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HYDRODYNAMICS AND CHAOS

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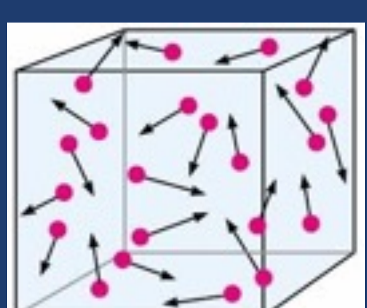
STATES OF MATTER

microscopic physics

symmetry breaking

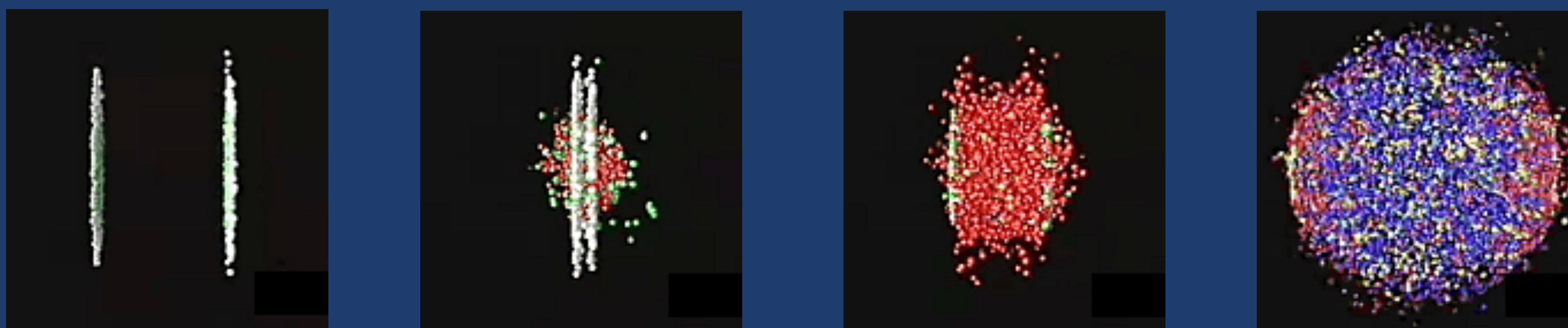


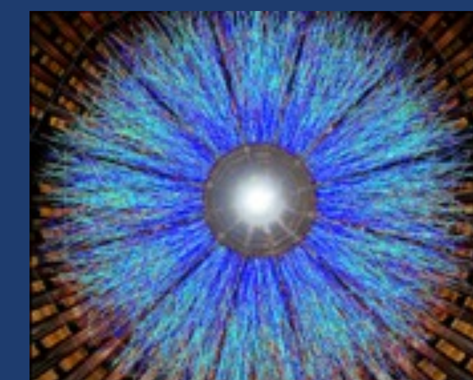
- solids
- liquids
- gases
- plasma

HYDRODYNAMICS

HYDRODYNAMICS

- from standard liquids and gases to quark-gluon plasma





- low-energy limit of QFTs (effective field theory)

$$T^{\mu\nu}\left(u^{\lambda},T,\mu\right) = (\varepsilon + P)\, u^{\mu} u^{\nu} + P g^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\nabla\cdot u \Delta^{\mu\nu} + \ldots$$

$$J^{\mu}(u^{\lambda},T,\mu) = n u^{\mu} - \sigma T \Delta^{\mu\nu}\nabla_{\nu}(\mu/T) + \ldots$$

$$\boxed{\nabla_{\mu} T^{\mu\nu} = 0 \qquad\qquad \nabla_{\mu} J^{\mu} = 0}$$

- tensor structures (phenomenological gradient expansions) with transport coefficients (microscopic)

HYDRODYNAMICS

- conformal (Weyl-covariant) hydrodynamics $\quad T^{\mu}_{\ \mu} = 0$

$$g_{\mu\nu} \to e^{-2\omega(x)} g_{\mu\nu} \qquad\qquad T^{\mu\nu} \to e^{6\omega(x)}\, T^{\mu\nu}$$

