SQCD meets SUGRA at Two Loops

work with H. Johansson & G. Mogull [1706.09381]
and G. Mogull & A. Ochirov [1811.09604]

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QCD meets Gravity

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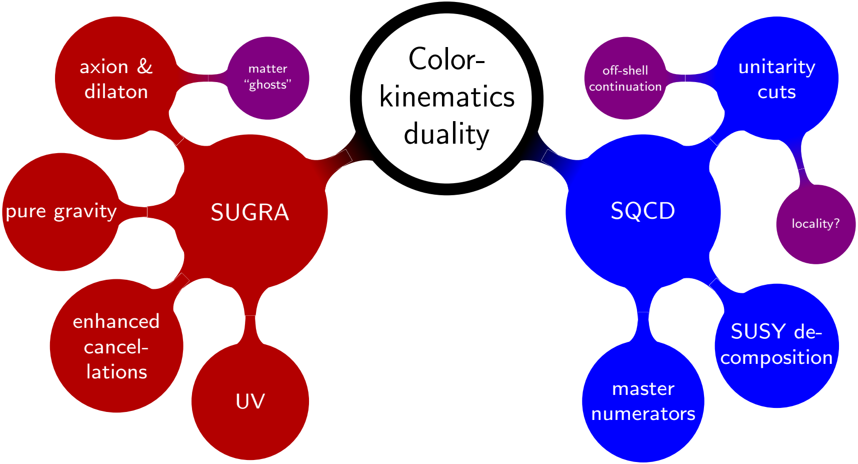
Motivation


Color-kinematics duality
SUGRA
axion & dilaton
matter "ghosts"
pure gravity
enhanced cancel-lations
UV
SQCD
unitarity cuts
off-shell continuation
locality?
SUSY de-composition
master numerators

Plan

- Factorizability Problem: Axion & Dilaton
- Color-Kinematics Duality with Matter
- Non-Maximal Pure Supergravities
- (S)QCD Amplitude Constructions
- What's next?

Problem: Axion & Dilaton

Double copy of 4D pure YM on-shell states:

$$A^{\pm} \otimes A^{\pm} = h^{++} \oplus h^{--} \oplus \underbrace{\phi \oplus a}_{\text{Axion \& Dilaton}}$$

In general for $(\mathcal{N} \leq 2 \text{ SYM}) \times (\mathcal{M} \leq 2 \text{ SYM})$:

$$\mathcal{V}_{\mathcal{N}} \otimes \mathcal{V}'_{\mathcal{M}} \equiv \mathcal{H}_{\mathcal{N}+\mathcal{M}} \oplus X_{\mathcal{N}+\mathcal{M}} \oplus \overline{X}_{\mathcal{N}+\mathcal{M}}$$

Examples

- $(\mathcal{N} = 4 \text{ SYM})^2 = (\mathcal{N} = 8 \text{ SUGRA})$: $\mathcal{V}_4 \otimes \mathcal{V}_4 \equiv \mathcal{H}_8$
- $(\mathcal{N} = 2 \text{ SYM})^2 = (\mathcal{N} = 4 \text{ SUGRA} + \text{matter})$: $\mathcal{V}_2 \otimes \mathcal{V}_2 \equiv \mathcal{H}_4 \oplus 2\mathcal{V}_4$

Can we prevent these additional states to propagate in loop amplitudes?

Double copy of matter states

[Johansson, Ochirov '14]

Solution

Use a double copy of matter multiplets ((half-)hyper, chiral, fermion)

$$\Phi_{\mathcal{N}} \otimes \overline{\Phi}'_{\mathcal{M}} \equiv X_{\mathcal{N}+\mathcal{M}},$$
$$\overline{\Phi}_{\mathcal{N}} \otimes \Phi'_{\mathcal{M}} \equiv \overline{X}_{\mathcal{N}+\mathcal{M}}$$

to cancel out unwanted states (ghost statistics!).

