An upper bound on diffusivity

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Locality implies finite propagation speed

Even in non relativistic systems

Lieb Robinson theorem, rigorous for local spin chains

$$\left\| \left[\, A(t,x), B(0,0)\, \right] \right\| \leq c_1 \exp[c_2(t - |x|/v)]$$

$$v \simeq \frac{Ja}{\hbar}$$

We will discuss other velocities later (butterfly, lightcone, etc.)

Conserved quantities diffuse

Generic systems reach local equilibrium after a short time t_{eq}

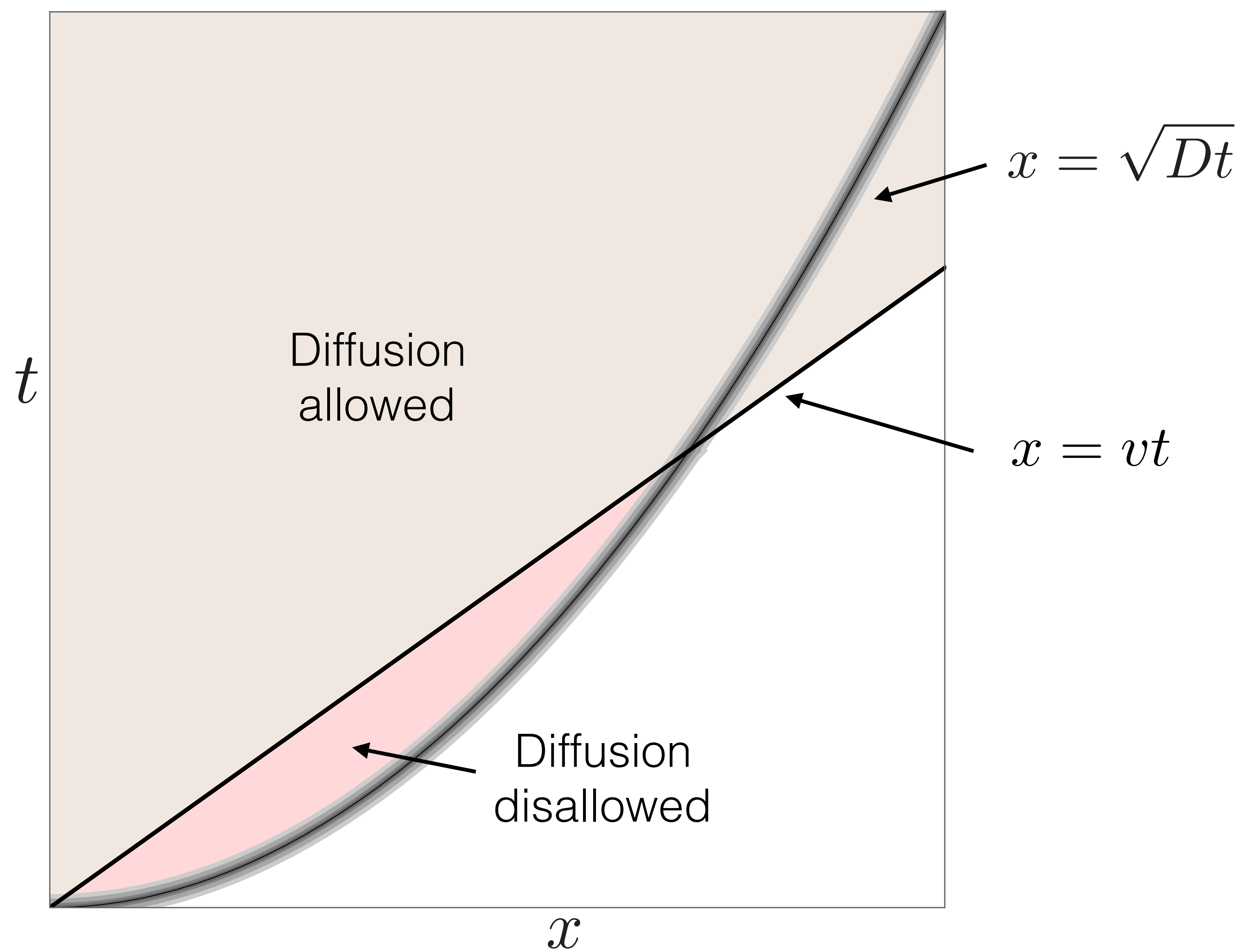
Hydrodynamics is the effective description after t_{eq}

$$\dot{n} = D\nabla^2 n$$

$$n(t, \mathbf{k}) = n(t = 0, \mathbf{k}) \exp[-Dk^2 t]$$

$$\langle [n(t, x), n(0, 0)] \rangle \propto \nabla^2 \frac{e^{-x^2/(4Dt)}}{t^{d/2}}$$

