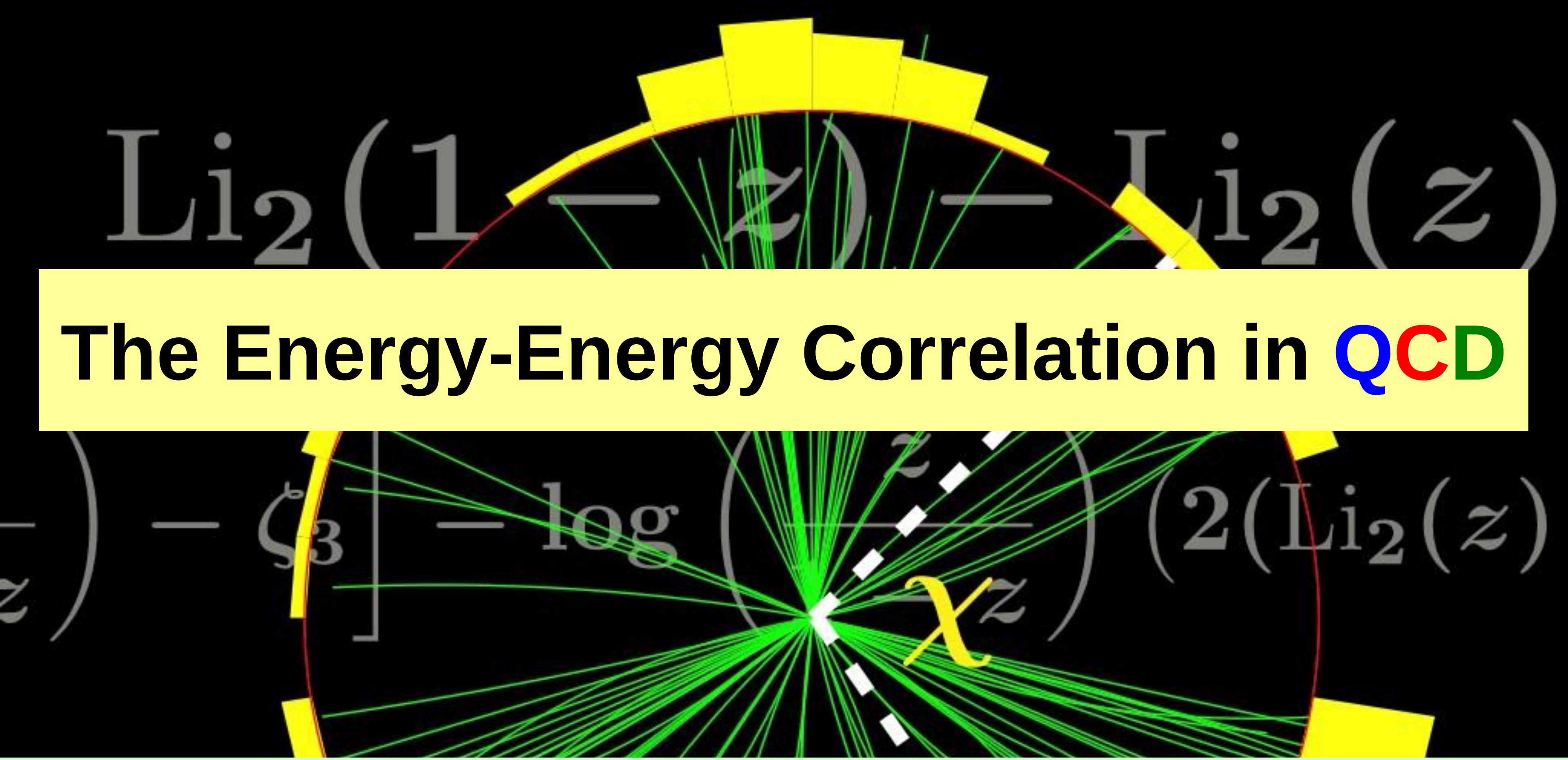


The Energy-Energy Correlation in QCD

Lance Dixon (SLAC)

