Gravitational Radiation from Color-kinematics Duality

Chia-Hsien Shen

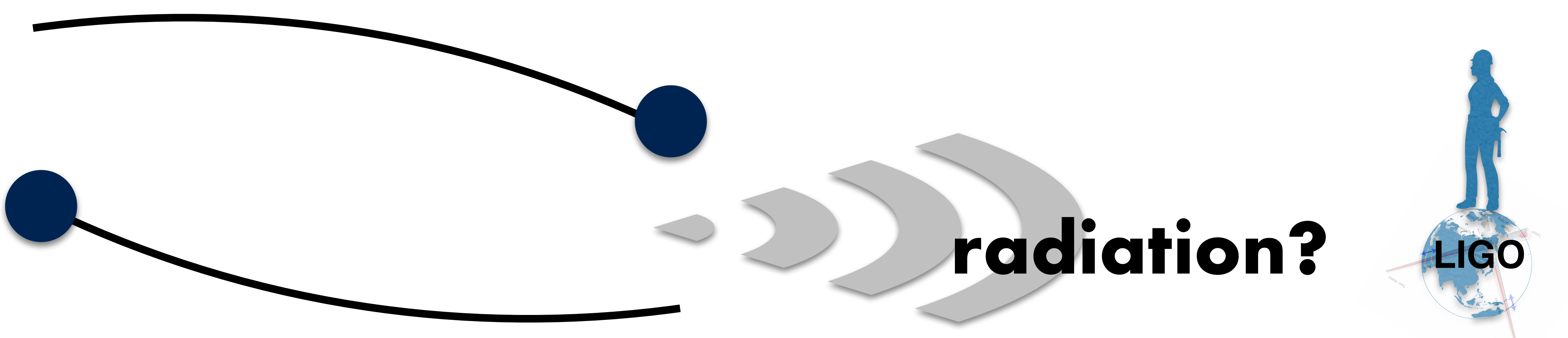
Mani L. Bhaumik Institute, UCLA

based on 1806.07388

UCLA Mani L. Bhaumik Institute for Theoretical Physics

Simple Problem

- Relativistic scattering of *classical point sources* in post-Minkowskian regime
  - blackholes, neutron stars, etc.
  - hyperbolic orbit + perturbation in G



Simple Strategy

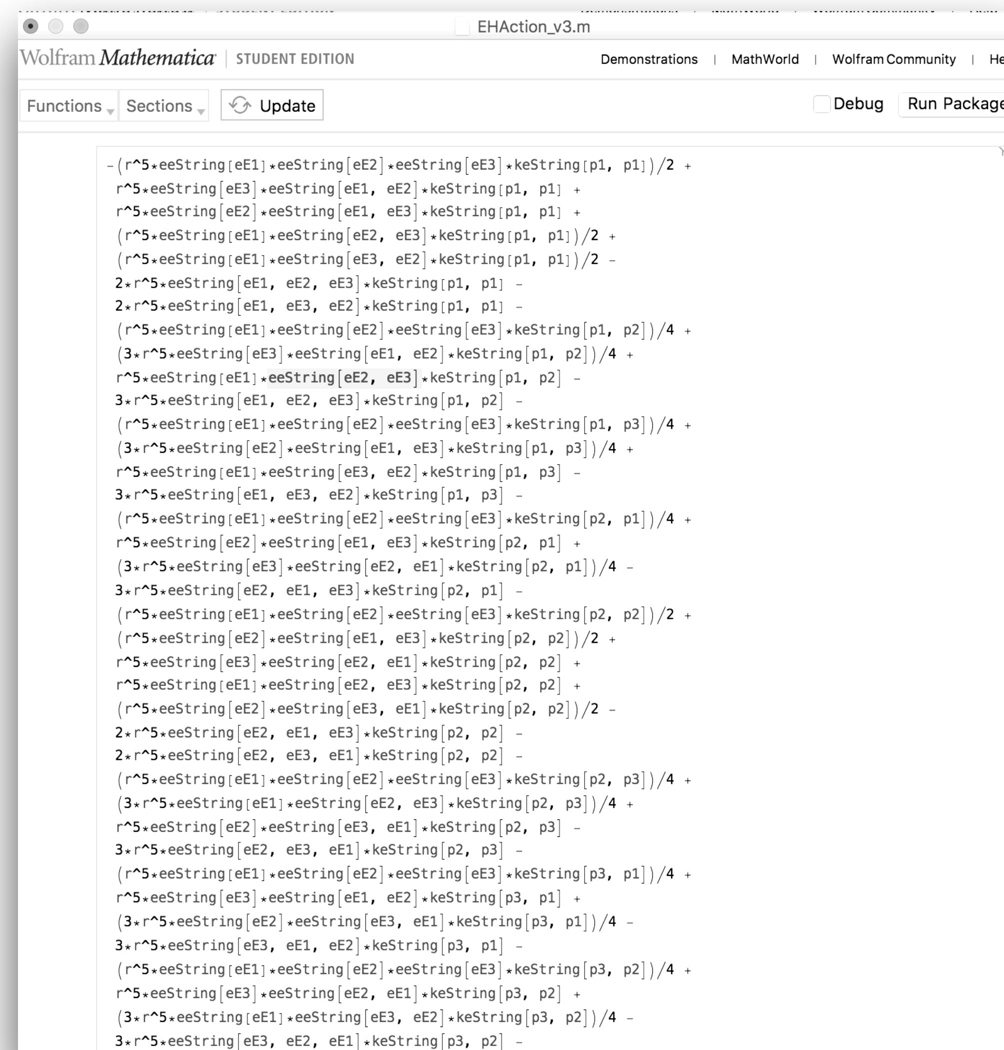
- Action:
    - Consistent with symmetries, diffeomorphism, reparametrization invariance, etc.
- Classical equations of motion
- Deviation from straight-line order by order

… not so simple calculation

- Even 3pt vertex ~ 100 terms
- *Infinite* tower

  of higher point vertices!
- Modern solution to

  this classical problem?



Simplicity in Gravity?

