Elliptic Integrable Systems, Special Functions and Quantum Field Theory

Nordita, Stockholm, 16-20 June 2019
All lectures are in room FB42, AlbaNova, Roslagstullsbacken 21

Sunday 16 June

09:30–10:30	registration, coffee
10:30–11:15	**Vyacheslav Spiridonov:** The endless beta integrals
11:25–12:10	**Oleg Lisovyy:** Painlevé functions, Fredholm determinants and combinatorics
12:10–14:00	lunch
14:00–14:30	**Arash Ardehali:** Elliptic hypergeometric integrals and black holes
14:30–15:00	**Rebecca Lodin:** Solving q-Virasoro constraints
15:00–15:30	coffee
15:30–16:00	**Jules Lamers:** The q-deformed Haldane–Shastry spin chain: from the affine Hecke algebra, via 'freezing', to exact eigenvectors
16:00–16:30	**Rob Klabbers:** Rationalizing Inozemtsev's elliptic spin chain with the Legendre relation

Monday 17 June

09:00–09:45	**Rinat Kashaev:** Quantum dilogarithms over LCA groups
09:45–10:15	coffee
10:15–11:00	**Nobutaka Nakazono:** Consistency around a cuboctahedron
11:10–11:40	**Andrew Kels:** Discrete integrability from hypergeometric integrals
11:40–12:10	**Ilmar Gahramanov:** The Bailey lemma and 3d mirror symmetry
12:10–14:00	lunch
14:00–14:30	**Yury Neretin:** An analog of the Dougall formula and of the de Branges–Wilson integral
14:30–15:30	coffee and poster session **Masayuki Fukuda:** Macdonald functions from topological vertex and its elliptic analogue **Daria Rudneva:** Integrable systems and BKP-equations **Paul Ryan:** Separated variables + factorized wave functions in gl(n) spin chains **Matteo Sacchi:** From 3d dualities to 2d free field correlators and back
15:30–16:00	**Masahiko Ito:** Elliptic extension of Gustafson's q-integral of type G_2
16:00–16:30	**Ole Warnaar:** On some theta-function conjectures

Tuesday 18 June

09:00–09:45 **Simon Ruijsenaars:** Painlevé–Calogero correspondence: The elliptic 8-parameter level
09:45–10:15 coffee
10:15–11:00 **Eric Rains:** Elliptic double affine Hecke algebras
11:10–11:40 **Carlos Arreche:** Differential transcendence of elliptic hypergeometric functions through Galois theory
11:40–12:10 **Shamil Shakirov:** Double elliptic generalization of Ruijsenaars-Schneider model
12:10–14:00 lunch
14:00–14:30 **Hitoshi Konno:** Elliptic quantum (toroidal) algebras and their representations
14:30–15:00 **Kohei Motegi:** Izergin–Korepin method to elliptic identities
15:00–15:30 coffee
15:30–16:00 **Farrokh Atai:** Source identities for relativistic models of Calogero–Moser–Sutherland type
16:00–16:30 **Bjorn Berntson:** Second quantization of elliptic Calogero–Sutherland models

Wednesday 19 June

09:00–09:45 **Oleg Chalykh:** From Dunkl and Cherednik operators to Lax pairs
09:45–10:15 coffee
10:15–11:00 **Junichi Shiraishi:** Affine screening operators, affine Laumon spaces, and conjectures concerning non-stationary Ruijsenaars functions
11:10–11:40 **Masatoshi Noumi:** Formal eigenfunctions for the elliptic Ruijsenaars difference operators
11:40–12:10 **Edwin Langmann:** Perturbative solution of the elliptic Ruijsenaars model
12:10–14:00 lunch
14:00–14:30 **Makoto Katori:** Two-dimensional elliptic determinantal point processes and related systems
14:30–15:00 **Martin Hallnäs:** Soliton scattering in the hyperbolic relativistic Calogero–Moser system
15:00–15:30 coffee
18:30–22:00 Boat tour and conference dinner.

