Perturbative Gravity from Gauge Theory

QCD Meets Gravity
December 10, 2018
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After a general introduction to subject, some new results:

Five-loop UV behavior of $N = 8$ supergravity

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng, arXiv:1804.09311

Classical 2 body conservative potential at 3 PM order

ZB, Cheung, Roiban, Shen, Solon, Zeng, arXiv:1812.xxxx

Outline

1. Nontriviality of gravity calculations.
2. Duality between color and kinematics and the associated double-copy construction of gravity amplitudes.
3. Classical double copy.

   Talks from Shen, Steinhoff, Carrillo Gonzalez, Nicholson, O'Connell, Casali, etc
4. Application: Constructing and evaluating supergravity.
   - Various supergravities.
   - High loop order UV studies. 5 loops

   Talks from Roiban, Kälin, Edison
5. New Application: High orders in the conservative 2-body classical potential.
   - Can we do something new? Yes! We have 3PM potential.

See Clifford Cheung's talk

Gravity vs Gauge Theory

Consider the Einstein gravity Lagrangian

curvature

Flat-space metric

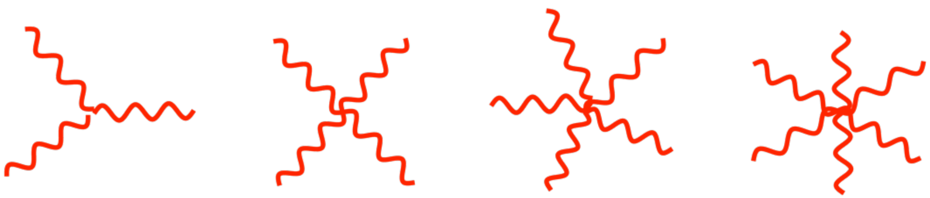
$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

graviton field

metric

$$\kappa^2 = 32\pi G_{\text{Newton}}$$

Infinite number of complicated interactions

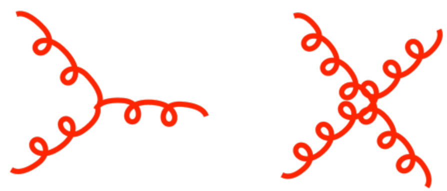


$+ \cdots$

terrible mess

Compare to gauge-theory Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$



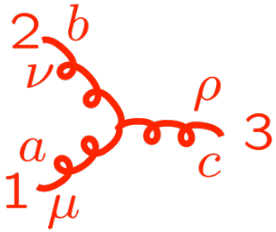
Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Does not look related!

Three Vertices

Standard Feynman diagram approach.



Three-gluon vertex:

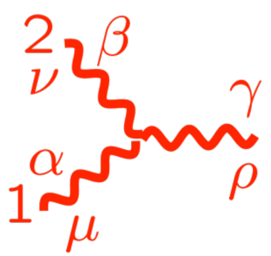
$$V^{abc}_{3\,\mu\nu\sigma} = -g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \eta_{\nu\rho}(k_1-k_2)_\mu + \eta_{\rho\mu}(k_1-k_2)_\nu)$$

Three-graviton vertex:

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\text{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})$$
$$+ P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})$$
$$+ P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})$$
$$+ 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]$$

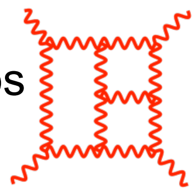


About 100 terms in three vertex

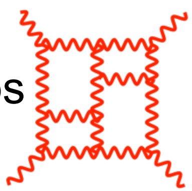
Naïve conclusion: Gravity is a nasty mess.

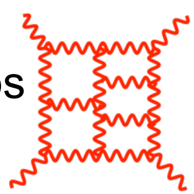
Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

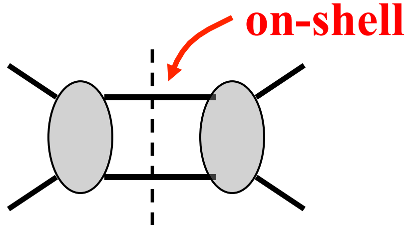
- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Modern Unitarity Method

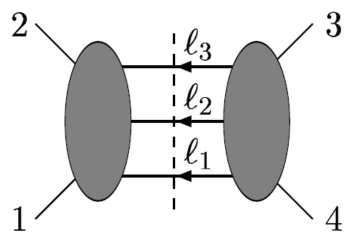
$$E^2 = p^2 + m^2$$

ZB, Dixon, Dunbar and Kosower (1994)

Two-particle cut:

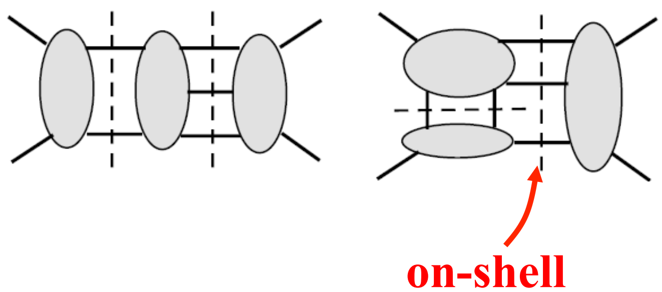


Three-particle cut:



- **Systematic assembly of complete amplitudes from other amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool.



ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger
and many others

Reproduces Feynman diagrams except intermediate steps of calculation based on physical quantities *not* unphysical ones.

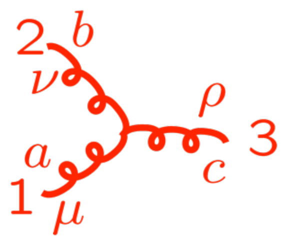
Simplicity of Gravity Amplitudes

People were looking at gravity amplitudes the wrong way.
On-shell viewpoint much more powerful.

***On-shell* three vertices contains all information:**

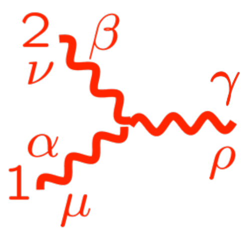
$$k_i^2 = 0$$

Yang-Mills gauge theory:



$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

Einstein gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic}) \times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

"square" of Yang-Mills vertex.

Using modern methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertex. Higher-point vertices irrelevant!

Very simple!

Gravity Amplitudes

KLT (1985)

Kawai-Lewellen-Tye string relations in low energy limit:

gravity → gauge theory color ordered

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34}A_5^{\rm tree}(1,2,3,4,5)\,A_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\,A_5^{\rm tree}(3,1,4,2,5)$$

Pattern gives explicit all-leg form.

