A Holographic toy Model for the Anomalous Scalings of the Cuprates

Sera Cremonini
Lehigh University

Nordita 2017

Based on recent and new work:

- 1707.01505 [SC, Anthony Hoover, Li Li]
- 1710.01326 [Erin Blauvelt, SC, Anthony Hoover, Li Li, Steven Waskie]

Today's arXiv:

arXiv.org > hep-th > arXiv:1710.01326

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High Energy Physics - Theory

A Holographic Model for the Anomalous Scalings of the Cuprates

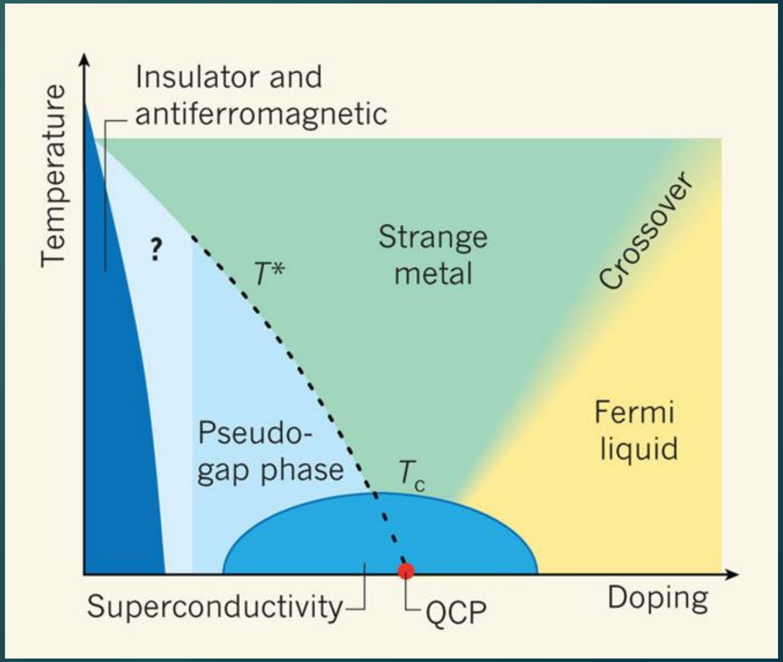
Erin Blauvelt, Sera Cremonini, Anthony Hoover, Li Li, Steven Waskie

(Submitted on 3 Oct 2017)

We examine transport in a holographic model in which the dynamics of the charged degrees of freedom is described by the non-perturbative Dirac--Born--Infeld (DBI) action. Axionic scalar fields are included to break translational invariance and generate momentum dissipation in the system. Scaling exponents are introduced by using geometries which are non-relativistic and hyperscaling-violating in the infrared. When the momentum

Strongly Correlated Electrons – Challenges

- Strong coupling
  - Breakdown of Fermi-liquid theory, no quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
  - Phases may compete but may also have a common origin and be intertwined [e.g. 1612.04385 and 1705.05390]
- Interplay between different scales in the system
- …



A Laundry List for Holography

Starting point:

Study models that may be in same universality class as QM systems of interest

Understand:

- Breaking symmetries
- Instabilities and phase transitions
- Interplay between phases and scales
- Minimal set of ingredients to obtain specific behaviors

Can we extract <u>any universal properties</u>? What drives this behavior?

Example of Intertwined Orders in Holography

Talk I am **not** going to give today

Pair and Charge Density Waves

arXiv.org > hep-th > arXiv:1705.05390

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High Energy Physics - Theory

Intertwined Orders in Holography: Pair and Charge Density Waves

Sera Cremonini, Li Li, Jie Ren

(Submitted on 15 May 2017)

Building on [1], we examine a holographic mo

arXiv.org > hep-th > arXiv:1612.04385

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High Energy Physics - Theory

Holographic Pair and Charge Density Waves

Sera Cremonini, Li Li, Jie Ren

(Submitted on 13 Dec 2016)

We examine a holographic model in which a U(1) symmetry and translational invariance are broken spontaneously at the same time. Our construction provides an example of a system with pair-density wave order, in which the superconducting order parameter is spatially modulated but has a zero average. In addition, the charge density oscillates at twice the frequency of the scalar condensate. Depending on the choice of parameters, the model also admits a state with co-existing superconducting and charge density wave orders, in which the scalar condensate has a uniform component.

