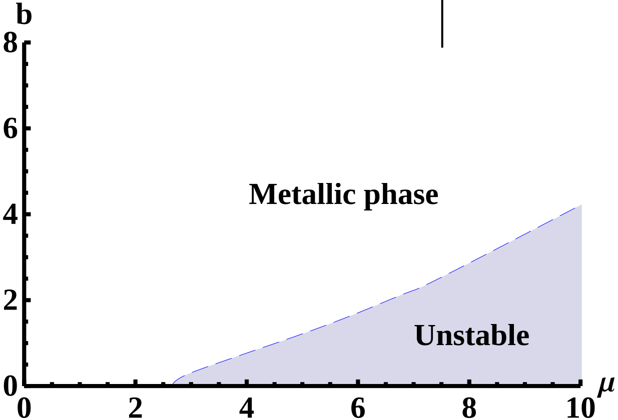
Striped instability

Analysis of quasinormal modes ⇒ instability (Nakamura-Ooguri-Park)

[Bergman,NJ,Lifschytz,Lippert arXiv:1106.3883]



- In the metallic phase at large charge density
- Finite wave number k ⇒ stripes



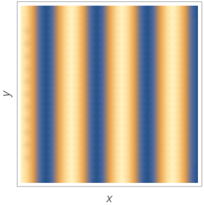
Striped ground state

D7 probe action

$$S = S_{\mathrm{DBI}} + S_{\mathrm{CS}} \quad = -T_7 \int d^8\xi \, e^{-\Phi} \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} - \frac{(2\pi\alpha')^2 T_7}{2} \int P[C_4] \wedge F \wedge F$$

Look for inhomogeneous ground state

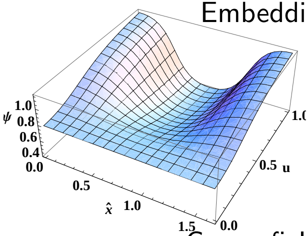
[Järvinen,NJ,Lippert arXiv:1408.1397]

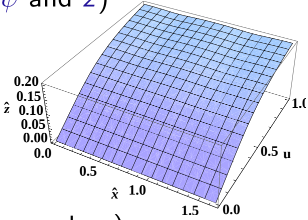


- Periodic with period L in x-direction (wave number $k = 2\pi/L$)
- Translation symmetry intact in y-direction
- Inhomogeneous embedding of the D7-brane $\psi(x,u)$, $z(x,u)$ and gauge fields $a_0(x,u)$, $a_y(x,u)$ with $u = r_T/r$.

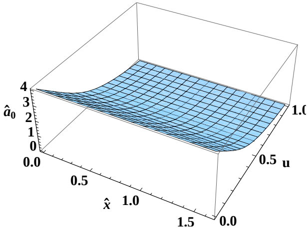
Example solution

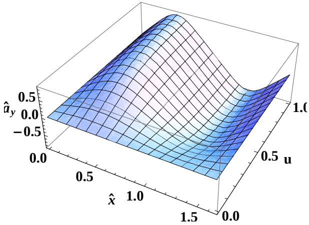
Embedding (ψ and z)





Gauge fields (a_0 and a_y)

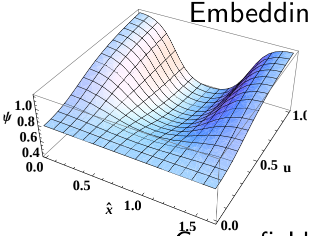


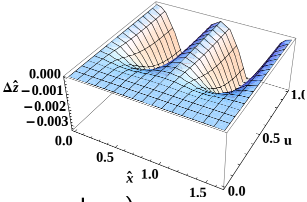


- "SDW" $\gg$ CDW
- Modulated persistent current $J_y(x)$

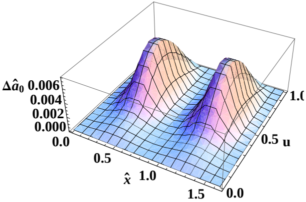
Zoom in to highlight modulation

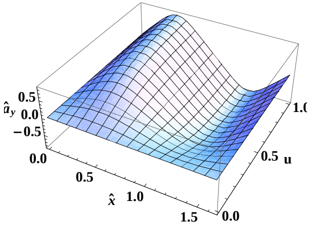
Embedding (ψ and Δz)