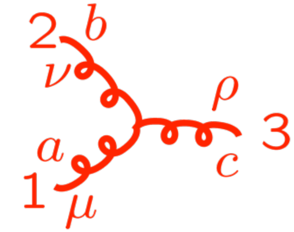
1. **Gravity is derivable from gauge theory. Standard QFT offers no hint why this is possible.**
2. **It looked very generally applicable.**

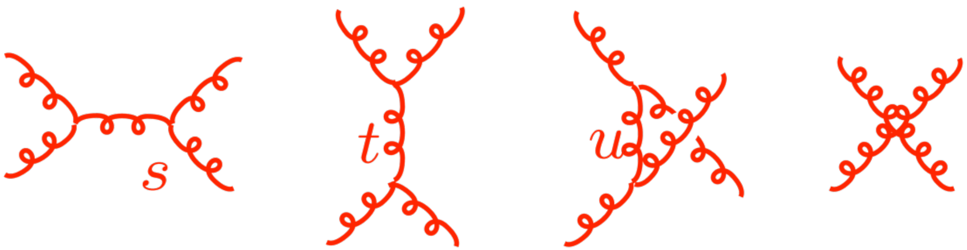
Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

coupling constant → **color factor** → **momentum dependent kinematic factor**

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$





Use 1 = *s*/*s* = *t*/*t* = *u*/*u* to assign 4-point diagram to others.

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}\right)$$

$$s = (k_1 + k_2)^2 \qquad u = (k_1 + k_3)^2$$
$$t = (k_1 + k_4)^2$$

polarization vector

$$n_s = \frac{i}{2}\Big\{\Big[(\varepsilon_1 \cdot \varepsilon_2)p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4)p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\Big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\Big]\Big\}, \qquad c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$$

$$c_t n_t = c_s n_s\Big|_{1\to2\to3\to1}, \qquad c_u n_u = c_s n_s\Big|_{1\to3\to2\to1}.$$

Double Copy and Gauge Invariance

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

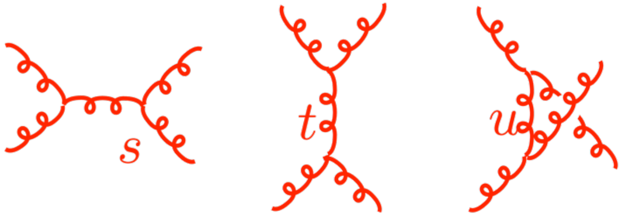
$$n_s = \frac{i}{2}\Big\{\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big] + s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\},$$

Take $\varepsilon_4 \to p_4$. **Should vanish by gauge invariance.**

$$n_s \to i\frac{s}{2}\Big[(\varepsilon_1\cdot\varepsilon_2)\big((\varepsilon_3\cdot p_2) - (\varepsilon_3\cdot p_1)\big) + \text{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon,p)$$

$$\left.\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right|_{\varepsilon_4\to p_4} = (c_s + c_t + c_u)\,\alpha(\varepsilon,p)$$

$$c_s + c_t + c_u = 0$$ **color Jacobi identity**



Gauge invariance follows from color Jacobi identitity

Double Copy and Gauge Invariance

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$$n_s = \frac{i}{2}\Big\{\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big] + s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\},$$

Easy to check: $n_s + n_t + n_u = 0$ **BCJ duality**

Double-copy construction: $c_i \longrightarrow n_i$ $\varepsilon^{\mu\nu} = \varepsilon^\mu\varepsilon^\nu$

Gauge invariance:

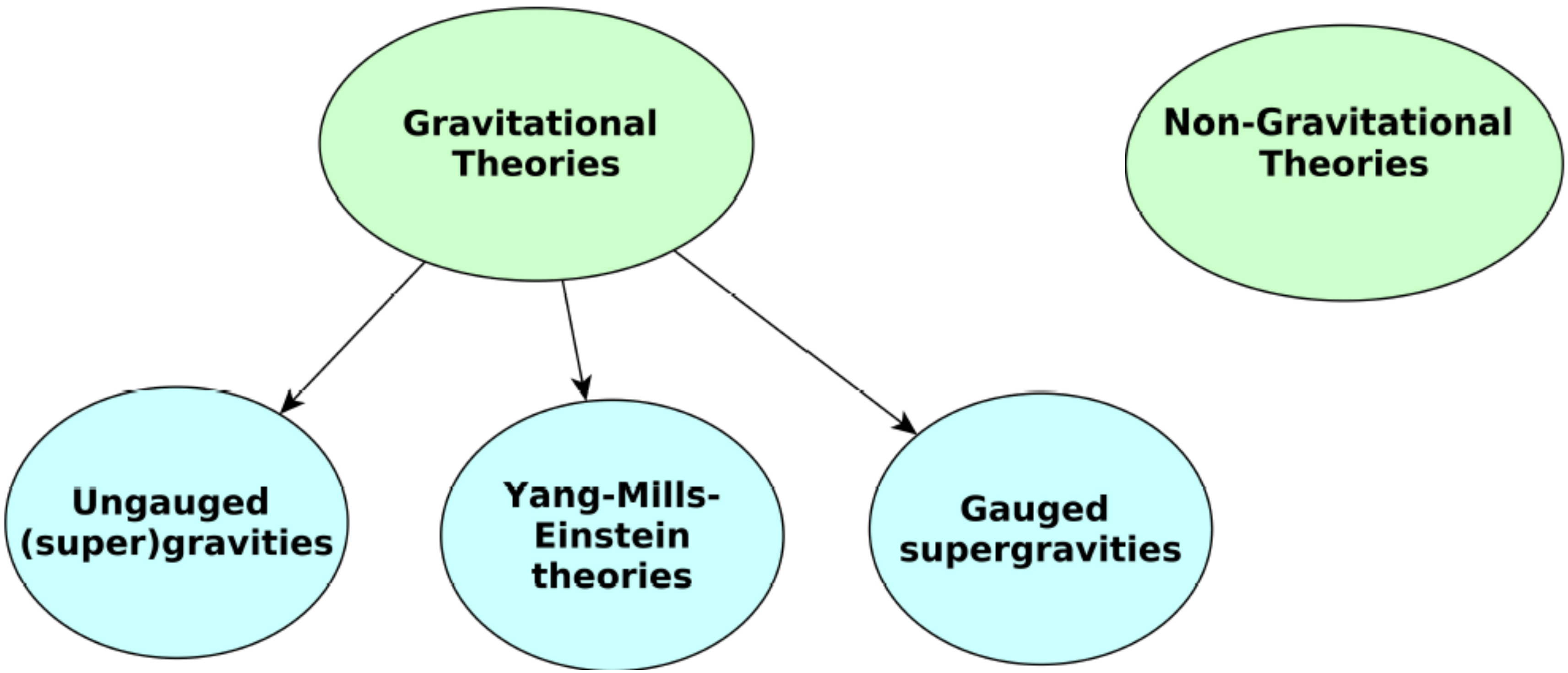
$$\mathcal{A}_4^{\text{tree}}\Big|_{c_i\to n_i} \equiv -i\mathcal{M}_4^{\text{tree}} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

$$\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\Big|_{\varepsilon_4^{\mu\nu}\to p_4^\mu\varepsilon_4^\nu + p_4^\nu\varepsilon_4^\mu} = 2(n_s + n_t + n_u)\,\alpha(\varepsilon, p) = 0$$

Double-copy construction generates gauge-invariant scattering amplitudes for spin 2 (i.e. for gravity). Generalized to higher points.

Generalizations

Ideas generalize to a variety of theories.



Generalizations of Double Copy

Supergravities:

- $N \geq 1$ pure supergravities.
- Supergravities with chiral, vector, and hypermultiplets.
- Maxwell-Einstein supergravities.
- Gauged supergravities
- Conformal supergravities
- Einstein gravity with matter
- Einstein gravity plus higher dimension operators
- Higgsed supergravities.
- Seems possible that *all* supergravities are double copies.

