Comments on the Wick-Cutkosky model

+ No-precession of supersymmetric BH's

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on work with Zahra Zahraee, 1810.04694

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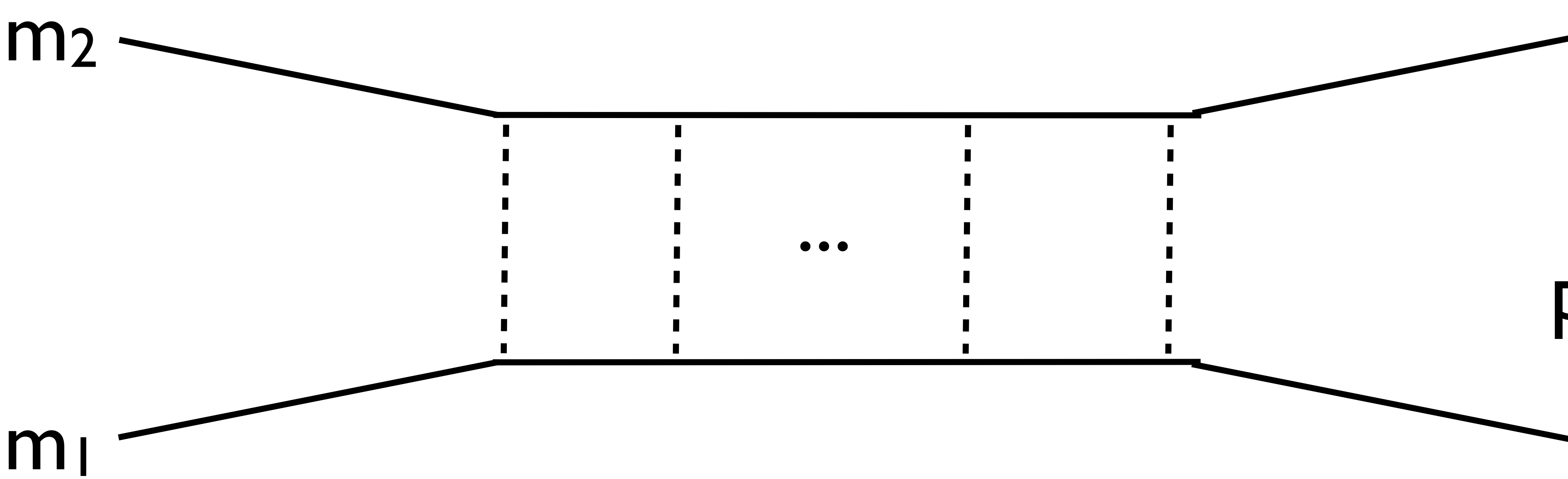
Wick-Cutkosky model = planar ladders for (scalar) ep scattering



'QCD-ish' part of talk

The model has interesting symmetries. [Cutkosky '54] showed that mass& CM energy enter only through:

$$u = \frac{s - m_1^2 - m_2^2}{2m_1 m_2}$$

- Solvable thanks to hidden SO(4,2) 'dual conformal symmetry' [Cutkosky '54 Itzykzon&Bender ~70]

- Obviously not a realistic model! (even for QED) Symmetry is not realistic either.

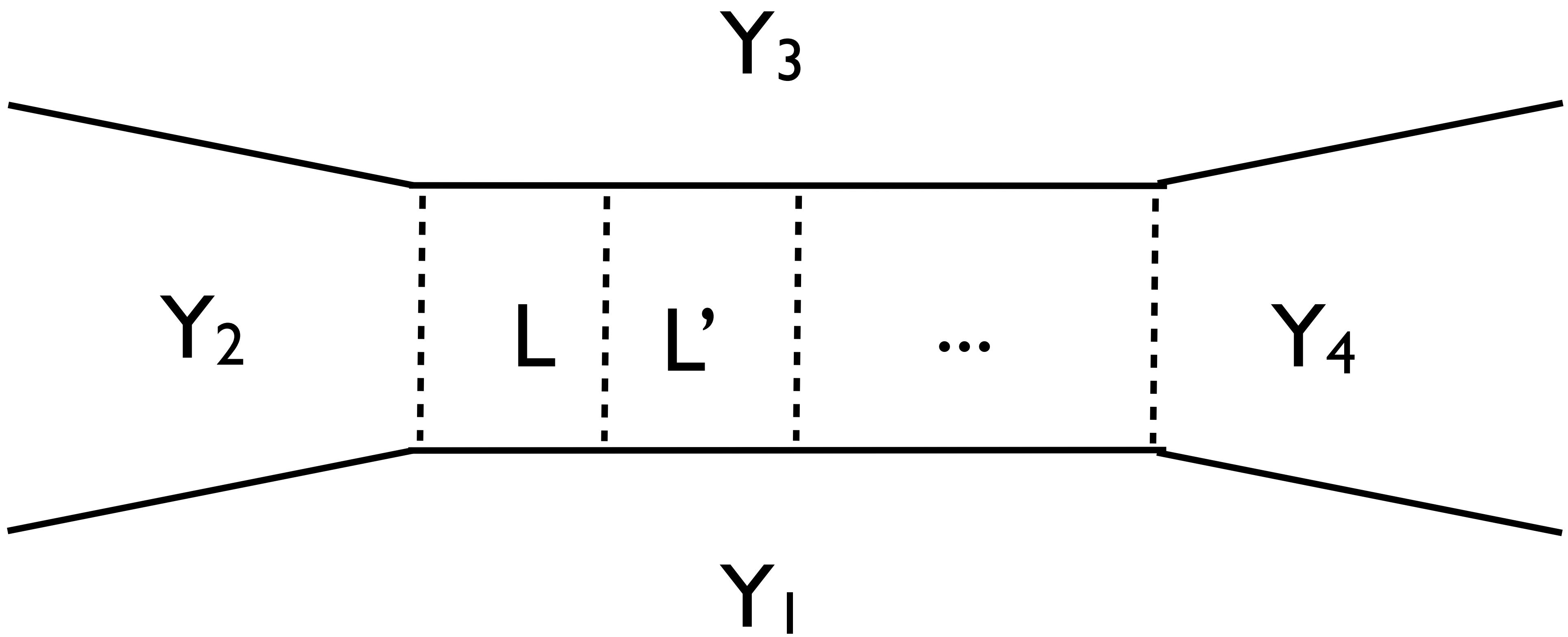
- Surprising fact: a SO(1,2) subgroup actually is a symmetry of GR @ 1PM (⇒EOB). [Buonanno& Damour '99]

  This is what I'll now comment on.