Thursday 20 June

09:00–09:45	**Marta Mazzocco:** Quantum uniformisation and Calabi–Yau algebras
09:45–10:15	coffee
10:15–11:00	**Alexander Varchenko:** Dynamical elliptic Bethe algebra, KZB eigenfunctions, and theta-polynomials
11:10–11:40	**Michael Schlosser:** Elliptic Stirling and Lah numbers
11:40–12:10	**Takashi Takebe:** Q-operators for higher spin eight vertex models with a rational anisotropy parameter
12:10–14:00	lunch
14:00–14:30	**Linnea Hietala:** A combinatorial description of certain polynomials related to the XYZ spin chain
14:30–15:00	**Hjalmar Rosengren:** Correlations for the XYZ spin chain and Painlevé VI
15:00–15:30	coffee
15:30–16:15	open problem session, concluding discussion

Abstracts

Arash Ardehali (Uppsala University)
Elliptic hypergeometric integrals and black holes
Elliptic Hypergeometric Integrals (EHIs) arise in physics roughly as generating functions of the number of states in certain quantum field theories. For those quantum field theories that holographically describe a theory of gravitation, the EHI is then expected to encode the states of the gravitational theory as well. Of particular interest among the states of a gravitational theory are the Black Hole states. We explain how recent investigations of asymptotics of certain EHIs have succeeded in holographically reproducing the number of states (and hence the entropy) of the corresponding Black Holes (computed previously from General Relativity), thus addressing an outstanding 13-year-old problem in mathematical physics.

Carlos Arreche (University of Texas at Dallas)
Differential transcendence of elliptic hypergeometric functions through Galois theory
Elliptic hypergeometric functions arose roughly 10 years ago as a generalization of classical hypergeometric functions and q-hypergeometric functions. These special functions enjoy remarkable symmetry properties, like their more classical counterparts, and find applications in representation theory and mathematical physics. After interpreting one of these symmetries as a linear difference equation over an elliptic curve, we apply differential Galois theory of difference equations to show that these functions are always differentially transcendental for "generic" values of the parameters. This is joint work with **Thomas Dreyfus** and **Julien Roques**.

Farrokh Atai (Kobe University)
Source identities for relativistic models of Calogero–Moser–Sutherland type
Kernel functions provide a powerful tool for studying the relativistic and non-relativistic models of Calogero–Moser–Sutherland (CMS) type, including elliptic models. A simple method for finding kernel functions systematically is by using so-called *source identities*. In this talk, I will present the source identities for the relativistic CMS models and show how this method allows us to find such kernel functions.

Bjorn Berntson (KTH Royal Institute of Technology, Stockholm)
Second quantization of elliptic Calogero-Sutherland models
We construct a second quantization of a shifted elliptic Calogero–Sutherland model defined by

$$H^{M,N}(x,y,p,q) = H^M(x,p) + H^N(y,q) + g(g-1)\sum_{j=1}^{M}\sum_{k=1}^{N}\wp(x_j - y_k + \omega_2),$$

constructed from the basic elliptic Calogero–Sutherland Hamiltonian

$$H^M(x,p) = \frac{1}{2}\sum_{j=1}^{M} p_j^2 + g(g-1)\sum_{j=1}^{M}\sum_{k=1,k\neq j}^{M} \wp(x_j - x_k),$$

where ω_2 is an imaginary half-period of the $\wp$-function, so the x and y particles interact through a non-singular potential. A second quantization of the basic elliptic Calogero–Sutherland model using an anyonic quantum field theory was presented by Langmann (Comm. Math. Phys. 2004); we extend this result to the shifted system. The equations of motion for (the classical limit of) the collective field theory Hamiltonian of the shifted elliptic Calogero–Sutherland model form a new two-component, elliptic Benjamin–Ono-type equation. We construct a Lax pair for this equation via a Riemann–Hilbert problem on the torus. This is joint work with **Edwin Langmann** and **Jonatan Lenells**.