- infinite-order asymptotic expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} T^{\mu\nu}_{(n)} \quad \longrightarrow \quad \omega = \sum_{n=0}^{\mathcal{O}_H} \alpha_n k^{n+1}$$

- classification of tensors beyond Navier-Stokes

first order: 2 (1 in CFT) - shear and bulk viscosities

second order: 15 (5 in CFT) - relaxation time, … [Israel-Stewart and extensions]

third order: 68 (20 in CFT) - [S. G., Kaplis, PRD 93 (2016) 6, 066012, arXiv:1507.02461]

HYDRODYNAMICS

- diffusion and sound dispersion relations in CFT

$$\text{shear:} \quad \omega = -i\frac{\eta}{\varepsilon+P}k^2 - i\left[\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{2}\frac{\theta_1}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

$$\text{sound:} \quad \omega = \pm c_s k - i\Gamma_c k^2 \mp \frac{\Gamma_c}{2c_s}\left(\Gamma_c - 2c_s^2\tau_\Pi\right)k^3 - i\left[\frac{8}{9}\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{3}\frac{\theta_1+\theta_2}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

- loop corrections break analyticity of the gradient expansion (long-time tails), but are *1/N* suppressed [Kovtun, Yaffe (2003)]
- entropy current, constraints on transport and new transport coefficients (anomalies, broken parity)
- non-relativistic hydrodynamics
- hydrodynamics from effective Schwinger-Keldysh field theory with dissipation
  [Nicolis, et. al.; S. G., Polonyi; Haehl, Loganayagam, Rangamani; de Boer, Heller, Pinzani-Fokeeva; Crossley, Glorioso, Liu]

(WEAK) CHAOS

OTOC'S AND CHAOS

- OTOC of local, unbounded operators grows exponentially

$$C(t,x) = -\frac{\langle[\hat{W}(t,x),\hat{V}(0)]^\dagger[\hat{W}(t,x),\hat{V}(0)]\rangle_\beta}{2\langle\hat{W}(t,x)\hat{W}(t,x)\rangle_\beta\langle\hat{V}(0)\hat{V}(0)\rangle_\beta} \simeq e^{2\lambda(t-x/v_B)}$$

- not all OTOC's can grow indefinitely
- think of a local spin chain or a fermionic theory [Kukuljan, S. G., Prosen, PRB 96 (2017) 6, 060301, arXiv:1701.09147]

triangular inequality: $C\left(x,t\right) \leq 4\left\|v\right\|^2\left\|w\right\|^2$

Lieb-Robinson theorem: $C\left(x,t\right) \leq 4\left\|v\right\|^2\left\|w\right\|^2 e^{-\mu\max\{0,|x|-v_{LR}t\}}$

- OTOC is heavily suppressed for early times $t \ll t^* = \frac{|x|}{v_{LR}}$ by the LR theorem
- after t^*, OTOC rapidly saturates; it does not grow exponentially after that!
- this is not chaos as normally defined; Lyapunov exponent requires a limit

$$t \to \infty$$

DOTOC'S AND WEAK CHAOS

- proposal: consider instead a density of OTOC of non-local (smeared) operators

$$V \equiv \sum_{x \in \Lambda} v_x \,, \qquad W \equiv \sum_{x \in \Lambda} w_x$$

$$c^{(N)}(t) := -\frac{1}{N} \left(\langle [W(t), V(0)]^2 \rangle_\beta - \langle [W(t), V(0)] \rangle_\beta^2 \right)$$

- unlike the OTOC, the DOTOC grows indefinitely, but is bounded polynomially

$$\boxed{c^{(N)}(t) \leq A t^{3d}}$$

EXAMPLE OF WEAK CHAOS

- kicked Ising model

$$H(t) = \sum_j J\sigma_j^x \sigma_{j+1}^x$$
$$+ \sum_j h\left(\sigma_j^z \cos\varphi + \sigma_j^x \sin\varphi\right)\delta\left(t-n\right)$$