- In '54, solution required to invent Wick rotation

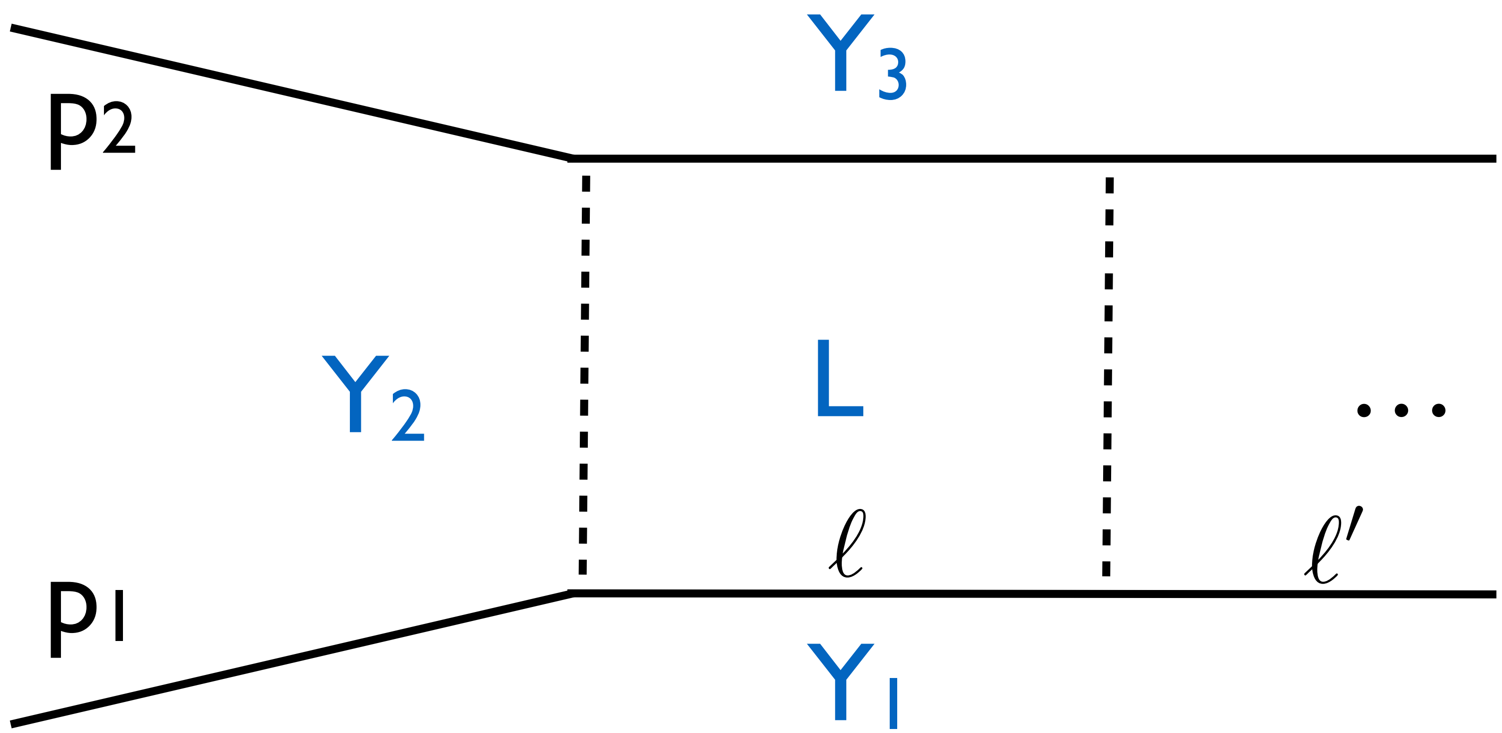
Dual conformal symmetry = conformal SYM in mom space

For any planar graph, introduce 6-vector in each region



Idea: propagators ⇒ dot products

$$L{\cdot}Y_1, \quad L{\cdot}Y_2, \quad L{\cdot}Y_3, \quad L{\cdot}L' \ldots$$



$$\int \frac{d^4\ell}{[\ell^2 - m_1^2](\ell - p_1)^2[(\ell - p_1 - p_2)^2 - m_2^2](\ell - \ell')^2}$$

$$= \int \frac{d^4 L^*}{(L{\cdot}Y_1)(L{\cdot}Y_2)(L{\cdot}Y_3)(L{\cdot}L')}$$

$$^* d^4 L \equiv \frac{d^6 L \;\delta(L^2)}{\mathrm{vol}(\mathrm{gl}(1))}$$

easy to find explicit 6-vectors ('Dirac embedding')

$$L_i^A = \begin{array}{cc} \begin{pmatrix} \ell_i^\mu \\ \frac{1}{2\Lambda}(\Lambda^2 - \ell_i^2) \\ \frac{1}{2\Lambda}(\Lambda^2 + \ell_i^2) \end{pmatrix} & \begin{array}{c} -+++ \\ + \\ - \end{array} \end{array} \qquad Y_i^A = \begin{pmatrix} y_i^\mu \\ \frac{1}{2\Lambda}(\Lambda^2 - y_i^2 - m_i^2) \\ \frac{1}{2\Lambda}(\Lambda^2 + y_i^2 + m_i^2) \end{pmatrix}$$

region momenta: $y_1^\mu = 0, \quad y_2^\mu = p_1^\mu, \quad y_2^\mu = (p_1 + p_2)^\mu, \ldots$

Properties:

$$-2L_i \cdot L_j = (\ell_i - \ell_j)^2$$

$$-2L_i \cdot Y_j = (\ell_i - y_j)^2 + m_j^2$$

$$-2Y_i \cdot Y_j = (y_i - y_j)^2 + m_i^2 + m_j^2$$

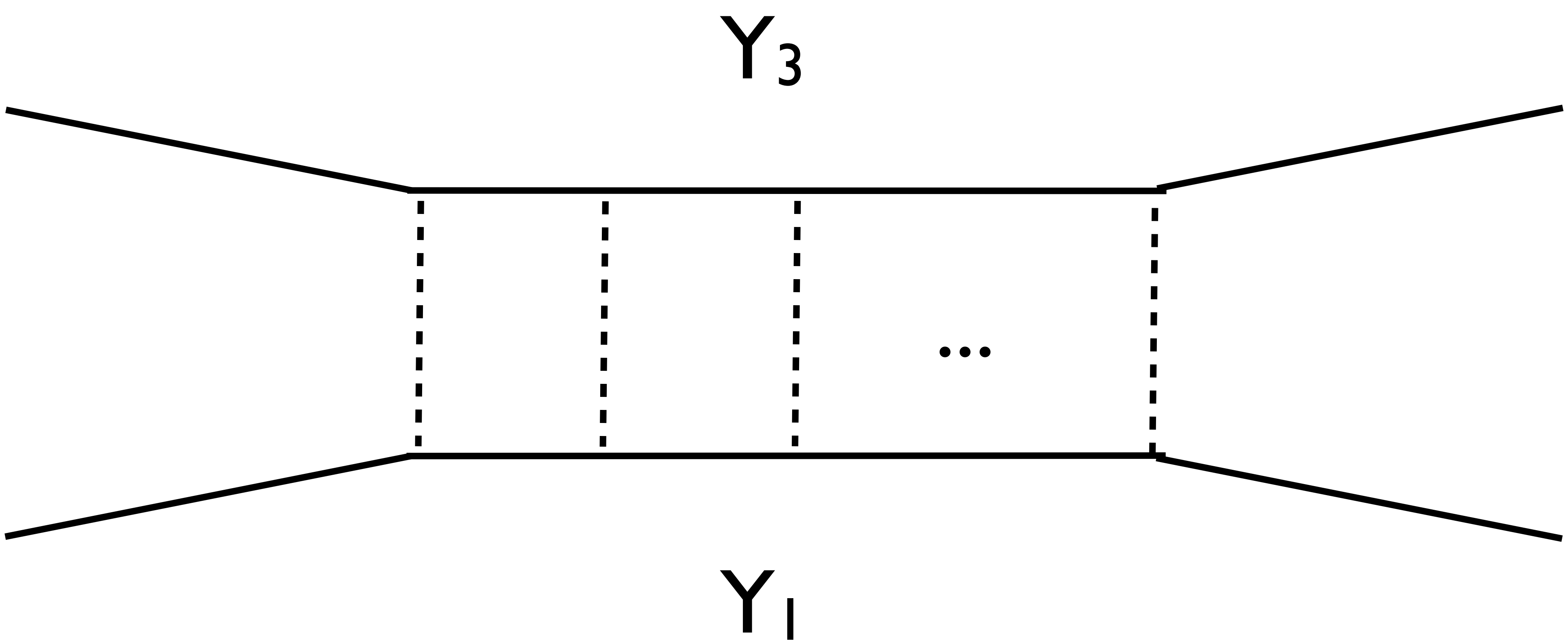
$$\overset{+}{Y_i^4} + \overset{-}{Y_i^5} = \Lambda$$

[cf App of Henn+SCH '18]

SO(4,2) =	-Lorentz SO(3,1)	6
	-Translations of y^μ	4
	-Dilatations	1
	-Special conformal of y^μ	4
		15

Since all denominators are 6D dot products, they transform nicely under SO(4,2) rotations

Obviously, dilatations and special conformal transformations are not symmetries when $m \neq 0$. They are covariances of the model.



Two interesting subgroups:

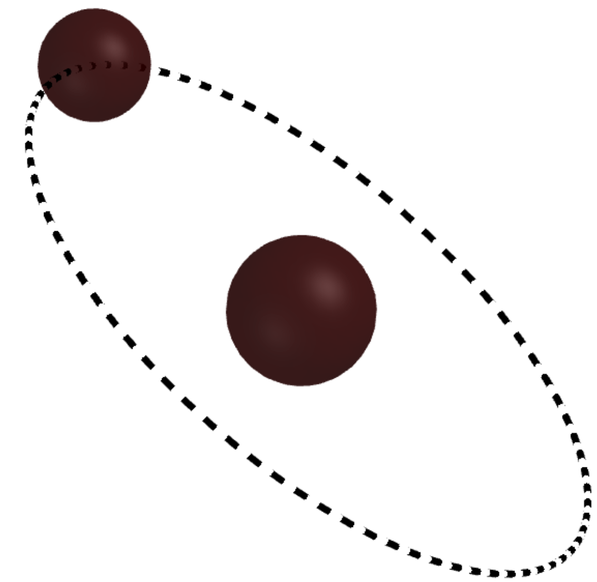
1. symmetries of bound states
   = SO(4) which preserves Y_1, Y_3 (thus m_1, m_2, E)

2. symmetries of rest frame
   = SO(3) x SO(1,2) which acts on m_1, m_2, E

The SO(4) symmetry is well-known.

$\vec{J}, \vec{A}$ = rotations+Laplace-Runge-Lenz

$$\vec{A} = \vec{p} \times \vec{L} - mk\frac{\vec{x}}{x} = \text{constant}$$



SO(4) $\Rightarrow$ -no perihelion precession (closed orbits)
-quantum degeneracies of H $(E_{n,\ell} = E_n)$

Is an exact (relativistic) symmetry of Wick-Cutkosky model

This SO(4) clearly is not a symmetry of GR,
and it won't be our focus today

Claim: SO(1,2) is a symmetry of GR @ 1PM

$$Y_1^{\ A} = \begin{pmatrix} 0 \\ \vec{0} \\ \frac{1}{2\Lambda}(\Lambda^2 - m_1^2) \\ \frac{1}{2\Lambda}(\Lambda^2 + m_1^2) \end{pmatrix} \begin{matrix} - \\ +++ \\ + \\ - \end{matrix} \qquad Y_3^{\ A} = \begin{pmatrix} E \equiv E_1 + E_2 \\ \vec{0} \\ \frac{1}{2\Lambda}(\Lambda^2 + E^2 - m_2^2) \\ \frac{1}{2\Lambda}(\Lambda^2 - E^2 + m_2^2) \end{pmatrix}$$

SO(1,2) acts on E, m_1, m_2, but preserves the cross-ratios:

$$\frac{Y_1 \cdot Y_3}{\sqrt{Y_1^2 Y_3^2}} = \frac{E_{\text{real}}^2 - m_1^2 - m_2^2}{2 m_1 m_2} \equiv \frac{E_{\text{eff}}}{\mu}$$

[Cutkosky '54]

Since translations of y^0 do not affect kinematics, what matters is actually the 2D coset SO(1,2)/SO(0,2)

One generator = (4,5)-plane boost, which is just dilatation:

$$\begin{pmatrix} 0_{4\times 4} & 0 & 0 \\ 0 & \cosh\alpha & -\sinh\alpha \\ 0 & -\sinh\alpha & \cosh\alpha \end{pmatrix} : \quad Y_i^A \mapsto \begin{pmatrix} y^\mu \\ \frac{1}{2\Lambda}(\Lambda^2 e^{-\alpha} - (y_i^2 + m_i^2)e^{\alpha}) \\ \frac{1}{2\Lambda}(\Lambda^2 e^{-\alpha} + (y_i^2 + m_i^2)e^{\alpha}) \end{pmatrix}$$