LD, M.-X. Luo, V. Shtabovenko, T.-Z. Yang, H.-X. Zhu,
1801.03219

work in progress with I. Moult, H.-X. Zhu

QCD Meets **Gravity** IV @ Nordita

December 12, 2018

Introduction

- Energy-energy correlation (EEC) in e^+e^- annihilation:

one of first infrared safe event-shapes defined in QCD, 40 years ago Basham, Brown, Love, S. Ellis, PRD, PRL 1978

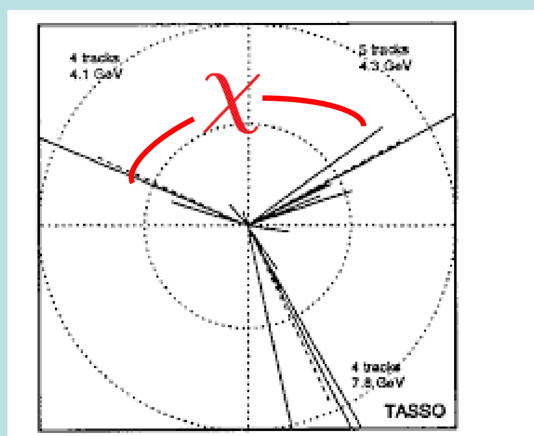
$$\boxed{\frac{d\Sigma}{d\cos\chi} = \sum_{\text{partons } i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta(\cos\theta_{ij} - \cos\chi)}$$

Collinear parton splitting

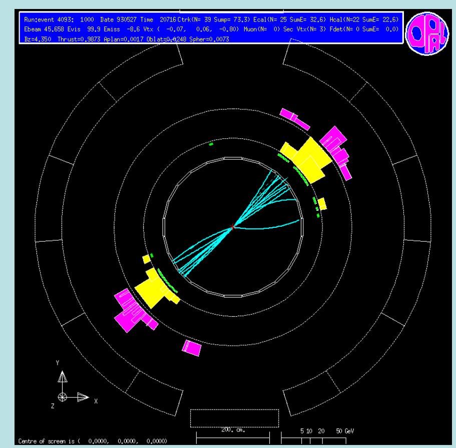
$$E_i \to x E_i + (1-x) E_i$$

preserves observable.
So does soft emission.