Upper bound on diffusivity



Intersection time must be less than the equilibration time

Upper bound on diffusivity

Intersection time = $\frac{D}{v^2}$

Thus $D \lesssim v^2 t_{eq}$

This is our main result

No claims about order one constants

EFT viewpoint

- Hydrodynamics is a low energy effective field theory
- The heavy modes set a scale t_{eq}
- One can expect that the coupling constants in the low energy EFT (in this case the diffusion constant) are upper bounded by this scale
- CEMZ analogy: If the higher-derivative corrections to GR are larger than the scale set by the higher-spin particles, causality is violated

Einstein Gravity

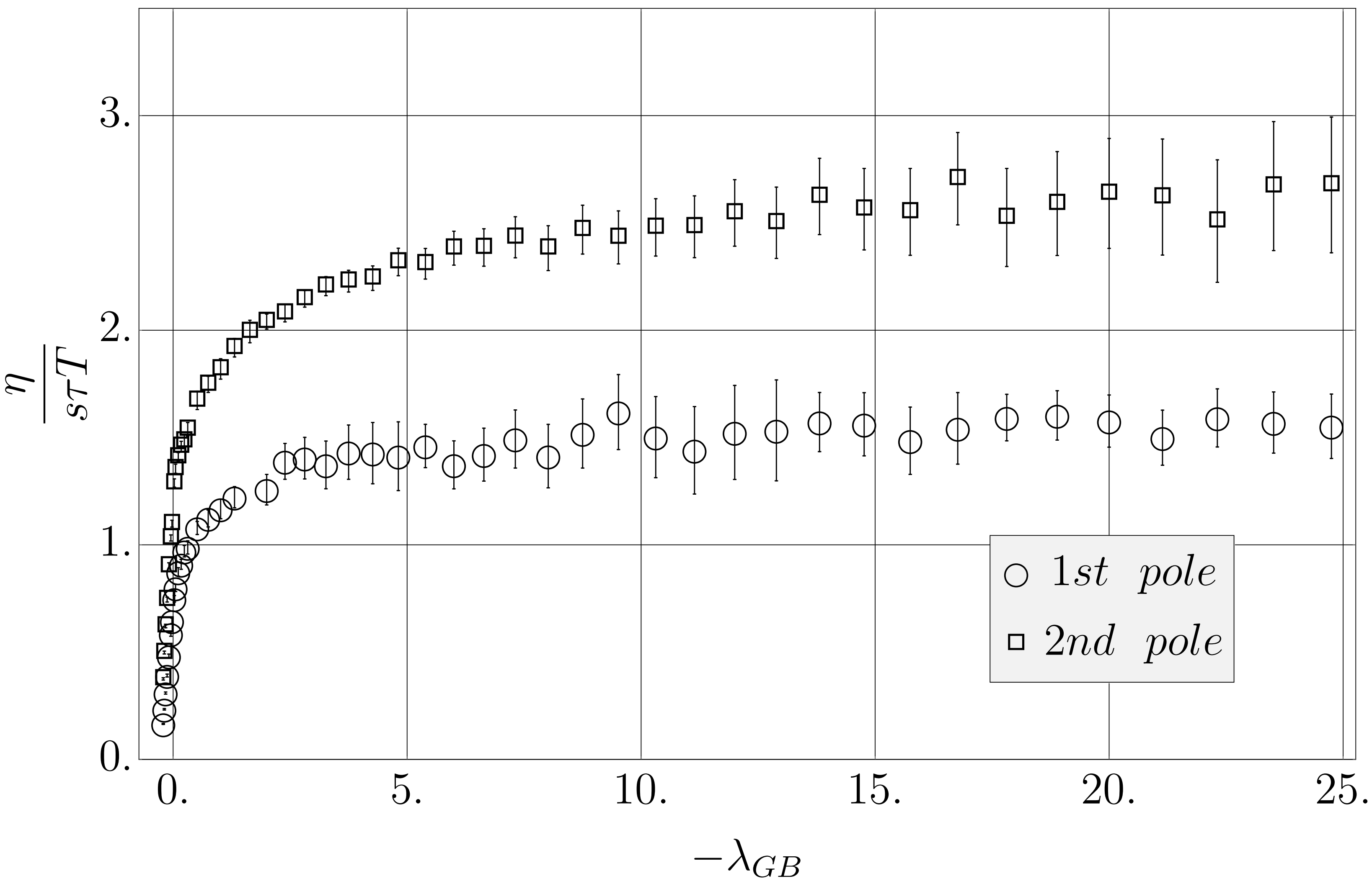
In this case, one has $D \sim v^2 t_{eq}$