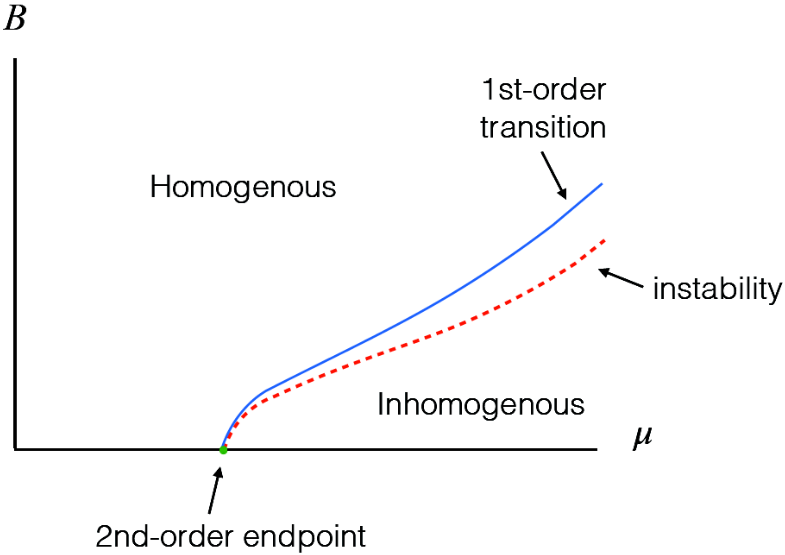
Gauge fields (Δa_0 and a_y)





- "SDW" $\gg$ CDW
- Modulated persistent current $J_y(x)$

Phase diagram



Fluctuations and Goldstone mode

Study time-dependent fluctuations of D7' fields

- $\delta f(t, x, u),\ f = \psi, z, a_t, a_x, a_y$
- All sources vanish, except δE_x or δE_y

Goldstone mode

- Translation symmetry spontaneously broken
- For any solution

$$f(t, x, u) \to f(t, x + \kappa, u)$$

- Time-independent fluctuation

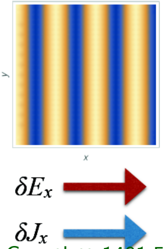
$$\delta f(t, x, u) = \kappa \partial_x f(x, u)$$

DC conductivity: $\sigma_{\rm xx}$

Ansatz:

- Turn on constant electric field δE_x
- Must allow stripes to slide

$$\delta f(t,x,u) = \delta f(x,u) - v_s t \partial_x f(x,u)$$



[Donos-Gauntlett 1401.5077]

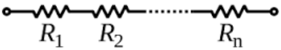
Follow "standard recipe"

- write bdry quantities (e.g. δJ_x) in terms of horizon data
- impose horizon regularity
- speed v_s not fixed

$$\sigma_{xx}(x) = \langle \hat\sigma_{xx}^{-1}\rangle^{-1} + \frac{v_s}{\delta E_x}\Big[\underbrace{d(x) - \langle d(x)\rangle}_{\sim CDW} + \underbrace{SDW}_{\gg CDW} + tiny\Big]$$

- "Local" conductivity

$$\langle \hat\sigma_{xx}^{-1}\rangle^{-1} \quad \leftrightarrow$$



$\delta a_x = -(\delta E_x - p'(x))t + \delta a_x(x,u), \delta a_t = p(x) + \delta a_t(x,u) - v_s t \partial_x a_t(x,u)$

DC conductivity: σ_{yx}

Turn on $\delta E_x, \delta J_y \Rightarrow$ Hall conductivity

- Sliding stripes with background current J_y

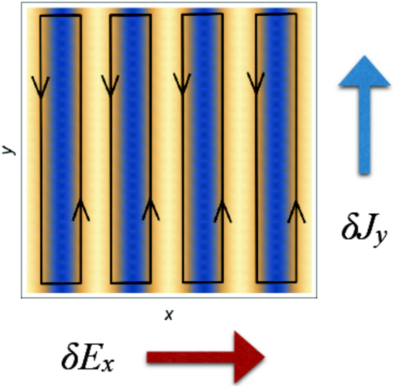
$$J_y(x - v_s t) \approx J_y(x) - v_s t J_y'(x)$$