- U(1) symmetry and translational invariance are broken spontaneously at the same time
- The orders have a common origin and are intertwined
- The model can reproduce features of a PDW as well as a co-existing SC and CDW

Today instead:

Examine magnetotransport in a holographic DBI model which describes <u>non-perturbative interactions</u> for the charged degrees of freedom

Focus on behavior of Hall angle and resistivity in this model

- Any clean scaling regimes? Can one obtain the scaling laws of the cuprates?
- So far challenging in holography (see e.g. EMD theories) →

$$\rho_{xx} \sim T\,, \qquad \cot\Theta_H \sim T^2$$

$$\cot\Theta_H = \frac{\sigma_{xx}}{\sigma_{xy}}\,, \qquad \rho_{xx} = \rho_{yy} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

The model

SC, Hoover, Li 1707.01505 (backreaction)

$$S = \int d^4x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Y(\phi)}{2}\sum_{I=1}^{2}(\partial\psi^I)^2\right] + S_{DBI}$$

$$S_{DBI} = -\int d^4x Z_1(\phi)\sqrt{-\det(g_{\mu\nu} + Z_2(\phi)F_{\mu\nu})}$$

Low-energy dynamics of D-branes (non-linear dynamics between charged d.o.f.)

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Axions break translational symmetry in simple way

Low-energy dynamics of D-branes (non-linear dynamics between charged d.o.f.)

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)(dx^2 + dy^2), \quad \phi = \phi(r)$$

$$\psi^1 = k\,x, \quad \psi^2 = k\,y, \quad A = A_t(r)\,dt + \frac{h}{2}(xdy - ydx)$$

k→momentum dissipation

The model

SC, Hoover, Li 1707.01505 (backreaction)

$$S = \int d^4x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Y(\phi)}{2} \sum_{I=1}^{2} (\partial\psi^I)^2 \right] + S_{DBI}$$

$$S_{DBI} = -\int d^4x Z_1(\phi) \sqrt{-\det(g_{\mu\nu} + Z_2(\phi) F_{\mu\nu})}$$

Scalar couplings can support scaling solutions (and generate simple T-dependence)

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)(dx^2 + dy^2), \quad \phi = \phi(r)$$

$$\psi^1 = k\,x, \quad \psi^2 = k\,y, \quad A = A_t(r)\,dt + \frac{h}{2}(xdy - ydx)$$

Magnetotransport

[see 1707.01505 for details]

- Conductivities can be computed using horizon method of Donos+Gauntlett (see 1707.01505 for details)

- For generic homogenerous and isotropic metrics in the presence of a magnetic field, they have a highly non-trivial form

$$\delta g_{ti} = C(r)h_{ti}(r)\,, \quad \delta g_{ri} = C(r)h_{ri}(r)$$
$$\delta A_i = -E_i\, t + a_i(r), \quad \delta\psi_1 = \chi_1(x), \quad \delta\psi_2 = \chi_2(x)$$

$$\sigma_{xx} = \sigma_{yy} = \frac{k^2 CY\left[\Omega(h^2\Omega + k^2 Y)(C^2 + h^2 Z_2^2)^2 + C^2 Q^2\right]}{(h^2\Omega + k^2 Y)^2(C^2 + h^2 Z_2^2)^2 + h^2 C^2 Q^2}\,,$$

$$\sigma_{xy} = -\sigma_{yx}$$
$$= \frac{hQ[(h^2\Omega + k^2Y)^2(h^2 Z_2^4 + 2C^2 Z_2^2) + (h^2\Omega + k^2 Y)C^4\Omega + C^2Q^2 - C^2k^2Y(C^2\Omega + k^2 Y Z_2^2)]}{(h^2\Omega + k^2 Y)^2(C^2 + h^2 Z_2^2)^2 + h^2 C^2 Q^2}$$

