There and Back

RG Boomerangs and Spatially Modulated Deformations

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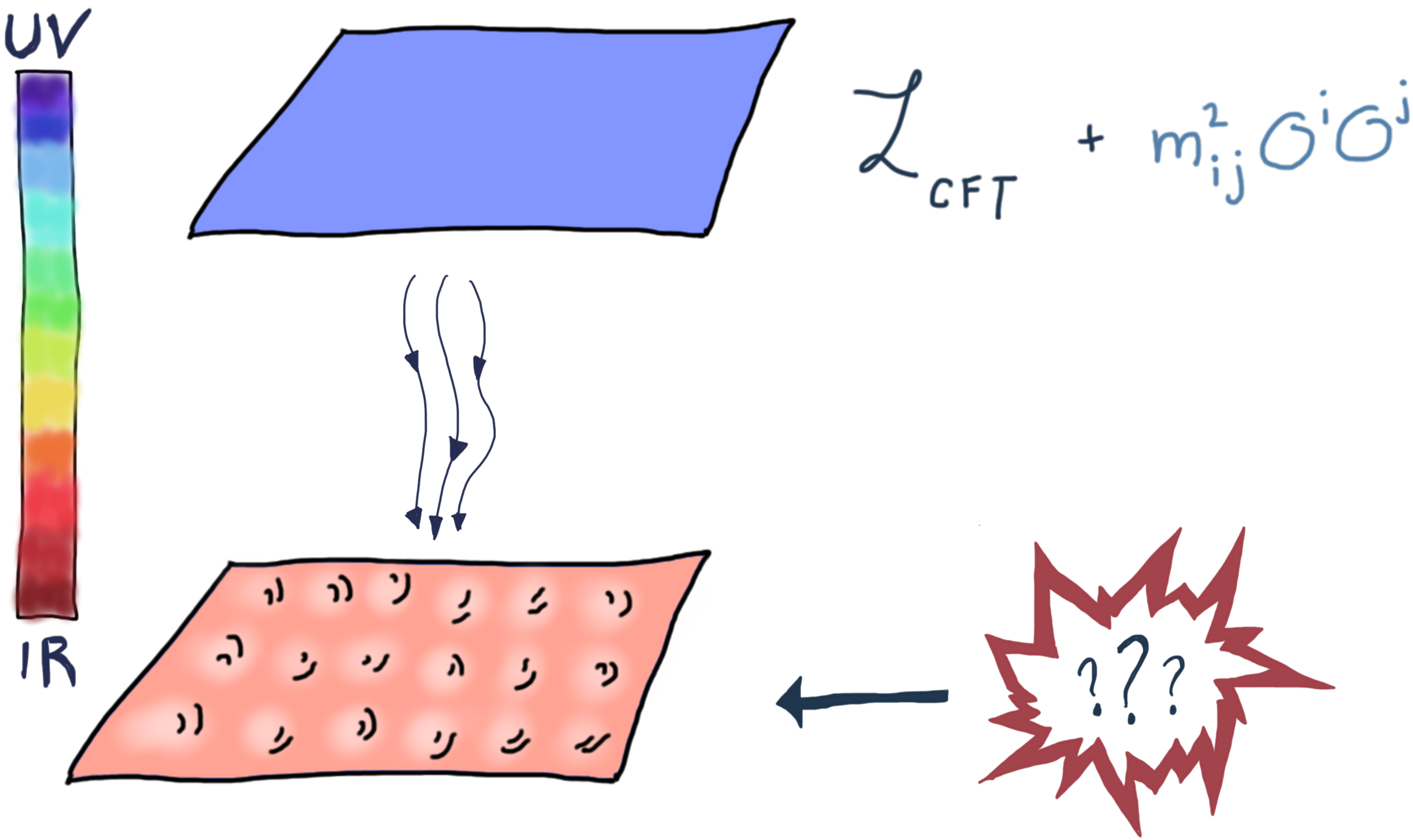
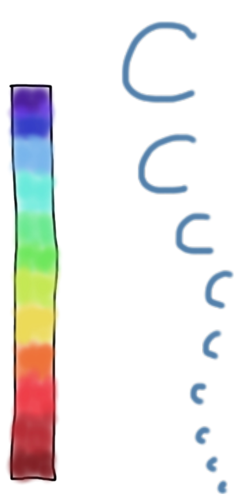

UV
IR
$\mathcal{L}_{CFT} + m^2_{ij} \mathcal{O}^i \mathcal{O}^j$
???

Why?

Chart the landscape of interacting fixed point theories

"Because it's there" -- Mallory

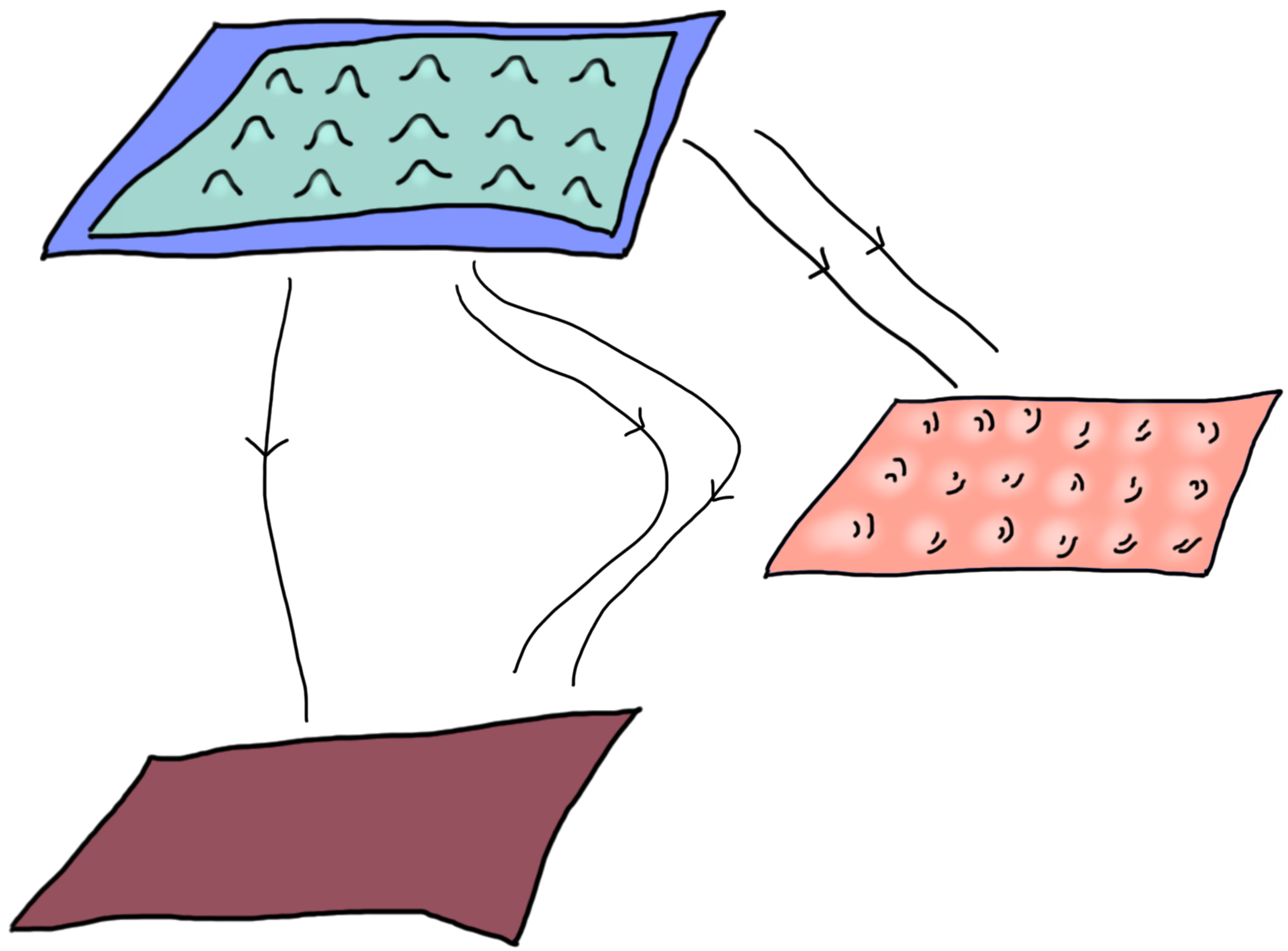
(but also to see what's possible, strengthen intuition, etc.)



Understand, criticize, develop RG results/lore

What is the validity/reach of a/F/c theorems, entanglement measures, etc.?

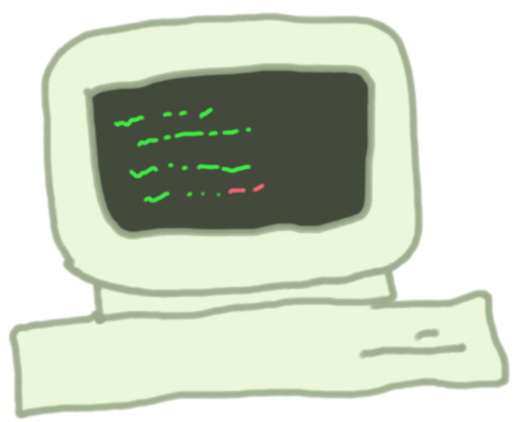
$$\mathcal{L}_{ABJM} + m(x)^2_{ij}\, \mathcal{O}^i \mathcal{O}^j$$