$$DT = \frac{\eta}{s} = \frac{1}{4\pi}$$

$$v = \text{speed of light}$$

$$t_{eq} = \frac{1}{\omega_{\rm qnm}} \sim \frac{1}{T}$$

Gauss Bonnet Gravity

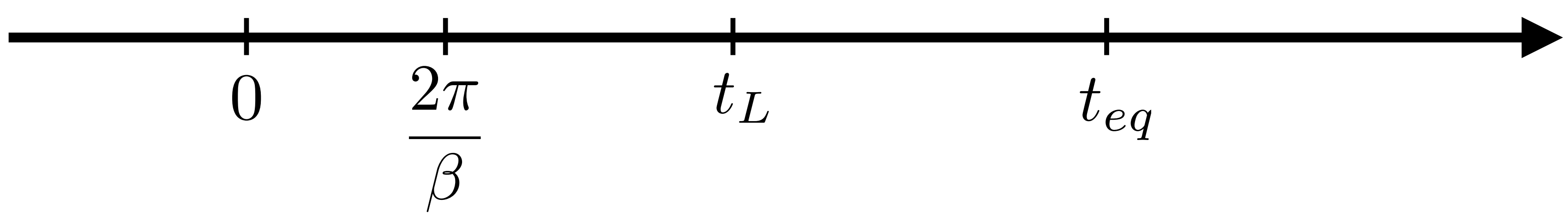


- Grozdanov et. al., 1605.02173. Diffusivity increases, but is compensated by the quasinormal mode.
- Indeed, the essence of our bound is that D can never be much much larger than $v^2 t_{eq}$

Linear axion spacetimes

- Translation invariance is broken at energy scale m (Mike Blake 1604.01754)

- When m/T is small, $D/t_{eq} \sim 1$
  This shows that one cannot use the Lyapunov time in our bound, since $D/t_L \gg 1$ in this case



- When m/T is large, $D/t_{eq} \sim T/m \ll 1$
  Our bound is obeyed but is far from saturation

Can we do better?

- Can we do better in cases where the bound is far from saturation? Possibly, but we don’t know yet.

- The issue is that we are using a microscopic, UV velocity to be absolutely safe, but thermal velocities are much smaller. Can we use the thermal velocity?

- The *holographic* “butterfly” speed is of order the thermal velocity. But its current definition is problematic in the causality context, since sound speed is larger than this butterfly speed.

