Boson-dominated quantum critical metals at the Lorentz-symmetric point

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Outline

- A quantum critical metal model in 2+1 dimensions
- Boson-dominated theories
- Equal boson and Fermi-velocity: Additional symmetry in the IR
- Fermion two-point function: described by single holomorphic function, $f(z)$
- Properties of $f(z)$ and implications on two-point function
- Explicit $f(z)$ for vanishing fermion flavor number quantum critical metal

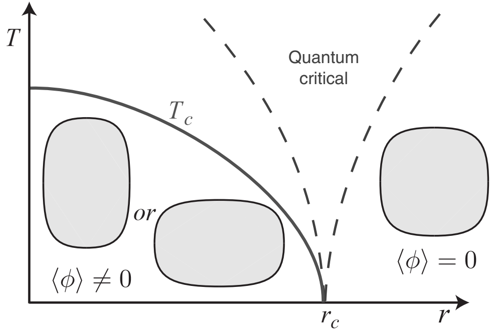
Quantum critical metals

Fermions at finite density coupled to critical bosons:

$$S = \int \mathrm{d}\tau \mathrm{d}x^d \left(\psi^\dagger (\partial_\tau - \frac{\nabla^2}{2m} - \mu)\psi + \frac{1}{2}\left(\partial_\tau \phi\right)^2 + \frac{1}{2}\left(\nabla \phi\right)^2 \right) + S_{\text{int}}$$

ϕ is an order parameter field of e.g. Ising-nematic transition

Putative model of some heavy fermion systems and high-T_c materials.



Quantum critical metals

$$S_{\mathrm{int}} = \int \mathrm{d}\tau \mathrm{d}x^d \left(\lambda \phi \psi^\dagger \psi + g \phi^4 \right)$$

- λ, fermion-boson interaction. Scaling dimension $(d-3)/2$
- g, boson self-interaction. Scaling dimension $d-3$

- $d=3$ is marginal. Admits perturbative treatment: Hertz-Millis theory
- $d=2$ is strongly coupled. Landau Fermi liquid theory breakdown[1]. Fermion sign-problem prohibits Monte-Carlo, except certain cases[2]. Full description still an open problem.

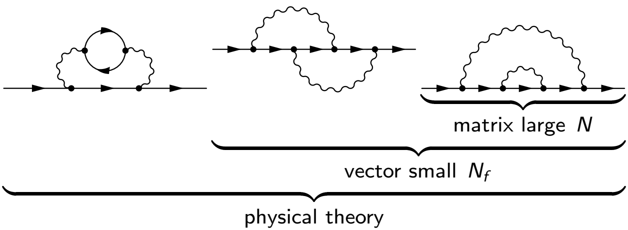
[1] P. A. Lee, Phys. Rev. Lett. **63** (1989) 680

[2] E. Berg, M. A. Metlitski, S. Sachdev, Science **338** (2012) 6114

Two boson-dominated limits

- Vector small N_f [3] [4]: $S_{\rm int} = \lambda \int \mathrm{d}x^{d+1} \phi \psi_i^\dagger \psi_i$
- Matrix large N [5]: $S_{\rm int} = \frac{\lambda}{\sqrt{N}} \int \mathrm{d}x^{d+1} \psi_i^\dagger \phi_{ij} \psi_j$

Fermion self-energy contributions at order λ^4:



[3] B. Meszena, PS, A. Bagrov, K. Schalm, Phys. Rev. B **94** (2016), 115134

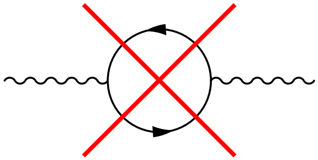
[4] PS, B. Meszena, K. Schalm, 1612.05326

[5] A. L. Fitzpatrick, S. Kachru, J. Kaplan, S. Raghu, Phys. Rev. B **89** (2014), 165114

General boson-dominated theories

In general we define a boson-dominated theory a generalization of the quantum critical metal such that

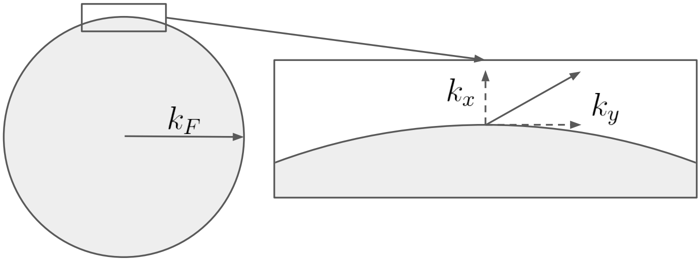
- All diagrams of the perturbative expansion have new prefactors $a_i \in \mathbb{R}$.
- All diagrams containing fermionic loops are suppressed, $a_i = 0$ for i a diagram with a fermionic loop.



