Spinor-helicity meets Petrov

Isobel Nicholson
University of Edinburgh
i.nicholson@sms.ed.ac.uk

Based on work done with Donal O'Connell, Andrés Luna and Ricardo Monteiro

Outline

1. The Petrov classification
2. Spinor helicity for GR?
3. Type D solutions and the Weyl double copy
4. A new frontier: five dimensional GR

1) The Petrov classification

- What is the Petrov classification - and why do we care about it?

- Separate Weyl tensor into 5 complex scalars with physical meaning

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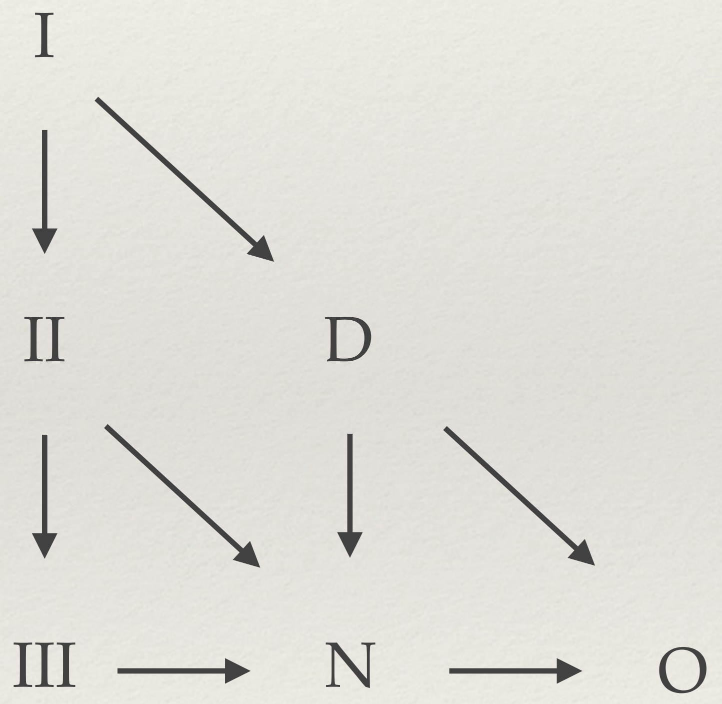

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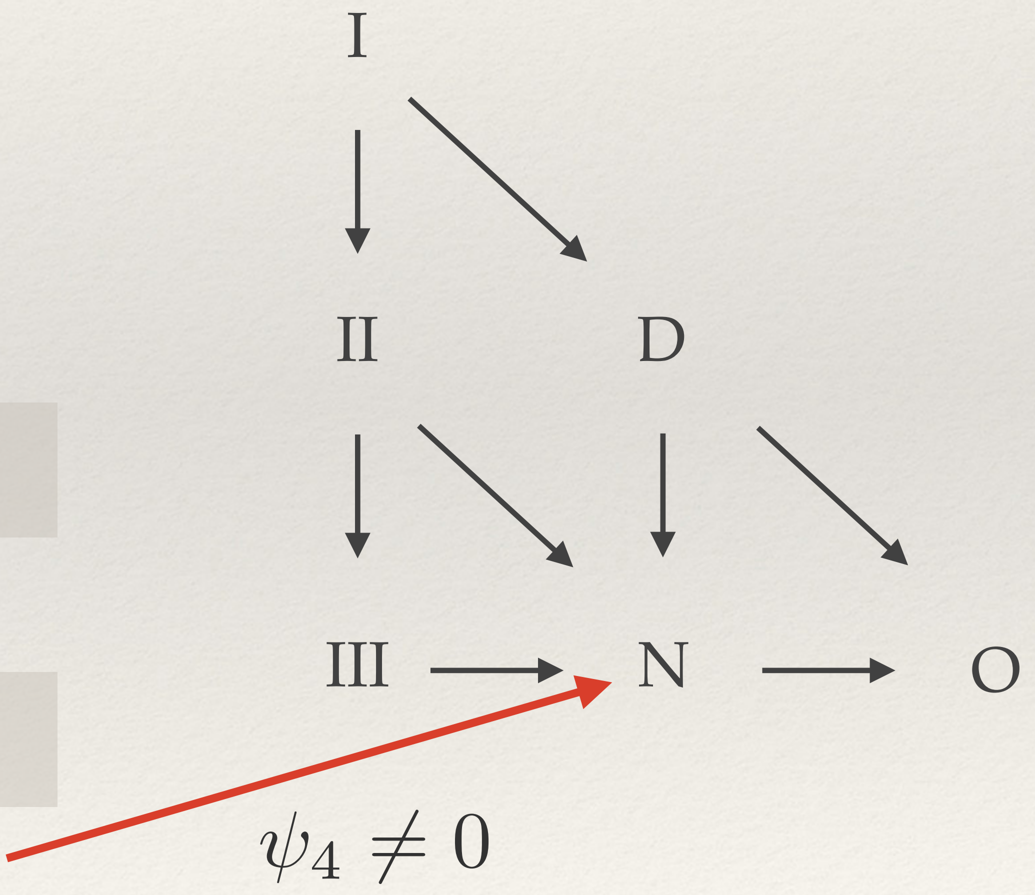

