Topics in supergravity

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a rather biased review

Five loop UV behavior of $\mathcal{N}= 8$ supergravity
with Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Zeng

and

Non-abelian gauged supergravity as double copies
with Gunaydin, Chiodaroli, Johansson

Supergravity has a long and stellar history

- Constructed as a stand-alone theory
- Low energy limit of string theory
- Exhibits U-duality, whose full consequences are yet to be understood
- Prominent place in the gauge/gravity holographic correspondence
- Unique if there is sufficient supersymmetry
- Less supersymmetry $\longleftrightarrow$ More data needed to specify theory

    e.g. - liftable to higher dimensions?
    - some interactions

- Powerful methods for constructing supergravities
- Not clear if all supergravities have been (in principle) constructed

S matrix -- tool for exploration of many aspects of perturbative supergravity
their finiteness properties: Is there a finite QFT with/of gravity?
toy model for Einstein gravity coupled to matter or gauge fields

1. Understand perturbative properties of supergravity theories
   - e.g. are there novel symmetries (in certain supergravity theories)?
   - e.g. is it possible to avoid anomaly effects in S-matrix elements?
   - e.g. is it possible to gauge an anomalous symmetry w/o penalty?
   - e.g. verify arguments based on known symmetries (UV properties)

3/2. Develop techniques to actually do higher-loop calculations

2. Understand whether all supergravities share an underlying simplicity

$$\mathrm{Gravity} = \mathrm{YM} \otimes \mathrm{YM}$$

5/2. Apply said techniques to other situations/fields

By trying to answer such questions we already learned and are likely to learn more about the structure and properties of gravity and supergravity theories

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their finiteness properties: Is there a finite QFT with/of gravity?

toy model for Einstein gravity coupled to matter or gauge fields

tool for applying perturbative (super)gravity methods

tool for constructing novel supergravities

A plan

Supergravity theories – a Lagrangian perspective

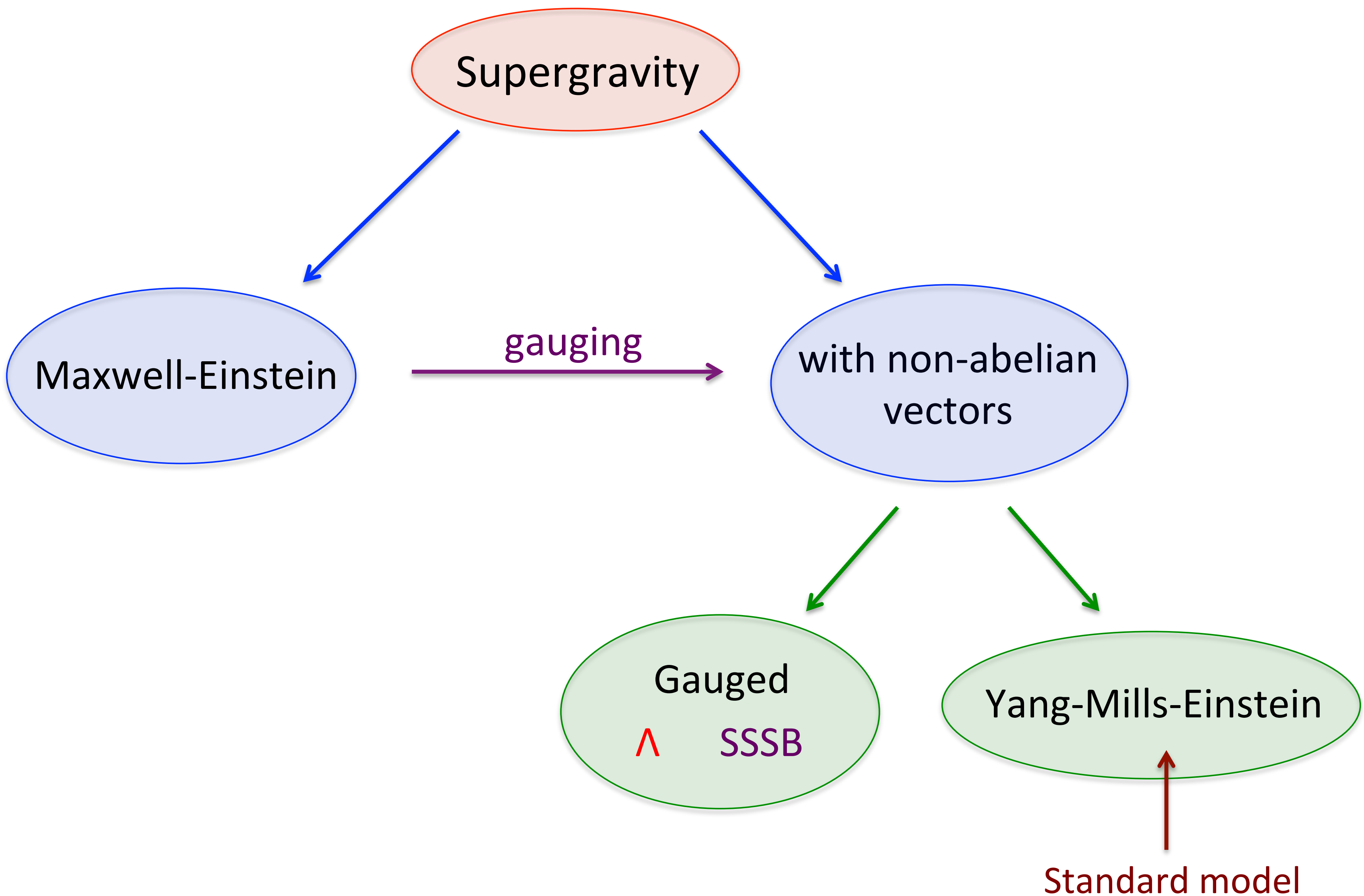
Supergravity theories – an S matrix perspective

Examples: Supergravities with gauged R symmetry

Maxwell-Einstein supergravities

Pure supergravities -- $\mathcal{N}$=8 SG at 5 loops

more in Alex Edison's talk


Supergravity
Maxwell-Einstein
gauging
with non-abelian vectors
Gauged
Λ
SSSB
Yang-Mills-Einstein
Standard model

Focusing on $\mathcal{N}$=2 Maxwell-Einstein theories...