Examples

- $(\mathcal{N} = 2\ \text{SQCD})^2$

$$\mathcal{V}_2 \otimes \mathcal{V}_2 \equiv \mathcal{H}_4 \oplus 2\mathcal{V}_4 \qquad (\Phi_2 \otimes \overline{\Phi}_2) \oplus (\overline{\Phi}_2 \otimes \Phi_2) = 2\mathcal{V}_4$$

- $(\mathcal{N} = 2\ \text{SQCD}) \times (\mathcal{N} = 0\ \text{QCD})$

$$\mathcal{V}_2 \otimes \mathcal{V}_0 \equiv \mathcal{H}_2 \oplus \mathcal{V}_2 \qquad (\Phi_2 \otimes \overline{\Psi}) \oplus (\overline{\Phi}_2 \otimes \Psi) = \mathcal{V}_2$$

Double copy of on-shell states

[Johansson, Ochirov '14]

(S)QCD $\otimes$ (S)QCD	$\mathcal{V} \otimes \mathcal{V}$	$(\Phi \otimes \overline{\Phi}) \oplus (\overline{\Phi} \otimes \Phi)$
$\mathcal{N} = 0 + 0$	$h^{++} \oplus h^{--} \oplus \phi \oplus a$	$\phi \oplus a$
$\mathcal{N} = 1 + 0$	$\mathcal{H}_{\mathcal{N}=1} \oplus \Phi_{\mathcal{N}=1} \oplus \overline{\Phi}_{\mathcal{N}=1}$	$\Phi_{\mathcal{N}=1} \oplus \overline{\Phi}_{\mathcal{N}=1}$
$\mathcal{N} = 2 + 0$	$\mathcal{H}_{\mathcal{N}=2} \oplus \mathcal{V}_{\mathcal{N}=2}$	$\mathcal{V}_{\mathcal{N}=2}$
$\mathcal{N} = 1 + 1$	$\mathcal{H}_{\mathcal{N}=2} \oplus \Phi_{\mathcal{N}=2} \oplus \overline{\Phi}_{\mathcal{N}=2}$	$\Phi_{\mathcal{N}=2} \oplus \overline{\Phi}_{\mathcal{N}=2}$
$\mathcal{N} = 2 + 1$	$\mathcal{H}_{\mathcal{N}=3} \oplus \mathcal{V}_{\mathcal{N}=4}$	$\mathcal{V}_{\mathcal{N}=4}$
$\mathcal{N} = 2 + 2$	$\mathcal{H}_{\mathcal{N}=4} \oplus 2\mathcal{V}_{\mathcal{N}=4}$	$2\mathcal{V}_{\mathcal{N}=4}$

All pure $\mathcal{N} = 0, 1, 2, 3, 4$ supergravities can be constructed from $\mathcal{N} = 0, 1, 2$ SQCD.

Color-kinematics duality with matter [Johansson, Ochirov '14]

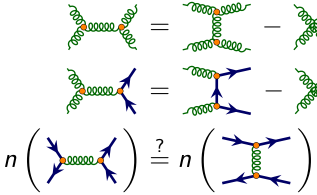
General L-loop gauge theory amplitude:

$$\mathcal{A}_m^{(L)} = i^{L-1} g^{m+2L-2} \sum_{\text{cubic graphs } \Gamma_i} \int \frac{\mathrm{d}^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{D_i}$$

We seek a duality between color and kinematics:

$$n_i - n_j = n_k \Longleftrightarrow c_i - c_j = c_k, \qquad n_i = -n_j \Longleftrightarrow c_i = -c_j$$

Expressed in a diagramatic language:



(Jacobi identity)

(commutation relation)

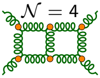
(two-term identity)

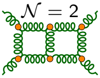
Double copy: Explicit construction

Cancel out unwanted states to obtain a pure SUGRA amplitude:

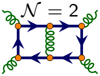
$$\mathcal{M}_m^{(L)} = i^{L-1} \left(\frac{\kappa}{2}\right)^{m+2L-2} \sum_{\text{cubic graphs } \Gamma_i} \int \frac{\mathrm{d}^{LD}\ell}{(2\pi)^{LD}} \frac{(N_f)^{|i|}}{S_i} \frac{n_i \bar{n}'_i}{D_i}$$

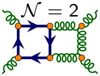
Ghosts statistics: $N_f = -1$; $N_f > -1$ supergravity theories with any number of matter multiplets.
In practice:

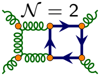
$N \Bigg($ 

$\Bigg) = \Bigg| n \Bigg($ 

$\Bigg) \Bigg|^2$

$+ 2N_f \Bigg(\Bigg| n \Bigg($ 

$\Bigg) \Bigg|^2 + \Bigg| n \Bigg($ 

$\Bigg) \Bigg|^2 + \Bigg| n \Bigg($ 

$\Bigg) \Bigg|^2 \Bigg)$

Status report: Two-loop

We have obtained:

- Color-dual $\mathcal{N} = 2$ SQCD numerators for all possible configurations of external states (vectors + hypers).
- Color-dual $\mathcal{N} = 1$ SQCD numerators for external vector states.
- Non-color-dual $\mathcal{N} = 0$ numerators for external gluon states using Feynman rules.