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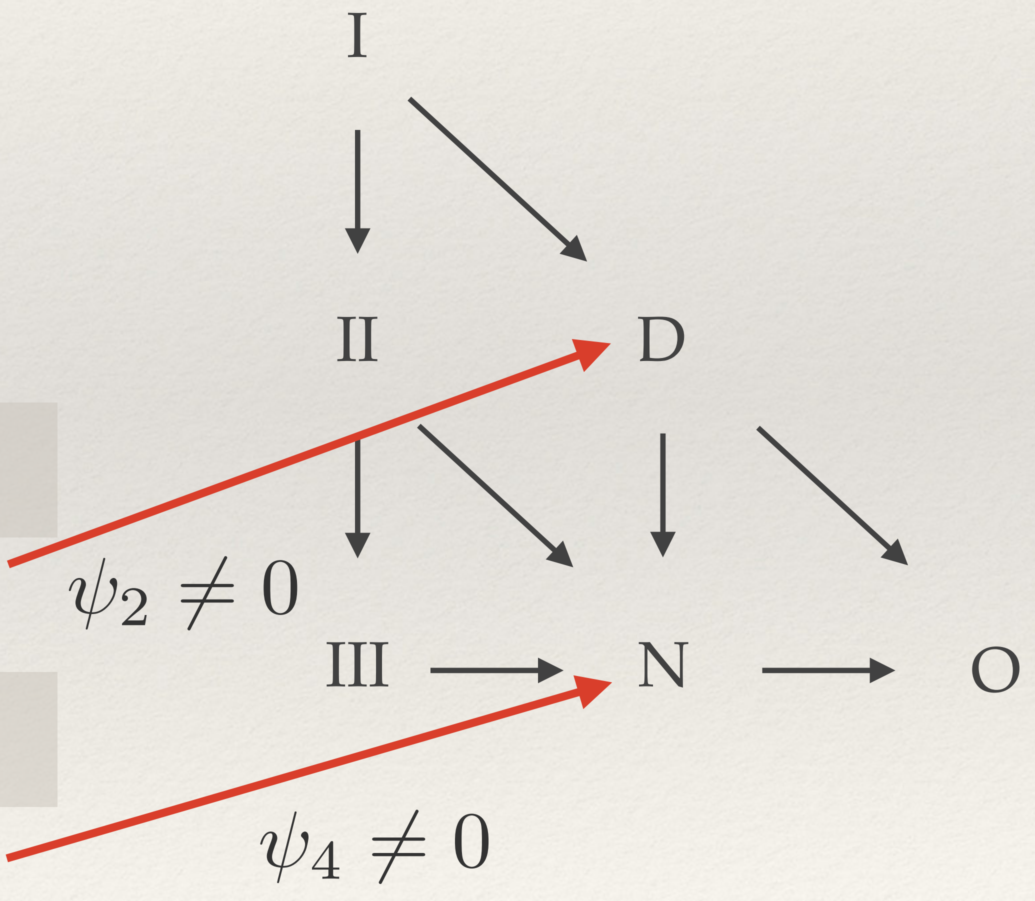

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- What is the Petrov classification - and why do we care about it?
  1. We want a non-perturbative formulation of the double copy

$$\mathcal{A}^{\rm YM} = \sum_{i\in{\rm diags}} \frac{n_i c_i}{D_i} \qquad\qquad \mathcal{A}^{\rm GR} = \sum_{i\in{\rm diags}} \frac{n_i n_i}{D_i}$$

