



arXiv:1411.1003

arXiv:1601.03384

arXiv:1708.08306

arXiv:1708.08477



Crete Center  
for Theoretical Physics

# ***Elasticity and (pseudo)-phonons In holography***

***Many-Body Quantum Chaos, Bad Metals and Holography***

**With O.Pujolas, M.Ammon, L.Alberte, A.Jimenez, (+ A.Krikun, T.Andrade, N.Poovuttikul )**

***Matteo Baggioli***

***UOC & Crete Center for Theoretical Physics***

arXiv:1708.08477



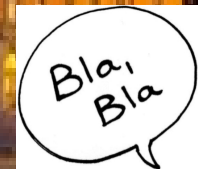
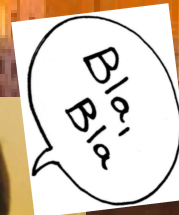
**COMING SOON**

# *Holographic transverse PHONONS*

With O.Pujolas, M.Ammon, L.Alberte, A.Jimenez

**MY FIRST  
TIME HAVING**

*Transverse phonons*



See also Daniele & Nick talks

# Out of Time Talk (OTT)

$$\langle [\text{CONCLUSIONS}(t, \vec{x}), \text{INTRO}(0, \vec{x})] \rangle_{\beta}$$

**HOLOGRAPHIC  
MASSIVE  
GRAVITY**

**=**

**BREAKING  
OF  
TRANSLATIONS**



**3-IN-ONE®**

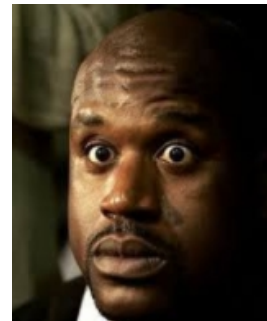
**1) Explicit**



**2) Spontaneous**



**3) Both**



**WHERE ARE THE  
PHONONS ??**



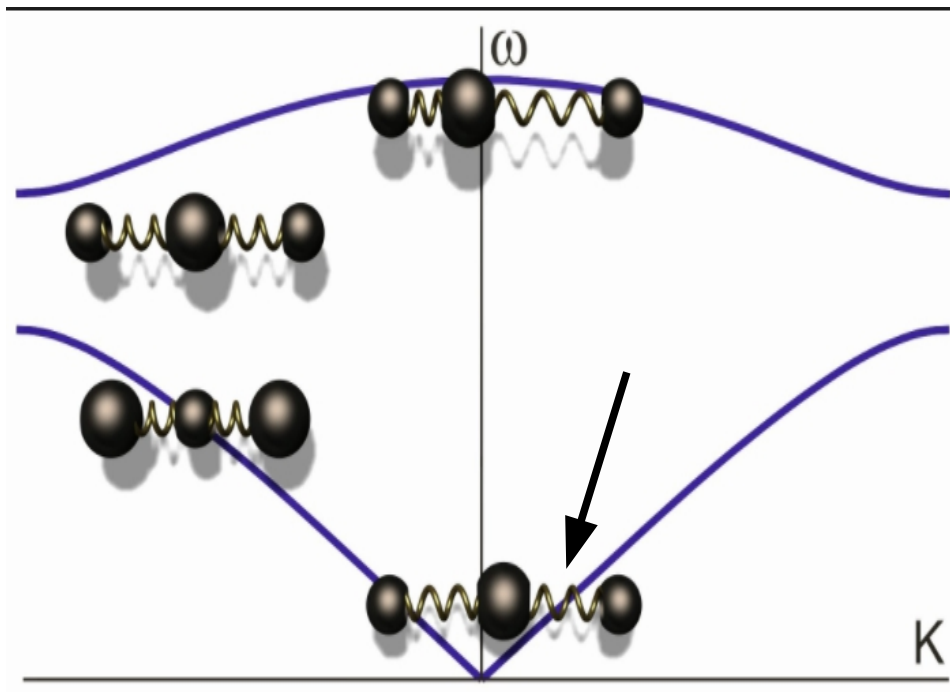
**IS HOLOGRAPHY  
ELASTIC ??**

**WHICH ARE THE EFFECTS  
ON TRANSPORT ??**





# Phonons as goldstone bosons



$$\lim_{k \rightarrow 0} E(k) = 0.$$

$$\omega_T = c_T k, \quad \omega_L = c_L k$$

Shear sound

"sound"

**PHONON = GOLDSTONE FOR TRANSLATIONAL SYMMETRY**

Old school EFT

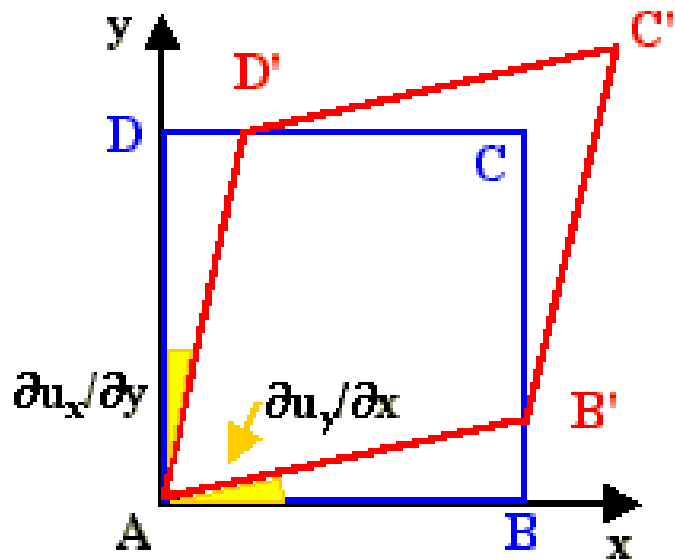
[arXiv:hep-ph/9609466](https://arxiv.org/abs/hep-ph/9609466)

$$\mathcal{L}_2 = \frac{1}{2} \rho_0 \dot{\xi}_a \dot{\xi}_a - \frac{1}{2} \mu \partial_a \xi_b \partial_a \xi_b - \frac{1}{6} (\mu + 3K) \partial_a \xi_a \partial_b \xi_b$$

Modern EFT : Sym. breaking patterns

[arXiv:1501.03845](https://arxiv.org/abs/1501.03845)

# Elasticity



**ELASTIC  
MODULI**

$$\lambda \stackrel{\text{def}}{=} \frac{\text{stress}}{\text{strain}}$$

**Bulk modulus**

$\kappa$

**Shear modulus**

$\mu$

**Strain tensor  $\longrightarrow$  Stress tensor**

$$u_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i) \longrightarrow T_{ij} = \kappa \delta_{ij} u_{kk} + 2\mu \left( u_{ij} - \frac{1}{3} \delta_{ij} u_{kk} \right)$$

**PHONONS**



**ELASTICITY**

$$c_L^2 = \frac{\kappa + \frac{4}{3}\mu}{\rho},$$

$$c_T^2 = \frac{\mu}{\rho}$$

# Gapped and Damped Goldstones arXiv:hep-th/9808176

**GOLDSTONE THEOREM**  $\langle G | \rho(\mathbf{r}, t) | \Omega \rangle \neq 0.$

**GOLDSTONE BOSON**  $\lim_{p \rightarrow 0} E(p) = 0.$

*Relativistic setups :*

$$\omega = v p$$

**PSEUDO - PHONONS**

**Now add a small  
Explicit breaking**

$$\mathcal{H} \rightarrow \mathcal{H} + \mathcal{H}_{EB}$$

Example: PIONS

$$\omega^2 = \omega_0^2 + v^2 p^2 - i \omega \Gamma + \dots$$

*See also Niko's and Daniele's talks*

**Mass gap ( pinning frequency )**

**Damping (relaxation rate )**

**Both have to be small compared to the typical energy scale !!**

# Implications on (electric) Transport

**Purely spontaneous**

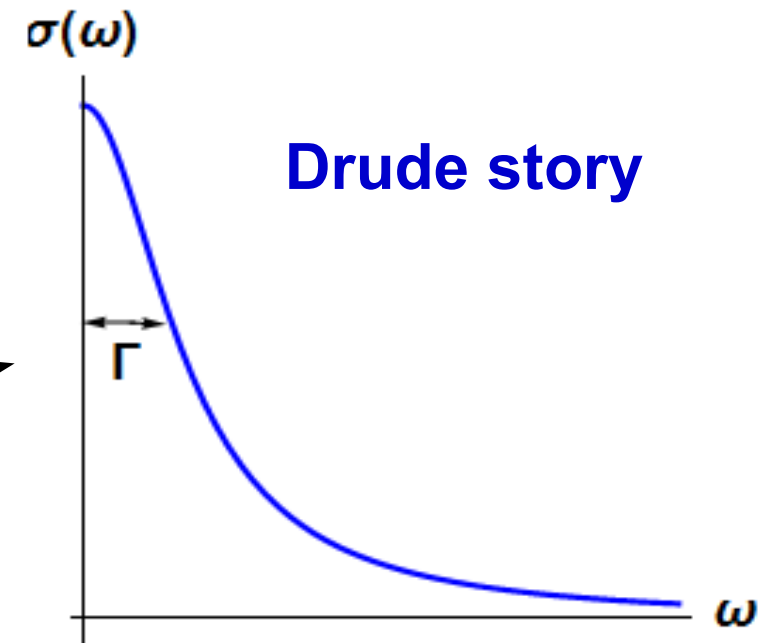
$$\sigma = \sigma_0 + \frac{\rho^2}{\chi_{PP}} \delta(\omega)$$

**Purely Explicit**

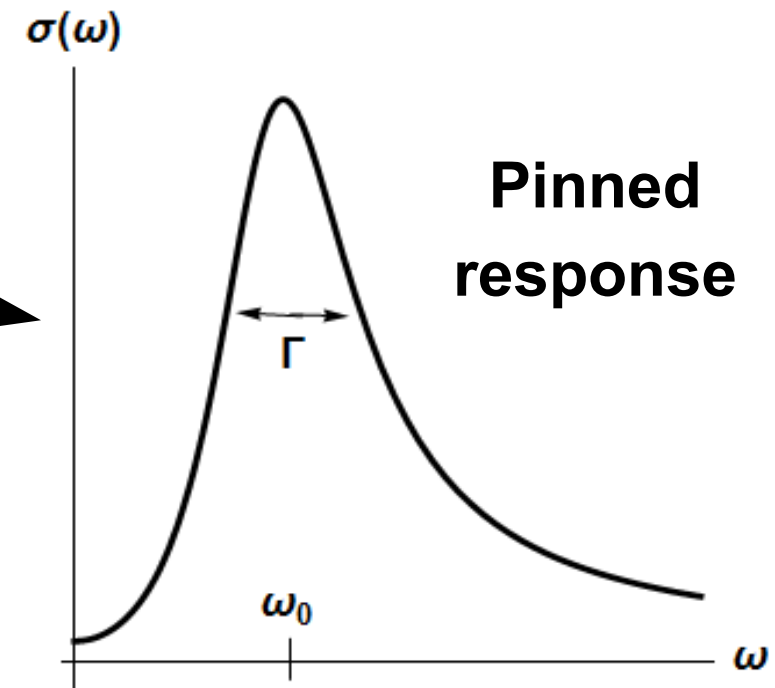
$$\sigma = \sigma_0 + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma - i\omega}$$

**Explicit + Spontaneous**

$$\sigma = \sigma_0 + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{-i\omega(\Gamma - i\omega) + \omega_0^2}$$



**Drude story**



**Pinned response**



# "Solid" Massive gravity

arXiv:1411.1003

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( R + \frac{6}{l^2} \right) - m^2 V(X) \right]$$

$$X \equiv \frac{1}{2} \text{tr}[\mathcal{I}^{IJ}], \quad \mathcal{I}^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J, \quad \hat{\phi}^I = \delta_i^I x^i$$

**LORENTZ VIOLATING**

**HOLOGRAPHIC MASSIVE GRAVITY**

Used a lot to implement momentum dissipation  
(explicit breaking of translations)

Is it just that??

