Out-of-time-ordered density correlators in Luttinger liquids

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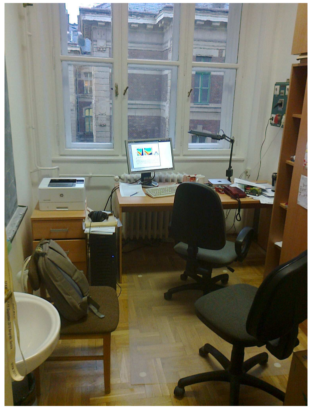
Outline:
- Out-of-time-ordered (OTO) commutator and correlator
- Luttinger liquids
- Universal features in OTOC: Short and long wavelength density
- Numerics on spinless fermions (XXZ chain)
- Non-universal short time dynamics



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Linear response

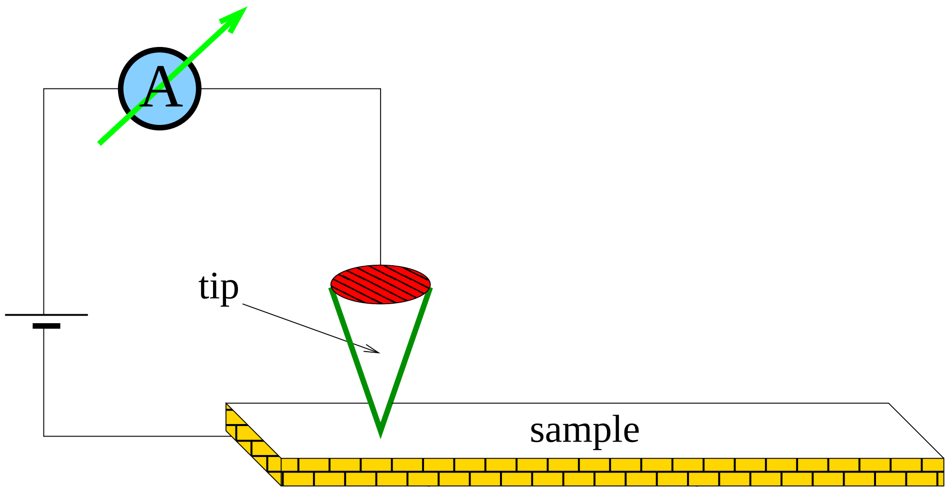
- An external field, F couples to an operator W, such as magnetic field and spin.

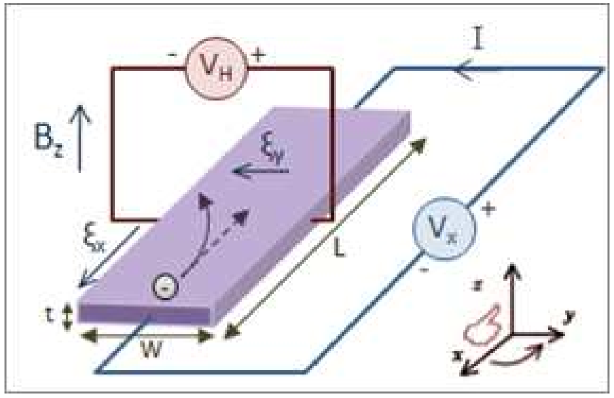
- How does the expectation value of operator V change?

- $\langle V(t)\rangle \sim \int_{-\infty}^{t} dt' \chi(t-t')F(t')$

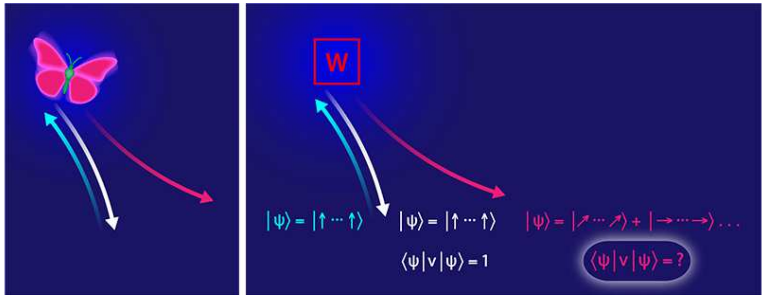
- Susceptibility: $\chi(t) = i\langle [V, W(t)]\rangle$, Kubo formula

- Many universal properties, fluctuation dissipation theorem(s), experimentally accessible, i.e. conductivity, neutron scattering etc.