ABJM Theory deformed by a “lattice” as a QFT toy

“Conventional” Methods

- Perturbative Methods



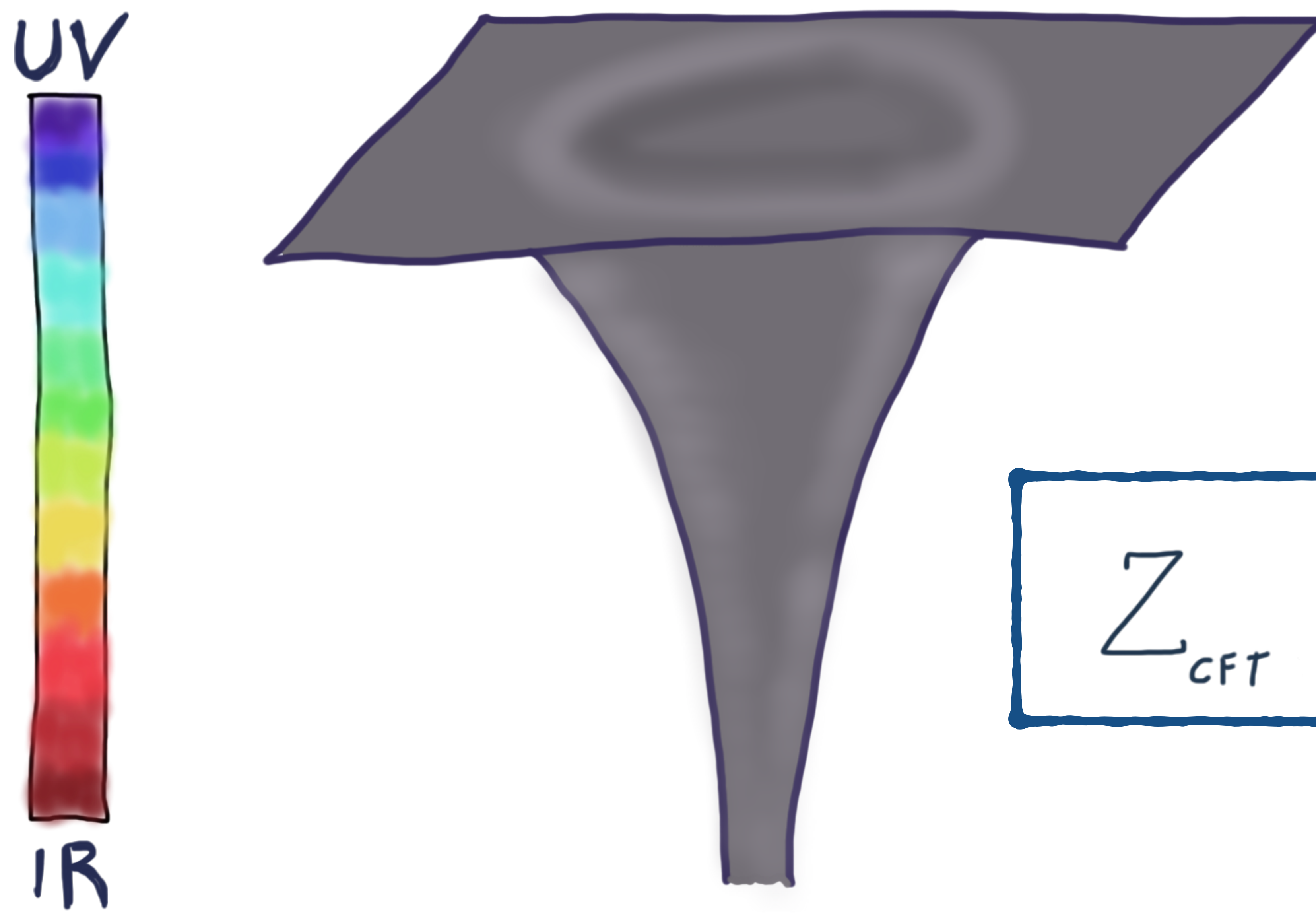
- Lattice Techniques

$$\{Q,\bar{Q}\} = P$$

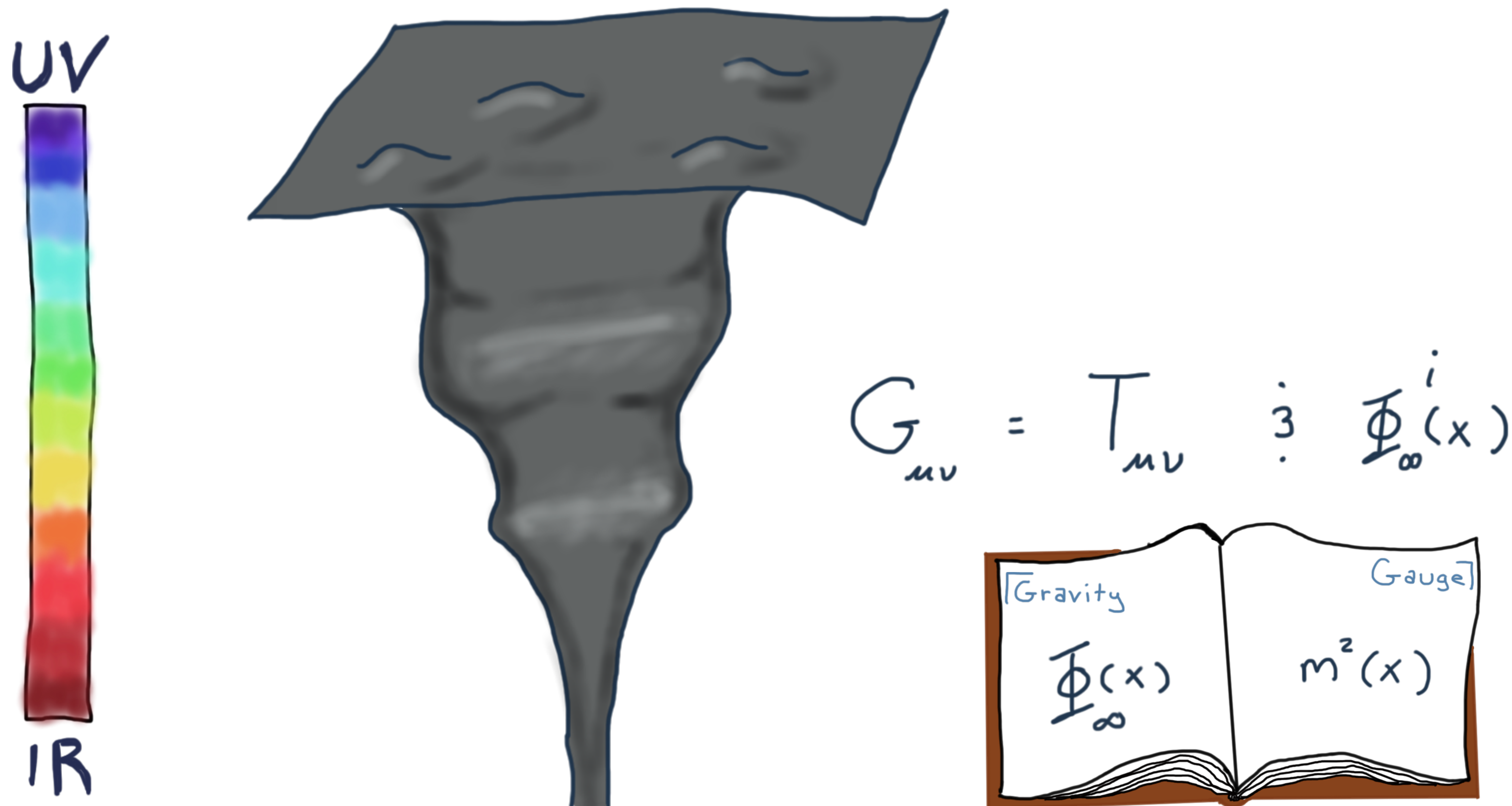
- SUSY Analysis

(... the list goes on...)

Use better variables…



$$\boxed{Z_{CFT} \simeq e^{-S_{SUGRA}}}$$



RG Geometrized

Ex.

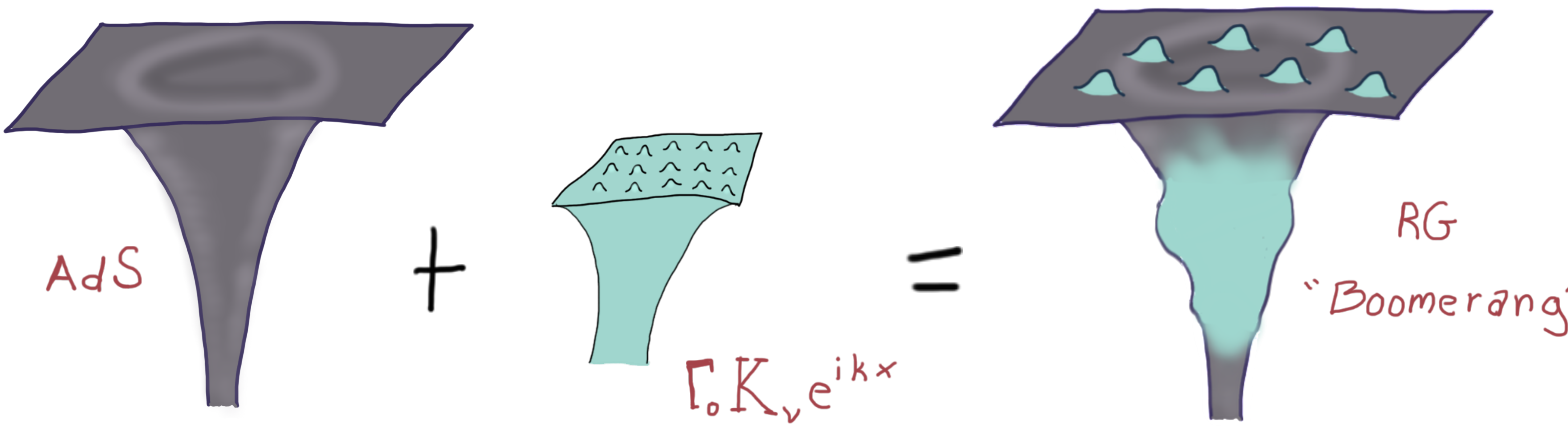
$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\left(R + 6 + \mathcal{L}_2\right) + \dots$$

$$\mathcal{L}_2 = -G_{\alpha\bar{\beta}}\,\partial_\mu z^\alpha \partial^\mu \bar{z}^{\bar{\beta}} - V(|z|)$$

$\alpha = 1, 2$

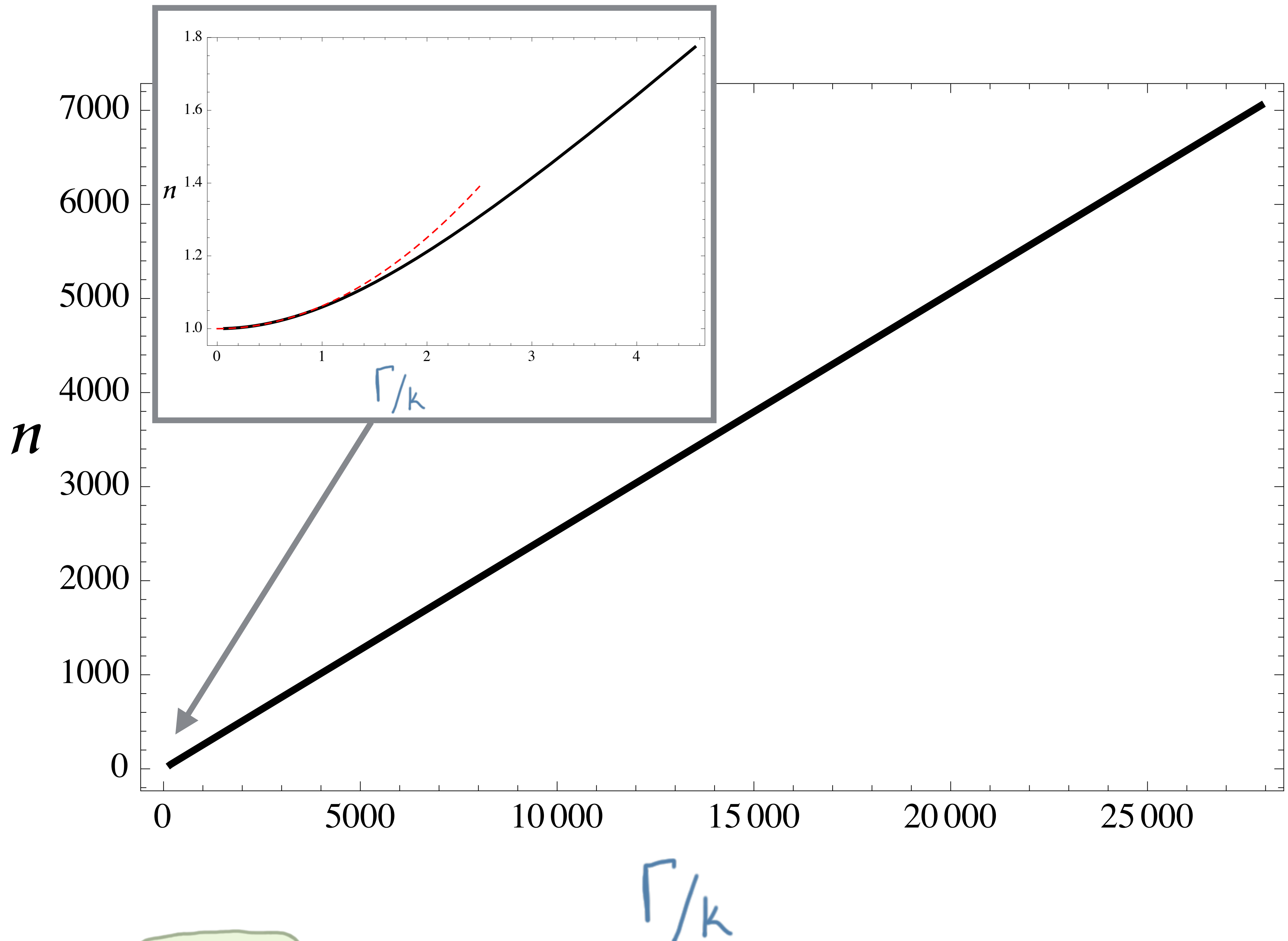
$\mathcal{G} = U(1)^2$

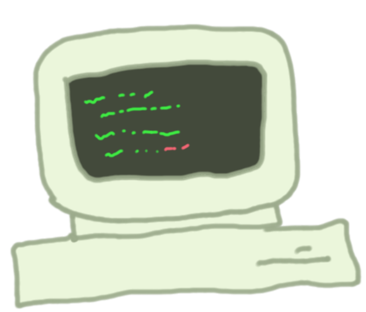
$$z^\alpha = \gamma(r)\,e^{ikx^\alpha} \qquad ds^2 = -U e^{-\chi} dt^2 + r^2 dx^\alpha dx^\alpha + \frac{dr^2}{U}$$



$$\eta \equiv \frac{c_{UV}}{c_{IR}} = \frac{L_{IR}}{L_{UV}}\, e^{\frac{1}{2}(\chi_{IR}-\chi_{UV})} = e^{\frac{1}{2}(\chi_{IR}-\chi_{UV})} \sim 1 + \#\left(\frac{\Gamma_0}{k}\right)^2 + \dots$$