Oleg Chalykh (University of Leeds)
From Dunkl and Cherednik operators to Lax pairs
We present a direct conceptual link between elliptic Dunkl operators and Lax pairs for the elliptic Calogero–Moser model. It works both for the classical and quantum models and for all root systems, including the BC_n-case with 5 couplings (Inozemtsev system). A similar method can be applied to the elliptic Ruijsenaars model and its generalisations, where very little was known beyond the A_n-case. In particular, this allows us to calculate a Lax matrix for the van Diejen system with full 9 couplings, which was an old open problem.

Masayuki Fukuda (University of Tokyo)
Macdonald functions from topological vertex and its elliptic analogue
We can obtain Macdonald functions as matrix elements of some compositions of the intertwining operators of Ding–Iohara–Miki algebra. By taking the trace of the intertwining operators, we obtain the "elliptic analogue" of the Macdonald functions. Although this is not the good elliptic deformation of the Macdonald functions, this function agree with the special case of the affine Macdonald functions, which is recently introduced by Shiraishi (2019).

Ilmar Gahramanov (Mimar Sinan University, Istanbul)
The Bailey lemma and 3d mirror symmetry
In this talk, we discuss the relationship between the integral Bailey lemma and mirror symmetry for three-dimensional supersymmetric gauge theories. Three-dimensional mirror symmetry relates the superconformal infrared fixed points of a certain class of quiver gauge theories. The simplest example of such a duality is the equivalence of 3d $N = 2$ supersymmetric quantum electrodynamics and the theory of three chiral multiplets X,Y, Z with a superpotential $W = XYZ$. One can check these dualities by computing the superconformal indices of dual theories. It happens that in some cases, starting with an index identity for a certain mirror duality one can construct a family of duality via the integral analog of the Bailey chains technique.

Martin Hallnäs (Chalmers University of Technology and University of Gothenburg)
Soliton scattering in the hyperbolic relativistic Calogero–Moser system

Integrable N-particle systems of relativistic Calogero–Moser type were first introduced by Ruijsenaars and Schneider (1986) in the classical and Ruijsenaars (1987) in the quantum case. In the hyperbolic regime they are closely related to several soliton equations, in particular the sine-Gordon equation.

In this talk, I will focus on the quantum case and present a proof of the long-standing conjecture that the particles in the relativistic Calogero–Moser system of hyperbolic type exhibit soliton scattering, i.e. conservation of momenta and factorization of scattering amplitudes. The talk is based on joint work with **Simon Ruijsenaars**.

Linnea Hietala (Chalmers University of Technology and University of Gothenburg)
A combinatorial description of certain polynomials related to the XYZ spin chain

Bazhanov and Mangazeev studied certain polynomials $q_n(z)$ which appear in the eigenvectors of the Hamiltonian of the XYZ spin chain. These polynomials seem to have positive integer coefficients, which suggests that there should be a combinatorial interpretation. By studying the bijection between the alternating sign matrices and the states of the six-vertex model with domain wall boundary conditions (DWBC), Kuperberg proved the alternating sign matrix conjecture of Mills, Robins and Rumsey. In this talk, we specialize the parameters in the partition function of the eight-vertex solid-on-solid model with DWBC and one reflecting end in Kuperberg's way, which yields an explicit combinatorial expression for $q_n(z)$ in terms of the partition function of the three-color model with the same boundary conditions.

Masahiko Ito (University of the Ryukyus)
Elliptic extension of Gustafson's q-integral of type G_2

I will present an evaluation formula for an elliptic beta integral of type G_2. The integral is expressed by a product of Ruijsenaars' elliptic gamma functions, and the formula includes that of Gustafson's q-beta integral of type G_2 as a special limiting case as $p \to 0$. The elliptic beta integral of type BC_1 by van Diejen and Spiridonov is effectively used in the proof of the evaluation formula. This is a joint work with **Masatoshi Noumi**.