Using these numerators we obtained $\mathcal{N} = (2+2)$, $\mathcal{N} = (2+1)$, $\mathcal{N} = (1+1)$, $\mathcal{N} = (2+0)$ and $\mathcal{N} = (1+0)$ pure supergravity numerators at two loops. This allows us to study their UV behavior in $D \leq 6$ (work in progress).

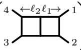
UV divergences and counterterms

- In $D = 4$ the R^3 counterterm is forbidden by supersymmetry.
- In $D = 5$ we reproduced the enhanced cancellation for $\mathcal{N} = 4$ supergravity using a double copy of the form ($\mathcal{N} = 2$ SQCD) $\times$ ($\mathcal{N} = 2$ SQCD).
- Enhanced cancellations also for $\mathcal{N} = 2$ supergravity? We are working on it.
- In $D = 4$ the first non-trivial UV behaviour starts at three loops:
  - Ansatz construction: possible, but hard.
  - Better:
    Reduce the number of master numerators (size of the Ansatz) using various constraints.
    Use off-shell lift of cuts to significantly simplify the Ansatz further or even guess the whole solution.

$\mathcal{N} = 2 + 2$ supergravity: Enhanced cancellations

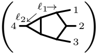
- Already previously obtained via $(\mathcal{N} = 4) \times (\mathcal{N} = 0)$ at two loops [Boucher-Veronneau, Dixon '11]
- Reproduces enhanced cancellations in $D = 5 - 2\epsilon$ [Bern, Davies, Dennen, Huang '12]

$$\mathcal{I}^{5-2\epsilon}\left[N^{[\mathcal{N}=4\text{ SG}]}\left(\;\cdot\;\right)\right] =$$

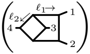


$$\frac{1}{(4\pi)^5}\left(\frac{(2-D_s)\pi}{70\epsilon}(\kappa_{12}^2+\kappa_{34}^2) - \frac{(29D_s-142)\pi}{210\epsilon}(\kappa_{13}^2+\kappa_{14}^2+\kappa_{23}^2+\kappa_{24}^2)\right) + \mathcal{O}(\epsilon^0)$$

$$\mathcal{I}^{5-2\epsilon}\left[N^{[\mathcal{N}=4\text{ SG}]}\left(\;\cdot\;\right)\right] = \frac{1}{(4\pi)^5}\frac{(D_s-2)\pi}{5\epsilon}\sum_{i<j}\kappa_{ij}^2 + \mathcal{O}(\epsilon^0)$$

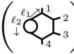


$$\mathcal{I}^{5-2\epsilon}\left[N^{[\mathcal{N}=4\text{ SG}]}\left(\;\cdot\;\right)\right] =$$



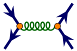
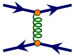
$$\frac{1}{(4\pi)^5}\left(\frac{(18-23D_s)\pi}{105\epsilon}(\kappa_{12}^2+\kappa_{34}^2) - \frac{2(4+5D_s)\pi}{105\epsilon}(\kappa_{13}^2+\kappa_{14}^2+\kappa_{23}^2+\kappa_{24}^2)\right) + \mathcal{O}(\epsilon^0)$$

$$\mathcal{I}^{5-2\epsilon}\left[N^{[\mathcal{N}=4\text{ SG}]}\left(\;\cdot\;\right)\right] = -\frac{1}{(4\pi)^5}\frac{(D_s-2)\pi}{6\epsilon}\sum_{i<j}\kappa_{ij}^2 + \mathcal{O}(\epsilon^0).$$

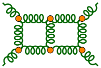
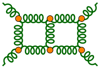
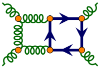


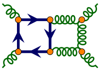
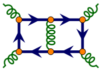
$\mathcal{N}=2$ SQCD: How to

1. Solve Jacobi identities to obtain master numerators.
2. Solve two-term identities to reduce the set of master numerators:

$n\Big($  $\Big) \stackrel{?}{=} n\Big($  $\Big)$

3. Implement supersymmetric decomposition identities to reduce the set of master numerators:

$n^{[\mathcal{N}=4]}\Big($  $\Big) = n^{[\mathcal{N}=2]}\Big($  $\Big) + 2n^{[\mathcal{N}=2]}\Big($  $\Big)$

$+ 2n^{[\mathcal{N}=2]}\Big($  $\Big) + 2n^{[\mathcal{N}=2]}\Big($  $\Big)$

4. Make Ansätze and implement constraints and further desirable properties
   - Unitarity cuts
   - Manifest CPT
   - Matter reversal: Φ and $\overline{\Phi}$ have the same particle content.

$\mathcal{N} = 2$ SQCD: How to

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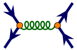
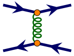
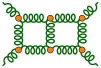
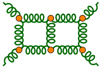
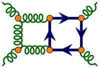
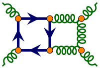
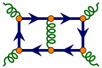
$n\Big($  $\Big) \stackrel{?}{=} n\Big($  $\Big)$

3. Implement supersymmetric decomposition identities to reduce the set of master numerators:

$n^{[\mathcal{N}=4]}\Big($  $\Big) = n^{[\mathcal{N}=2]}\Big($  $\Big) + 2n^{[\mathcal{N}=2]}\Big($  $\Big)$

$+ 2n^{[\mathcal{N}=2]}\Big($  $\Big) + 2n^{[\mathcal{N}=2]}\Big($  $\Big)$

4. Make Ansätze and implement constraints and further desirable properties
   - Unitarity cuts
   - Manifest CPT
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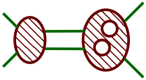
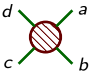
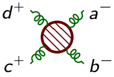
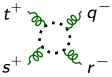
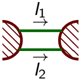
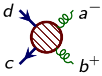
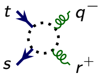
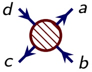
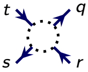
$\mathcal{N} = 2$ SQCD: Iterated two-particle cuts (4D)

Idea 1: Systematize supersum computations [Bern, Carrasco, Johansson, Roiban '12]
Idea 2: Iteratively glue four-point blobs together:

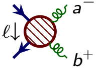


General rules	Internal rules	External rules
 $\to -\frac{i}{s_{ab}s_{bc}}$	 $\to \langle ab\rangle[cd]$	 $\to [qr]\langle st\rangle\hat{\kappa}_{(qr)(qr)}$
 $\to s_{l_1 l_2}$	 $\to \langle a\|c\|b]$	 $\to [q\|s\|r\rangle\hat{\kappa}_{(qs)(qt)}$
	 $\to s_{ac} = s_{bd}$	 $\to s_{rt}\hat{\kappa}_{(qs)(rt)}$

$$\hat{\kappa}_{(qr)(st)} \equiv \frac{st A^{\text{tree}}}{s_{qr}s_{st}}$$

Cut properties

- Factorized rules for each blob
- Poles are handled transparently (locality!).
  - Physical poles from each tree-level blob
  - Spinor-helicity remnants in $\hat{\kappa}$
- Regulated matter legs in the IR, e.g.

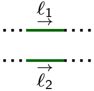


$\rightarrow \langle a|\ell|b]$

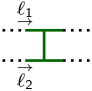
- Natural two-term identity
- Numerators form dirac traces

Off-shell continuation ($\sim$ the rung rule)

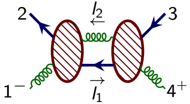
Idea: motivated by the two-particle cut in $\mathcal{N}=4$ numerators are constructed via a *rung rule* [Bern, Rozowsky, Yan '97; Bern, Dixon, Dunbar, Perelstein, Rozowsky '98]



$$\rightarrow -i(\ell_1+\ell_2)^2 \times$$

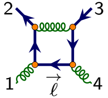


Can we also lift $\mathcal{N}=2$ cuts off-shell? E.g.



$$= \frac{\mathrm{tr}_+(4 l_1 l_2 l_1 12)}{s\, s_{4l_1} s_{2l_2}} \hat{\kappa}_{(12)(13)} = -\frac{\mathrm{tr}_+(4 l_1 12)}{s_{4l_1} s_{2l_2}} \hat{\kappa}_{(12)(13)}$$

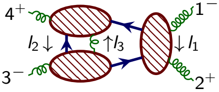
$$\Longrightarrow n \left(\right) = \mathrm{tr}_+(4\ell 12)\hat{\kappa}_{(12)(13)} + \mathrm{tr}_-(4\ell 12)\hat{\kappa}_{(24)(34)}$$



When does it work?