Nongravitational theories:

- Dirac Born-Infeld as NLSM x (gauge theory).
- Special Galileon as NLSM x NLSM

Forthcoming review: "The Duality Between Color and Kinematics and its Applications"

ZB, Carrasco, Chiodaroli, Johansson and Roiban (to appear soon)

ZB, Carrasco, Chiodaroli, Johansson and Roiban

$\mathcal{N}=2$ supergravities with vector/ hypermultiplets	• $\mathcal{N}=1$ sYM theory with chiral multiplets • $\mathcal{N}=1$ sYM-scalar with chiral multiplets	[69, 70, 71]	• construction known in particular cases
$\mathcal{N}=1$ supergravities with vector multiplets	• $\mathcal{N}=1$ sYM theory with chiral multiplets • YM-scalar theory with fermions	[69, 70, 71, 65]	• fields in matter representations • construction known in particular cases
$\mathcal{N}=1$ supergravities with chiral multiplets	• $\mathcal{N}=1$ sYM theory with chiral multiplets • YM-scalar with extra matter scalars	[69, 70, 71, 65]	• fields in matter representations • construction known in particular cases
Einstein gravity with matter	• YM theory with matter • YM theory with matter	[65]	• construction known in particular cases
Einstein gravity $+\,\phi R^2 + R^3$	• YM theory $+ F^3 + \ldots$ • YM theory $+ F^3 + \ldots$	[72]	
Conformal (super)gravity	• DF^2 theory • (s)YM theory	[73, 74]	• up to $\mathcal{N}=4$ supersymmetry • involves specific gauge theory with dimension-six operators
$3D$ maximal supergravity	• BLG theory • BLG theory	[76, 77]	• $3D$ only

3 A WEB OF DOUBLE-COPY-CONSTRUCTIBLE THEORIES DRAFT

Gravity	Gauge theories	Refs.	Variants and notes
$\mathcal{N}>4$ supergravity	• $\mathcal{N}=4$ sYM theory • sYM theory ($\mathcal{N}=1,2,4$)	[1, 2, 62, 49, 63]	
$\mathcal{N}=4$ supergravity with vector multiplets	• $\mathcal{N}=4$ sYM theory • YM-scalar theory from dimensional reduction	[1, 2, 62, 64]	• $\mathcal{N}=2\times\mathcal{N}=2$ construction is also possible
pure $\mathcal{N}<4$ supergravity	• (s)YM theory with ghosts • (s)YM theory with ghosts	[65]	• ghost fields in fundamental rep
$\mathcal{N}=2$ Maxwell-Einstein supergravities (generic family)	• $\mathcal{N}=2$ sYM theory • YM-scalar theory from dimensional reduction	[66]	• truncations to $\mathcal{N}=1,0$ • only adjoint fields
$\mathcal{N}=2$ Maxwell-Einstein supergravities (homogeneous theories)	• $\mathcal{N}=2$ sYM theory with half hypermultiplet • YM-scalar theory from dimensional reduction with matter fermions	[67]	• fields in pseudo-real reps • include Magical Supergravities
$\mathcal{N}=2$ supergravities with hypermultiplets	• $\mathcal{N}=2$ sYM theory with half hypermultiplet • YM-scalar theory from dimensional reduction with extra matter scalars	[67, 68]	• fields in matter representations • construction known in particular cases

Gravity	Gauge theories	Refs.	Notes
Yang-Mills-Einstein supergravities	• sYM theory • YM + ϕ^3 theory	[93, 66, 85, 86, 88, 89, 90, 91, 92, 87]	• trilinear scalar couplings • $\mathcal{N}=0,1,2,4$ possible
Higgsed supergravities	• sYM theory (Coulomb branch) • YM + ϕ^3 theory with extra massive scalars	[94]	• $\mathcal{N}=0,1,2,4$ possible • massive fields in supergravity
$U(1)_R$ gauged supergravities	• sYM theory (Coulomb branch) • YM theory with supersymmetry broken by fermion masses	[95]	• $0\le\mathcal{N}\le 8$ possible • supersymmetry is spontaneously broken

Double copy	Starting theories	Refs.
Dirac-Born-Infeld theory	• Nonlinear sigma model • (s)YM theory	[86, 87, 78, 81, 82, 83]
Special Galileon theory	• Nonlinear sigma model • Nonlinear sigma model	[86, 87, 79]
Dirac-Born-Infeld + YM theory	• Nonlinear sigma model + ϕ^3 • YM theory	[80, 86, 87, 78, 81, 82, 83]

See Radu Roiban's talk

References and detail will be given in our soon to appear review.

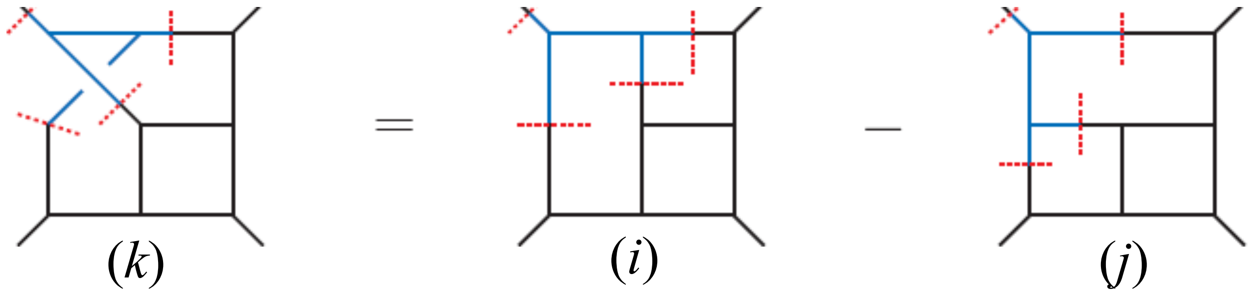
ZB, Carrasco, Chiodaroli, Johansson and Roiban

Applications of On-shell Methods to Quantum Gravity

BCJ

Gravity loop integrands are free!

Ideas conjectured to generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying numerators.

To get:

gauge theory → gravity theory

simply take

color factor → kinematic numerator

$$c_k \longrightarrow n_k$$

Double copy construction

Gravity loop integrands follow from gauge theory!

$\mathcal{N} = 5$ Supergravity at Four Loops

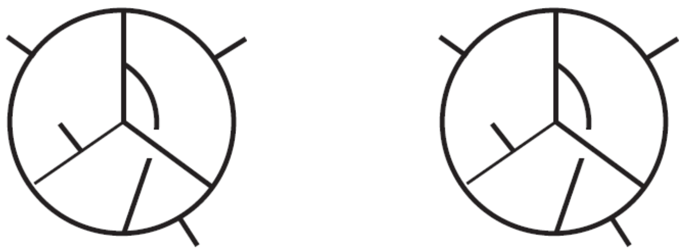
ZB, Davies and Dennen

A good example is four-loop divergence in $\mathcal{N} = 5$ supergravity in $D = 4$.

$\mathcal{N} = 5$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 1$ sYM)

Crucial help from FIRE5 and (Smirnov)²

$\mathcal{N} = 4$ sYM **$\mathcal{N} = 1$ sYM**



Diagrams necessarily UV divergent.