**Radially dependent graviton mass  
(gauge inv. Order parameter)**

**VS fluid**

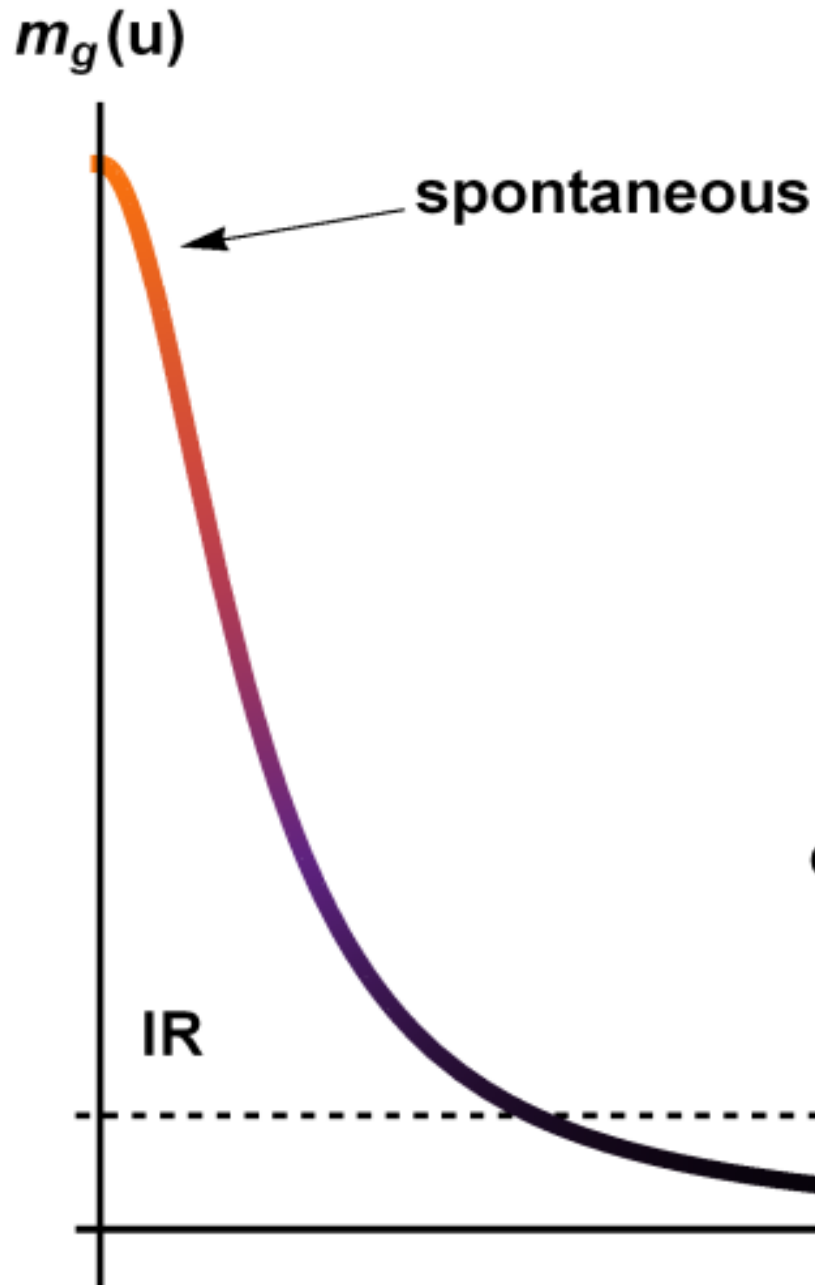
**Massive gravity**

$$Z = \det [\mathcal{I}^{IJ}]$$

arXiv:1510.09089

$$M^2(z) \equiv \tilde{m}^2 V_X$$

# Explicit + Spontaneous (part I)



We want to localize the graviton  
Mass in the IR (close to the horizon)

Example :  $V(X) = X + \beta X^5$

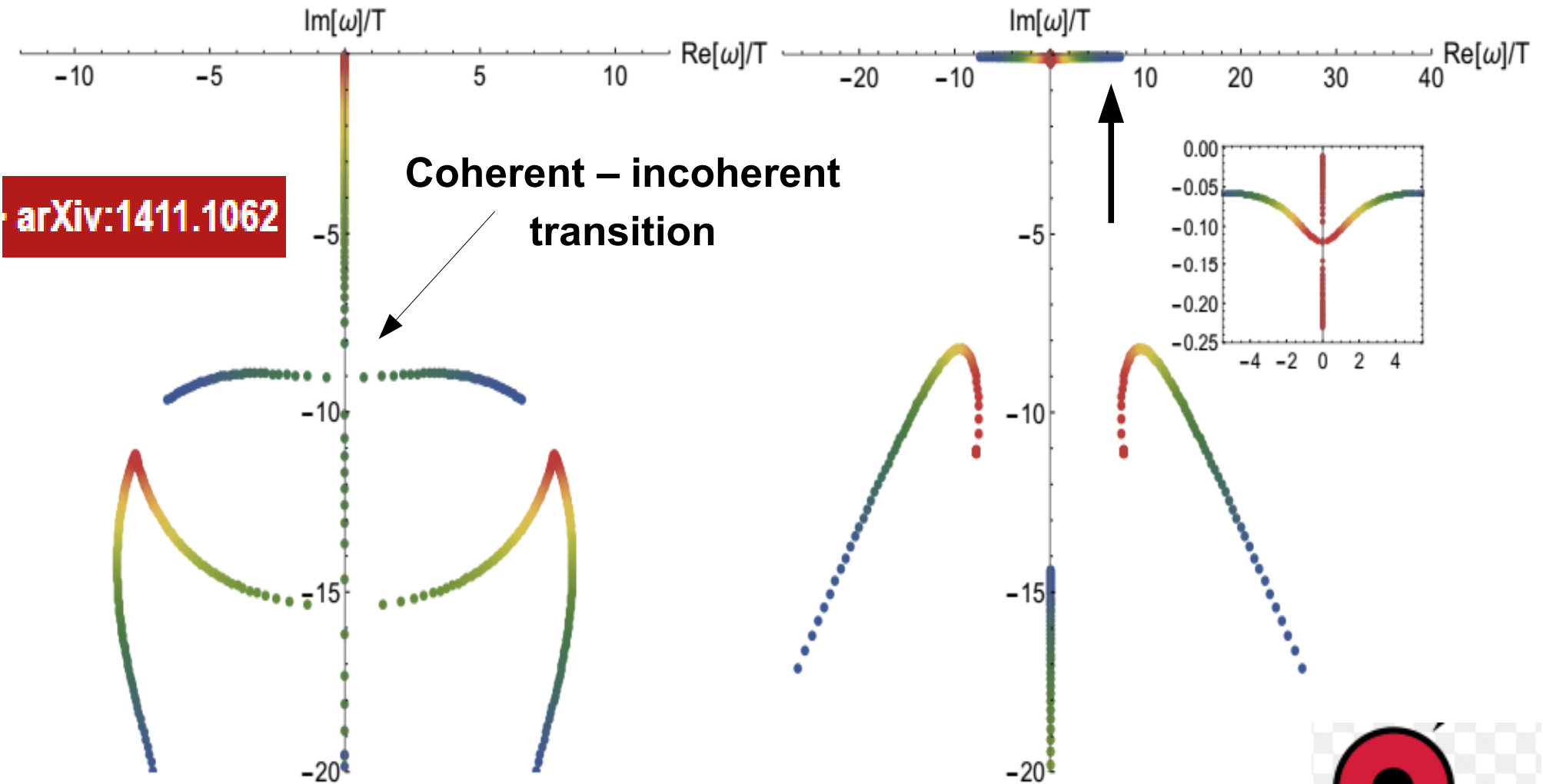
$$\frac{m_{UV}^2}{m_{IR}^2} = \frac{1}{1 + 5\beta \left(\frac{u_h}{l}\right)^8}$$

Large "beta" Realizes that !!

KEEP  
CALM  
AND  
LET'S CHECK

# The Transverse (Pseudo)-Phonon

arXiv:1708.08477

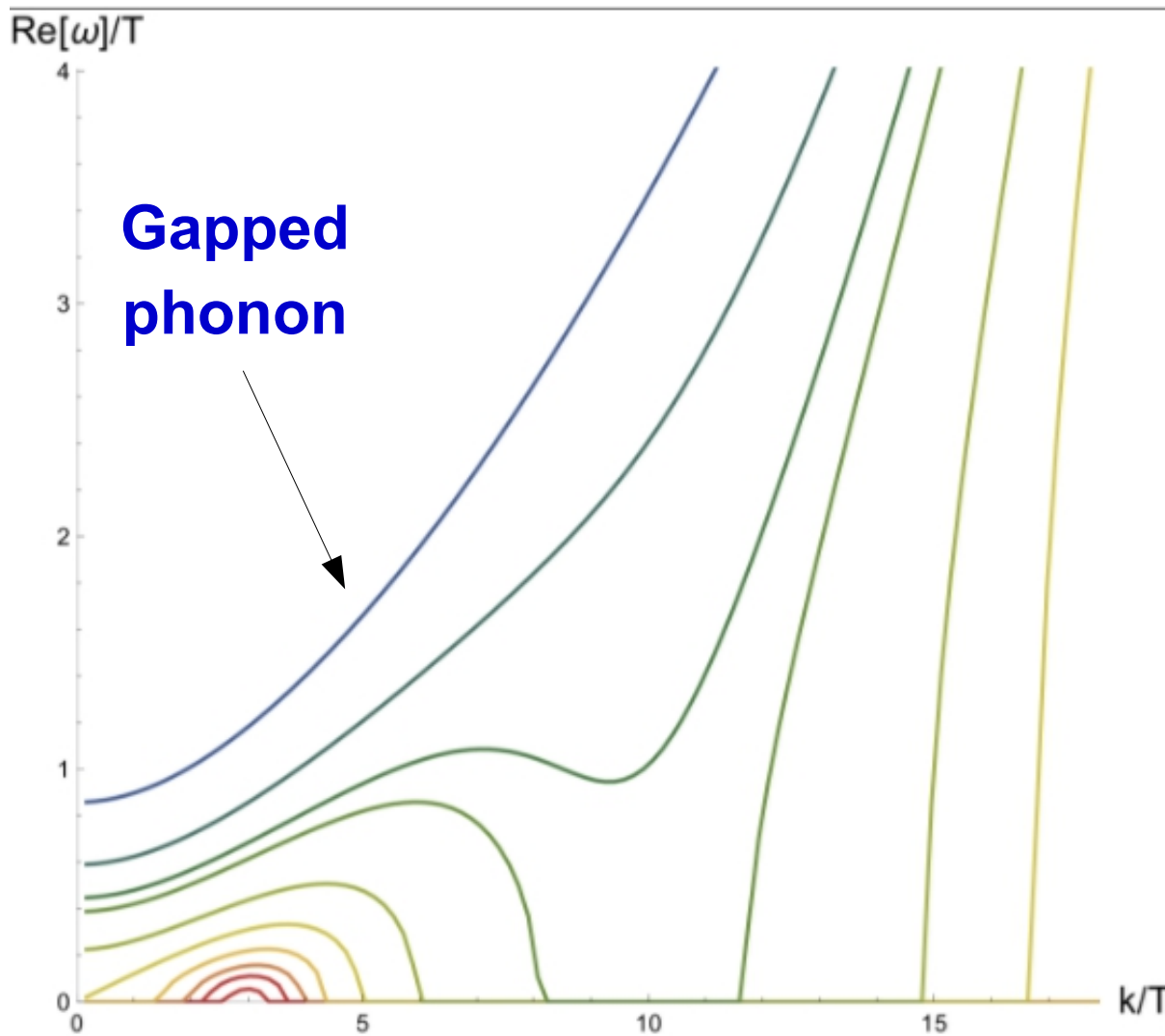


**A light and well separated mode  
appears in the transverse excitations**



**TRANSVERSE (GAPPED AND DAMPED) PHONON**

# The Gapped and Damped Phonon



At small  $m/T$  we have

$$(\text{Re}[\omega])^2 = a_1 + a_2 k^{a_3},$$

$$a_3 = 2 \pm 10^{-4}$$

We can fit and get:

The mass gap  $\omega_0$

$$\omega = \frac{1}{2} \left( -i\Gamma + \sqrt{4w_0^2 + c_T^2 k^2 - \Gamma^2} \right)$$

The speed  
Of the  
Shear sound !!  $c_T^2$



# Shear Sound and Elastic modulus

Shear elastic  
modulus :

$$\langle T_{ij} \rangle = \mathcal{G}_{T_{ij} T_{ij}}^R \gamma_{ij}^{(0)}$$

$$\mu = \text{Re } \mathcal{G}_{T_{ij} T_{ij}}^R$$

**SHEAR SOUND SPEED**  $c_T^2 = \frac{\mu}{\chi_{PP}}$

[arXiv:1601.03384](#)