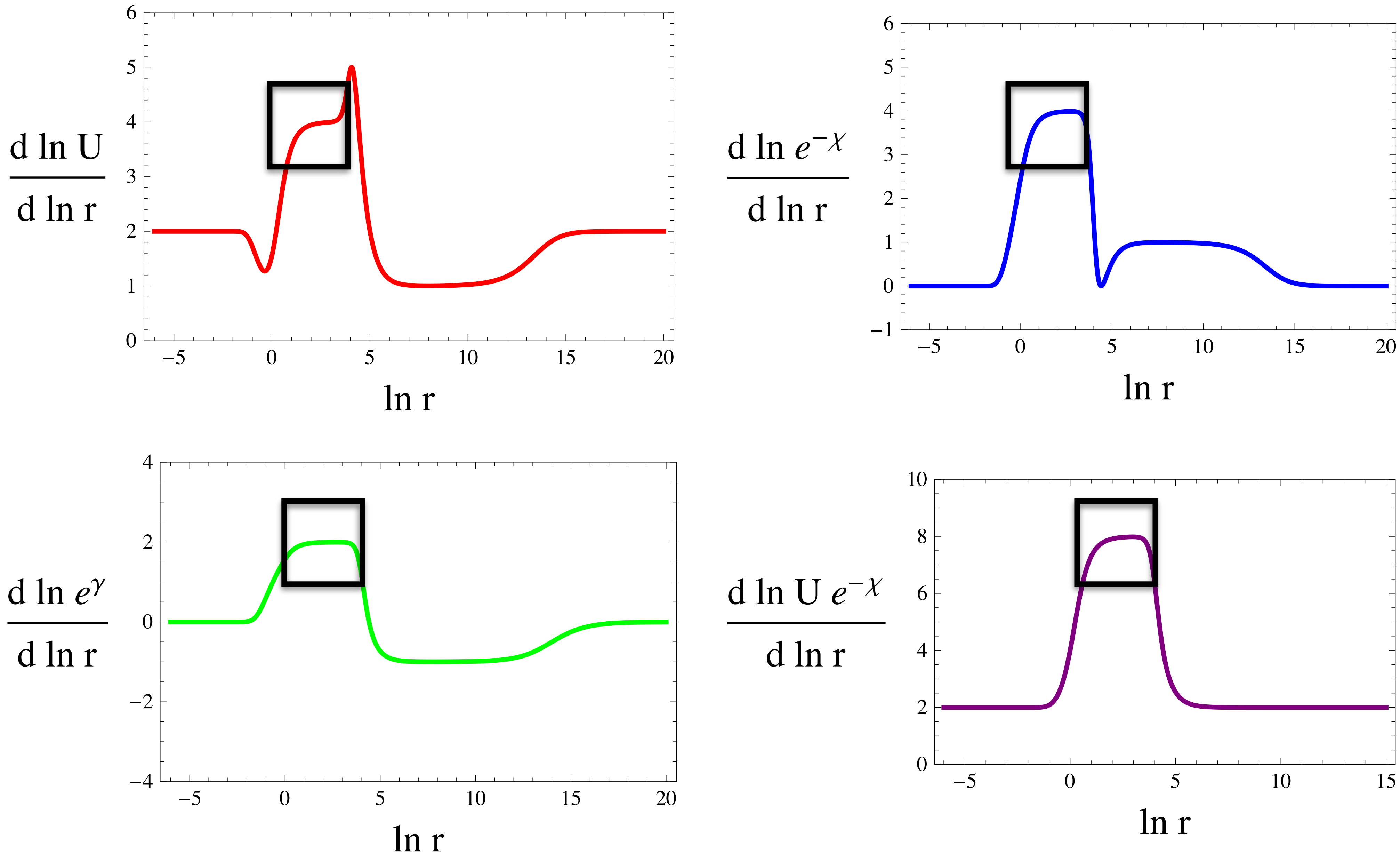
Non-Perturbative Solutions





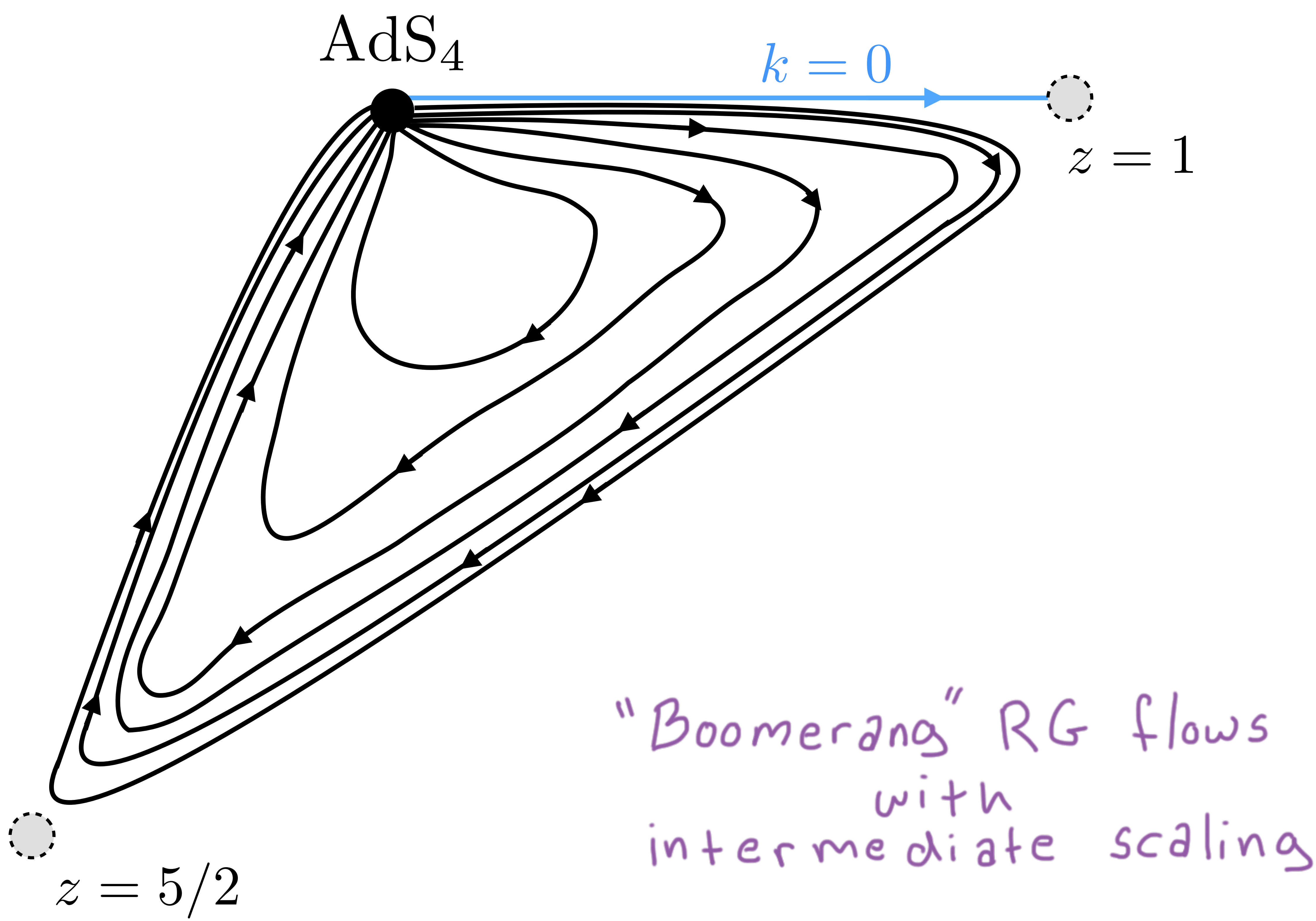
Boomerangs persist to BIG deformations

Non-Perturbative Solutions



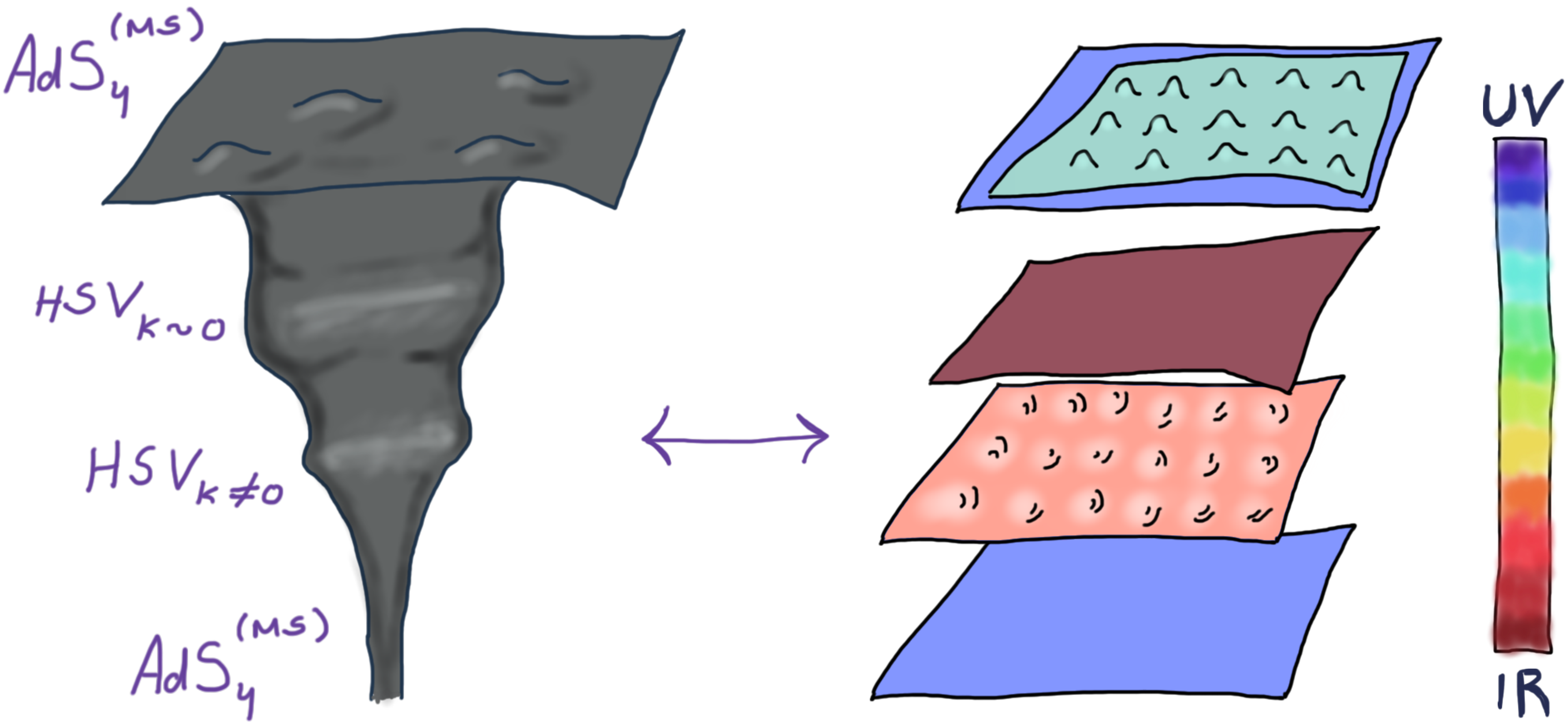
$$ds^2 = \frac{1}{\rho}\left(-\frac{1}{\rho^3}dt^2 + d\vec{x}^2 + d\rho^2\right)$$