- Gives modulated divergence

$$\sigma_{yx} = i v_s J_y'(x) \times \infty + \text{finite}$$

- Spatial average vanishes

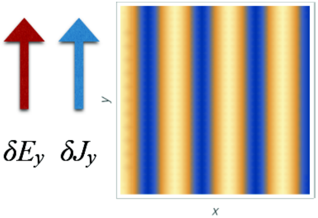
$$\langle \sigma_{yx} \rangle = 0$$



DC conductivity: σ_{yy} and σ_{xy}

Turn on δE_y

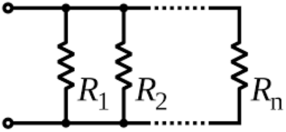
- Stripes don't move
- Parity $\Rightarrow \sigma_{xy}(x) = 0$
- No conserved bulk current for $\delta a_y(x)$



- But, spatially averaged current conserved

$$\langle \sigma_{yy} \rangle = \langle \hat{\sigma}(1 + \mathrm{small}) \rangle + \underbrace{\langle \sigma_{yy}^{SDW} \rangle}_{\text{dominant}}$$

$\langle \hat{\sigma}(x) \rangle \quad \leftrightarrow$



Numerics precisely match analytics.
Hold your breath!

Pinning

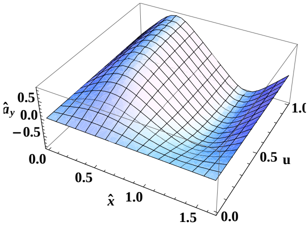
- Mimic impurities via explicit deformed bdry conditions:

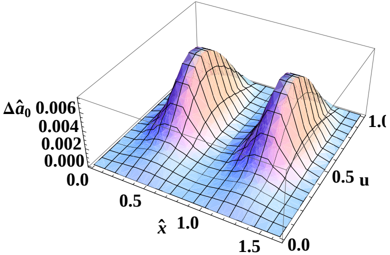
$$\text{Magnetic lattice}: \quad a_y(x, u=0) = bx + \alpha_b \sin(k_0 x)$$

OR

$$\text{Ionic lattice}: \quad a_t(x, u=0) = \mu + \alpha_\mu \cos(2k_0 x)$$

- Fixed lattice wavelength = dynamical wavelength

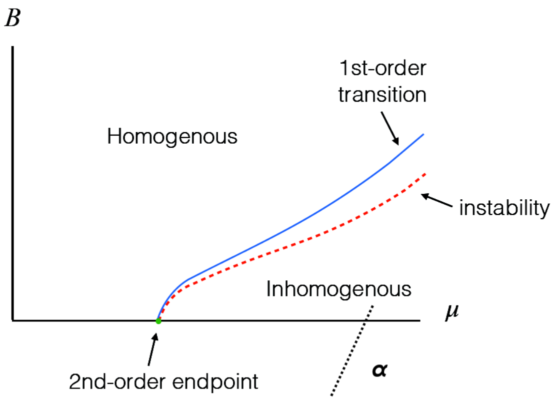




- Commensurability by hand, work beyond, see

[Krikun's talk]

Pinning



- For simplicity, focus on

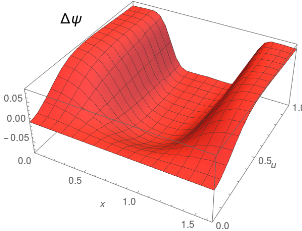
$$\mu = 4, b = 0, m = 0$$

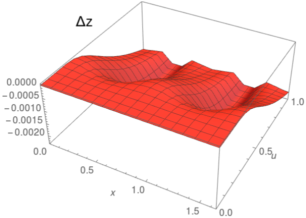
- Expect to be in preferred phase for small α's

Pinning

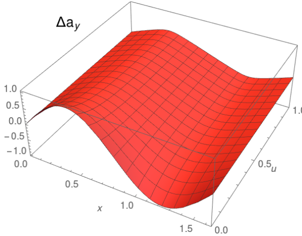
Exemplified solutions ($\Delta f = f_{\alpha_b=1} - f_{\alpha_b=0}$):

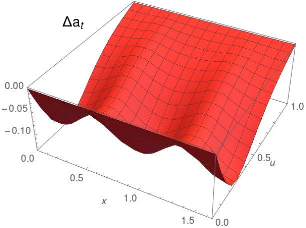
Embedding ($\Delta\psi$ and Δz)