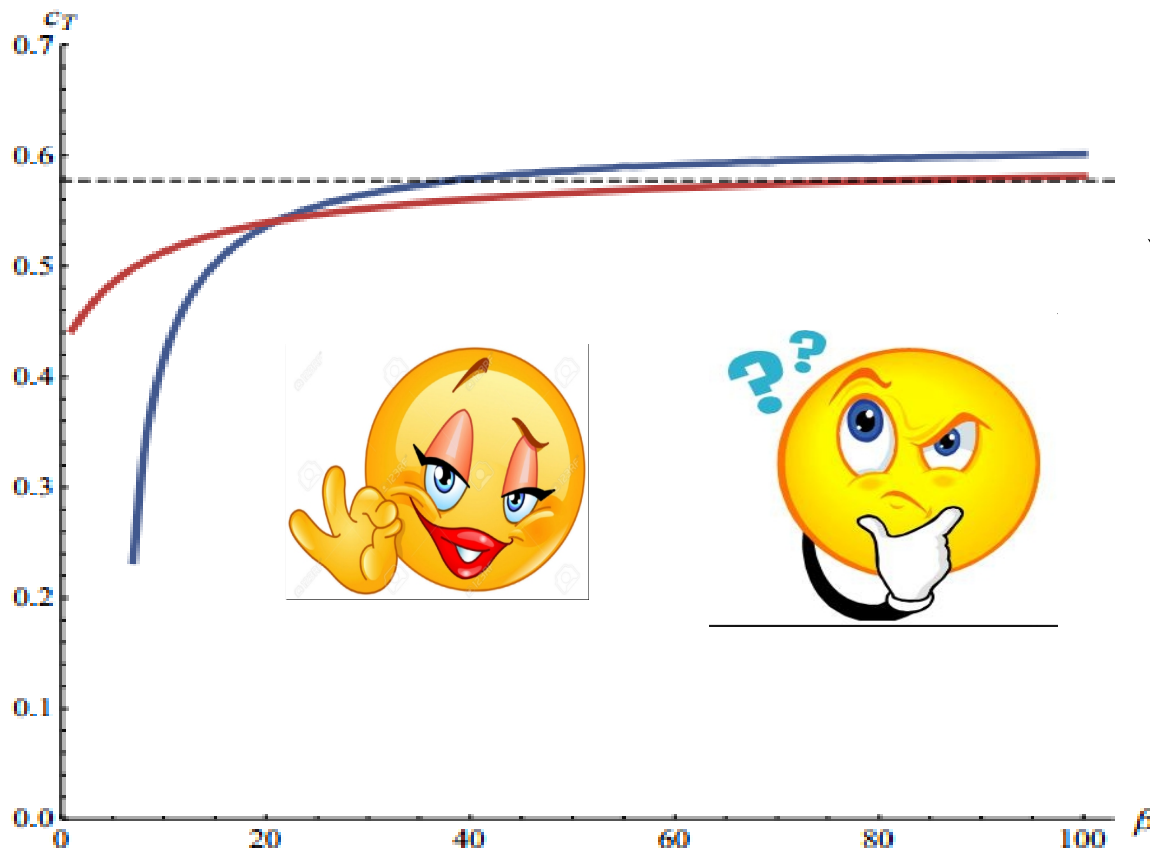
$$\chi_{PP} = T_{00} + T_{xx}$$

**Good agreement  
(but not perfect...)**

$$\approx \frac{1}{\sqrt{3}}$$

See also :

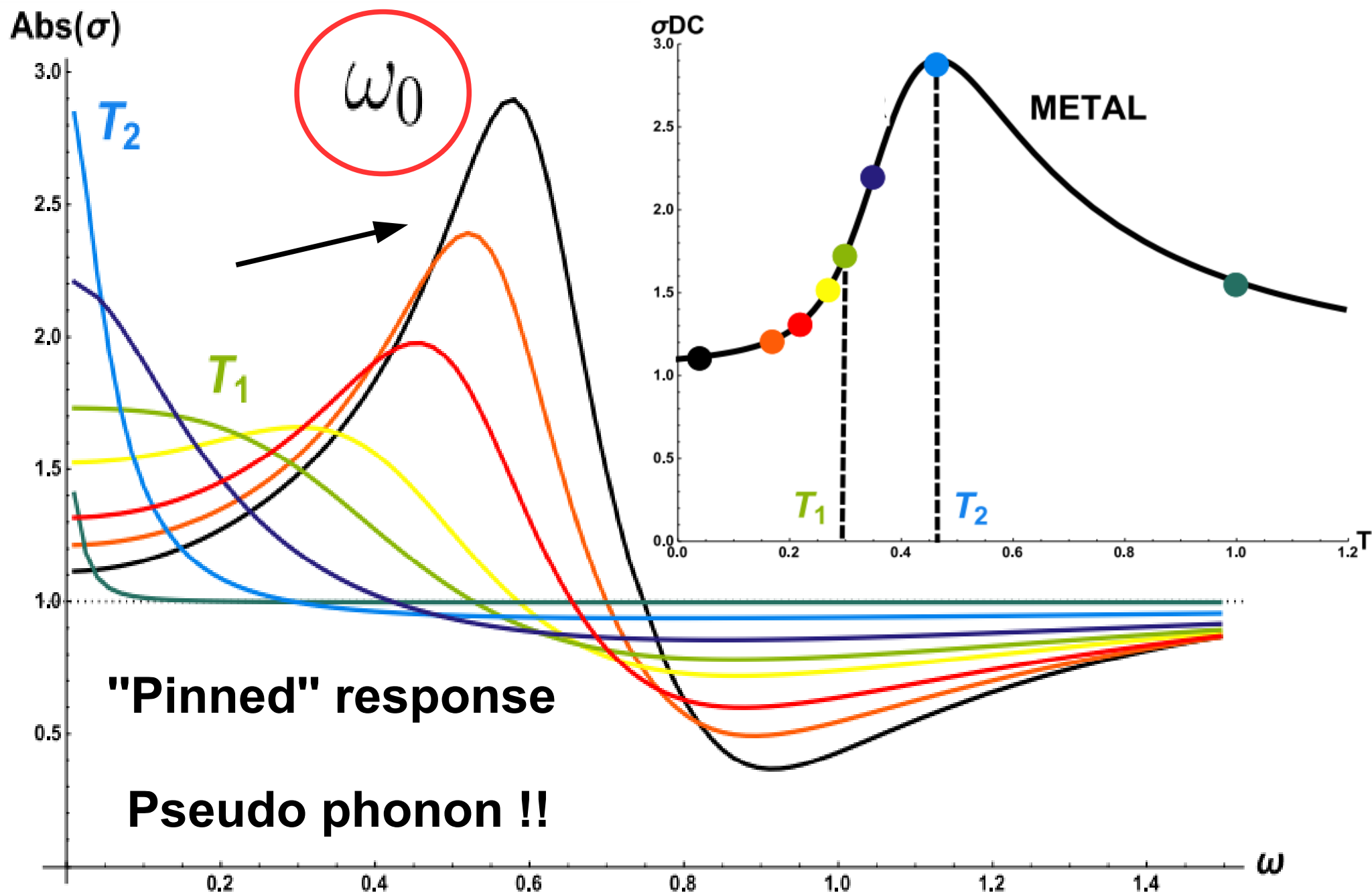
[arXiv:1708.09391](#)