- Mondrian-type diagrams can in many cases directly be read off.
- Otherwise (if cuts do not agree): significantly reduced ansatz.
- Choose master numerators that we can get from the cuts like this.
- Jacobi relations, two-term identities, SUSY decomposition identities and further constraints give all other diagrams.
- The off-shell continuation clashes with the color-kinematics duality for some numerators:
  - Use the construction to find local (S)QCD numerators.
- Missing μ terms are in general much simpler: use 6D spinor helicity formalism [Cheung, O'Connell '09]

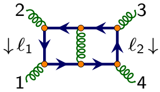
$\mathcal{N} = 2$ SCQD: Example results and their properties I



$$= -\frac{\langle 3|l_2|4]}{(l_2+p_4)^2(l_2-p_3)^2} \times s \times \frac{\langle 1|l_1|2]}{s\, l_1^2} \times [13]\langle 24\rangle \hat{\kappa}_{13}$$

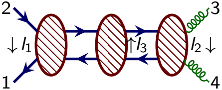
$$= \frac{\mathrm{tr}_-(1 l_1 2 4 l_2 3)}{l_1^2(l_2+p_4)^2(l_2-p_3)^2}\hat{\kappa}_{13}\,,$$

$\Rightarrow n\Big($



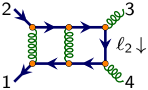
$\Big)$

$$= \hat{\kappa}_{13}\,\mathrm{tr}_-(1\ell_1 24\ell_2 3) + \hat{\kappa}_{14}\,\mathrm{tr}_-(1\ell_1 23\ell_2 4) + \hat{\kappa}_{23}\,\mathrm{tr}_+(1\ell_1 23\ell_2 4) + \hat{\kappa}_{24}\,\mathrm{tr}_+(1\ell_1 24\ell_2 3)$$



$$= -\frac{\mathrm{tr}_-(4 l_2 3 1)(s+l_3^2)(s+l_1^2)}{s l_1^2 l_2^2 l_3^2}\hat{\kappa}_{(41)(42)} - \frac{\mathrm{tr}_+(4 l_2 3 1)(s+l_3^2)(s+l_1^2)}{s l_1^2 l_2^2 l_3^2}\hat{\kappa}_{(31)(32)}\,,$$

$\Rightarrow n\Big($



$\Big)$

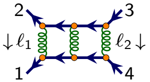
$$= -s\,\mathrm{tr}_-(4\ell_2 31)\hat{\kappa}_{(41)(42)} - s\,\mathrm{tr}_+(4\ell_2 31)\hat{\kappa}_{(31)(32)}$$

$\mathcal{N} = 2$ SCQD: Example results and their properties II

The above numerators

- are directly read off from cut
- have regulated soft & collinear limit for hyper lines
- have all previously discussed desirable properties: two-term identity, SUSY decomposition, matter-reversal symmetry, manifest CPT

Slightly more difficult:

$n\Bigg($ 

$\Bigg) = -s^2 \left[(\ell_1 + p_1)\cdot(\ell_1 - p_2) + (\ell_2 + p_4)\cdot(\ell_2 - p_3)\right] \hat{\kappa}_{(12)(34)}$

- Cuts do not agree, but Ansatz is reduced to 3 terms (from initially ~ 300)
- regulates soft & collinear limit for hyper lines

What's next?

- UV properties of all obtainable theories
- Three loops
  - color-dual representation: $\mathcal{N} = 2 \otimes \mathcal{N} = 2$
  - vs. local representation: study SQCD IR properties
  - Iterated multi-particle cuts
- Less supersymmetry
  - Iterated (local) cut formulae also exist for $\mathcal{N} = 0, 1$
  - Use rung-rule techniques to compute both color-dual and local representation
- Challenges:
  - Systematize off-shell continuation: Can we make all cuts to agree? Minimal set of terms for an Ansatz?
  - Color-kinematics duality vs. locality
- Simplified gravity cuts from the double copy
- External matter in SQCD and SUGRA