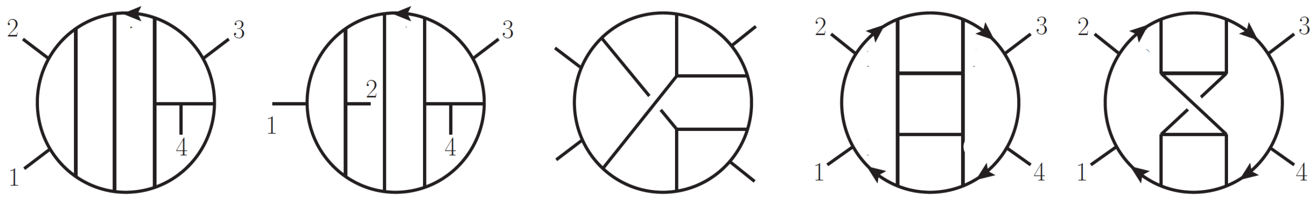
$\mathcal{N} = 5$ supergravity in $D = 4$ has no divergence at four loops.

An example of an "enhanced cancellation".
No standard symmetry explanation is known.

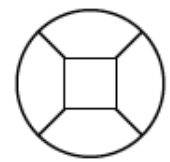
Freedman, Kallosh, Yamada (2018)

Five-loop UV in $N = 8$ supergravity

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)



To make a long story short, in $D = 24/5$, where symmetry arguments suggest an ultraviolet divergence, we do find it diverges:

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left(\frac{1}{48}\right.$$  $$+\frac{1}{16}$$  $$\left.\right)$$

See Radu Roiban's and Alex Edison's talks

Positive definite vacuum integrals containing UV divergences

Unlike $N = 5$ supergravity at 4 loops here no "enhanced cancellation".

What is the difference? $D = 4$? Need 7 loops to resolve this.

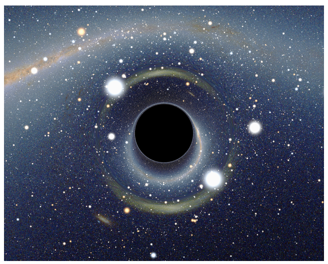
Double copy + unitarity allow us to carry out "impossible" multi-loop computations in (super)gravity.

Applications to Classical Gravity

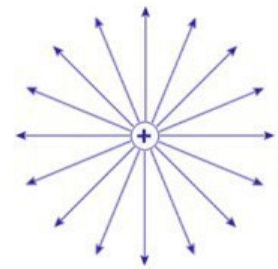
Applications to Black Hole Physics

Wouldn't it be really cool if every classical solution in gravity could be mapped to a double copy of classical solutions?

Where to start? **Obviously the coolest place possible: black holes.**



black hole



point charge

Monteiro, O'Connell and White

Special coordinates: Kerr-Schild coordinates:

Schwarzschild black hole

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu \qquad \phi(r) = \frac{2m}{r}$$

k is null

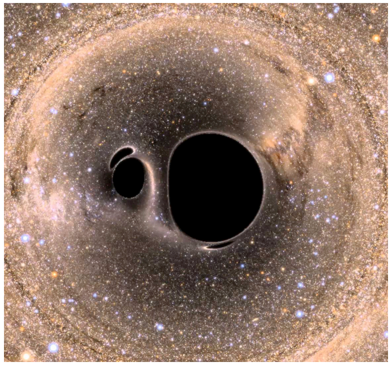
Coulomb point charge

$$A_\mu = \phi k_\mu \qquad \phi(r) = \frac{Q}{r}$$

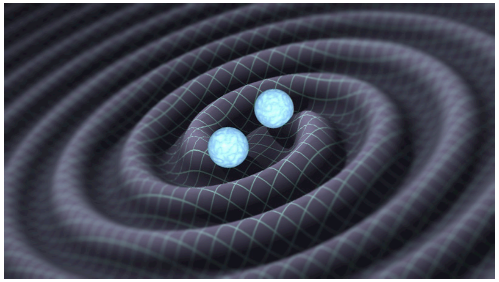
Schwarzschild ~ (Coulomb)^2

Double Copy for Classical Solutions

A variety of cases:



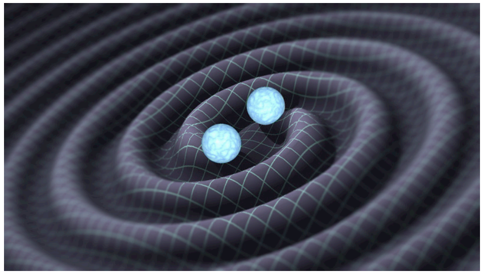
- **Kerr (rotating) black hole.**
- **Taub-NUT space.**
- **Solutions with cosmological constant.**
- **Radiation from accelerating black hole.**
- **Maximally symmetric space times.**
- **Plane wave background.**
- **Gravitational radiation.**



Luna, Monteiro, O'Connell and White;
Luna, Monteiro, Nicholsen, O'Connell and White;
Ridgway and Wise; Carrillo González, Penco, Trodden;
Adamo, Casali, Mason, Nekovar;
Goldberger and Ridgway; Chen
Luna, Monteiro, Nicholson, Ochirov;
Bjerrum-Bohr, Donoghue, Vanhove;
O'Connell, Westerberg, White;
Kosower, Maybee, O'Connell, etc

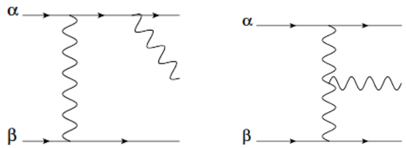
Double Copy and Gravitational Radiation

Can we simplify the types of calculations needed for LIGO?



A small industry has developed to study this.

- **Connection to scattering amplitudes.**
  Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Luna, Nicholson, O'Connell, White
  Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon; Damour;
  Plefka, Steinhoff, Wormsbecher



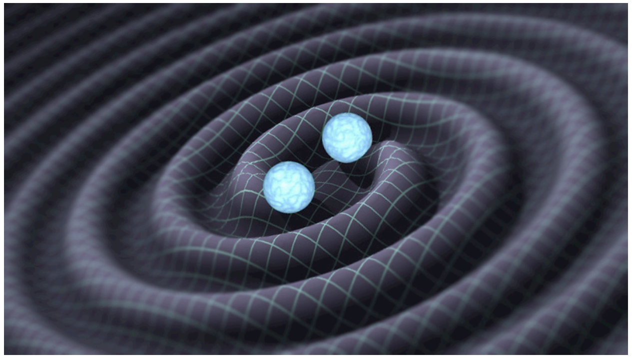
- **First quantized approach for radiation.**
  Goldberger and Ridgway; Goldberger, Li, Prabhu, Thompson; Chester
- **BCJ duality and double copy at NLO in grav. coupling.**
  Enormous simplification. Chia-Hsien Shen

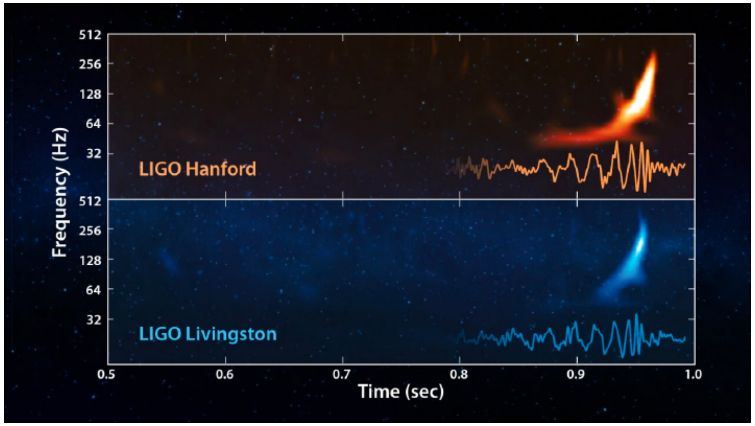
- **Removing the dilaton contamination.**
  Luna, Nicholson, O'Connell, White; Johansson, Ochirov; Johansson, Kalin;
  Henrik Johansson; Gregor Kälin, Mogull.

