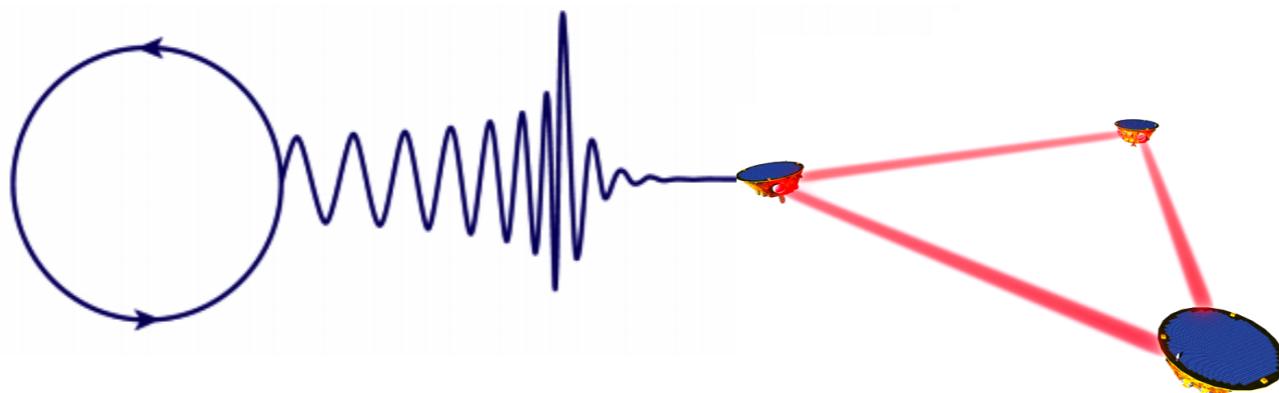


# The Gravitational Collider: The future of Precision GW physics



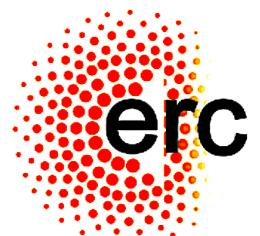
Rafael A. Porto

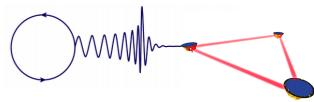
QCD meets Gravity  
Nordita 2018



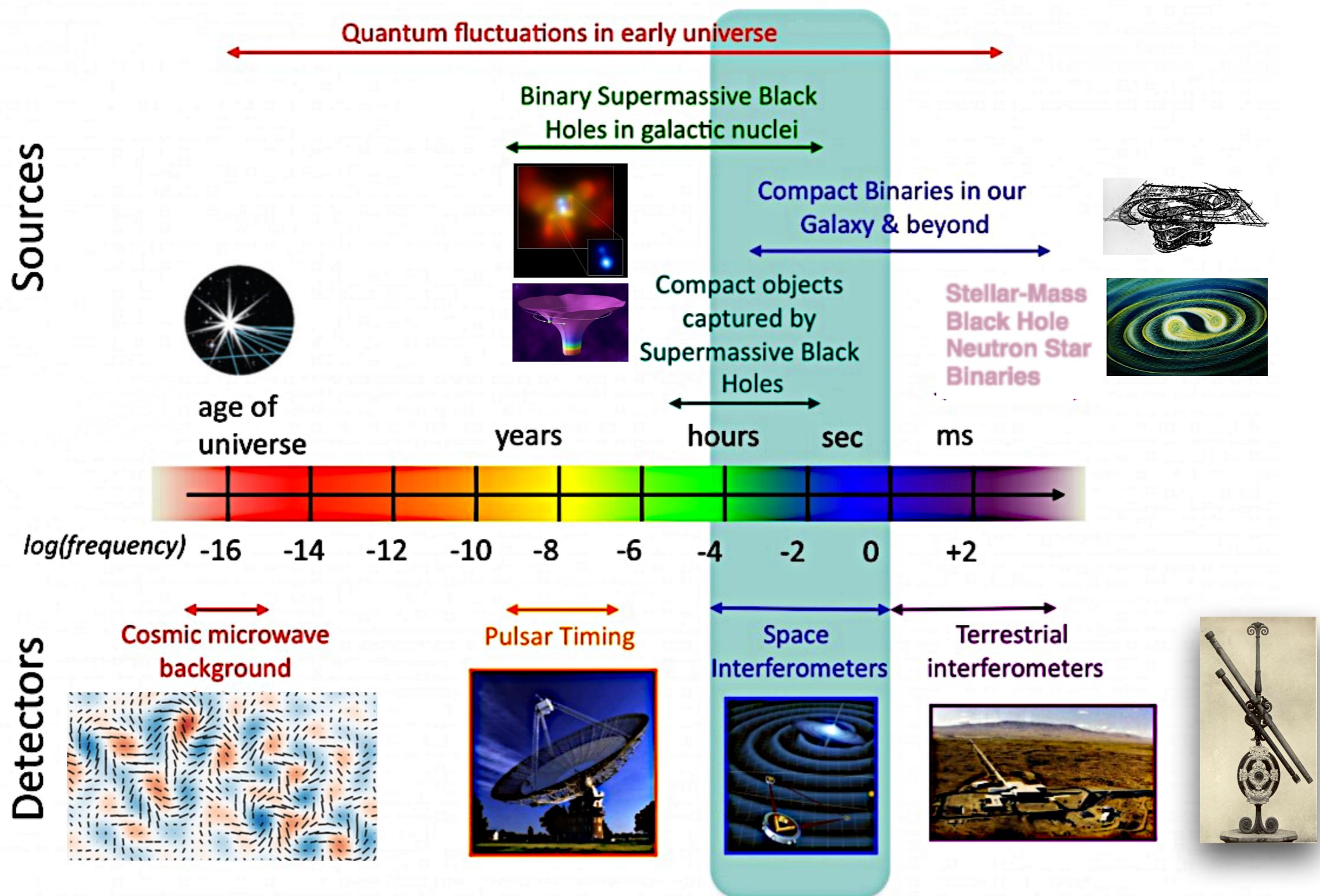
SIMONS FOUNDATION  
Advancing Research in Basic Science and Mathematics

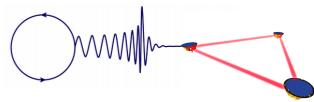
DFG Deutsche  
Forschungsgemeinschaft



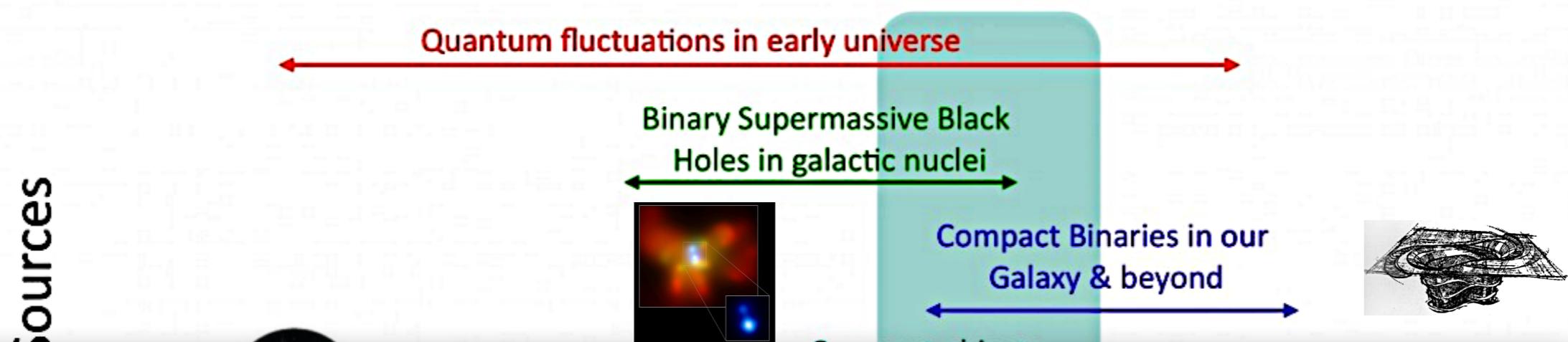


# The Gravitational Wave Spectrum

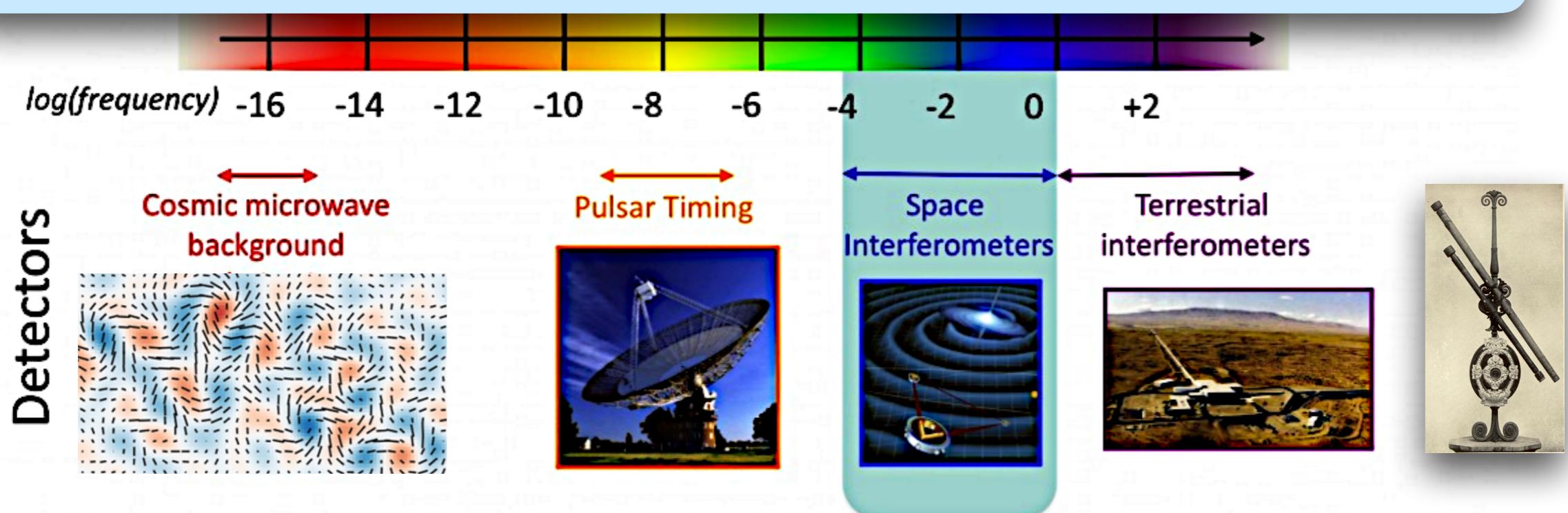


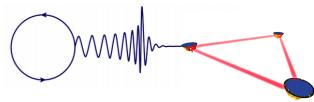


# The Gravitational Wave Spectrum



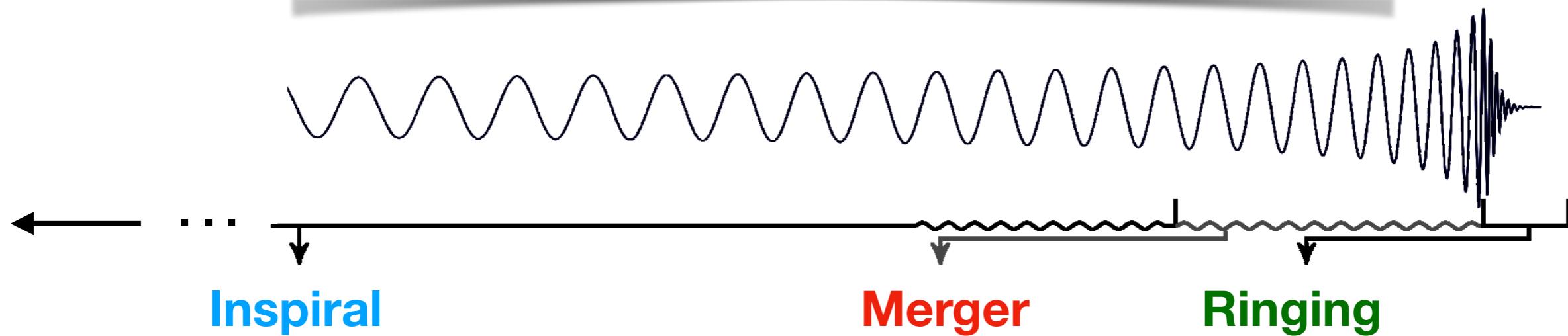
The discovery potential in GW Science relies on  
**Precise Theoretical Predictions**



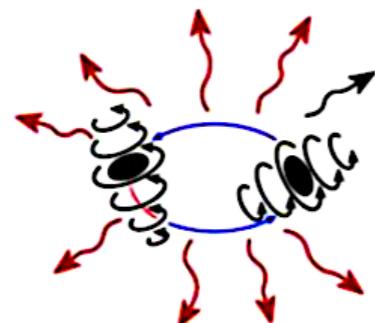


# Challenge: Two-body problem in General Relativity

$$R_{im} = \sum_i \frac{\partial \Gamma_{im}^i}{\partial x_i} + \sum_{j \neq i} \Gamma_{ij}^i \Gamma_{ji}^j = -x \left( T_{im} - \frac{1}{2} g_{im} T \right)$$

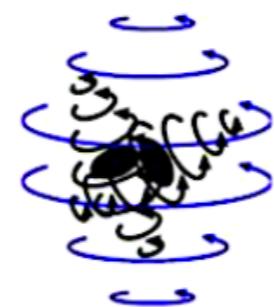


Inspiral



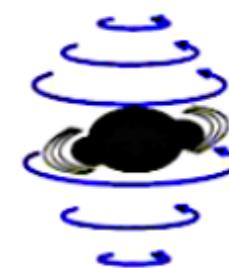
Analytic/Perturbative  
(Approx. but fast)