Rinat Kashaev (University of Geneva)
Quantum dilogarithms over LCA groups

The notion of a quantum dilogarithm over a (Pontryagin self dual) locally compact abelian group, originating from quantum Teichmüller theory, admits various examples of realization at it leads to applications in quantum topology and integrable systems. I will describe those applications and some of the known examples of realization.

Makoto Katori (Chuo University, Tokyo)
Two-dimensional elliptic determinantal point processes and related systems

First I will explain a general framework of determinantal point processes (DPPs) based on partial isometries between a pair of Hilbert spaces. Then as examples of DPPs con-

structed by this method, seven kinds of DPPs on a torus are introduced using the R_N-theta functions studied by Rosengren and Schlosser (2006) for the seven irreducible reduced affine root systems. In the bulk scaling limit, they are degenerated into the three types of Ginibre DPPs on a complex plane with an infinite number of points. The present talk is based on the paper to be published in Commun. Math. Phys. (https://doi.org/10.1007/s00220-019-03351-5) and on a joint work with **Tomoyuki Shirai** (https://arxiv.org/abs/1903.04945).

Andrew Kels (SISSA, Trieste)
Discrete integrability from hypergeometric integrals

I will talk about a connection between hypergeometric integrals and two different types of integrable equations. The first type of integrable equation is the star-triangle relation (Yang–Baxter equation) for integrable Ising-type models of statistical mechanics, and the second types are the discrete soliton equations that satisfy an integrability condition known as 3D-consistency (or multi-dimensional consistency/consistency-around-a-cube). This connection relates hypergeometric integrals which can be put in a star-triangle form, to 3D-consistent equations which arise in a certain asymptotic limit. This has led to the discovery of new types of integrable equations, as well as new types of hypergeometric integral formulas. I will present some examples, such as a sum/integral extension of Spiridonov's elliptic beta integral formula, and new multi-component 3D-consistent equations.

Rob Klabbers (Humboldt Universität zu Berlin)
Rationalising Inozemtsev's elliptic spin chain with the Legendre relation

In the world of elliptic integrability Inozemtsev's spin chain is quite a strange bird: this model of long-range-interacting electrons with elliptic interactions links the Heisenberg XXX spin chain, the Haldane–Shastry spin chain and the elliptic CSM model and is exactly solvable, but whether it is integrable remains an open question.

In its current form the solution of this elliptic spin chain looks complicated and does not promote further investigation. In an effort to change this, I will present a reparametrisation of this solution in the form of an extended Bethe ansatz that simplifies things considerably: most importantly, I will show that the M-particle energy can be written in (quasi-)additive form and the Bethe ansatz equations can be written in a familiar form. Moreover, written in the right variables the spectral problem becomes fully rational.

Hitoshi Konno (Tokyo University of Marine Science and Technology)
Elliptic quantum (toroidal) algebras and their representations

Reviewing previous works on a geometric action of $U_{q,p}(\widehat{sl}_N)$ on equivariant elliptic cohomology, we discuss a triality among the weight function, the stable envelope and the Hopf algebroid twistor. We then discuss a construction of the level-$(0,1)$ representations of the elliptic quantum toroidal algebras $U_{q,p}(g_{tor})$ for $g = gl_1$, gl_N on semi-infinite tensor product spaces (the q-Fock spaces) and make some conjectures on their geometric interpretations and corresponding elliptic cohomologies.

Jules Lamers (University of Melbourne)

The q-deformed Haldane–Shastry spin chain: from the affine Hecke algebra, via 'freezing', to exact eigenvectors

The Haldane–Shastry model is an exactly-solvable long-range spin chain whose remarkable properties include Yangian symmetry already at finite system size. Its q-deformation was found by D. Uglov in '95 in an e-print that seems to have gone by unnoticed. Last year I managed to express Uglov's Hamiltonian in a more friendly, pairwise form.