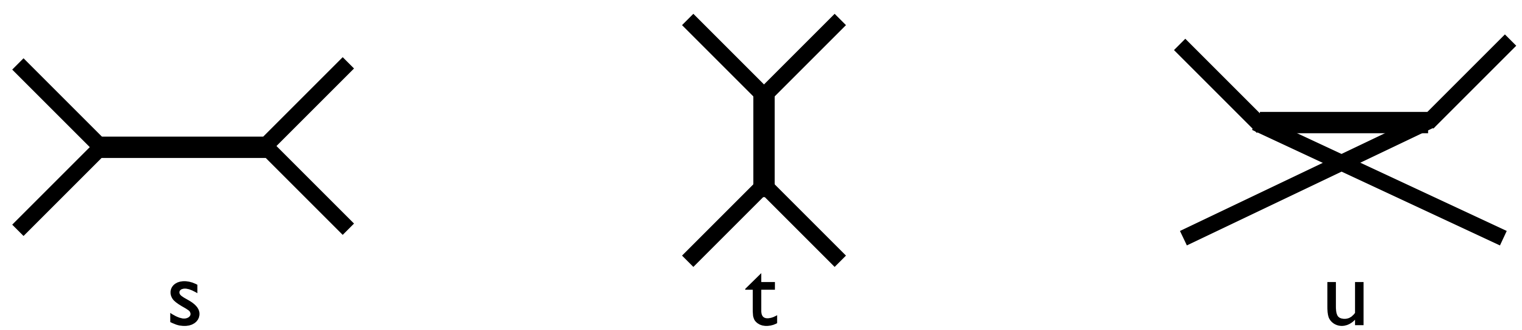
- On-shell 3pt amplitudes: *simple!*

$$\mathcal{A}_{\mathrm{GR}} = (p_1 e_3 + p_2 e_1 + p_3 e_2)^2$$

$$\mathcal{A}_{\mathrm{YM}} = \underbrace{f^{abc}}_{\text{color}} \times \underbrace{(p_1 e_3 + p_2 e_1 + p_3 e_2)}_{\text{kinematics}}$$

Gravity = Yang-Mills^2

- **Color-kinematics** duality: [See Bern's talk]



$$\begin{cases} C_s + C_t + C_u = 0 \\ N_s + N_t + N_u = 0 \end{cases}$$

YM **GR**

$$\mathcal{A} = \sum_{\text{cubic}} \frac{C_i\, N_i}{\prod P^2} \quad \Rightarrow \quad \mathcal{M} = \sum_{\text{cubic}} \frac{N_i\, \tilde{N}_i}{\prod P^2}$$

[Bern, Carrasco, Johansson '08]

[Bern, Carrasco, Johansson; +Dennen, Huang, Kiermaier '10]

QCD meets **Classical** Gravity?

- Double Copy in Exact Classical Solutions
  [Monteiro, O'Connell, White; Luna, Nicholson, Berman, Chacon, Bahjat-Abbas; Ridgway, Wise; Carrillo-Gonzalez, Penco, Trodden,...]

See Bern's review;
talks by Steinhoff, Gonzalez, Nicholson, Casali

- Double Copy in Perturbative Classical Radiation
  [Goldberger, Ridgway, Prabhu, Thompson, Li; Luna, Nicholson, Ochirov, O'Connell, Westerberg, White; Chester; Carrillo-Gonzalez, Penco, Trodden; CHS; Adamo, Casali, Mason, Nekovar]
  - Accessible to study higher order/non-linear effects
  - Accessible to study broader range of theories/sources

Outline

- Setup and Review
- Next-to-leading Order
- Double Copy
- Conclusion & Outlook

Setup & Review

Theories

$$A = \sum \frac{N_i \tilde{N}_i}{D_i}$$

Gravity = kinematics^2

Yang-Mills = color x kinematics

Bi-adjoint scalar = color^2

See Bern's and Roiban's talk for the full zoology

Theories

Theory	Radiation	Point sources DoF
Dilaton Gravity	graviton+dilaton	trajectory
Yang-Mills	gluon	trajectory + charge
Bi-adjoint	bi-adjoint scalar	trajectory + charge/dual charge

[Goldberger, Ridgway '16]
[Goldberger, Prabhu, Thompson '17]

Theories

$$S_{pp} = -m^2 \int d\tau' e^{\kappa\phi}$$

$$S_{\rm GR} = \int d^dx \sqrt{g} \left[-\frac{2}{\kappa^2} R + 2(d-2)\, g^{\mu\nu} D_\mu \phi D_\nu \phi \right]$$

$$S_{pp} = -m^2 \int d\tau' + \int d\tau' \, \psi^\dagger i p \cdot D \psi$$

$$S_{\rm YM} = -\frac{1}{4} \int d^dx \, F^{a\mu\nu} F^a_{\mu\nu}$$

$$S_{pp} = -m^2 \int d\tau' + \int d\tau' \, \left(\psi^\dagger i p \cdot \partial \psi + y \phi^{a\tilde{a}} c^a \tilde{c}^{\tilde{a}} \right)$$

$$S_{\rm BS} = \int d^dx \left(\frac{1}{2} (\partial_\mu \phi^{a\tilde{a}})^2 - \frac{y}{3} f^{abc} \tilde{f}^{\tilde{a}\tilde{b}\tilde{c}} \phi_{a\tilde{a}} \phi_{b\tilde{b}} \phi_{c\tilde{c}} \right)$$

$\Rightarrow$

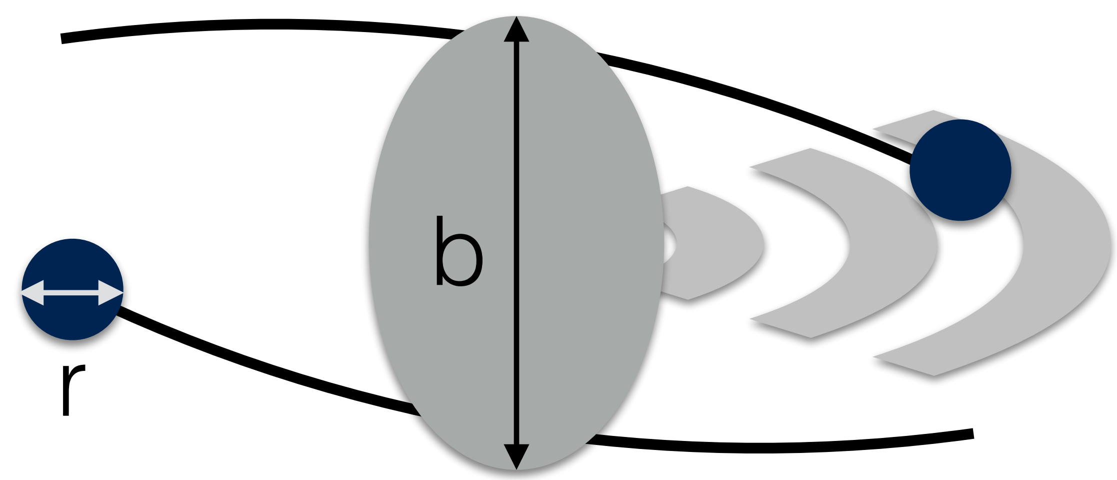
EoM for

$x^\mu(\tau), c^a(\tau), \tilde{c}^{\tilde{a}}(\tau)$

$h_{\mu\nu}, A^{a\mu}, \phi^{a\tilde{a}}$

Scales

Internal size	$r \gtrsim GM$
Orbital size	$b \sim 1/q$
Mass	M
Coupling	G



- Point source approximation: $r \sim GM \ll b$

  Classical angular momentum: $M\, b \gg 1 \quad (M \gg q)$

Observables

- Observables: radiation at spatial infinity (r → ∞) **on-shell** (k^2=0) and **transverse** components

$$A^{\pm a}(t, \vec{n}) = \frac{1}{4\pi r} \int \frac{d\omega}{2\pi} e^{-i\omega t_r} \lim_{k^2 \to 0} \left[-k^2 \left(e^{\pm}_{\mu} \cdot A^{\mu a}(k) \right) \right]$$

can be viewed as the amplitude

- GR=YM^2 only refer to this observable part!