Simplest to describe in D=5:

Gunaydin, Sierra, Townsend

graviphoton + matter

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^{I}_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}A^{K}_{\lambda}$$

functions of φ

constants

Maxwell/Einstein 5d supergravity theories w. vectors

Gunaydin, Sierra, Townsend

graviphoton + matter

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^{I}_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

functions of φ constants

Everything is determined by a prepotential

$$N(\xi) = \left(\frac{2}{3}\right)^{3/2} C_{IJK}\xi^I\xi^J\xi^K \qquad \xi = (\varphi^x, \rho) \qquad a_{IJ} = -\frac{1}{2}\partial_I\partial_J \ln N(\xi)$$

$$\mathring{a}_{IJ}(\varphi) = a_{IJ}\big|_{N(\xi)=1} \qquad g_{xy}(\varphi) = \mathring{a}_{IJ}\partial_x\xi^I\partial_y\xi^J$$

$\mathring{a}_{IJ}$ in terms of vielbeine on the scalar manifold:

- canonical basis for C : $C_{000} = 1 \quad C_{00i} = 0 \quad C_{0ij} = -\frac{1}{2}\delta_{ij} \quad i,j = 1,\ldots,n$
  all other entries C_{ijk} arbitrary

D=5 $\mathcal{N}$=2 SG w/ vector multiplets: spectrum does not fix theory uniquely; interactions can be different

- If scalar manifold is symmetric space (incomplete summary)

  - $$\mathcal{M}_{gJf} = \frac{SO(n-1,1)}{SO(n-1)} \times SO(1,1)$$
  $$N_{gJf}(\xi) = \sqrt{2}\xi^0\Big((\xi^1)^2 - (\xi^2)^2 - \ldots - (\xi^n)^2\Big)$$
  - $$\mathcal{M}_{g-NJf} = \frac{SO(n,1)}{SO(n)}$$
  $$N_{g-NJf}(\xi) = \frac{3\sqrt{3}}{2\sqrt{2}}\Big(\sqrt{2}\xi^0(\xi^1)^2 - \xi^1\Big((\xi^2)^2 + \ldots + (\xi^n)^2\Big)\Big)$$
  - Magical cases for $n = 4, 8, 14, 26$
  $$\mathcal{M}_{\text{magical}} \in \left\{\frac{SL(3,\mathbf{R})}{SO(3)}, \ \frac{SL(3,\mathbf{C})}{SU(3)}, \ \frac{SU^*(6)}{USp(6)}, \ \frac{E_{6(-26)}}{F_4}\right\}$$

- More general – homogeneous scalar manifolds

de Wit, van Proeyen

$$N_{\text{hom}}(\xi) = \sqrt{2}\xi^0((\xi^1)^2 - (\xi^i)^2) + \xi^1(\xi^\alpha)^2 + \tilde{\Gamma}^i_{\alpha\beta}\xi^i\xi^\alpha\xi^\beta$$

- Totality of global symmetries = U-duality
- Scalar manifolds need not be homogeneous spaces or have any symmetries;
  When present, some symmetries can be gauged
- Hypermultiplets can be included as well

Dimensional reduction to D=4

Gunaydin, McReynolds, Zagerman

- one extra vector multiplet

$$
\begin{aligned}
e^{-1}\mathcal{L}^{(4)} \;=\;& -\frac{1}{2}R-\frac{1}{2}e^{3\sigma}W_{\mu\nu}W^{\mu\nu}-\frac{3}{4}\partial_\mu\sigma\partial^\mu\sigma \\
& -\frac{1}{4}e^{\sigma}\,\mathring{a}_{IJ}(F^I_{\mu\nu}+2W_{\mu\nu}A^I)(F^{J\mu\nu}+2W^{\mu\nu}A^J) \\
& -\frac{1}{2}e^{-2\sigma}\,\mathring{a}_{IJ}\partial_\mu A^I\partial^\mu A^J-\frac{3}{4}\mathring{a}_{IJ}\partial_\mu h^I\partial^\mu h^J \\
& +\frac{e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\Big\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K+2F^I_{\mu\nu}W_{\rho\sigma}A^JA^K+\frac{4}{3}W_{\mu\nu}W_{\rho\sigma}A^IA^JA^K\Big\}
\end{aligned}
$$

- Fermion terms are known
- U-duality group grows; e/m duality may or may not be factorized subgroup

If factorized:

- all vectors $F^A=\{W,F^I\}$ and their duals $\tilde{F}^A$ in rep. of 4d G
- $\begin{pmatrix}F^A\\ \tilde{F}^A\end{pmatrix}$ doublet of $SU(1,1)$
- dilaton/axion parametrize $SU(1,1)/U(1)$
- $SU(1,1)$ is typically anomalous; anomaly can sometimes be assigned to group elements that do not affect S matrix

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    do not affect S matrix

    Bern, Parra-Martinez, RR

Example: $\mathcal{N}$=4 supergravity

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \frac{1}{4}\left(\partial_\mu\varphi\partial^\mu\varphi + e^{2\varphi}\partial_\mu\chi\partial^\mu\chi - e^{-\varphi}F^A_{\mu\nu}F^{\mu\nu\,A} - i\chi F^A_{\mu\nu}\tilde{F}^{\mu\nu\,A}\right)$$

$$= -\frac{1}{2}R + \frac{1}{4}\left(\frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{(Im(\tau))^2} + i\tau\, F^{A+}_{\mu\nu}F^{A+}_{\mu\nu} - i\bar{\tau}\, F^{A-}_{\mu\nu}F^{A-}_{\mu\nu}\right)$$

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\,g_{rs}(\Phi)\partial_\mu\Phi^r\partial^\mu\Phi^s + \frac{1}{4}\left(i\bar{\mathcal{N}}_{\Lambda\Sigma}F^{-\Lambda}_{\mu\nu}F^{-\Sigma|\mu\nu} - i\mathcal{N}_{\Lambda\Sigma}F^{+\Lambda}_{\mu\nu}F^{+\Sigma|\mu\nu}\right)$$

$$G = SO(n,6)\times SU(1,1) \qquad\qquad \mathcal{M}_s = \frac{SO(n,6)}{SO(n)\times SO(6)}\times\frac{SU(1,1)}{U(1)}$$

de Roo

Gauging of global symmetries – a Lagrangian perspective

- Global symmetries: R symm & symm's of scalar space which are symm's of $\mathcal{L}$

  5d is advantageous: U-duality is a symmetry of Lagrangian

- Gauging: deformation of the Maxwell-Einstein theory

  some of the *existing* vector fields are given interactions that gauge some of the global symm's; vectors can become self-interacting

- Several types of gaugings with gauge group *K*

$K \subseteq R$ $\qquad$ $K \subseteq \mathrm{Compact}(U)$ $\qquad$ $K \subseteq \mathrm{Noncompact}(U)$