Merger



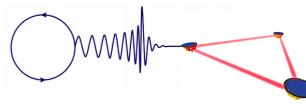
Numerical  
(exact but slow)

Ringing



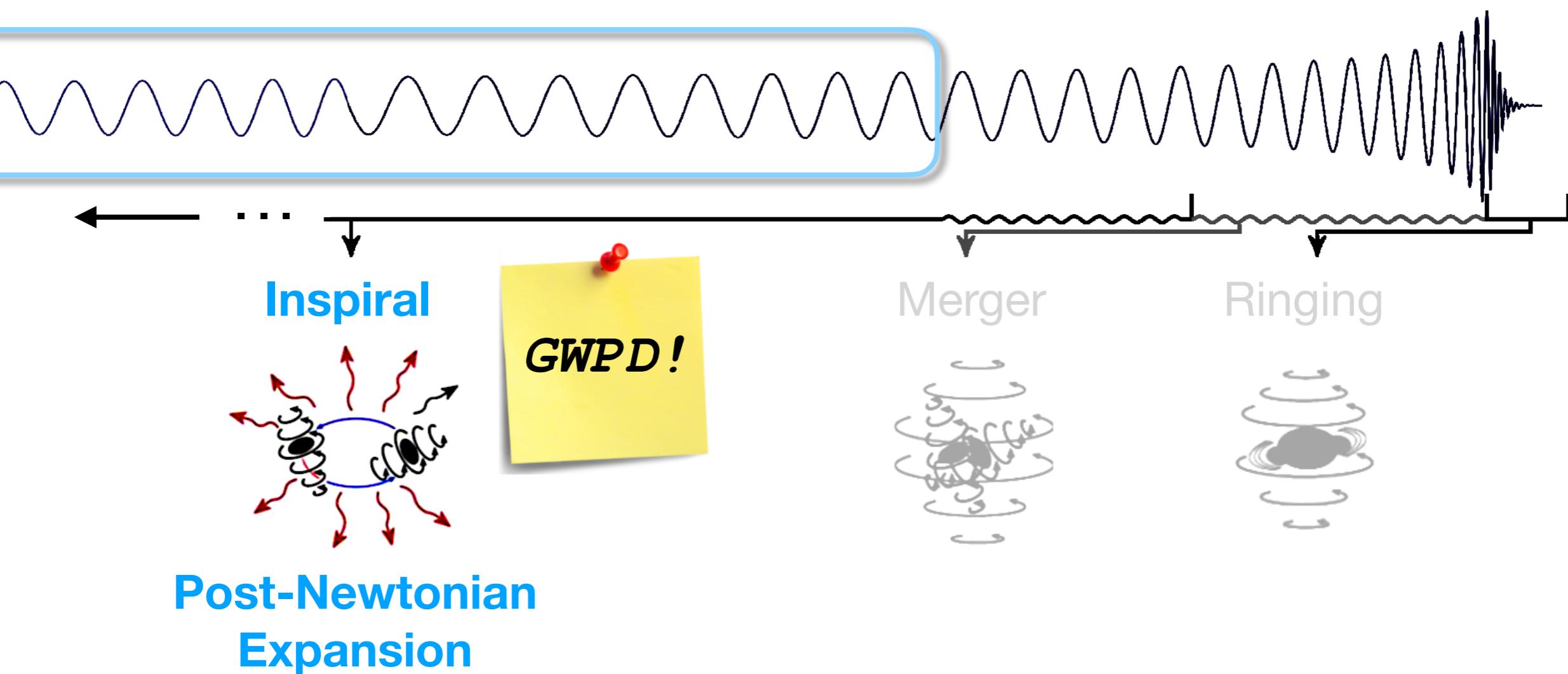
Analytic/  
Perturbative

Waveforms need to **match data over all cycles**

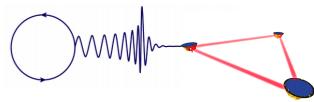


# Main Goal: Extremely accurate Post-Newtonian waveforms

1000+ cycles in band @ Design-Sensitivity  
100+ events per year!

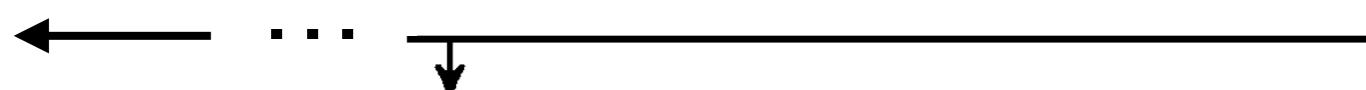


‘New Physics’ searches through GW Precision Data (GWPD)™

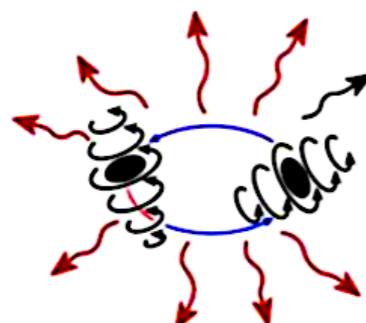


# State-of-the-art

**1000+ cycles in band @ Design-Sensitivity**  
**100+ events per year!**



**Inspiral**



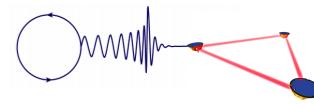
**3.5PN order**

$$\frac{\dot{\omega}}{\omega^2} = \underbrace{\frac{96}{5}\nu x^{5/2}}_{\text{Quadrupole Formula}} \left\{ 1 + \overbrace{\cdots + [\cdots]}^{\text{3.5PN order}} x^{7/2} \right\}$$

Quadrupole Formula

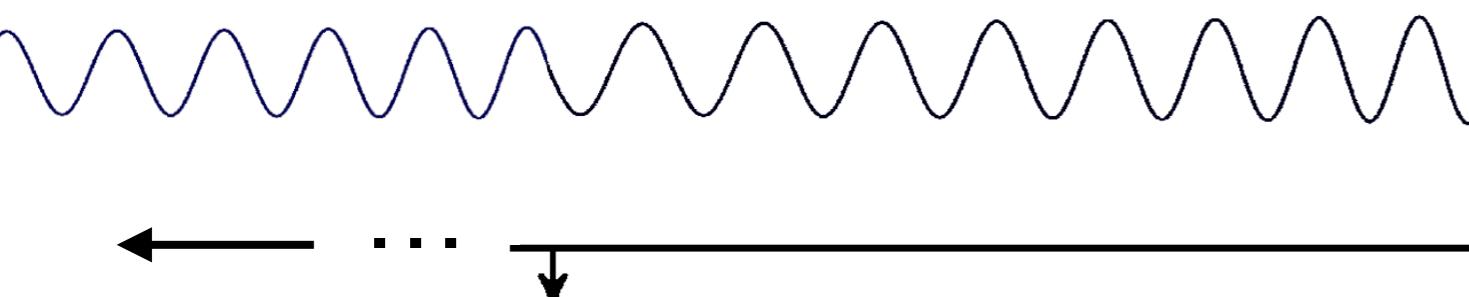
$$\begin{aligned} \nu &\sim m_2/m_1 \\ x &\sim (v/c)^2 \end{aligned}$$

$$4\pi R^2 \bar{G} = \frac{x}{40\pi} \left[ \sum_{\mu\nu} \bar{j}_{\mu\nu}^2 - \frac{1}{3} \left( \sum_{\mu} \bar{j}_{\mu\mu} \right)^2 \right]$$

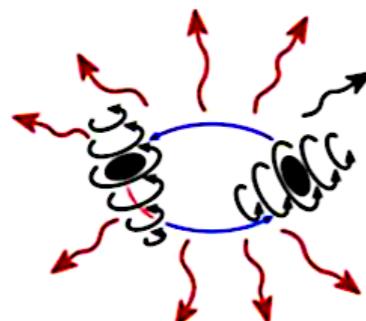


# Are we ready for the future?

Theoretical uncertainties may dominate over planned empirical reach



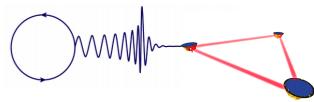
Inspiral



Not GOOD  
ENOUGH

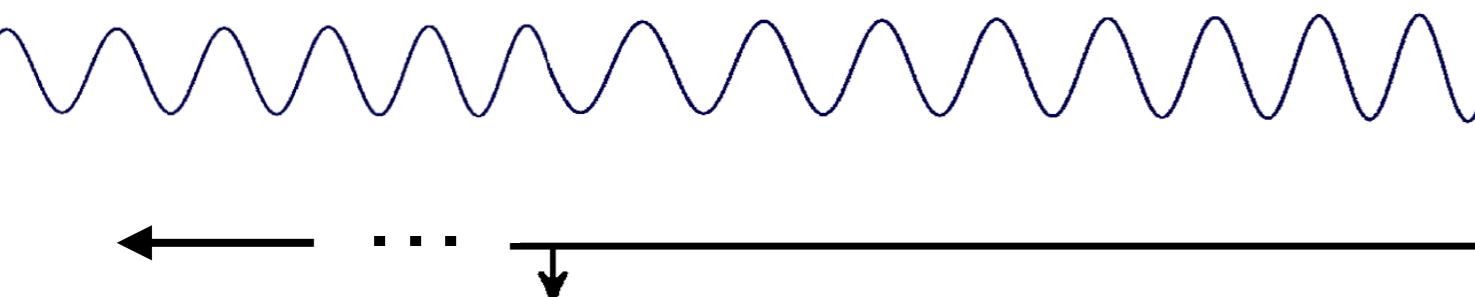
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \overbrace{\cdots + [\cdots]}^{x^{7/2}} x^{7/2} \right\}$$

SNR: LIGO/VIRGO ~ 30 but ET & LISA ~ 100-1000!



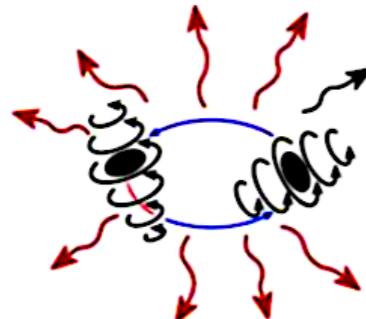
# Are we ready for the future?

We haven't reached the precision  
to distinguish (from PN) between compact bodies!



New Physics  
Threshold

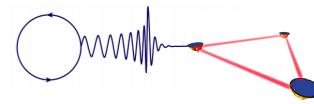
Inspiral



$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \dots + [\dots] x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5) \right\}$$

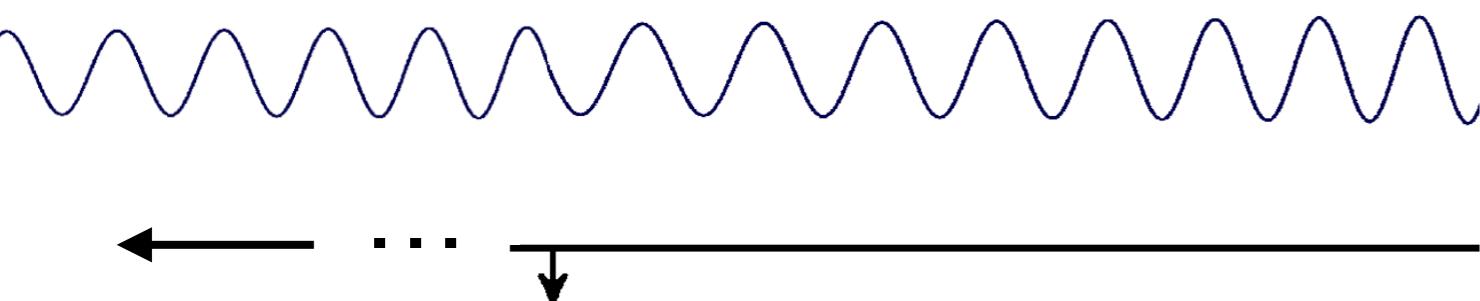
*N<sup>5</sup>LO*  
*5PN*

Inner structure  
of compact objects

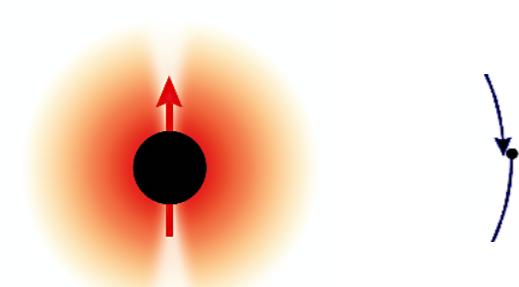


# Finite-size Threshold

**Impact:** ‘New Physics’  
searches with **GWPD**



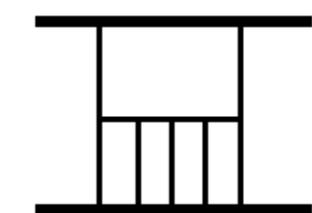
- **Strong Interaction**  
(Neutron star’s state)
- **Spacetime**  
(Black holes in GR)
- **Dark Matter**  
(Axions, Exotics)

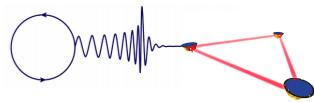


$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \dots + [\dots] x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5) \right\}$$

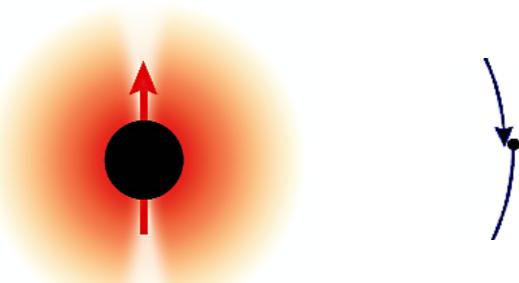
*N<sup>5</sup>LO  
5PN*

sample diagrams  
to ‘5 loops’