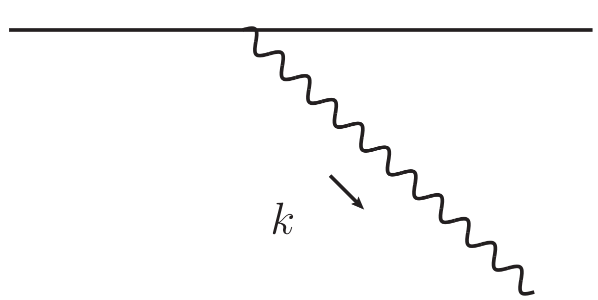
Example: YM

- EoM for radiation:

$$\Box A^{\mu a}(x) = -\frac{\delta S_{int}}{\delta A^a_\mu} + g\, J^{\mu a}(x)$$

Berends-Giele Recursion in QCD

$$\begin{aligned} A^\mu_a(k) &= -\frac{g}{k^2}\int d\tau'\, e^{ik\cdot(b+p\tau')}\, c^a p^\mu \\ &= -\frac{g}{k^2}\, 2\pi\delta(k\cdot p)\, e^{ik\cdot b}\, c^a p^\mu, \end{aligned}$$



Example: YM

- EoM for radiation:

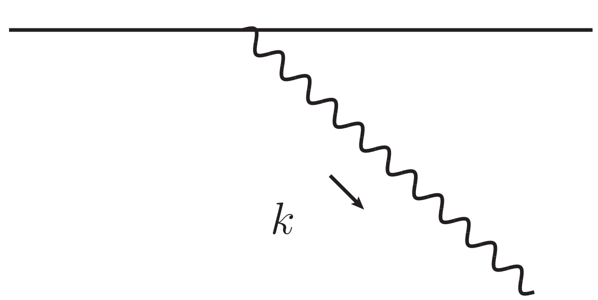
$$\Box A^{\mu a}(x) = -\frac{\delta S_{int}}{\delta A^a_\mu} + g\, J^{\mu a}(x)$$

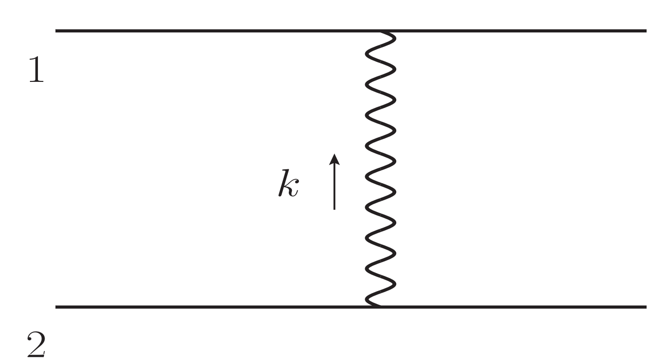
- EoM for worldline:

$$\frac{d^2 x^\mu}{d\tau'^2} = g\, p^\nu\ \left(c \cdot F^\mu{}_\nu\right)$$
$$\frac{dc^a}{d\tau'} = ig\, p^\mu\, [c, A_\mu]$$

$$A^\mu_a(k) = -\frac{g}{k^2}\int d\tau'\, e^{ik\cdot(b+p\tau')}\, c^a p^\mu$$
$$= -\frac{g}{k^2}\, 2\pi\delta(k\cdot p)\, e^{ik\cdot b}\, c^a p^\mu,$$

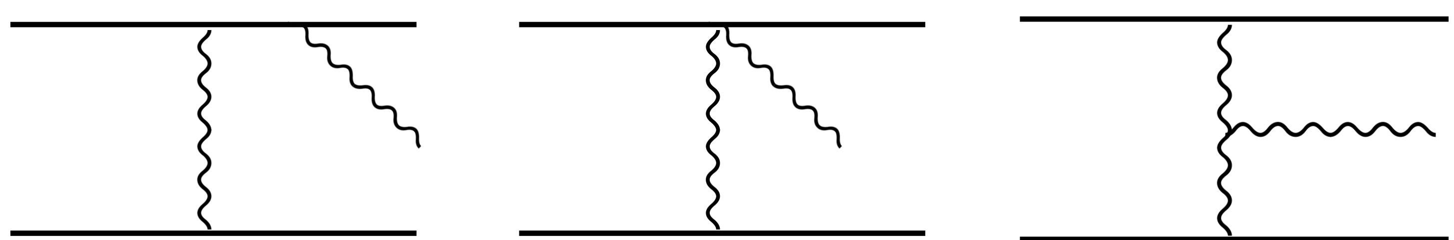
$$\delta x^\mu \propto \frac{e^{-ik\cdot(b+p\tau')}}{(k\cdot p)^2}\, p^\nu\ \left(c \cdot F^\mu{}_\nu\right)$$
$$\delta c^a \propto \frac{e^{-ik\cdot(b+p\tau')}}{(ik\cdot p)}\, p^\nu\, [c, A_\nu]$$



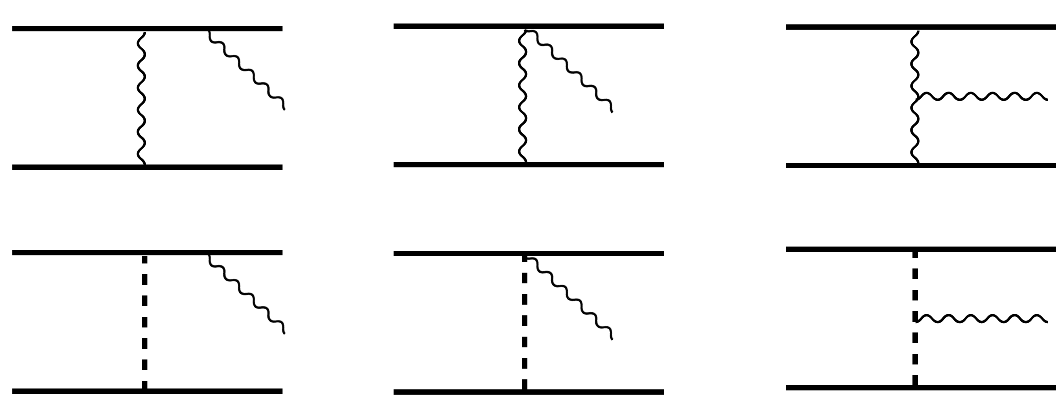


Leading Order

- Pioneered by Goldberger and Ridgway
- YM:



- Gravity+dilaton:



Leading Order

- Goldberger and Ridgway: beautiful relation from YM to GR

$$\boxed{\begin{aligned} c^a &\to p^\mu \\ f^{abc} &\to V_{\mathrm{YM}}^{\mu\nu\rho} \end{aligned}}$$

- Reproduced by S-matrix double copy in classical limit
  [Luna, Nicholson, Monteiro, O'Connell, White '17]

- Generalization to bound states: quadrupole & potential
  [Goldberger and Ridgway '17; Steinhoff, Plefka, Wormsbecker'18]

- Generalization to Einstein gravity, spinning sources, scalar theories
  [Goldberger, Prabhu, Thompson, Li; Chester '17; Carrillo-Gonzalez, Penco, Trodden '18]

See Steinhoff's and Gonzalez's talk

QCD meets Classical Gravity?

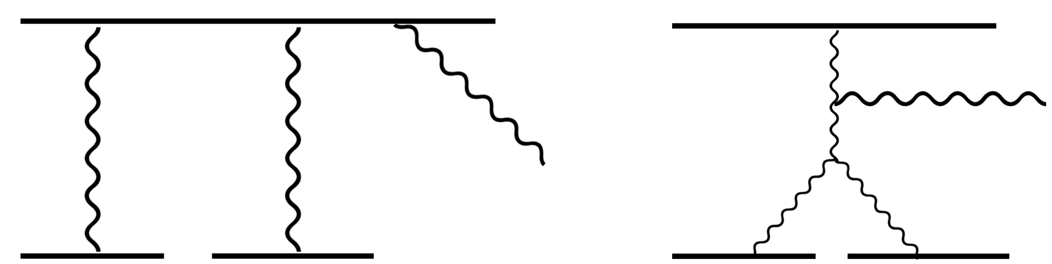
- Higher orders are crucial for connection to LIGO
- Challenges at NLO
  - Systematic construction? Gravity for *free*?
  - What are “propagators” ?
  - Color-kinematics duality?

UCLA Mani L. Bhaumik Institute for Theoretical Physics

Next-to-leading Order

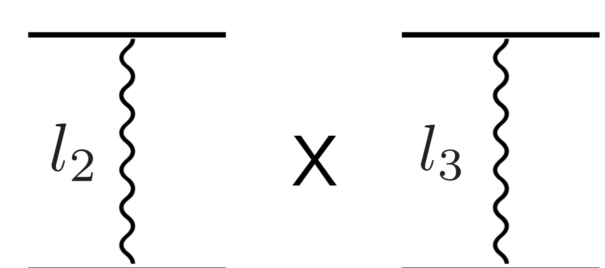
NLO

- ~ one-loop process



- For classical EoM, one more source inserted
    - Convenient to imagine this as a 3-body problem
- No clean diagrammatic expression

$$\delta(e^{ikx}) = ikx^{(2)} + \frac{1}{2!}\left(ikx^{(1)}\right)^2$$



Colors

- 21 color structures

$C_1 = (c_1 \cdot c_2)(c_1 \cdot c_3)c_1^a$	$C_7 = (c_1 \cdot c_2)(c_2 \cdot c_3)c_1^a$	$C_{13} = (c_1 \cdot c_3)\,[c_1, c_2]^a$	$C_{19} = [c_1,\, [c_2,\, c_3]]^a$
$C_2 = (c_1 \cdot c_2)(c_2 \cdot c_3)c_2^a$	$C_8 = (c_1 \cdot c_3)(c_2 \cdot c_3)c_1^a$	$C_{14} = (c_2 \cdot c_3)\,[c_1, c_2]^a$	$C_{20} = [c_2,\, [c_3,\, c_1]]^a$
$C_3 = (c_1 \cdot c_3)(c_2 \cdot c_3)c_3^a$	$C_9 = (c_1 \cdot c_2)(c_1 \cdot c_3)c_2^a$	$C_{15} = (c_1 \cdot c_2)\,[c_2, c_3]^a$	$C_{21} = [c_3,\, [c_1,\, c_2]]^a$
$C_4 = (c_1 \cdot [c_2, c_3])\; c_1^a$	$C_{10} = (c_1 \cdot c_3)(c_2 \cdot c_3)c_2^a$	$C_{16} = (c_1 \cdot c_3)\,[c_2, c_3]^a$	
$C_5 = (c_1 \cdot [c_2, c_3])\; c_2^a$	$C_{11} = (c_1 \cdot c_2)(c_2 \cdot c_3)c_3^a$	$C_{17} = (c_1 \cdot c_2)\,[c_3, c_1]^a$	
$C_6 = (c_1 \cdot [c_2, c_3])\; c_3^a$	$C_{12} = (c_1 \cdot c_2)(c_1 \cdot c_3)c_3^a$	$C_{18} = (c_2 \cdot c_3)\,[c_3, c_1]^a$	

- Jacobi identity

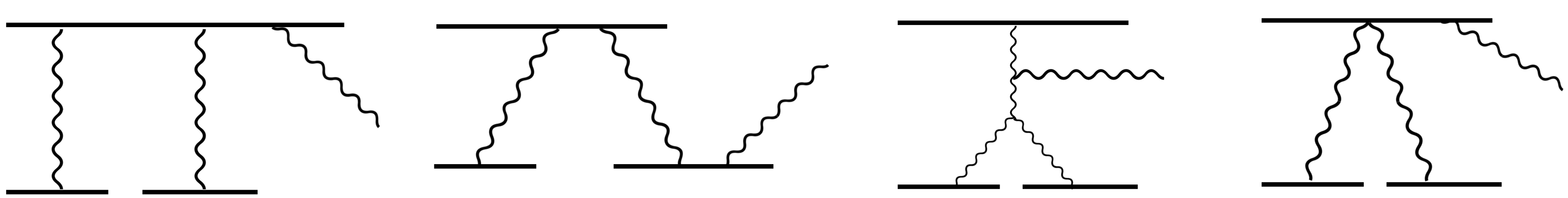
$$[c_1,\, [c_2, c_3]]^a + [c_2,\, [c_3, c_1]]^a + [c_3,\, [c_1, c_2]]^a = 0$$