Highly non-trivial structure

Main Features:

$$\Omega(r) = \frac{Z_2}{C^2 + h^2 Z_2^2}\sqrt{Q^2 + Z_1^2 Z_2^2 (C^2 + h^2 Z_2^2)}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{k^2 C Y\left[\Omega(h^2\Omega + k^2 Y)(C^2 + h^2 Z_2^2)^2 + C^2 Q^2\right]}{(h^2\Omega + k^2 Y)^2 (C^2 + h^2 Z_2^2)^2 + h^2 C^2 Q^2},$$

Physical scales k, h, T, Q
k: momentum dissipation

$$\sigma_{xy} = -\sigma_{yx}$$
$$= \frac{hQ\left[(h^2\Omega + k^2 Y)^2 (h^2 Z_2^4 + 2C^2 Z_2^2) + (h^2\Omega + k^2 Y) C^4 \Omega + C^2 Q^2 - C^2 k^2 Y (C^2 \Omega + k^2 Y Z_2^2)\right]}{(h^2\Omega + k^2 Y)^2 (C^2 + h^2 Z_2^2)^2 + h^2 C^2 Q^2}$$

- Controlled by C(r) and three scalar couplings Z_1, Z_2, Y
- For a background with a running scalar, these are generically T-dependent
  → can provide <u>different temperature scales</u>
- T-dependence of Hall angle will generically be different from that of DC conductivity

Working with Backreacted geometries

SC, Hoover, Li 1707.01505

- Metallic or insulating behavior

  working definition →

$$\text{Metal}: \quad \frac{dR_{xx}}{dT} > 0\,, \qquad \text{Insulator}: \quad \frac{dR_{xx}}{dT} < 0$$

- New magnetic field driven metal-insulator crossovers

- Fully backreacted analytical (dyonic) black branes

- …

Can we identify clean scaling regimes?

arXiv:1710.01326 [E. Blauvelt, SC, A. Hoover, S. Waskie]

Can we obtain the scalings of the Hall angle and resistivity of the cuprates in some temperature range?

Can we identify clean scaling regimes?

arXiv:1710.01326 [E. Blauvelt, SC, A. Hoover, S. Waskie]

- Work in regimes in which these expressions naturally simplify

  → focus on **strong momentum dissipation** limit (probe DBI)

- Lessons learned in simpler cases can help **build intuition for generalizations**

  → away from strong momentum dissipation limit

- Use geometries that are **non-relativistic and hyperscaling violating in the IR**

  → scaling exponents will be tunable paremeters

Strong Momentum Dissipation (probe DBI)

Basic Idea:

when the momentum dissipation k is strongest scale in the system, the conductivities simplify greatly

$$\cot \Theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{C}{hQZ_2}\sqrt{Q^2 + Z_1^2 Z_2^2 (C^2 + h^2 Z_2^2)}$$

$$\rho_{xx} = \frac{C}{Z_2}\frac{\sqrt{Q^2 + Z_1^2 Z_2^2 (C^2 + h^2 Z_2^2)}}{Q^2 + C^2 Z_1^2 Z_2^2}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

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Q and h terms are small in large k regime (and appropriate T range)

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Strong Momentum Dissipation (probe DBI)

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leading terms

Leading behavior: $$\cot\Theta_H = \frac{C^2 Z_1}{hQ}, \qquad \rho_{xx} = \frac{1}{Z_1 Z_2^2}$$ **when** $h << \frac{C}{Z_2}$ $\quad Q << Z_1 Z_2 C$

temperature range

Strong Momentum Dissipation (probe DBI)

The couplings Z_1, Z_2, and g_{xx} = C control the T-dependence of the conductivities

$$\cot\Theta_H = \frac{C^2 Z_1}{hQ}\,, \qquad \rho_{xx} = \frac{1}{Z_1 Z_2^2}$$

Generically provide different temperature scales in the system (note: compare structure to EMD)