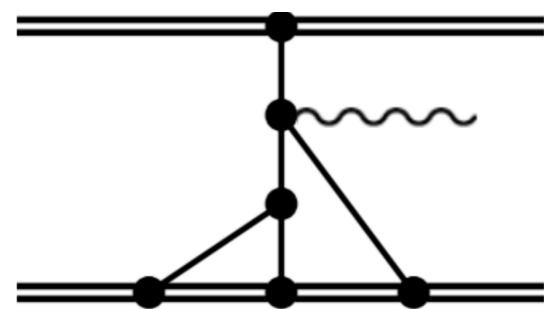




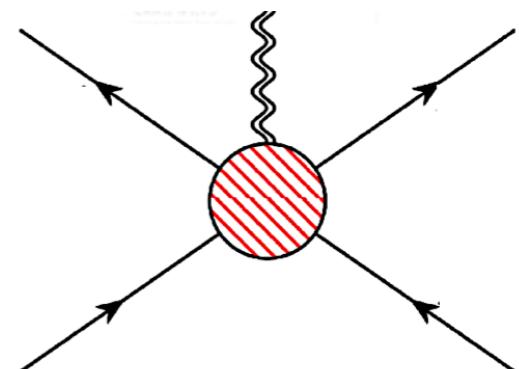
- **Opportunities:** ‘Future of GW Science’

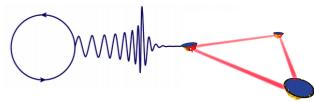


- **Now:** Feynman

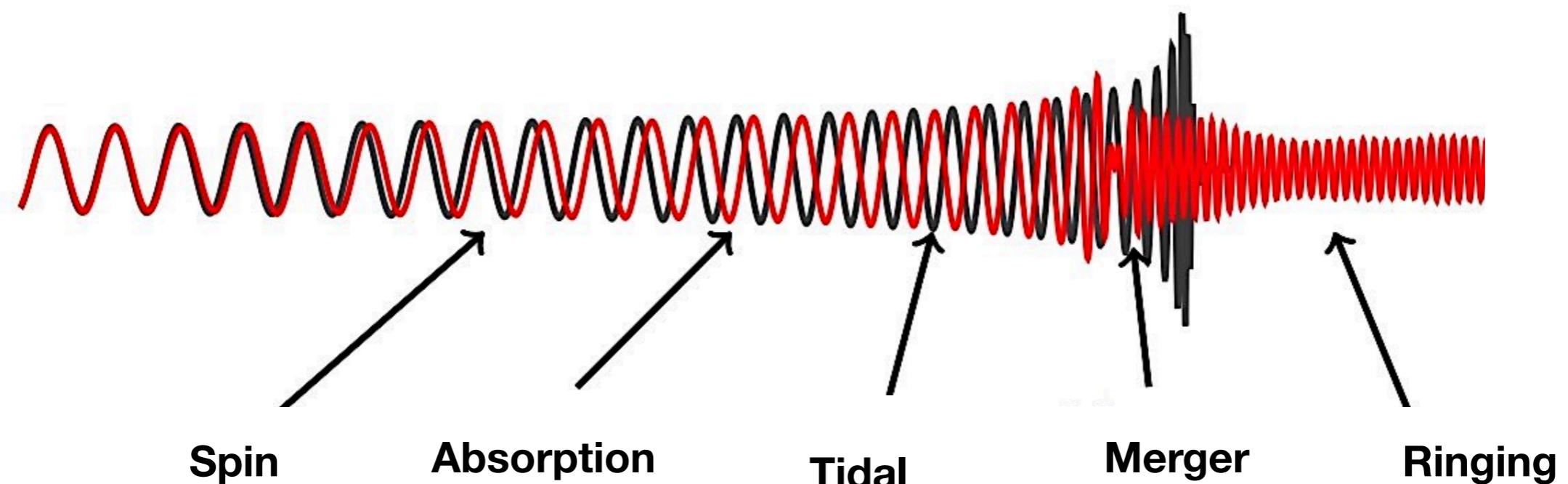


- **Challenges:** New tools?

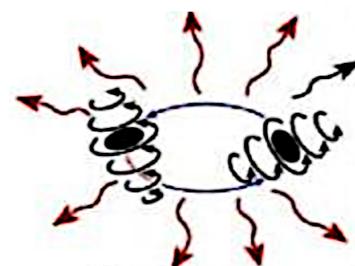
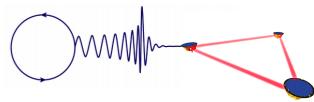




# Opportunities

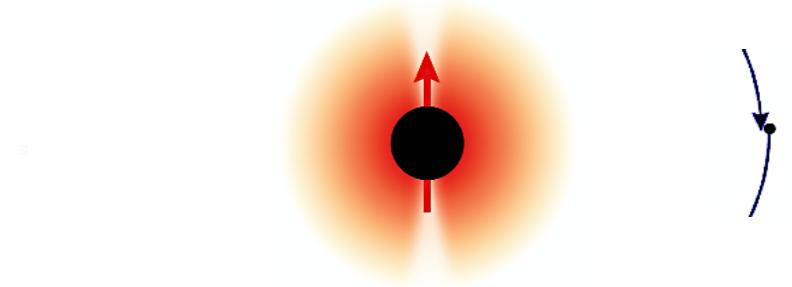


- **Strong Interaction** (Neutron stars' EOS)
- **Spacetime** (Black holes in General Relativity)
- **Dark Matter** (Axions, Exotic Compact Objects)
- **Unknown Unknowns!**



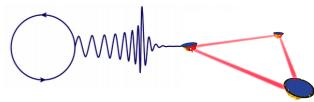
Inspiral

Tidal  
(5PN)



$$\Psi(v) = \Psi_{\text{PP}}(v) + \Psi_{\text{tidal}}(v)$$

Clean **analytic** control for the majority of cycles ( $10^4+!$ )  
during the **inspiral** phase (many astrophysical sources)



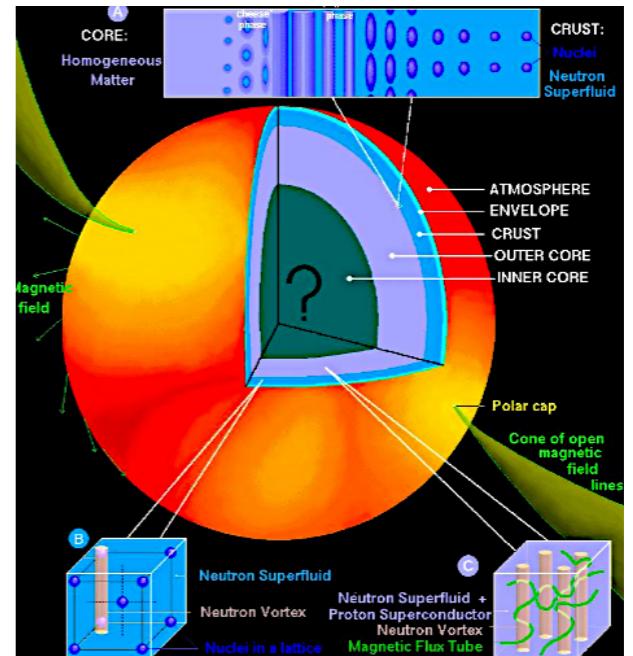
## GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

Static Limit:

$$\int dt Q^{ij} E_{ij}$$

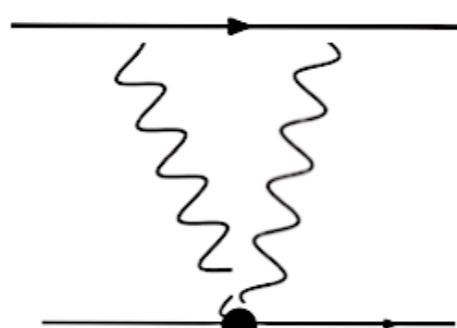
$$Q_{ij} = C_E E_{ij}$$

**Tidal  
(5PN)**



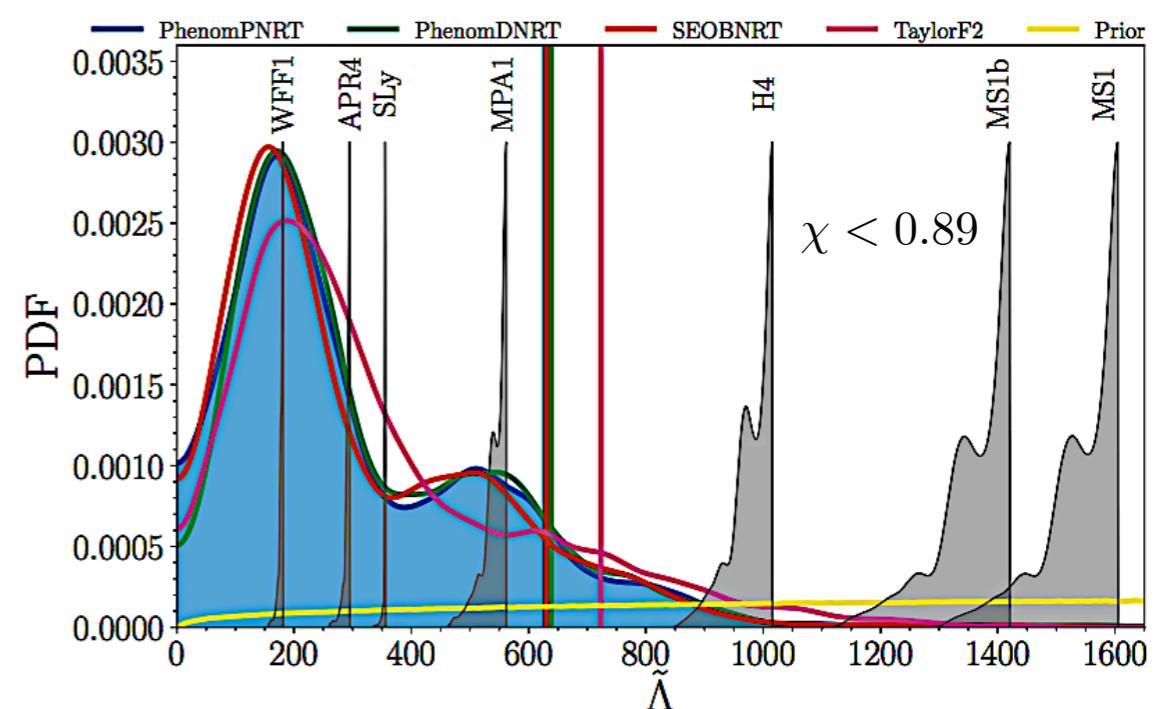
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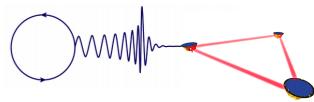
$$\left(\frac{R}{r}\right)^5 \sim v^{10}$$



$$C_E \sim R^5$$

$$\tilde{\Lambda} \sim C_E/M^5$$





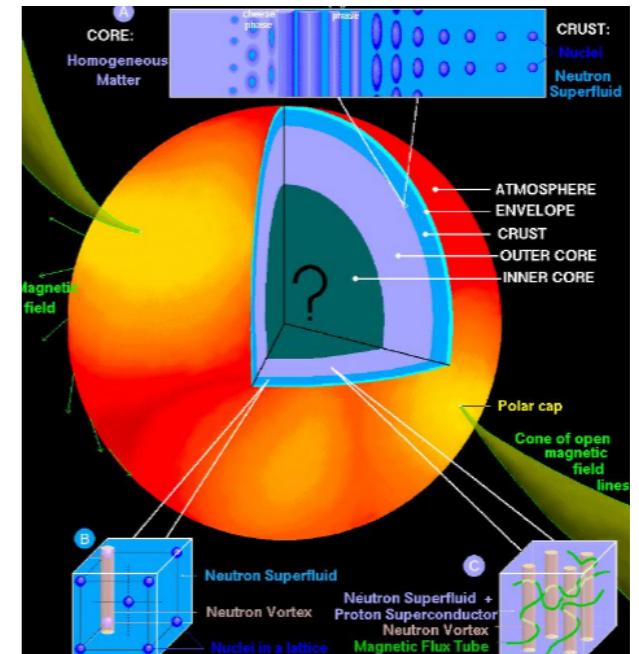
## GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

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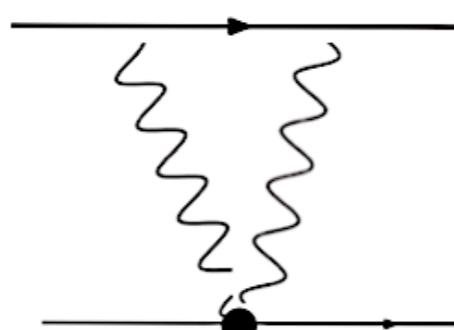
$$Q_{ij} = C_E E_{ij}$$