Weakly coupled gases having quasiparticles

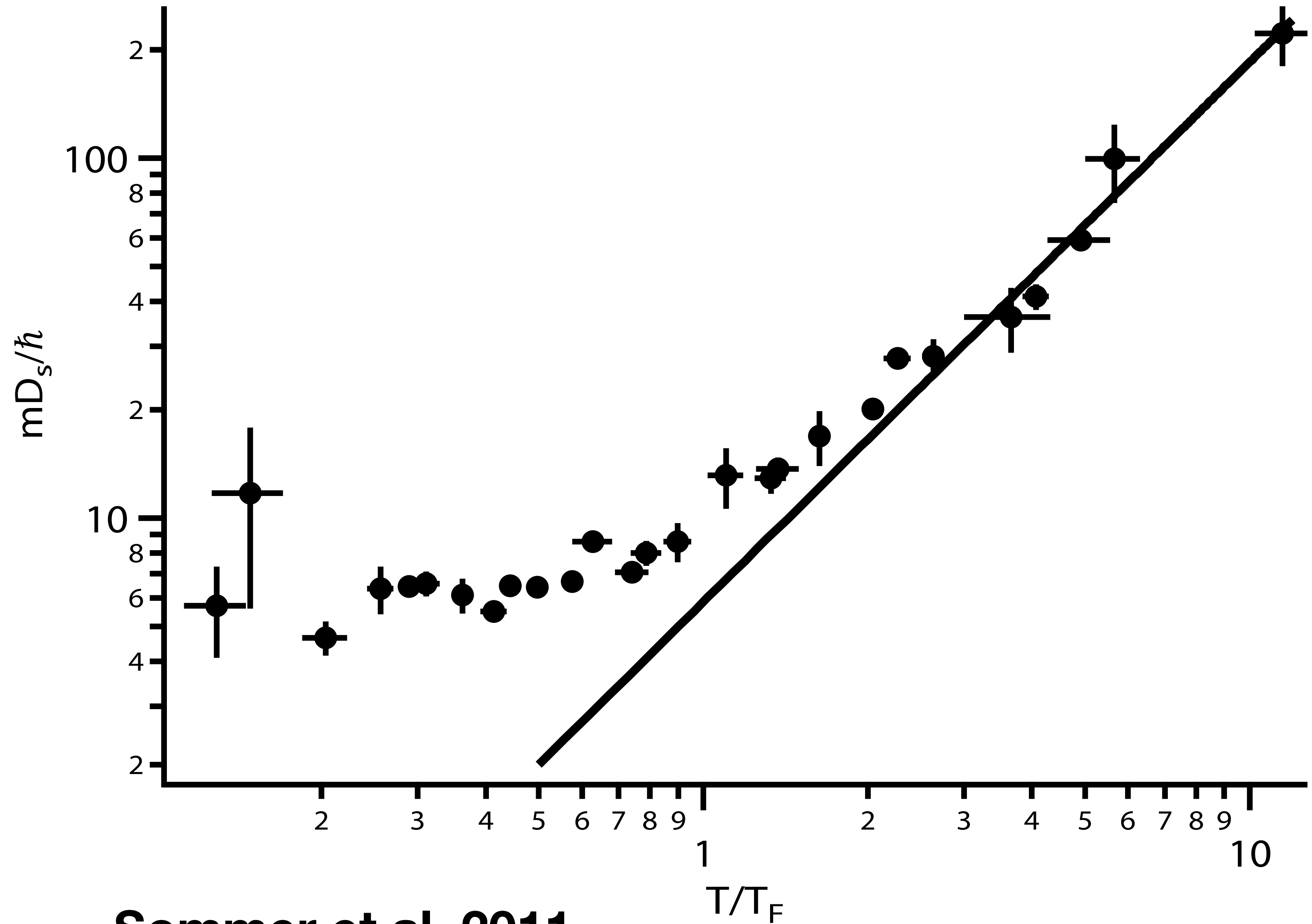
- One expects from Boltzmann equation that $D \sim v_{\rm qp}^2 t_{\rm mf}$
- Also, it is clear that $v_{\rm qp} \lesssim v$ and $t_{\rm mf} \lesssim t_{eq}$
- Thus, our bound is obeyed and is close to saturation

Strongly coupled systems

- Cold atomic gases at unitarity
- Bad metals violating the Mott-Ioffe-Regal bound
- Both have $\ell_{\mathrm{mf}} \sim a \sim \dfrac{1}{k_F}$
- Hence quasiparticles are not well defined
- *Yet much of the transport data obeys naive single particle formulas* for diffusion and resistivity after you properly calculate single particle parameters from data