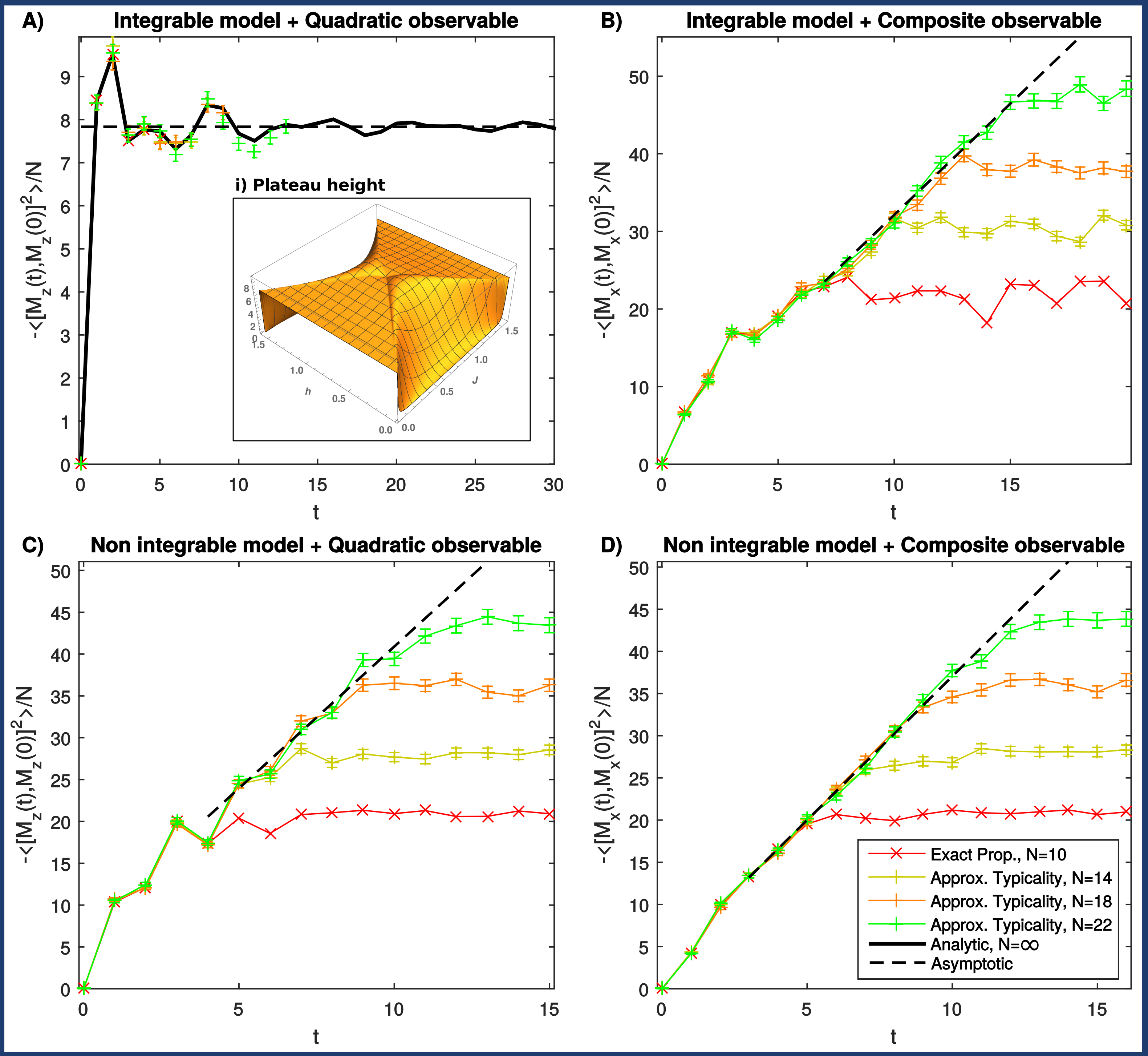
- it is chaotic [Pineda, Prosen]

- integrable $\varphi = 0$

  vs.

  non-integrable $\varphi > 0$

$$W = V = M_\alpha = \sum_{j=1}^{N} \sigma_j^\alpha$$



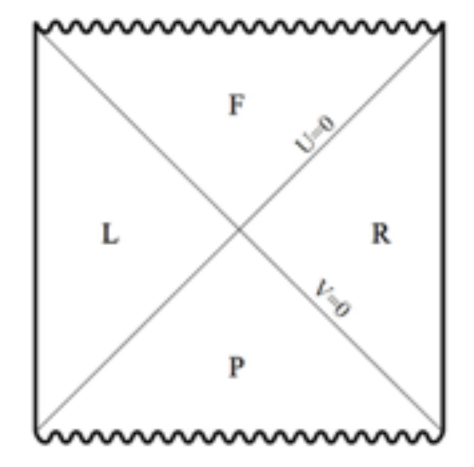
- what is the holographic dual of the DOTOC?