HSV: $\theta = 1$, $z = 5/2$



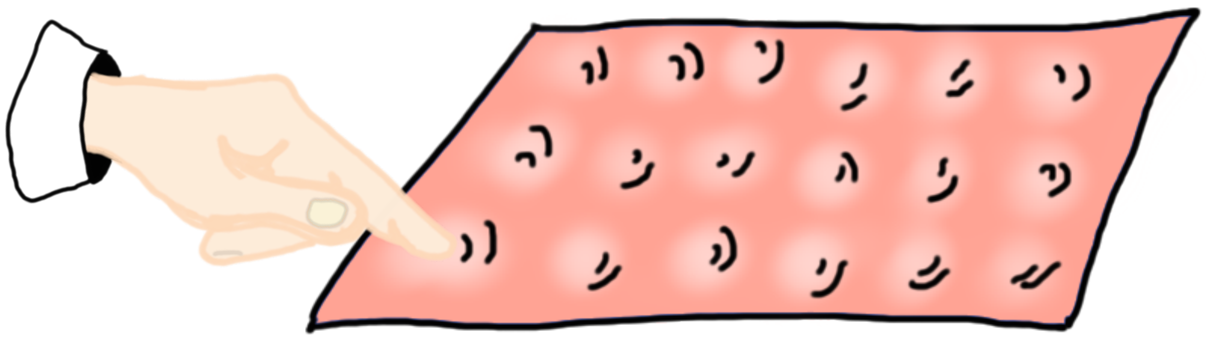
AdS_4
$k = 0$
$z = 1$
$z = 5/2$
"Boomerang" RG flows
with
intermediate scaling

The Big Picture

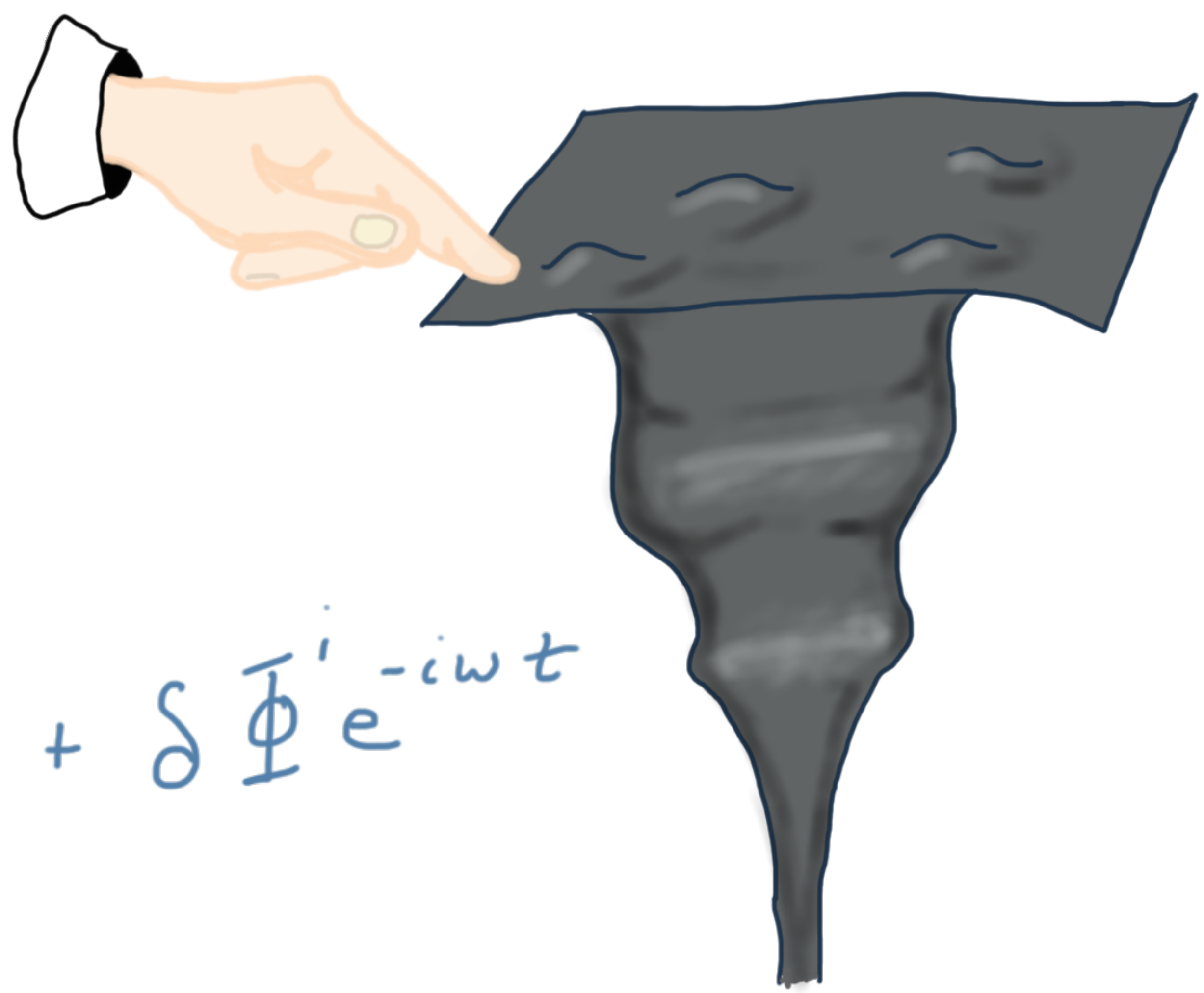


How is intermediate energy scaling realized in ABJM?

Scaling Signatures: Linear Response

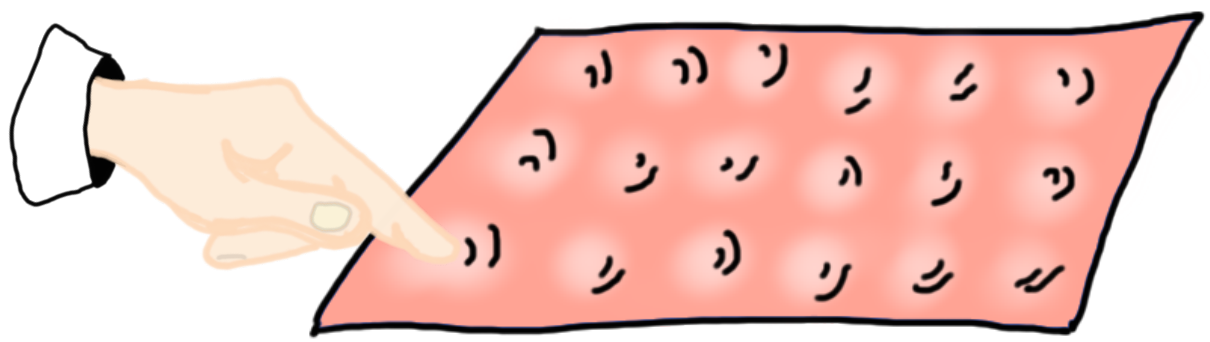


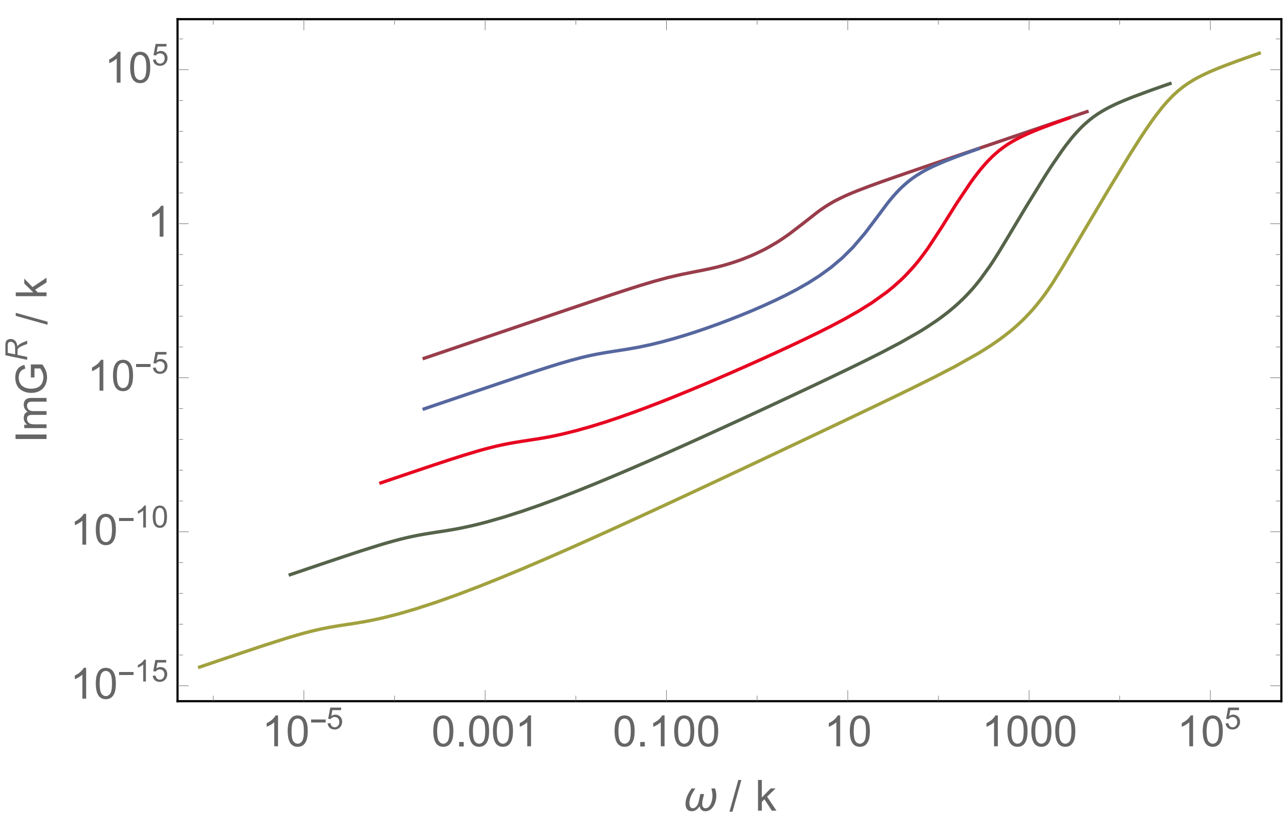
$$\hat{P}(\omega) \propto \mathrm{Im}\, G_R^{OO}(\omega)$$

