Do We Kerr?
Or rather do we get
Spin in Gravity?

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Outline

Consider Kerr as Particle

Mass + *Spin* =
The 2 unique features of black holes in nature.

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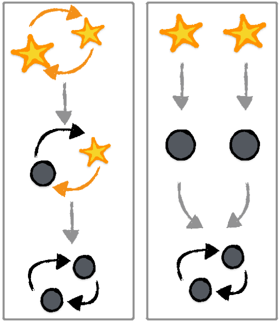


Crucial to consider the *rotating* black hole case:

- For black holes each higher post-Newtonian correction significantly affects the accuracy of the theoretical modeling of GW signal
- For rapidly rotating/near-extremal Kerr black holes corrections due to spin enter as low as the 1.5PN order [nPN$\equiv (v/c)^{2n}$]

Can we give an effective description of a rotating black hole?

Where is our center?

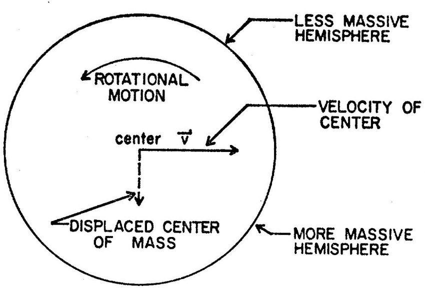
Spin is FAT:

- Special Relativity already requires a minimal finite measure, S/Mc, for the rotational velocity not to exceed the speed limit!
- Also in General Relativity, where this is the ring singularity of Kerr

In Newtonian physics a unique notion of a center, with the 3 nice properties:

1. 3-vector
2. Forms canonical pair together with the total momentum
3. Center of spatial mass distribution

No unique 'center' in relativistic physics!



Fleming 1965

Degrees of freedom

1. *Gravitational field*
   - The metric $g_{\mu\nu}(x)$
   - The tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$

2. *Particle coordinate*

   $y^{\mu}(\sigma)$ a function of an arbitrary affine parameter σ

   Particle worldline position does not in general coincide with object's 'center'

3. *Particle rotational DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma)$ + conjugate spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Lorentz matrices, $\eta^{AB}\Lambda_A{}^{a}(\sigma)\Lambda_B{}^{b}(\sigma) = \eta^{ab}$ + conjugate local spin, $S_{ab}(\sigma)$

Symmetries

1. General coordinate invariance, and parity invariance
2. Worldline reparametrization invariance
3. Internal Lorentz invariance of local frame field
4. SO(3) invariance of 'body-fixed' spatial triad
5. Spin gauge invariance, that is invariance under the choice of completion of body-fixed spatial triad through timelike vector
6. Assume isolated object has no intrinsic permanent multipole moments beyond mass (monopole) and spin (dipole), e.g. Kerr black hole

Spin as particle DOF in action

Effective action of a spinning particle

- Coordinate velocity $u^\mu \equiv dy^\mu / d\sigma$
- Worldline tetrad $e^\mu_A(\sigma)$
  $\Rightarrow$ Angular velocity $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma}$
  similar to the flat $\Omega^{ab} \equiv \Lambda^a_A \frac{d\Lambda^{Ab}}{d\sigma}$
- $\Rightarrow L_{\rm pp}\left[u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source

$$\Rightarrow S_{\rm pp} = \int d\sigma \left[-m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm NMC}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}\left(y^\mu\right)\right]\right]$$

- *Linear* momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(S^2)$
- Start with the 'covariant' gauge with spin supplementary condition (SSC):

$$e_{[0]\mu} = \frac{p^\mu}{\sqrt{p^2}}, \qquad S_{\mu\nu}p^\nu = 0$$

Introduce gauge freedom in tetrad and spin

Transform gauge of worldline tetrad

From a condition

$$e_{A\mu} q^{\mu} = \eta_{[0]A} \Leftrightarrow e_{[0]\mu} = q_{\mu}$$

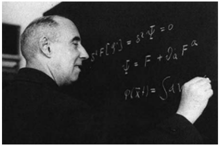
to

$$\hat{e}_{A\mu} w^{\mu} = \eta_{[0]A} \Leftrightarrow \hat{e}_{[0]\mu} = w_{\mu}$$

with a boost-like transformation in covariant form

$$\hat{e}^{A\mu} = L^{\mu}{}_{\nu}(w, q) e^{A\nu}$$

with q_{μ}, w_{μ} timelike unit 4-vectors



Ernst Stueckelberg

$\Rightarrow$ Generic gauge for the tetrad satisfies the generic SSC

$$\hat{e}_{[0]\mu} = w_{\mu}, \qquad \hat{S}^{\mu\nu}\left(p_{\nu} + \sqrt{p^2} w_{\nu}\right) = 0$$