CHAOS FROM HYDRODYNAMICS

OTOC AND SHOCK WAVES

- Lyapunov exponent and butterfly velocity follow from the holographic shock wave on the horizon of a two-sided black hole

$$ds^2 = A(UV)dUdV + B(UV)d\vec{x}^2 - A(UV)\delta(U)h(x)dU^2$$



- Lyapunov exponent for black holes saturates a conjectured bound [Maldacena, Shenker, Stanford]

$$\lambda_L \leq 2\pi T$$

- butterfly velocity may play a role in bounding diffusion [Hartnoll; Blake]

$$D \sim v_B^2/\lambda_L \geq \text{``?''}$$

- we want to understand these claims from the point of view of hydrodynamics (sound modes)

CHAOS FROM HYDRODYNAMICS

- reconstruct the solution in terms of linearised longitudinal (sound) waves at infinite coupling and infinite *N* [S.G., Schalm, Scopelliti, arXiv:1710.00921]

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} + \mathcal{L}_{matter}\right]$$

- in the sound channel, look for a (radially) null solution at the horizon

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + b(r)\left(dx^2+dy^2+dz^2\right) - \left[f(r)H_1dt^2 - 2H_2dtdr + \frac{H_3dr^2}{f(r)} + H_4\left(dx^2+dy^2\right)\right]$$

- use the ansatz

$$H_1 = H_3 = \left(C_+W_+(t,z,r) + C_-W_-(t,z,r)\right)$$
$$H_2 = \left(C_+W_+(t,z,r) - C_-W_-(t,z,r)\right)$$
$$H_4(r_h) = 0$$

- impose regularity and solve (in pure *N*=4)

$$W_\pm(t,z,r) = e^{-i\omega\left[t\pm\int^r\frac{dr'}{f(r')}\right]+ikz}\,h_\pm(r)$$
$$h_\pm(r) = e^{\int^r\frac{k^2\pm 9i\omega r' - 12r'^2}{3r'f(r')}dr'}$$

diffusion

$$\boxed{\omega_\pm \equiv \pm i\mathfrak{D}k^2 = \pm i\frac{1}{3\pi T}k^2}$$

CHAOS FROM HYDRODYNAMICS

- regularity demands a single mode solution $k^2 \equiv -\mu^2 = -6\pi^2 T^2$
- we recover known properties of the holographic butterfly effect

$$\boxed{\begin{aligned} &\omega_\pm \equiv \mp i\lambda_L\,, \quad \lambda_L = 2\pi T \\ &v_B \equiv \left|\frac{\omega_\pm}{k}\right| = \sqrt{\lambda_L \mathfrak{D}} \end{aligned}}$$

$$e^{-i\omega t + ikz} = e^{\lambda_L (t - z/v_B)}$$

- consider more general theories (with bulk matter content) and an important assumption of horizon decoupling
- (advanced and retarded) diffusion equation with horizon diffusion constant of Blake