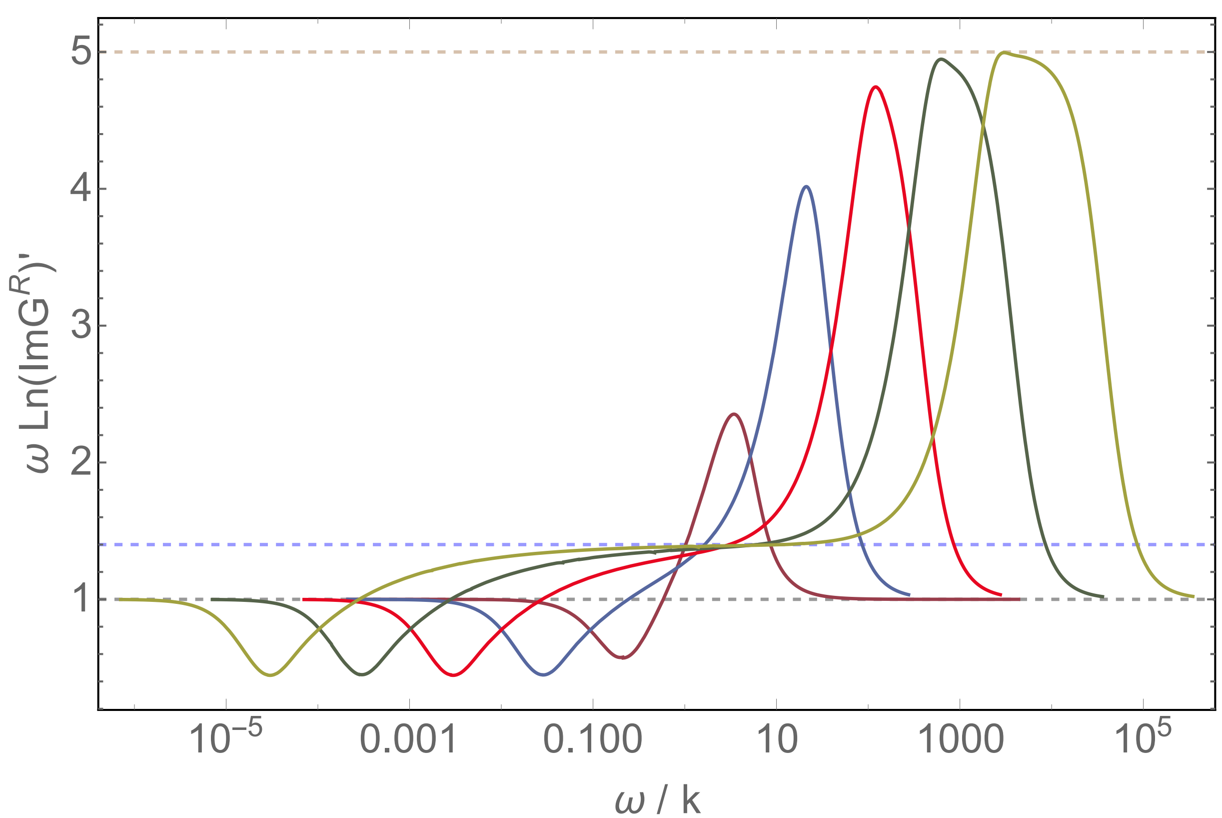


$$\Phi^i = \Phi^i_0 + \delta \Phi^i e^{-i\omega t}$$

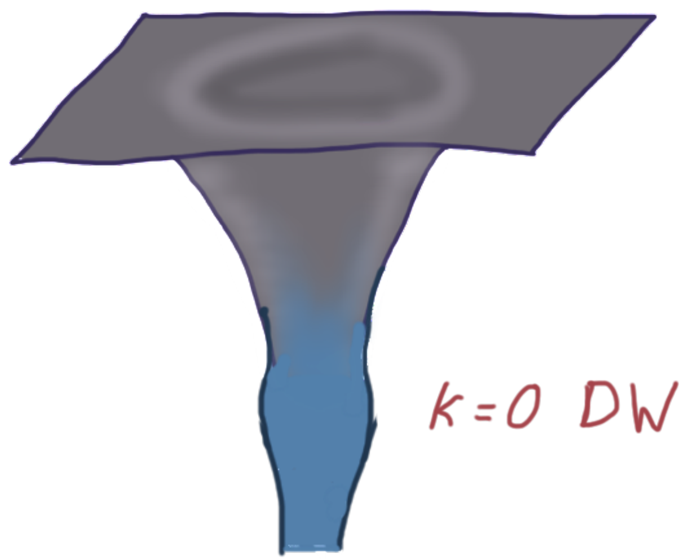
Scaling Signatures: Linear Response

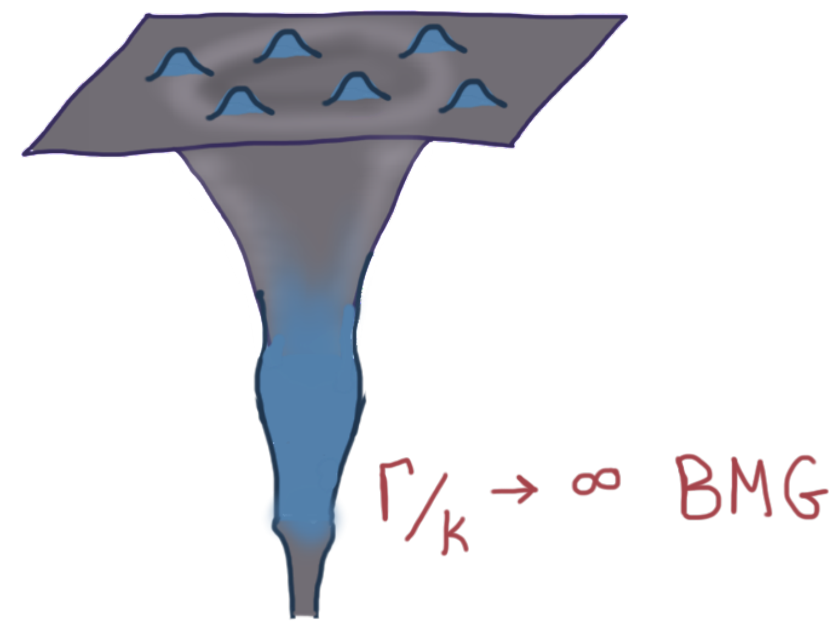


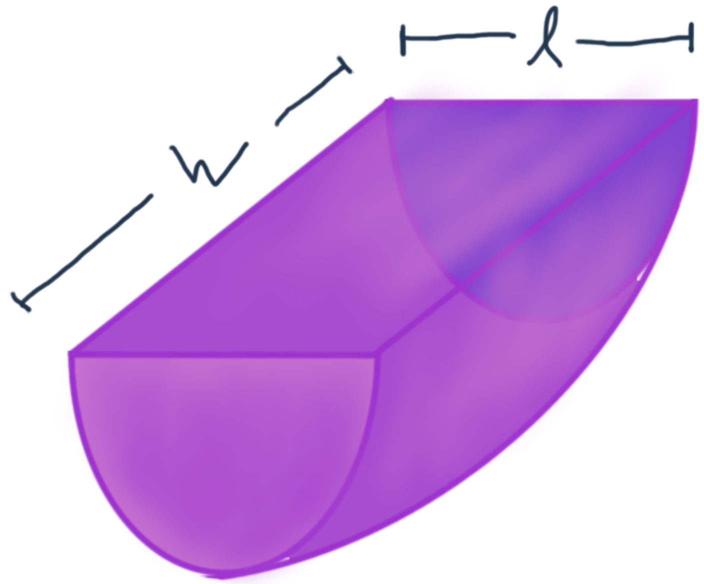




Scaling Signatures: Entropic c-Functions





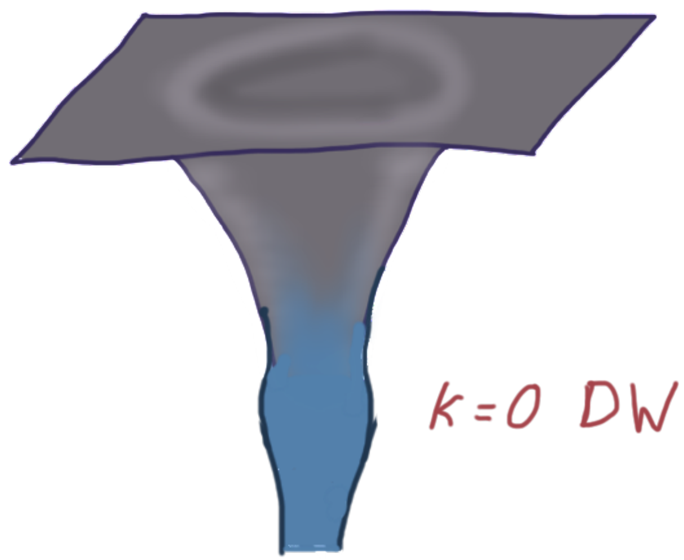


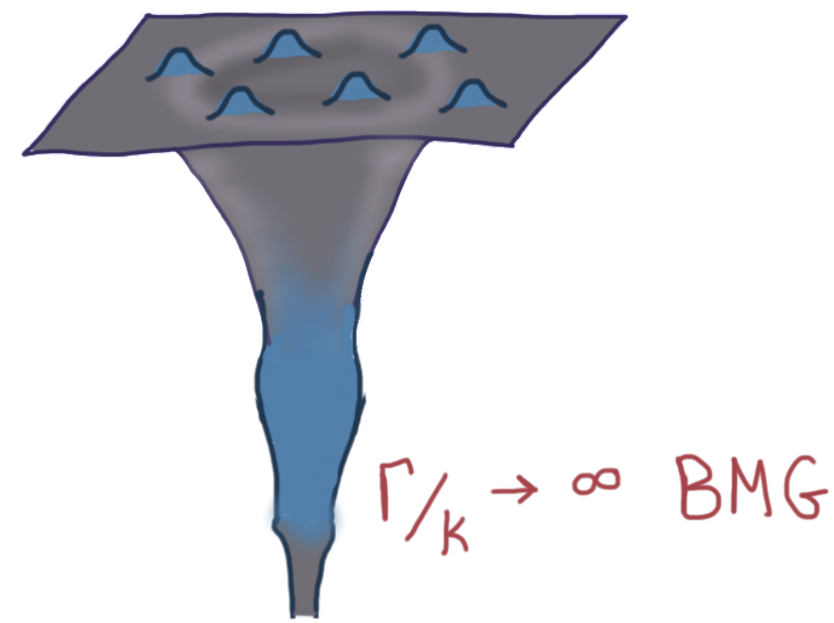
$$S_{\mathcal{A}} = 4\pi \mathcal{A}$$

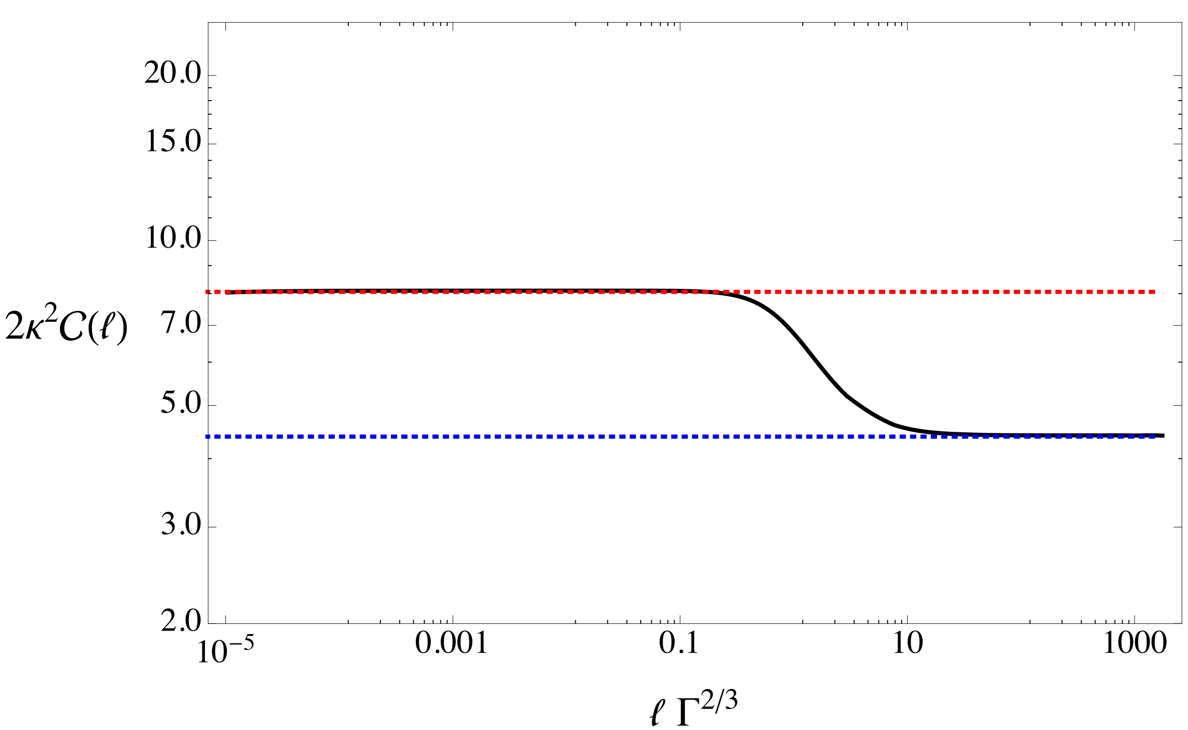
$$C(\ell) = \frac{\ell^3}{w^2} \frac{dS_{\mathcal{A}}}{d\ell}$$