Results

- All the trajectories, colors, and radiation are calculated at NLO
  - Complexity:

BS	YM	GR
20kb	80kb	

  - Checked Ward identity, conservation laws



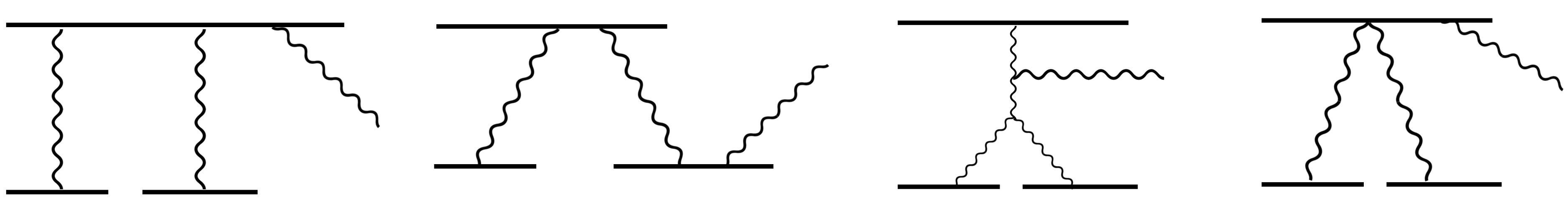
....

Results

- All the trajectories, colors, and radiation are calculated at NLO
  - Complexity:

BS	YM	GR
20kb	80kb	70Mb

  - Checked Ward identity, conservation laws



....

Bi-adjoint amplitudes

- Partial amplitudes in bi-adjoint scalar theory

$$\mathcal{J}^{a\tilde{a}}_{\mathrm{BS}}(2,C_7) = \frac{l_{123}\cdot l_{23}}{l_3^2 l_{23}^2\,(l_3\cdot p_2)\,(p_1\cdot l_{23})^2}\left(\tilde{C}_4+\frac{l_3\cdot l_{23}}{l_3\cdot p_2}\,\tilde{C}_7\right)+\frac{1}{l_3^2 l_{23}^2\,(l_3\cdot p_2)\,(p_1\cdot l_{23})}\left(\tilde{C}_{19}+\frac{l_3\cdot l_{23}}{l_3\cdot p_2}\tilde{C}_{14}\right), \tag{52}$$

$$\begin{aligned}\mathcal{J}^{a\tilde{a}}_{\mathrm{BS}}(2,C_{13}) = &-\frac{1}{l_2^2 l_3^2\,(l_2\cdot p_1)\,(l_3\cdot p_1)}\left(\tilde{C}_{17}-\frac{l_{123}\cdot l_3}{l_3\cdot p_1}\tilde{C}_1\right)-\frac{1}{l_2^2 l_3^2\,(l_3\cdot p_1)\,(p_1\cdot l_{23})}\left(\tilde{C}_4+\frac{l_2\cdot l_3}{l_3\cdot p_1}\tilde{C}_1\right)\\ &+\frac{1}{l_3^2 l_{13}^2\,(l_3\cdot p_1)\,(p_2\cdot l_{13})}\left(\tilde{C}_5-\frac{l_3\cdot l_{13}}{l_3\cdot p_1}\,\tilde{C}_9\right)-\frac{2}{l_2^2 l_3^2 l_{13}^2\,(l_3\cdot p_1)}\left(\tilde{C}_{20}+\frac{l_3\cdot l_{13}}{l_3\cdot p_1}\,\tilde{C}_{13}\right).\end{aligned} \tag{53}$$

- Double poles, simple poles, composite poles,… kinematic numerators…

Double Copy

Double Copy

- Gauge invariance:
  - Need to impose color-kinematics duality

$$C_i \pm C_j = 0 \quad \leftrightarrow \quad N_i \pm N_j = 0$$

$$C_i \pm C_j \pm C_k = 0 \quad \leftrightarrow \quad N_i \pm N_j \pm N_k = 0$$

Double Copy

- Gauge invariance: *color-kinematics duality*
- Locality:
    - Fuzzy for classical radiation, even nontrivial structure for bi-adjoint theory!
    - Still, the structure should be universal

$$\Box A^{\mu a}(x) = -\frac{\delta S_{int}}{\delta A^a_\mu} + g\, J^{\mu a}(x) \qquad \frac{d^2 x^\mu}{d\tau'^2} = g\, p^\nu\ (c \cdot F^\mu{}_\nu)$$

$$\frac{dc^a}{d\tau'} = ig\, p^\mu\, [c, A_\mu]$$

Double Copy

- Gauge invariance: *color-kinematics duality*
- Locality: *build from bi-adjoint scalar theory*
  - Fuzzy for classical radiation, even nontrivial structure for bi-adjoint theory!
  - Still, the structure should be universal

$$\boxed{\Box A^{\mu a}(x)} = -\frac{\delta S_{int}}{\delta A^a_\mu} + g\, J^{\mu a}(x)$$

$$\boxed{\begin{aligned} \frac{d^2 x^\mu}{d\tau'^2} \\ \frac{dc^a}{d\tau'} \end{aligned}} \begin{aligned} &= g\, p^\nu \;(c \cdot F^\mu{}_\nu) \\ &= ig\, p^\mu \,[c, A_\mu] \end{aligned}$$

Simple Strategy

1. Build the propagator matrix from bi-adjoint theory

$$A_{\mathrm{BS}}(n) = (-1)^n \sum_{i,j} C_i \, P_{ij} \, \tilde{C}_j$$

not just $1/p^2$

Simple Strategy

1. Build the propagator matrix from bi-adjoint theory

$$A_{\mathrm{BS}}(n) = (-1)^n \sum_{i,j} C_i \, P_{ij} \, \tilde{C}_j$$

$$A_{\mathrm{YM}}(n) = \sum_{i,j} C_i \, P_{ij} \, N_j$$

2. Impose color-kinematics duality for gauge invariance

$$C_i \pm C_j = 0 \quad \leftrightarrow \quad N_i \pm N_j = 0$$

$$C_i \pm C_j \pm C_k = 0 \quad \leftrightarrow \quad N_i \pm N_j \pm N_k = 0$$

Simple Strategy

3. Gravity for free from YM^2

$$A_{\mathrm{BS}}(n) = (-1)^n \sum_{i,j} C_i \, P_{ij} \, \tilde{C}_j$$

$$A_{\mathrm{YM}}(n) = \sum_{i,j} C_i \, P_{ij} \, N_j$$

$$C_i \pm C_j = 0 \quad \leftrightarrow \quad N_i \pm N_j = 0$$

$$C_i \pm C_j \pm C_k = 0 \quad \leftrightarrow \quad N_i \pm N_j \pm N_k = 0$$

$$A_{\mathrm{GR}}(n) \stackrel{?}{=} (-1)^n \sum_{i,j} N_i \, P_{ij} \, \tilde{N}_j ,$$