Assuming power law T-dependence for the scalar couplings, cuprates' scalings follow when:

Condition for the cuprates

$$\frac{C}{Z_2} = \frac{T^{3/2}}{\ell_0^{1/2}} \quad \text{and} \quad Z_1 Z_2^2 = \frac{z_0}{T}$$

$\hookrightarrow \rho_{xx} \sim T \qquad \cot\Theta_H \sim T^2$

Provided

$Q \ll T^{1/2}$

$h \ll T^{3/2}$

Explicit realization:
non-relativistic, hyperscaling violating IR geometries

Ingredients

- Want clean scaling regime in IR of theory
  - Couplings should be simple powers of T
- Natural way to achieve this is to use Lifshitz-like, hyperscaling violating geometries
- Well-known how to generate these in EMD theories (scalar couplings are simple exponentials)
- Present even in DBI theories → same structure for couplings

  (with and without backreaction)

Background Geometries in strong k limit

Scaling solutions in the IR of the geometry

$$V(\phi) \sim -V_0\, e^{\eta\,\phi}, \quad Y(\phi) \sim e^{\alpha\,\phi}$$

$$ds^2 = r^\theta \left[-\frac{f(r)}{r^{2z}} dt^2 + \frac{L^2}{r^2 f(r)} dr^2 + \frac{dx^2 + dy^2}{r^2} \right]$$

$$\phi = \kappa \ln(r), \quad \psi^1 = k\, x, \quad \psi^2 = k\, y\,, \quad A = A_t(r)\, dt + \frac{h}{2}(x dy - y dx)$$

Well-known Lifshitz-like, hyperscaling violating black branes

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Well-known Lifshitz-like, hyperscaling violating black branes

$$f(r) = 1 - \left(\frac{r}{r_h}\right)^{2+z-\theta}, \quad z = \frac{\alpha^2 - \eta^2 + 1}{\alpha(\alpha + \eta)}, \quad \theta = \frac{2\eta}{\alpha}, \quad \kappa = -\frac{2}{\alpha};$$

$$L^2 = \frac{(z + 2 - \theta)(\theta - 2z)}{V_0}, \quad k^2 L^2 = 2(z - 1)(z + 2 - \theta)\,,$$

Simple form

$$T \sim r_h^{-z}$$

Keep in mind constraints on range of z,θ (NEC, no singularities, thermo etc)

Probe DBI Limit

- The strong momentum dissipation limit coincides with the probe DBI limit
- Geometries are seeded by scalar and axions only – backreaction of DBI interactions on the geometry can be neglected
- One must ensure that DBI stress tensor is subleading compared to remaining matter stress tensor

$$Z_1^2 Z_2^2 r_0^{2\theta} L^2 \ll Z_2 r^2 \sqrt{Q^2 + Z_1^2 Z_2^2 (C^2 + h^2 Z_2^2)} \ll Z_2^2 / L^2$$

- This ultimately corresponds to a certain temperature range (see paper)

Temperature Scalings

Recall we had $V(\phi) \sim -V_0\, e^{\eta\,\phi}, \quad Y(\phi) \sim e^{\alpha\,\phi}$

$T \sim r_h^{-z}$

Choose simple couplings $Z_1 \sim e^{\gamma\phi}$ and $Z_2 \sim e^{\delta\phi}$, and the scalar was log-running

$$C(r_h) = r_h^{\theta-2} \Rightarrow C(T) \sim T^{\frac{2-\theta}{z}} \qquad Z_1 \sim T^{\frac{2\gamma}{z\alpha}} \quad \text{and} \quad Z_2 \sim T^{\frac{2\delta}{z\alpha}}$$

Temperature Scalings

Recall we had $V(\phi) \sim -V_0\, e^{\eta\,\phi}, \quad Y(\phi) \sim e^{\alpha\,\phi}$