After rescaling $Y_i \mapsto e^{\alpha} Y_i$ to restore $Y_4 + Y_5 = \Lambda$

$$(y_i^\mu, m_i) \mapsto (e^{\alpha} y_i^\mu, e^{\alpha} m_i)$$

The other SO(1,2) boost is more interesting:
can use it to map any kinematics to probe limit $m_1 \to \infty$

two-body $\qquad\qquad$ one-body

$$P_1^\mu, P_2^\mu \longrightarrow P_{\text{eff}}^\mu = (E_{\text{eff}}, \vec{p}_{\text{eff}})$$

$$m_1, m_2 \qquad\qquad m_{\text{eff}}$$

This is the EOB map, realized as dual conformal transformation

[Buoannano& Damor '99]

$$E_{\text{eff}} = \frac{E_{\text{real}}^2 - m_1^2 - m_2^2}{2(m_1+m_2)}, \quad \vec{p}_{\text{eff}} = \vec{p_1}\frac{E_1+E_2}{m_1+m_2}, \quad m_{\text{eff}} = \frac{m_1 m_2}{m_1+m_2}$$

Consider Post-Minkowski EFT:

$$H_{\rm real}|\vec{p_1}\rangle = (\sqrt{\vec{p_1}^{\,2}+m_1^2}+\sqrt{\vec{p_1}^{\,2}+m_2^2})|\vec{p_1}\rangle + \int_{\vec{p}_1'} V(\vec{p}_1,\vec{p}_1')|\vec{p}_1'\rangle$$

in form of [Cheung,Rothstein&Solon '18]

Apply EOB change of variable:

$$|\vec{p}_1\rangle \mapsto \left|\frac{\partial \vec{p_{\rm eff}}}{\partial \vec{p}_1}\right|^{-1/2} |\vec{p}_{\rm eff}\rangle$$

$$H_{\rm eff}|\vec{p}_{\rm eff}\rangle = \sqrt{\vec{p}^{2}_{\rm eff}+\mu^2}|\vec{p}_{\rm eff}\rangle + \int_{p'_{\rm eff}} V_{EOB}(\vec{p}_{\rm eff},\vec{p}'_{\rm eff})|\vec{p}'_{\rm eff}\rangle$$

In Wick-Cutkowski model, $V_{EOB} = V$

What about GR?

$$V^{1PM}(p_1, p_2) \sim G_N \frac{\cosh 2\eta}{q^2} = G_N \frac{2(\beta_1 \cdot \beta_2)^2 - \beta_1^2 \beta_2^2}{q^2}$$

[on-shell: $p_1^2 = p_1'^2$]

$$-2\beta_1 \cdot \beta_2 \equiv \frac{-2p_1 \cdot p_2}{m_1 m_2} = \frac{E^2 - m_1^2 - m_2^2}{m_1 m_2} = \frac{2E_{\text{eff}}}{\mu} = \text{SO(1,2)-invariant}$$

In the EOB frame, $\beta_1^\mu = (1, \vec{0})$:

1PM Hamiltonian = probe of mass μ in Schwartzschild

There is nothing special about GR here;
we could have a dilaton and/or electric charges

At 1PM, the EOB-frame potential is always the same as the original one, up to some rescaling of the charges.

SO(1,2) = symmetry of 1PM approximation.

Generalizes that 'physics depends only on reduced mass'.

Q: -Does it first break at 2PM or higher? [Damour et al]
-How does SO(1,2) acts on spin?

Part II: N=8 no-precession

Is there a gravity theory with hidden (SO(4,2)?) symmetry?

In planar N=4 SYM, SO(4,2) is exact

[Bern,Dixon&Smirnov(+Kosower,Anastasiou); Drummond,Henn,Smirnov&Sokatchev; Alday&Maldacena,…]

Can get masses via Higgs mechanism.
Hydrogen-like bound states have SO(4) LRL symmetry.

[SCH& Henn, '14]

integrability of planar N=4 SYM

dual conformal + conformal = Yangian

$\Updownarrow$ LRL vector / SO(4)
No precession

$\Updownarrow$ theory integrable

[Arkani-Hamed, Cachazo& Kaplan ’08]

Best candidate for gravity with hidden symmetries:

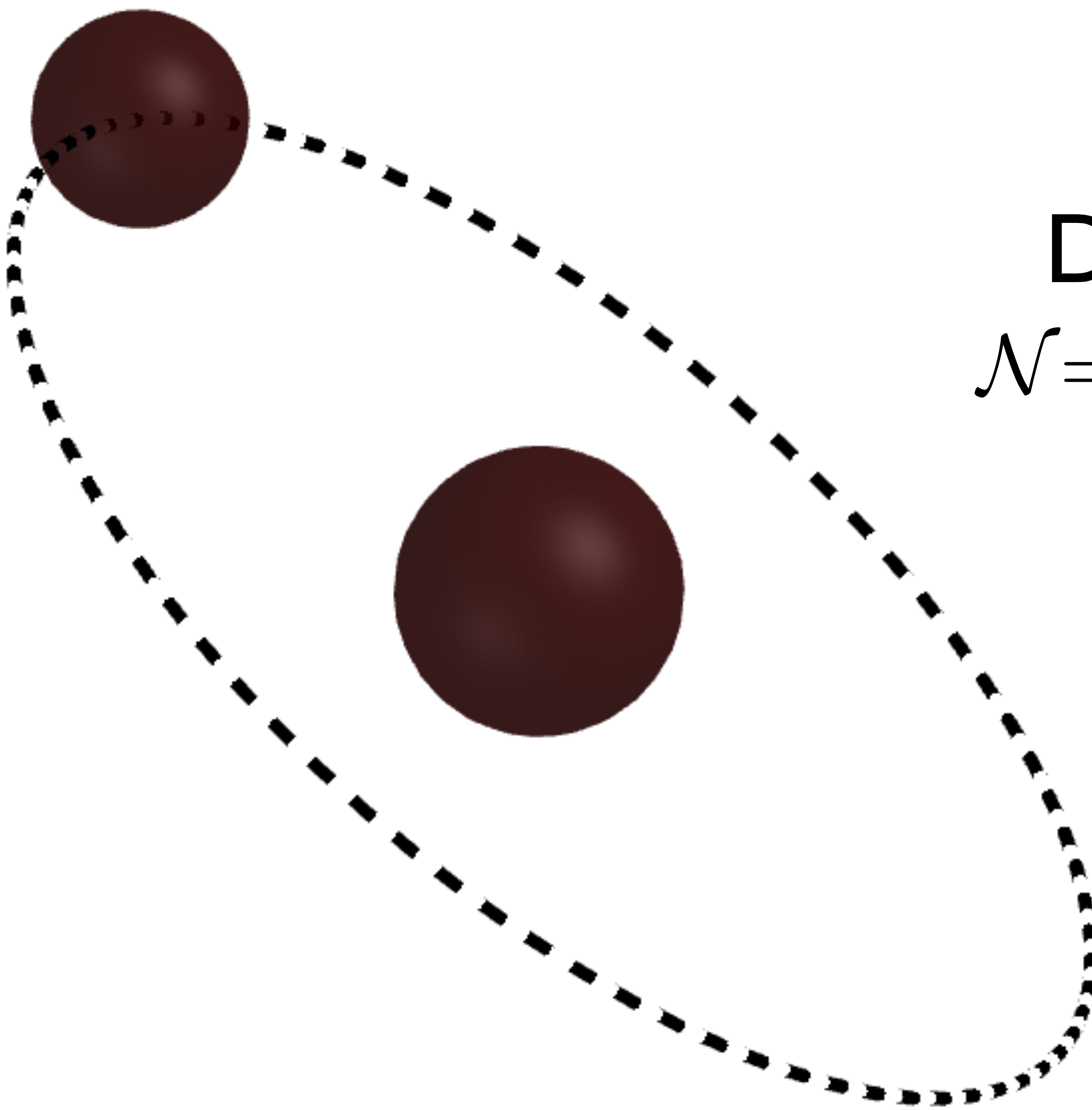
$$\underset{\text{sugra}}{\mathcal{N}=8} \quad = \quad \underset{\text{sym}}{(\mathcal{N}=4)^2}$$