Extra term in minimal coupling

$$\Rightarrow \hat{S}^{\mu\nu} = S^{\mu\nu} - \delta z^{\mu} p^{\nu} + \delta z^{\nu} p^{\mu}$$

$$\delta z^{\mu} = \frac{S^{\mu\rho} w_{\rho}}{\sqrt{p^2} + pw} = -\frac{\hat{S}^{\mu\rho} p_{\rho}}{p^2}, \qquad \delta z^{\mu} p_{\mu} = 0$$

⇒ Extra term in action appears!

- For minimal coupling

$$\frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} = \frac{1}{2} \hat{S}_{\mu\nu} \hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho} p_{\rho}}{p^2} \frac{D p_{\mu}}{D\sigma}$$

- Extra term with covariant derivative of momentum, contributes to finite size effects, yet carries no Wilson coefficient

- Beyond minimal coupling

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho} p^{\rho} p_{\nu}}{p^2} + \frac{\hat{S}_{\nu\rho} p^{\rho} p_{\mu}}{p^2}$$

Disentangling field and particle DOFs

Worldline tetrad contains both particle and field DOFs

- $\hat{e}_A{}^\mu = \hat{\Lambda}_A{}^b \tilde{e}_b{}^\mu$: $\eta^{AB}\hat{\Lambda}_A{}^a\hat{\Lambda}_B{}^b = \eta^{ab}$, tetrad field $\eta_{ab}\tilde{e}^a{}_\mu \tilde{e}^b{}_\nu = g_{\mu\nu}$
- For the 1st term of minimal coupling:

$$\frac{1}{2}\hat{S}_{\mu\nu}\hat{\Omega}^{\mu\nu} = \frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{LocFlat}} + \frac{1}{2}\hat{S}_{ab}\omega_\mu{}^{ab}u^\mu$$

with Ricci rotation coefficients $\omega_\mu{}^{ab} \equiv \tilde{e}^b{}_\nu D_\mu \tilde{e}^{a\nu}$

$\Rightarrow$ New rotational variables: $\hat{\Omega}^{ab}_{\text{LocFlat}} = \hat{\Lambda}^{Aa}\frac{d\hat{\Lambda}_A{}^b}{d\sigma}$, $\hat{S}_{ab}$

Separation of field from particle DOFs is not complete

- $\hat{\Lambda}_{[0]}{}^a = w^a = \tilde{e}^a{}_\mu w^\mu$ may contain further field dependence
- $\hat{S}_{0i}$ components contain further field dependence

$\Rightarrow$ Field completely disentangled from particle DOFs only once gauge for rotational variables is fixed

Fixing gauge of rotational variables

$$\hat{\Lambda}_{[0]a} = w_a, \qquad \hat{S}^{ab}\left(p_b + \sqrt{p^2}w_b\right) = 0$$

3 sensible gauges

1. The 'covariant' gauge

$$\hat{\Lambda}_{[0]a} = \frac{p_a}{\sqrt{p^2}} \Rightarrow \qquad \hat{S}^{ab}p_b = 0$$

2. The 'canonical' gauge

$$\hat{\Lambda}_{[0]}{}^{a} = \delta^a_0 \Rightarrow \qquad \hat{S}^{ab}\left(p_b + \sqrt{p^2}\delta_{0b}\right) = 0$$

   Generalization of Pryce-Newton-Wigner from flat spacetime

3. The 'no mass dipole' gauge

$$\hat{\Lambda}_{[0]}{}^{a} = \frac{2p_0\delta^a_0 - p^a}{\sqrt{p^2}} \Rightarrow \qquad \hat{S}_{a0} = 0$$

Head is spinning?