**We will improve it...**

# Electric (transport)

arXiv:1411.1003

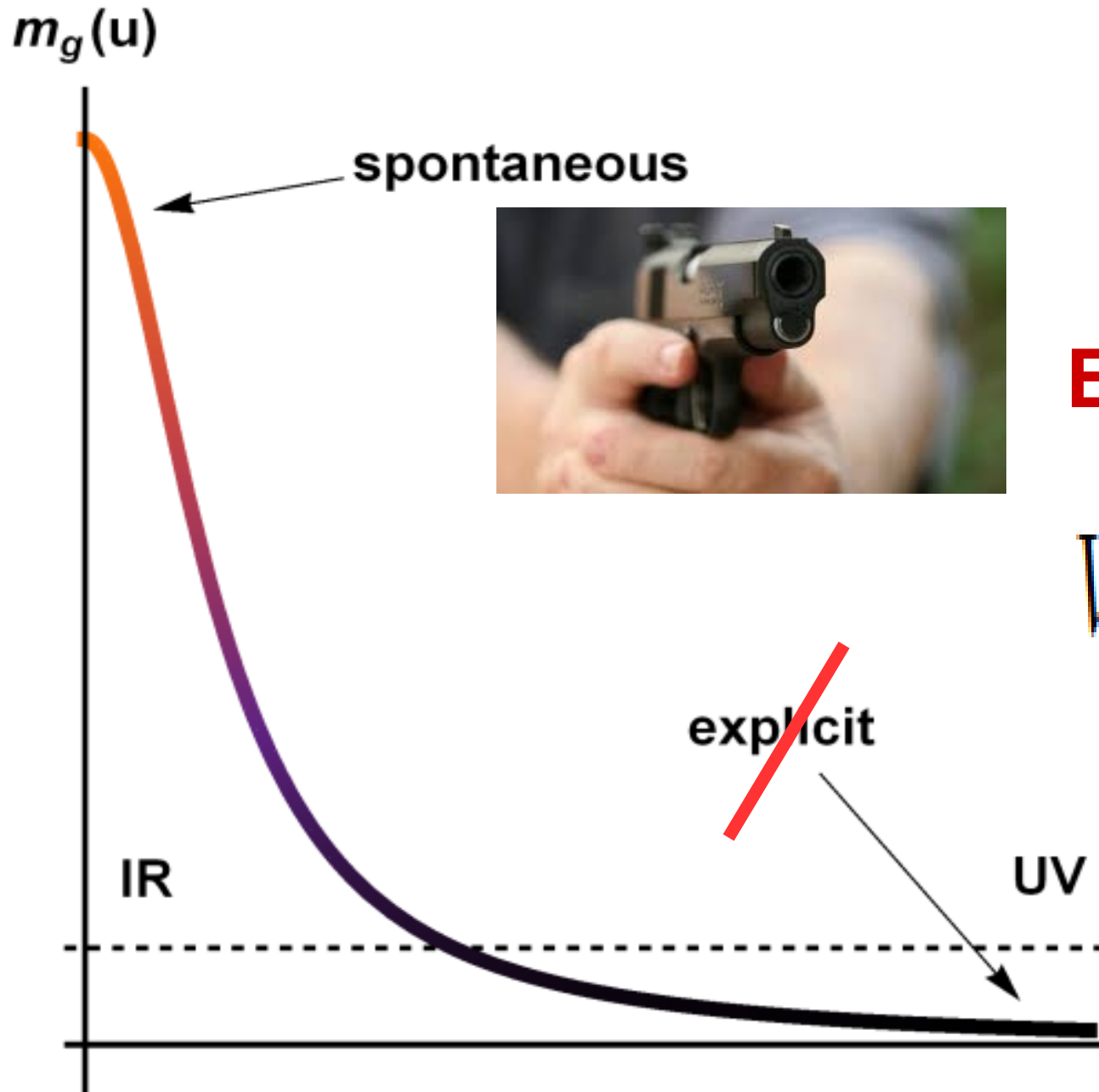




**MASSLESS  
"TRUE"  
PHONON**



# Purely Spontaneous (part II)



$$M^2(z) \equiv \tilde{m}^2 V_X$$

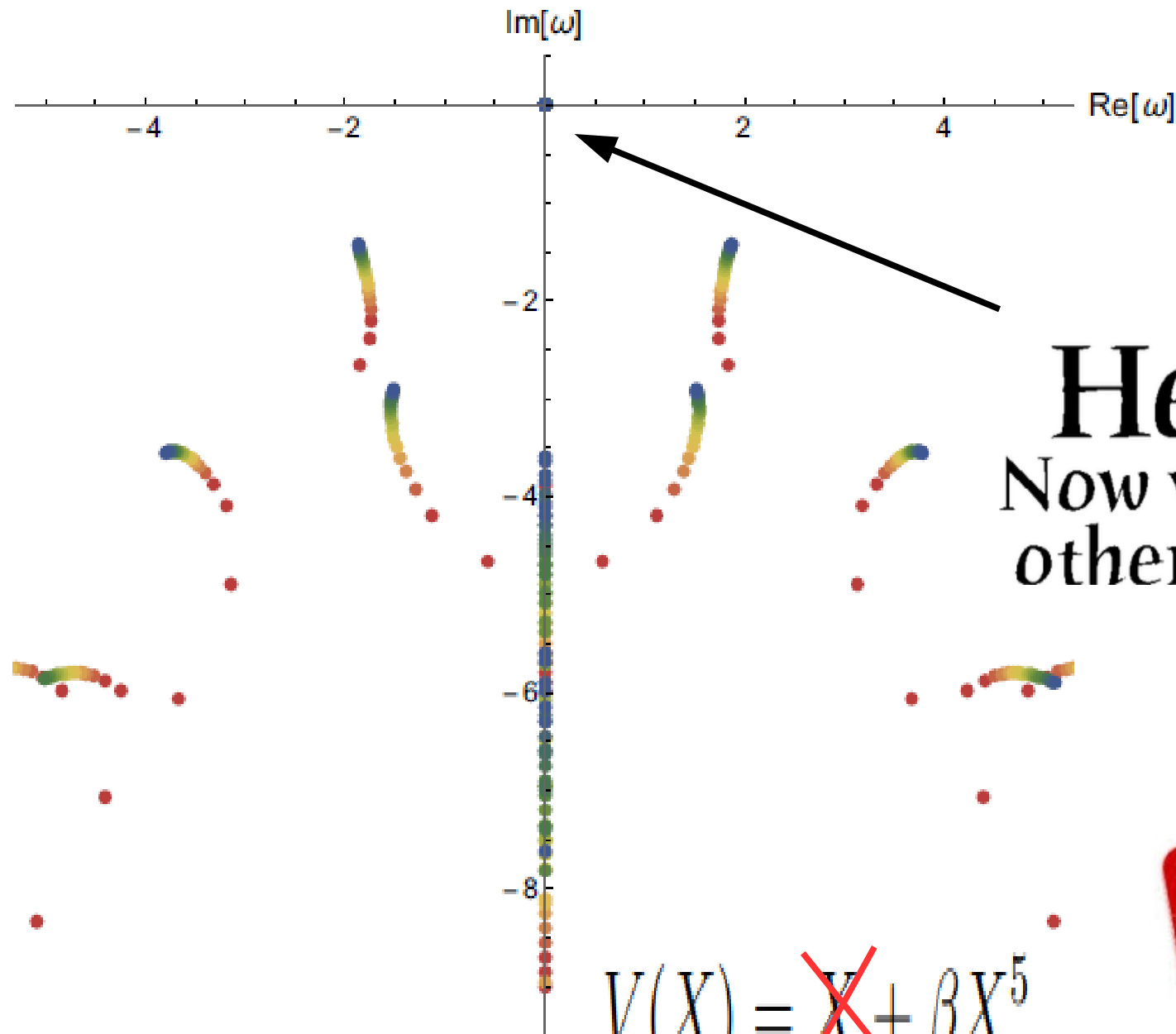
**Killing the  
Explicit breaking**

$$V(X) = \cancel{m^2 X^2} + \beta X^5$$

**UV MASS  
BECOMES  
ZERO !!!**



# THE TRANSVERSE PHONON



Here I am  
Now what are your  
other two wishes?

$$V(X) = \cancel{X} + \beta X^5$$

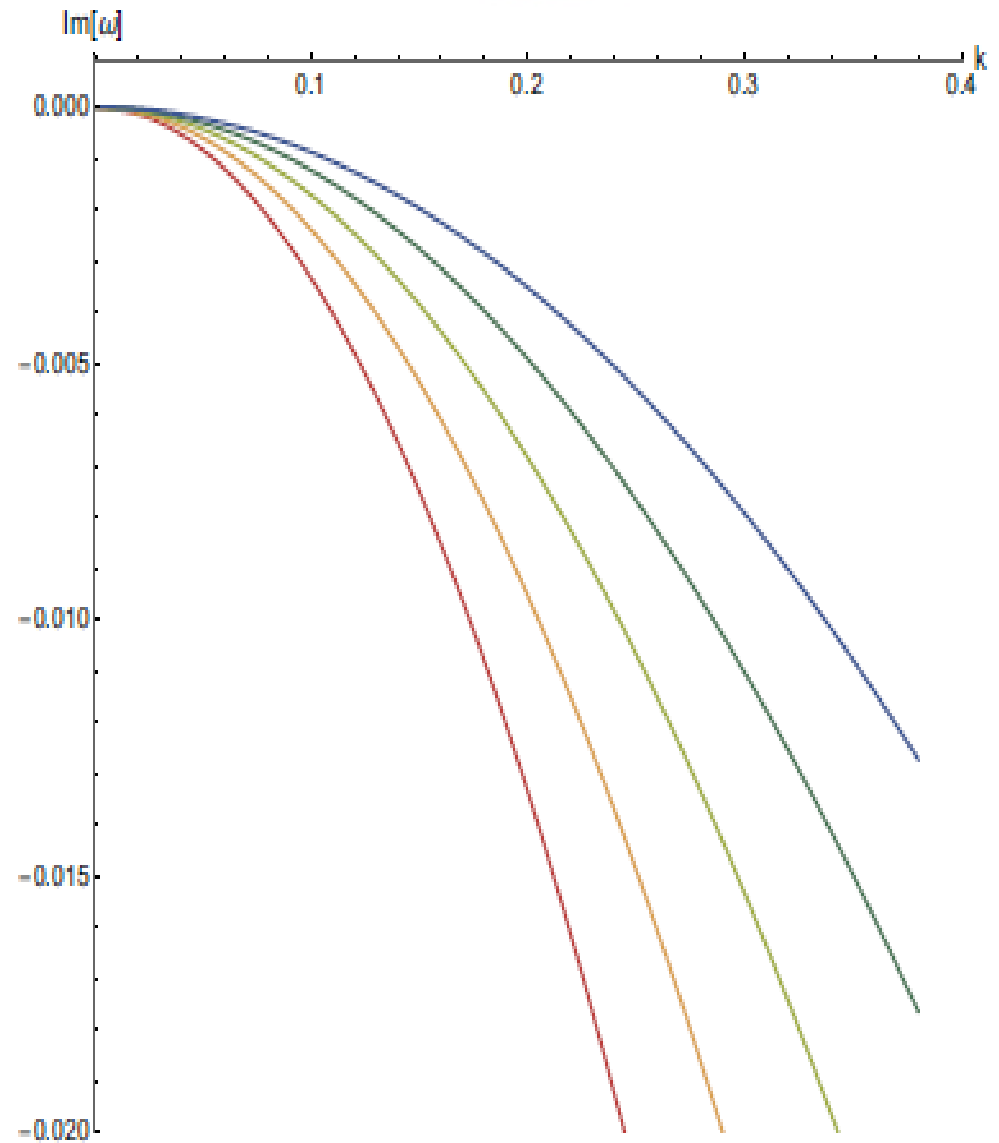
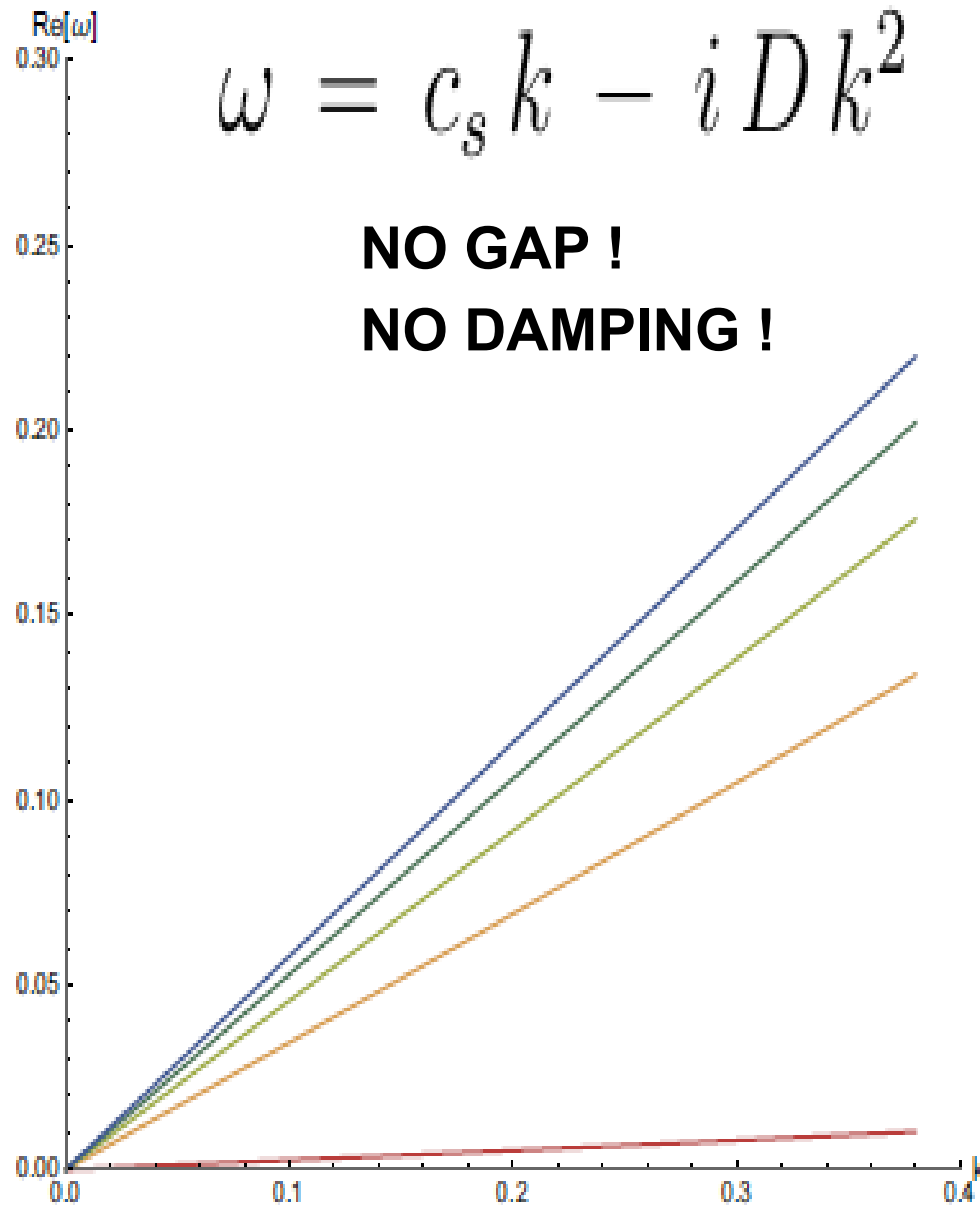
**COMING  
SOON!**