**Tidal  
(5PN)**



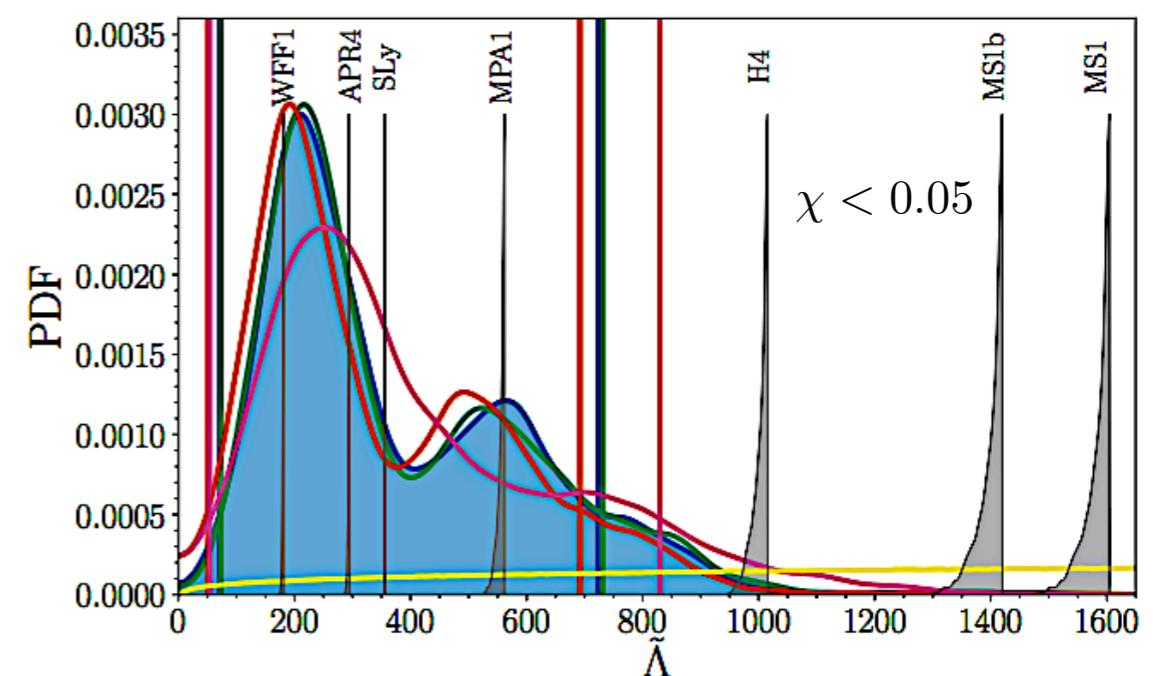
$$\Psi(v) = \Psi_{\text{PP}}(v) + \Psi_{\text{tidal}}(v)$$

$$\left(\frac{R}{r}\right)^5 \sim v^{10}$$



$$C_E \sim R^5$$

$$\tilde{\Lambda} \sim C_E/M^5$$

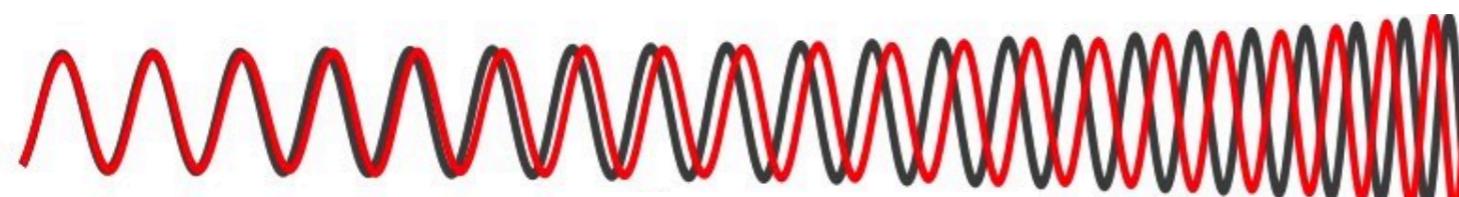




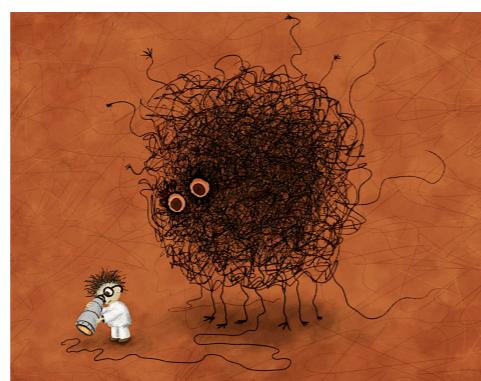
Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

## The tune of love and the nature(ness) of spacetime

Rafael A. Porto\*



???



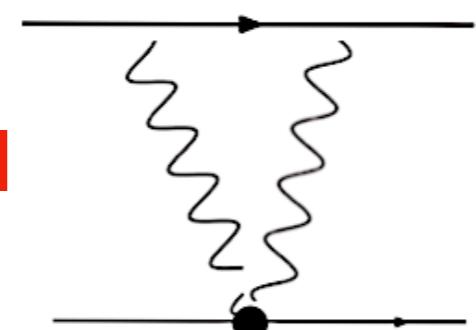
Tidal  
(5PN)



$$\Psi(v) = \Psi_{\text{PP}}(v) +$$



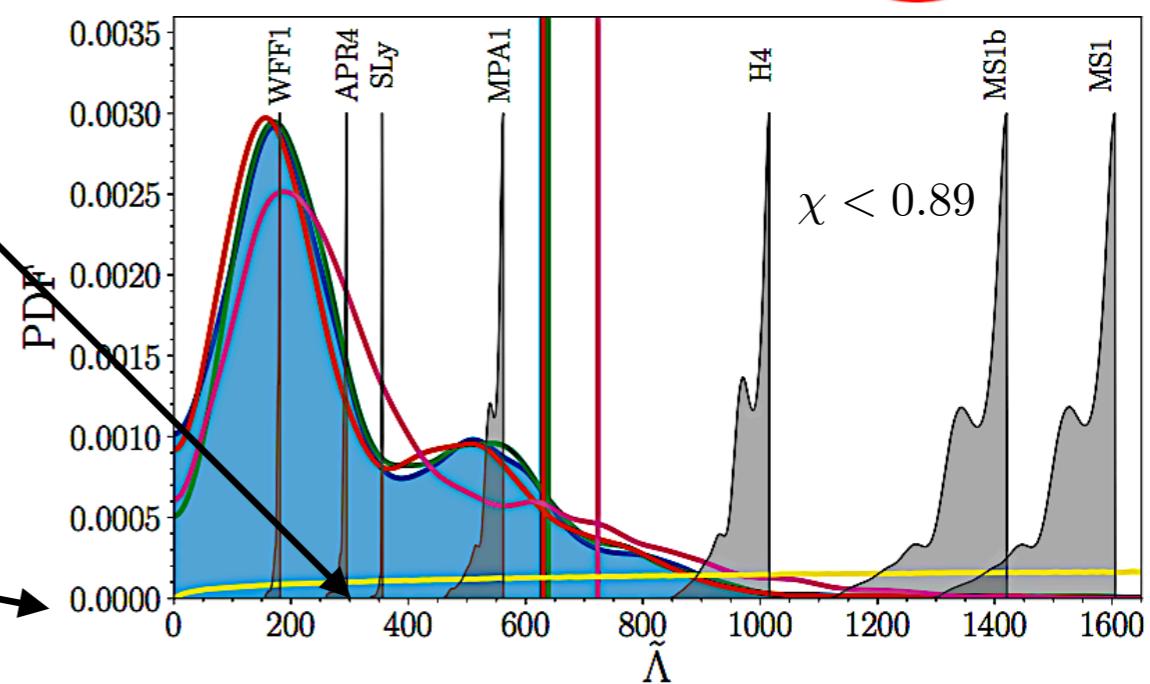
**NO  
'Standard Model  
Background'!**

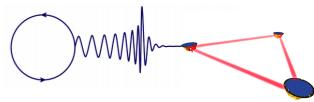


$$C_{E(B)}^{\text{bh}}(\mu) = 0$$

Unrelated to no-hair!  
(only zero in d=4)

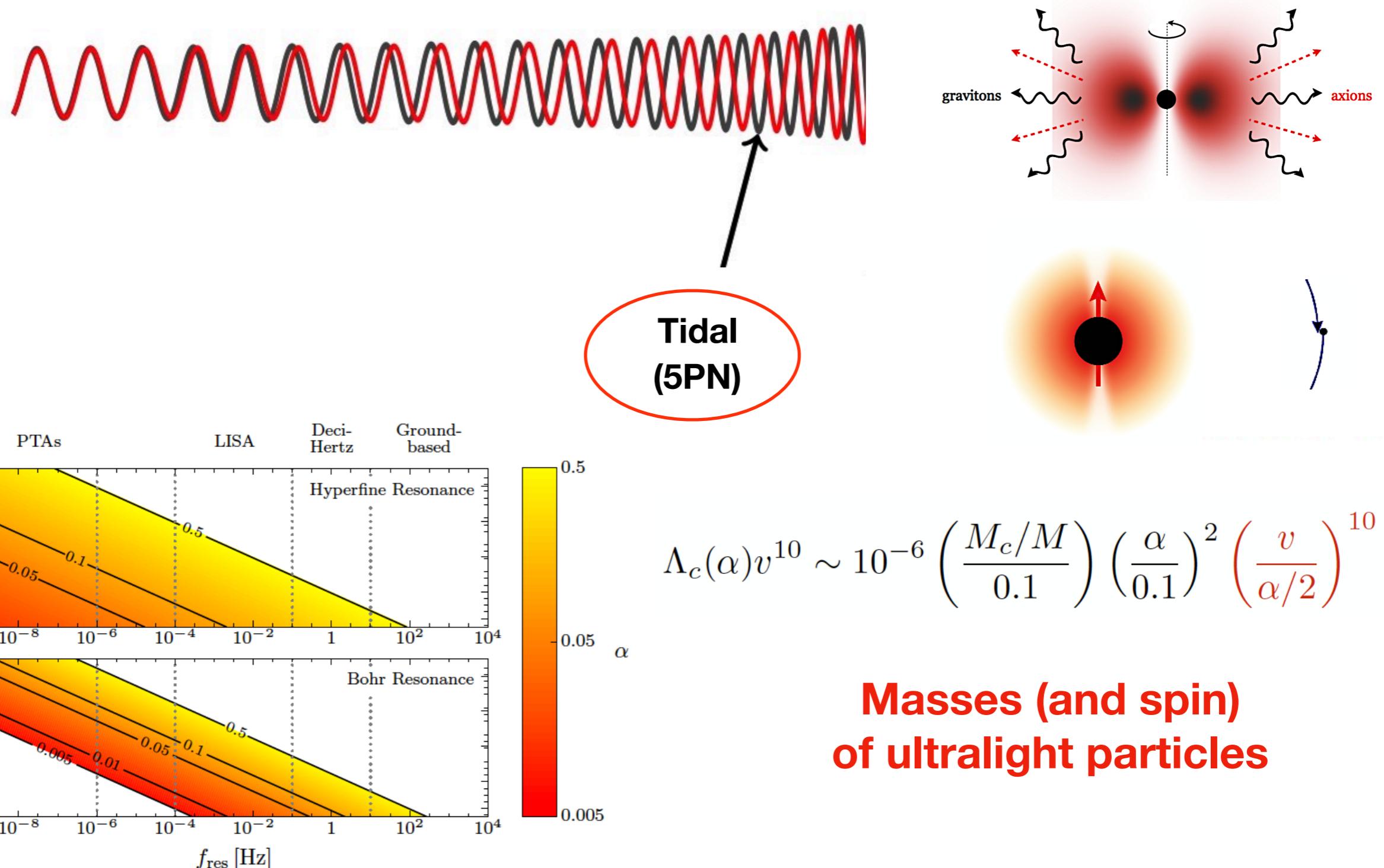
Damour Nagar  
Bennington Poisson  
Kol Smolkin

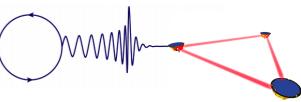




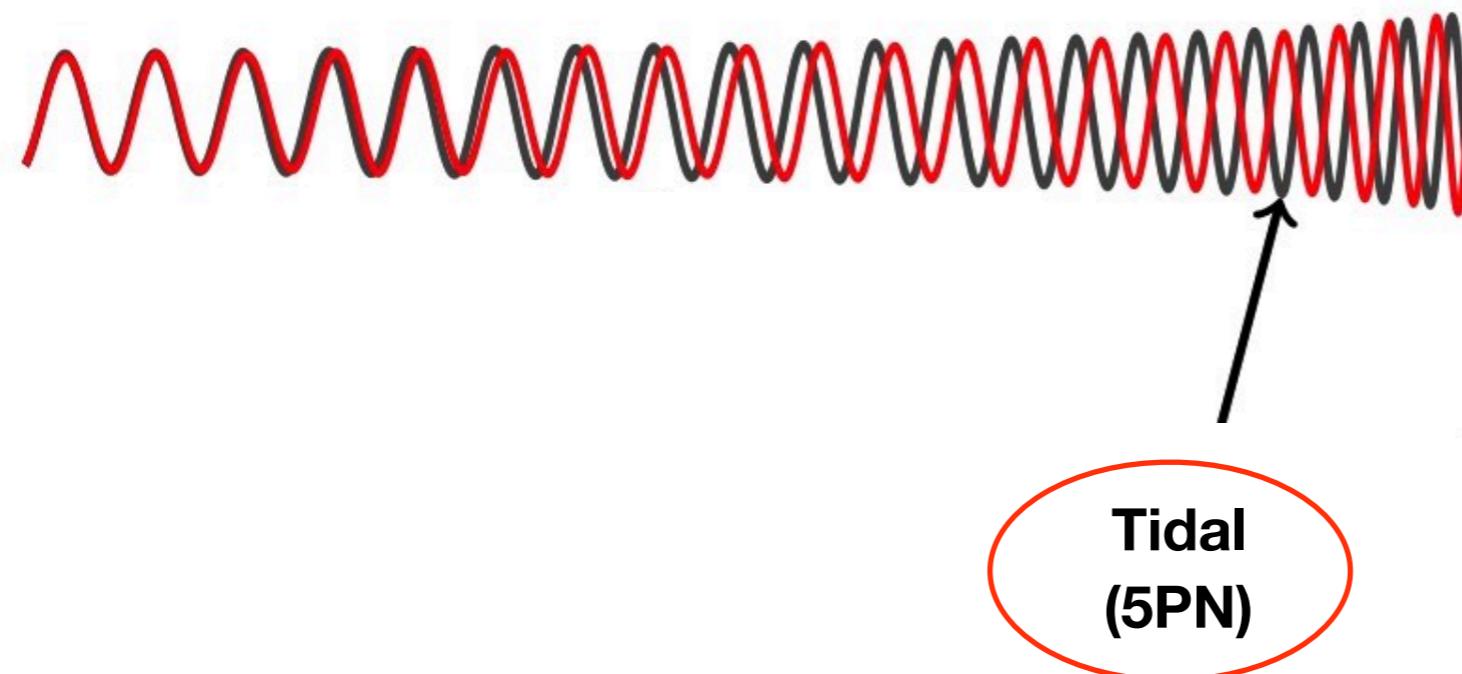
# Probing Ultralight Bosons with Binary Black Holes

Daniel Baumann,<sup>1</sup> Horng Sheng Chia,<sup>1</sup> and Rafael A. Porto<sup>2,3,4</sup>





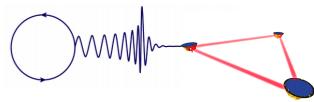
# GW Precision Data



- **Opportunities:**  
New physics threshold
- **More accurate templates!**  
(Feynman, New tools...)