After introducing the model I will indicate how it is obtained by 'freezing' the spin generalisation of the Ruijsenaars–Schneider model (Macdonald operators). The generalised model enjoys quantum-affine symmetries that, by construction, are inherited by the spin chain upon freezing.

Our main new result is an exact closed-form expression for the (highest-weight) eigenvectors at finite size. It has a simple component that involves the symmetric square of the q-Vandermonde times a Macdonald polynomial (q-deformed zonal polynomial). This is ongoing work together with **Vincent Pasquier** and **Didina Serban**.

Edwin Langmann (KTH Royal Institute of Technology, Stockholm)

Perturbative solution of the elliptic Ruijsenaars model

Shiraishi recently presented remarkable explicit functions which, as he conjectured, provide an explicit solution of the elliptic Ruijsenaars model. In my talk I present an explicit perturbative solution of the elliptic Ruijsenaars model which allows to check Shiraishi's conjecture. I also plan to discuss related results for a non-stationary variant of the elliptic Calogero–Sutherland model. (Based on ongoing work with **Masatoshi Noumi** and **Junichi Shiraishi**.)

Oleg Lisovyy (LMPT, Tours)

Painlevé functions, Fredholm determinants and combinatorics

I will explain how to associate a tau function to the Riemann–Hilbert problem set on a union of non-intersecting smooth closed curves with generic jump matrix. The main focus will be on the one-circle case, relevant to the analysis of Painlevé VI equation and its degenerations to Painlevé V and III. The tau functions in question will be defined as block Fredholm determinants of integral operators with integrable kernels. They can be alternatively represented as combinatorial sums over tuples of Young diagrams which coincide with the dual Nekrasov–Okounkov instanton partition functions for Riemann–Hilbert problems of isomonodromic origin.

Rebecca Lodin (Uppsala University)
Solving q-Virasoro constraints
The Virasoro constraints - arising from Ward identities - are a key component in understanding the relation between matrix models and conformal field theories; they provide the set of equations constraining the generating function which can then be solved using CFT methods. These Virasoro constraints can be derived either using differential operators or by using the so-called free field representation of the Virasoro algebra. In this talk I will discuss what happens when these constraints are q-deformed. In particular, I will outline how such q-Virasoro constraints can be derived for a large class of deformed eigenvalue matrix models by an elementary trick of inserting certain q-difference operators under the integral. These q-Virasoro constraints can then be solved recursively and they also have applications for gauge theories.

Marta Mazzocco (University of Birmingham)
Quantum uniformisation and Calabi–Yau algebras
In this talk I will explain a few facts about quantum del Pezzo surfaces belonging to a certain class. In particular I will introduce the generalised Sklyanin–Painlevé algebra that contains as limiting cases the generalised Sklyanin algebra, Etingof–Ginzburg and Etingof–Oblomkov–Rains quantum del Pezzo and the quantum monodromy manifolds of the Painlevé equations.

Kohei Motegi (Tokyo University of Marine Science and Technology)
Izergin–Korepin method to elliptic identities
We give an example of studying a partition function of an elliptic integrable model, with an application to an elliptic identity. We analyze a variation of the domain wall boundary partition functions (OS boundary) based on the elliptic version of the Izergin–Korepin method. We find two expressions using elliptic Pfaffians, based on two different versions of Korepin's method. As a consequence, we find an identity between two different elliptic Pfaffians.

Nobutaka Nakazono (Tokyo University of Agriculture and Technology)
Consistency around a cuboctahedron
In the theory of discrete integrable systems, the classification of lattice equations using the consistency around a cube (CAC) property [Adler–Bobenko–Suris 2003&2009, Boll 2011] are well known. The resulting equations are collectively called ABS equations. In this talk, we show new classification of lattice equations using a consistency around a cuboctahedron (CACO) property.