# THE TRANSVERSE PHONON



$$\omega = c_s k - i D k^2$$

**NO GAP !**  
**NO DAMPING !**



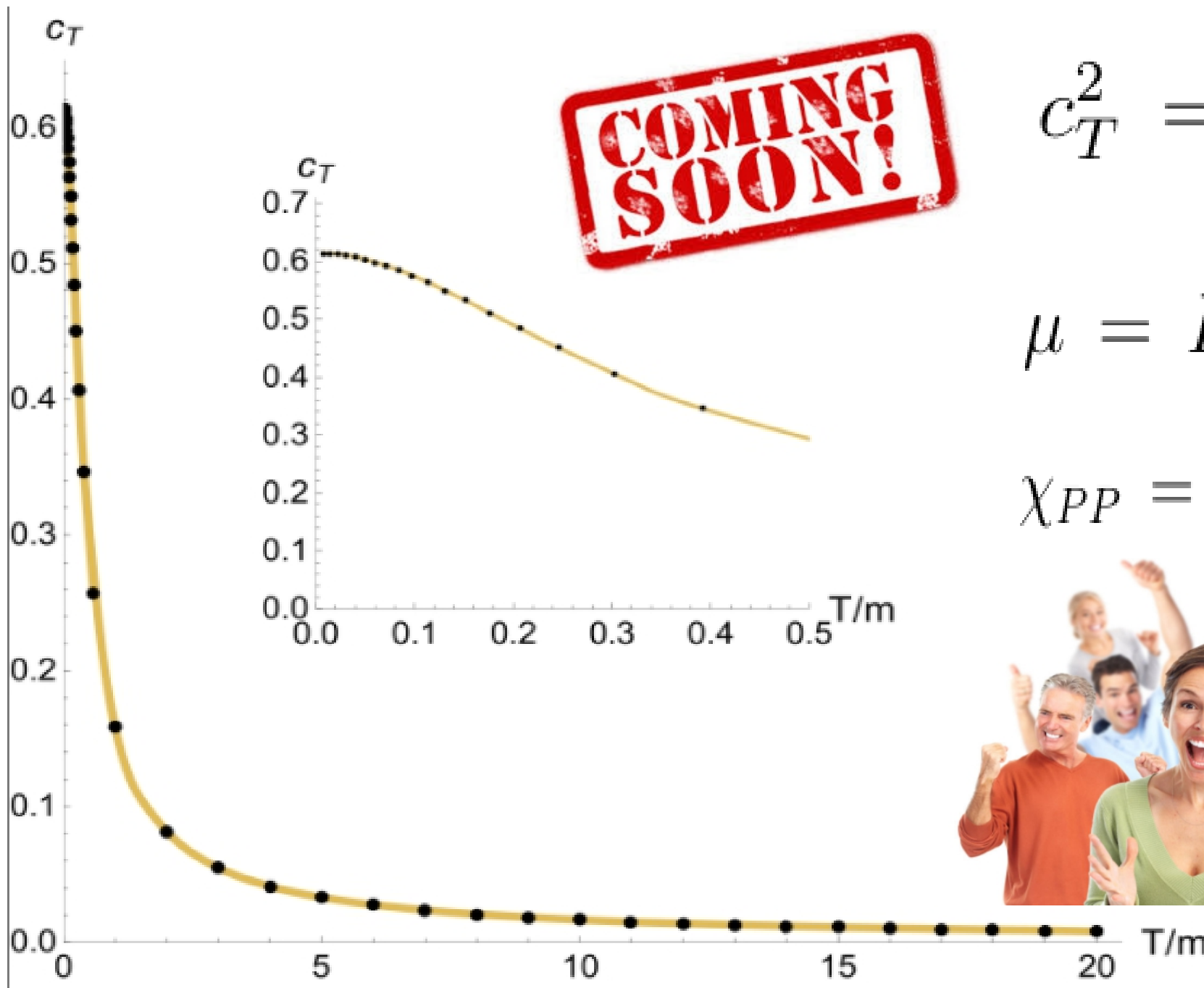
# SHEAR SOUND AND ELASTICITY

**COMING SOON!**

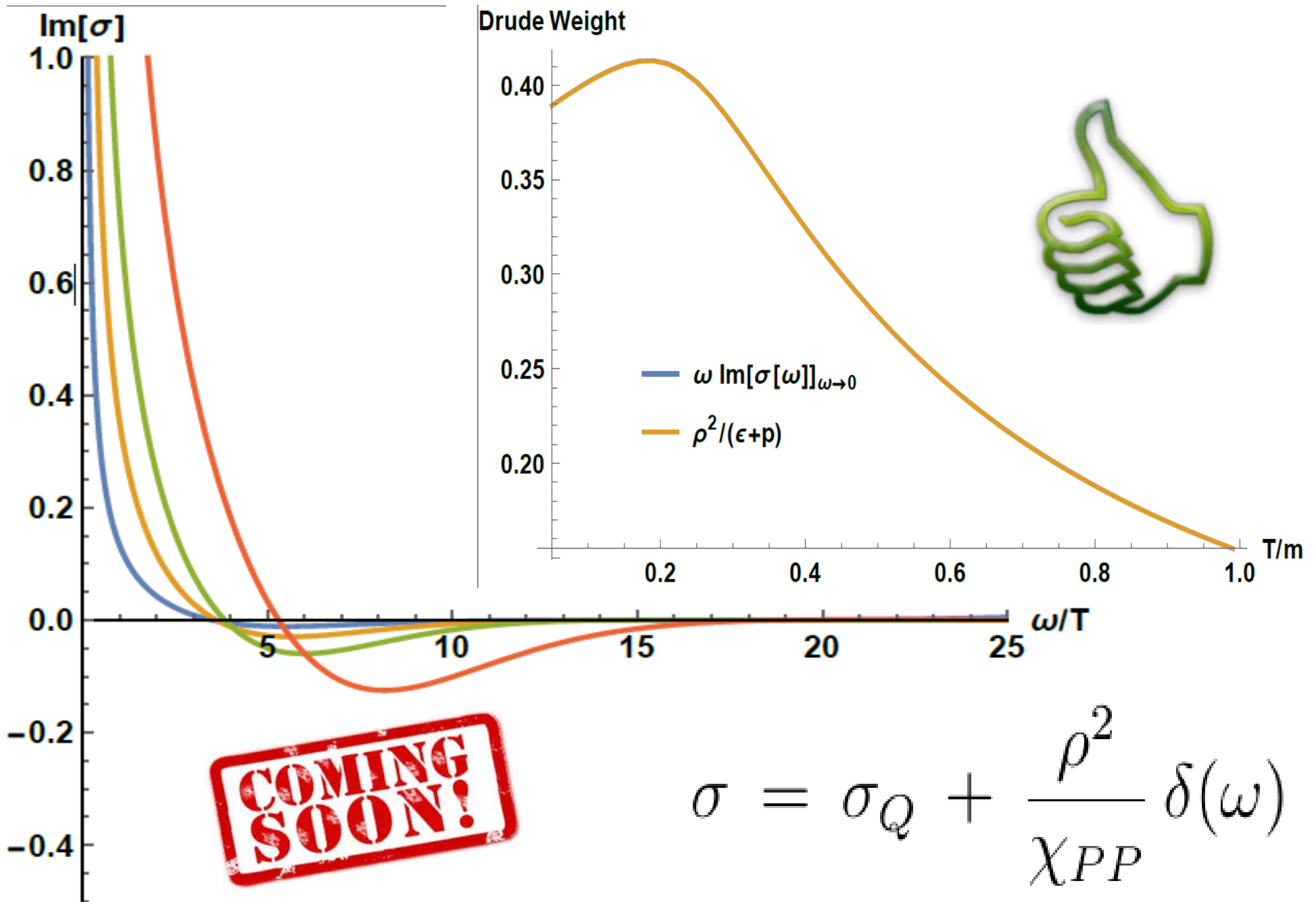
$$c_T^2 = \frac{\mu}{\chi_{PP}}$$

$$\mu = \text{Re } \mathcal{G}_{T_{ij}T_{ij}}^R$$

$$\chi_{PP} = T_{00} + T_{xx}$$



# (ELECTRIC) TRANSPORT I





*fast*

SUMMARY



***HABEMUS  
PHONONS***

**HOLOGRAPHIC SOLIDS !!!**

**(VISCO) ELASTIC HOLOGRAPHY**



**We have an holographic model which implements :**

**1) Explicit breaking of translations (momentum relaxation)**

**2) Explicit + spontaneous breaking of translations (pseudophonons)**

**3) Spontaneous breaking of translations (phonons)**

$$1) V(X) = X, \quad 2) V(X) = X + X^5, \quad 3) V(X) = X^5$$



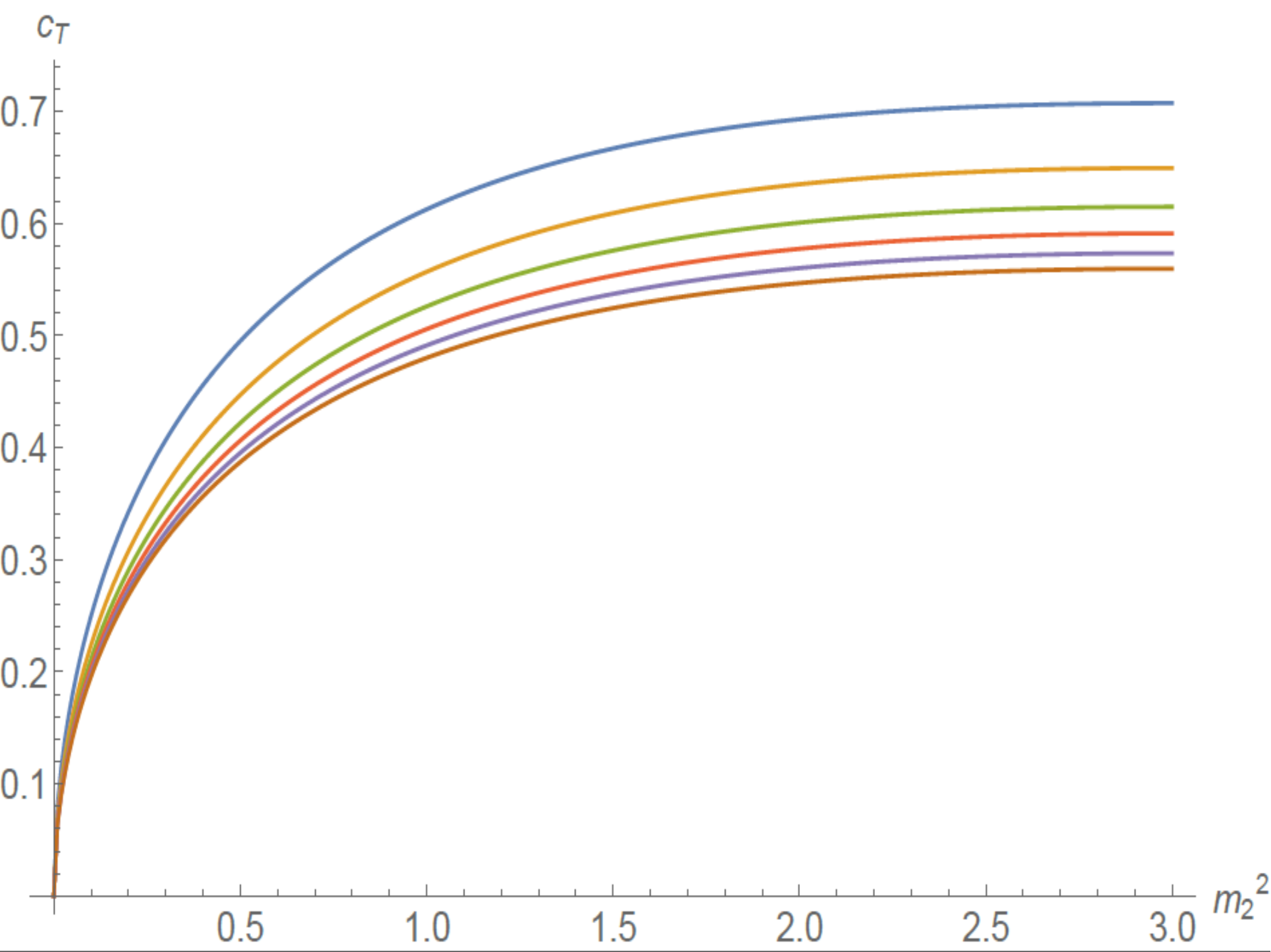


NORDITA

# Tack så Mycket





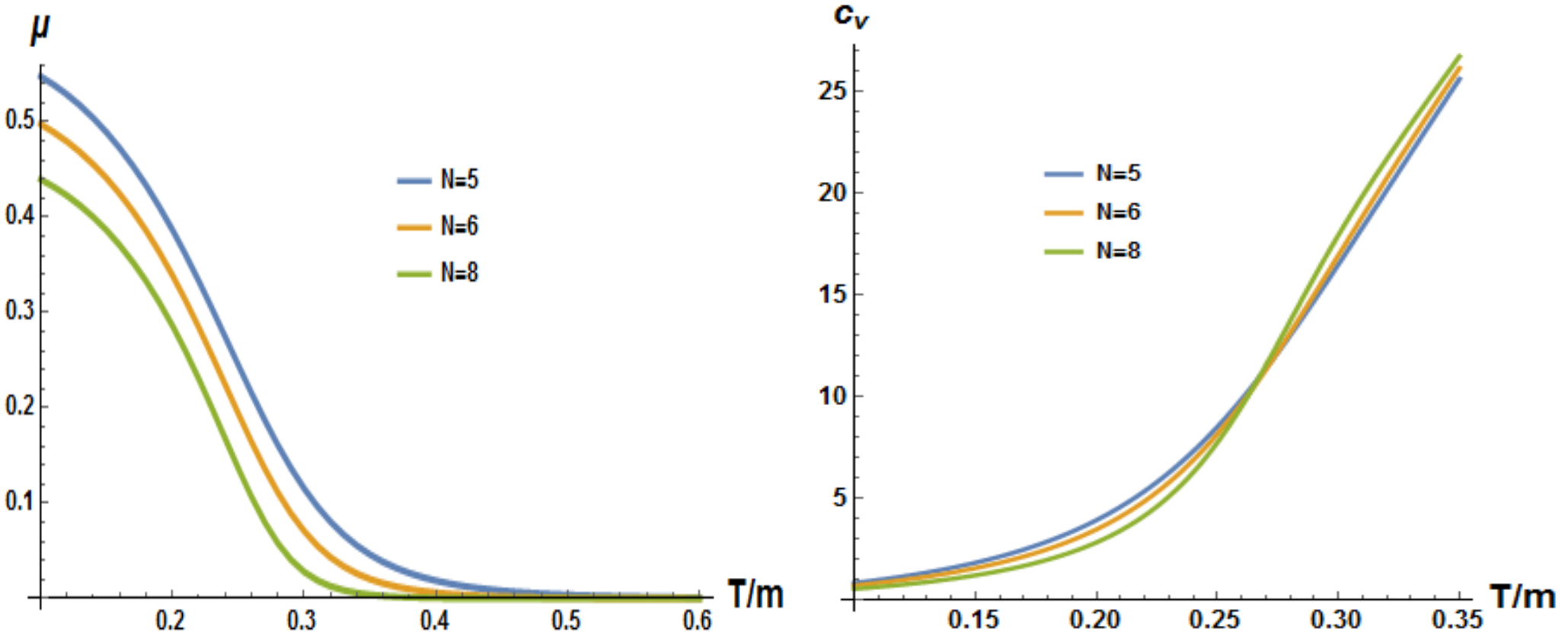


# Elastic Holography

**IN PROGRESS**

$$V(X) = m^2 X^N$$

Massive gravity models with large N

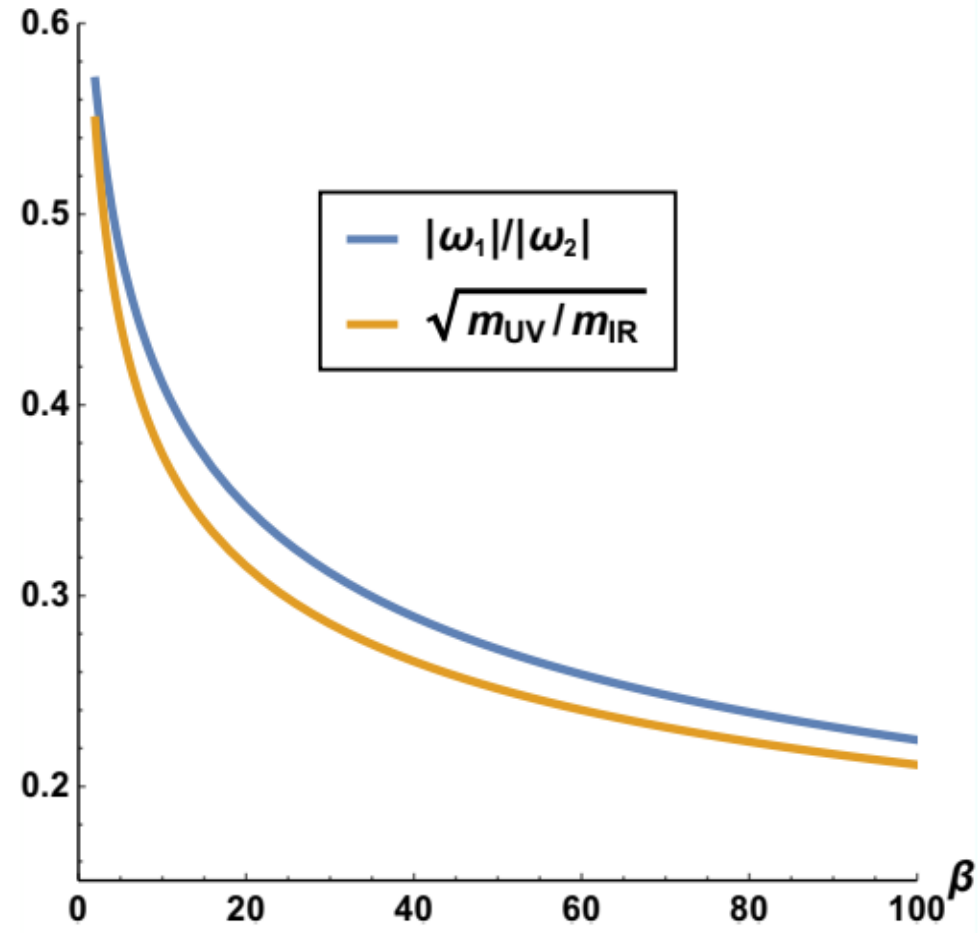
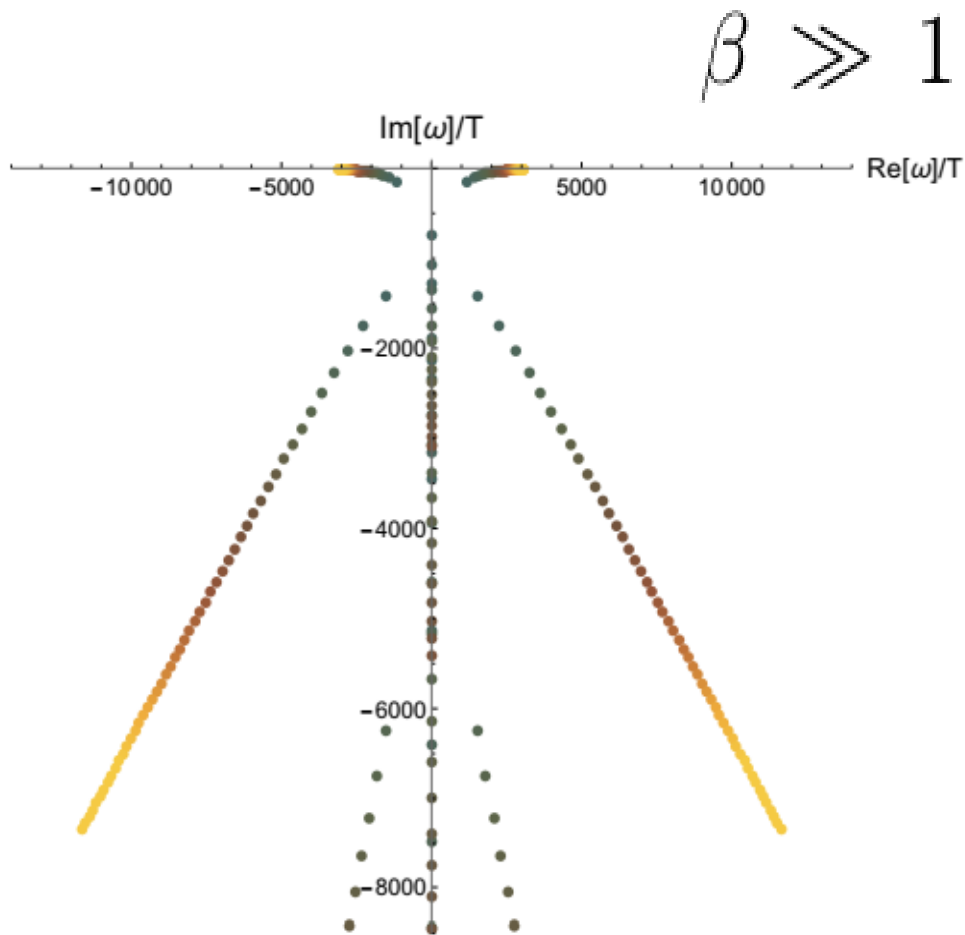