Double Copy

- Perfectly works at next-to-leading order

$$A_{\mathrm{BS}}(n) = (-1)^n \sum_{i,j} C_i\, P_{ij}\, \tilde{C}_j$$

$$A_{\mathrm{YM}}(n) = \sum_{i,j} C_i\, P_{ij}\, N_j$$

$$A_{\mathrm{GR}}(n) = (-1)^n \sum_{i,j} N_i\, P_{ij}\, \tilde{N}_j,$$

$$C_i \pm C_j = 0 \quad \leftrightarrow \quad N_i \pm N_j = 0$$

$$C_i \pm C_j \pm C_k = 0 \quad \leftrightarrow \quad N_i \pm N_j \pm N_k = 0$$

Double Copy

- Perfectly works at NLO: *hidden simplicity*

$$A_{\mathrm{BS}}(n) = (-1)^n \sum_{i,j} C_i\, P_{ij}\, \tilde{C}_j$$

$$A_{\mathrm{YM}}(n) = \sum_{i,j} C_i\, P_{ij}\, N_j$$

$$A_{\mathrm{GR}}(n) = (-1)^n \sum_{i,j} N_i\, P_{ij}\, \tilde{N}_j,$$

$$\begin{aligned}
N_1 &= (p_1 \cdot p_2)(p_1 \cdot p_3)(p_1 \cdot e) \\
N_4 &= \Big((p_1 \cdot [p_3, p_2] \cdot l_{23}) + \tfrac{1}{2}(p_2 \cdot p_3)(p_1 \cdot q_{23})\Big)\,(p_1 \cdot e) + \tfrac{1}{2}\,(p_1 \cdot l_{23})\,(p_1 \cdot [p_2, p_3] \cdot e) \\
N_7 &= (p_1 \cdot p_2)(p_2 \cdot p_3)(p_1 \cdot e) \\
N_{13} &= (p_1 \cdot p_3)\,((l_{123} \cdot [p_1, p_2] \cdot e) - (p_1 \cdot p_2)(l_2 \cdot e)) \\
N_{19} &= \Big((l_1 \cdot [p_2, p_3] \cdot l_{23}) - \tfrac{1}{2}(p_2 \cdot p_3)(l_{123} \cdot q_{23})\Big)\,(p_1 \cdot e) + (p_1 \cdot l_{23})\,(l_{23} \cdot [p_2, p_3] \cdot e) \\
&\quad + (p_1 \cdot [p_2, p_3] \cdot l_{23})\,(l_{23} \cdot e) - (p_2 \cdot p_3)\,(p_1 \cdot [l_2, l_3] \cdot e) - \tfrac{1}{2}(l_{23} \cdot l_{123})\,(p_1 \cdot [p_2, p_3] \cdot e)
\end{aligned}$$

Color-kinematics Duality

- Leading order:

$$c^a \to p^\mu$$

$$f^{abc} \to V_{\mathrm{YM}}^{\mu\nu\rho}$$

- Non-trivially generalized at NLO

C_1	$= (c_1 \cdot c_2)(c_1 \cdot c_3) c_1^a$
C_4	$= (c_1 \cdot [c_2, c_3])\ c_1^a$
C_7	$= (c_1 \cdot c_2)(c_2 \cdot c_3) c_1^a$
C_{13}	$= (c_1 \cdot c_3)\,[c_1, c_2]^a$
C_{19}	$= [c_1,\ [c_2,\ c_3]]^a$

$N_1 =$	$(p_1 \cdot p_2)(p_1 \cdot p_3)(p_1 \cdot e)$
$N_4 =$	$\big((p_1 \cdot [p_3, p_2] \cdot l_{23}) + \frac{1}{2}(p_2 \cdot p_3)(p_1 \cdot q_{23})\big)\,(p_1 \cdot e) + \frac{1}{2}\,(p_1 \cdot l_{23})\,(p_1 \cdot [p_2, p_3] \cdot e)$
$N_7 =$	$(p_1 \cdot p_2)(p_2 \cdot p_3)(p_1 \cdot e)$
$N_{13} =$	$(p_1 \cdot p_3)\,((l_{123} \cdot [p_1, p_2] \cdot e) - (p_1 \cdot p_2)(l_2 \cdot e))$
$N_{19} =$	$\big((l_1 \cdot [p_2, p_3] \cdot l_{23}) - \frac{1}{2}(p_2 \cdot p_3)(l_{123} \cdot q_{23})\big)\,(p_1 \cdot e) + (p_1 \cdot l_{23})\,(l_{23} \cdot [p_2, p_3] \cdot e)$ $+ (p_1 \cdot [p_2, p_3] \cdot l_{23})\,(l_{23} \cdot e) - (p_2 \cdot p_3)\,(p_1 \cdot [l_2, l_3] \cdot e) - \frac{1}{2}(l_{23} \cdot l_{123})\,(p_1 \cdot [p_2, p_3] \cdot e)$

Color-kinematics Duality

- Leading order:

$$c^a \to p^\mu$$

$$f^{abc} \to V_{\text{YM}}^{\mu\nu\rho}$$

- Non-trivially generalized at NLO

$$
\begin{aligned}
C_1 &= (c_1 \cdot c_2)(c_1 \cdot c_3) c_1^a \\
C_4 &= (c_1 \cdot [c_2, c_3])\ c_1^a \\
C_7 &= (c_1 \cdot c_2)(c_2 \cdot c_3) c_1^a \\
C_{13} &= (c_1 \cdot c_3)\, [c_1, c_2]^a \\
C_{19} &= [c_1,\ [c_2,\ c_3]]^a
\end{aligned}
$$

$$
\begin{aligned}
N_1 &= (p_1 \cdot p_2)(p_1 \cdot p_3)(p_1 \cdot e) \\
N_4 &= \Big((p_1 \cdot [p_3, p_2] \cdot l_{23}) + \tfrac{1}{2}(p_2 \cdot p_3)(p_1 \cdot q_{23})\Big)\,(p_1 \cdot e) + \tfrac{1}{2}\,(p_1 \cdot l_{23})\,(p_1 \cdot [p_2, p_3] \cdot e) \\
N_7 &= (p_1 \cdot p_2)(p_2 \cdot p_3)(p_1 \cdot e) \\
N_{13} &= (p_1 \cdot p_3)\,((l_{123} \cdot [p_1, p_2] \cdot e) - (p_1 \cdot p_2)(l_2 \cdot e)) \\
N_{19} &= \Big((l_1 \cdot [p_2, p_3] \cdot l_{23}) - \tfrac{1}{2}(p_2 \cdot p_3)(l_{123} \cdot q_{23})\Big)\,(p_1 \cdot e) + (p_1 \cdot l_{23})\,(l_{23} \cdot [p_2, p_3] \cdot e) \\
&\quad + (p_1 \cdot [p_2, p_3] \cdot l_{23})\,(l_{23} \cdot e) - (p_2 \cdot p_3)\,(p_1 \cdot [l_2, l_3] \cdot e) - \tfrac{1}{2}(l_{23} \cdot l_{123})\,(p_1 \cdot [p_2, p_3] \cdot e)
\end{aligned}
$$