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \dots + [\dots] x^{7/2} + \mathcal{O}(x^4) + \textcolor{red}{\mathcal{O}(x^5)} \right\}$$

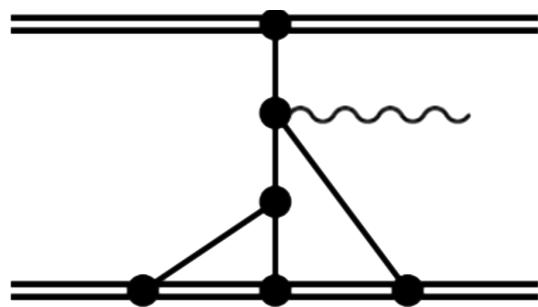
*N<sup>5</sup>LO*  
*5PN*



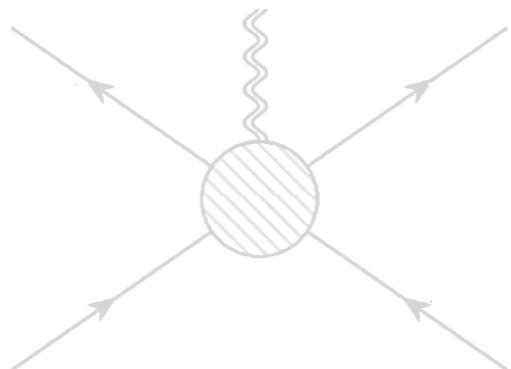
- **Opportunities:** ‘Future of GW Science’



- **Now:** Feynman



- **Challenges:** New tools?



# Now: Feynman (thus far)

\* General Relativity and Gravitation:  
A Centennial Perspective

Chapter 6: Sources of Gravitational Waves: Theory and  
Observations

*Alessandra Buonanno and B.S. Sathyaprakash*

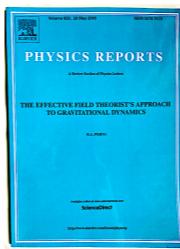
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \underbrace{\dots + [\dots]}_{\text{higher order terms}} x^{7/2} \right\}$$

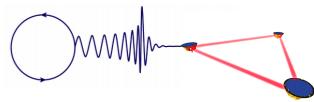
\* the EFT approach has extended the knowledge of the conservative dynamics and multipole moments to high PN orders [134–145].

- [134] Porto, R. A. 2006. *Phys. Rev. D*, **73**, 104031.
- [135] Porto, R. A., Rothstein, I. Z. 2006. *Phys. Rev. Lett.*, **97**, 021101.
- [136] Kol, B., Smolkin, M. 2008. *Class. Quant. Grav.*, **25**, 145011.
- [137] Porto, R. A., Rothstein, I. Z. 2008. *Phys. Rev. D*, **78**, 044013.
- [138] Porto, R. A., Rothstein, I. Z. 2008. *Phys. Rev. D*, **78**, 044012.
- [139] Porto, R. A., Ross, A., Rothstein, I. Z. 2011. *JCAP*, **1103**, 009.
- [140] Porto, R. A. 2010. *Class. Quant. Grav.*, **27**, 205001.
- [141] Levi, M. 2010. *Phys. Rev. D*, **82**, 104004.
- [142] Levi, M. 2012. *Phys. Rev. D*, **85**, 064043.
- [143] Herkt, S., Steinhoff, J., Schaefer, G. 2012. *Annals Phys.*, **327**, 1494–1537.
- [144] Herkt, S., Steinhoff, J., Schaefer, G. 2014. *J.Phys.Conf.Ser.*, **484**, 012018.
- [145] Porto, R. A., Ross, A., Rothstein, I. Z. 2012. *JCAP*, **1209**, 028.

The effective field theorist's approach to gravitational dynamics  
Physics Reports

Rafael A. Porto Volume 633, 20 May 2016, Pages 1-104





# Brand new: Binding energy to 4PN

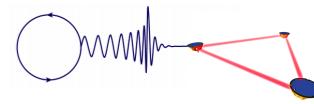
$$\begin{aligned}
 E^{4\text{PN}} = & -\frac{\mu c^2 x}{2} \left\{ 1 + \left( -\frac{3}{4} - \frac{\nu}{12} \right) x + \left( -\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \right. \\
 & + \left( -\frac{675}{64} + \left[ \frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
 & + \left( -\frac{3969}{128} + \left[ -\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x) \right] \nu \right. \\
 & \left. \left. + \left[ -\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right] \nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right) x^4 \right\}
 \end{aligned}$$

Damour Jaranowski Schaefer (2014, 2016)

Blanchet, Faye et al. (2015, 2017)

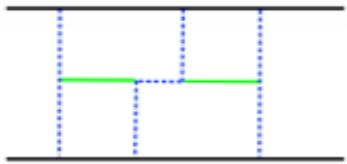
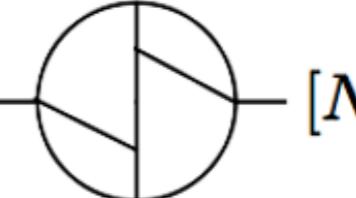
$$\nu \sim m_2/m_1$$

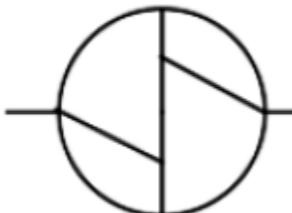
$$x \sim (v/c)^2$$



## Brand new: Binding energy to 4PN

$$\begin{aligned}
 E^{4\text{PN}} = -\frac{\mu c^2 x}{2} \left\{ 1 + \left( -\frac{3}{4} - \frac{\nu}{12} \right) x + \left( -\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \right. \\
 + \left( -\frac{675}{64} + \left[ \frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
 + \left( -\frac{3969}{128} + \left[ -\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x) \right] \nu \right. \\
 \left. \left. + \left[ -\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right] \nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right) x^4 \right\}
 \end{aligned}$$


 $= -2 i (8\pi G_N)^5 \left( \frac{(d-2)}{(d-1)} m_1 m_2 \right)^3$ 

 $[N_{49}]$


 $[N_{49}] \equiv \int_{k_1, k_2, k_3, k_4} \frac{N_{49}}{k_1^2 p_2^2 k_3^2 p_4^2 k_{12}^2 k_{13}^2 k_{23}^2 k_{24}^2 k_{34}^2},$

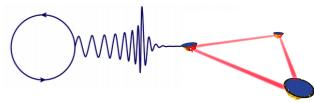
Foffa Sturani (2012)

Galley Porto Leibovich Ross (2015)

Foffa Sturani Mastrolia Sturm (2016)

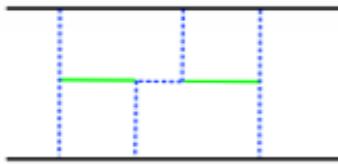
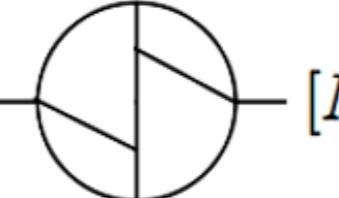
Foffa Sturani (to appear)

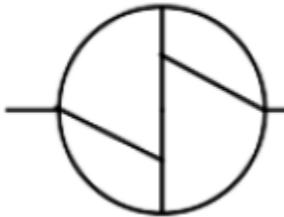
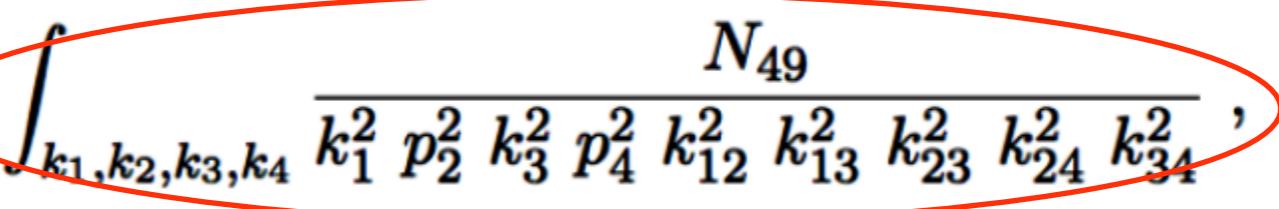
Foffa Porto Sturani Rothstein (to appear)



# Challenging computations!

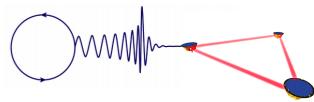
$$\begin{aligned}
 E^{4\text{PN}} = & -\frac{\mu c^2 x}{2} \left\{ 1 + \left( -\frac{3}{4} - \frac{\nu}{12} \right) x + \left( -\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \right. \\
 & + \left( -\frac{675}{64} + \left[ \frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
 & + \left( -\frac{3969}{128} + \left[ -\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x) \right] \nu \right. \\
 & \left. \left. + \left[ -\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right] \nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right) x^4 \right\}
 \end{aligned}$$


 $= -2 i (8\pi G_N)^5 \left( \frac{(d-2)}{(d-1)} m_1 m_2 \right)^3$ 

 $[N_{49}]$


 $[N_{49}] \equiv$ 


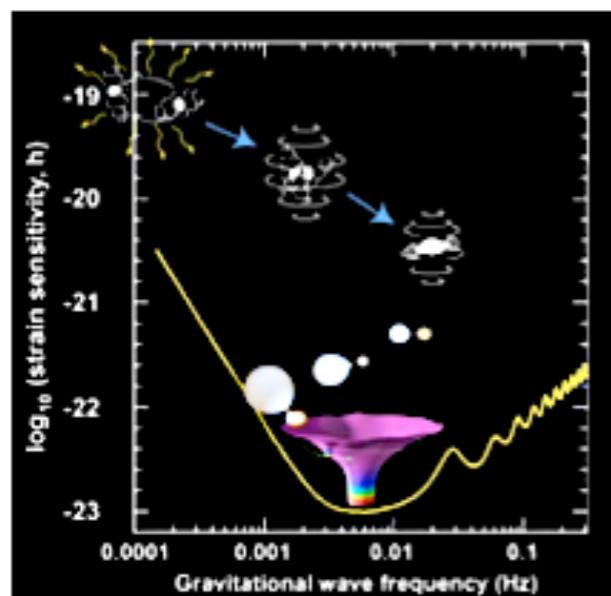
$$\int_{k_1, k_2, k_3, k_4} \frac{N_{49}}{k_1^2 p_2^2 k_3^2 p_4^2 k_{12}^2 k_{13}^2 k_{23}^2 k_{24}^2 k_{34}^2},$$

Complexity escalates quickly



# Different expansion parameters

$$\begin{aligned}
 E^{4\text{PN}} = & -\frac{\mu c^2 x}{2} \left\{ 1 + \left( -\frac{3}{4} - \frac{\nu}{12} \right) x + \left( -\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \right. \\
 & + \left( -\frac{675}{64} + \left[ \frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
 & + \left( -\frac{3969}{128} + \left[ -\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x) \right] \nu \right. \\
 & \left. \left. + \left[ -\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right] \nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right) x^4 \right\}
 \end{aligned}$$

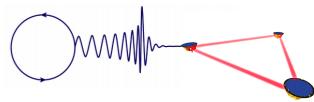


eLISA  
Super-massive BHs (EMRIs)

**Gravitational self-force in the ultra-relativistic limit:  
the “large- $N$ ” expansion**

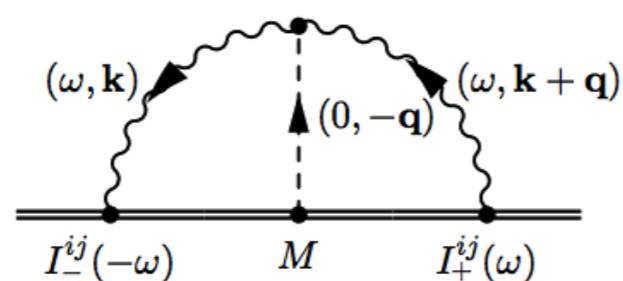
Chad R. Galley<sup>a</sup> and Rafael A. Porto<sup>b</sup>

$$\begin{array}{c}
 \text{---} \sim \frac{L}{N}, \quad \text{---} \sim \frac{\lambda L}{N} \\
 \text{---} \sim \frac{\lambda^2 L}{N}, \quad \text{---} \sim \frac{\lambda^2 L}{N^2} \quad N \equiv \gamma^2 \\
 \lambda = \epsilon N
 \end{array}$$



## There are logs!

$$\begin{aligned}
 E^{4\text{PN}} = & -\frac{\mu c^2 x}{2} \left\{ 1 + \left( -\frac{3}{4} - \frac{\nu}{12} \right) x + \left( -\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \right. \\
 & + \left( -\frac{675}{64} + \left[ \frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
 & + \left( -\frac{3969}{128} + \left[ -\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15} \ln(16x) \right] \nu \right. \\
 & \left. \left. + \left[ -\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right] \nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right) x^4 \right\}
 \end{aligned}$$