Yury Neretin (ITEP, Moscow and University of Vienna)
An analog of the Dougall formula and of the de Branges–Wilson integral
We derive a beta-integral over $\mathbb{Z} \times \mathbb{R}$ including ${}_{10}H_{10}$-summation. Our tool is a counterpart of the Jacobi transform related to representations of the Lorentz group.

Masatoshi Noumi (Kobe University)

Formal eigenfunctions for the elliptic Ruijsenaars difference operators

I will give some remarks on the existence of formal eigenfunctions of the elliptic Ruijsenaars difference operators which are p-deformations of Macdonald polynomials and asymptotically free eigenfunctions of the trigonometric cases.

Eric Rains (Caltech, Pasadena)

Elliptic double affine Hecke algebras

One of the primary tools used in the study of Macdonald and Koornwinder polynomials has been the "double affine Hecke algebra", but despite the existence of elliptic analogues of Koornwinder polynomials, the DAHA perspective has lacked an elliptic version. I'll explain what the elliptic DAHA looks like, and some of its implications, which in addition to some new results on biorthogonal functions includes a new integrability proof for van Diejen's BC_n-symmetric Hamiltonian and a (conjectural, but now proved by Chalykh) generalization to a family of higher-order Hamiltonians.

Hjalmar Rosengren (Chalmers University of Technology and University of Gothenburg)

Correlations for the XYZ spin chain and Painlevé VI

We compute nearest neighbour correlations for the XYZ spin chain at the supersymmetric or combinatorial line $J_xJ_y + J_xJ_z + J_yJ_z = 0$. They can be expressed in terms of the polynomials s_n and $\bar{s}_n$ introduced by Bazhanov and Mangazeev, which are known to be tau functions of Painlevé VI. Curiously, the correlations can be obtained by evaluating one Painlevé VI Hamiltonian at the solution of another instance of Painlevé VI, with shifted parameter values. This is joint work with **Christian Hagendorf**.

Daria Rudneva (NRU HSE, Moscow)

Integrable systems and BKP-equations

We derive equations of motion for poles of elliptic solutions to the BKP equation. The basic tool is the auxiliary linear problem for the Baker-Akhiezer function. We also discuss integrals of motion for the pole dynamics which follow from the equation of the spectral curve.

Simon Ruijsenaars (University of Leeds)

Painlevé–Calogero correspondence: The elliptic 8-parameter level

The 8-parameter elliptic Sakai difference Painlevé equation admits a Lax pair formulation. We sketch how a suitable specialization of one of the Lax equations gives rise to the time-independent Schrödinger equation for the BC_1 8-parameter relativistic Calogero-Moser Hamiltonian due to van Diejen. This amounts to a generalization of previous results concerning the Painlevé–Calogero correspondence to the highest level of the two hierarchies. This talk is based on joint work with **Masatoshi Noumi** and **Yasuhiko Yamada**.

Paul Ryan (Trinity College, Dublin)
Separated Variables + Factorised Wave Functions in gl(n) *spin chains*
We propose a basis for rational gl(n) spin chains in an arbitrary rectangular representation (S^A) that factorises the Bethe vectors into products of Slater determinants in Baxter Q-functions. This basis is constructed by repeated action of fused transfer matrices on a suitable reference state. We show that it diagonalises the so-called B-operator, hence the operatorial roots of the latter are the separated variables. The spectrum of the separated variables is also explicitly computed and it turns out to be labelled by Gelfand–Tsetlin patterns, providing a direct link between SoV and Yangian representation theory.