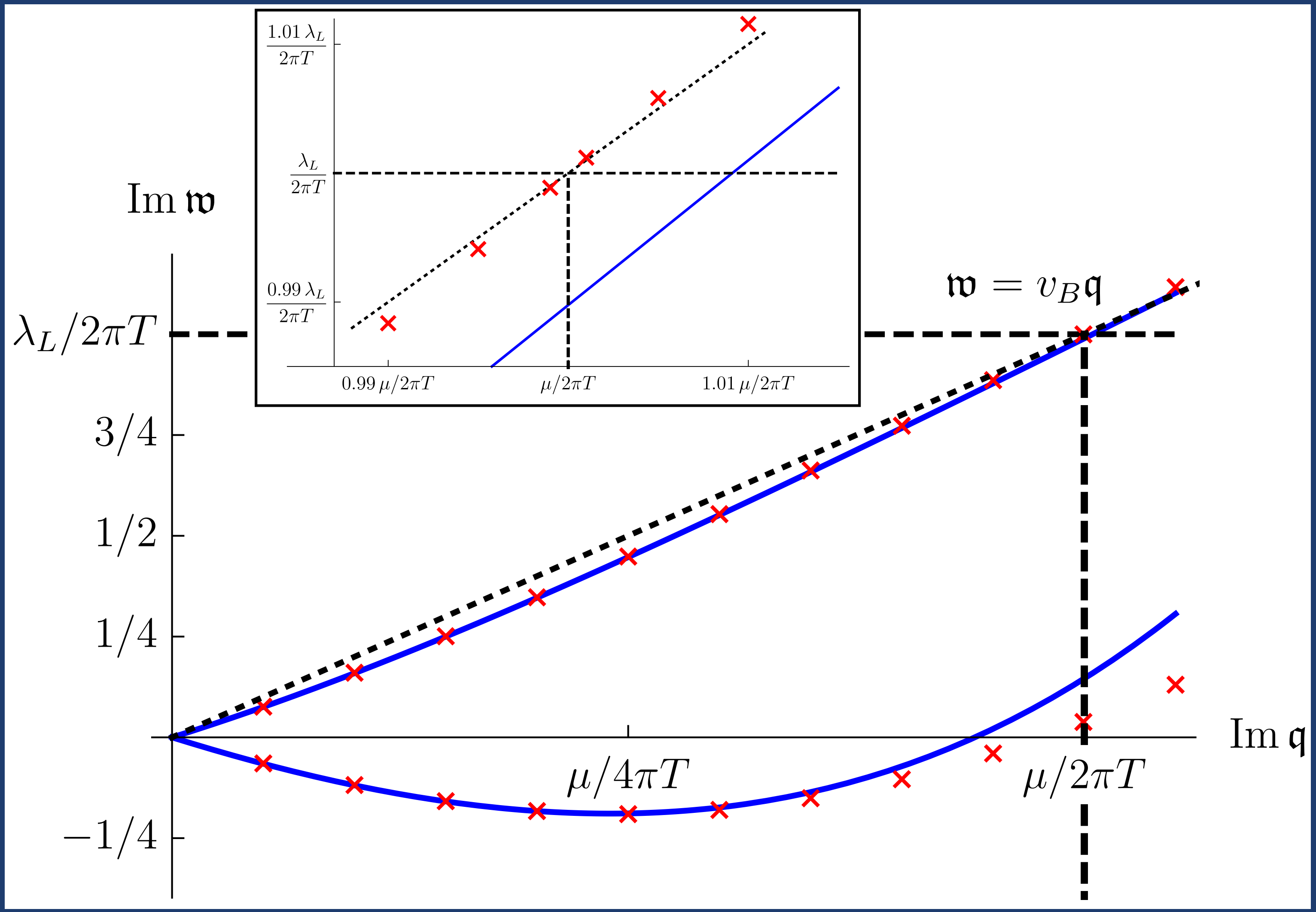
$$\boxed{\begin{aligned} &\partial_t W_\pm = \mp \mathfrak{D}\, \partial_z^2 W_\pm \\ &\mathfrak{D} = \frac{v_B^2}{\lambda_L} = \frac{2}{3}\frac{1}{b'(r_h)} = \frac{1}{12}\frac{f'(r_h)}{b(r_h)} \end{aligned}}$$

CHAOS FROM HYDRODYNAMICS

- the sound mode is a smeared shock wave (in KS coordinates)

$$ds^2 = A(UV)\,dU\,dV + B(UV)d\vec{x}^2 - A(UV)\,e^{ikz}\left(C_+\frac{dU^2}{U} - C_-\frac{dV^2}{V}\right)$$