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Propagators

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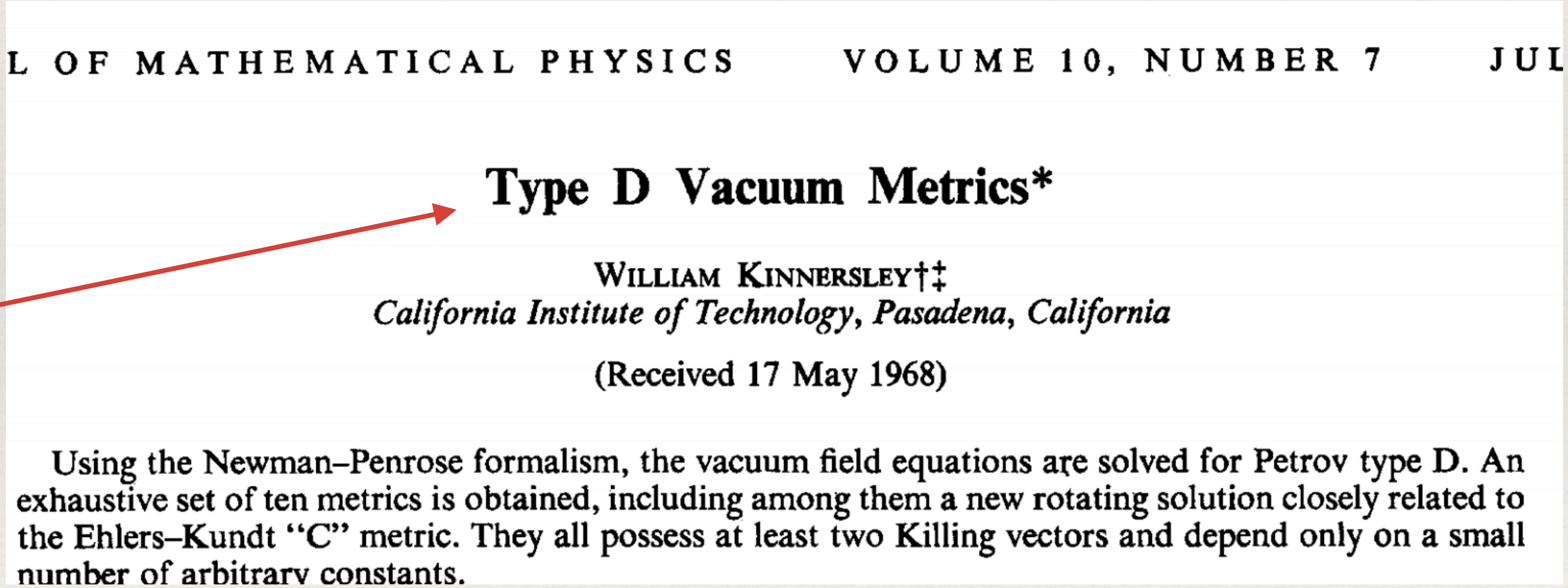
   2. We want to find new GR solutions



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Type D Vacuum Metrics*

WILLIAM KINNERSLEY†‡
California Institute of Technology, Pasadena, California

(Received 17 May 1968)

Using the Newman–Penrose formalism, the vacuum field equations are solved for Petrov type D. An exhaustive set of ten metrics is obtained, including among them a new rotating solution closely related to the Ehlers–Kundt "C" metric. They all possess at least two Killing vectors and depend only on a small number of arbitrary constants.

Complete set of 4d type D solutions

- 5d is an interesting place…

2) Spinor-helicity for GR?