See Gregor Kälin's talk for one way to control dilatons in loops

Also talks from Carrillo Gonzalez, Cheung, Nicholson, O'Connell, Shen, Steinhoff

Double Copy and Gravitational Radiation





Big Question: **Can double copy help us calculate something with direct interest for LIGO physics that is *beyond* current calculations?**

PN versus PM expansion for conservative two-body dynamics

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\ldots\right] + \frac{1}{c^4}\left[\ldots\right] + \ldots$$

From Buonanno Amplitudes 2018

$$E(v) = -\frac{\mu}{2} v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Justin Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results — current known PM results — overlap between PN & PM results — unknown

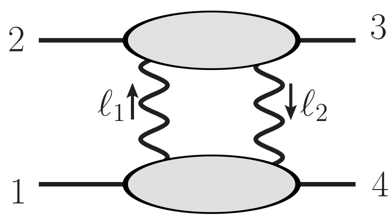
- **PM results** (Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)

Generalized unitarity is the right tool

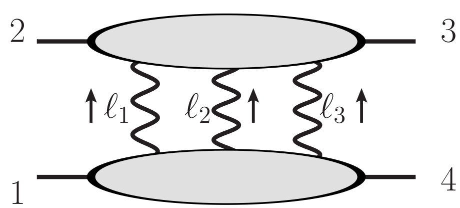
- **Long range force: Two matter lines must be separated by cut propagators**
- **Classical limit: 1 matter line per loop is cut.**

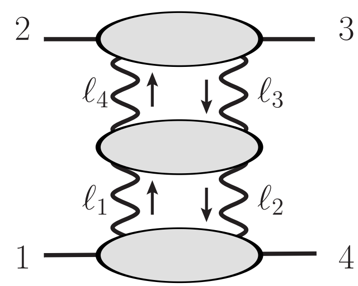
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove
Cheung, Rothstein and Solon

Only independent unitarity cut for 2 PM.



Independent generalized unitarity cuts for 3 PM.





Game plan:

- **Apply double copy and unitarity method to compute the two-loop integrand.**
- **Extract 3PM potential via EFT methods of Cheung, Rothstein and Solon.**

What about the dilaton?

ZB, Cheung, Shen, Roiban, Solon, Zeng (to appear)

(gluon) x (gluon) = graviton + dilaton + axion

For physical gravitational waves need to eliminate dilaton and axion.

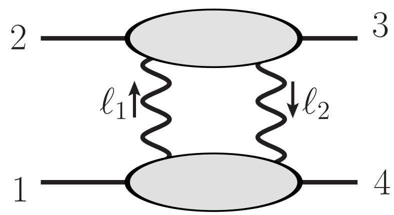
Will now explain that these are *not* a problem, merely an annoyance.

Real issues:

1. **Scalability of integrand construction. Will the methods work as we reach higher orders?** **This talk.**
2. **Scalability of integration methods.** **Clifford Cheung's talk.**

Graviton Projector

External Scalars (black holes with no spin)



Exposed inner lines are all on shell gravitons

Bad idea to use the graviton physical state projector:

$$\sum_\lambda \varepsilon^{-\lambda}_{\mu\alpha}\varepsilon^{\lambda}_{\nu\beta} = \frac{1}{2}\left(\eta_{\mu\alpha} - \frac{p_\mu q_\alpha + q_\mu p_\alpha}{p\cdot q}\right)\left(\eta_{\nu\beta} - \frac{p_\nu q_\beta + q_\nu p_\beta}{p\cdot q}\right)$$

← **double copy**

antisymmetric tensor removed →

$$+\frac{1}{2}\left(\eta_{\nu\alpha} - \frac{p_\nu q_\alpha + q_\nu p_\alpha}{p\cdot q}\right)\left(\eta_{\mu\beta} - \frac{p_\mu q_\beta + q_\mu p_\beta}{p\cdot q}\right)$$

dilaton removed →

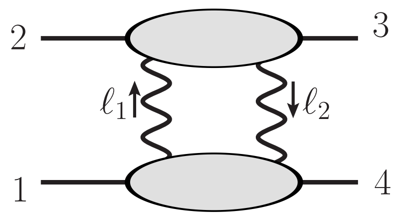
$$-\frac{1}{D-2}\left(\eta_{\mu\nu} - \frac{p_\mu q_\nu + q_\mu p_\nu}{p\cdot q}\right)\left(\eta_{\alpha\beta} - \frac{p_\alpha q_\beta + q_\alpha p_\beta}{p\cdot q}\right)$$

We need to project out axion (antisymmetric tensor) and dilaton.
This makes a mess, especially as we go to higher orders

Are there a smarter ways?

Taming the Graviton Projector

External Scalars



Gauge-theory physical state projector same as closed ghost loop. Antisymmetric tensor only in closed loop. No contribution!

Bad idea to use the graviton physical state projector:

$$\sum_\lambda \varepsilon^{-\lambda}_{\mu\alpha}\varepsilon^{\lambda}_{\nu\beta} = \frac{1}{2}\left(\eta_{\mu\alpha} - \frac{p_\mu q_\alpha + q_\mu p_\alpha}{p\cdot q}\right)\left(\eta_{\nu\beta} - \frac{p_\nu q_\beta + q_\nu p_\beta}{p\cdot q}\right)$$

← **Ghosts are quantum**

Antisymmetric tensor is quantum →

$$+\frac{1}{2}\left(\eta_{\nu\alpha} - \frac{p_\nu q_\alpha + q_\nu p_\alpha}{p\cdot q}\right)\left(\eta_{\mu\beta} - \frac{p_\mu q_\beta + q_\mu p_\beta}{p\cdot q}\right)$$