Color-kinematics Duality

- Leading order:

$$c^a \to p^\mu$$

$$f^{abc} \to V^{\mu\nu\rho}_{\rm YM}$$

- Non-trivially generalized at NLO

- Still, it can be chosen s.t.

$$c_i \cdot c_j \to p_i \cdot p_j$$

Another example of kinematic algebra?

[Monteiro, O'Connell '11, '13; Cheung, CHS, '16; Cheung, Remmen, CHS, Wen '17; Fu, Vanhove, Wang '18]

BCJ and KLT relations

- BCJ relations: non-trivial consequence of the duality

$$A_{\mathrm{YM}}(n) = \sum_{i,j} C_i \, P_{ij} \, N_j$$

- **e.g. LO:** $(p_1 \cdot l_2) A(1, C_1) + (p_2 \cdot l_1) A(1, C_2) + \frac{1}{2}\left(l_1^2 - l_2^2\right) A(1, C_3) = 0.$

- KLT relations: GR= YM^2 at physical level

$$\begin{aligned} A_{\mathrm{GR}}(n) &= (-1)^n \sum_{i,j} \underline{N_k \, P_{ki}} \, P^{-1}_{ij} \, \underline{P_{jl} \, N_l} \\ &= (-1)^n \sum_{i,j} \underline{A_{\mathrm{YM}}(n, C_i)} \, S_{ij} \, \underline{A_{\mathrm{YM}}(n, C_j)}, \end{aligned}$$

- **e.g. LO:** $S_{11} = l_2^2 (p_1 \cdot l_2)^2 / (l_2 \cdot l_{12}), \; S_{22} = l_1^2 (p_2 \cdot l_1)^2 / (l_1 \cdot l_{12}),$

Conclusion & Outlook

Conclusion

- In the **classical** radiation @ **NLO**, we find

$$A_{\mathrm{BS}} \xrightarrow{\tilde{C}_i \to N_i} A_{\mathrm{YM}} \xrightarrow{C_i \to N_i} A_{\mathrm{GR}}$$

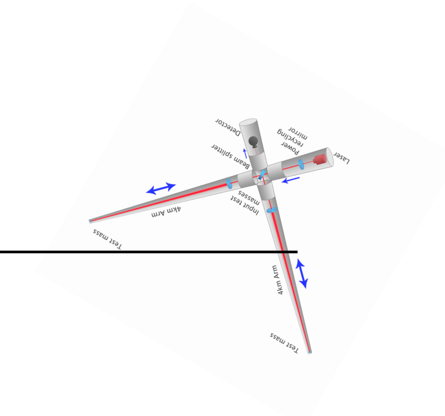
$$C_i \pm C_j \pm C_k = 0 \quad \leftrightarrow \quad N_i \pm N_j \pm N_k = 0$$

☑ Gravity for free; Systematic approach works

☑ Color-kinematics duality w/ BCJ + KLT relations

☑ Hidden simplicity

From LHC to LIGO

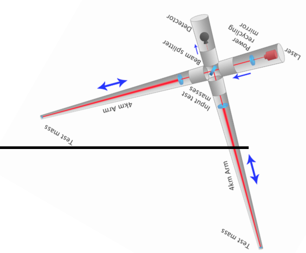


- Double copy for Einstein gravity?

- Bound orbit?

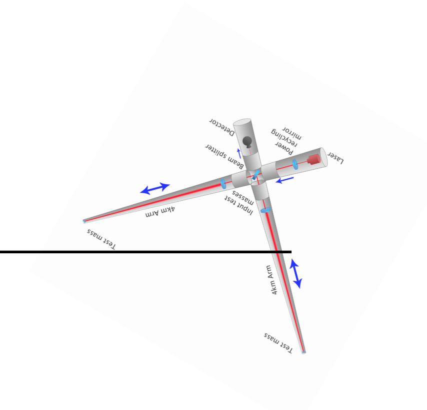
- Higher order?

From LHC to LIGO



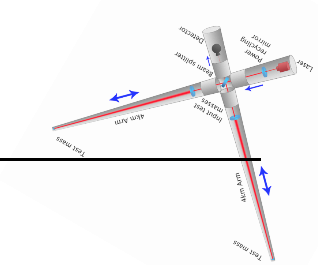
- Double copy for Einstein gravity?
    - Done at LO by adding “ghost” in YM
      [Luna, Nicholson, Monteiro, O’Connell, White ’17]
- Bound orbit?
- Higher order?

From LHC to LIGO



- Double copy for Einstein gravity?
  - Done at LO by adding “ghost” in YM
    [Luna, Nicholson, Monteiro, O’Connell, White ’17]
- Bound orbit?
  - Direct confirmation at LO [Goldberger and Ridgway ’17]
  - Universality: match to EFT action via S-matrix — see Cheung’s talk
    [Neill, Rothstein ’13; Vaidya ’14; Cheung, Rothstein, Solon ’18; Bern, Cheung, Roiban, CHS, Solon, Zeng]
- Higher order?

From LHC to LIGO



- Double copy for Einstein gravity?
  - Done at LO by adding "ghost" in YM
    [Luna, Nicholson, Monteiro, O'Connell, White '17]
- Bound orbit?
  - Direct confirmation at LO [Goldberger and Ridgway '17]
  - Universality: match to EFT action via S-matrix — see Cheung's talk
    [Neill, Rothstein '13; Vaidya '14; Cheung, Rothstein, Solon '18; Bern, Cheung, Roiban, CHS, Solon, Zeng]
- Higher order?
  - Understand the connection to S-matrix — see O'Connell's talk; Proof?
    [Luna, Nicholson, Monteiro, O'Connell, White '17; Kosower, Maybee, O'Connell '18]