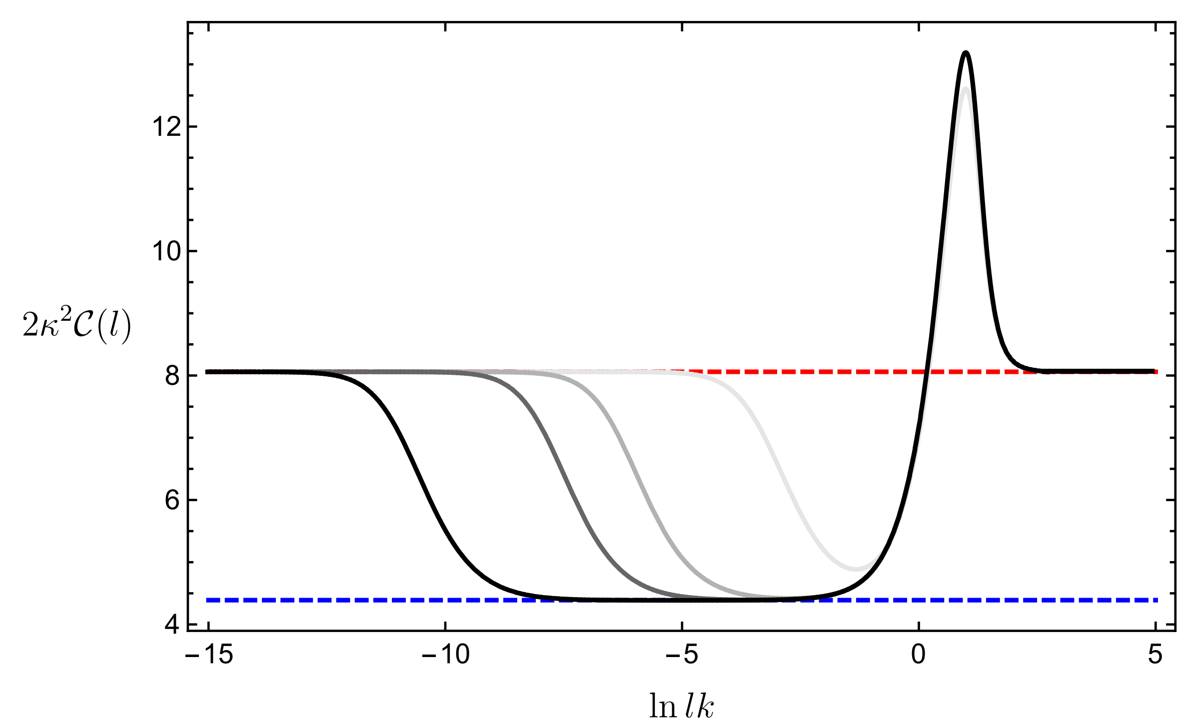
RG Monotone?

Scaling Signatures: Entropic c-Functions

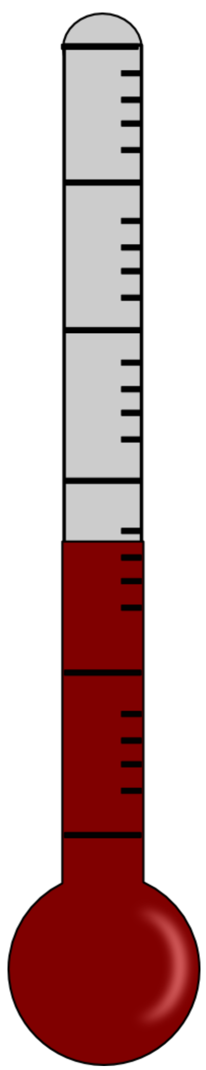


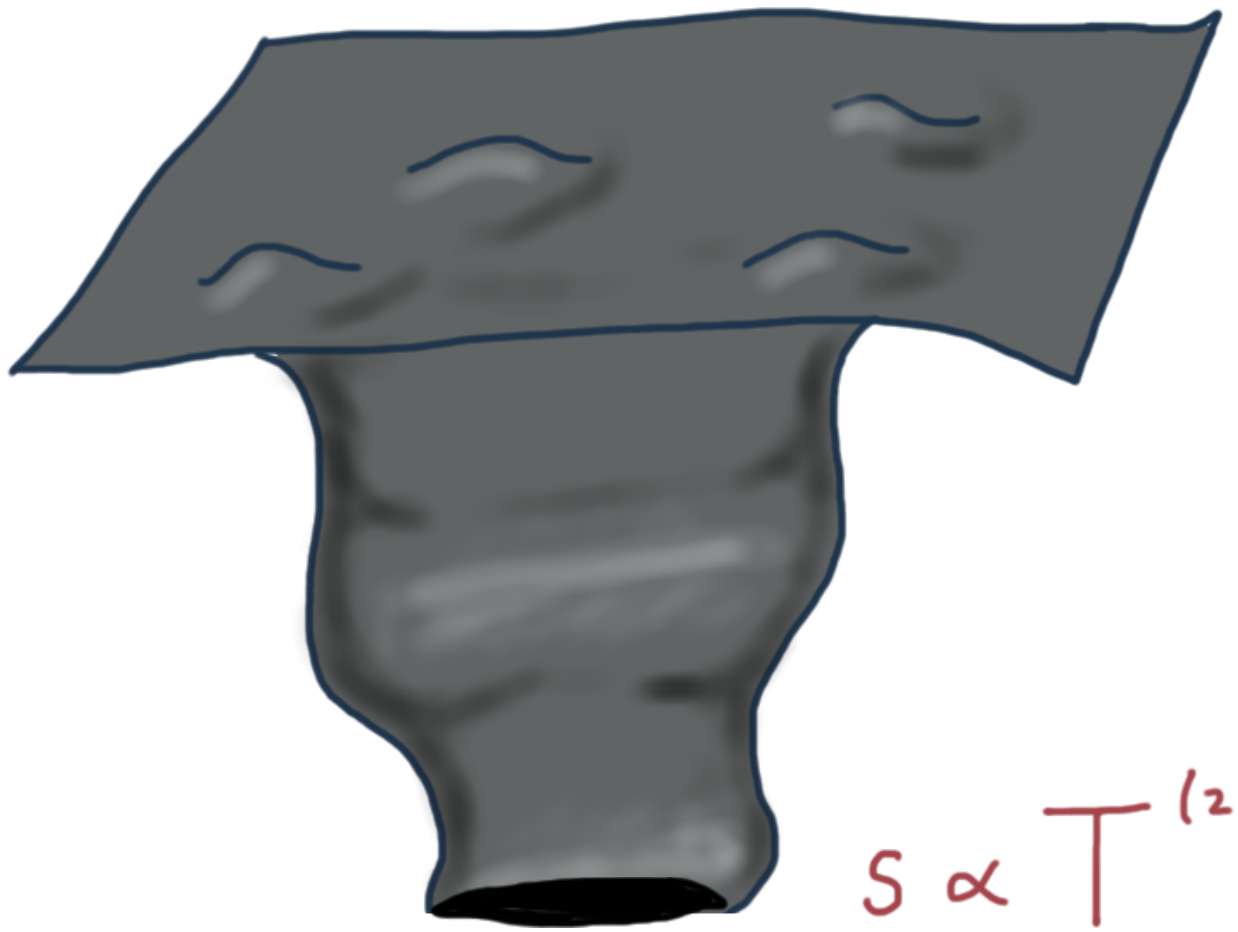






Scaling Signatures: Thermodynamics

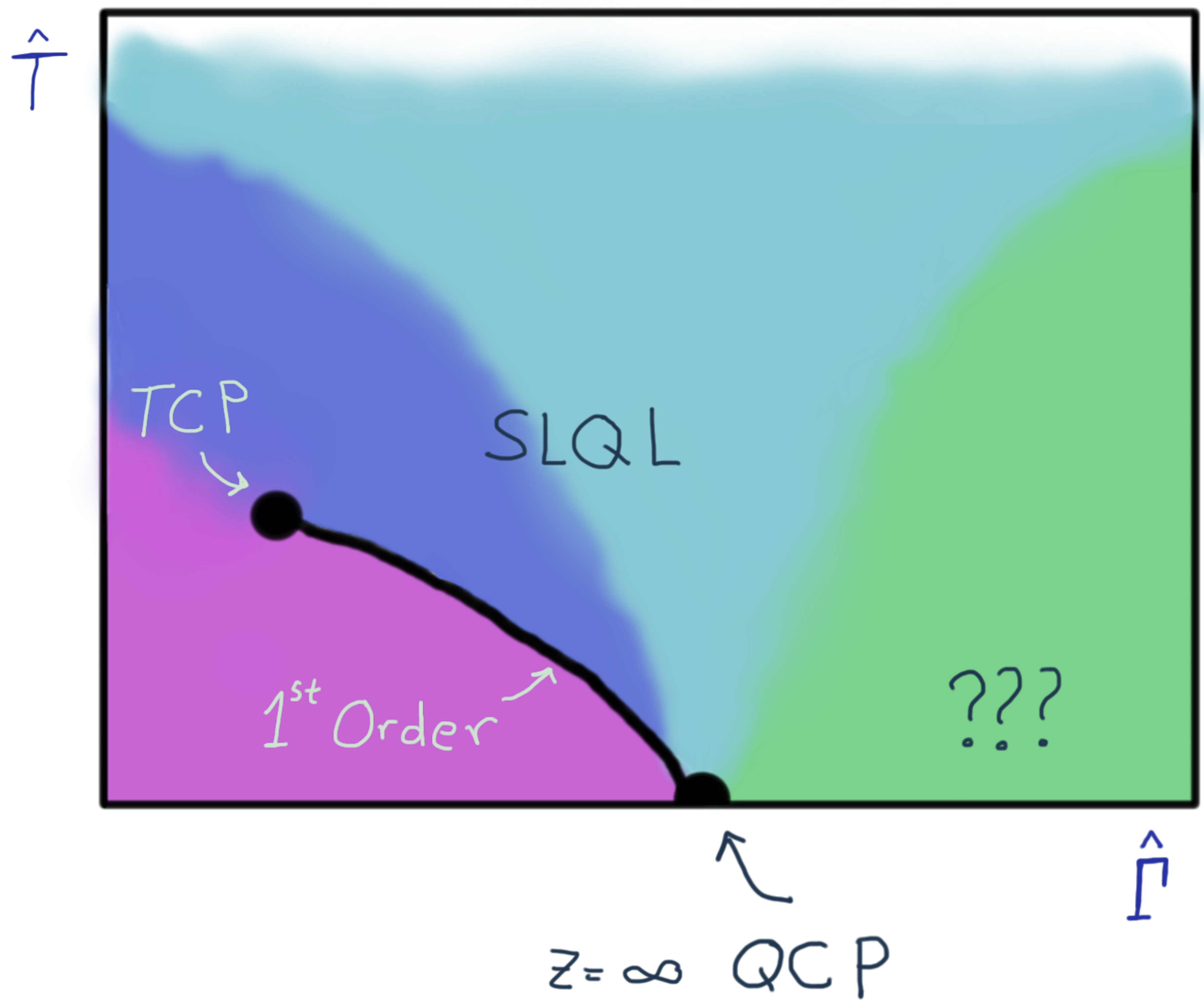




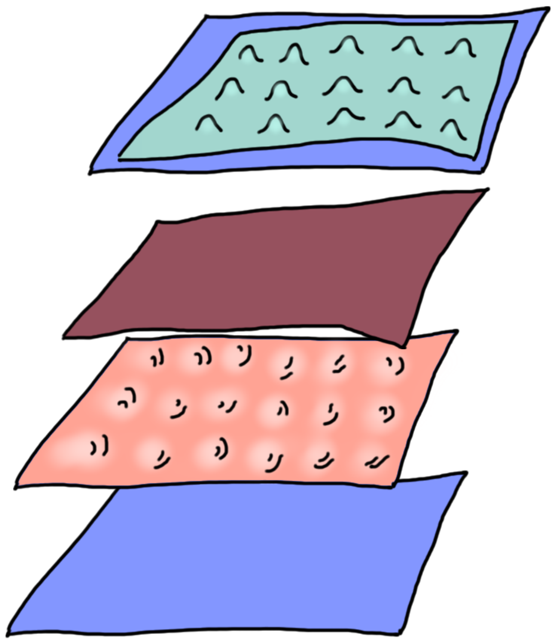
$S \propto T^{(2-\theta)/z}$ “✓”

Can we exploit these lessons to engineer interesting holographic phase diagrams?