$T \sim r_h^{-z}$

Choose simple couplings $Z_1 \sim e^{\gamma\phi}$ and $Z_2 \sim e^{\delta\phi}$, and the scalar was log-running

$$C(r_h) = r_h^{\theta-2} \Rightarrow C(T) \sim T^{\frac{2-\theta}{z}} \qquad Z_1 \sim T^{\frac{2\gamma}{z\alpha}} \quad \text{and} \quad Z_2 \sim T^{\frac{2\delta}{z\alpha}}$$

Strong momentum dissipation → $\cot\Theta_H = \dfrac{C^2 Z_1}{hQ}, \qquad \rho_{xx} = \dfrac{1}{Z_1 Z_2^2}$

Scaling behavior

$$\rho_{xx} \sim T^{-\frac{2}{z}\left(\frac{\gamma}{\alpha}+2\frac{\delta}{\alpha}\right)} \quad \text{and} \quad \cot\Theta_H \sim \frac{1}{hQ} T^{\frac{2}{z}\left(2-\theta+\frac{\gamma}{\alpha}\right)}$$

provided $h << T^{\frac{2}{z}\left(1-\frac{\theta}{2}-\frac{\delta}{\alpha}\right)}$ and $Q << T^{\frac{2\gamma+2\delta+\alpha(\theta-2)}{z\alpha}}$

Anomalous Scalings of the cuprate strange metal

Scaling behavior in strong momentum dissipation limit:

$$\rho_{xx} \sim T^{-\frac{2}{z}\left(\frac{\gamma}{\alpha}+2\frac{\delta}{\alpha}\right)} \quad \text{and} \quad \cot\Theta_H \sim \frac{1}{hQ} T^{\frac{2}{z}\left(2-\theta+\frac{\gamma}{\alpha}\right)}$$

To obtain the cuprates you need to take

$$\frac{\gamma}{\alpha} = z+\theta-2 \quad \text{and} \quad \frac{\delta}{\alpha} = 1-\frac{\theta}{2}-\frac{3}{4}z \quad \Rightarrow \quad \rho_{xx} \sim T\,, \qquad \cot\Theta_H \sim T^2$$

Anomalous Scalings of the cuprate strange metal

Scaling behavior in strong momentum dissipation limit:

$$\rho_{xx} \sim T^{-\frac{2}{z}\left(\frac{\gamma}{\alpha}+2\frac{\delta}{\alpha}\right)} \quad \text{and} \quad \cot\Theta_H \sim \frac{1}{hQ} T^{\frac{2}{z}\left(2-\theta+\frac{\gamma}{\alpha}\right)}$$

To obtain the cuprates you need to take

$$\frac{\gamma}{\alpha} = z + \theta - 2 \quad \text{and} \quad \frac{\delta}{\alpha} = 1 - \frac{\theta}{2} - \frac{3}{4} z \quad \rightarrow \quad \rho_{xx} \sim T\,, \qquad \cot\Theta_H \sim T^2$$

Note: Interesting special case (cuprates): $z = 4/3, \theta = 0$

Recall Hartnoll and Karch
1501.03165
(purely QFT analysis)

"Minimal model"

$$S_{DBI} \sim \int d^4x\, e^{-\frac{2}{3}\phi} \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}$$

Reminiscent of dilaton coupling in string theory. Top down realization? All parameters fixed!

Recap:

- the Hall angle and DC conductivity generically display a different temperature dependence (different T scales) AND cuprate scaling laws can be obtained without violating NEC
- Non-trivial dynamics of D-brane action encoded by Z_1, Z_2 plays crucial role

  → Suggests that non-perturbative interactions between charged d.o.f. may be important
- IR scaling regime
- DBI provides another example in which σ_{DC} is not simply $\sigma_{ccs} + \sigma_{diss}$

  (two contributions are mixed in non-trivial way)

Away from strong k limit? Full backreaction

- Harder to identify specific scalings but ONLY TECHNICAL OBSTACLE, not conceptual
- Most of the ingredients that we need are still present → expect similar behavior
- BUT: analytical fully backreacted solutions are harder to find and rely on a number of (simplifying) relations between theory parameters
- When only a single quantity controls the T-behavior of the system, one can not "decouple" the various conductivities from each other → explicit example next