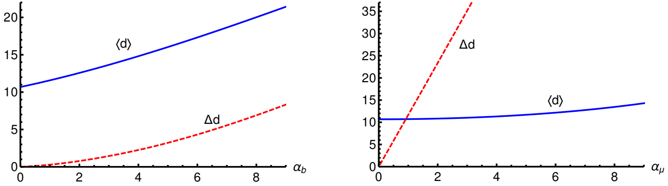
Gauge fields (Δa_y and Δa_t)





Pinning

- Deformed bdry conditions add in more charge



- DC conductivities (from slides 18 & 20):

$$\langle \sigma_{xx} \rangle = \langle \hat{\sigma}_{xx}^{-1} \rangle^{-1} + \delta_{\alpha_b,\alpha_\mu,0} \frac{v_s}{\delta E_x} \left[SDW + tiny \right]$$

$$\langle \sigma_{yy} \rangle = \langle \hat{\sigma}(1 + small) \rangle + \langle \sigma_{yy}^{SDW} \rangle$$

$$\langle \sigma_{xy} \rangle = 0 = \langle \sigma_{yx} \rangle$$

- Parametrically large wrt δE_x: stripes pinned w/ any α's

Optical conductivities

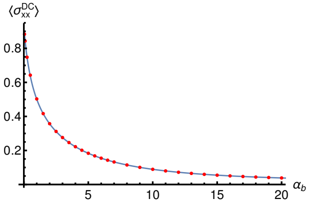
- Turn on electric field

$$\delta E_x e^{-i\omega t}$$
$$OR$$
$$\delta E_y e^{-i\omega t}$$

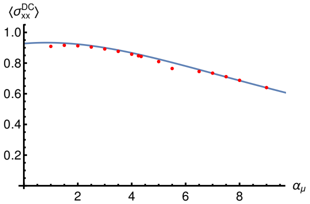
- Solve fluctuation EOM numerically w/ pseudospectral method
- Extract AC conductivities $\sigma_{ij}(\omega, x)$, $i, i = x, y$
- DC conductivities as $\omega \to 0$ limits of AC

DC limits

Magnetic lattice



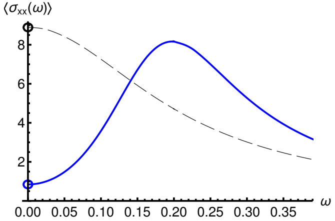
Ionic lattice

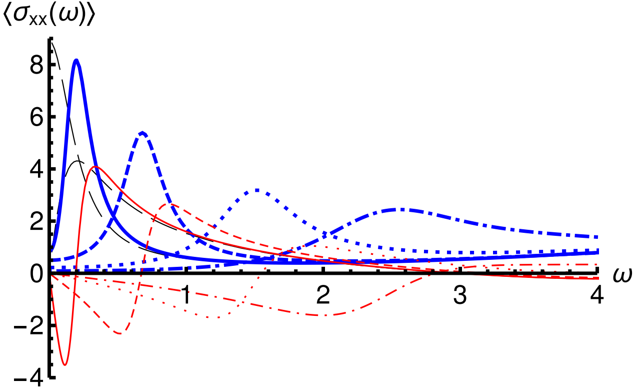


- Recall SDW $\gg$ CDW thus effect w/ $\alpha_b \gg \alpha_\mu$
- $\langle\sigma_{xx}\rangle \ll \langle\sigma_{yy}\rangle$; pinning
- $\langle\sigma_{xx}\rangle$ decreases with α; stripes are more strong
- $\langle\sigma_{yy}\rangle$ increases with α; more charge carriers around

Magnetic lattice: σ_{xx}

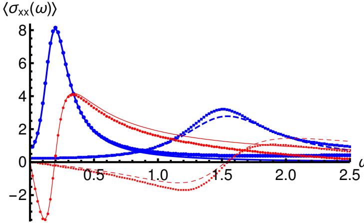
- Lifting of the Goldstone mode: $\alpha_b = 0 \to \alpha_b \neq 0$
- The DC conductivity drops by a decade
- Height of the peak shrinks and width broadens