- Newman-Penrose formulation: key GR technique

$$g_{\mu\nu} = -2k_{(\mu}n_{\nu)} + 2m_{(\mu}\bar{m}_{\nu)}\,, \qquad k\cdot n = -1, \quad m\cdot\bar{m} = 1$$

$$k^{\mu} = \sigma^{\mu}{}_{\alpha\dot{\alpha}}\lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}, \quad n^{\mu} = \sigma^{\mu}{}_{\alpha\dot{\alpha}}\mu^{\alpha}\tilde{\mu}^{\dot{\alpha}}$$

- Complete basis:

$$m^{\mu} = \sigma^{\mu}{}_{\alpha\dot{\alpha}}\lambda^{\alpha}\tilde{\mu}^{\dot{\alpha}} \;\sim\; \epsilon^{\mu}_{+}, \quad \bar{m}^{\mu} = \sigma^{\mu}{}_{\alpha\dot{\alpha}}\mu^{\alpha}\tilde{\lambda}^{\dot{\alpha}} \;\sim\; \epsilon^{\mu}_{-}$$

Penrose (1960) & Newman, Penrose (1962)

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$$\lambda_\alpha \mu^\alpha = 1$$

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Penrose (1960) & Newman, Penrose (1962)

2) Spinor-helicity for GR?

- Build a "Weyl spinor":

$$\Psi_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\sigma^{\mu\nu}{}_{\alpha\beta}\sigma^{\rho\sigma}{}_{\gamma\delta}\,, \qquad \sigma^{\mu\nu}{}_{\alpha}{}^{\beta} = \frac{1}{2}\left(\sigma^{\mu}_{\alpha\dot{\alpha}}\tilde{\sigma}^{\nu\dot{\alpha}\beta} - \sigma^{\nu}_{\alpha\dot{\alpha}}\tilde{\sigma}^{\mu\dot{\alpha}\beta}\right)$$

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- Use $\{\lambda, \mu\}$ to isolate components

$\psi_0 = \Psi_{\alpha\beta\gamma\delta}\,\lambda^\alpha\lambda^\beta\lambda^\gamma\lambda^\delta$	transverse incoming radiation
$\psi_1 = \Psi_{\alpha\beta\gamma\delta}\,\lambda^\alpha\lambda^\beta\lambda^\gamma\mu^\delta$	incoming longitudinal field
$\psi_2 = \Psi_{\alpha\beta\gamma\delta}\,\lambda^\alpha\lambda^\beta\mu^\gamma\mu^\delta$	$1/r^2$ potential (Coulomb-like)
$\psi_3 = \Psi_{\alpha\beta\gamma\delta}\,\lambda^\alpha\mu^\beta\mu^\gamma\mu^\delta$	outgoing longitudinal field
$\psi_4 = \Psi_{\alpha\beta\gamma\delta}\,\mu^\alpha\mu^\beta\mu^\gamma\mu^\delta$	transverse outgoing radiation

2) Spinor-helicity for GR?

- Handy fact: symmetric 2-component spinors factorize

$$\Psi_{\alpha\beta\gamma\delta} = A_{(\alpha}B_{\beta}C_{\gamma}D_{\delta)}$$

$$\begin{tikzcd}
\mathrm{I} \arrow[d] \arrow[dr] & & \\
\mathrm{II} \arrow[d] \arrow[dr] & \mathrm{D} \arrow[d] \arrow[dr] & \\
\mathrm{III} \arrow[r] & \mathrm{N} \arrow[r] & \mathrm{O}
\end{tikzcd}$$

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Petrov Type	
I	*A*, *B*, *C*, *D* distinct
II	*A*=*B*
D	*A*=*C*, *B*=*D*
III	*A*=*B*=*C*
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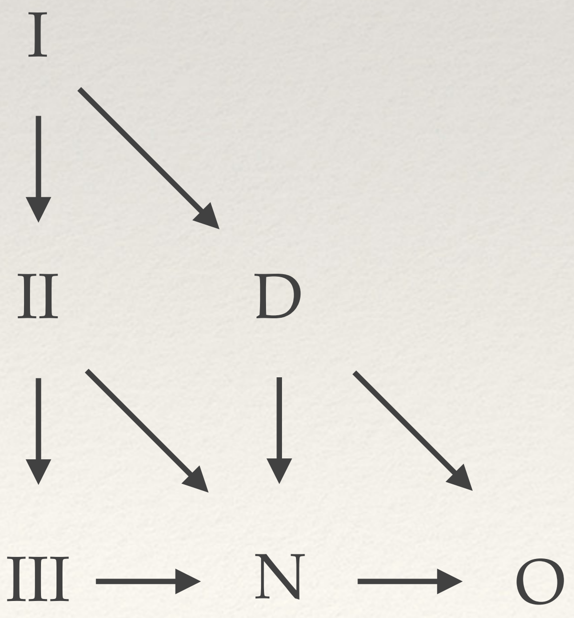