Shear elastic modulus goes to zero increasing temperature

There is no first order phase transition!

It is not melting ! Glass transition ?? Viscoelasticity ??

# Spontaneous breaking and EFT



Separation of scales  
 Breaking mostly “spontaneous”  
 Small damping

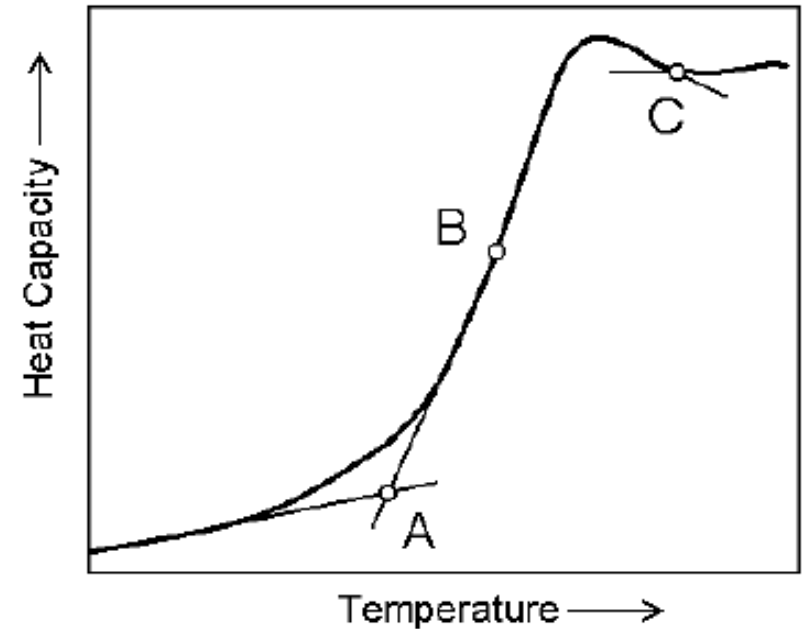
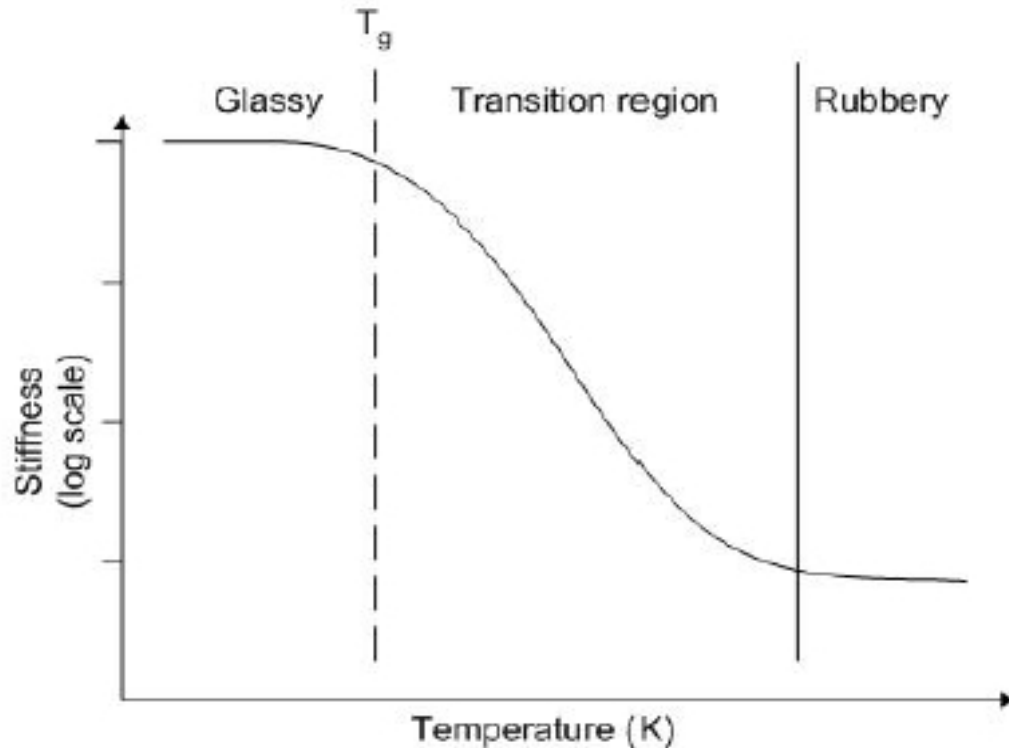


Gapped and damped  
 Transverse phonon  
 ?????

# Glassy transition ??

**IN PROGRESS**

If it is not melting what is it ???



Also the speed of transverse phonons goes to zero with  $T \dots$

Is that a GLASS TRANSITION ?

Are these VISCOELASTIC "black holes" ?

Please  
Leave a  
Comment

# Holographic viscoelasticity

arXiv:1510.09089 arXiv:1601.03384

**Lorentz violating  
Massive gravity**

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( R + \frac{6}{L^2} \right) - \frac{L^2}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{L^2} V(X, Z) \right]$$

$$X \equiv \frac{1}{2} \text{tr}[\mathcal{I}^{IJ}], \quad Z \equiv \det[\mathcal{I}^{IJ}], \quad \mathcal{I}^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J,$$