our strategy: check a physical observable

do two-body subsystems have a conserved LRL?

[more precise: degeneracy of S-matrix poles;
such exclusive amplitudes suppress radiation]

we'll compute the analog of mercury precession of perihelion for half-BPS black holes in $\mathcal{N}=8$ supergravity

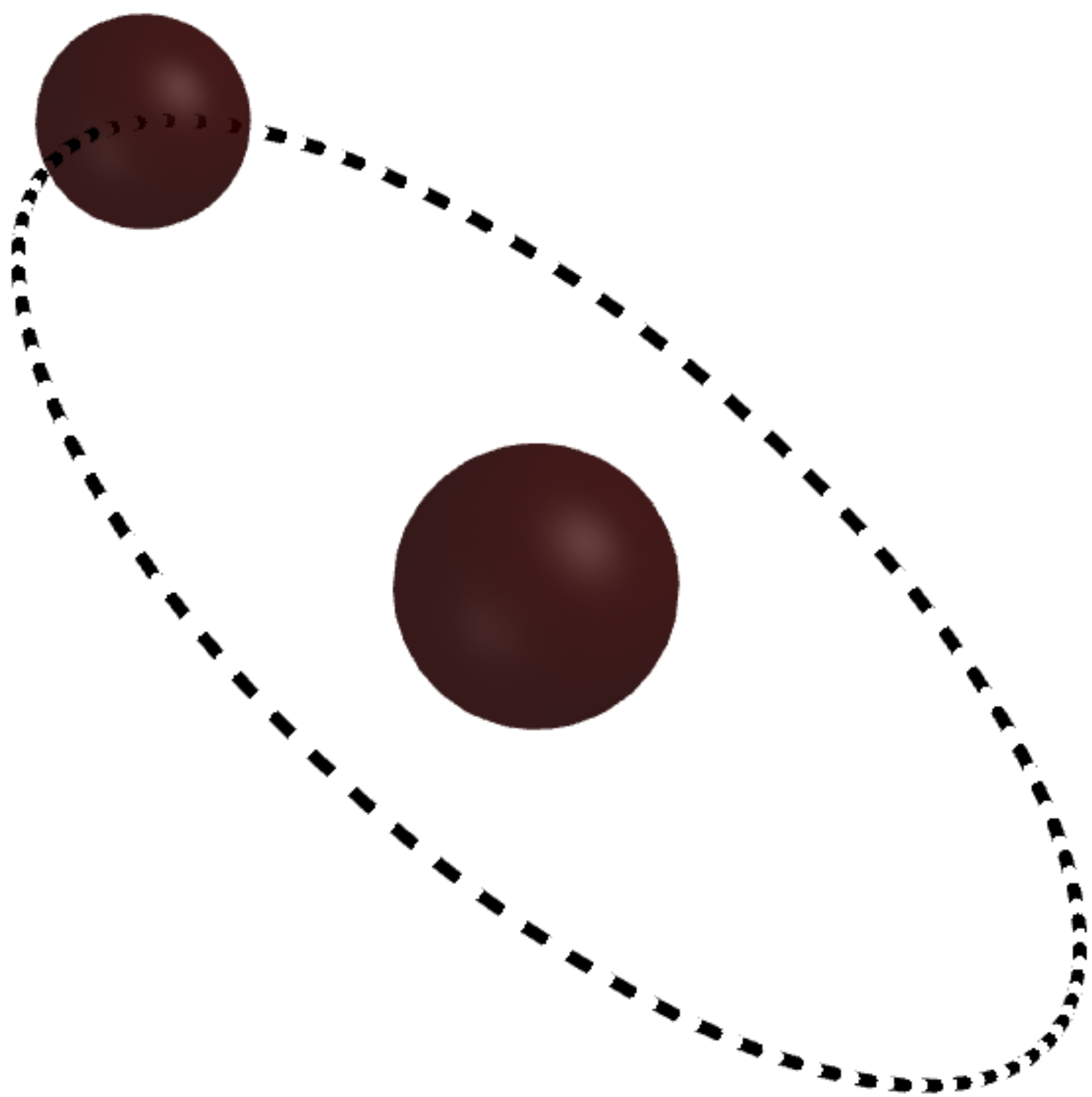


Differs from GR since $\mathcal{N}=8$ has other light DOFs

Result: ≈ 0

Setup

two extremal (half-BPS) black holes



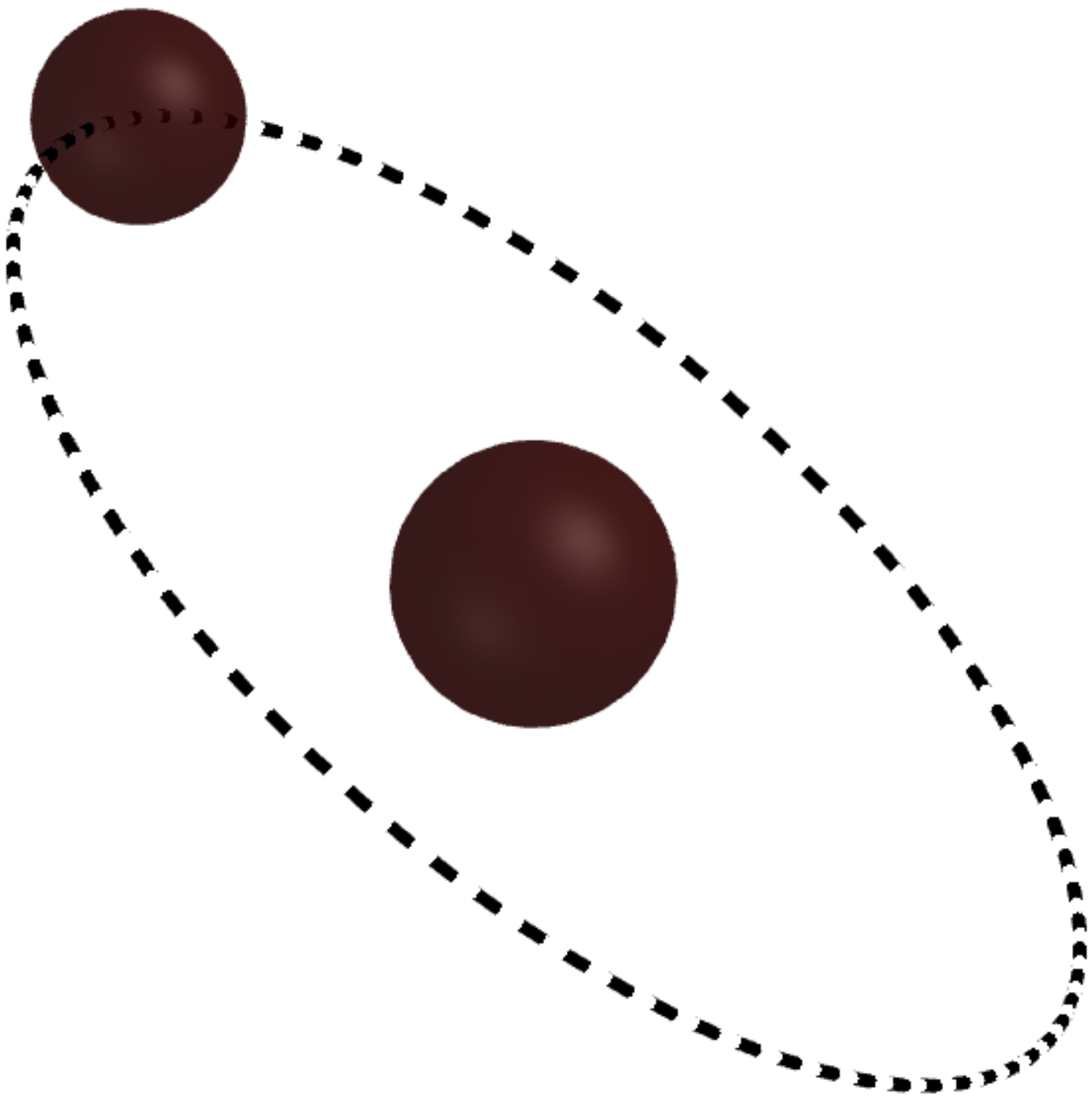
Total force:

  gravity
+ electric
+ scalar

0

Setup

two extremal (half-BPS) black holes



Total force:

gravity
+ electric
+ scalar

0

$\times$ "$\cos\theta$"

$\neq 0$

need misaligned charge vectors

Classifying misalignments

Recall $\mathcal{N} = 8$ graviton multiplet:

1	g^{++}