Spin Diffusion

Fermionic Lithium-6 atoms at unitarity

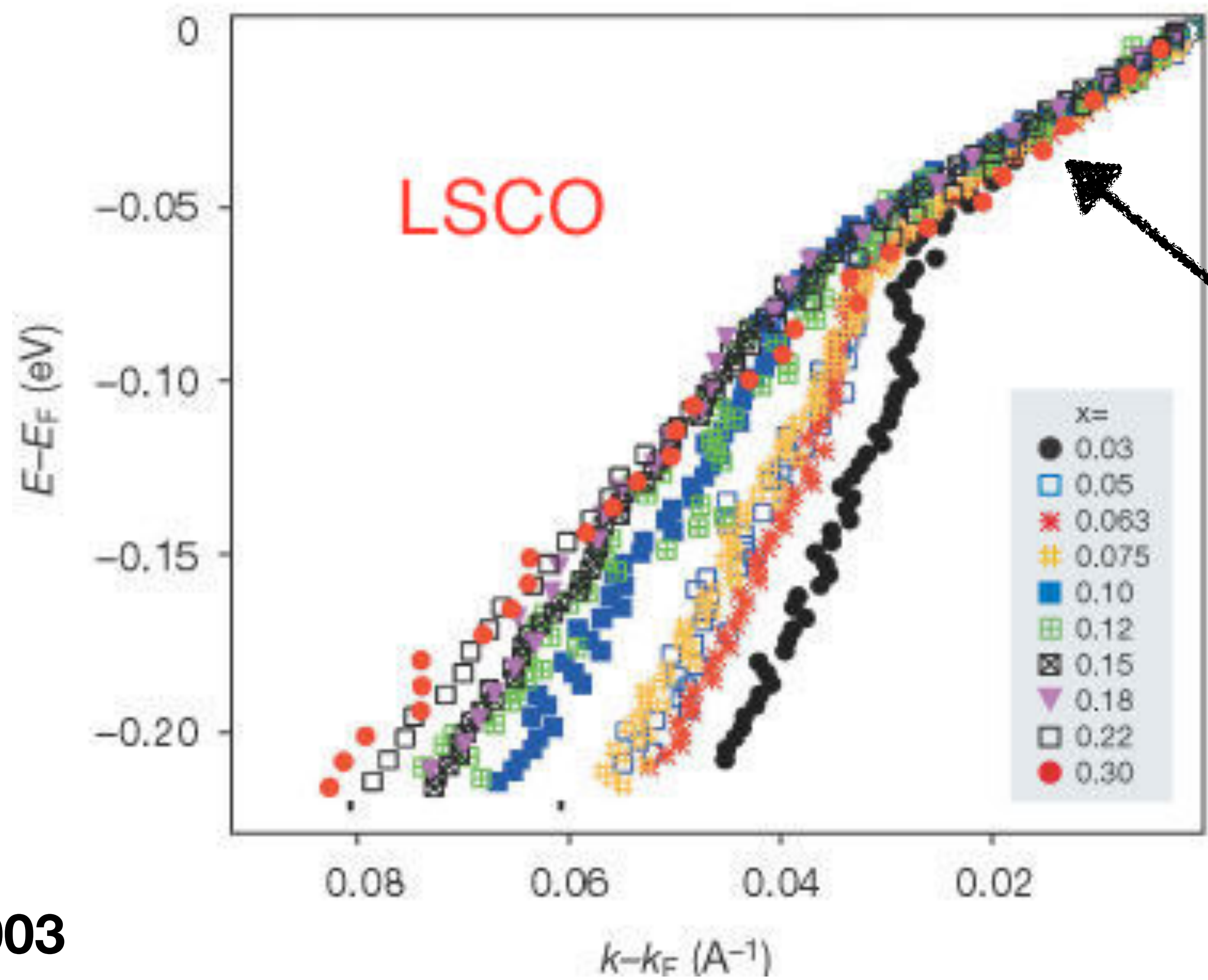


$$D \sim v_F^2 \tau_{\mathrm{mf}}$$
$$\sim v_F \ell_{\mathrm{mf}}$$
$$\sim v_F / k_F$$
$$\sim 1/m$$

Sommer et al, 2011

Fermi velocity in cuprates

Can be deduced from ARPES measurements
Set by microscopic couplings, hence similar order as Lieb-Robinson velocity