Patch theory

The low-energy limit of a boson-dominated theory ($\omega \ll k_F$) can be studied by considering a single patch of the "Fermi surface" for calculating the fermion two-point function.

$$G_0(\omega, k) = \frac{1}{i\omega - k^2/2m + \mu} \approx \frac{1}{i\omega - v_F k_x}$$



$$k_F = \sqrt{2\mu m}$$
$$v_F = \sqrt{2\mu/m}$$

Equal boson velocity and Fermi-velocity: "Lorentz"-symmetry

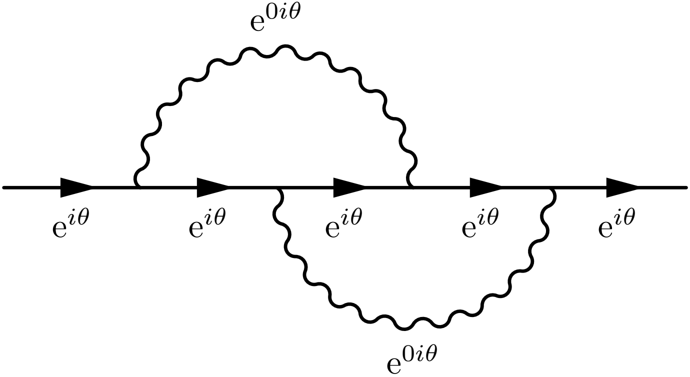
In addition to the boson-dominated limit, we consider the special case $v_F = 1$.

$$
\begin{aligned}
G_0(\omega, k_x) =& \frac{1}{\bar{w}} \\
D(\omega, k_x, k_y) =& \frac{1}{w\bar{w} + k_y^2}
\end{aligned}
$$

where $w = i\omega + k_x$. The free propagators transform only with a phase under a θ rotation in the complex w-plane:

$$
\begin{aligned}
w &\to \mathrm{e}^{i\theta} w \\
G_0(\omega, k_x) &\to \mathrm{e}^{i\theta} G_0(\omega, k_x) \\
D(\omega, k_x, k_y) &\to D(\omega, k_x, k_y)
\end{aligned}
$$

"Lorentz transformation" of patch coordinates:



Total phase change: $e^{5i\theta}$

Implications for fermion two-point function

All two-point function diagrams at order λ^{2n} have $2n+1$ fermion propagators. This implies a phase dependence $\mathrm{e}^{(2n+1)i\theta}$ upon rotating the external $w = i\omega + k_x$.

By dimensional analysis we find that the magnitude of corrections at order λ^{2n} are proportional to $\lambda^{2n}|w|^{-1-n}$.

$$G(w) = \sum_{n=0}^{\infty} f_n \lambda^{2n} \frac{w^{n/2}}{\bar{w}^{3n/2+1}} \equiv f\left(\lambda^2 \frac{\sqrt{w\bar{w}}}{\bar{w}^2}\right) \frac{1}{\bar{w}}$$

(pause for questions)

Implications for fermion two-point function

$$G(\omega, k_x) = f\left(\lambda^2 \frac{\sqrt{\omega^2 + k_x^2}}{(i\omega - k_x)^2}\right) \frac{1}{i\omega - k_x}$$

We define the mapping $z : \mathbb{R}^2 \to \mathbb{C}$

$$z : (\omega, k_x) \mapsto \frac{\sqrt{\omega^2 + k_x^2}}{(i\omega - k_x)^2}$$

Analytically continue z in ω to find retarded two-point function

$$z(-i\omega_R + 0^+, k_x) = \begin{cases} \frac{\sqrt{k_x^2 - \omega_R^2}}{(\omega_R - k_x)^2} + 0^+ \operatorname{sgn}(k_x) & \omega_R^2 < k_x^2, \text{ ``timelike''} \\ -i \operatorname{sgn}(\omega_R) \frac{\sqrt{\omega_R^2 - k_x^2}}{(\omega_R - k_x)^2} & \omega_R^2 > k_x^2, \text{ ``spacelike''} \end{cases}$$

Light-cone coordinates: $\alpha = \omega_R - k_x$ and $\beta = \omega_R + k_x$

$$G_R(\alpha, \beta) = s G_R(s\alpha, s^3\beta)$$

cf. actual Lorentz invariance:

$$G_R(\alpha, \beta) = G_R(s\alpha, s^{-1}\beta)$$

Properties of characteristic function f

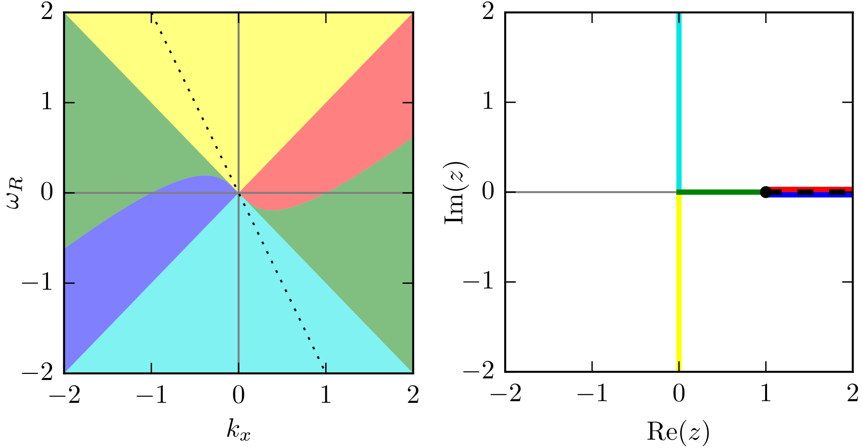
Under quite general assumptions

- Unitarity
- Causality
- Dispersion relation not identical to free theory
- Theory approaches free theory in the UV

we find that

- $f(0) = 1$
- $f(z)$ is analytic apart from a branch cut from $z = r_0 > 0$ to $+\infty$.
- $f(\bar{z}) = \overline{f(z)}$
- $$f(z) = \frac{C}{(-z)^{2/3}} + \text{subleading}, \quad C \in \mathbb{R}$$