8	$\chi_I^{+3/2}$
28	A_{IJ}^{+}
56	$\psi_{IJK}^{+1/2}$
70	ϕ_{IJKL}
56	
28	$\cdots$
8	
1	g^{--}

electric charges is
8x8 antisymmetric matrix

charges are 8x8 matrices;
3 angles between 1/2-BPS charges

[SCH+Zarahee '18]

1. Special Special case: 1 angle

$$\phi_3 = \phi_4 = -\phi_1 = -\phi_2$$

equivalent to KK graviton from 10D IIA on T^6

⇒integrand known to 5 loops!

✓

2. Special case: 2 angles

$$\phi_3 = -\phi_1, \quad \phi_4 = -\phi_2$$

T-even so can be realized without mag charge ✓

3. General case: 3 angles

Re C_B = electric

Im C_B= magnetic

has both!

[partial results]

HUGE literature on BPS states

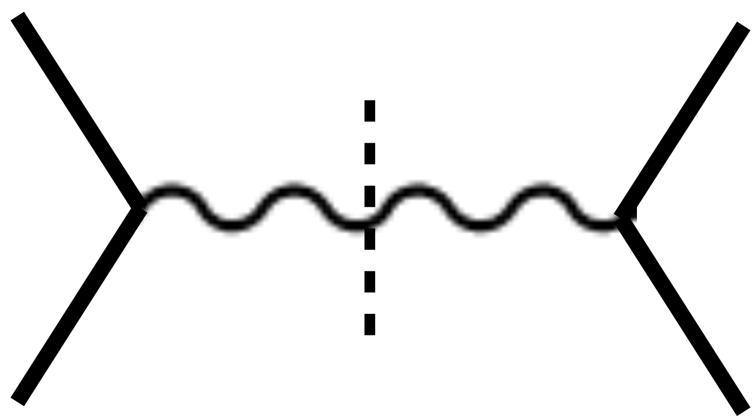
- BPS bound states [static] ['wall crossing']
- Motion at low velocities [Denef& Anynos, ...]
- velocity expansion for D0 branes, etc [...]
- ...