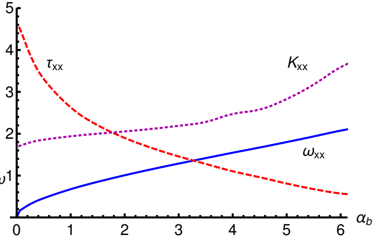




Captured by fitting to Drude-Lorentz model

$$\langle \sigma_{xx} \rangle = \frac{\langle \sigma_{xx}^{DC} \rangle}{1 - i\tau_{xx}\omega} + \frac{iK_{xx}\omega}{\omega^2 - \omega_{xx}^2 + i\omega/\tau_{xx}}$$



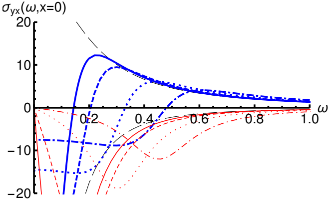


- Three parameter $K_{xx}, \tau_{xx}, \omega_{xx}$ fit; $\langle \sigma_{xx}^{DC} \rangle$ from analytics
- Small α_b, $\omega_{xx} \sim \alpha_b^{1/2}$ as in driven harmonic oscillator model

Magnetic lattice: $\sigma_{\rm yx}$

- Captured by a modified Lorentzian

$$\sigma_{yx} = \frac{K_{yx}(x)/\tau_{yx}}{\omega^2 - \omega_{yx}^2 + i\omega/\tau_{yx}}$$



- Three parameter $K_{yx}(x) \sim \cos(2\pi x/L), \tau_{yx}, \omega_{yx}$ fit
- Find $\omega_{yx} \approx \omega_{xx}$ and $\tau_{yx} \approx \tau_{xx}$
- Notice that as $\omega_{yx} \to 0$:

$$\sigma_{yx}\Big|_{\alpha_b=0} = \frac{\tau_{yx} K_{yx}(x)}{1 - i\tau_{yx}\omega} - K_{yx}(x)\left(\frac{i}{\omega} + \delta(\omega)\right)\ ,\ K_{yx}(x) = v_s J'_y(x)$$

Magnetic lattice: σ_{yy} and σ_{xy}

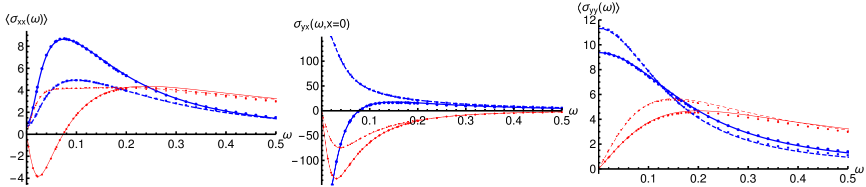
- Electric field in the y-direction: no pinning effects
- Captured by simple Drude

$$\langle \sigma_{yy} \rangle = \frac{\langle \sigma_{yy}^{DC} \rangle}{1 - i\tau_{yy}\omega}$$

- Parity: $\sigma_{xy} = 0$.

Ionic lattice: AC conductivities

- Small α_μ analogous to magnetic lattice; pinning much weaker
- Captured again by σ_{xx}: Drude-Lorentz , σ_{yx}: modified Lorentz, σ_{yy}: Drude
- Find Goldstone mode parameters $\omega_{xx} = \omega_{yx}$, $\tau_{xx} = \tau_{yx}$

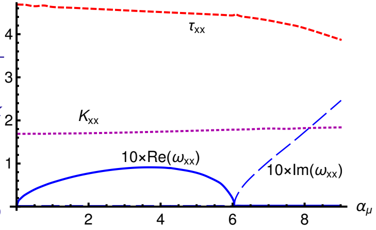


Ionic lattice: instability

$$\langle \sigma_{xx} \rangle = \frac{\langle \sigma_{xx}^{DC} \rangle}{1 - i\tau_{xx}\omega} + \frac{iK_{xx}\omega}{\omega^2 - \omega_{xx}^2 + i\omega/\tau_{xx}}$$

- Poles from the second term

$$\omega = -\frac{i}{2\tau_{xx}} \pm i\sqrt{\frac{1}{4\tau_{xx}^2} - \omega_{xx}^2}$$



- Get an instability for $\alpha_\mu \gtrsim 6$
- Interpretation?

To-do list

- Phase diagram for $\alpha \neq 0$: away from locking
- QNM, especially the y-dependent fluctuations: bubble phase?
- Construct stripes w/ finite E_x
- Finite B field
- Transport of anyonic stripes
- Backreaction