$K \subseteq R \times \mathrm{Compact}(U)$ $\qquad$ $K \subseteq R \times \mathrm{Noncompact}(U)$

Gauging of global symmetries – a Lagrangian perspective

- **Global symmetries:** R symm & Symm's of scalar space which are symm's of $\mathcal{L}$

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- **Several types of gaugings**

  $K \subseteq R$ — Gauged SG

  $K \subseteq \mathrm{Compact}(U)$ — Compact YME

  $K \subseteq \mathrm{Noncompact}(U)$ — Noncompact YME

  $K \subseteq R \times \mathrm{Compact}(U)$ — Gauged compact YME

  Λ

  $K \subseteq R \times \mathrm{Noncompact}(U)$ — Gauged noncompact YME

  Unitarity is preserved for noncompact gaugings

- Gauging can lead to scalar-field potentials; vacuum structure and vacuum symm's can be different between the gauged and ungauged theories (e.g. Minkowski vacua may have broken supersymmetry)

YME gauging in d=5:

Gunaydin, Sierra, Townsend

- Gauge group generators: $(M_r)_I{}^J$, $[M_r, M_s] = f_{rs}{}^t M_t$

  Lagrangian symmetry: $(M_r)_{(I}{}^L C_{JK)L} = 0$

- Covariantization:

Killing vector: $K_s^x = -\sqrt{\frac{3}{2}}(M_s)^J{}_I h_J h^{Ix}$

$$\partial_\mu \varphi^x \to \partial_\mu \varphi^x + g A_\mu^s K_s^x$$

$$L_t^{ab} = (M_t)^J{}_I h_J^{[a|} h^{I|b]} - \Omega_x^{ab} K_t^x$$

Spin connection on $\mathcal{M}$

$$\nabla_\mu \lambda^{ia} \to \nabla_\mu \lambda^{ia} + g L_t^{ab} A_\mu^t \lambda^{ib}$$

$$F_{\mu\nu}^I \to \mathcal{F}_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + g f^I{}_{JK} A_\mu^J A_\nu^K$$

Zero unless indices are in gauge group

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- Modifications to the Chern-Simons coupling:

$$\frac{e^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K \longrightarrow$$

$$\frac{e^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} \left\{ F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K + \frac{3}{2} g f^K{}_{J'K'} F_{\mu\nu}^I A_\rho^J A_\sigma^{J'} A_\lambda^{K'} + \frac{3}{5} g^2 A_\mu^I f^J{}_{I'J'} A_\nu^{I'} A_\rho^{J'} f^K{}_{K'L'} A_\sigma^{K'} A_\lambda^{L'} \right\}$$

- Tensors: tensor $\longleftrightarrow$ vector in d=5, *but* vectors in nontrivial rep of gg $\longrightarrow$ tensors
- Fermion-scalar couplings: $\mathcal{L}_Y = -\frac{i}{2} g \bar{\lambda}^{ia} \lambda_i^b K_{r[a} h_{b]}^r$
- No scalar potential $\longrightarrow$ there is a Minkowski ground state with unbroken susy

- 4d from 5d by dimensional reduction — Gunaydin, McReynolds, Zagermann
- Scalar potential appears; nonetheless, Minkowski vacuum survives

$$
\begin{aligned}
e^{-1}\mathcal{L}^{(4)} \;=\;& -\frac{1}{2}R - \frac{3}{4}\overset{\circ}{a}_{IJ}(\mathcal{D}_\mu \tilde{h}^I)(\mathcal{D}^\mu \tilde{h}^J) - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{IJ}(\mathcal{D}_\mu A^I)(\mathcal{D}^\mu A^J) \\
& -\frac{1}{4}e^{\sigma}\overset{\circ}{a}_{IJ}(\mathcal{F}^I_{\mu\nu} + 2W_{\mu\nu}A^I)(\mathcal{F}^{J\mu\nu} + 2W^{\mu\nu}A^J) - \frac{1}{2}e^{3\sigma}W_{\mu\nu}W^{\mu\nu} \\
& +\frac{e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\left\{\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma}A^K + 2\mathcal{F}^I_{\mu\nu}W_{\rho\sigma}A^J A^K + \frac{4}{3}W_{\mu\nu}W_{\rho\sigma}A^I A^J A^K\right\} \\
& -g^2 P_4\ ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_\mu A^I \;&\equiv\; \partial_\mu A^I + g A^J_\mu f^I_{JK} A^K \\
\mathcal{F}^I_{\mu\nu} \;&\equiv\; 2\partial_{[\mu}A^I_{\nu]} + g f^I_{JK} A^J_\mu A^K_\nu
\end{aligned}
$$

$$
P_4 = \frac{3}{4}e^{-3\sigma}\overset{\circ}{a}_{IJ}(A^r (M_r)^I{}_K h^K)(A^s (M_s)^J{}_L h^L)
$$

Gauging of R symmetry

First gauging: *U(1)* and *SU(2)* of pure $\mathcal{N}$=2 and $\mathcal{N}$=3 SGs. Freedman, Das

Minimal vector-gravitino coupling $\longrightarrow$ gravitino mass
cosmological constant

$\longrightarrow$ Typical vacuum is supersymmetric and not Minkowski
Minkowski vacua have (partial) spontaneous supersymmetry breaking

First gauging of $\mathcal{N}$=8 supergravity: *SO(8)* De Wit, Nicolai

$E_{7(7)}$ is a large group; other options should be available

The game: 1) formulate the theory such that a (desired) subgroup of U duality is realized off shell
2) try to gauge it

SU(3,1) Gunaydin, Romans, Warner

SO(p, 6-p) in d=5

SO(p,8-p) Hull

Many further examples, $2 \leq \mathcal{N} < 8$

More systematic approach: embedding tensor formalism
De Wit, Nicolai, Samtleben, Trigiante

More systematic approach to gauging: embedding tensor formalism

De Wit, Nicolai, Samtleben, Trigiante

- Start with U-duality-covariant formulation of the action

- Replace $\partial_\mu \to D_\mu = \partial_\mu - g A_\mu^M \Theta_M{}^\alpha T_\alpha$

U-duality generators

All vectors

Embedding of the gauge group in *U*-duality group

- Constrain $\Theta_M{}^\alpha$ by demanding:
  - algebra closure (quadratic relation)
  - supersymmetry (linear relation)