Data from wide range of CM energies →

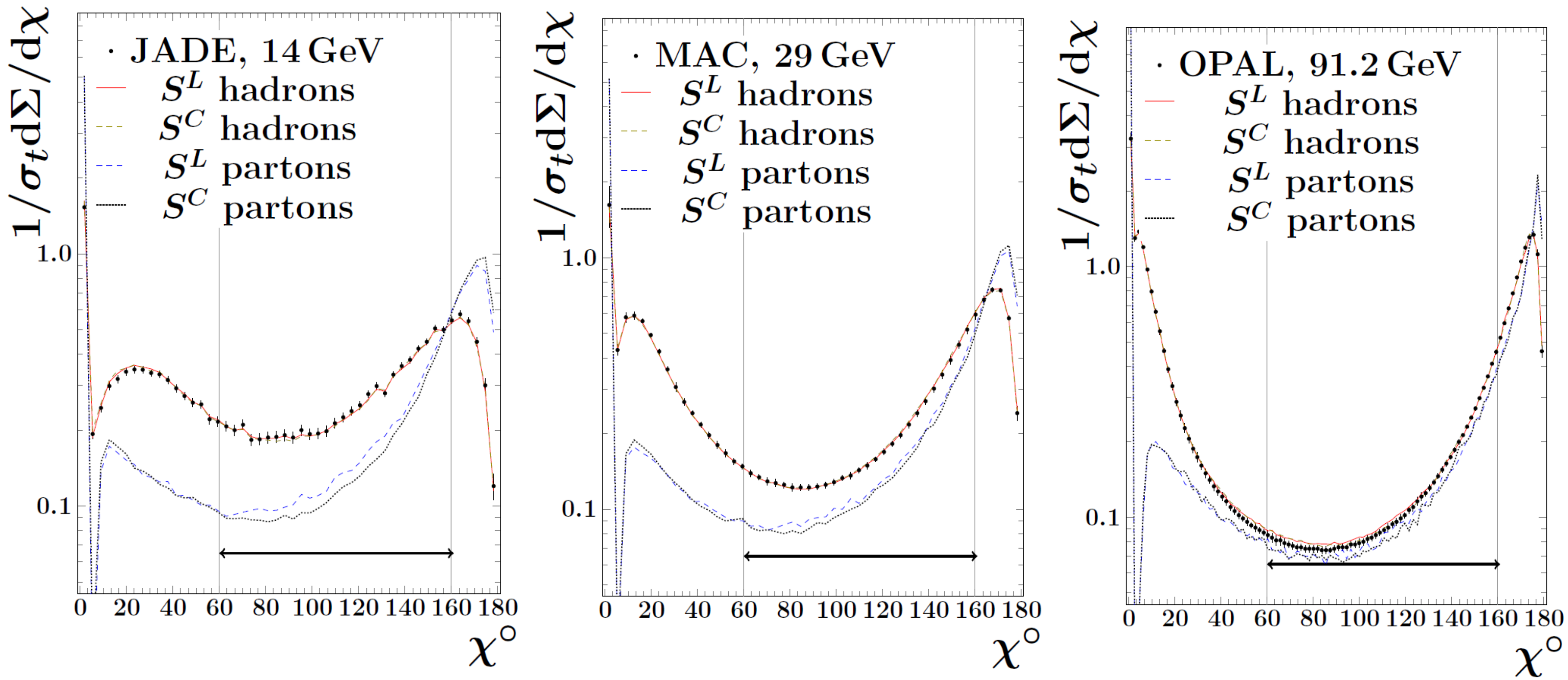


$Q \approx 22$ GeV



$Q = 91$ GeV

Evolution with energy clearly visible



data collected in Kardos et al, 1804.09146

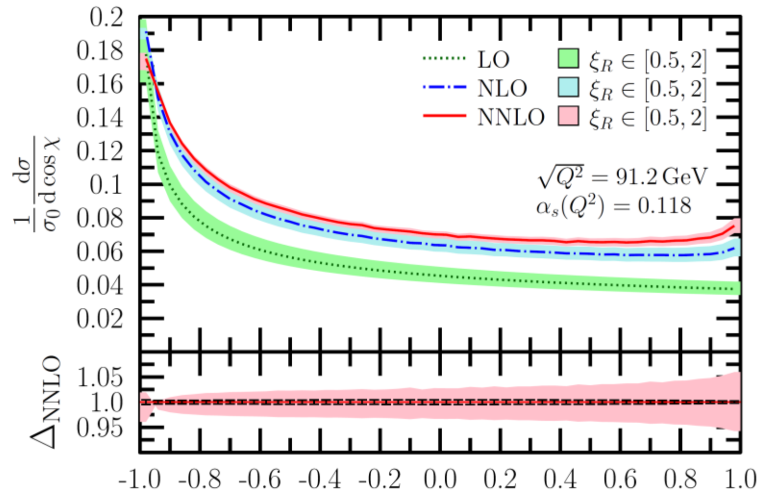
Why the EEC?

- Many event-shape variables to choose from:
  thrust, oblateness, C parameter, heavy jet mass, angularity, jet rates, …
- EEC among the simplest analytically
- Angle χ lives on a compact domain, $[0,\pi]$:
  large logarithms on **both** ends can be resummed
- As $\chi \rightarrow 0$, probe jet substructure.
  → computable LHC jet substructure variables?
  Moult, Necib, Thaler, 1609.07483
- **Gravitons** couple to **energy**, so AdS/CFT holography can be used to compute at strong gauge coupling (in planar N=4 SYM). QCD **meets Gravity**!
  Hofman, Maldacena, 0803.1467

Numerical results

- EEC computed at NLO numerically in 1980s and 1990s
  Richards, WJ Stirling, Ellis, 1982, 1983; Ali, Barreiro, 1982, 1984; Schneider, Kramer, Schierholz, 1984; Falck, Kramer, 1989; Kunszt, Nason, Marchesini, Webber, LEP Yellow Book, 1989; Glover, Sutton, 1994; Clay, Ellis, 1995; Kramer, Spiesberger, 1996; Catani, Seymour, 1996 [EVENT2].
- Computed numerically at NNLO only 2 years ago

  Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, 1603.08927

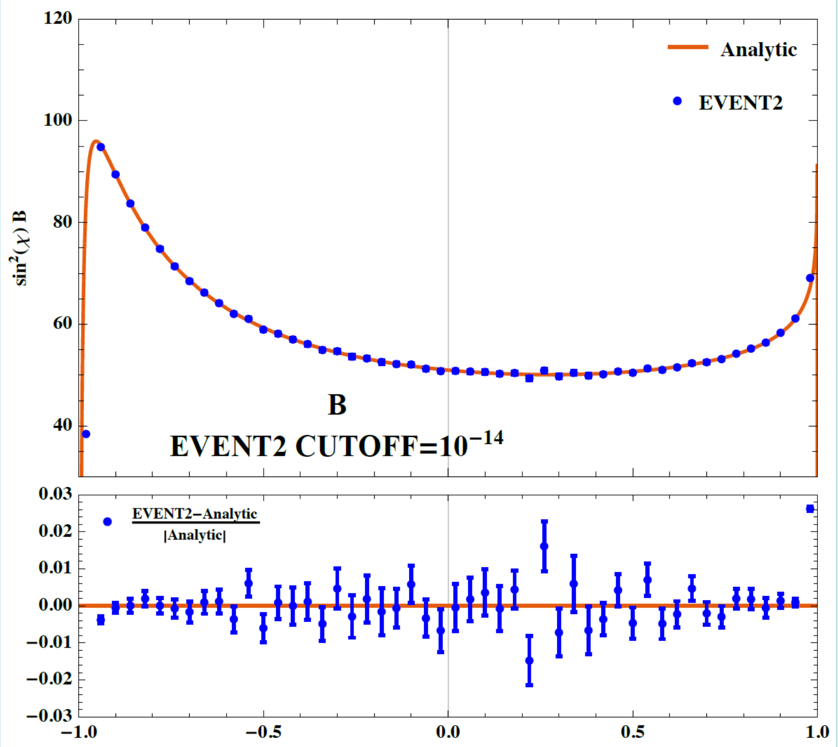


- Can it be computed analytically beyond LO?

Why analytic?

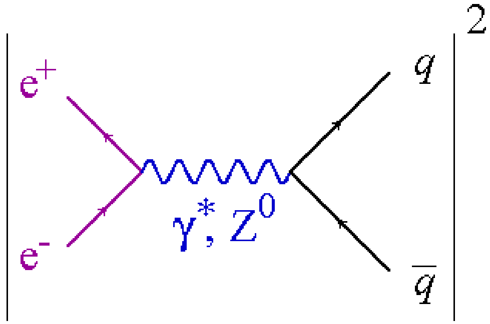
- Validate accuracy of numerical QCD results.
- Compare with analytic result in N=4 SYM

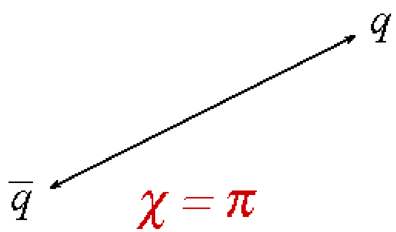
Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 1309.0769, 1309.1424, 1311.6800



- Study limits as $\chi \to 0,\pi$ to aid resummation of large logarithms there.

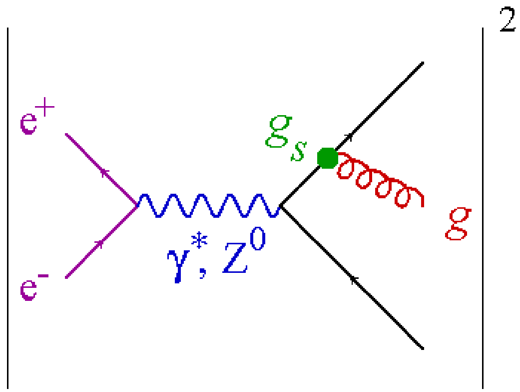
LO EEC for $0 < \chi < \pi$ is $O(\alpha_s)$

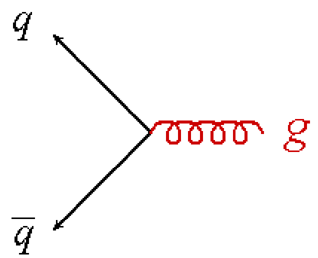




$\chi = \pi$

$$\alpha_s = \frac{g_s^2}{4\pi}$$





$0 < \chi < \pi$

How to compute at NLO?

