Radiation of scalar modes and the classical double copy

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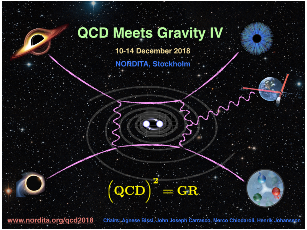
Based on:
1809.04611 : MCG, R.Penco, M. Trodden

December 10, 2018
QCD meets Gravity IV



Penn
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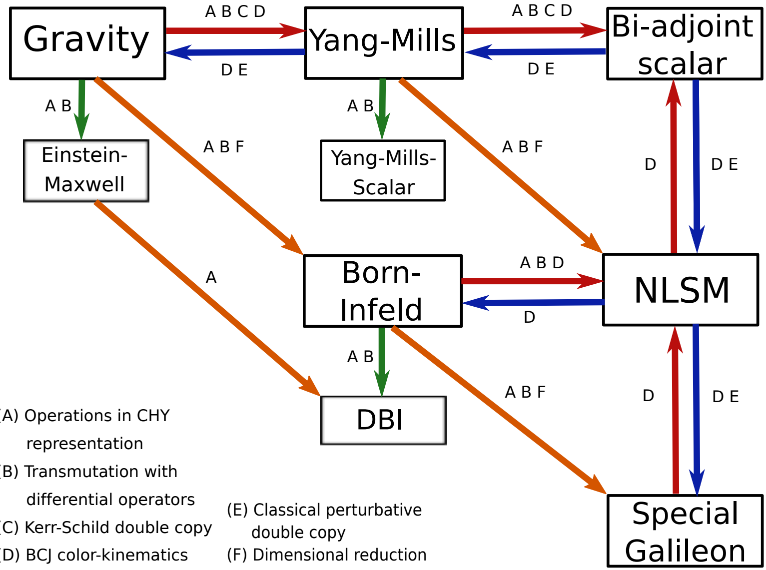
Web of Amplitudes



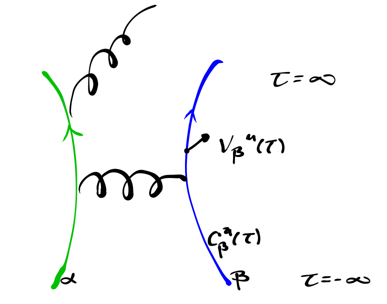
(A) Operations in CHY representation
(B) Transmutation with differential operators
(C) Kerr-Schild double copy
(D) BCJ color-kinematics
(E) Classical perturbative double copy
(F) Dimensional reduction

Classical realizations of the double copy

- Exact Solutions:
  - Kerr-Schild spacetimes
    - Black holes (Monteiro, O'Connell, White; 2014)
    - Taub - NUT spacetime (Luna, Monteiro, O'Connell, White; 2015)
    - Stress tensors, energy conditions (Ridgway, Wise; 2015)
    - Accelerating black holes (Luna, Monteiro, Nicholson, O'Connell, White; 2016)
    - In curved space (Bahjat-Abbas ,Luna, White; 2017), (MCG, Penco, Trodden; 2017)
  - Weyl Double Copy
    - Type D spacetimes (Luna, Monteiro, Nicholson, O'Connell; 2018)
- Perturbative case: Double copy of radiation
  - For color charges (Goldberger, Ridgway; 2016), (Goldberger, Prabhu, Thompson; 2017)
  - For bound states (Goldberger, Ridgway; 2017)
  - Including spin (Goldberger, Li, Prabhu; 2017)
  - NLO (Chia-Hsien Shen; 2018)
  - Relation to amplitudes (Luna, Nicholson, O'Connell, White; 2017),(Plefka, Steinhoff, Wormsbecher; 2018),(Kosower, Maybee, O'Connell: 2018)
- ...