Non-minimal coupling: Under construction

Spin-induced higher multipoles

- Consider the spin vector

$$S^{\mu} \equiv *S^{\mu\nu}\frac{p_{\nu}}{\sqrt{p^2}} \simeq *S^{\mu\nu}\frac{u_{\nu}}{\sqrt{u^2}}, \qquad *S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}S^{\mu\nu}$$

$$\Rightarrow\; S_{\mu}S^{\mu} = -\frac{1}{2}S_{\mu\nu}S^{\mu\nu} \equiv -S^2$$

- Consider dependence of higher powers of spin:

$$S^{\alpha}{}_{\mu}S^{\mu}{}_{\beta} = -S^{\alpha}S_{\beta} - S^2\left(\delta^{\alpha}{}_{\beta} - \frac{u^{\alpha}u_{\beta}}{u^2}\right)$$

$$S^{\alpha}{}_{\mu}S^{\mu}{}_{\nu}S^{\nu}{}_{\beta} = -S^2 S^{\alpha}{}_{\beta}$$

$$\Rightarrow X(X+iS)(X-iS) = 0$$

$\Rightarrow$ Independent combinations: S^{μ}, $S^{\mu\nu}$, $S^{\alpha}{}_{\mu}S^{\mu}{}_{\beta} \sim S^{\alpha}S_{\beta}$.

Non-minimal coupling: Under construction

Even/odd spin-induced multipoles couple to even/odd parity electric/magnetic curvature tensors, and their covariant derivatives

Spin-induced higher multipoles

- Considering body-fixed frame: Spin multipoles are SO(3) irreps tensors
- Recall we start from 'covariant' gauge: $e_{[0]}{}^{\mu} = u^{\mu}/\sqrt{u^2}$, $e_{[i]}{}^{\mu} u_{\mu} = 0$

$\Rightarrow$ Spin-induced higher multipoles are symmetric, traceless, and spatial, constant tensors in body-fixed frame

Curvature

- Electric component $E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^{\alpha} u^{\beta}$
- Magnetic component $B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^{\gamma} u^{\delta}$

$\Rightarrow$ In vacuum they are symmetric, traceless, and orthogonal to u^{μ}, also when projected to body-fixed frame, where they are spatial

Non-minimal coupling: Under Construction

Adding covariant derivatives

- Covariant derivatives of electric/magnetic tensors also projected to body-fixed frame $D_{[i]} = e^{\mu}_{[i]} D_{\mu}$
- Time derivative $D_{[0]} = u^{\mu} D_{\mu} \equiv D/D\sigma$ can be ignored at linear order in curvature, i.e. for non-tidal effects
- In analogy to Maxwell's equations

$$\epsilon_{[ikl]} D_{[k]} E_{[lj]} = \dot{B}_{[ij]} \simeq 0$$
$$\epsilon_{[ikl]} D_{[k]} B_{[lj]} = -\dot{E}_{[ij]} \simeq 0$$

$$\Rightarrow D_{[i]} E_{[ij]} = D_{[i]} B_{[ij]} = 0, \quad \Box E_{[ij]} = \Box B_{[ij]} = 0$$

$\Rightarrow$ Indices of covariant derivatives would be symmetrized with respect to indices of electric/magnetic tensors

$\Rightarrow$ Covariant derivatives of these tensors also traceless

LO non-minimal couplings to all orders in spin

New spin-induced Wilson coefficients:

$$L_{\rm NMC} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

LO spin couplings up to 4PN order

- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu,$ Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda,$ Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa,$ Hexadecapole @4PN


WITH ROY KERR! 😍😍😍

Yes We Kerr!

What have we learned by now?

- Spin may be tricky, so handle with Kerr
- How to control our rotational DOFs
- How to disentangle field and particle DOFs
- We got the spin-induced non-minimal couplings
- How to get next the useful basics: EOMs, Hamiltonians...

Why we must learn more!

- Coupling gravity to spin differentiates candidate theories of gravity
- Kerr black hole $\leftrightarrow$ massive spin particle – to use amplitudes power


рахмат
danke
謝謝
spas
ngiyabonga
teşekkür ederim
Баярлалаа
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kia ora
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