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Types D and N: natural double copies

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Petrov Type		
I	A, B, C, D distinct	$\psi_0 = 0$
II	A=B	$\psi_0 = \psi_1 = 0$
D	A=C, B=D	$\psi_2 \neq 0$
III	A=B=C	$\psi_3, \psi_4 \neq 0$
N	A=B=C=D	$\psi_4 \neq 0$

Types D and N: natural double copies

3) The Weyl double copy

$$A_\mu = \phi\, k_\mu \qquad g_{\mu\nu} = \eta_{\mu\nu} + \phi\, k_\mu k_\nu$$

- Many ideas for exact double copies
  - Monteiro, O'Connell, White (2014) + Luna (2015)
  - Carrillo González, Penco, Trodden (2017)
  - Adamo, Casali, Mason (2017) + Nekovar (2018)
  - Gurses, Tekin (2018)
  - Ridgeway, Wise (2016)
  - Lee (2018)
- Move beyond Kerr-Schild using field strengths $C_{\mu\nu\rho\sigma} \sim F_{\mu\nu} F_{\rho\sigma}$

  Cardoso, Nagy, Nampuri (2017)

  - Need spinors to match gauge redundancies

    Anastasiou, Borsten, Duff, Hughes, Nagy (2014)

3) The Weyl double copy

- **All** type D vacuum solutions can be expressed in double Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi_k\, k_\mu k_\nu + \phi_l\, l_\mu l_\nu$$

Plebanski, Demianski (1976)

- Find a_l and a_k s.t. $A_\mu = a_k\, k_\mu + a_l\, l_\mu$ solves flat vacuum MEs
- Construct field strength spinor $\Phi_{\alpha\beta} = F_{\mu\nu}\sigma^{\mu\nu}{}_{\alpha\beta}$

$$\Rightarrow \Psi_{\alpha\beta\gamma\delta} = \frac{1}{S}\Phi_{(\alpha\beta}\Phi_{\gamma\delta)}$$

- Propagator S solves wave equation on flat background

Luna, Monteiro, IN, O'Connell (2018)