- Reverse unitarity, e.g.

Anastasiou, Melnikov, hep-ph/0207004;
Anastasiou, LD, Melnikov, Petriello, hep-ph/0312266

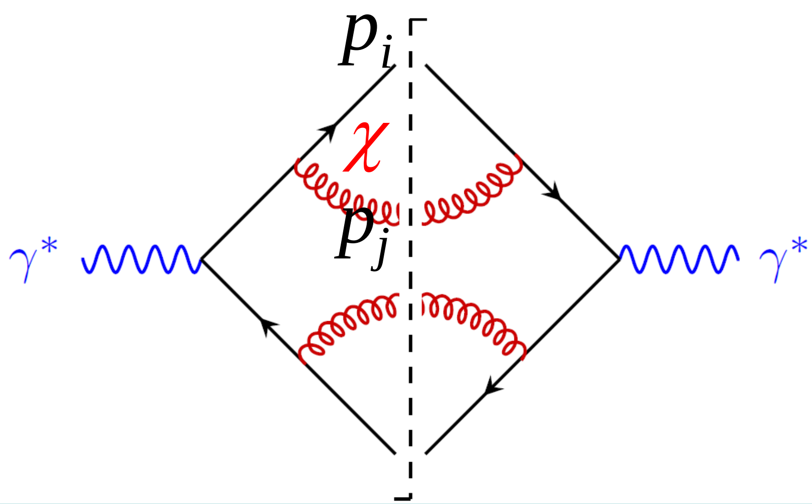
- Phase space integral over final-state partons with extra delta function which can be turned into a propagator:

$$\delta[\mathcal{M}_{ij}(\chi)] = \frac{1}{2\pi i}\left[\frac{1}{\mathcal{M}_{ij}(\chi) - i\varepsilon} - \frac{1}{\mathcal{M}_{ij}(\chi) + i\varepsilon}\right]$$

where
$$\begin{aligned} \mathcal{M}_{ij}(\chi) &= (p_i \cdot Q\, p_j \cdot Q)(\vec{n}_i \cdot \vec{n}_j - \cos\chi) \\ &= (p_i \cdot Q\, p_j \cdot Q)(1 - \cos\chi) - p_i \cdot p_j \end{aligned}$$

- Nonlinear in parton momenta p_i , p_j
- Sum over i,j

Sample NLO real contribution



- Feynman diagrams suffice.
- Treat all momenta as loop momenta, put all cut momenta on shell and impose one more $\delta[\mathcal{M}_{ij}(\chi)]$
- IBPs/Laporta algorithm Chetyrkin, Tkachov (1981), Laporta (2000)
- Diff. equations for master integrals Gehrmann, Remiddi (2000)

can all be solved in terms of polylogarithms.

Structure of QCD result

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi}A(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \left(\beta_0 \log\frac{\mu}{Q}A(z) + B(z)\right) + \mathcal{O}(\alpha_s^3)$$