- quasinormal mode (pole of the retarded stress-energy tensor correlator)



sound to third order in the hydro expansion

[S.G., Kaplis]

$$\omega = \pm\frac{1}{\sqrt{3}}k - \frac{1}{6\pi T}k^2 \pm \frac{3-2\ln 2}{24\sqrt{3}\pi^2T^2}k^3 - \frac{\pi^2 - 24 + 24\ln 2 - 12\ln^2 2}{864\pi^3T^3}k^4$$

$$\omega(i\mu) \approx 0.990 \times i\lambda_L$$

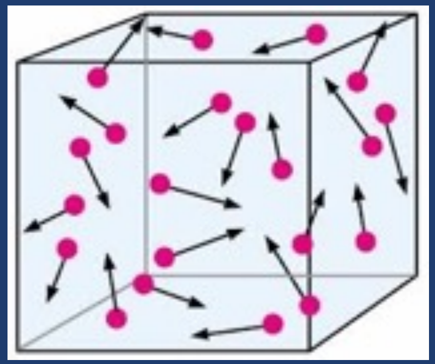
IMPLICATIONS

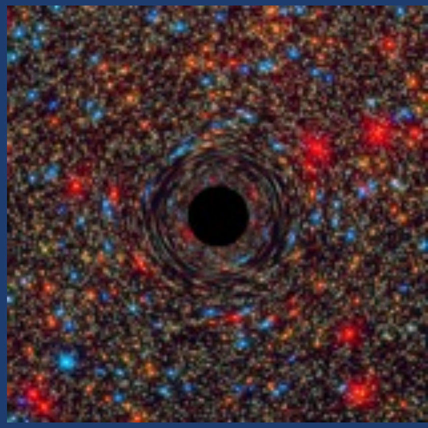
- momentum diffusion for these theories can be computed from the horizon membrane paradigm [Kovtun, Son, Starinets]

$$\frac{D}{\mathfrak{D}} = \frac{3\, b'(r_h)}{8\pi T}$$

- this ratio can be anything in the presence of additional scales or coupling constant(s)
- however, the same physics controls scrambling and hydrodynamics
- similar to the dilute gas

$$\eta_{cdg} = \frac{1}{3} m \frac{\sqrt{\langle v^2(T)\rangle}}{\sigma_{2-2}}$$
$$\lambda_L \simeq \rho(T) \sqrt{\langle v^2(T)\rangle}\sigma_{2-2}$$