Set-up

Consider N weakly interacting particles



- No point-particles are created or annihilated → $E_{\text{CM}} \ll 1$
- Perturbative solution requires: small deviations $\Delta x, \Delta c \ll 1$ → small impact parameter

Map scalar radiation at spatial infinity

E.g. for biadjoint scalar: $\mathcal{A}^{a\,\tilde{a}}(k) = y\,\mathcal{J}^{a\,\tilde{a}}(k)\Big|_{k^2=0}, \quad \Box\phi^{a\tilde{a}} = y\mathcal{J}^{a\tilde{a}}$

At large observation time T: $\frac{\mathrm{d}P}{\mathrm{d}\Omega\mathrm{d}|\mathbf{k}|} = \left|\mathcal{A}^{a\,\tilde{a}}\right|^2 \frac{|\mathbf{k}|^2}{2(2\pi)^2\,T}$

The scalar theories & their couplings to point particles

Biadjoint Scalar (Zeroth Copy)

$$\mathcal{L}_{BS} = \frac{1}{2}\left(\partial\varphi^{a\,\tilde{a}}\right)^2 - \frac{y}{3} f^{abc}\,\tilde{f}^{\tilde{a}\tilde{b}\tilde{c}}\,\varphi^{a\,\tilde{a}}\varphi^{b\,\tilde{b}}\varphi^{c\,\tilde{c}}$$

$$S_{\rm pp} = -\frac{1}{2}\sum_\alpha \int \mathrm{d}\lambda \left[\eta^{-1}(\lambda)\,\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda}\cdot\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda} + \eta(\lambda)\left(m_\alpha^2 - 2\,y\,\varphi^{a\,\tilde{a}}\,c_\alpha^a \tilde{c}_\alpha^{\tilde{a}}\right)\right]$$

Non-Linear Sigma Model-NLSM (Single Copy)

$$\mathcal{L}^{(2)}_{\rm NLSM} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^{-1}\right)\ , U = e^{i\frac{\sqrt{2}}{F}\phi^a T^a}$$

$c_\alpha^a p_\mu A^{\mu a} \to c_\alpha^a p^\mu\, \partial_\mu \phi^a + \cdots$ $\Rightarrow$ No radiation

Special Galileon (Double Copy)

$$\mathcal{L}_{\rm SG} = \frac{1}{2}(\partial\pi)^2 - \frac{1}{12\Lambda^6}(\partial\pi)^2\left[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right]$$

$h_{\mu\nu} \to \partial_\mu\partial_\nu\phi + \cdots$ $\Rightarrow$ No radiation

The scalar theories & their couplings to point particles

Biadjoint Scalar (Zeroth Copy)

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$$S_{\rm pp} = -\frac{1}{2}\sum_{\alpha}\int \mathrm{d}\lambda \left[\eta^{-1}(\lambda)\,\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda}\cdot\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda} + \eta(\lambda)\left(m_\alpha^2 - 2\,y\,\varphi^{a\,\tilde{a}}\,c_\alpha^a\tilde{c}_\alpha^{\tilde{a}}\right)\right]$$

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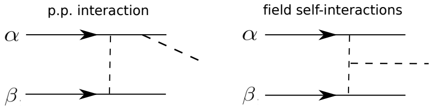
$$S_{\rm pp} = -\sum_{\alpha} m_\alpha \int \mathrm{d}\lambda \sqrt{1 + 2\frac{\pi}{\Lambda}}\sqrt{\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda}\cdot\frac{\mathrm{d}x_\alpha}{\mathrm{d}\lambda}}$$

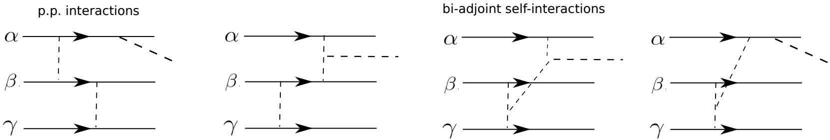
Perturbative calculation for $\mathcal{J}^{a\,\tilde{a}}$