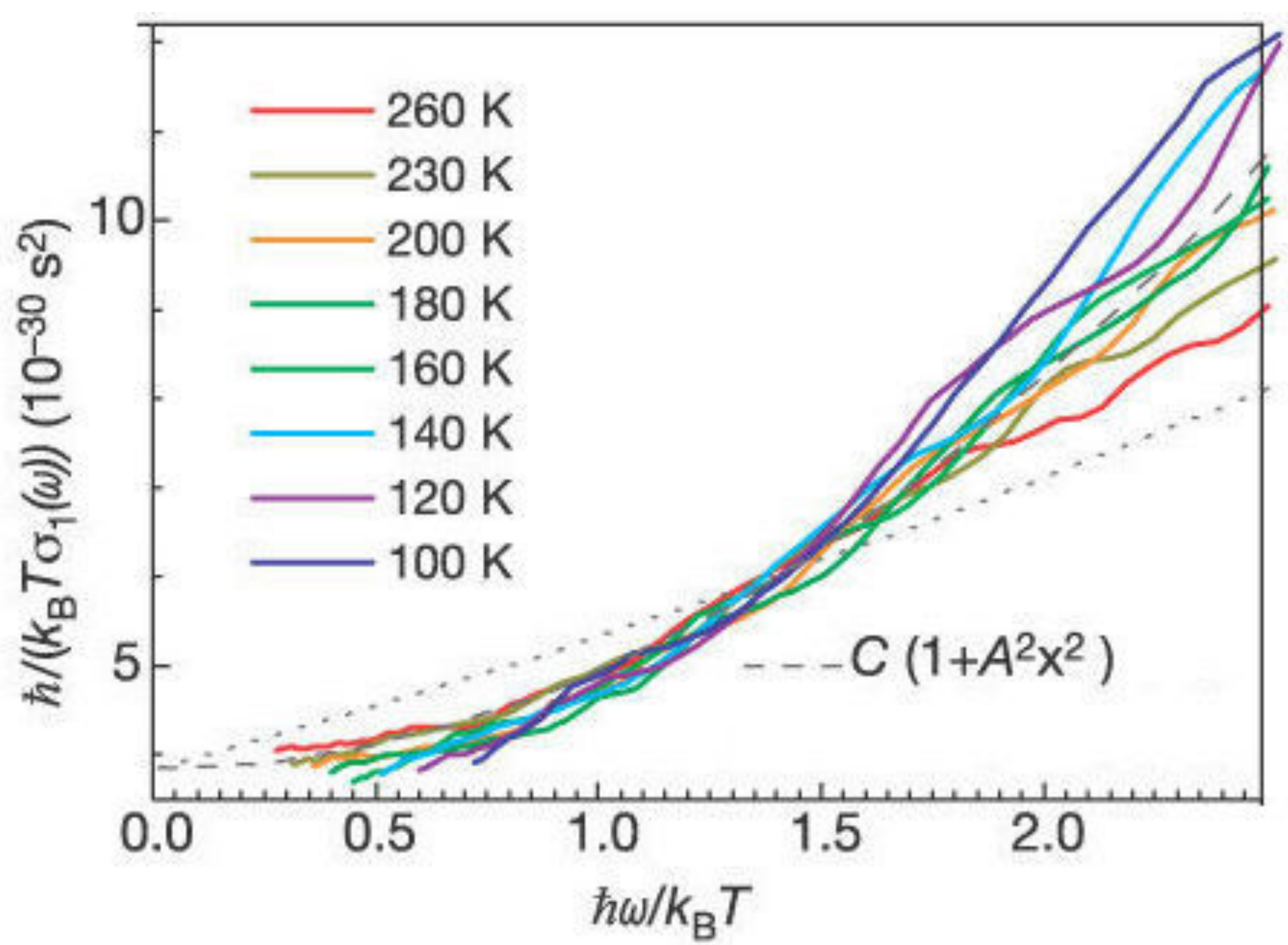
$$v_F = \frac{\partial \epsilon}{\partial k}$$