Matteo Sacchi (University of Milano–Bicocca)
From 3d dualities to 2d free field correlators and back
Supersymmetric localization allowed for a new prolific connection between Physics and Mathematics. Thanks to it, we can reduce partition functions of supersymmetric quantum field theories to ordinary matrix integrals of some special functions and dualities can be translated into non-trivial integral identities. In particular, 3d dualities imply integral identities involving hyperbolic or hypergeometric functions, depending on the geometry of the background. We discuss how a particular 2d limit of some known Seiberg-like 3d dualities on $S^2 \times S^1$ yields identities for complex integral, also known as Dotsenko–Fateev (DF) integrals, that admit a physical interpretation in terms of correlators of 2d CFTs in the free field realization. Then, we reverse the process and find new 3d dualities and hyperbolic integral identities by uplifting some identities for DF integrals that were already known in the CFT literature. Based on `arXiv:1903.10817` and `arXiv:1905.05807`, the project is supported by the ERC-STG grant 637844-HBQFTNCER.

Michael J. Schlosser (University of Vienna)
Elliptic Stirling and Lah numbers
We present elliptic generalizations of the Stirling numbers of the second and first kind. These extend the corresponding q-Stirling numbers considered by Carlitz in 1933 and 1948, and (using a slightly different normalization) by Gould in 1961, by two extra independent parameters a, b and a nome p. Analogous to the classical case, the elliptic Stirling numbers of the two kinds are doubly indexed integer sequences being inverses of each other, they appear as coefficients connecting powers of elliptic numbers with elliptic shifted factorials and happen to satisfy nice recursions. The elliptic Stirling numbers of the second kind and the unsigned elliptic Stirling numbers of the first kind can be given easy combinatorial interpretations involving elliptic weight functions. Convolution of elliptic Stirling numbers of the two different kinds give rise to elliptic Lah numbers with properties analogous to the classical case. This is based on joint work with **Zsófia R. Kereskényiné Balogh.**

Shamil Shakirov (Uppsala University)

Double elliptic generalization of Ruijsenaars–Schneider model

Junichi Shiraishi (University of Tokyo)

Affine screening operators, affine Laumon spaces, and conjectures concerning non-stationary Ruijsenaars functions

Vyacheslav P. Spiridonov (BLTP JINR, Dubna and NRU HSE, Moscow)

The endless beta integrals

Euler's beta integral is a germ for a long series of univariate exactly computable integrals of hypergeometric type. A top known level is described by the elliptic beta integral yielding a kind of universal formula working even beyond the original definition using the standard elliptic gamma function. I will give a survey of old and new results around this universal formula together with a brief description of its applications to superconformal indices of 4d and 3d quantum field theories and 2d statistical mechanics models.

Takashi Takebe (NRU HSE, Moscow)

Q-operators for higher spin eight vertex models with a rational anisotropy parameter

A Q-operator is a linear operator first introduced by Baxter to solve the eight vertex model and recently attracts attention from representation theoretical viewpoint. Baxter's constructions of his Q-operators, which fully exploit the structure of the elliptic R-matrix, seem to be quite technical, but it turned out that they can be generalised to higher spin cases, which is an evidence of existence of mathematical background of the constructions. Two years ago at this conference I reported generalisation of Baxter's second construction. This time, I shall report generalisation of Baxter's very first construction and Fabricius's construction.

Alexander Varchenko (University of North Carolina at Chapel Hill)

Dynamical elliptic Bethe algebra, KZB eigenfunctions, and theta-polynomials

The dynamical elliptic Bethe algebra is a commutative algebra of differential operators acting on vector-valued functions on the Cartan subalgebra of a Lie algebra $\mathfrak{sl}_N$. The algebra is generated by values of the coefficient of a certain differential operator, defined by V. Rubtsov, A. Silantyev, D. Talalaev in 2009. We study the eigenfunctions of the dynamical elliptic Bethe algebra for $\mathfrak{sl}_2$ by the Bethe ansatz method. We show that such Bethe eigenfunctions are in one-to-one correspondence with ordered pairs of theta-polynomials of certain degree.

Ole Warnaar (Univesity of Queensland, Brisbane)

On some theta-function conjectures

We will discuss some mysterious identities for Jacobi theta functions that have arisen in our recent joint work with **Adam Walsh** on modular analogues of the Nekrasov–Okounkov formula.