$\mathcal{O}(y^2)$:



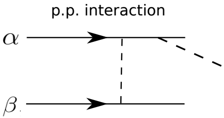
$$\mathcal{J}^{a\,\tilde a}(k)\big|_{\mathcal{O}(2)} = y^2 \sum_{\alpha,\,\beta\neq\alpha}\int_{q_\alpha,\,q_\beta}\Bigg(-\frac{q_\alpha^2}{(k\cdot p_\alpha)}\Big[c_\alpha^a(c_\alpha\cdot c_\beta)\,\tilde c_\alpha^{\tilde a}(\tilde c_\alpha\cdot\tilde c_\beta)\frac{k\cdot q_\beta}{k\cdot p_\alpha} - i f^{abc}c_\alpha^c c_\beta^b\, \tilde c_\alpha^{\tilde a}(\tilde c_\alpha\cdot\tilde c_\beta) - i\tilde f^{\tilde a\tilde b\tilde c}\tilde c_\alpha^{\tilde c}\tilde c_\beta^{\tilde b}c_\alpha^a(c_\alpha\cdot c_\beta)\Big] - f^{abc}\tilde f^{\tilde a\tilde b\tilde c}c_\alpha^b c_\beta^c\tilde c_\alpha^{\tilde b}\tilde c_\beta^{\tilde c}\Bigg)\Bigg(\prod_{i=\alpha,\beta}(2\pi)\delta(q_i\cdot p_i)\frac{e^{i\, q_i\cdot b_i}}{q_i^2}\Bigg)(2\pi)^d\delta^d(k-q_\beta-q_\alpha)$$

$\mathcal{O}(y^4)$:



Perturbative calculation for $\mathcal{J}^a$ and $\mathcal{J}$

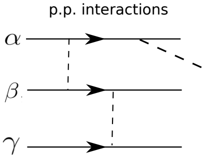
$\mathcal{O}(g^2)$:

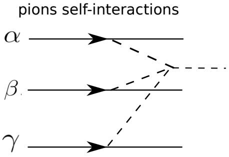


$$\mathcal{J}^a(k)\big|_{\mathcal{O}(2)} = i\sqrt{2}\frac{4}{F^2}\sum_{\alpha,\,\beta\neq\alpha}\int_{q_\alpha,\,q_\beta} m_\alpha^2 m_\beta^2 \Bigg(m_\alpha^2\,(k\cdot M_\alpha^a)(q_\beta\cdot M_\alpha^b)\,(q_\beta\cdot M_\beta^b)\,\frac{k\cdot q_\beta}{(k\cdot p_\alpha)^2} + i\,f^{abc}(q_\alpha\cdot M_\alpha^b)(q_\beta\cdot M_\beta^c)\Bigg) q_\alpha^2 \Bigg(\prod_{i=\alpha,\beta}(2\pi)\delta(q_i\cdot p_i)\frac{e^{i\,q_i\cdot b_i}}{q_i^2}\Bigg)(2\pi)^d\,\delta^d(k-q_\beta-q_\alpha)$$

$$\mathcal{J}(k)\big|_{\mathcal{O}(2)} = \frac{1}{\Lambda^2}\sum_{\alpha,\,\beta\neq\alpha}\int_{q_\alpha,\,q_\beta} m_\alpha^2 m_\beta^2 \Big(-m_\alpha^2 q_\alpha^2\,\frac{k\cdot q_\beta}{(k\cdot p_\alpha)^2}\Big)\Bigg(\prod_{i=\alpha,\beta}(2\pi)\delta(q_i\cdot p_i)\frac{e^{i\,q_i\cdot b_i}}{q_i^2}\Bigg)(2\pi)^d\,\delta^d(k-q_\beta-q_\alpha)$$

$\mathcal{O}(g^4)$:





Double Copy

Single copy: $\tilde{C}(\tilde{c}^{\tilde{a}}) \to \tilde{N}(\{q\})$, $C(\{q\}; c^a) \to C(\{q\} \cdot M^a)$

Double Copy: $C(\{q\} \cdot M^a) \to N(\{q\})$

1. **Coupling constants:**

$$y \to \frac{\sqrt{2}}{F} \to \frac{1}{\Lambda}$$

2. **Color charges:**

$$\tilde{c}^{\tilde{a}} c^a \to q \cdot M^a \to 1$$

3. **Three-point vertex:**

$$f \cdot c \cdot c \to 0$$

4. **Color-kinematics duality for the double copy:**

$$i\, 4\sqrt{2}\, f^{abc} f^{bde} (q_\beta \cdot M_\beta)^d (q_\gamma \cdot M_\gamma)^e (q_\alpha \cdot M_\alpha)^c \to \frac{(q_\beta + q_\alpha)^2 - (q_\gamma + q_\alpha)^2}{3}$$

Both satisfy Jacobi identity

Examples of color-kinematic replacements: Single copy

$$c_\alpha^a (c_\alpha \cdot c_\beta)(c_\alpha \cdot c_\gamma) \to i\,4\sqrt{2}\, m_\alpha^6 m_\beta^2 m_\gamma^2 (k\cdot M_\alpha)^a (q_\beta \cdot M_\alpha)\cdot(q_\beta \cdot M_\beta)(q_\gamma \cdot M_\alpha)\cdot(q_\gamma \cdot M_\gamma)\ ,$$

$$\frac{(c_\alpha \cdot c_\gamma) f^{abc} c_\alpha^b c_\beta^c}{q_\beta \cdot p_\alpha} \to i\,4\sqrt{2}\, m_\alpha^4 m_\beta^2 m_\gamma^2 (q_\gamma \cdot M_\alpha)\cdot(q_\gamma \cdot M_\gamma) f^{abc} ((k - q_\beta)\cdot M_\alpha)^b (q_\beta \cdot M_\beta)^c)\ ,$$

$$\frac{f^{abc} f^{bde} c_\beta^d c_\gamma^e c_\alpha^c}{(q_\beta \cdot p_\gamma)(q_\delta \cdot p_\alpha)} \to i\,4\sqrt{2}\, m_\alpha^2 m_\beta^2 m_\gamma^2 f^{abc} f^{bde} (q_\beta \cdot M_\beta)^d (q_\gamma \cdot M_\gamma)^e \times$$

$$\times \left[\left(k - q_\delta + q_\alpha \frac{q_\delta^2}{12 q_\alpha^2 q_\gamma^2} n(\alpha,\beta,\gamma)\right)\cdot M_\alpha\right]^c\ ,$$

$$\ldots$$

$$\tilde{c}_\alpha^{\tilde{a}} (\tilde{c}_\alpha \cdot \tilde{c}_\beta)(\tilde{c}_\alpha \cdot \tilde{c}_\gamma) \to 1\ ,$$

$$\tilde{c}_\alpha^{\tilde{a}}\ \ (\tilde{f}\cdot \tilde{c}_\alpha \cdot \tilde{c}_\beta \cdot \tilde{c}_\gamma) \to 0\ ,$$

$$(\tilde{c}_\alpha \cdot \tilde{c}_\gamma) \tilde{f}^{\tilde{a}\tilde{b}\tilde{c}} \tilde{c}_\alpha^{\tilde{b}} \tilde{c}_\beta^{\tilde{c}} \to 0\ ,$$

$$\ldots$$

$$\mathcal{J}^{a\,\tilde{a}}(k)\Big|_{k^2=0} \to \mathcal{J}^{a}(k)\Big|_{k^2=0}$$