We want: relativistic velocities and non-aligned charges

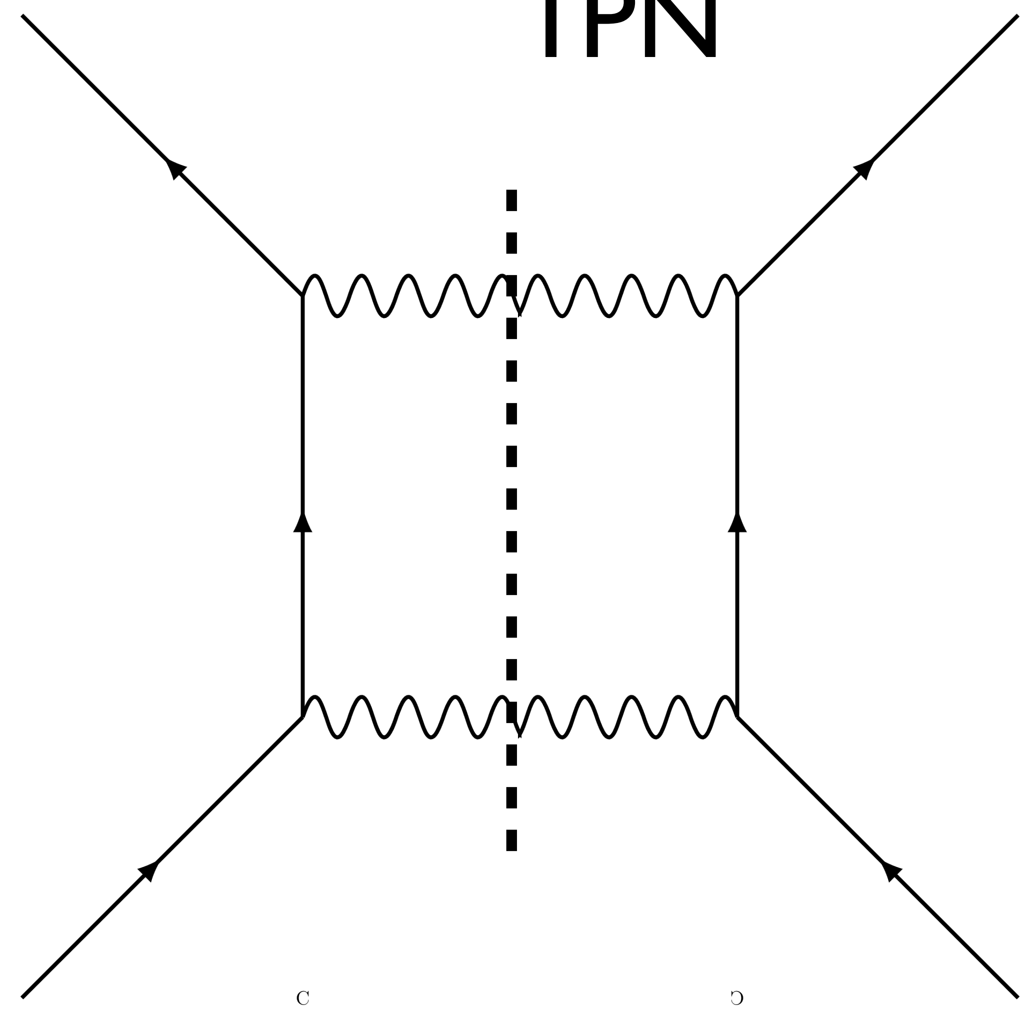
Instead of piecing results together,
we just computed from scratch

Long range forces = exchange of massless particles

0PN+1PN



1PN



…

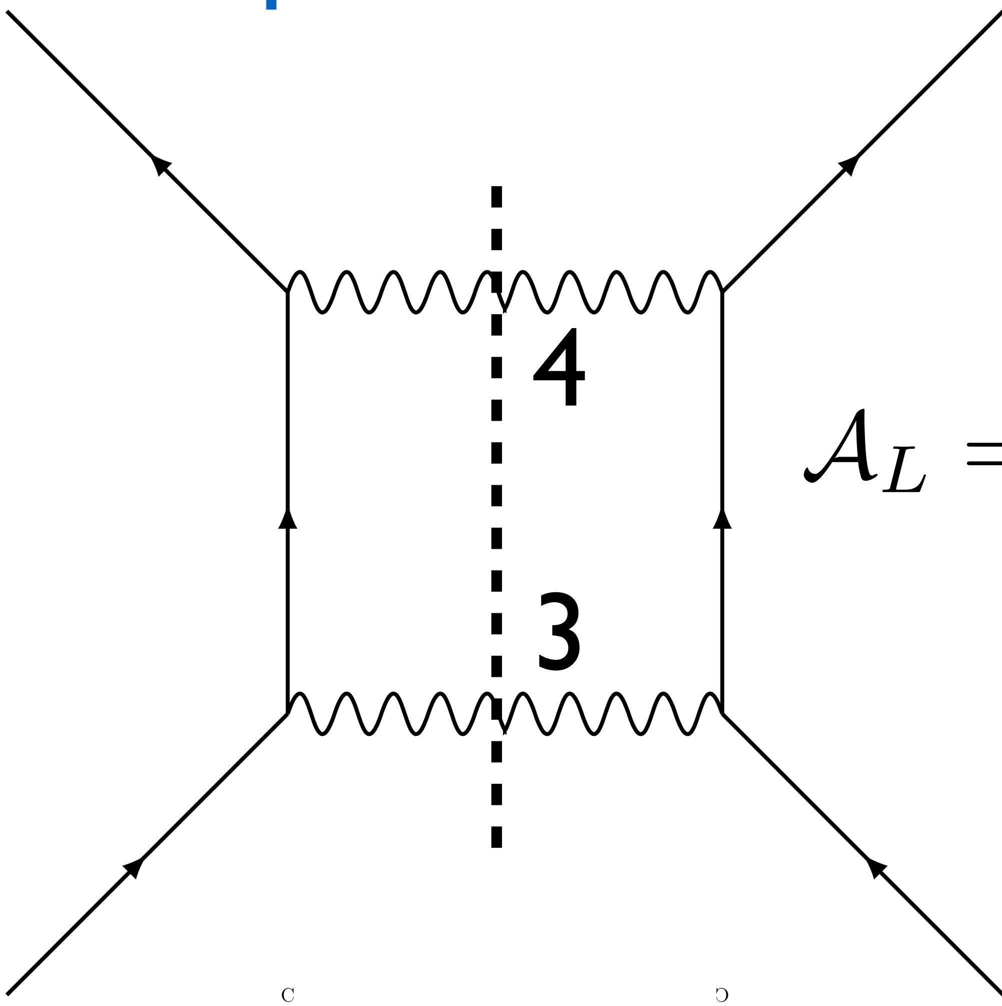
On-shell susy Ward identities fix 3&4-point amplitudes

long distances = t-channel cut

[Feinberg& Sucher, ’80s;
Bjerrrum-Bohr, Donoghue, Holstein, Vanhove, …]

One loop

(subtlety if mag charge)