Information scrambling



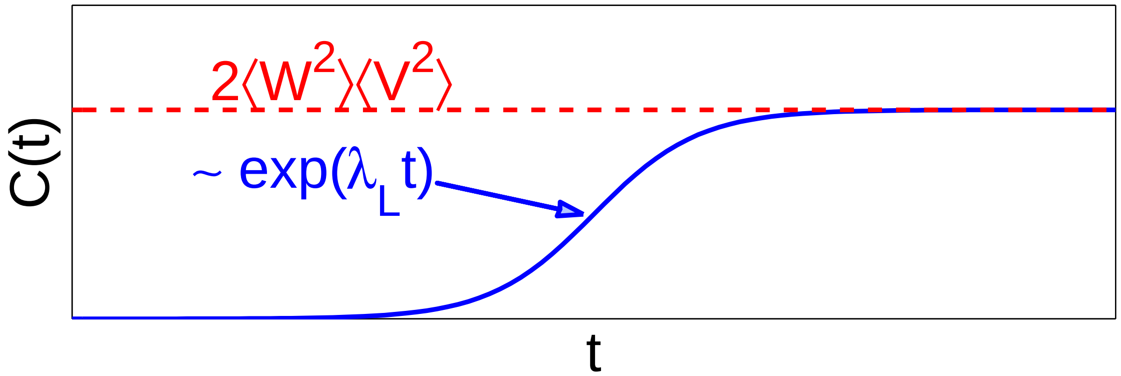
The quantum butterfly effect, i.e. sensitivity of a system to small perturbations and information scrambling occurs in chaotic models.

OTO commutator: $C(t) = \langle [V, W(t)]\,[W^+(t), V^+] \rangle \geq 0$ (Larkin&Ovchinnikov)

V and W local operators (separated spatially), $W(t) = \exp(iHt)W\exp(-iHt)$.

It contains conventional, $-\langle VW(t)W^+(t)V^+\rangle$ and OTO correlator, $-\langle VW(t)V^+W^+(t)\rangle$.

Measures the spread of information, signals quantum chaos with a growth bounded by a thermal Lyapunov exponent, $\lambda_L \leq 2\pi T$.

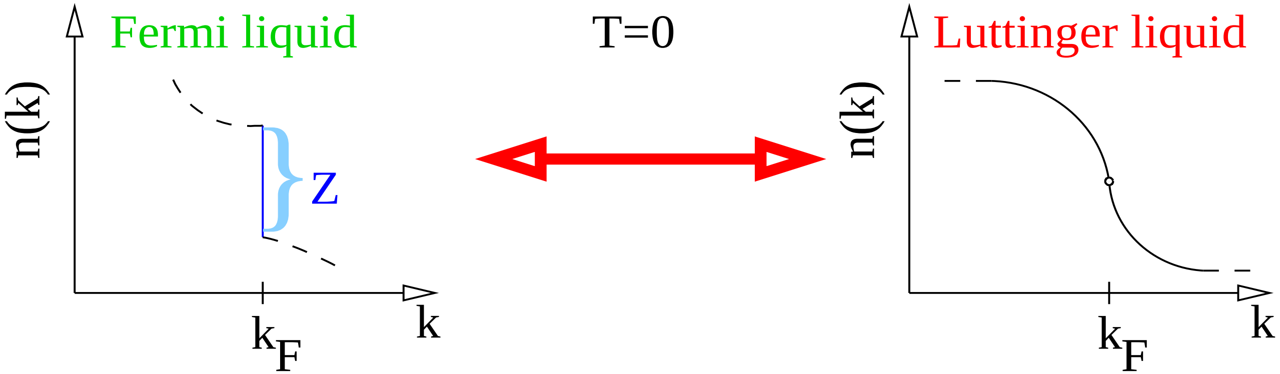


Luttinger liquid in equilibrium

Interacting 1D electron gas: metallic or gapped, Fermi liquid picture breaks down.