**Helicity 2 graviton mass  
(different from helicity 1 !!)**

$$M^2(r) \equiv \frac{1}{2r^2} \hat{V}_X(r).$$



**FLUIDS :**  $\phi^I \mapsto \psi^I(\phi^J), \quad \det \left[ \frac{\partial \psi^I}{\partial \phi^J} \right] = 1.$

$$V_{fluids} = V(Z) \longrightarrow M^2(r) = 0.$$

$$\boxed{\mu = 0,}$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$



*It's a Match!*

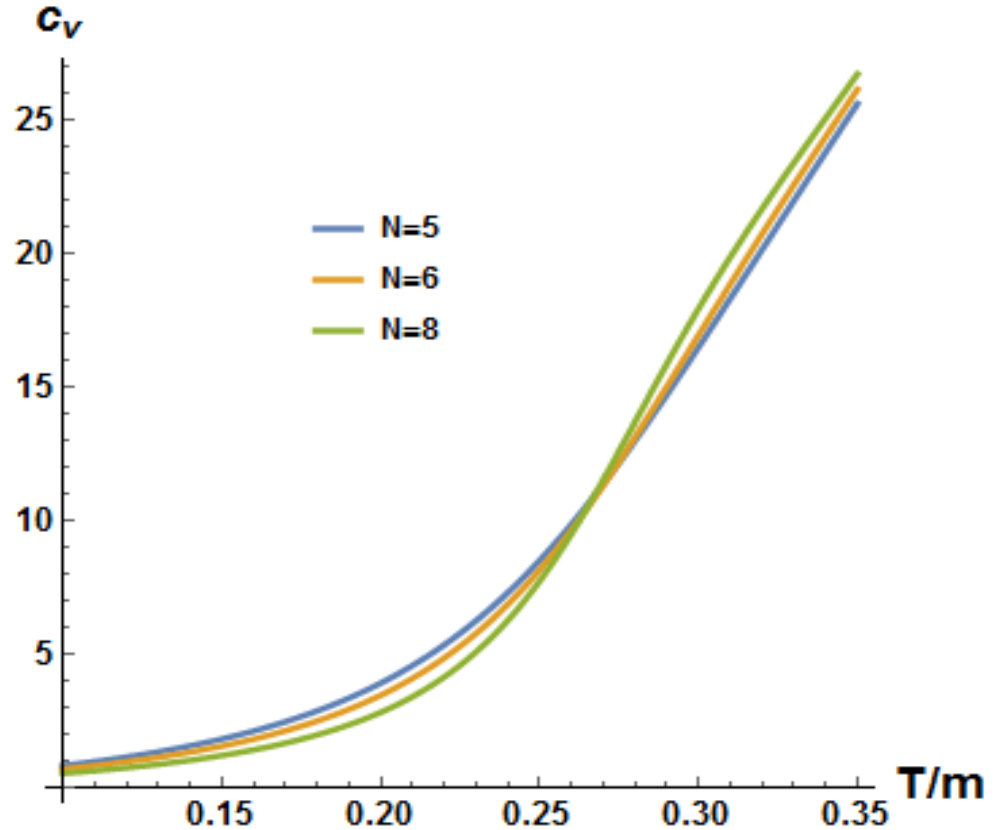
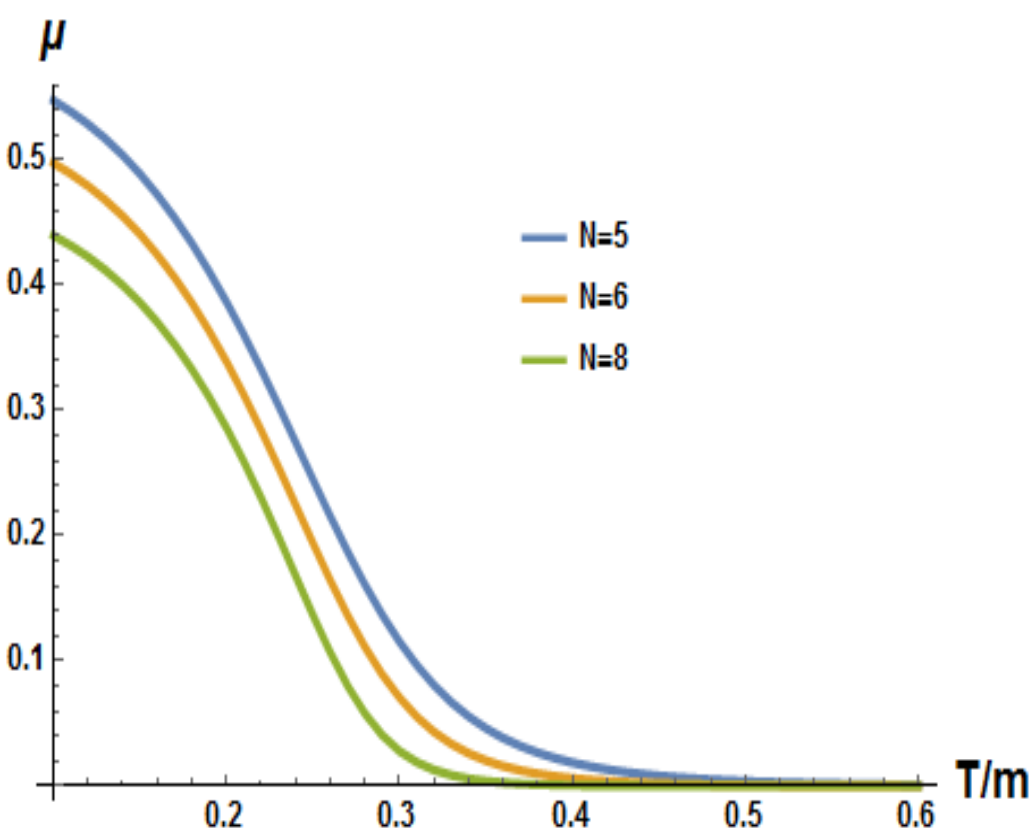


# Elastic Holography : part II

**IN PROGRESS**

$$V(X) = m^2 X^N$$

Massive gravity models with large N



Shear elastic modulus goes to zero increasing temperature

There is no first order phase transition!

It is not melting ! Glass transition ?? Viscoelasticity ??

# Viscoelasticity

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**ELASTIC + VISCOUS**

$$T_{ij}^{(T)} = \mu u_{ij}^{(T)} + \eta \dot{u}_{ij}^{(T)}$$

Two arrows point from the text "ELASTIC + VISCOUS" to the terms  $\mu u_{ij}^{(T)}$  and  $\eta \dot{u}_{ij}^{(T)}$  in the equation above.

$$\eta \propto \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} \mathcal{G}_{T_{ij}T_{ij}}^R$$

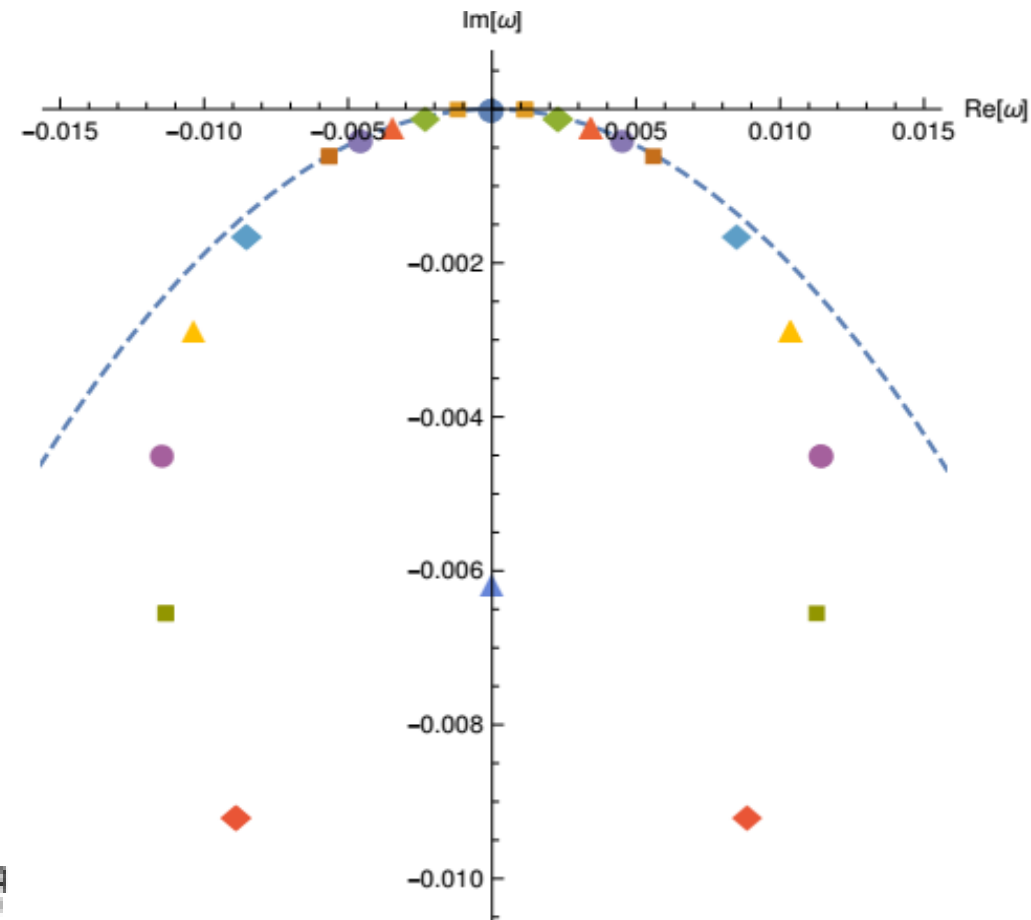
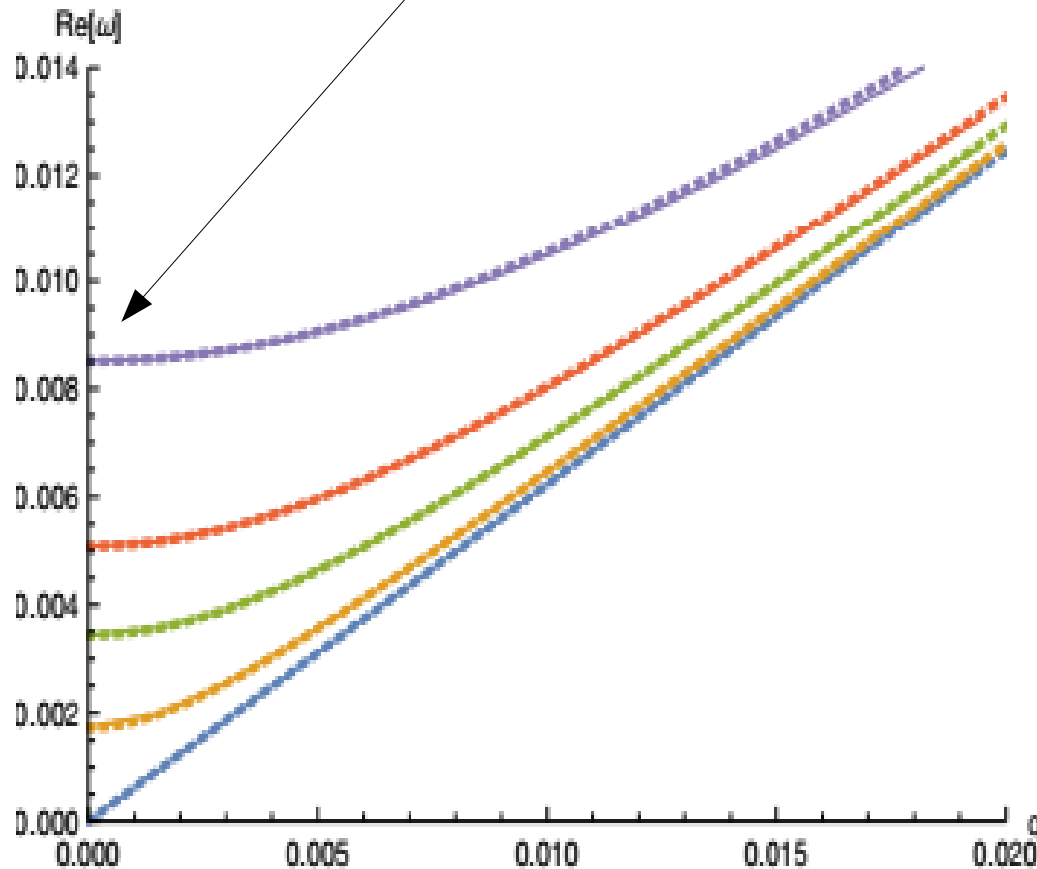
**VISCOSITY**

$$\mu \propto \lim_{\omega \rightarrow 0} \text{Re} \mathcal{G}_{T_{ij}T_{ij}}^R$$

**SHEAR MODULUS**

# Gapping and Damping the Phonon

$$\omega^2 = \underline{-i\omega\Gamma} + \underline{\omega_0^2} + c_s^2 q^2 + \dots$$