- Solve them

Examples of new gaugings of $\mathcal{N}$=8 SG with Minkowski vacua

Gauge group	Residual susy	Unbroken gauge group
SO*(8)	$\mathcal{N}$=0	SU(4)xU(1)
CSO*(6,2)	$\mathcal{N}$=2	SU(3)xU(1)
► CSO*(4,4)	$\mathcal{N}$=4	SU(2)xU(1)
(SO(4) x SO(2,2)) x T^{16}	$\mathcal{N}$=0	SU(2)^{2}xU(1)^2
(SO*(4) x U(1)) x N^{20}	$\mathcal{N}$=2	SU(2)xU(1)^2

Borghese, Dibitetto, Guarino, Roest, Varela; Catino, Dall'Agata, Inverso, Trigiante, Zwirner; Hull

Supergravity theories – an S matrix perspective

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- Probes aspects of the theory without complications due to certain nonlinearities and possible field choices
- Less transparent for properties that do not have a linearized approximation

approaches: Arkani-Hamed, Cachazo, Kaplan; etc
Kallosh

The main tools for gravitational S-matrix elements: $\mathrm{Gravity} = \mathrm{YM} \otimes \mathrm{YM}$

- The double-copy construction/color-kinematics   Bern, Carrasco, Johansson

$$\mathcal{M}_m^{L-\mathrm{loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i \in \mathcal{G}_3} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- Diffeomorphism invariance is a consequence of gauge invariance of the two gauge theories entering the construction   Chiodaroli, Gunaydin, Johansson, RR
Arkani-Hamed, Rodina, Trnka

Diffeomorphism invariance from gauge invariance

1. Linearized YM gauge transformations: $\epsilon^\mu(p) \mapsto p^\mu$

$$\mathcal{A} = \sum_\Gamma \frac{n_\Gamma(\epsilon_1(p_1), \epsilon_2, \ldots) c_\Gamma}{D_\Gamma} \longrightarrow 0 = \sum_\Gamma \frac{n_\Gamma(p_1, \epsilon_2, \ldots) c_\Gamma}{D_\Gamma}$$

1. structure of n_Γ 2. Algebra relations for c_Γ

2. Linearized diffeomorphisms: $\epsilon^{\mu\nu}(p) \mapsto p^{(\mu} q^{\nu)}$

$$\epsilon^{\mu\nu}(p) \equiv \epsilon^{(\mu}(p)\epsilon'^{\nu)}(p) \mapsto p^{(\mu}\epsilon'^{\nu)}(p) + p^{(\nu}\epsilon^{\mu)}(p)$$

$$\mathcal{M} = \sum_\Gamma \frac{n_\Gamma(\epsilon_1(p_1), \epsilon_2, \ldots)\tilde{n}_\Gamma(\epsilon'_1(p_1), \epsilon'_2, \ldots)}{D_\Gamma}$$

$$\delta\mathcal{M} = \sum_\Gamma \frac{n_\Gamma(p_1, \epsilon_2, \ldots)\tilde{n}_\Gamma(\epsilon'_1(p_1), \epsilon'_2, \ldots)}{D_\Gamma} + (n \leftrightarrow \tilde{n})$$

$n_\Gamma, \tilde{n}_\Gamma \& c_\Gamma$ have the same properties $\Longrightarrow$ $\delta\mathcal{M} = 0$ for the same reasons as in YM theory

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- Further subtleties when matter in non-adjoint representation is present
  - correlation between representations of fields on graph's edges
  - 2-copy spectrum $\longleftrightarrow$ (ir)reducibility of matter representations

- Generalized double-copy construction   Bern, Carrasco, Chen, Johansson, Zeng, RR
(more later)

Supergravities with a double-copy structure

$\mathcal{N} > 4$ SG	$\mathcal{N}$=4 sYM $\mathcal{N}$≤4 sYM	Bern, Boucher-Veronneau, Davies, Dennen, Dixon, Carrasco, Johansson, RR
$\mathcal{N} = 4$ + vectors	$\mathcal{N}$=4 sYM $\mathcal{N}$=0 sYM or $\mathcal{N}$=2 × $\mathcal{N}$=2	
pure $\mathcal{N} < 4$	YM+ghosts YM+ghosts	Johansson, Ochirov
$\mathcal{N}$=2 MESG generic Jordan	$\mathcal{N}$=2 sYM $\mathcal{N}$=0 sYM	Chiodaroli, Gunaydin, Johansson, RR
$\mathcal{N}$=2 MESG homogeneous	$\mathcal{N}$=2 sYM+½hyper $\mathcal{N}$=0 sYM+fermions	
$\mathcal{N}$=2 MESG +hypers special cases	$\mathcal{N}$=2 sYM+½hyper $\mathcal{N}$=0 sYM+scalars	Chiodaroli, Gunaydin, Johansson, RR; Anastasiou, Borsten, Duff, Marrani, Nagy, Zoccali
$\mathcal{N}$=2 MESG +vectors+hypers special cases	$\mathcal{N}$=1 sYM+chiral $\mathcal{N}$=1 sYM+chiral	Carrasco, Chiodaroli, Gunaydin, RR; Damgaard, Huang, Sondergaard, Zhang; Anastasiou, Borsten, Hughes, Nagy
0≤$\mathcal{N}$≤4 YME generic Jordan	$\mathcal{N}$=2 sYM $\mathcal{N}$=0 sYM + Φ^3	Chiodaroli, Gunaydin, Johansson, RR; Cachazo, He, Yuan; Du, Teng, Feng; Cheung, Shen, Wen; Casali, Geyer,Mason, Monteiro, Roehrig; Nandan, Plefka, Schlotterer, Wen

$\mathcal{N}$=2 U(1) gauged special cases	$\mathcal{N}$=2 sYM + ✗ $\mathcal{N}$=0 sYM SSB	Chiodaroli, Gunaydin, Johansson, RR
$\mathcal{N}\geq$4 U(1) and G-gauged special cases	$\mathcal{N}$=4 sYM SSB $\mathcal{N}$=4 sYM+mass + ✗	
$\mathcal{N}$ = 1 SG +vectors special cases	$\mathcal{N}$=1+chiral YM+scalars+fermions	Chiodaroli, Gunaydin, Johansson, RR; Anastasiou, Borsten, Duff, Marrani, Nagy, Zoccali ; Johansson, Ochirov
$\mathcal{N}$ = 1 SG +chirals special cases	$\mathcal{N}$=1+chiral YM+scalars+scalars	
$\mathcal{N}$ = 0 SG +matter special cases	YM+matter YM+matter	Johansson, Ochirov
$\mathcal{N}$=0 SG $+\Phi R^2 + R^3$	YM+F^3 +… YM+$F^{3+\ldots}$	Broedel, Dixon
$\mathcal{N}\leq$4 conformal (S)G	DF^2 (s)YM	Johansson, Nohle; Johansson, Mogull, Teng
3D maximal SG	BLG × BLG or sYM × sYM	Huang, Johansson; Bargeer, He, McLoughlin

So many of them... Is it possible that all supergravities are double copies?