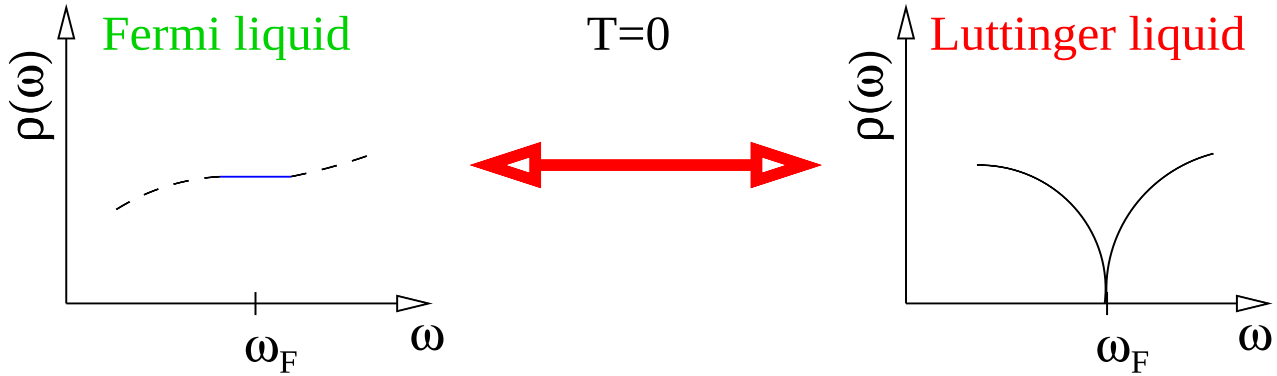
$$\langle \Psi(x,t)\Psi^{+}(0,t)\rangle \sim \quad \frac{Z}{x} \quad \Longleftrightarrow \quad \frac{1}{x^{1+\gamma^2}}$$

momentum distribution:



$$\langle \Psi(x,t)\Psi^{+}(x,0)\rangle \sim \quad t^{-1} \quad \Longleftrightarrow \quad t^{-(1+\gamma^2)}$$

density of states:



Experimental realization of Luttinger liquids

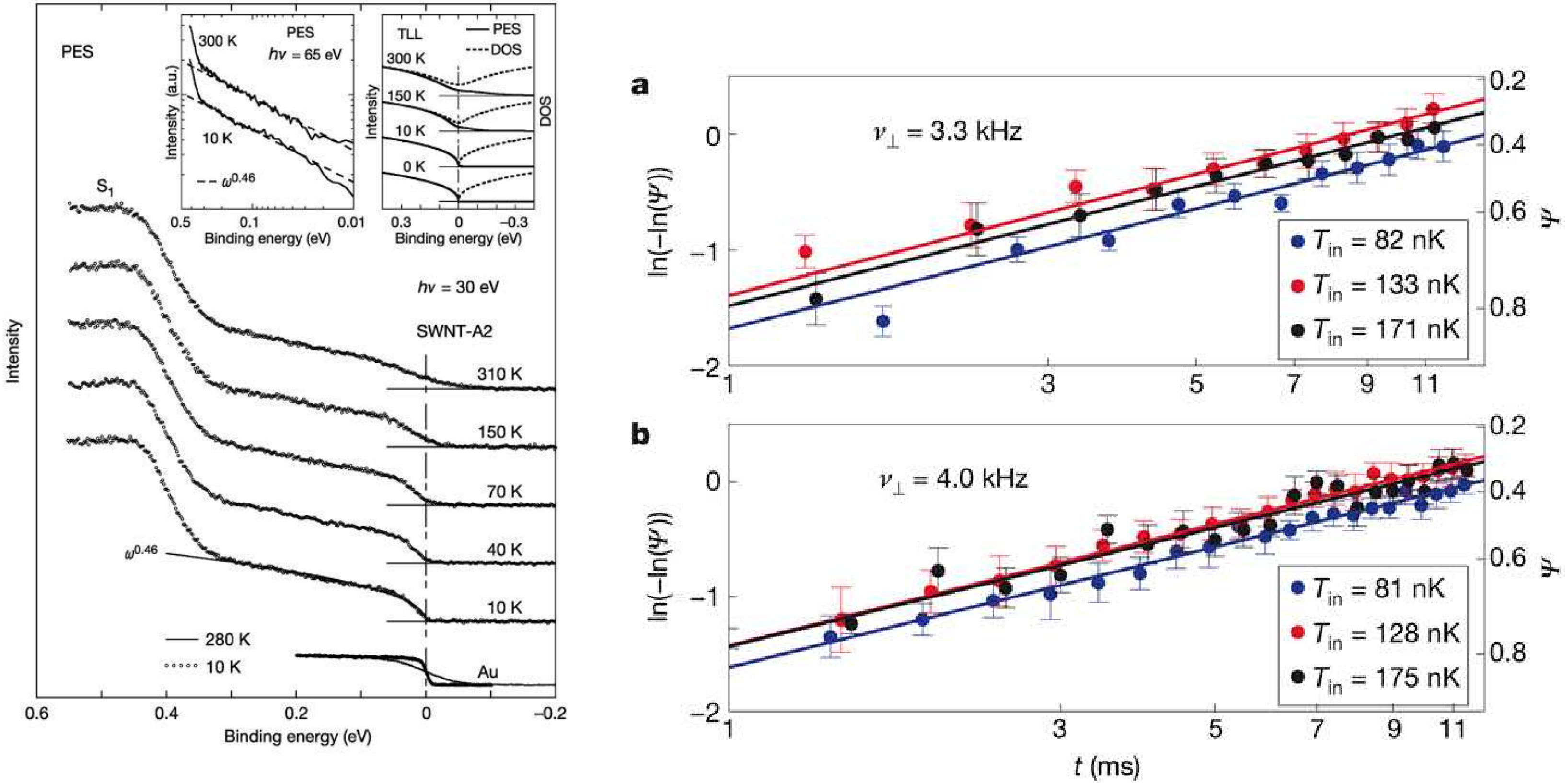
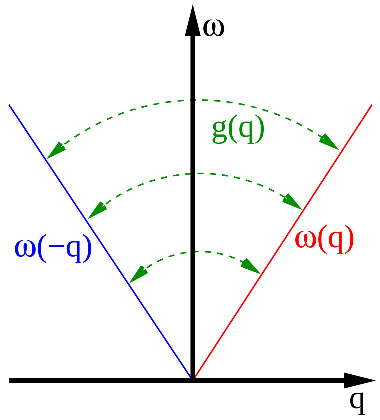