Scaling Signatures: Quantum Critical Behavior



Some Open Questions and Outlook



Related solutions with more/less symmetry

Options abound...

- Sol$^{\underline{n}}$s in different dimensions/theories
- Sol$^{\underline{n}}$s dual to systems @ finite-μ
- Sol$^{\underline{n}}$s away from the Q-lattice
- SUGRA higher curvature corrections
- More, more, more

Flows are (pertubatively) stable in IR & UV

The HSV solutions likely are not -- what about full flow?

Other probes may yield further surprises

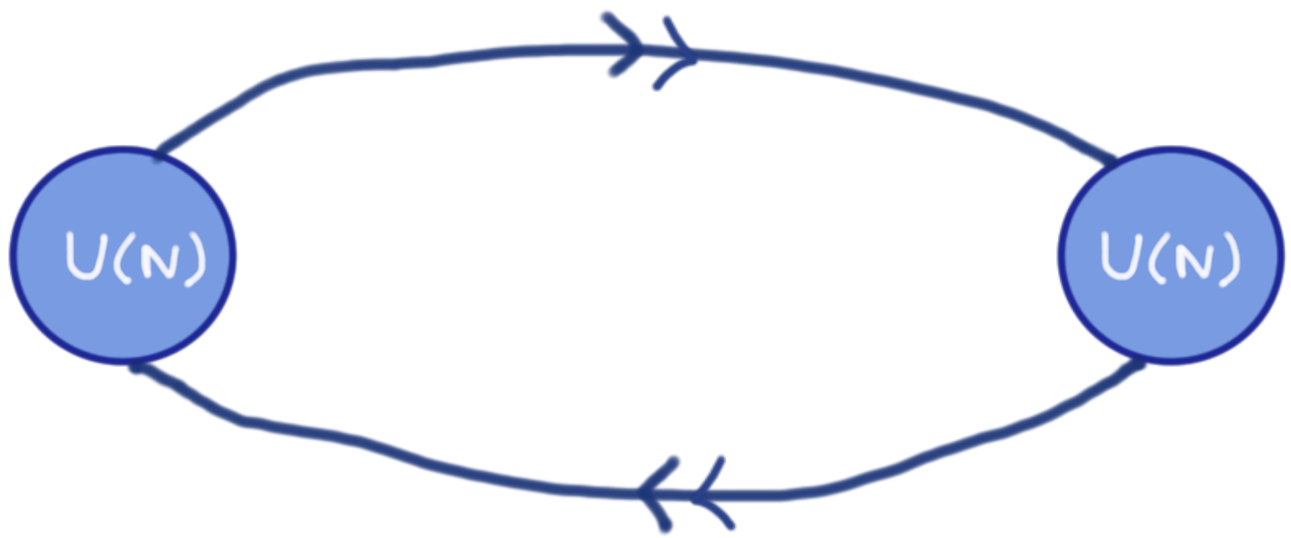
Top down fermionic spectral functions a natural candidate

Thank You

* DETAILS AND ASSORTED EXTRAS BEYOND THIS POINT

ABJM as a toy model of d=2+1 strongly interacting matter

$\mathcal{N}=6$ SUSY

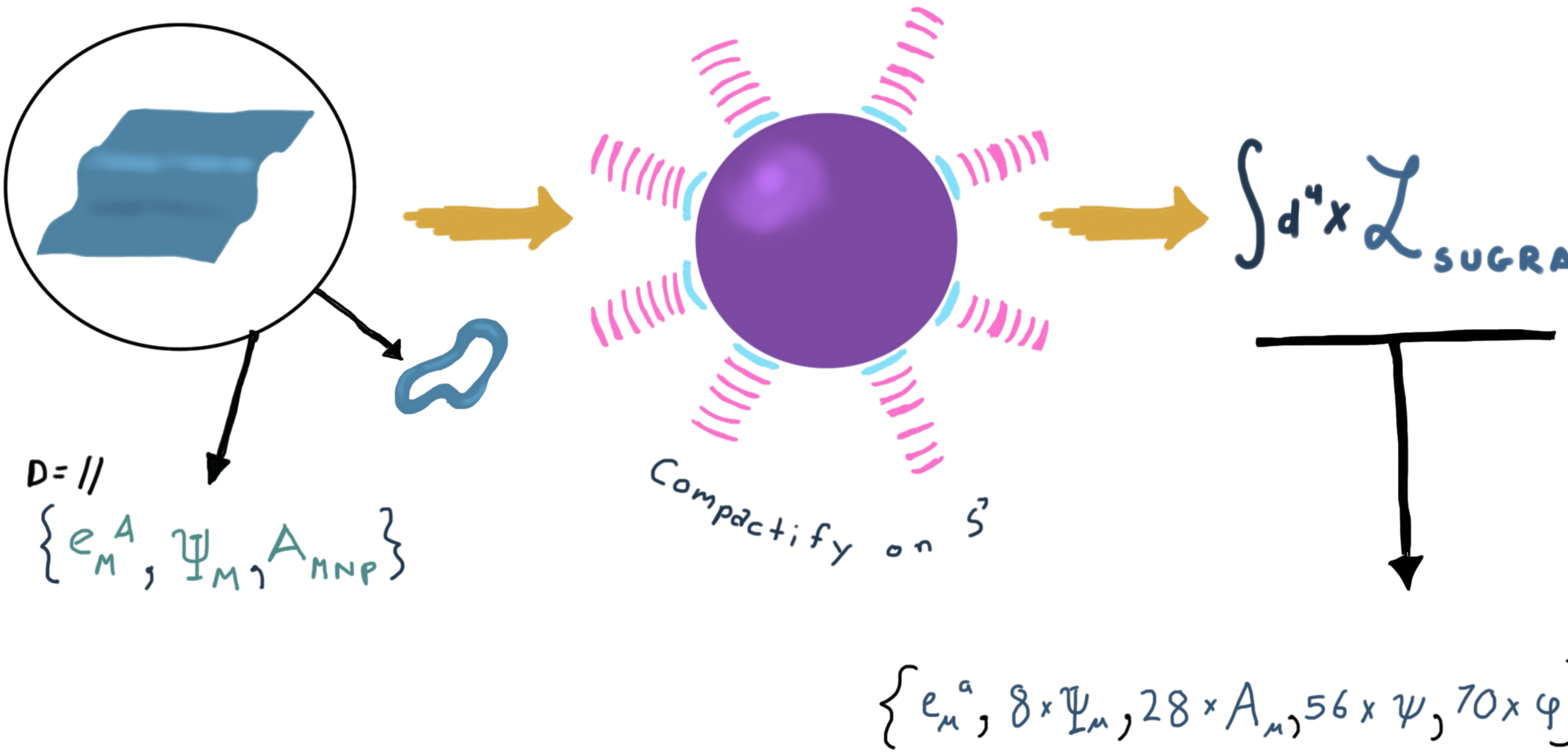


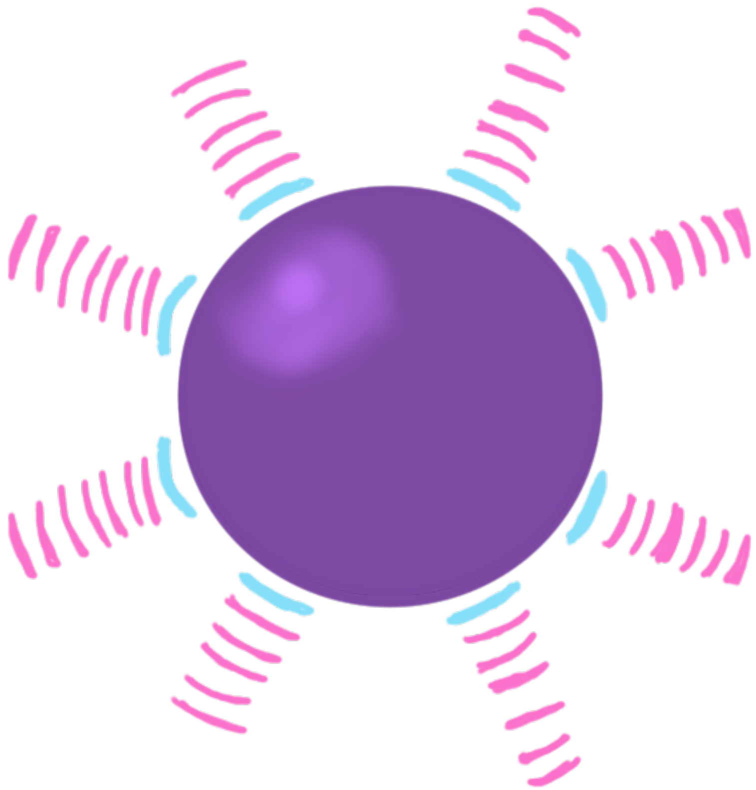
$SU(4)_R \times U(1)_b$
global symmetry

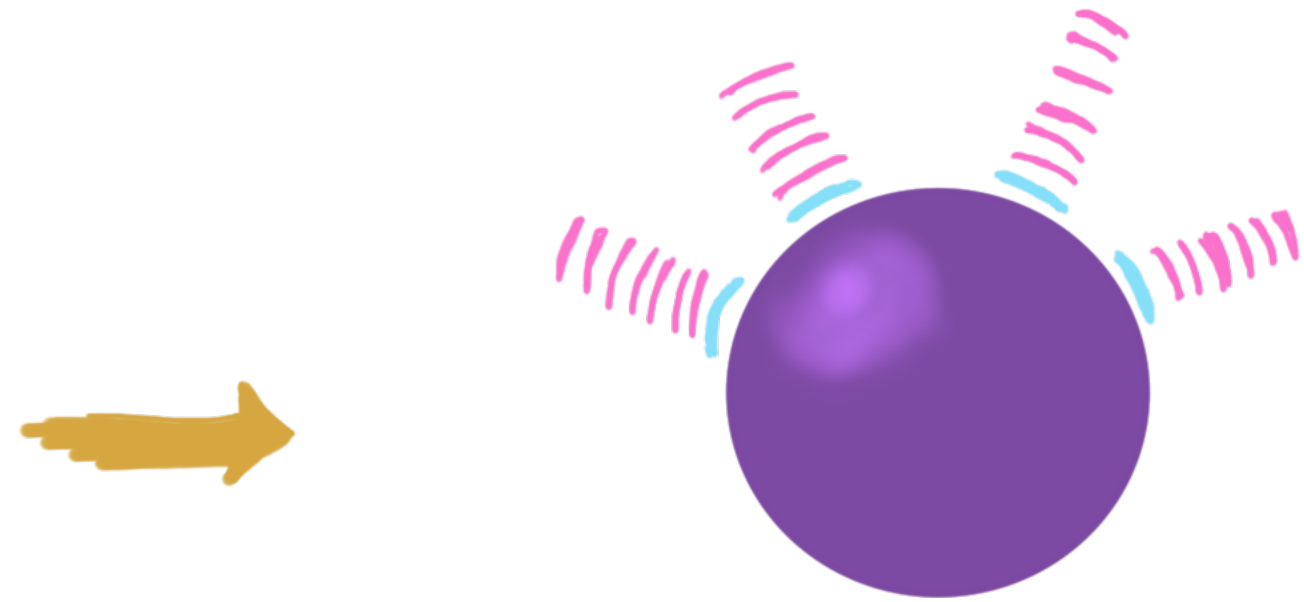
$$(Y_A)^{a}{}_{\hat{a}}\,,\ (\psi^A)^{a}{}_{\hat{a}}$$

$$\boxed{S_{ABJM} \sim \int A\wedge dA + \tfrac{2}{3}A^3 - \hat{A}\wedge d\hat{A} - \tfrac{2}{3}\hat{A}^3 - (DY)^2 + \bar{\psi}\not{D}\psi + Y^2\psi^2 - V(Y^6)}$$