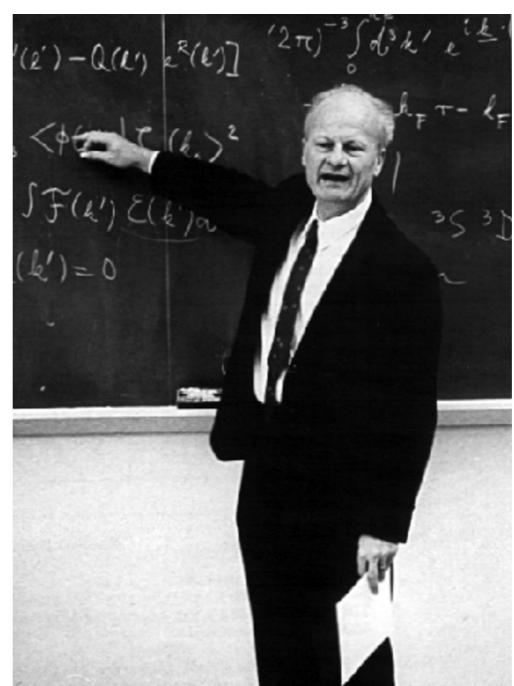


PHYSICAL REVIEW D 93, 124010 (2016)

**Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution**

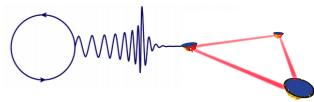
Chad R. Galley,<sup>1</sup> Adam K. Leibovich,<sup>2</sup> Rafael A. Porto,<sup>3</sup> and Andreas Ross<sup>4</sup>

$$\overbrace{\quad}^{1\text{-loop}} = \overbrace{\quad} + 2 \overbrace{\quad} + \overbrace{\quad}$$



PHYSICAL REVIEW D 96, 024063 (2017)

**Lamb shift and the gravitational binding energy for binary black holes**

PHYSICAL REVIEW D **89**, 064058 (2014)**Nonlocal-in-time action for the fourth post-Newtonian  
conservative dynamics of two-body systems**

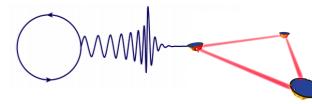
T. Damour, P. Jaranowski, and G. Schäfer,

$$H_{\text{4PN}}^{\text{near-zone (s)}}[\mathbf{x}_a, \mathbf{p}_a] = H_{\text{4PN}}^{\text{loc0}}[\mathbf{x}_a, \mathbf{p}_a] + F[\mathbf{x}_a, \mathbf{p}_a] \left( \ln \frac{r_{12}}{s} + C \right)$$

Ambiguity parameter  
associated to IR divergences  
(Similar to Lamb shift)

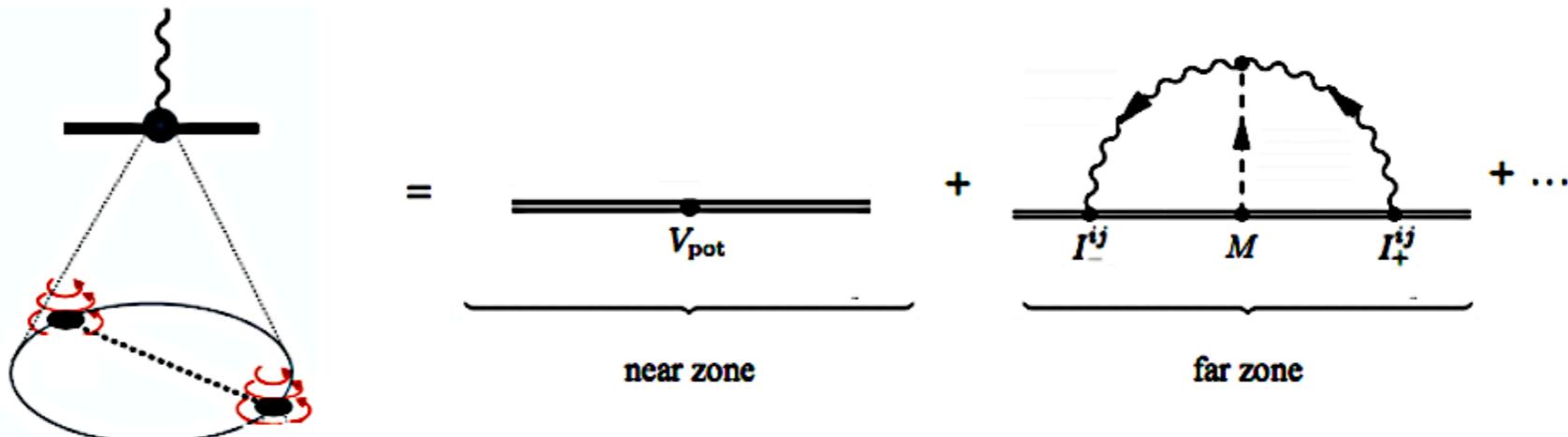
$$C = -\frac{1681}{1536}$$

Originally not determined from first principles within PN framework.



PHYSICAL REVIEW D 96, 024062 (2017)

## Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto<sup>1</sup> and Ira Z. Rothstein<sup>2</sup>

$$\frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left( -\frac{1}{\epsilon_{IR}} + 2 \log(\mu r) + \dots \right) + \left( \frac{1}{\epsilon_{UV}} + 2 \log(\Omega/\mu) + \dots \right)$$

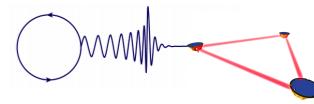
**“IR/UV” cancellation  
There are no  
ambiguities!**

PHYSICAL REVIEW D 93, 124010 (2016)

## Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

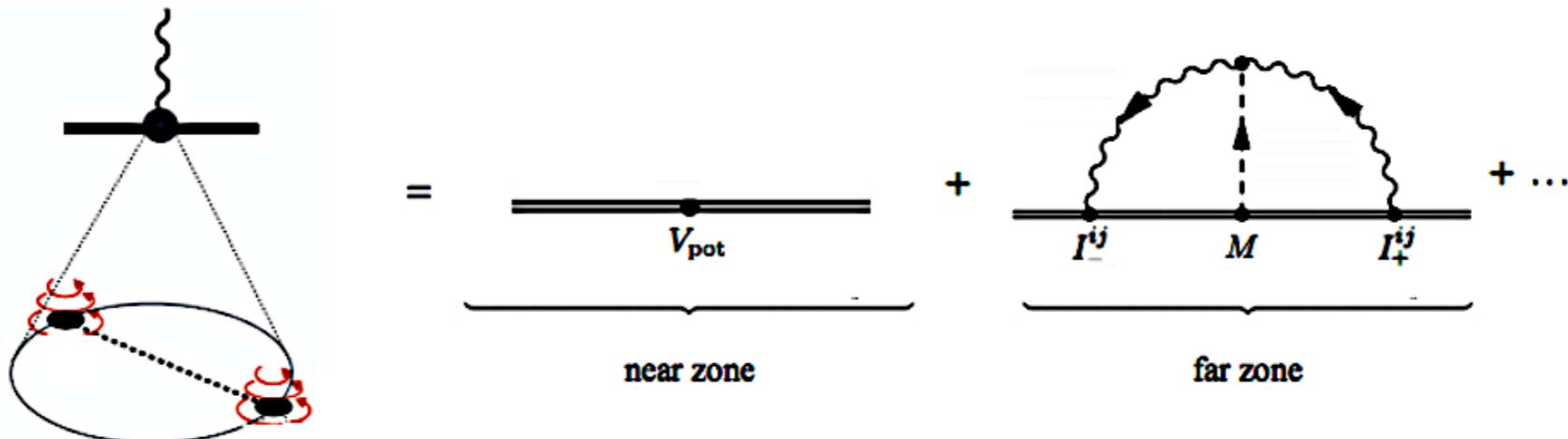
Chad R. Galley,<sup>1</sup> Adam K. Leibovich,<sup>2</sup> Rafael A. Porto,<sup>3</sup> and Andreas Ross<sup>4</sup>

The divergence is due to split into regions  
The ambiguity arises from independent  
regularizations of the near/far zone



PHYSICAL REVIEW D 96, 024062 (2017)

## Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto<sup>1</sup> and Ira Z. Rothstein<sup>2</sup>

PHYSICAL REVIEW D 96, 024063 (2017)

## Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

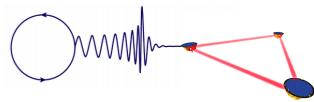
$$\delta E_{n,\ell} = (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cV} + \dots$$

$$= \frac{2\alpha_e}{3\pi} \left[ \frac{5}{6} e^2 \frac{|\psi_{n,\ell}(x=0)|^2}{2m_e^2} - \sum_{m \neq n, \ell} \left\langle n, \ell \left| \frac{\mathbf{p}}{m_e} \right| m, \ell \right\rangle^2 (E_m - E_n) \log \frac{2|E_n - E_m|}{m_e} \right] +$$

**correct  
value w/out  
ambiguities!**

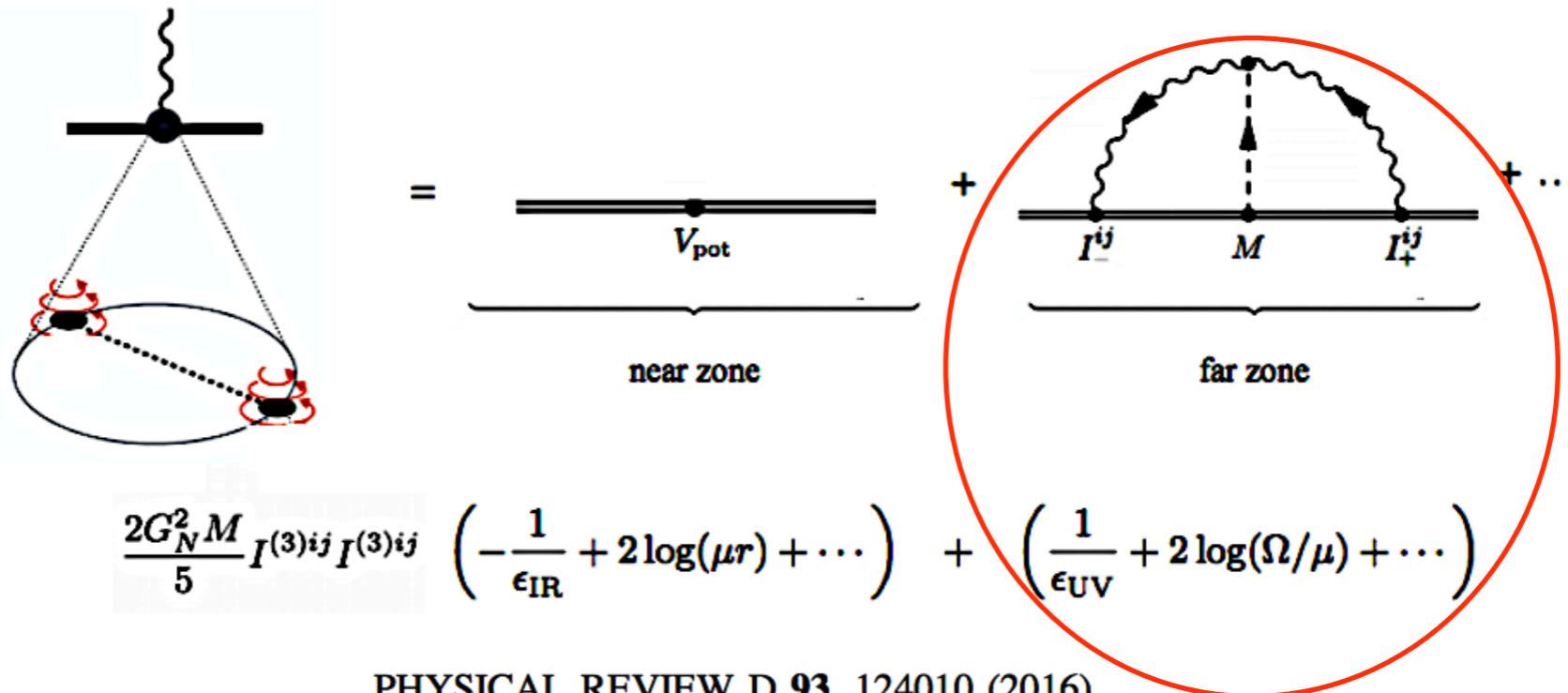
$$+ \frac{4\alpha_e^2}{3m_e^2} \left( \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) |\psi_{n,\ell}(x=0)|^2.$$

**IR/UV cancelation  
in dim. reg.  
(non-trivial in  
other schemes)**



PHYSICAL REVIEW D 96, 024062 (2017)

## Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto<sup>1</sup> and Ira Z. Rothstein<sup>2</sup>

PHYSICAL REVIEW D 93, 124010 (2016)

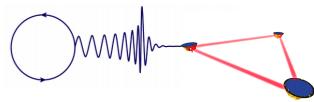
## Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,<sup>1</sup> Adam K. Leibovich,<sup>2</sup> Rafael A. Porto,<sup>3</sup> and Andreas Ross<sup>4</sup>

$$\mu \frac{d}{d\mu} V_{\text{ren}}(\mu) = \frac{2G_N^2 M}{5} I^{ij(3)}(t) I^{ij(3)}(t)$$

$$E_{\log} = -2G_N^2 M \langle I^{ij(3)}(t) I^{ij(3)}(t) \rangle \log v$$

**Logarithmic contribution to energy!**



PHYSICAL REVIEW D 97, 044023 (2018)

## Ambiguity-free completion of the equations of motion of compact binary systems at the fourth post-Newtonian order

Tanguy Marchand,<sup>1,2,\*</sup> Laura Bernard,<sup>3,†</sup> Luc Blanchet,<sup>1,‡</sup> and Guillaume Faye<sup>1,§</sup>

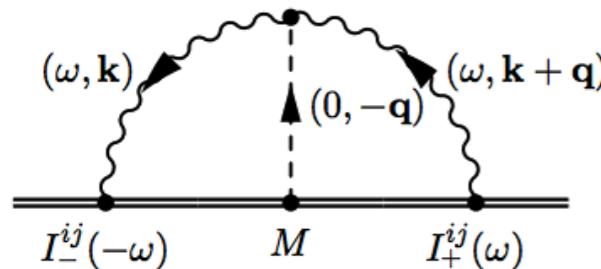
## V. DETERMINATION OF THE AMBIGUITY PARAMETERS

Remarkably, the value  $\kappa = \frac{41}{60}$  we have obtained in our result for the tail [see Eq. (4.13)], agrees with the result found by Galley *et al* [10] in their computation of the tail term in  $d$