# (Real World) Pinned Charge Density Wave

## The dynamics of charge-density waves

G. Grüner

Rev. Mod. Phys. **60**, 1129 – Published 1 October 1988

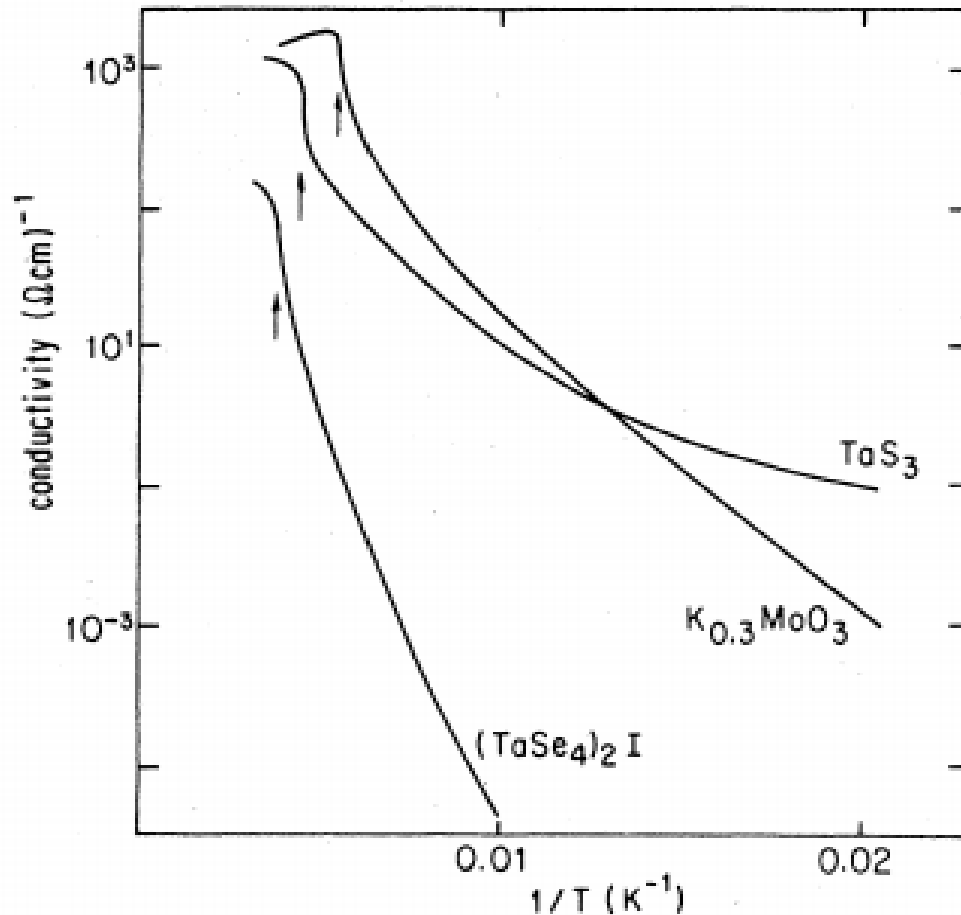


FIG. 4. Temperature dependence of the electrical conductivity in *o*-TaS<sub>3</sub>, in (TaSe<sub>4</sub>)<sub>2</sub>I, and in K<sub>0.3</sub>MoO<sub>3</sub>. The arrows represent the Peierls transition temperatures; they are evident by examining the temperature derivatives  $dR/dT$ .

$$\sigma(\omega) = \frac{n_c e^2}{i\omega m^*} \frac{\omega^2}{\omega_0^2 - \omega^2 - i\omega/\tau}$$

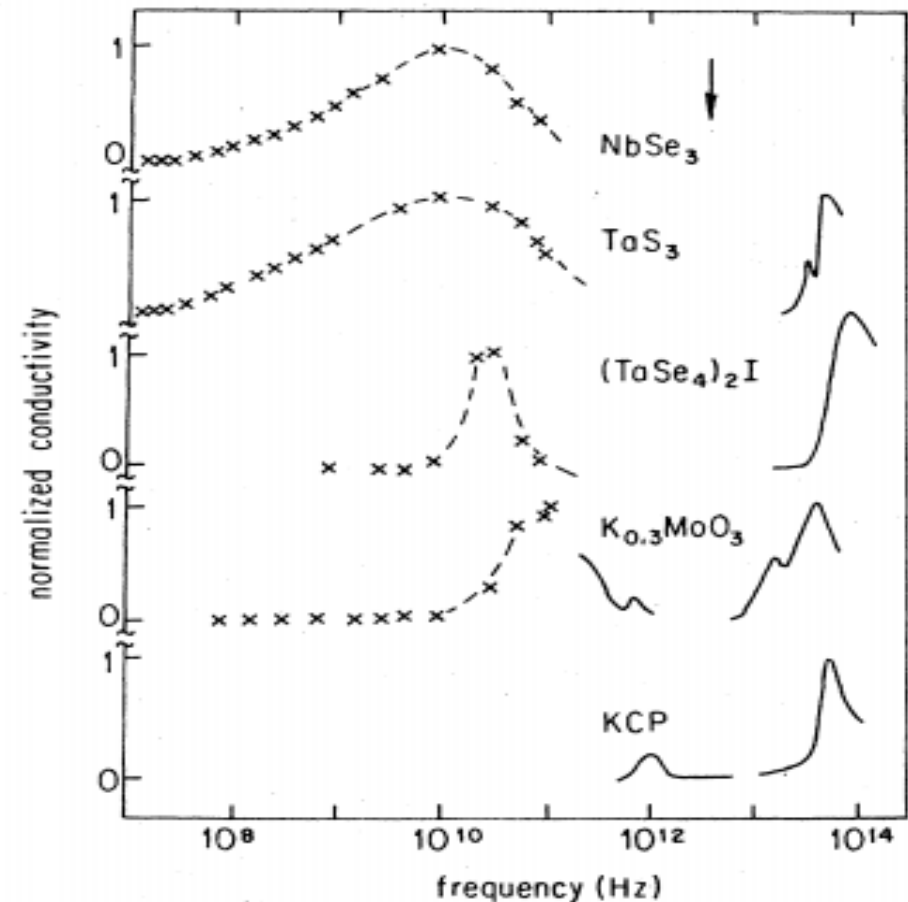
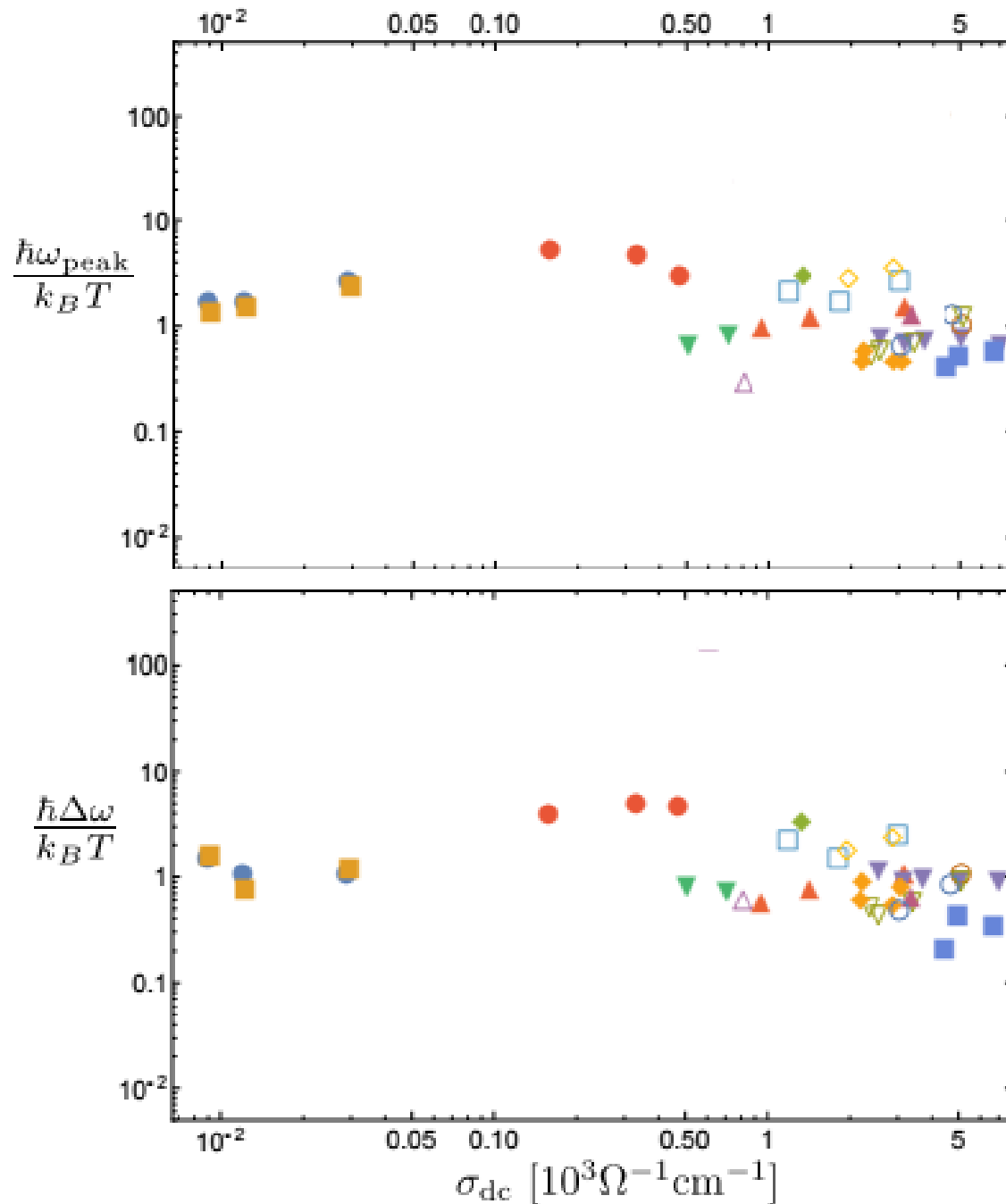


FIG. 9. The frequency-dependent conductivity of NbSe<sub>3</sub>, *o*-TaS<sub>3</sub> and (TaSe<sub>4</sub>)<sub>2</sub>I, and K<sub>0.3</sub>MoO<sub>3</sub>. The solid lines represent the regions where the drop signals the single-particle gaps; the strong peaks in the millimeter wave spectral range are due to the response of the pinned collective mode.

# The Bad Metal Proposal

arXiv:1612.04381



$$\sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}.$$

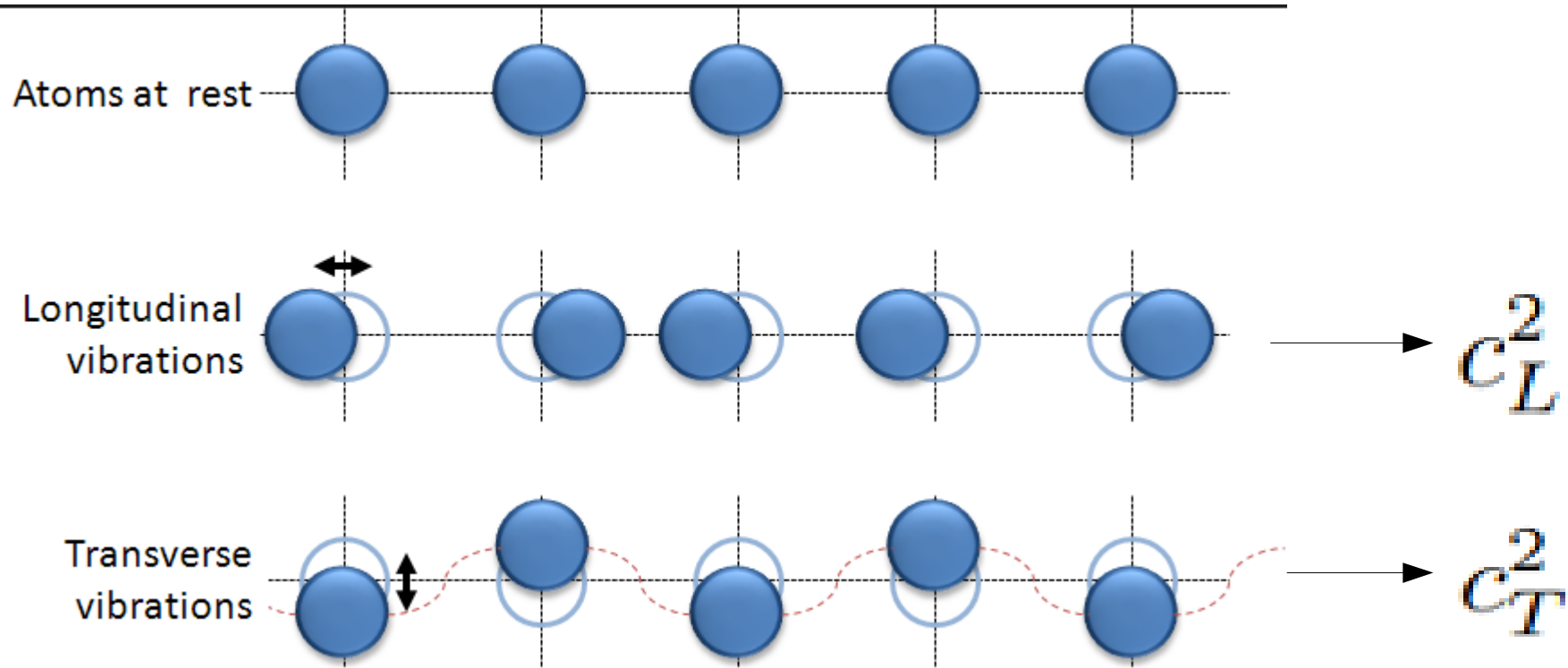
$$\hbar \omega_{\text{peak}} = \alpha(T) k_B T,$$

$$\hbar \Delta \omega = \beta(T) k_B T$$

**Spontaneous + explicit  
Could explain transport  
In bad metals**

**The pinning frequency  
Is controlled by the same  
"universal" scale  
Of the relaxation time**

# Solids VS Fluids



**In fluids**

$$\rightarrow \mu = 0 \rightarrow c_T = 0.$$

**NO SHEAR SOUND!**