Implications for two-point function



(ω_R, k_x)-space (left) mapped onto the complex z plane (right).

Vector small N_f result

The two-point function can be found non-perturbatively in the $N_f \to 0$-limit for $0 < v_F < 1$ [6].

Taking $v_F \to 1$ we find

$$3f^3z^2 - 16\pi^2 (f+2)(f-1)^2 = 0$$

f is analytic everywhere except a branch-cut from $z = 4\pi/\sqrt{3}$ to $+\infty$.

The main features inferred solely from the extra symmetry at $v_F = 1$ are actually there for all $0 < v_F < 1$ in the $N_f \to 0$-theory.

[6] B. Meszena, PS, A. Bagrov, K. Schalm, Phys. Rev. B **94** (2016), 115134

Matrix large N

This has been studied in $D = 4 - \epsilon$ dimensions [7].

$$G(\omega, \mathbf{k}) = A\omega^{2\gamma - 1}$$

where A is a constant and the anomalous dimension $\gamma = \epsilon/4$. This does not agree with the symmetry we found so ϵ expansions down to $D = 3$ do not work.

[7] A. L. Fitzpatrick, S. Kachru, J. Kaplan, S. Raghu, Phys. Rev. B **89** (2014), 165114

Summary

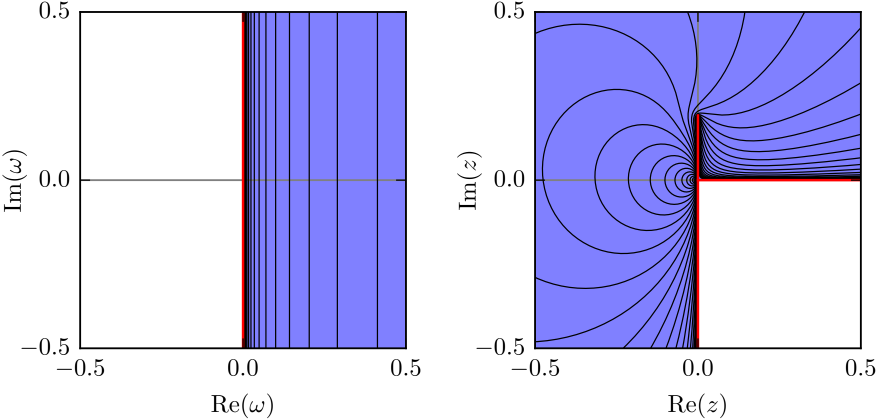
- We define boson-dominated models of quantum critical metals.
- We consider the IR of such theories at special point $v_F = 1$.
- We find strong constraint on fermion two-point function.
- Dispersion is non-monotonic under general assumptions.
- Result agrees with earlier $N_f \to 0$ result. Main predictions also agree for $v_F < 1$.
- Incompatible with matrix large-N result from dimensional continuation.

Outlook

- Find explicit f in matrix large N limit
- Consider boson four point interactions/different boson dynamical critical exponent.

Properties of f

Demanding causality, the retarded two-point function is analytic in UHP. Analytic continuation of mapping z for $k_x = 1$:



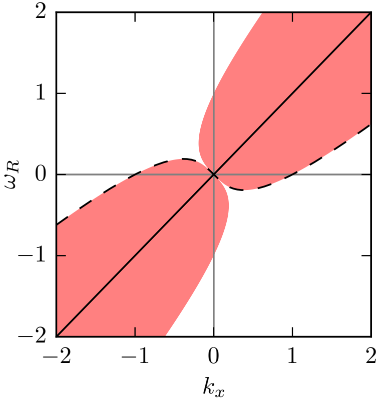
This means that f is analytic everywhere except possibly $\mathbb{R}^+$.

Three separate cases

r is the radius of convergence for the power-series defining f.

- $r = 0$: Typical asymptotic perturbation theory.
  $G_R(\omega, k_x)$ is not analytic at $\omega = -k_x$
- $r =$ finite: Convergent perturbation theory.
  Singularity at $0 < z$. See next slide.
- $r = \infty$: f is entire.
  True for free theory, $f(z) = 1$
  Dispersion same as free theory.

f with a single singularity



The red area indicates where perturbation theory diverges for $r_0 = 1$. Perturbation theory converges outside of this area. The dashed line indicates where the singularity at $z = r_0$ shows up in ω_R, k_x-space, the solid line indicates $\omega_R = k_x$.

“Gap equation”