PHYSICAL REVIEW D 93, 124010 (2016)

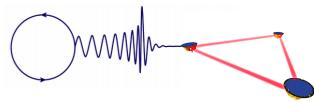
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$$W_{\text{tail}}[\mathbf{x}_a^\pm] = \frac{2G_N^2 M}{5} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^6 I_-^{ij}(-\omega) I_+^{ij}(\omega) \left[ -\frac{1}{(d-4)_{\text{UV}}} - \gamma_E + \log \pi - \log \frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi \text{sign}(\omega) \right].$$

The 41/30 \*only makes sense\* if IR/UV poles (and  $\mu$ ) are properly removed as in the Lamb shift (Otherwise you have scheme dependence = ambiguity)



PHYSICAL REVIEW D 97, 044023 (2018)

## Ambiguity-free completion of the equations of motion of compact binary systems at the fourth post-Newtonian order

Tanguy Marchand,<sup>1,2,\*</sup> Laura Bernard,<sup>3,†</sup> Luc Blanchet,<sup>1,‡</sup> and Guillaume Faye<sup>1,§</sup>

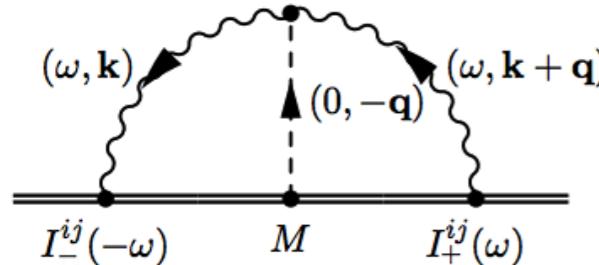
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PHYSICAL REVIEW D 93, 124010 (2016)

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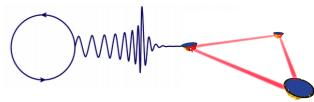


This includes  
everything

$$W_{\text{tail}}[\mathbf{x}_a^\pm] = \frac{2G_N^2 M}{5} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^6 I_-^{ij}(-\omega) I_+^{ij}(\omega) \left[ -\frac{1}{(d-4)_{\text{UV}}} - \gamma_E + \log \pi - \log \frac{\omega^2}{\mu^2} + \frac{41}{30} - i\pi \text{sign}(\omega) \right].$$

$$\frac{P_{\text{tail}}}{P_{LO}} = 4\pi x^{3/2}$$

Accounts for  
non-conservative part



PHYSICAL REVIEW D 97, 044023 (2018)

**Ambiguity-free completion of the equations of motion of compact binary systems at the fourth post-Newtonian order**

Tanguy Marchand,<sup>1,2,\*</sup> Laura Bernard,<sup>3,†</sup> Luc Blanchet,<sup>1,‡</sup> and Guillaume Faye<sup>1,§</sup>

The elimination of **IR divergences** in Marchand et al. isn't fully *kosher*. It works because physical logs appear at higher orders.

$$\xi_1 = \frac{11}{3} \frac{G^2 m_1^2}{c^6} \left[ \frac{1}{\varepsilon} - 2 \ln \left( \frac{\bar{q}^{1/2} r'_1}{\ell_0} \right) - \frac{327}{1540} \right] \mathbf{a}_{1,N}^{(d)} + \frac{1}{c^8} \xi_{1,4\text{PN}}$$

Some of those poles are infrared.

\***Must not**\* be absorbed into WL (short-distance)



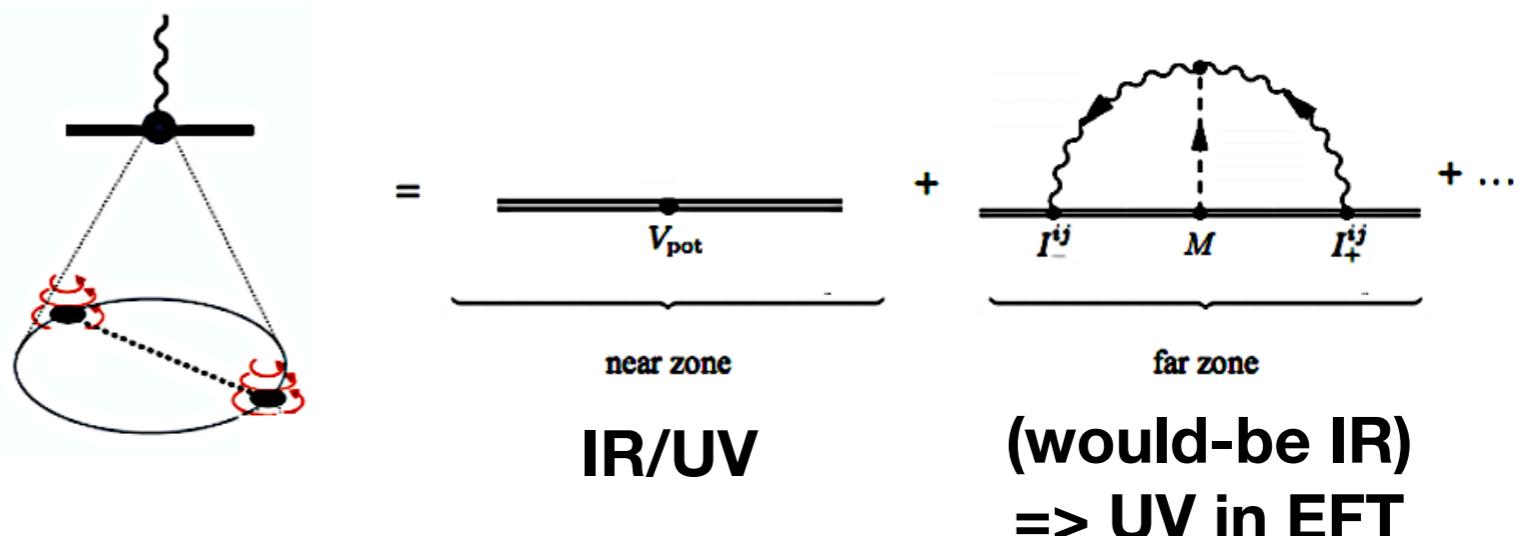
# **Unambiguous/Consistent derivation in EFT**

(UV renormalization and IR/UV identification in dim. reg.)

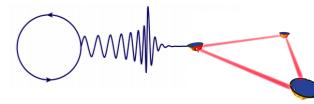
$$S_{\text{PP}}[x_a^\alpha(\tau_a)] = \sum_a \int d\tau_a \left( -m_a + \sum_i c_i \mathcal{O}_i[x_a^\alpha(\tau_a), \dot{x}_a^\alpha(\tau_a), \dots; g_{\mu\nu}, \partial_\beta g_{\mu\nu}, \dots] \right)$$

UV counter-terms (can be removed by field-redef. at 4PN)

The cancelation is  
by construction.  
Entirely due to regions.  
It is not there in  
PM expansion!



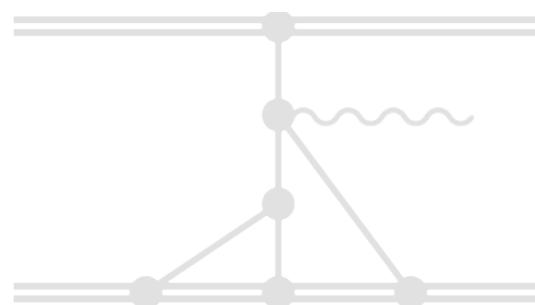
# Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Ambiguity-free renormalization and physical results



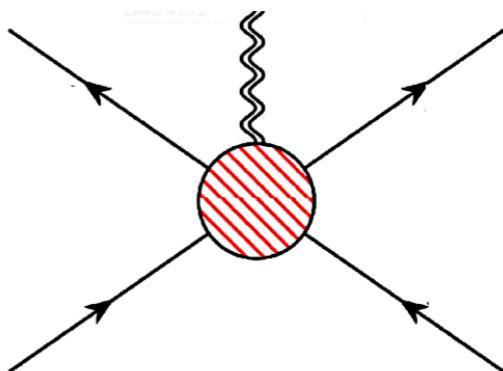
- **Opportunities:** ‘Future of GW Science’

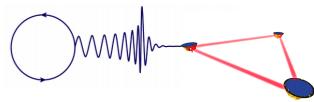


- **Now:** Feynman

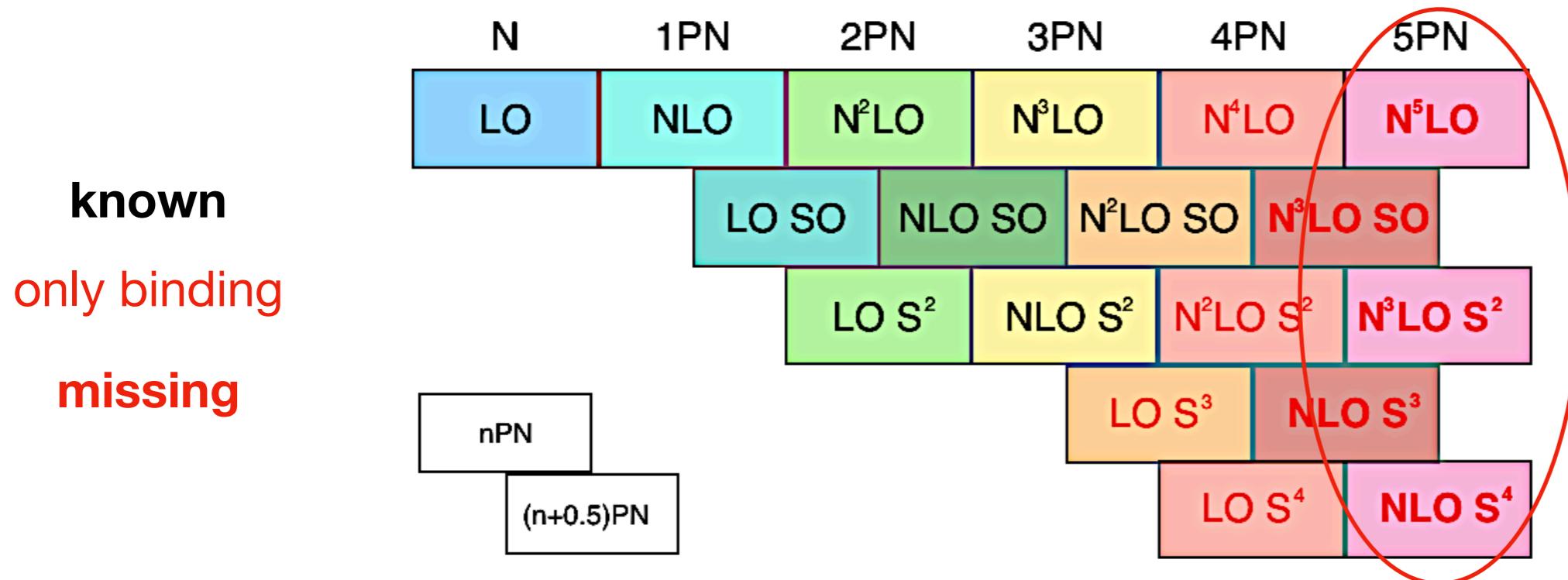


- **Challenges:** New tools?





## Challenges: New tools?



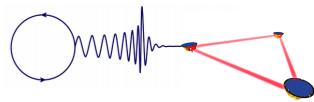
We know (EFT) how to compute the integrals to reach the 5PN threshold (See Pier-Paolo's talk)

(If we know  $G^n$  integrals we know  $(n+1)PN$ )

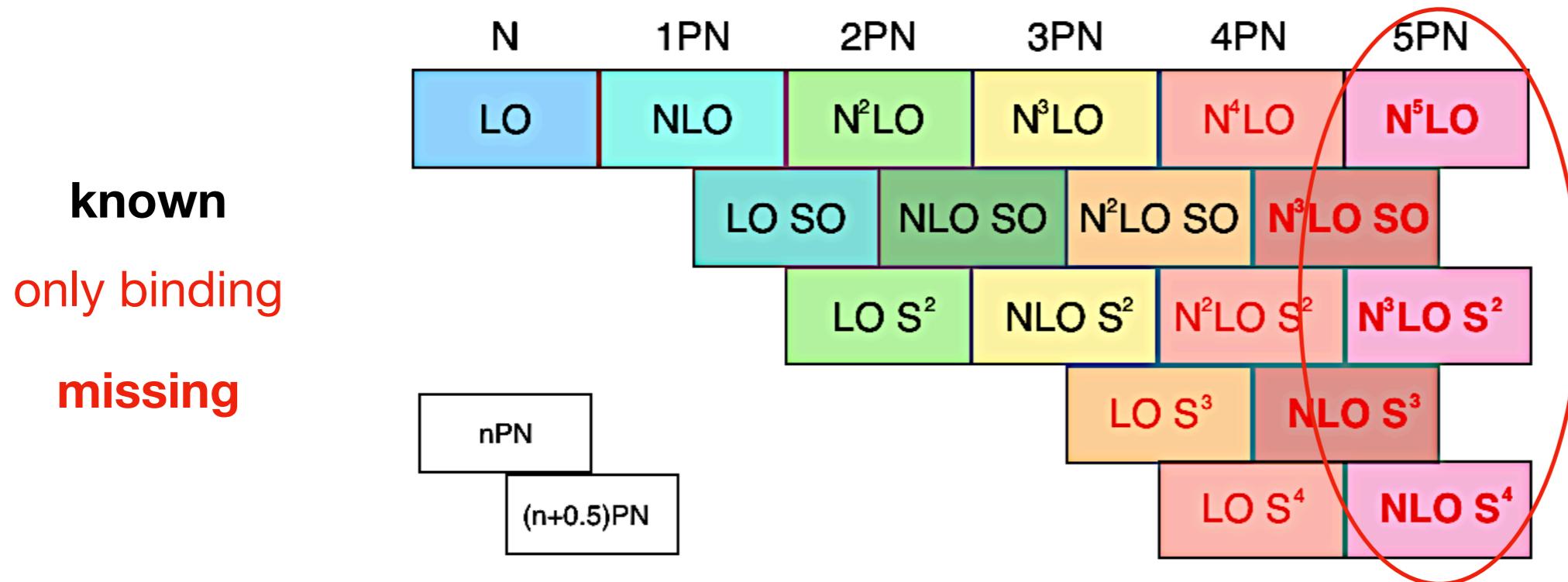
There is a  $Z_2$  bulk sym. for the static case ( $\phi \rightarrow -\phi$ )

$$S[\phi, \gamma_{ij}] = \frac{1}{16\pi G} \int \sqrt{-\gamma} d^d x \left[ - \left( 1 + \frac{1}{\hat{d}} \right) \gamma^{ij}(x) \partial_i \phi \partial_j \phi + R[\gamma] \right]$$