$$z = \frac{1}{2}(1-\cos\chi) \;\in\; [0,1]$$

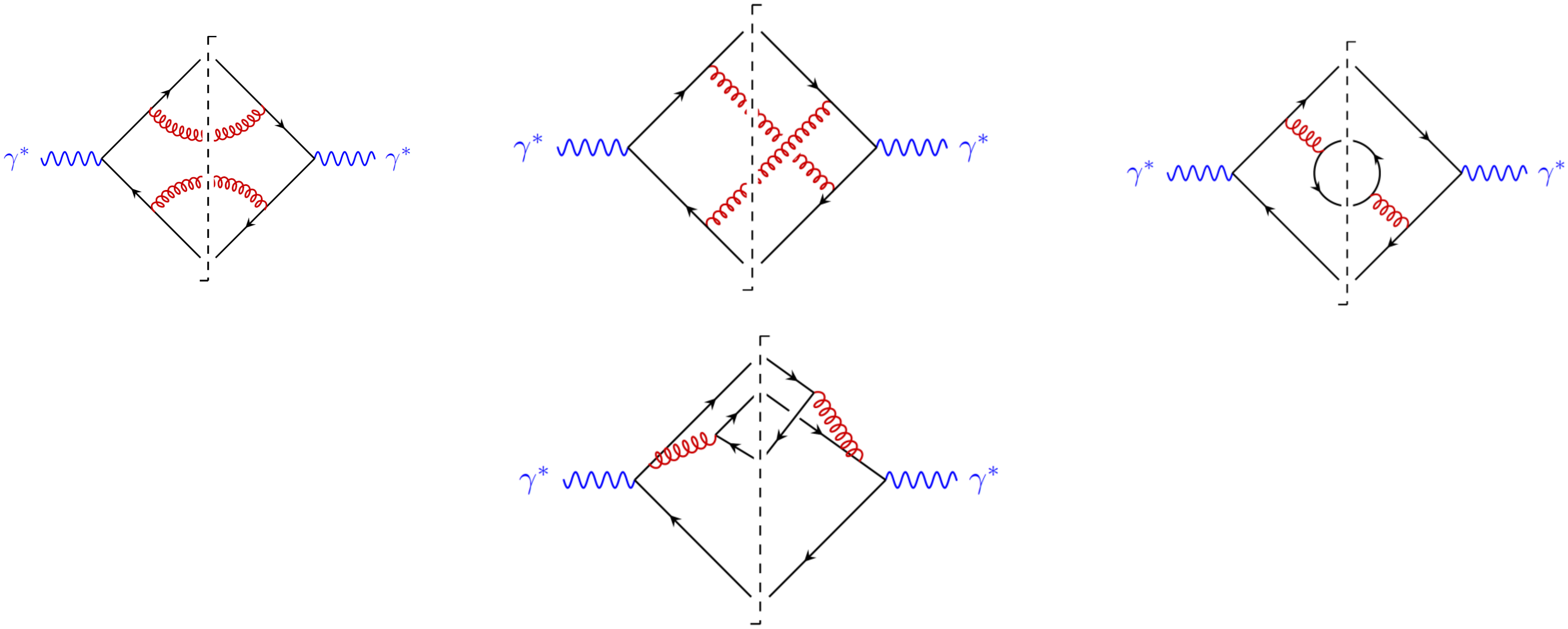
LO result fits on one line:

Basham, Brown, Love, S. Ellis, 1978

$$A(z) = C_F \frac{3-2z}{4(1-z)z^5}[3z(2-3z) + 2(2z^2 - 6z + 3)\log(1-z)]$$

Color structure of NLO QCD result

$$B(z) = C_F^2 B_{\mathrm{lc}}(z) + C_F(C_A - 2C_F) B_{\mathrm{nlc}}(z) + C_F N_f T_f B_{N_f}(z)$$



Leading color coefficient fits on one page

$$
\begin{aligned}
B_{\rm lc} = &+ \frac{122400z^7 - 244800z^6 + 157060z^5 - 31000z^4 + 2064z^3 + 72305z^2 - 143577z + 63298}{1440(1-z)z^4} \\
&- \frac{-244800z^9 + 673200z^8 - 667280z^7 + 283140z^6 - 48122z^5 + 2716z^4 - 6201z^3 + 11309z^2 - 9329z + 3007}{720(1-z)z^5} g_1^{(1)} \\
&- \frac{244800z^8 - 550800z^7 + 422480z^6 - 126900z^5 + 13052z^4 - 336z^3 + 17261z^2 - 38295z + 19938}{720(1-z)z^4} g_2^{(1)} \\
&+ \frac{4z^7 + 10z^6 - 17z^5 + 25z^4 - 96z^3 + 296z^2 - 211z + 87}{24(1-z)z^5} g_1^{(2)} \\
&+ \frac{-40800z^8 + 61200z^7 - 28480z^6 + 4040z^5 - 320z^4 - 160z^3 + 1126z^2 - 4726z + 3323}{120z^5} g_2^{(2)} \\
&- \frac{1-11z}{48z^{7/2}} g_3^{(2)} - \frac{120z^6 + 60z^5 + 160z^4 - 2246z^3 + 8812z^2 - 10159z + 4193}{120(1-z)z^5} g_4^{(2)} \\
&- 2\left(85z^4 - 170z^3 + 116z^2 - 31z + 3\right) g_1^{(3)} + \frac{-4z^3 + 18z^2 - 21z + 5}{6(1-z)z^5} g_2^{(3)} + \frac{z^2+1}{12(1-z)} g_3^{(3)} ,
\end{aligned}
$$