UCLA Mani L. Bhaumik Institute for Theoretical Physics

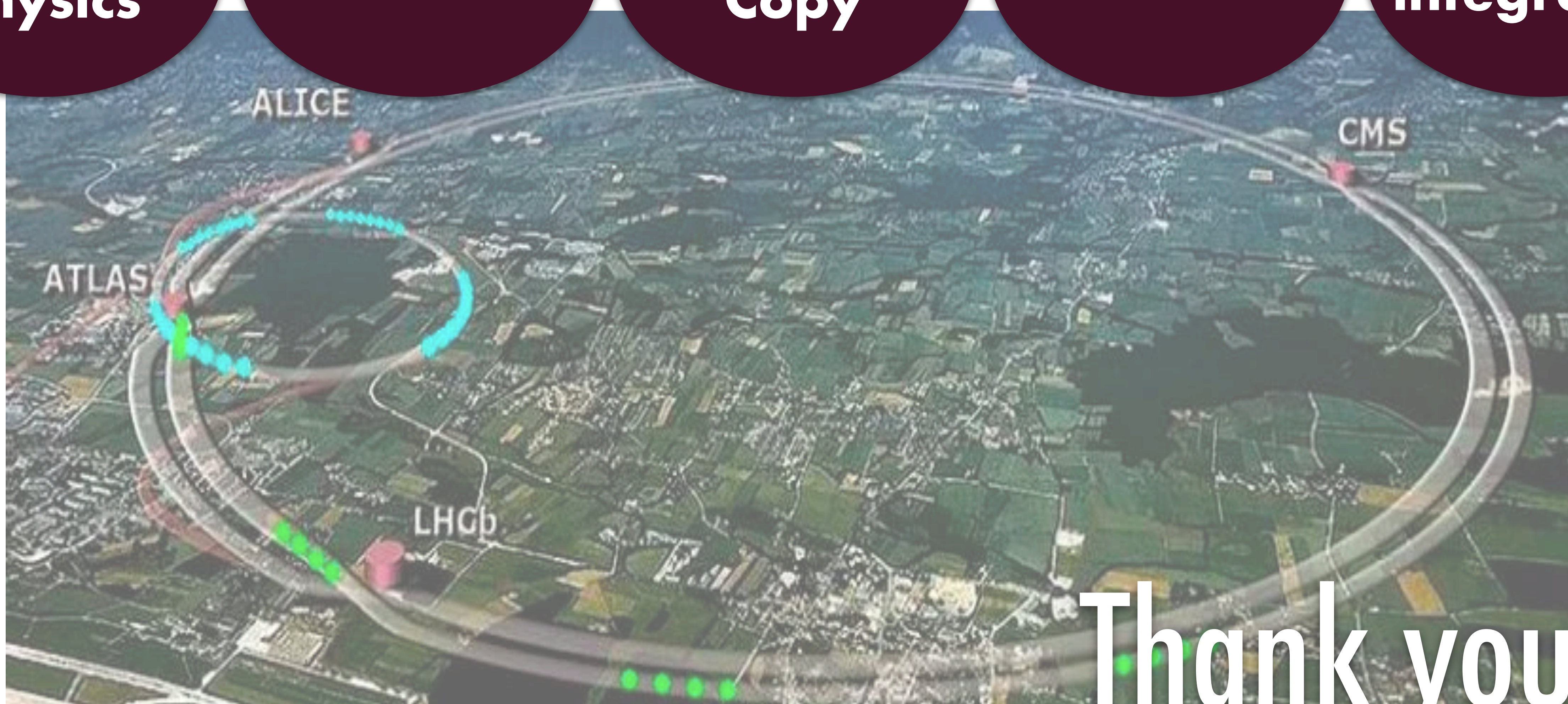
Soft physics

EFT

Double Copy

Unitarity

Loop integration



Thank you

Backups

Perturbative Solutions

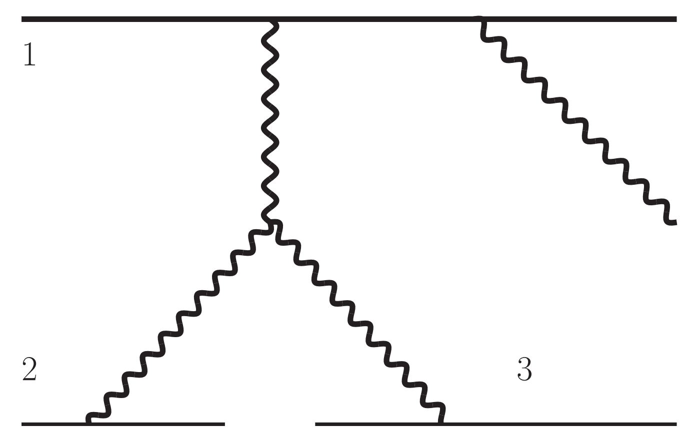
- General structure:

$$A^{\mu a}(k) = \sum_n \left(-\frac{g^{2n+1}}{k^2} \sum_{\text{perm}} \underbrace{\int_1 \int_2 \cdots \int_{n+1}}_{\text{phase space}} \underbrace{(2\pi)^d \delta^d\Big(k - \sum_{i=1}^{n+1} l_i\Big)}_{\text{Mom. conservation}} \underbrace{\mathcal{J}^{\mu a}(n)}_{\textbf{“Integrand”}} \right)$$

$$\int_i = \int \frac{d^d l_i}{(2\pi)^d} \underbrace{2\pi\delta(p_i \cdot l_i)}_{\text{“on-shell”}} \underbrace{e^{i l_i b_i}}_{\text{phase shift.}}$$

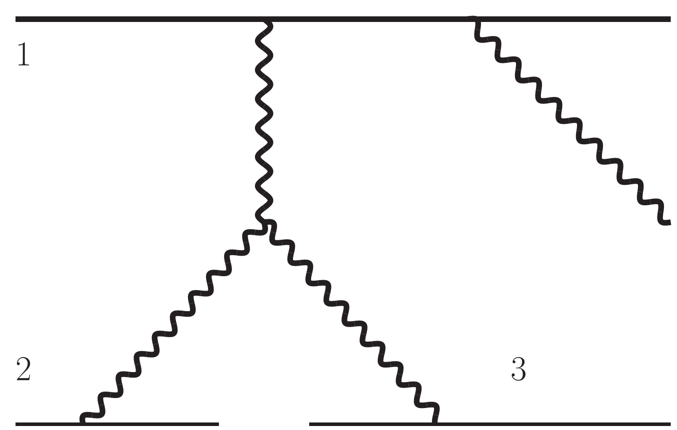
Color-kinematics Duality

- Zero in YM but not zero in GR?



Color-kinematics Duality

- Zero in YM but not zero in GR?



- Well-known examples in multi-loop gravity
  [Bern, Carrasco, Dixon, Johansson, Roiban, '12]

$$\text{YM: } C \times N = 0 \times N \rightarrow \text{GR: } N * N \neq 0$$

- 3-body treatment naturally avoid this degeneracy