Figure 1: Left: The PES spectra of SWCNT. The spectral function, $|\omega|^\alpha$ ($\alpha = 0.46$), broadened by the energy resolution, is indicated by a thick solid line in the spectrum at 10 K. The spectra of Au (3D conventional metal) are also shown. The left inset shows the PES, which were measured with an energy resolution of 15 meV at $h\nu = 65$ eV. The right inset shows the photoemission spectra and the densities of states (DOS) calculated for the LL state in the metallic nanotubes. Right: Logarithmic plot of the coherence factor, Ψ as a function of time for decoupled 1D condensates. The slopes of the linear fits are in good agreement with a 2/3 exponent, coming from a LL description.

The Luttinger model (in equilibrium and after a quench)



The Luttinger model with an interaction quench:

$$H(t) = \sum_{q\neq 0} \omega(q) b_q^+ b_q + \frac{g(q)}{2}[b_q b_{-q} + b_q^+ b_{-q}^+]$$

b_q: bosonic density wave, $g(q) = g_2|q|$, g_2 the interaction strength, $\omega(q) = v|q|, v$ the sound velocity of the non-interacting system.

Luttinger parameter: $K = \sqrt{(v - g_2)/(v + g_2)}$.
Bogoliubov rotation: $b_q = \cosh(\theta) B_q + \sinh(\theta) B_{-q}^+$, $[\cosh(\theta), \sinh(\theta)] = \frac{K\pm 1}{2\sqrt{K}}$.

The time dependence of boson field

$$b_q(t) = u_q(t) b_q + v_q(t)^* b_{-q}^+,$$

$$u_q(t) = \cos(\omega_q t) - i \sin(\omega_q t) \cosh(2\theta), \qquad v_q(t) = -i \sin(\omega_q t) \sinh(2\theta)$$

Equilibrium evolution: same K for Bogoliubov rotation and t-dependence.

Quench: no Bogoliubov rotation (non-interacting to interacting quench)

Operators in OTOC

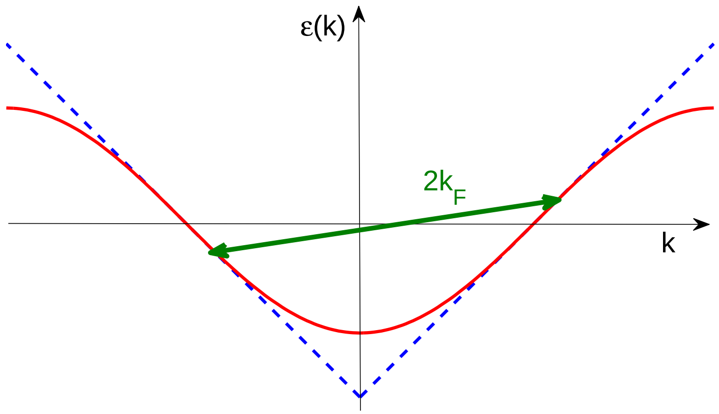
Long wavelength ($q \sim 0$) density fluctuations and a vertex operator:

$$n_0(x) = -\frac{1}{\pi}\frac{\partial \phi(x)}{\partial x}$$
$$V_n(x) = \exp(in\phi(x))$$

n integer, $V_{\pm 2}/2\pi\alpha$ is the $\pm 2k_F$ Fourier component of the density.

V_n is responsible for

- density wave phase transitions,
- Friedel oscillations,
- phase fluctuations in a 1D quasi condensate,
- superconductivity.



The mapping closes with

$$\phi(x) = \sum_{q \neq 0} \sqrt{\frac{\pi}{2|q|L}} \left(\exp(iqx) b_q + \exp(-iqx) b_q^+ \right) .$$

Long wavelength OTOC

Density response function: $\chi(t) = \langle [n_0(x,t), n_0(0,0)] \rangle$.
The bare commutator is a c-number due to the linear dispersion: $\chi(t) = \frac{iK}{\pi^2}\left(\frac{\alpha(v_f t+x)}{(\alpha^2+(v_f t+x)^2)^2} + (x \to -x)\right)$.

OTO commutator:

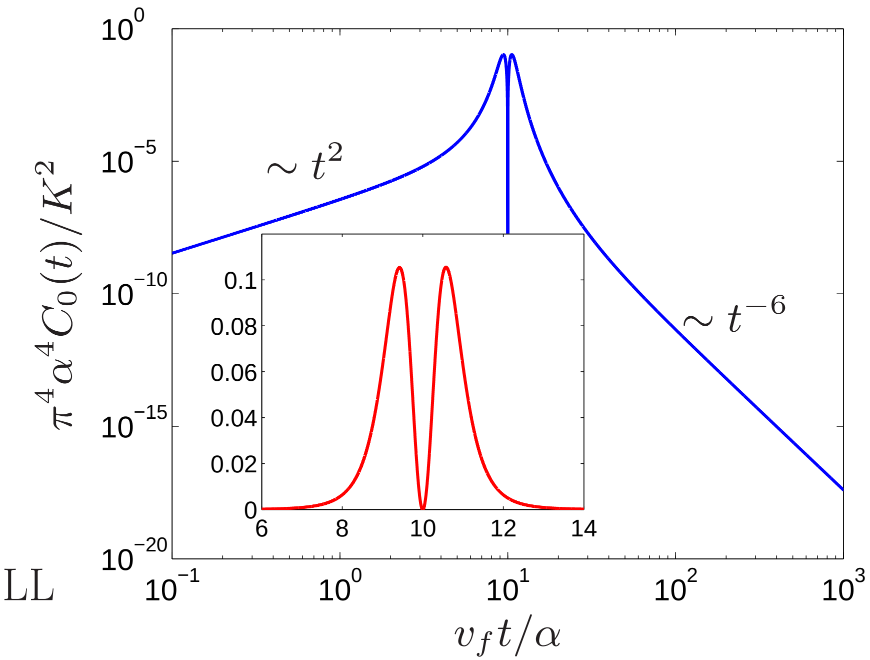
$$C_0(t) = \left\langle |[n_0(x,t), n_0(0,0)]|^2 \right\rangle = |\chi(t)|^2.$$

Properties:

a.) no influence on the OTO correlator.

b.) the bare commutator is c-number, and depends *only* on the time evolution operator, independent of the initial state.
No distinction between quench and equilibrium.

c.) $C_0(t) \sim t^2$ initially: lowest possible power with prefactor $\langle |\, [[H,W]\,,V]\, |^2\rangle$.