3) The Weyl double copy

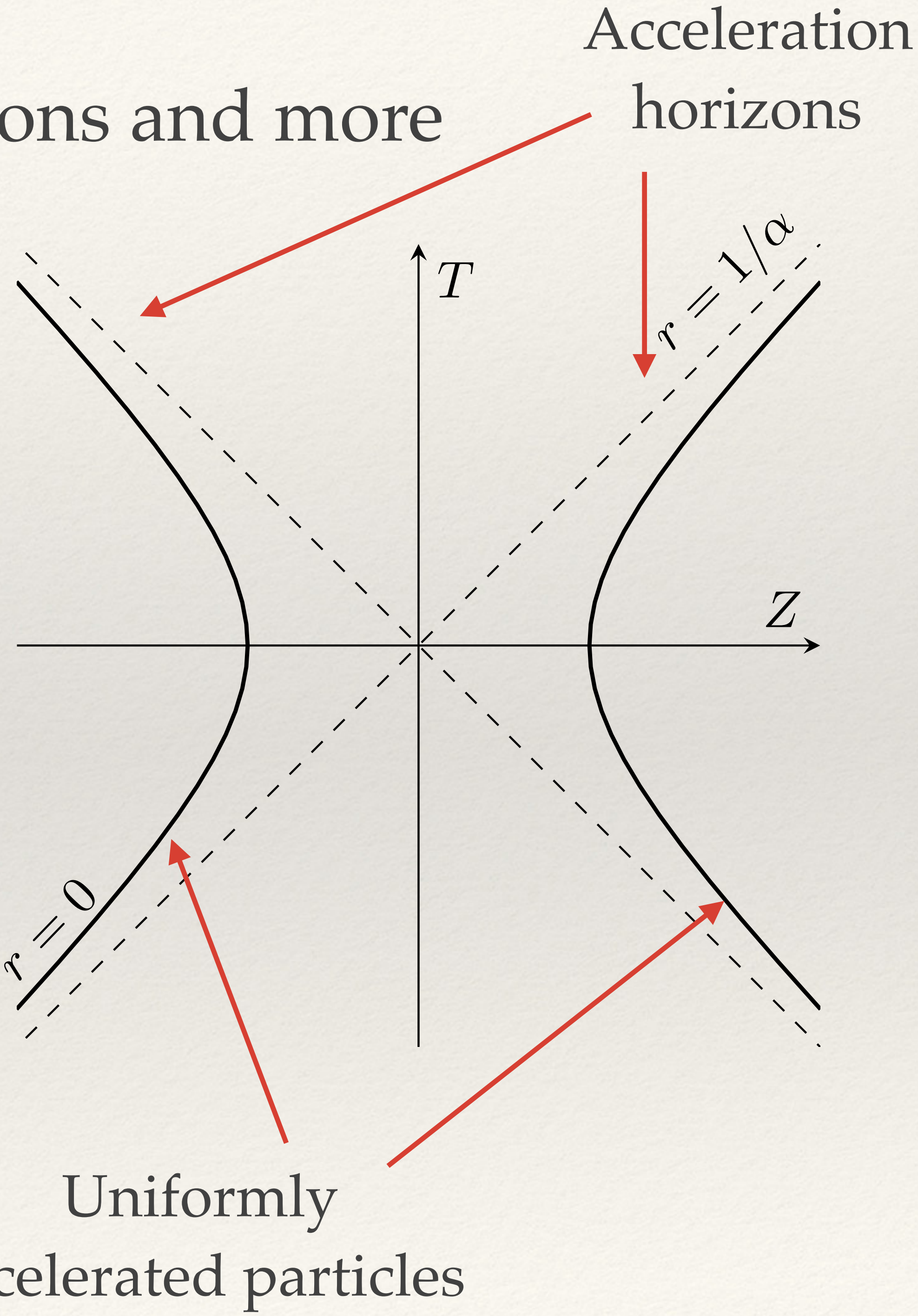
- Includes all previous Kerr-Schild solutions and more
- Example: accelerating C-metric

$$\Psi_{\alpha\beta\gamma\delta} = \frac{3}{2} m (x+y)^3 \xi_{\alpha\beta}\xi_{\gamma\delta}$$

$$\Phi_{\alpha\beta} = \frac{1}{2} \mathfrak{m} (x+y)^2 \xi_{\alpha\beta}$$

$$S = \frac{\mathfrak{m}^2}{6m}(x+y)$$

$$\xi_{\alpha\beta} = \lambda_\alpha \lambda_\beta - \mu_\alpha \mu_\beta$$



3) The Weyl double copy

- GR explanation:
  - Write type D metric as $\Psi_{\alpha\beta\gamma\delta} = |\chi|^{-5}\overset{\overset{A_{(\alpha}B_{\beta}A_{\gamma}B_{\delta)}}{\downarrow}}{\chi_{(\alpha\beta}\chi_{\gamma\delta)}}, \quad |\chi| = \sqrt{\chi_{\alpha\beta}\chi^{\alpha\beta}}$
  - Then $\chi_{\alpha\beta}$ is Killing spinor $\nabla_{(\alpha}{}^{\dot{\alpha}}\chi_{\beta\gamma)} = 0$
  - $\Phi_{\alpha\beta} = |\chi|^{-3}\chi_{\alpha\beta}$ solves curved background MEs

$$\begin{aligned} \Psi_{\alpha\beta\gamma\delta} &= |\chi|^{-5}\chi_{(\alpha\beta}\chi_{\gamma\delta)} \\ \Phi_{\alpha\beta} &= |\chi|^{-3}\chi_{\alpha\beta} \\ S &= |\chi|^{-1} \end{aligned}$$

- Double Kerr-Schild form maps to flat space

Penrose, Walker (1970) + Hughston, Sommers (1972)

4) 5d spinor-helicity

- Can we develop a 5d Petrov classification?
- Require $SO(3)$ little group: $\epsilon^{\mu}_{(ab)}$

$$g^{\mu\nu} = -2k^{(\mu}n^{\nu)} + \varepsilon^{ac}\varepsilon^{bd}\,\epsilon^{\mu}{}_{ab}\epsilon^{\nu}{}_{cd}, \quad k\cdot n = -1, \quad \epsilon^{\mu}{}_{ab}\epsilon_{\mu cd} = \varepsilon_{a[c}\varepsilon_{d]b}$$