## Challenges: New tools?

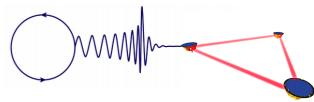


**We know** (EFT) how to compute the integrals to reach the 5PN threshold (See Pier-Paolo's talk)

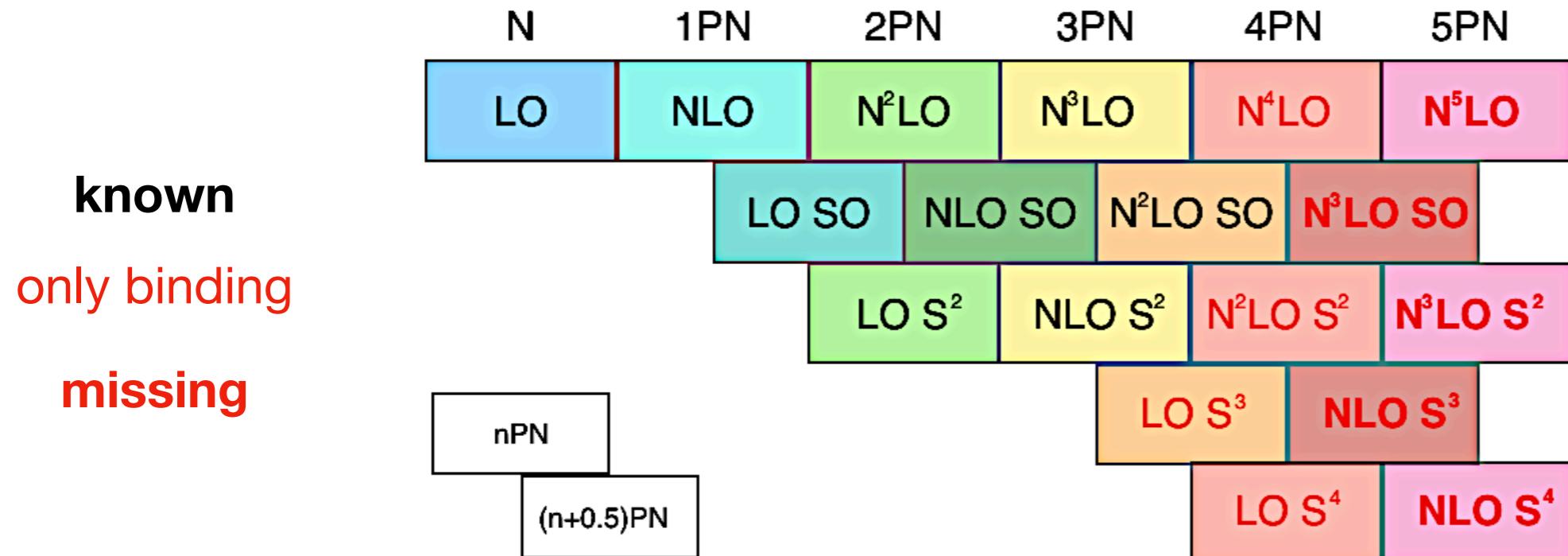
(If we know  $G^n$  integrals we know  $(n+1)PN$ )

**Challenge:** Combinatorial & 5loops for nPN with  $n>5$

(Spinning part is 'easier': New vertices in the Feynman rules)



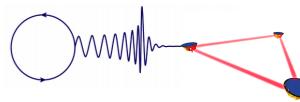
## Challenges: New tools?



Radiation moments are \*simpler\* than computing the binding energy



The  $N^{(n+1)}$  LO multipoles depend on  $N^n$  LO integrals



## Challenges for New tools

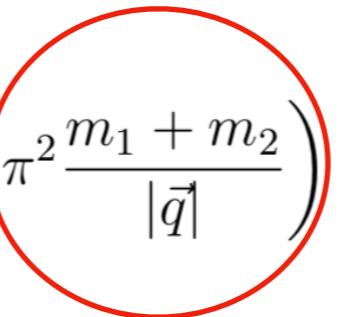
- Gauge (in)variant quantities

$$V^{\text{one-loop}}(q) = \frac{M^{\text{non-rel.}}(q)}{4m_1m_2} = \frac{e^4}{8\pi^2 m_1 m_2} \left( \frac{7}{3} \log(\vec{q}^2) - \pi^2 \frac{m_1 + m_2}{|\vec{q}|} \right)$$

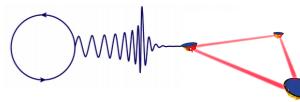
Darwin  
Feinberg Sucher

**Gauss' law?**

$$m_2 \gg m_1 \quad \frac{e^4}{mr^2}$$



Bern-Carrasco-Johansson  
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Cachazo et al.  
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Goldberger et al.  
Ladha Sen  
O'Connell et al.  
O'Connell Kosower Maybee  
Neill Rothstein  
Plefka Steinhoff  
Vines  
...



## Challenges for New tools

- Deflection angle (integrated vs instantaneous)

$$\begin{aligned} \Delta p_1^{\mu, (1)} \Big|_{m_2 \rightarrow \infty} &= \int \hat{d}^4 \bar{q} \hat{\delta}(\bar{q} \cdot u_1) \hat{\delta}(\bar{q} \cdot u_2) e^{-i\bar{q} \cdot b} \frac{e^4 Q_1^2 Q_2^2}{2m_1} \int \hat{d}^4 \bar{\ell} \frac{\hat{\delta}(\bar{\ell} \cdot u_2)}{\bar{\ell}^2 (\bar{\ell} - \bar{q})^2} \\ &\times \left[ i\bar{q}^\mu \left( 1 + \frac{\bar{\ell} \cdot (\bar{\ell} - \bar{q})(u_1 \cdot u_2)^2}{(\bar{\ell} \cdot u_1 + i\epsilon)^2} \right) + \bar{\ell}^\mu \bar{\ell} \cdot (\bar{\ell} - \bar{q})(u_1 \cdot u_2)^2 \hat{\delta}'(\bar{\ell} \cdot u_1) \right]. \end{aligned}$$



Non-instantaneous correction (shift in WL)

$$\Delta^{(1)} x_1^\mu(\tau_1) = -i \frac{e^2 Q_1 Q_2}{m_1} \int \hat{d}^4 \bar{q} \hat{\delta}(\bar{q} \cdot u_2) e^{-i\bar{q} \cdot (b + u_1 \tau_1)} \frac{\bar{q}^\mu u_1 \cdot u_2 - u_2^\mu \bar{q} \cdot u_1}{\bar{q}^2 (\bar{q} \cdot u_1 + i\epsilon)^2}.$$

IN-IN classical formalism vs. IN-OUT Scattering Amplitudes

Galley  
Leibovich

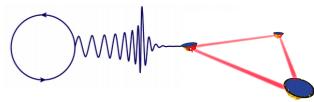


$$(a_a^i)_{rr}(t) = -\frac{2G_N}{5} I^{(5)ij}(t) x_a^j(t).$$

IN-IN double(double)-copy?

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...

RR force includes everything!  
(Modulo Schott terms)  
Maia Galley Leibovich Porto



## Challenges for New tools

- **Radiation (Flux vs Multipoles)**

‘large N’?

❖ Find:

$$\langle k^\mu \rangle = \sum_X \int Dp_1 Dp_2 Dk \ k^\mu \left| \int Dp'_1 Dp'_2 e^{ib \cdot p'_1} \right. \begin{array}{c} p_1 \\ k \\ X \\ \phi(p'_1) \end{array} \left. \delta^4(\sum p) \right|^2$$

O’Connell

Multipole PN expansion in EFT:

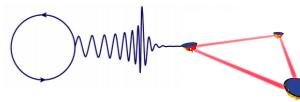
$$iA_h(\omega, \mathbf{k}) = \overbrace{\text{---}}^{I^{ij}} + \overbrace{\text{---}}^{J^{ij}} + \overbrace{\text{---}}^{I^{ijk}} + \dots$$

$$\mathcal{F}_{\text{inst}} = \frac{G}{c^5} \left\{ \frac{1}{5} I_{ij}^{(3)} I_{ij}^{(3)} + \frac{1}{c^2} \left[ \frac{1}{189} I_{ijk}^{(4)} I_{ijk}^{(4)} + \frac{16}{45} J_{ij}^{(3)} J_{ij}^{(3)} \right] + \dots \right\}$$

Amplitude:  
Imaginary part +  
optical theorem (?)

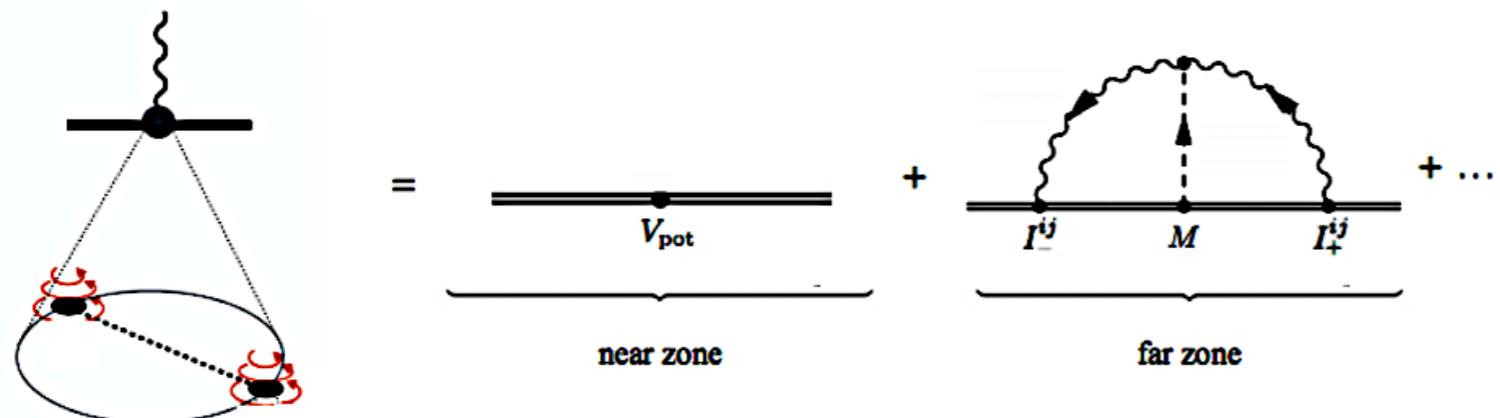
$$2 \times \overbrace{\text{---}}^{\text{wavy}} = \overbrace{\text{---}}^{\text{wavy}} \times \overbrace{\text{---}}^{\text{wavy}}$$

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## Challenges for New tools

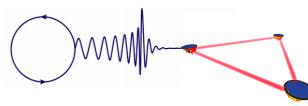
- ‘Gravitational Lamb shift’ (PN vs PM)



$$G^4 v^2 \frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left( -\frac{1}{\epsilon_{IR}} + 2\log(\mu r) + \dots \right) + \left( \frac{1}{\epsilon_{UV}} + 2\log(\Omega/\mu) + \dots \right)$$

- There are **no** IR divergences in S-matrix (PM does not have zones!)
- **Classically**, region which contributes to kick (real part) is ‘potential’  $q_0 \sim v \cdot q$  (with  $q \sim 1/b$ )
- Yet, ‘Lamb-shift’ logarithm(s) must show up — resummed — at 4PM ( $O(G^4 p^2 + \dots)$ ) from ‘soft modes’  $p_0 \sim p$ .

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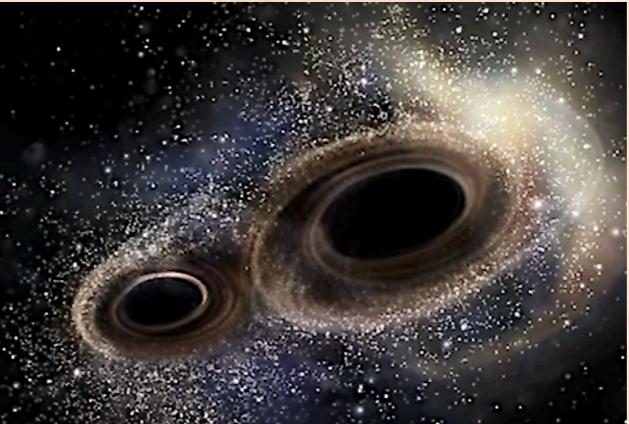


*Thank you...*

*no.203.078*

## Precision Gravity: from LHC to LISA

New era of foundational investigations  
established—through GWPD.

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5} \nu x^{5/2} \left\{ 1 + \dots + [\dots]x^{7/2} + [\dots]x^4 + [\dots]x^5 \right\}$$


*New particles discovered!*  
*New objects found!*  
*Neutron stars unveiled!*

PRECISION GRAVITY: FROM THE LHC TO LISA

26 August - 20 September 2019

**MIAPP** Munich Institute for  
Astro- and Particle Physics

John Joseph Carrasco, Ilya Mandel, Donal O'Connell, Rafael Porto,  
Fabian Schmidt