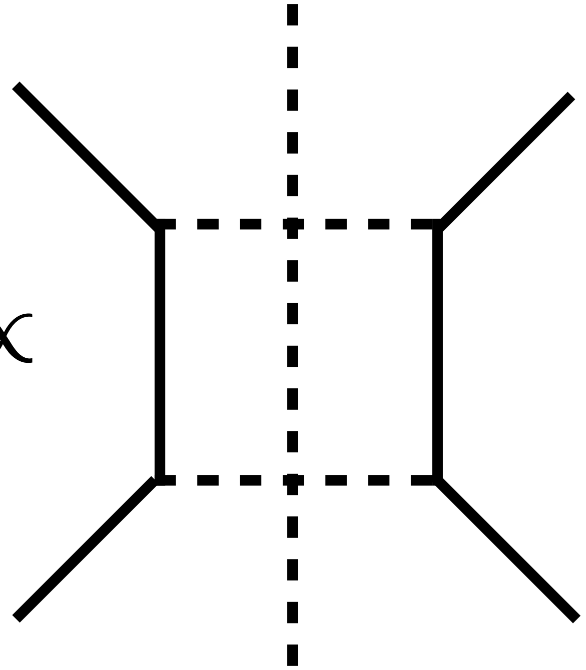


$$\mathcal{A}_L = \frac{16\pi G_N [34]^4}{(p_A \cdot k_3)(p_A \cdot k_4)(k_3 \cdot k_4)} \times \exp\left(\sum(\ldots)(\eta^3\eta^3, \eta^3\eta^4, \eta^4\eta^4\right)$$

square and integrate over η_3,η_4.

No triangle!!

$\text{Disc}_t\, \mathcal{A}_{\phi\phi\bar{\phi}\bar{\phi}} \propto$



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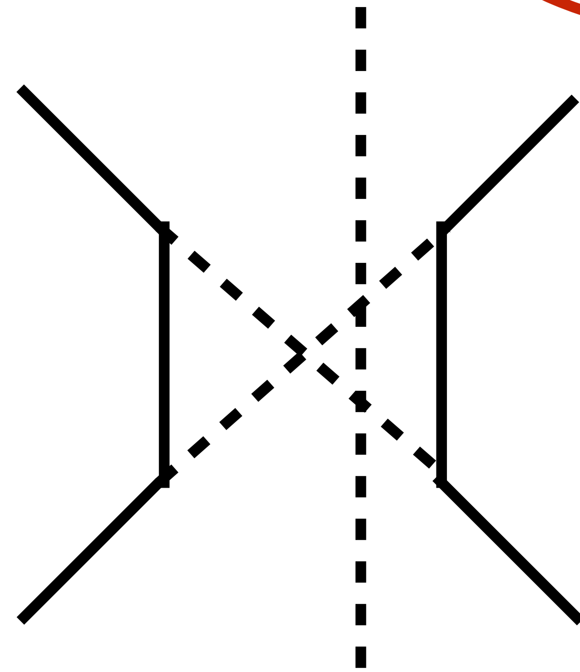

Precession?

Link between scattering problem & closed orbits: EFT

$$H_{\text{eff}} = \frac{p^2}{2\mu} - \frac{G_N M m}{r} \qquad \text{0PN}$$

$$+ A p^4 + B p^2 \frac{G_N}{r} + C p_r^2 \frac{G_N}{r} + D \frac{G_N^2}{r^2} \qquad \text{1PN}$$

$$+ \dots$$

Precession?

Link between scattering problem & closed orbits: EFT

$$H_{\rm eff} = \frac{p^2}{2\mu} - \frac{G_N M m}{r}$$ 0PN

$$+ A p^4 + B p^2 \frac{G_N}{r} + C p_r^2 \frac{G_N}{r} + D \frac{G_N^2}{r^2}$$ 1PN

$$+ \ldots$$

EFT matching: free tree 1-loop

[same logic as pNRQCD!]

EFT matching: $$\mathcal{A}^{(1)}\Big|_{1/\sqrt{-t}} = \left[4A\alpha^2 + \alpha(2B+C) + D\right]$$

same!

precession: $$\frac{d}{dt}\vec{A}^{(0)} = \frac{d}{dt}(\ldots) + \{1/L_z, \vec{A}\}\times\left[4A\alpha^2 + \alpha(2B+C) + D\right]$$

$-\vec{A}^{(1)}$

[Kol]

No triangle $\Leftrightarrow$ No $\mathcal{A}^{(1)}\Big|_{1/\sqrt{-t}}$ $\Leftrightarrow$ No precession

Clearer link would be desirable between these observables

Probe limit

N=8 sugra = 10D IIA compactified on T^6.
'large' black hole: take D6-brane, preserves symmetries

$$ds^2 = -\frac{1}{\sqrt{H(r)}}dt^2 + \sqrt{H(r)}(dr^2 + r^2 d\Omega^2) + \frac{1}{\sqrt{H(r)}}\sum_{i=1}^{6} dx^i dx^i$$

$$e^{-2\Phi} = H(r)^{3/2} \qquad H(r) = 1 + \frac{4G_N m_A}{r}$$

probe action:

$$S_p = m\int d\tau e^{-\Phi}\sqrt{-g_{MN}\partial_\tau X^M \partial_\tau X_M} + q\int A_M \frac{dx^M}{d\tau}$$

Example effective metrics for the probe:

D0-D6 $$d\tilde{s}^2 = -H(r)dt^2 + H^2(r)(dr^2 + r^2 d\Omega^2)$$

$$H(r) = 1 + \frac{4G_N m_A}{r} \qquad \vec{A}(r) = \frac{N_6}{2r}\frac{[\vec{r}\times\vec{n}]}{r - (\vec{r}\cdot\vec{n})}$$

We also looked at D6-D6 with fluxes

Using conservation laws, we compute, in all cases:

$$\Delta\phi = \int_{r_{min}}^{r_{max}} dr \frac{d\phi}{dr} = \int_{r_{min}}^{r_{max}} dr \frac{1}{r\sqrt{\alpha r^2 + \beta r + 1}} = 2\pi$$

No Precession!

Summary

1. 1PM approximation has so(1,2) dual conformal symmetry, even in GR

(⇔EOB)

2. • Extended no-triangle to 1/2BPS black holes in $\mathcal{N} = 8$

no precession ⇔ no-triangle

• Hope: reverse the logic : 'no-triangle' at higher-loops?

• -put higher-PM potential in N=8 in EOB form;
  can we make 'no-precession' manifest?
  - Can the symmetry be formulated at higher-points?