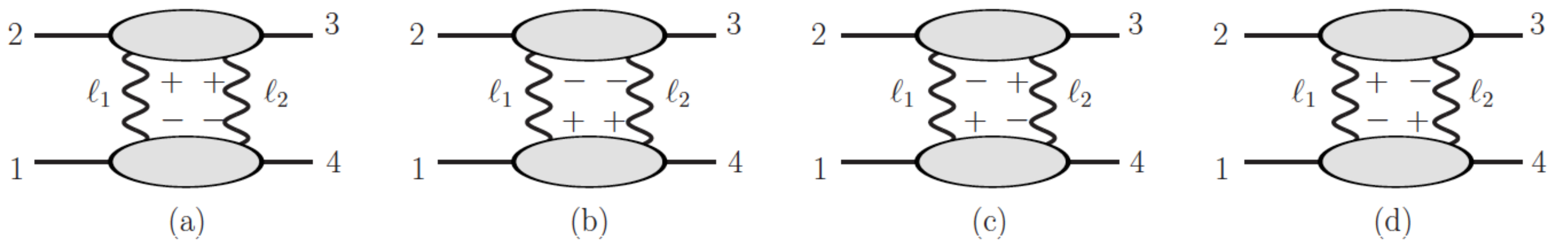
Choose special form of trees →

$$-\frac{1}{D-2}\left(\eta_{\mu\nu} - \frac{p_\mu q_\nu + q_\mu p_\nu}{p\cdot q}\right)\left(\eta_{\alpha\beta} - \frac{p_\alpha q_\beta + q_\alpha p_\beta}{p\cdot q}\right)$$

$$\sum_\lambda \varepsilon^{-\lambda}_{\mu\alpha}\varepsilon^{\lambda}_{\mu\alpha} \to \eta_{\mu\alpha}\eta_{\nu\beta} - \frac{1}{D-2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

Same game works at 2 loops which is needed for 3PM order.
What about 4PM and beyond? Want a method with better scaling.

A Better Way: Helicity



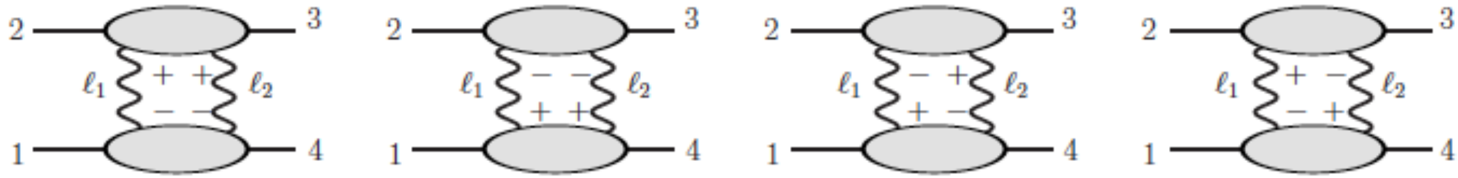
$$C_{\rm GR} = \sum_{h_1,h_2=\pm} M^{\rm tree}(3,\ell_2^{h_2},-\ell_1^{h_1},2)\times M^{\rm tree}(1,\ell_1^{-h_1},-\ell_2^{-h_2},4)$$
$$= -\sum_{h_1,h_2=\pm} s_{23}^2[A^{\rm tree}(3,\ell_2^{h_2},-\ell_1^{h_1},2)\times A^{\rm tree}(1,\ell_1^{-h_1},-\ell_2^{-h_2},4)]$$
$$\times\,[A^{\rm tree}(2,\ell_2^{h_2},-\ell_1^{h_1},3)\times A^{\rm tree}(4,\ell_1^{-h_1},-\ell_2^{-h_2},1)]$$

By correlating gluon helicities, removing dilaton is trivial.

$h^-_{\mu\nu} \to A^-_\mu A^-_\mu$ $\quad$ $h^+_{\mu\nu} \to A^+_\mu A^+_\mu$ $\quad$ **Forbid:** $A^+_\mu A^-_\mu$

Problem of computing the generalized cuts in gravity is reduced multiplying and summing gauge-theory tree amplitudes.

One loop gauge-theory warmup



(a) (b) (c) (d)

color ordered cut:

$$C_{\rm YM} = \sum_{h_1,h_2=\pm} A^{\rm tree}(3,\ell_2^{h_2},-\ell_1^{h_1},2)\times A^{\rm tree}(1,\ell_1^{-h_1},-\ell_2^{-h_2},4)$$

configuration (c):

$$\begin{aligned} C^{(c)}_{\rm YM} &= A^{\rm tree}(3,\ell_2^{+},-\ell_1^{-},2)\times A^{\rm tree}(1,\ell_1^{+},-\ell_2^{-},4)\\ &= \frac{\langle -\ell_1|2|\ell_2]^2}{2(\ell_1-\ell_2)^2\ell_1\cdot k_2}\times\frac{2\langle -\ell_2|4|\ell_1]^2}{(\ell_1-\ell_2)^2\ell_1\cdot k_1}\\ &= \frac{1}{s_{23}^2}\frac{{\rm tr}_-^2[\ell_1 4\ell_2 2]}{4\ell_1\cdot k_1\,\ell_1\cdot k_2}\end{aligned}$$

$$\begin{aligned} {\rm tr}_\pm[\ldots] &\equiv \frac{1}{2}{\rm tr}[(1\pm\gamma_5)\ldots]\\ {\rm tr}_-[\ell_1 4\ell_2 2] &\equiv \mathcal{E}-\mathcal{O}\\ t_{ij} &= 2p_i\cdot p_j\\ s_{23} &= (p_2+p_3)^2\end{aligned}$$

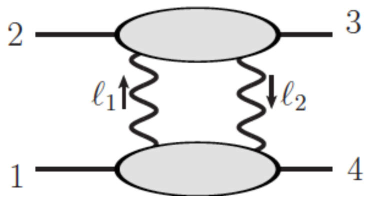
Add the helicities and evaluate gamma traces:

$$C_{\rm YM} = 2\left(\frac{\mathcal{E}^2+\mathcal{O}^2}{s_{23}^2}+m_1^2m_2^2\right)\frac{1}{t_{1\ell_1}\,t_{2\ell_1}}$$

$$\mathcal{E}^2 = \frac{1}{4}\Big[-t_{12}s_{23}+s_{23}t_{1\ell_1}-s_{23}t_{2\ell_1}+2t_{1\ell_1}t_{2\ell_1}\Big]^2$$

$$\mathcal{O}^2 = \mathcal{E}^2-(s_{23}m_1^2+s_{23}t_{1\ell_1}+t_{1\ell_1}^2)(s_{23}m_2^2-s_{23}t_{2\ell_1}+t_{2\ell_1}^2)$$

One loop gravity



Apply unitarity and KLT relations.
Import gauge theory cuts.

$$C_{\rm GR} = 2\left[\frac{1}{t^4}\left(\mathcal{E}^4 + \mathcal{O}^4 + 6\mathcal{E}^2\mathcal{O}^2\right) + m_1^4 m_2^4\right]\left[\frac{1}{t_{1\ell_1}} + \frac{1}{t_{4\ell_1}}\right]\left[\frac{1}{t_{2\ell_1}} + \frac{1}{t_{3\ell_1}}\right]$$

- **Same building blocks as gauge theory!**
- **Double copy is visible even though we have removed dilaton and axion.**

We can extract classical scattering angles or potentials following literature

Damour; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

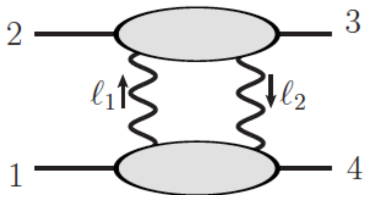
We get 2 PM this way

Potential Subtlety

Might we drop pieces by using four-dimensional helicity and kinematics?

$$\mathrm{Det}(p_i \cdot p_j) = 0$$ **For 5 independent momenta in $D = 4$**

External scalars

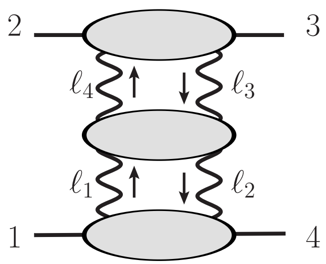


One loop:

Only 4 independent momenta in problem.
No lost pieces.