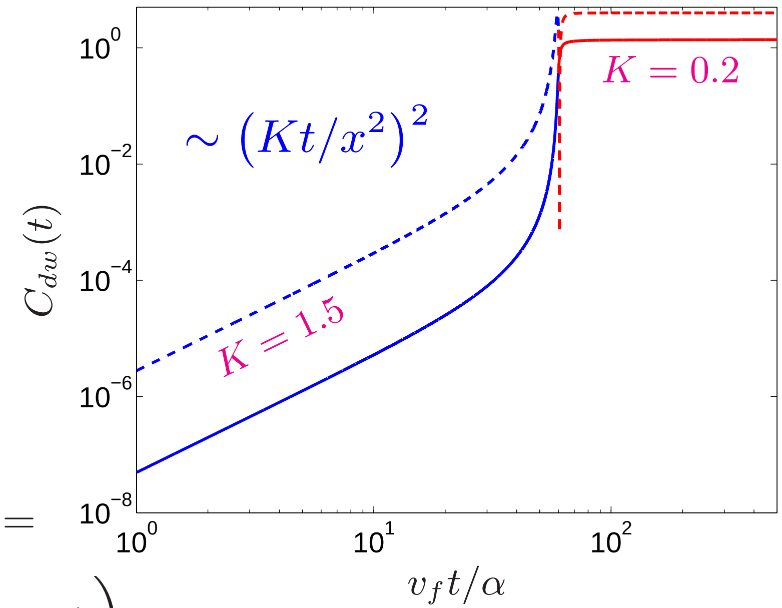


Warning: $\chi(t) \to S(\omega, q) \sim K|q|\delta(\omega - v_f|q|)$,
infinite lifetime due to the linearized dispersion: nonlinear LL

Short wavelength OTOC

Simple commutator:

$[V_n(x_1), V_{-m}(x_2)] = 2e^{i(n\phi_1 - m\phi_2)} \sinh\left(\frac{nm}{2}[\phi_1, \phi_2]\right)$
a vertex-like operator + commutator (c-number):



$[\phi(x,t), \phi(0,0)] = -i\frac{K}{2} \arctan\left(\frac{v_f t + x}{\alpha}\right) + (x \to -x).$
After some contraction, point splitting and merging:

$$C_{dw}(t) = \left\langle |[V_2(x,t), V_{-2}(0,0)]|^2 \right\rangle =$$
$$= 4\sin^2\left(K \arctan\left(\frac{v_f t + x}{\alpha}\right) + (x \to -x)\right)$$
$$\xrightarrow{t\to\infty} 4\sin^2(K\pi).$$

The long time value vanishes vanishes for $K = 1$.
From Cauchy-Schwarz inequality ($\|ab\| \leq \|a\|\|b\|$) and $\|V_n(x)\| = 1$: $C_{dw}(t) \leq 4.$

In a suitably chaotic system, the late time value is expected to be 2.

Influence of OTO correlator albeit independent from temperature and initial state !

Density OTOC for interacting fermions: -$\langle [n_1(t), n_1]^2 \rangle$

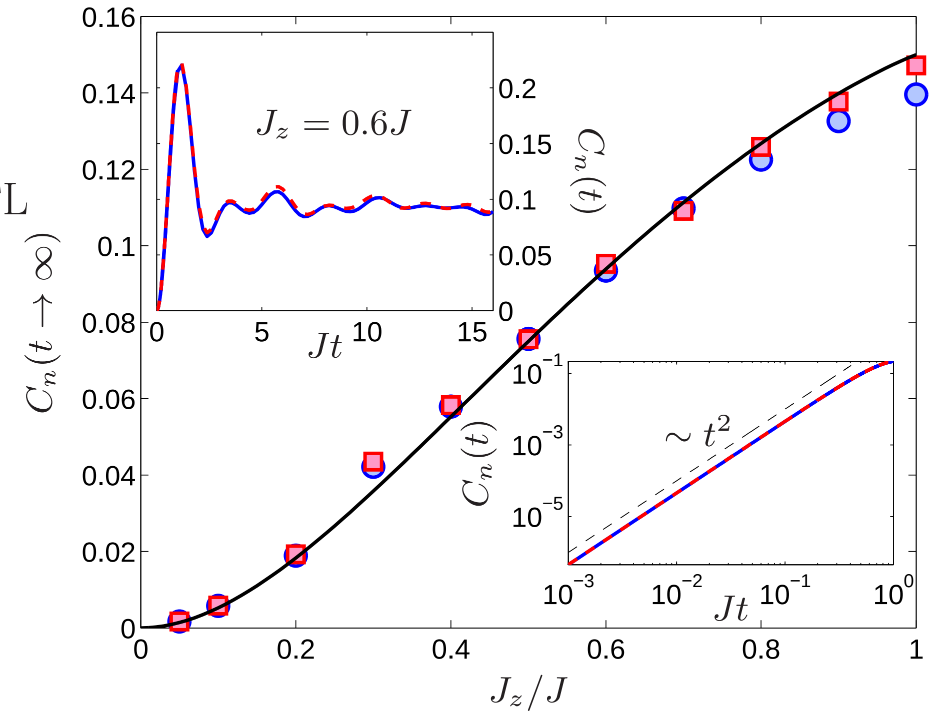
Numerics (PBC, $N = 10$, 14...26), LL for $J_z < J$ with LL parameter $K = \pi/2[\pi - \arccos(J_z/J)]$.