Truncations we know and love







$U(1)^4 \subset SO(8)$

$$\Phi_i = \lambda_i e^{i\sigma_i}$$

$$\boxed{e^{-1}\mathcal{L}_{SUGRA} = R - \frac{1}{2}\sum_{i}^{3}\left((\partial\lambda_i)^2 + \sinh^2\lambda_i(\partial\sigma_i)^2\right) - \mathrm{Re}\left(F^{+a}\mathcal{M}_{ab}F^{+b}\right) + 8g^2\sum_{i}^{3}\cosh\lambda_i}$$

($\mathcal{N}=2$ w/ gravity + 3x Vector) ("STU" model)

A SUGRA Q-lattice Ansatz

$$e^{-1}\mathcal{L}_{SUGRA} = R - \frac{1}{2}\sum_{i}^{3}\left((\partial\lambda_i)^2 + \sinh^2\lambda_i(\partial\sigma_i)^2\right) + 8g^2\sum_{i}^{3}\cosh\lambda_i$$

Periodic Spatial Modulation

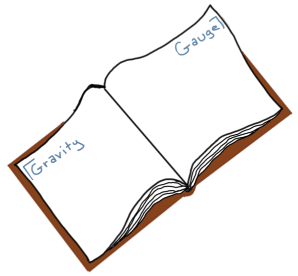
$$\lambda_1 = \sigma_1 = 0 \qquad \lambda_2 = \lambda_3 = \gamma(r) \qquad \boxed{\sigma_2 = kx \quad \sigma_3 = ky}$$

$$F_1 = F_2 = F_3 = F_4 = 0$$

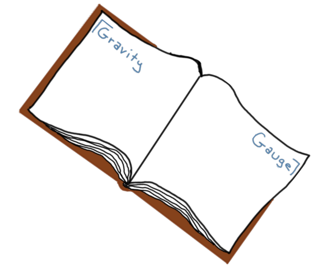
$$ds^2 = \boxed{-Ue^{-\chi/2}dt^2 + r^2 d\vec{x}^2} + \frac{dr^2}{U}$$

Broken Poincare Invariance

ODEs!!



Operator Matching



A top-down perk...

SUGRA $\longleftrightarrow$ **ABJM**

$$ds^2_{S^7} = (d\psi + \omega)^2 + ds^2_{\mathbb{CP}^3}$$

$$SU(4)_R \times U(1)_b$$

$$\lambda e^{i\sigma} = \chi + i\, Y_{35_v \to 15_0 \oplus \dots}$$

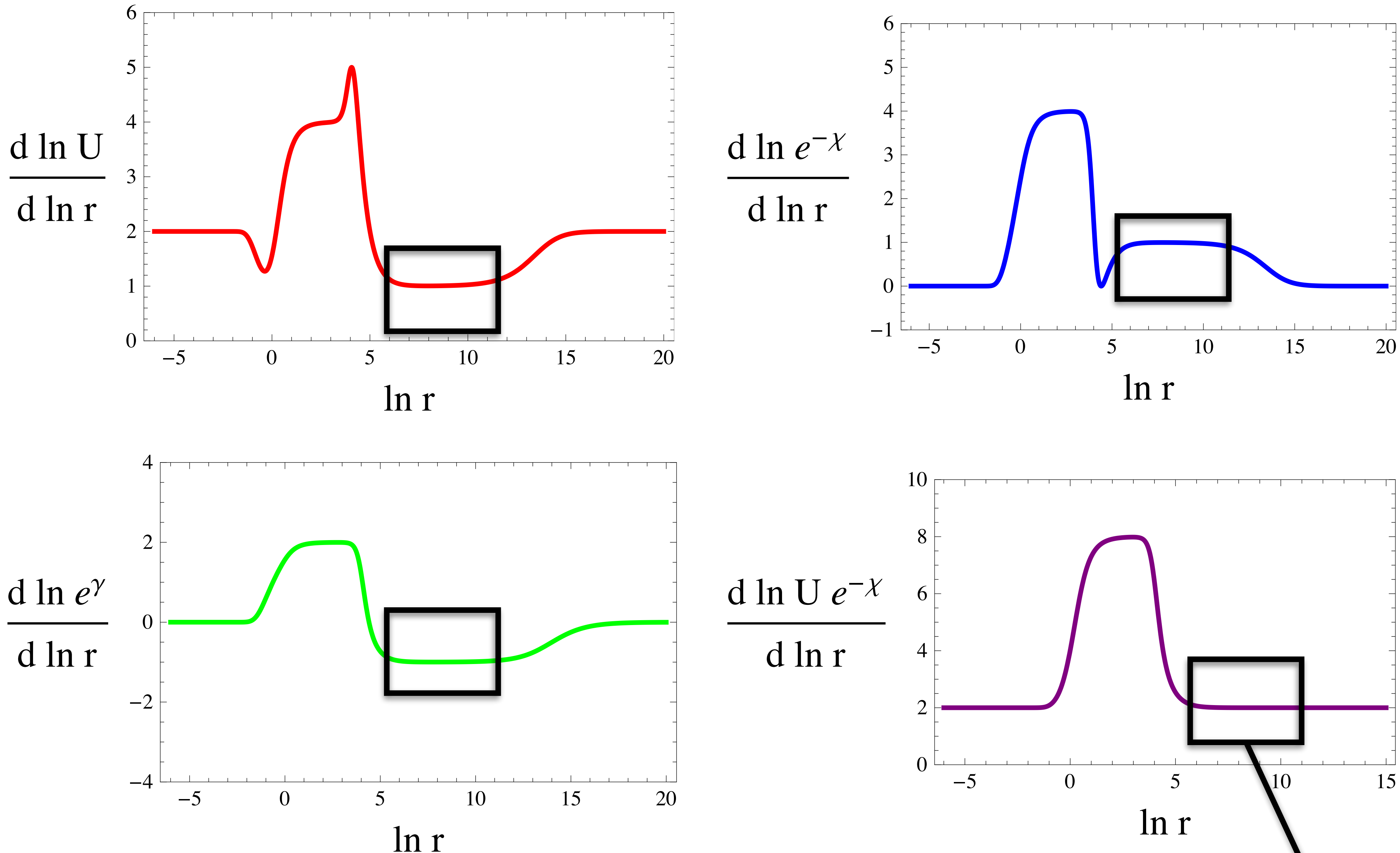
$$35_c \to 15_0 \oplus \dots$$

$$\mathcal{O}^{\Delta=1}_{Y^2} \sim \mathrm{tr}\left(Y^\dagger_A Y^B - \tfrac{1}{4}\delta^B_A Y^2\right)$$

$$\mathcal{O}^{\Delta=2}_{\psi^2} \sim \mathrm{tr}\left(\psi^\dagger_A \psi^B - \tfrac{1}{4}\delta^B_A \psi^2\right)$$

$$\boxed{\Delta\mathcal{L} \sim \hat{\Gamma}\cos kx\, \mathcal{O}_{Y^2} + \hat{\Gamma}\cos ky\, \mathcal{O}'_{Y^2} + \Gamma \sin kx\, \mathcal{O}_{\psi^2} + \Gamma \sin ky\, \mathcal{O}'_{\psi^2}}$$

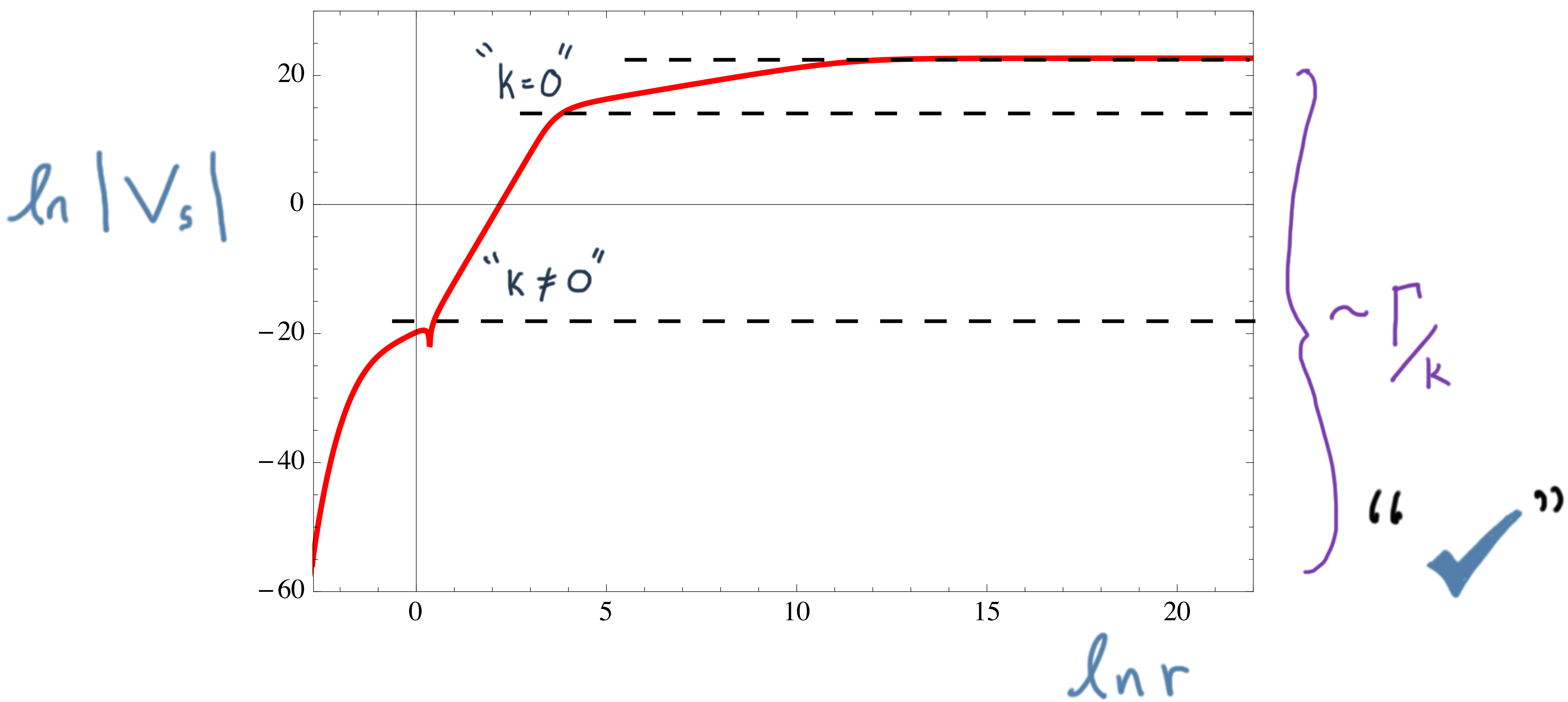
Nonlinear Solutions



$$ds^2 = \frac{1}{\rho^4}\left(-dt^2 + d\vec{x}^2 + d\rho^2\right)$$

HSV: $\theta = -2$, $z = 1$

Scaling Signatures: Linear Response



$$\left(\Box + \ldots\right)_{ij} \delta\Phi^{j} = 0 \Longrightarrow -\Psi'' + V_s \Psi = \omega^2 \Psi$$

but details matter...