General patterns:

Manifest global symmetries are inherited from single copies; enhancements

Global symmetries are gauged with single-copy cubic biadjoint coupling

(minimal vector self-coupling)

Explicit global symmetry breaking $\longrightarrow$ spontaneous symmetry breaking

Double-copy states $\longleftrightarrow$ gauge invariant bilinears; gauge theory representations need not be irreps; organization in representations correlated to target spectrum

Upcoming review, to appear

Bern, Carrasco, Chiodaroli, RR

Lots of new results

- Graviton-matter amp's in EYM generic Jordan family — Plefka, Wormsbecher
- Rational one-loop amp's in EYM generic Jordan family — Nandan, Plefka, Travaglini
- One-loop effective action in dilaton gravity for massive scalars — Plefka, Steinhoff, Wormsbecher
- One-loop amplitudes and UV properties of homogeneous ME supergravities with hypermultiplets — Ben-Shahar, Chiodaroli
- Cancellation of duality anomaly in S matrix in $\mathcal{N}$=4 SG and possibly elsewhere — Bern, Parra-Martinez, RR
- Progress on off-shell matching of DOF — Cardoso, Inverso, Nagy, Nampuri
- Rational map interpretation of YM and gravity amplitudes in d=6; YM coulomb branch amplitudes — Cachazo, Guevara, Heydeman, Mizera, Schwarz, Wen
- Double-copy approach of certain SGs with nonabelian gauged R symmetry — Chiodaroli, Gunaydin, Johansson, RR
- 2-loop SQCD matter amplitudes — Kälin, Mogull, Ochirov
- UV properties of $\mathcal{N}$=8 SG at 5 loops; new structures in the small momentum exp. — Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, RR, Zeng
- Novel/enhanced properties of $\mathcal{N}$=8 SG cuts in D=4 — Herrmann, Trnka

Lots of new results

- Conformal symmetry in tree-level gravity amplitudes — Loebbert, Mojaza, Plefka
  - relation to hidden superconformal symm. conj. in $\mathcal{N}$=4 SG? — Ferrara, Kallosh, van Proeyen
- Progress on scattering in curved space — Adamo, Casali, Mason, Nekovar

Classical gravity from S matrix

- BH scattering in N=8 SG; integrability of BH orbits — Caron-Huot, Zaharee
- Double-copy for self-dual solutions as a differential operator on harmonic functions in biadjoint scalar theory — Berman, Chácon, Luna, White
- Certain gravitational waves as Kerr-Schild double copies — Ilderton
- The Weyl double-copy – larger class of 2-copy s.-t. — Luna, Monteiro, Nicholson, O'Connell
  – generalizes Kerr-Schild double-copy, reduces to it in app'te limits
- Perturbiner for EFT; possible new approach to classical 2-copy — Mizera, Skrzypek
- Leading order gravitational radiation in EYM generic Jordan family — Chester
- Next-to-leading order radiation for gravity-coupled scalar fields — CH Shen
- Systematics of gravitational effective action in classical limit from scattering amplitudes — Cheung, Rothstein, Solon
- Systematics of certain classical gravitational observables from scattering amplitudes — Kosower, Maybee, O'Connell

Example: $\mathcal{N}$=8 supergravity at 5 loops

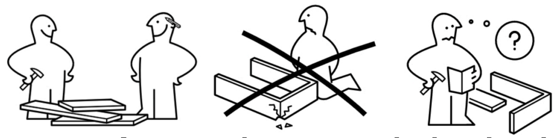
Symmetry-based predictions and refinements for UV div's of pure 4D SGs

3-loop $\mathcal{N}=8$	Green, Schwarz, Brink ('82); Marcus, Sagnotti ('85) Howe, Stelle ('89)	✗	Summary of results of Bern, Carrasco, Davies, Dennen, Kosower, Dixon, Johansson, RR, 2 x Smirnov
5-loop $\mathcal{N}=8$	Bern, Dixon, Dunbar, Perelsein, Rozowski ('98) Howe, Stelle ('03, '09)	✗	
6-loop $\mathcal{N}=8$	Howe, Stelle ('12)	✗	
7-loop $\mathcal{N}=8$	Grisaru, Siegel ('82); Bossard, Howe, Stelle ('09); Vanhove; Björnsson, Green ('10); Kiermaier, Elvang, Freedman('10); Ramond, Kallosh ('10); Biesert et al ('10); Bossard, Howe, Stelle, Vanhove ('11)	?	← Goal is to eventually check this
8-loop $\mathcal{N}=8$	Kallosh ('80); Howe, Lindstrom ('80)	?	
9-loop $\mathcal{N}=8$	Berkovits, Green, Russo, Vanhove ('09)	?	← retracted
4-loop $\mathcal{N}=5$	Bossard, Howe, Stelle, Vanhove ('11)	✗	← unexplained
3-loop $\mathcal{N}=4$	Bossard, Howe, Stelle, Vanhove ('11)	✗	
4-loop $\mathcal{N}=4$	Vanhove, Tourkine ('12)	√ but	← SU(1,1) anomaly

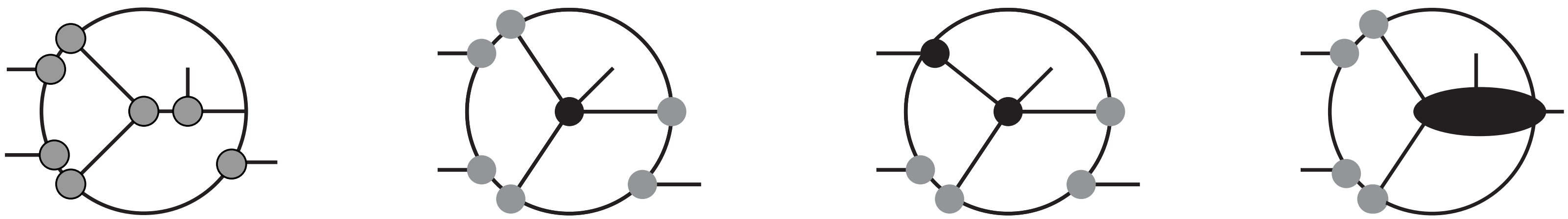
Symmetry-based predictions are as good as one's understanding of symmetries

Generalized unitarity/contact term method:

Bern, Carrasco, Dixon, Johansson, RR
Bern, Carrasco, Chen, Johansson, RR



0. Organize cuts into classes, labeled by the number of uncut propagators



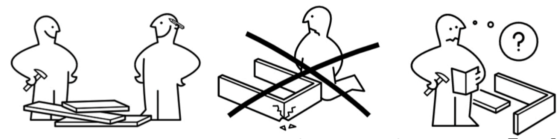
maximal cut | next-to-maximal cut | (next-to)^2-maximal cut | (next-to)^3-maximal cut

All exposed propagators are cut

- Each cut gives new information on the amplitude, fixing terms in which *all* the uncut propagators are absent

- There is a finite number of cuts, determined by the powercounting of the theory, which should be expected to give new contributions

Generalized unitarity/the contact term method/generalized double-copy



Bern, Carrasco, Dixon, Johansson, RR
Bern, Carrasco, Chen, Johansson, RR

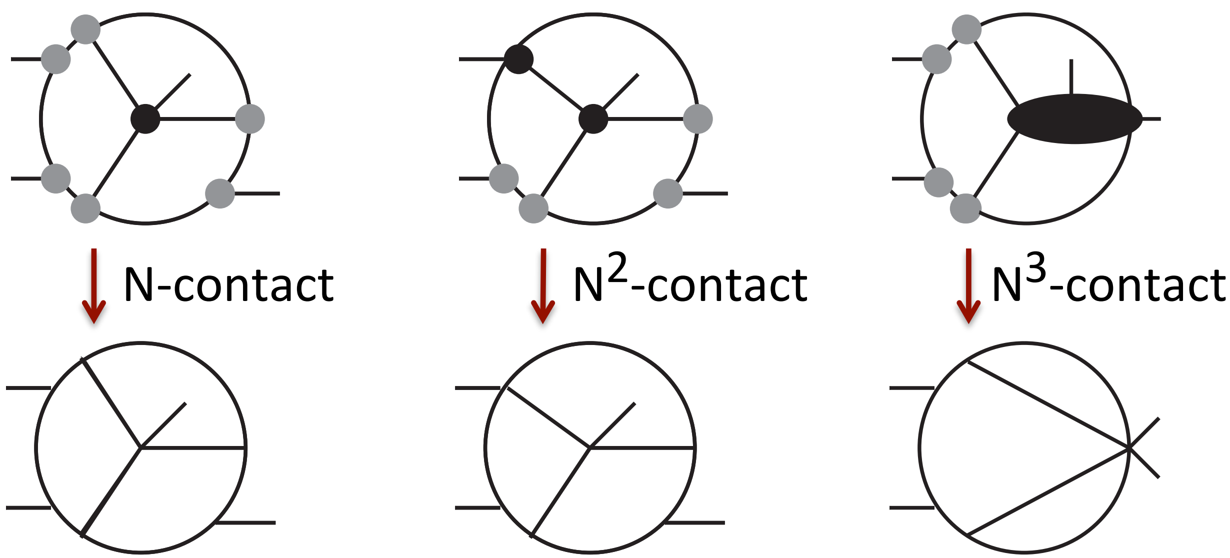
1. Start with some approximation of the supergravity amplitude, organized in terms of the graphs of φ^3 theory, which has the correct maximal cuts, e.g.

a naïve double-copy: $$\mathcal{M}^{(S)G} = \sum_{\Gamma} \int \frac{n_\Gamma \tilde{n}_\Gamma}{D_\Gamma}$$

2. Iteratively correct it w/ graphs w/ higher-pt. vert's to satisfy such that N^k-Max cuts

N^k-contact = N^k-max cut – (N^k-max cut of approximation of amp.)

E.g.

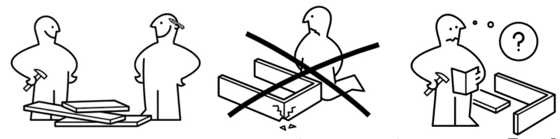


- Each cut gives an independent contrib. to amplitude
- Some freedom in choosing each of them
- Lots of cuts
- But a finite number!
- $\mathcal{N}$=8 SG: N^{2L-4}-contact

Magic: existence of BCJ rep ➡ contacts are bilinears in $J_{ijk} = n_i + n_j + n_k$