Cheung, O'Connell (2009)

4) 5d spinor-helicity

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$a, b = 1, 2$

$\mathfrak{so}(3) \sim \mathfrak{su}(2)$

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- Appropriate definitions:

$A, B = 1, 2, 3, 4$

$$k^{\mu} = \frac{1}{2}\gamma^{\mu}{}_{AB}\Lambda^{A}{}_{a}\Lambda^{Ba} \qquad\qquad n^{\mu} = \frac{1}{2}\gamma^{\mu}{}_{AB}M^{A}{}_{a}M^{B}{}_{b}$$

$$\epsilon^{\mu}_{ab} = \gamma^{\mu}{}_{AB}\Lambda^{A}{}_{(a}M^{B}{}_{b)}$$

Cheung, O'Connell (2009)

4) 5d spinor-helicity

- Choose $k^\mu = (k^0, k^1, k^2, k^3, 0), \quad n^\mu = (n^0, n^1, n^2, n^3, 0)$

- Build ${\gamma^\mu}_{AB}$ from 4d sigma matrices

$$
{\gamma^\mu}_{AB} = \begin{pmatrix} 0 & \sigma^{\mu\alpha}{}_{\dot\beta} \\ -\tilde\sigma^\mu{}_{\dot\alpha}{}^\beta & 0 \end{pmatrix}, \quad \mu = 0, \ldots, 3, \quad {\gamma^4}_{AB} = -i \begin{pmatrix} \varepsilon^{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta} \end{pmatrix}
$$

- Appropriate basis is:

$$
{\Lambda^A}_a = \begin{pmatrix} 0 & \lambda_\alpha \\ \tilde\lambda^{\dot\alpha} & 0 \end{pmatrix}, \quad {M^A}_a = \begin{pmatrix} \mu_\alpha & 0 \\ 0 & -\tilde\mu^{\dot\alpha} \end{pmatrix}
$$

4) 5d spinor-helicity

- Example: consider formula for $k^\mu, \quad \mu = 0,1,2,3$

$$\gamma^\mu{}_{AB}\Lambda^A{}_a\Lambda^{Ba} = \mathrm{Tr}\left[\begin{pmatrix} 0 & \lambda_\alpha \\ \tilde{\lambda}^{\dot{\alpha}} & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^{\mu\alpha}{}_{\dot{\beta}} \\ -\tilde{\sigma}^\mu{}_{\dot{\alpha}}{}^\beta & 0 \end{pmatrix}\begin{pmatrix} \lambda_\beta & 0 \\ 0 & -\tilde{\lambda}^{\dot{\beta}} \end{pmatrix}\right]$$

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$$= \sigma^\mu{}_{\alpha\dot\beta}\lambda^\alpha\tilde{\lambda}^{\dot\beta} + \tilde{\sigma}^\mu{}_{\dot\alpha\beta}\tilde{\lambda}^{\dot\alpha}\lambda^\beta \quad = 2k^\mu$$

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- And $\mu = 4:$

$$\gamma^4{}_{AB}\Lambda^A{}_a\Lambda^{Ba} = -i\mathrm{Tr}\left[\begin{pmatrix} 0 & \lambda_\alpha \\ \tilde\lambda^{\dot\alpha} & 0 \end{pmatrix}\begin{pmatrix} \varepsilon^{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta} \end{pmatrix}\begin{pmatrix} \lambda_\beta & 0 \\ 0 & -\tilde\lambda^{\dot\beta} \end{pmatrix}\right] = 0$$

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$$= \sigma^\mu{}_{\alpha\dot\beta}\lambda^\alpha\tilde{\lambda}^{\dot\beta} + \tilde{\sigma}^\mu{}_{\dot\alpha\beta}\tilde{\lambda}^{\dot\alpha}\lambda^\beta \quad = 2k^\mu$$