Illustrative case: backreacted dyonic scaling solutions

With single-exponential couplings (as before), dyonic solutions exist for $\gamma = -2\delta, \quad \eta = \alpha - \delta$

Much uglier:

$$f(r) = 1 - \left(\frac{r}{r_h}\right)^{2+z-\theta} + \frac{2(z-1)(2+z-\theta)(h^2 z_1^2 + Q^2)}{k^2(z-2)(2z+\theta-6)\sqrt{(1+h^2)z_1^2+Q^2}} \left(\frac{r}{r_h}\right)^{4-\theta} \left[1 - \left(\frac{r}{r_h}\right)^{z-2}\right]$$

$$\kappa = -\frac{2}{\alpha}, \quad z = \frac{1+\alpha^2-\eta^2}{\alpha(\alpha-\eta)}, \quad \theta = 2\frac{\eta}{\alpha}, \quad k^2 = \frac{V_0}{1 - \frac{(\eta-\alpha)^2-1}{(\eta^2-\alpha\eta-1)}\sqrt{(1+h^2)+\frac{Q^2}{z_1^2}}},$$

$$A_t'(r) = \frac{LQ}{z_1}\frac{r^{1-z}}{\sqrt{(1+h^2)+\frac{\rho^2}{z_1^2}}}, \quad L^2 = \frac{2(z-1)(2+z-\theta)}{k^2}.$$

Very non-trivial temperature

$$T = \frac{1}{4\pi}\sqrt{\frac{2+z-\theta}{2(z-1)}}\left[k\, r_h^{-z} + \frac{2(z-1)}{k(\theta+2z-6)}\frac{h^2 z_1^2 + Q^2}{\sqrt{1+h^2 z_1^2+Q^2}}\, r_h^{4-z-\theta}\right]$$

Illustrative case: backreacted dyonic scaling solutions

- These geometries are supported by a running scalar $\phi = \kappa \log r$
- Can still identify temperature regimes in which $T \sim r_h^p$
- Can still obtain regimes in which the resistivity is linear
- But the Hall angle in that particular regime will typically scale like the conductivity (unless there is severe fine tuning)

These particular solutions are “too simple”:

$$C(r) = Z_2(r) = r^{\theta-2}\,, \quad Z_1 = z_1 r^{4-2\theta} = z_1 C^2$$

example in which only one coupling effectively survives ~ CY → only temperature scale

Illustrative case: backreacted dyonic scaling solutions

- Only one quantity controls temperature dependence of conductivities

$$\rho_{xx} = \frac{a_1 CY + a_2(CY)^2 + a_3(CY)^3 + a_4(CY)^4}{a_5 + a_6 CY + a_7(CY)^2 + a_8(CY)^3 + a_9(CY)^4}$$

$$\cot\Theta_H = \frac{b_1 CY + b_2(CY)^2}{b_3 + b_4 CY + b_5(CY)^2},$$

- Although their T-dependence is different, no clear separation between conductivity and Hall angle in T-ranges in which they exhibit clean scalings
- **Severe fine-tuning** (coefficients) is only way to achieve different scalings for this particular background geometry

Numerically this should not be an issue:
expect to find non-trivial scaling solutions which should be controlled by two or more independent couplings → different temperature scales

To wrap up:

- DBI theory: **toy model** providing concrete example in which the DC conductivity and Hall angle behave differently with T → **<u>different temperature scales in the system</u>**

- **<u>proof of principle</u>** that cuprate scalings can be obtained in some range of T

  → can provide <u>intuition for ingredients needed and ways to generalize</u>

- We saw **<u>clean scalings in the large k regime</u>** (probe DBI limit)
  - Expect the same for the arbitrary k case (full backreaction), just harder to find

    → no conceptual obstacle (numerics)

- **Hint:** important to take into account **non-trivial dynamics between charged degrees of freedom?** In addition to competition between scales (already stressed).

Thank you

Come visit Lehigh!