Carrasco talk at QCD meets Gravity III & Amplitudes 2018

Generalized unitarity/the contact term method/generalized double-copy



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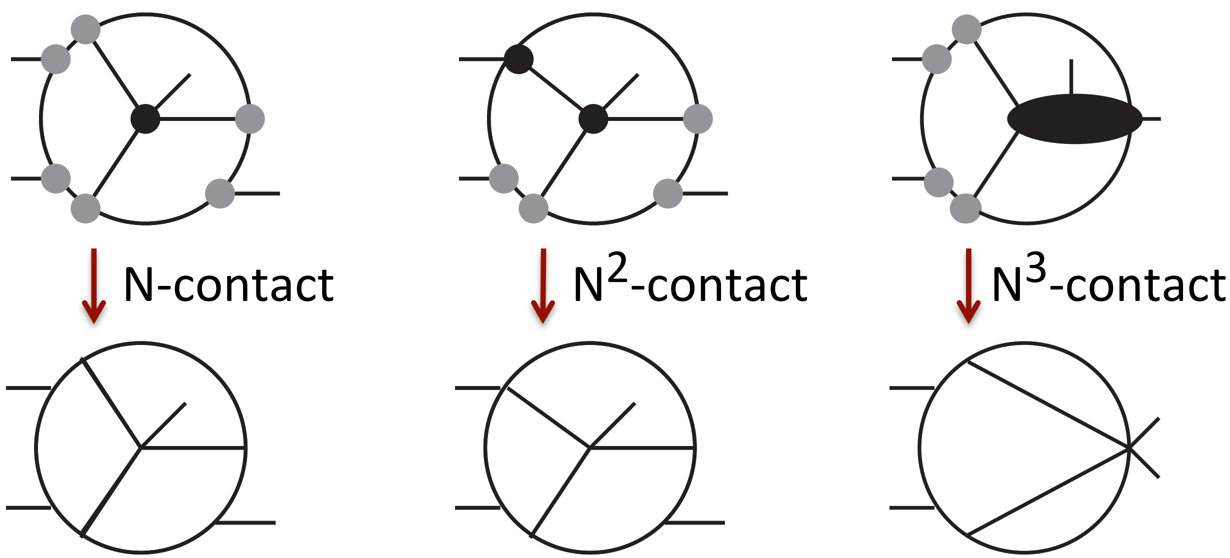
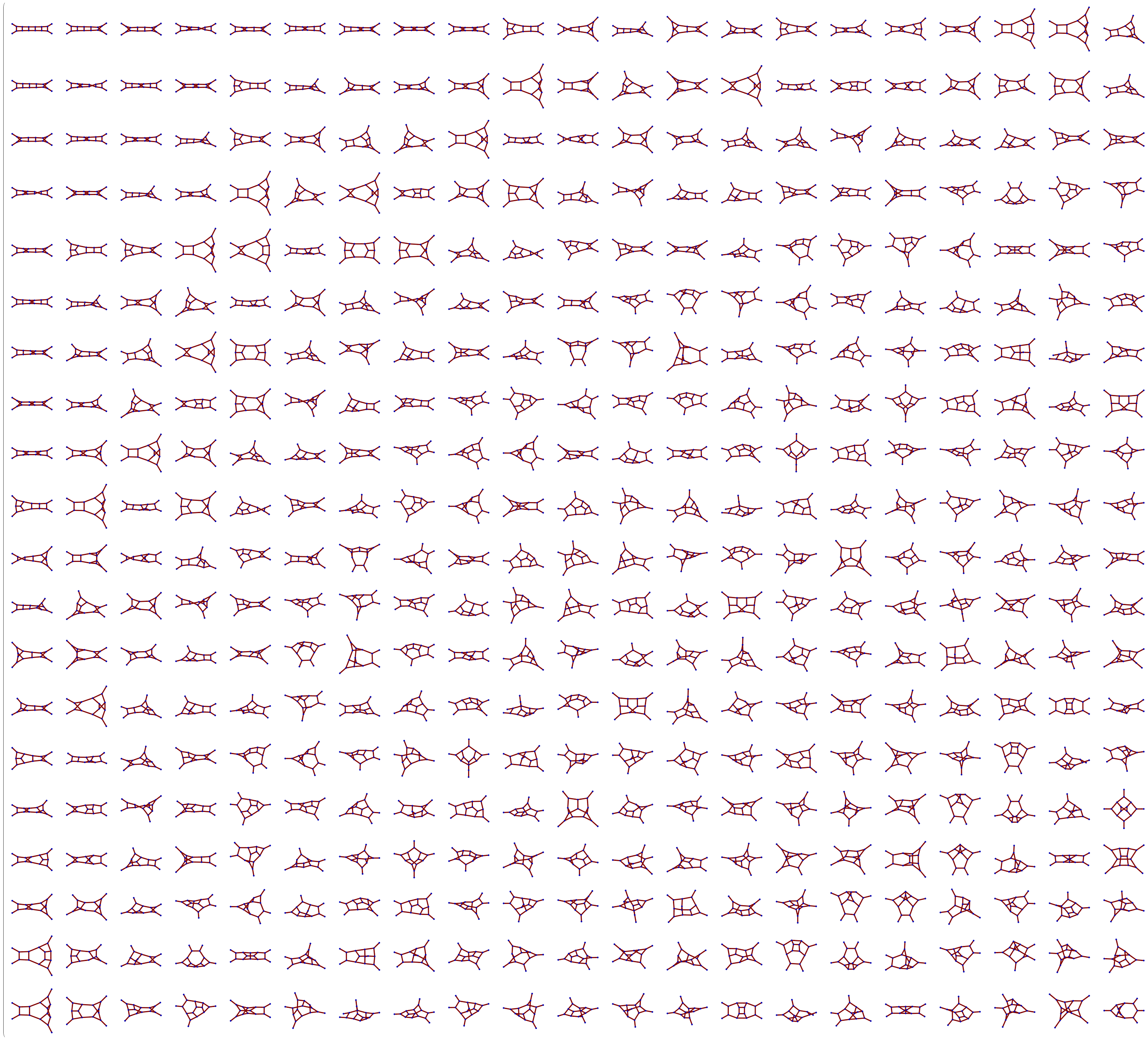


- Each cut gives an independent contrib. to amplitude
- Some freedom in choosing each of them
- Lots of cuts
- But a finite number!
- $\mathcal{N}$=8 SG: N^{2L-4}-contact

- Effectively a tree-level calculation
- Ideal if cuts are organized in terms of cubic tree graphs

Constructed the 4-point 5-loop integrand of $\mathcal{N} = 8$ supergravity

Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, RR, Zeng



together with 2-, 3-, 4-, 5-, and 6-collapsed propagator graphs:

N^0 : 752 /649
N^2 : 9007 /1306
N^3 : 17479/2457
N^4 : 22931/2470
N^5 : 20657/1335
N^6 : 13071/256

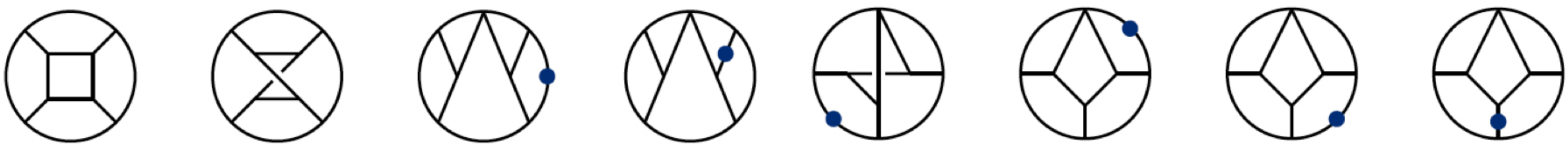
Exhibited power ct depends on that of the representation of YM amplitude

UV properties/critical dimension at 5 loops

Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, RR, Zeng

$$\mathcal{M}_4^{(5)} \sim (stu\mathcal{M}_4^{(0)})\, s^2 \int d^{5D}l\, \left[F_{-10}(l_i\cdot l_j) + sF_{-11}(l_i\cdot l_j) + s^2 F_{-12}(l_i\cdot l_j) + \ldots\right]$$

- Eight master integral topologies



(a) (b) (c) (d) (e) (f) (g) (h)

- $F_{-11}(l_i \cdot l_j)$ integrates to zero in *D* = 22/5
- $F_{-12}(l_i \cdot l_j)$ integrates to

positive-definite integrals

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2\, stu M_4^{\text{tree}} \Bigg(\frac{1}{48}$$  $$+ \frac{1}{16}$$  $$\Bigg)$$

See Alex Edison's talk for details and implications

- $\mathcal{N}$=8 SG has no enhanced cancellations in *D* = 24/5; diverges in this dimension
- $\mathcal{N}$=5 SG does have enhanced cancellations in *D* = 4
- Analysis of cuts vs. UV $\longrightarrow$ *D* = 4 can be responsible