Zhou et al, 2003

Scattering rates in cuprates

$$\mathrm{Re}\ \sigma(\omega) = \frac{\alpha}{1 + \beta(\omega/T)^2}$$

Optical conductivity fits Drude form with width of order T

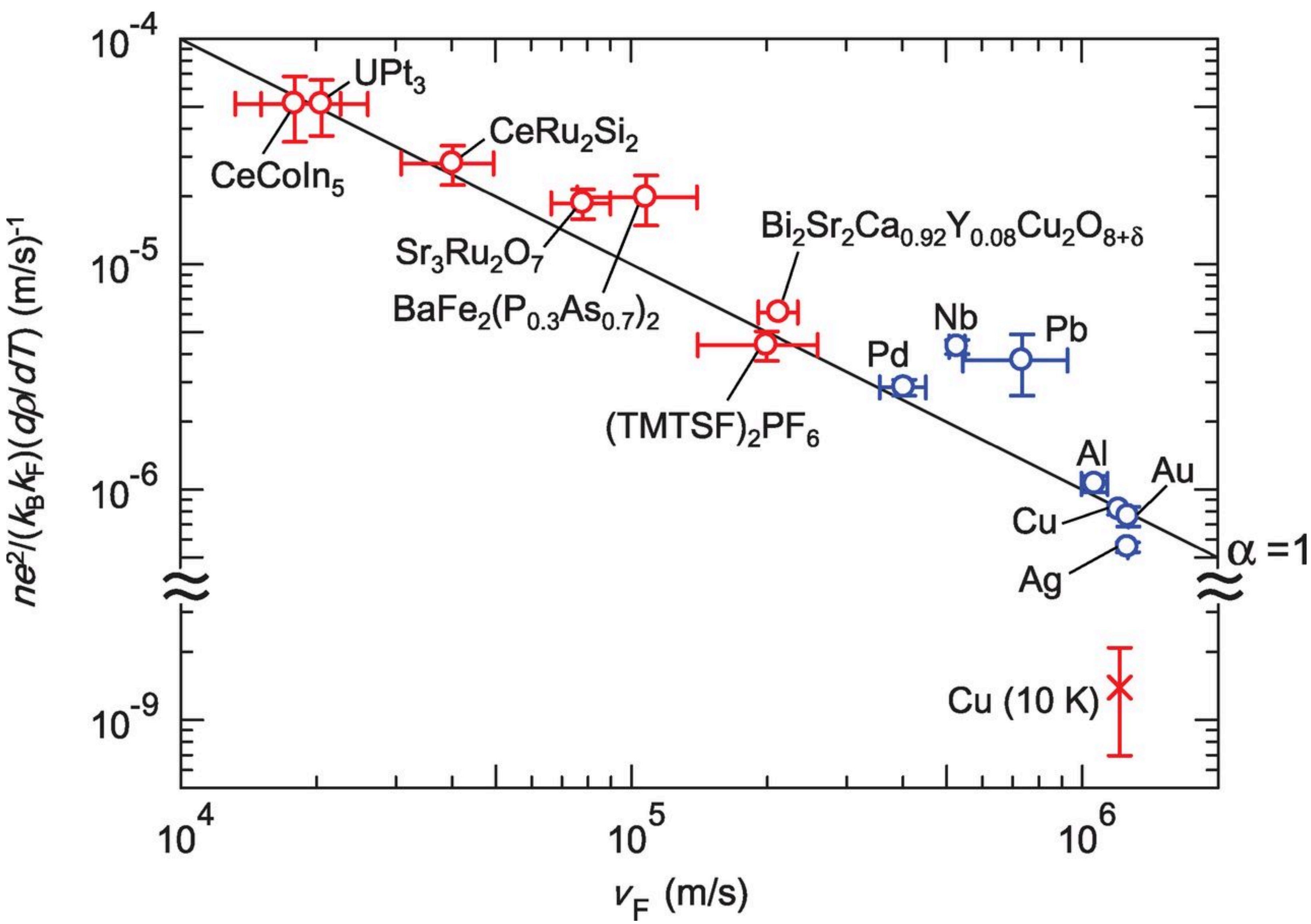


van der Marel et al, 2003

Scattering rates in T-linear materials

$$\frac{\rho}{T} = \frac{k_f/v_F}{ne^2}\frac{1}{\tau T}$$

Drude formula for DC conductivity



- Copper, gold and silver (at room temperatures) are also on the same plot!!
- Do we understand e-ph coupling as well as we think we do?

Bruin et al, 2013

Unreasonable effectiveness of single particle formulas

Why are the single particle Drude type formulas so effective? Theory needs to explain this.

- Maybe the way out is that there is a bound. What else could make Copper and Gold similar to Strontium Ruthenate and other fancy materials?

- Our bound is a step forward, but incomplete. Need to understand under what conditions is the bound far from saturation, and when is it almost saturated.

- Maybe there is also a lower bound? As has been conjectured in the literature starting from KSS.

Conclusion: UV Constraints on hydrodynamics

- Consistency with UV physics places constraints on IR effective field theories.
- What are such constraints for hydrodynamics?
- Effective actions for hydrodynamics, and corrections thereof might be very interesting in this regard (Loganayagam, Rangamani, Liu and collaborators).