Two loops:

5 independent momenta in problem. Potential lost pieces.



These have no contribution in classical limit:
Gram det gives (D - 4) x dimension shifted integral.
Wrong form to contribute in the classical limit, as expected.

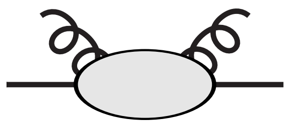
Seems likely $D = 4$ sufficient to all loop orders, but we need to keep an eye out for subtleties.

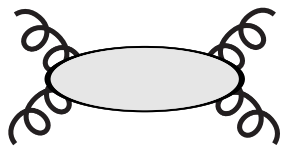
Gauge-Theory Building Blocks for 3 PM Gravity

$$A^{\rm tree}(1^s,2^+,3^+,4^s) = i\frac{m_1^2[23]}{\langle 23\rangle t_{12}}$$

$$A^{\rm tree}(1^s,2^+,3^-,4^s) = -i\frac{\langle 3|1|2]^2}{\langle 23\rangle [23] t_{12}}$$

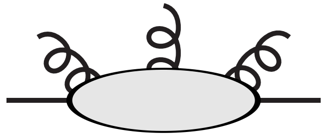
color-ordered gauge-theory tree amplitudes





$$A^{\rm tree}(1^-,2^-,3^+,4^+) = i\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$A^{\rm tree}(1^-,2^+,3^-,4^+) = i\,\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$



$$A(1^s,2^+,3^+,4^+,5^s) = i\,\frac{m_1^2\,[4|(2+3)1|2]}{t_{12}\,\langle 23\rangle\,\langle 34\rangle\,t_{45}}$$

$$A(1^s,2^+,3^+,4^-,5^s) = i\,\frac{t_{45}[34]\,\langle 4|51|4\rangle\,\langle 4|1|2] + m_1^2\,[23]\langle 34\rangle\,([43]\langle 4|51|4\rangle - s_{23}\langle 4|5|3\rangle)}{t_{12}\,\langle 23\rangle\,s_{34}\,t_{45}\,s_{51}}$$

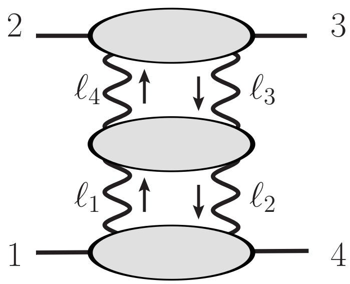
$$A^{\rm tree}(1^s,2^+,3^-,4^+,5^s) = \frac{-[23][34]\,\langle 3|1|2]\,\langle 3|5|4]\,\langle 3|51|3\rangle + m_1^2\,\langle 23\rangle\langle 34\rangle[24]^2\,([34]\langle 3|1|2] - [24]t_{45})}{t_{12}\,s_{23}\,s_{34}\,t_{45}\,s_{51}}$$

- **This is all you need for 3 PM! U(1) decoupling gives other orderings.**
- **Scaling with number of external legs is brilliant.**
- **Follows same procedure as at one-loop. Feed through KLT and unitarity.**

ZB, Dixon, Dunbar, Perelstein and Rozowsky (1998)

Two Loops for 3 PM

$$s_{23} = (p_2 + p_3)^2$$
$$t_{ij} = 2p_i \cdot p_j$$



ZB, Cheung, Shen, Roiban, Solon, Zeng (to appear)

- **Use KLT and sum over helicities**
- **Very similar to one loop**

$$C_{\rm GR}^{22\ {\rm cut}} = -2i\left[s_{23}^2 m_1^4 m_2^4 + \frac{1}{s_{23}^6}\left(\mathcal{E}_1^4 + \mathcal{O}_1^4 + 6\mathcal{O}_1^2\mathcal{E}_1^2 + \mathcal{E}_2^4 + \mathcal{O}_2^4 + 6\mathcal{O}_2^2\mathcal{E}_2^2\right)\right]$$
$$\times\left[\frac{1}{t_{3\ell_4}} + \frac{1}{t_{2\ell_4}}\right]\left[\frac{1}{(\ell_1-\ell_4)^2} + \frac{1}{(\ell_2+\ell_4)^2}\right]\left[\frac{1}{t_{1\ell_1}} + \frac{1}{t_{4\ell_4}}\right]$$

$$\mathcal{E}_1^2 = \frac{1}{4}\left[s_{23}\Big((t_{1\ell_1} - t_{1\ell_4})(t_{2\ell_1} - t_{2\ell_4}) + (t_{12} - t_{1\ell_1} + t_{2\ell_4})t_{\ell_1\ell_4}\Big) + 2t_{1\ell_1}t_{2\ell_4}t_{\ell_1\ell_4}\right]^2$$

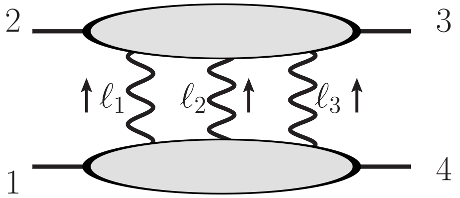
$$\mathcal{O}_1^2 = \mathcal{E}_1^2 - t_{\ell_1\ell_4}^2(m_1^2\, s_{23} + s_{23}\, t_{1\ell_1} + t_{1\ell_1}^2)(m_2^2\, s_{23} - s_{23}\, t_{2\ell_4} + t_{2\ell_4}^2)$$

$$\mathcal{O}_2^2 = \mathcal{O}_1^2\Big|_{\ell_3\leftrightarrow\ell_4} \qquad \mathcal{E}_2^2 = \mathcal{E}_1^2\Big|_{\ell_3\leftrightarrow\ell_4}$$

- **Same functional building blocks as gauge theory.**
- **Remarkably simple, given it is two-loop gravity.**

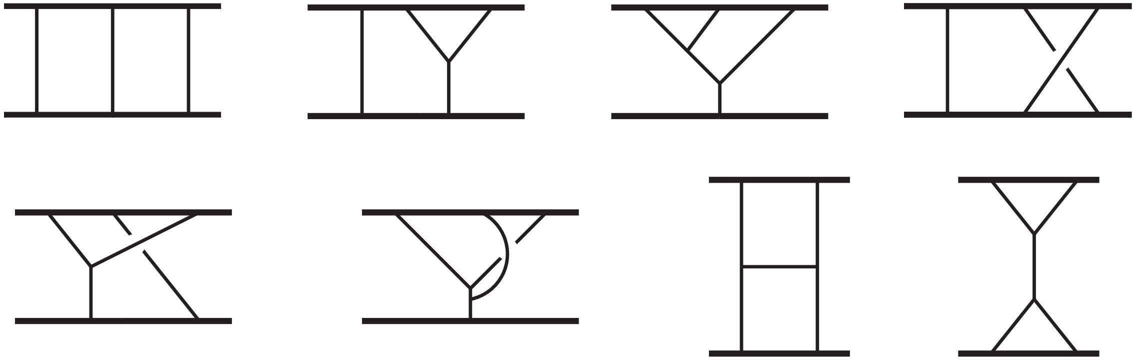
Two loops and 3 PM

Also need contributions from 3-particle cuts.