Herrmann, Trnka

Example: theories with gauged nonabelian *R* symmetry

Example: theories with gauged nonabelian R symmetry

Spontaneous supersymmetry breaking in the Minkowski vacuum

- gravitino minimal coupling $\longrightarrow$ $\mathcal{M}_3(1\overline{\psi}_i, 2\psi_j, 3A^a) = ig_{\rm R} t^a_{ij} \bar{v}_1^{\mu} \not{\epsilon}_3 v_{2\mu} + \mathcal{O}(g^0_{\rm R})$
- linearized susy transformation $v_{l\mu} \to v_{l\mu} + k_{l\mu}\epsilon$ is not a symmetry

- massive gravitini: constructed from a massive vector and massive fermion
- reduce to ungauged theory in massless limit; supersymmetry is restored

Does/can $\mathcal{M}_3$ factorize? $\mathcal{M}_3 = \mathcal{A}_3\, \mathcal{A}'_3$

From dim-$\frac{3}{2}(d-2)$ trilinear vector coupling w/ 2 massive vectors $\longrightarrow$ spontaneously-broken gauge theory

From dim-d Yukawa w/ massive fermions and massless scalar $\longrightarrow$ C/K-preserving massive deformation or/and truncation thereof

- (at least) one gauge theory has trilinear vector couplings (nonabelian gauge symm)

Massive deformations of $\mathcal{N}$=4 sYM

Chiodaroli, Gunaydin, Johansson, RR
to appear

Action ansatz:

$$\mathcal{L} = -\frac{1}{4}(F^{\hat{a}}_{\mu\nu})^2 + \frac{1}{2}(D_\mu \phi^{\hat{a}I})^2 - \frac{1}{2}m^2_{IJ}\phi^{\hat{a}I}\phi^{\hat{a}J} - \frac{g^2}{4}f^{\hat{a}\hat{b}\hat{e}}f^{\hat{c}\hat{d}\hat{e}}\phi^{\hat{a}I}\phi^{\hat{b}J}\phi^{\hat{c}I}\phi^{\hat{d}J} - \frac{g^2}{3!}f^{\hat{a}\hat{b}\hat{c}}F^{IJK}\phi^{\hat{a}I}\phi^{\hat{b}J}\phi^{\hat{c}K}$$
$$+ \frac{i}{2}\bar{\psi}\not{D}\psi - \frac{1}{2}\bar{\psi}M\psi + \frac{g}{2}\phi^{\hat{a}I}\bar{\psi}^a\Gamma^I t^{\hat{a}}_R\psi$$

with $\{M, \Gamma_{11}\} = 0$

Color-kinematics-duality constraints:

$$\left(\Gamma^I\Gamma^J M + \Gamma^I M \Gamma^J - M\Gamma^J\Gamma^I - \Gamma^J M \Gamma^I + i\lambda F^{IJK}\Gamma^K\right) = 0$$
$$\{\Gamma^I, \Gamma^J\} = -2\delta^{IJ}$$

A particular solution: $M = i\frac{g}{4}\Gamma^{789}$, $F^{789} = 1$; SU(2) symmetry

Further constraints related to spectrum

1) No undesired/undesirable states in the spectrum

2) Recover $\mathcal{N}$=8 spectrum in massless/ungauged limit

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Further constraints related to spectrum demand:

SU(3N) gauge group; orbifold with

$$m_{55} = m = m_{66} \ , \qquad m_{IJ} = 0$$

$$\lambda \to e^{\frac{2\pi}{5}\Gamma_{56}} g^\dagger \lambda g \ , \qquad \phi^a \to R^{ab}\left(\frac{4\pi}{5}\right) g^\dagger \phi^b g, \qquad g = \mathrm{diag}\left(I_N, e^{i\frac{2\pi}{5}} I_N, e^{i\frac{4\pi}{5}} I_N\right)$$

Assign representations as

$$\begin{pmatrix} A_\mu, \phi^i & \lambda^r & \phi^+ \\ \tilde{\lambda}^{r'} & A_\mu, \phi^i & \lambda^r \\ \phi^- & \tilde{\lambda}^{r'} & A_\mu, \phi^i \end{pmatrix} \longrightarrow \begin{pmatrix} G & R_1 & R_2 \\ \bar{R}_1 & G & R_1 \\ \bar{R}_2 & \bar{R}_1 & G \end{pmatrix}$$

The other side: $\mathcal{N}$=4 on Coulomb branch

$$\langle \phi^4 \rangle = \mathrm{diag}\big(u_1 I_N, u_2 I_N, u_3 I_N\big) \ , \qquad u_1 + u_2 + u_3 = 0$$

Masses of irreps combined into a single rep should be equal $\begin{pmatrix} G & R_1 & R_2 \\ \bar{R}_1 & G & R_1 \\ \bar{R}_2 & \bar{R}_1 & G \end{pmatrix}$

$$u_1 - u_2 = u_2 - u_3 \implies u_2 = 0$$

Matching spectra of gauge theories: $M^2 = -u_1^2 \qquad m^2 = 4u_1^2$

Supergravity spectrum:

Rep.	R	L	Sugra fields	mass^2
G	$A_\mu \oplus \phi^i$	$\mathcal{V}^0_{\mathcal{N}=4}$	$\mathcal{H}_{\mathcal{N}=4} \oplus 4\mathcal{V}^0_{\mathcal{N}=4}$	0
R_1	λ^r	$\mathcal{V}^m_{\mathcal{N}=4}$	$2\Psi^m_{\mathcal{N}=4}$	u_1^2
$\bar{R}_1$	$\tilde{\lambda}^{r'}$	$\mathcal{V}^m_{\mathcal{N}=4}$	$2\Psi^m_{\mathcal{N}=4}$	u_1^2
R_2	ϕ^+	$\mathcal{V}^m_{\mathcal{N}=4}$	$\mathcal{V}^m_{\mathcal{N}=4}$	$4u_1^2$
$\bar{R}_2$	ϕ^-	$\mathcal{V}^m_{\mathcal{N}=4}$	$\mathcal{V}^m_{\mathcal{N}=4}$	$4u_1^2$

Symmeries: - unbroken SU(2) x U(1)
- sufficient vectors for CSO*(4) spontaneously broken to SU(2) x U(1)

- Spectrum and interactions $\longrightarrow$ CSO*(4) theory of Dall'Agata et al

◆ Construction follows the pattern: explicit breaking $\longrightarrow$ spontaneous breaking

Many exciting developments
and
we should be looking forward to more
in the future