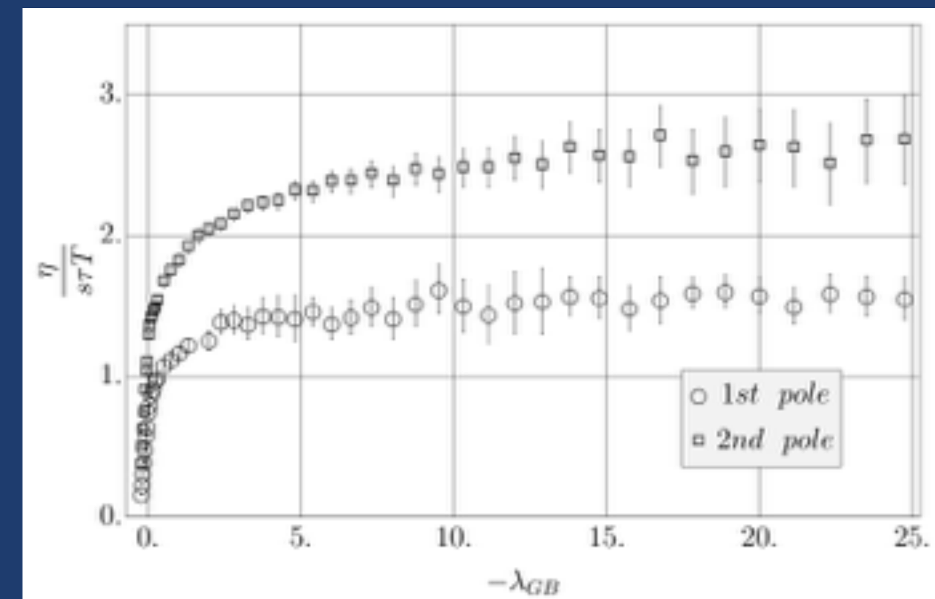


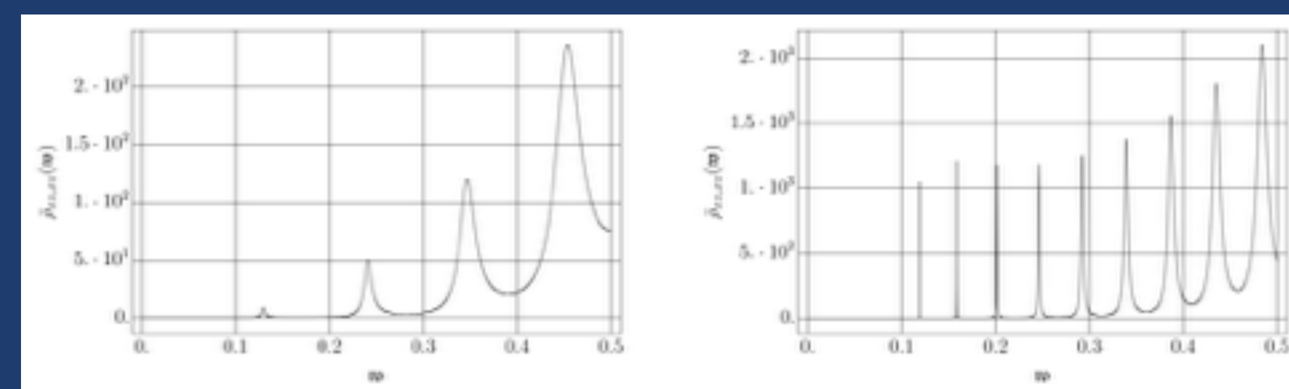


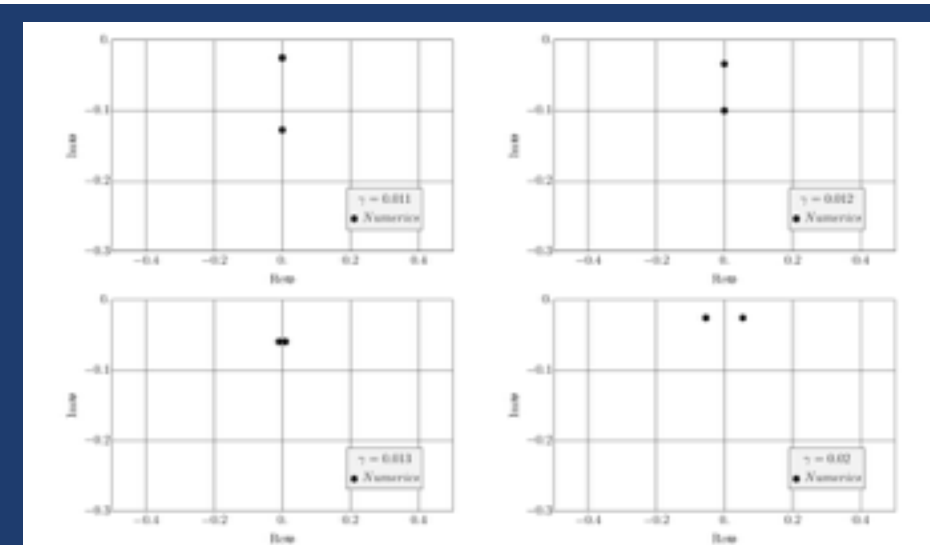
- a form of BBGKY hierarchy truncation for large-*N*, infinitely strongly coupled CFTs
- turbulence?

HOLOGRAPHY AT WEAK(ER) COUPLING

- higher-derivative gravity (IIB supergravity, Gauss-Bonnet, …) gives rise to expected weakly coupled physics extremely quickly [S.G., Kaplis, Starinets, …]
- kinetic theory, quasiparticles, formation of branch cuts, … destruction of hydrodynamics
- the same construction of a smeared shock waves works in GB
- what does this mean for hydrodynamics/chaos when hydro is less robust?







$$q_c/T \sim \lambda^{3/2}$$

THANK YOU!