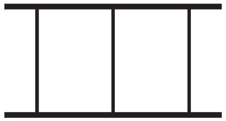


- **These unitarity cuts are more complicated that the interated 2 particle cuts.**
- **To interface easily with EFT approach of Cheung, Rothstein and Solon to extract potential we merged unitarity cuts into diagrams to get integrand.**

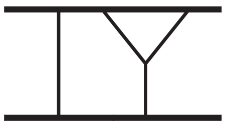
Integrand organized into 8 independent diagrams that can contribute in classical limit:



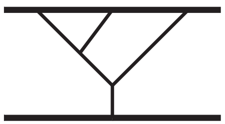
Two Loop Diagram Numerators



$$(A_4^{\text{tree}})^3$$



$$2m_2^4\, t_{47}^2\, A_4^{\text{tree}}$$



$$\begin{aligned}&2m_2^4(s_{23}^4 + s_{23}^3(2t_{12} + 2t_{15} - t_{47} - 2t_{67}) - 2m_1^2 m_2^2 (s_{23} - t_{67})^2 + (t_{15}t_{56} + (t_{12} - t_{47})t_{67})^2\\
&+ s_{23}^2(t_{12}^2 + t_{15}^2 + t_{47}^2 - t_{47}t_{56} + t_{12}(4t_{15} - 2t_{47} + t_{56} - 4t_{67}) + t_{15}(-2t_{47} + t_{56} - 2t_{67})\\
&+ 2t_{47}t_{67} + t_{67}^2) + s_{23}(t_{15}(t_{56}^2 + 2(-2t_{12} + t_{47})t_{67} - t_{56}t_{67})\\
&+ t_{67}(-2t_{12}^2 + t_{47}(-2t_{47} + t_{56} - t_{67}) + t_{12}(4t_{47} - t_{56} + 2t_{67}))))\end{aligned}$$

etc. Remaining 5 diagrams somewhat more complicated but not a big deal.

- **Even prior to classical limit, total is under 100KB in Mathematica**
- **Very simple compared to the usual Feynman diagram explosion.**
- **Higher-loop integrand constructions definitely possible!**

Integration + Extraction of Potential

ZB, Cheung, Shen, Roiban, Solon, Zeng arxiv:1812.xxxx

To integrate follow methods of Cheung, Rothstein and Solon.

- **Efficiently targets the classical pieces we want.**
- **Incorporates matching to effective field theory.**
- **Good scaling with perturbative order.**

See Clifford Cheung's talk

Checks on integrals using standard tools of QCD:

- **Mellin-Barnes integration.** V. Smirnov; Czakon
- **Sector decompostion.** Binoth and Heinrich, A. Smirnov
- **Integration by parts.** K. G. Chetyrkin and F. V. Tkachov, Laporta; A. Smirnov; Maierhöfer, Usovitsch, Uwer
- **Differential equations.** ZB, Dixon, Kosower; Remiddi and Gehrmann
- **Method of regions.** Beneke, V. Smirnov; A. Smirnov.

Conservative 3 PM Potential

ZB, Cheung, Roiban, Shen, Solon, Zeng arXiv:1812.xxxx

Clifford Cheung will explain the extraction of the potential.

Follows strategy of Cheung, Rothstein, Solon (2018)

The 3PM potential:

Contained in 4PN of Damour, Jaranowski, Schäfer;
Bernard, Blanchet, Bohé, Faye, Marsat; up to gauge transformations.

$$\frac{V_{3\text{PM}}}{\mu} = \frac{1}{\rho^3}\Bigg\{\left(-\frac{1}{4} - \frac{3\nu}{2}\right) + u^2\left(-\frac{25}{8} - \frac{65\nu}{4} - \frac{107\nu^2}{8}\right) + u^4\left(\frac{105}{32} - \frac{4049\nu}{160} - \frac{2589\nu^2}{32} - \frac{487\nu^3}{16}\right)$$

New

$$u^6\left(-\frac{273}{64} + \frac{178553\nu}{4480} - \frac{30993\nu^2}{640} - \frac{1527\nu^3}{8} - \frac{6607\nu^4}{128}\right)$$

$$u^8\left(\frac{2805}{512} - \frac{1947527\nu}{32256} + \frac{3093791\nu^2}{17920} + \frac{5787\nu^3}{320} - \frac{168131\nu^4}{512} - \frac{19425\nu^5}{256}\right)$$

$$u^{10}\left(-\frac{7007}{1024} + \frac{2354633\nu}{26880} - \frac{23190389\nu^2}{64512} + \frac{3013571\nu^3}{7168} + \frac{340279\nu^4}{1024} - \frac{237639\nu^5}{512} - \frac{104655\nu^6}{1024}\right) + \cdots\Bigg\}$$

$$u = p/\mu$$

$$M = m_A + m_B, \qquad \nu = \mu/M, \qquad \mu = m_A m_B/M, \qquad \rho = r/GM$$

Preliminary: Have full resummation of 3PM. Still checking...

PN versus PM expansion for conservative two-body dynamics

From Buonanno Amplitudes 2018

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\ldots\right] + \frac{1}{c^4}\left[\ldots\right] + \ldots$$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	$\ldots$
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	$\ldots$
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	$\ldots$
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	$\ldots$
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	$\ldots$
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	$\ldots$
$\ldots$						$\ldots$	$\ldots$	

(credit: Justin Vines)

Looks doable.

3 PM under control. Just the beginning!

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results

current known PM results

overlap between PN & PM results

- PM results *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

Outlook for Gravitational Wave Physics

- **Double-copy ideas give compact expressions!**
- **Combining unitarity with newly developed effective field theory methods gives efficient construction of potential.**

Old questions:

- **Is the dilaton a problem? No, just an annoyance.**
- **Can we use ideas from amplitudes to do something interesting for LIGO/VIRGO, *beyond* current calculations? Yes. 3PM**

New Question:

- **How far can we go? Still to be determined, but expect it to be far…**

Summary

- **Double-copy idea gives us a powerful new way to think about gravity.**
- **Remarkable connection between gauge and gravity theories:**
  - **— color ⟷ kinematics.**
  - **— gravity ~ (gauge theory)^2**
- **Web of theories. Duality and double copy look very general.**
- **Pure supergravities surprisingly tame in the UV. Very high loop order calculations: 5 loops. New phenomenon: *Enhanced cancellations.***
- **Obtained 3PM conservative 2-body potential. New result. Dilaton is *not* a problem, merely an annoyance.**

We can expect many new results in coming years

See upcoming review on duality between color and kinematics

ZB, Carrasco, Chiodaroli, Johansson and Roiban (to appear soon)