$$H = \sum_{m=1}^{N} \frac{J}{2} \left(c^+_{m+1} c_m + \text{h.c.} \right) + J_z n_{m+1} n_m,$$

OTOC of local charge density, $n_1 = c_1^+ c_1$, $n(0) = -\partial_x \phi(0)/\pi + n_{2k_F} \cos(2\phi(0))$

a.) equilibrium and quench: $\sim$ identical
b.) time independent value after a transient time
c.) $\sim t^2$ at short times

Any finite J_z destroys the Fermi gas and induces NFL (LL) behaviour, and the bare fermionic excitations are replaced by collective bosonic modes. The density operator naturally decomposes into collective modes during the time evolution.



OTOC of n_1 and n_{1+x}: -$\langle [n_1(t), n_{1+x}]^2 \rangle$

The late time behaviour after hitting the light cone agrees with our previous results.

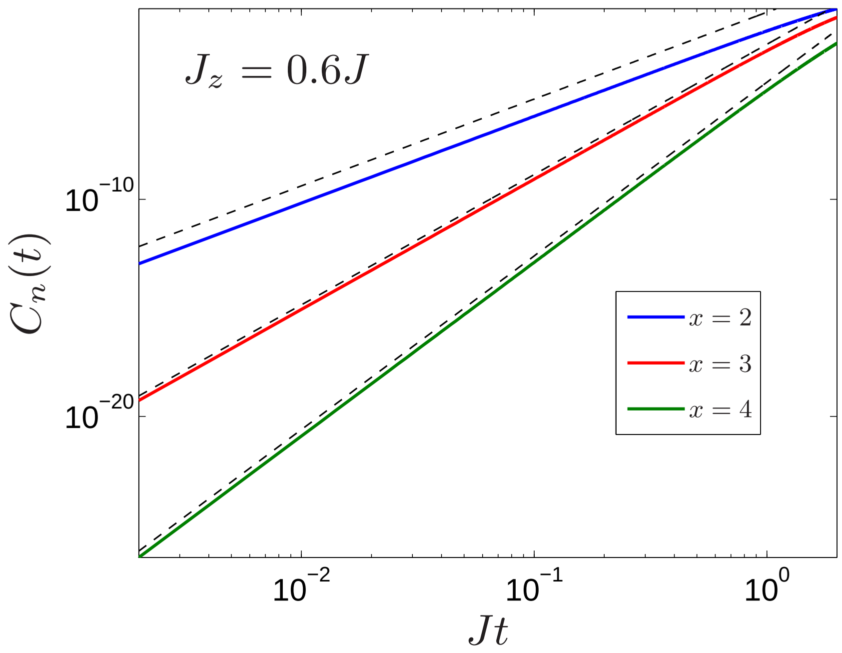
For shorter times, $\sim t^{2x}/(2x)!$. Baker-Campbell-Hausdorff expansion of $W(t)$ with the nested commutators

$$W(t) = W + it\,[H, W] + \frac{(it)^2}{2!}\,[H, [H, W]] + \ldots .$$

For $W = n_{x+1}$, the coefficient of t^2 term, $\langle [[H, n_{x+1}]\,, n_1]^2 \rangle$, vanishes for $x > 1$.

At distance x, the commutators start contributing at xth order nesting.

Non-integrable extension with $J'_z \sum_m n_{m+2} n_m$: same central features!



Summary

- Universal features in the OTOC: t^2 initial growth.

- Non-universal feature: the short time growth exhibits a distance dependent power.

- Temperature and initial state independence.

- Large late time value even in non-chaotic systems.

- Non-Fermi liquids behave as chaotic systems through the lense of OTOC.

- Similar behaviour in the 1 vs. 2 channel Kondo impurity model with Fermi vs. non-Fermi liquid ground states.