Color-kinematic replacements: Double copy

At order g^4:

$$i\,4\sqrt{2}\,(k\cdot M_\alpha)^a(q_\beta\cdot M_\alpha)\cdot(q_\beta\cdot M_\beta)(q_\gamma\cdot M_\alpha)\cdot(q_\gamma\cdot M_\gamma)\to -1\ ,$$

$$i\,4\sqrt{2}\,(k\cdot M_\alpha)^a(q_\delta\cdot M_\alpha)\cdot(q_\delta\cdot M_\beta)(q_\gamma\cdot M_\beta)\cdot(q_\gamma\cdot M_\gamma)\to -1\ ,$$

$$i\,4\sqrt{2}\,(k\cdot M_\alpha)[f\cdot(q_\gamma\cdot M_\alpha)\cdot(q_\beta\cdot M_\beta)\cdot(q_\gamma\cdot M_\gamma)]\to 0\ ,$$

$$i\,4\sqrt{2}\,(k\cdot M_\alpha)[f\cdot(q_\delta\cdot M_\alpha)\cdot(q_\beta\cdot M_\beta)\cdot(q_\gamma\cdot M_\gamma)]\to 0\ ,$$

$$i\,4\sqrt{2}\,(q_\gamma\cdot M_\alpha)\cdot(q_\gamma\cdot M_\gamma)f^{abc}(q\cdot M_\alpha)^b(q_\beta\cdot M_\beta)^c\to 0\ ,$$

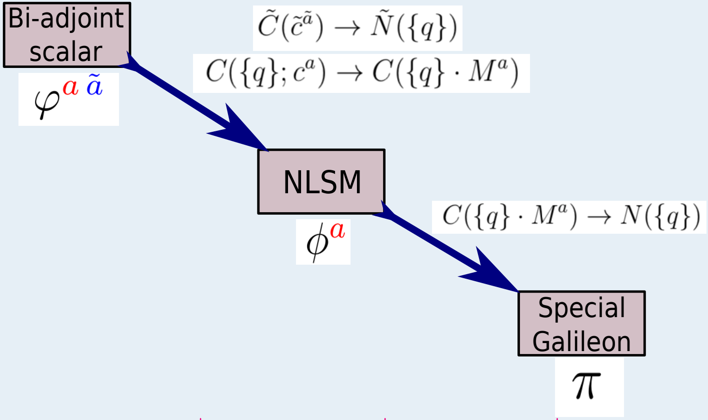
$$i\,4\sqrt{2}\,(q_\gamma\cdot M_\alpha)\cdot(q_\gamma\cdot M_\gamma)\,f^{abc}(q\cdot M_\beta)^b(q_\delta\cdot M_\alpha)^c\to 0\ ,$$

$$i\,4\sqrt{2}\,f^{abc}f^{bde}(q_\delta\cdot M_\alpha)^d(q_\beta\cdot M_\beta)^e(q_\gamma\cdot M_\gamma)^c\to -2\,n(\gamma,\alpha,\beta)\left(1+\frac{q_\beta q_\gamma^2}{q_\delta q_\alpha^2}+\frac{n(\gamma,\alpha,\beta)}{8q_\gamma^2}\right)\ ,$$

$$i\,4\sqrt{2}\,f^{abc}f^{bde}(q_\beta\cdot M_\beta)^d(q_\gamma\cdot M_\gamma)^e(q_\alpha\cdot M_\alpha)^c\to n(\alpha,\beta,\gamma)\ ,$$

$$\mathcal{J}^a(k)\Big|_{k^2=0}\to\mathcal{J}(k)\Big|_{k^2=0}$$

Summary



$$\mathcal{J}^{a\,\tilde{a}}(k)\Big|_{k^2=0} \to \mathcal{J}^{a}(k)\Big|_{k^2=0} \to \mathcal{J}(k)\Big|_{k^2=0}$$

Future directions

- Born Infeld radiation from NLSM $\times$ YM
- Higher derivative corrections to the EFTs allowing a double copy