- And $\mu = 4:$

$$\gamma^4{}_{AB}\Lambda^A{}_a\Lambda^{Ba} = -i\mathrm{Tr}\left[\begin{pmatrix} 0 & \lambda_\alpha \\ \tilde{\lambda}^{\dot\alpha} & 0 \end{pmatrix}\begin{pmatrix} \varepsilon^{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta} \end{pmatrix}\begin{pmatrix} \lambda_\beta & 0 \\ 0 & -\tilde{\lambda}^{\dot\beta} \end{pmatrix}\right] = 0$$

$$\Rightarrow k^\mu = (k^0, k^1, k^2, k^3, 0)$$

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- Construct Weyl spinor as before:

$$\Psi_{ABCD} = C_{\mu\nu\rho\sigma}\gamma^{\mu\nu}{}_{AB}\gamma^{\rho\sigma}{}_{CD}$$

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$\psi^{(2)}_{abcd} = \Psi_{ABCD}\,\Lambda^A{}_a\Lambda^B{}_b M^C{}_c M^D{}_d$	$1/r^2$ potential (Coulomb-like)
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$$\begin{tikzcd}
\mathrm{I} \arrow[d] \arrow[dr] & & \\
\mathrm{II} \arrow[d] \arrow[dr] & \mathrm{D} \arrow[d] \arrow[dr] & \\
\mathrm{III} \arrow[r] & \mathrm{N} \arrow[r] & \mathrm{O}
\end{tikzcd}$$

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Coley, Milson, Pravda, Pravdová (2004)

4) 5d spinor-helicity

- Decompose little group spinors to highlight sub-structure

	$\underline{5}$	$\underline{3}$	$\underline{1}$
$\Psi^{(0)}_{(abcd)}$	$\psi^{(0)}_{abcd}$		
$\Psi^{(1)}_{(abc)d}$	$\psi^{(1)}_{abcd}$	$\chi^{(1)}_{ab}$	
$\Psi^{(2)}_{(ab)(cd)}$	$\psi^{(2)}_{abcd}$	$\chi^{(2)}_{ab}$	$\Psi^{(2)}_{\mathrm{tr}}$
$\Psi^{(3)}_{a(bcd)}$	$\psi^{(3)}_{abcd}$	$\chi^{(3)}_{ab}$	
$\Psi^{(4)}_{(abcd)}$	$\psi^{(4)}_{abcd}$		

- Spinor-helicity is effective probe for 5d GR

Monteiro, IN, O'Connell (2018)

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Type D

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Monteiro, IN, O'Connell (2018)

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$\Psi^{(0)}_{(abcd)}$	$\psi^{(0)}_{abcd}$		
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$\Psi^{(2)}_{(ab)(cd)}$	$\psi^{(2)}_{abcd}$	$\chi^{(2)}_{ab}$	$\boxed{\Psi^{(2)}_{\text{tr}}}$
$\Psi^{(3)}_{a(bcd)}$	$\psi^{(3)}_{abcd}$	$\chi^{(3)}_{ab}$	
$\Psi^{(4)}_{(abcd)}$	$\psi^{(4)}_{abcd}$		

Tangherlini-Schwarzschild

$$\Psi^{(2)}_{\text{tr}} = \frac{m^2}{r^4}$$

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Monteiro, IN, O'Connell (2018)

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$\Psi^{(0)}_{(abcd)}$	$\psi^{(0)}_{abcd}$		
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$\Psi^{(2)}_{(ab)(cd)}$	$\boxed{\psi^{(2)}_{abcd}}$	$\chi^{(2)}_{ab}$	$\boxed{\Psi^{(2)}_{\text{tr}}}$
$\Psi^{(3)}_{a(bcd)}$	$\psi^{(3)}_{abcd}$	$\chi^{(3)}_{ab}$	
$\Psi^{(4)}_{(abcd)}$	$\psi^{(4)}_{abcd}$		

Black string

$$\Psi^{(2)}_{\text{tr}} = \frac{m}{r^3}$$

$$\psi^{(2)} = \frac{m}{r^3}\,\text{diag}\{1, 1, -2\}$$

- Spinor-helicity is effective probe for 5d GR

Monteiro, IN, O'Connell (2018)

Conclusions & outlook

- Petrov type D and N: natural exact double copies
- Complete set of Weyl double copies found for 4d type D
- Spinor helicity: a great language for classifying spacetimes - and double copies

- Next steps:
    - “Asymmetric” double copy?
    - Convert 4d type D double copy for 5d
        - Find new GR solutions in 5d - using the double copy?