where

$$
g_1^{(1)} = \log(1-z) , \qquad g_2^{(1)} = \log(z) , \qquad g_1^{(2)} = 2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z) ,
$$

$$
g_2^{(2)} = \mathrm{Li}_2(1-z) - \mathrm{Li}_2(z) , \qquad g_3^{(2)} = -2\,\mathrm{Li}_2\left(-\sqrt{z}\right) + 2\,\mathrm{Li}_2\left(\sqrt{z}\right) + \log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z) , \qquad g_4^{(2)} = \zeta_2
$$

$$
g_1^{(3)} = -6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right) - \zeta_3\right] - \log\left(\frac{z}{1-z}\right)\left(2(\mathrm{Li}_2(z) + \zeta_2) + \log^2(1-z)\right) ,
$$

$$
g_2^{(3)} = -12\left[\mathrm{Li}_3(z) + \mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right] + 6\,\mathrm{Li}_2(z)\log(1-z) + \log^3(1-z) ,
$$

$$
g_3^{(3)} = 6\log(1-z)\left(\mathrm{Li}_2(z) - \zeta_2\right) - 12\,\mathrm{Li}_3(z) + \log^3(1-z) .
$$

Observations

- Other QCD color coefficients similar in complexity
- See 1801.03219 or https://www.youtube.com/watch?v=WVC1ygsjZNc
- Around both $z = 0$ and $z = 1$, expansion is in integer powers of z (and $\ln z$ or $\ln(1\text{-}z)$)
- Individual real/virtual terms have polylog argument $\frac{i\sqrt{z}}{\sqrt{1-z}}$
- Rational function prefactors have no singularities at spurious locations, but their singularities at $z = 0,\ 1,\ \infty$ are "too strong" and cancel among different terms
- N=4 SYM result (next page) is considerably simpler than QCD, but mainly in rational function prefactors, not transcendental functions.

EEC for N=4 SYM at NLO

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 1311.6800

- Correlator is for scalar source instead of electromagnetic current (but the precise source doesn't matter much)

$$F(z;a) = aF_1(z) + a^2\left[(1-z)F_2(z) + F_3(z)\right]$$

where $a = g_{\rm YM}^2 N/(4\pi^2)$

$$F_1(z) = -\ln(1-z)$$

$$F_2(z) = 4\sqrt{z}\left[\mathrm{Li}_2\left(-\sqrt{z}\right) - \mathrm{Li}_2\left(\sqrt{z}\right) + \frac{\ln z}{2}\ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\mathrm{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + z\frac{\pi^2}{3}$$

$$\begin{aligned}F_3(z) = \frac{1}{4}\Big\{&(1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\mathrm{Li}_3(z)\\ &+ 6(3+3z-4z^2)\mathrm{Li}_3\left(\frac{z}{z-1}\right) - 2z(1+4z)\zeta_3 + 2\left[2(2z^2-z-2)\ln(1-z) + (3-4z)z\ln z\right]\mathrm{Li}_2(z)\\ &+ \frac{1}{3}\ln^2(1-z)\left[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\right] + \frac{\pi^2}{3}\left[2z^2\ln z - (2z^2+z-2)\ln(1-z)\right]\Big\}\end{aligned}$$

- No uniform or maximal transcendentality principle – except for $\chi \rightarrow \pi$

Belitsky et al. method

- Very different from “QCD method”.
- Exploit conformal invariance of 4-point function with two “energy flow operators”

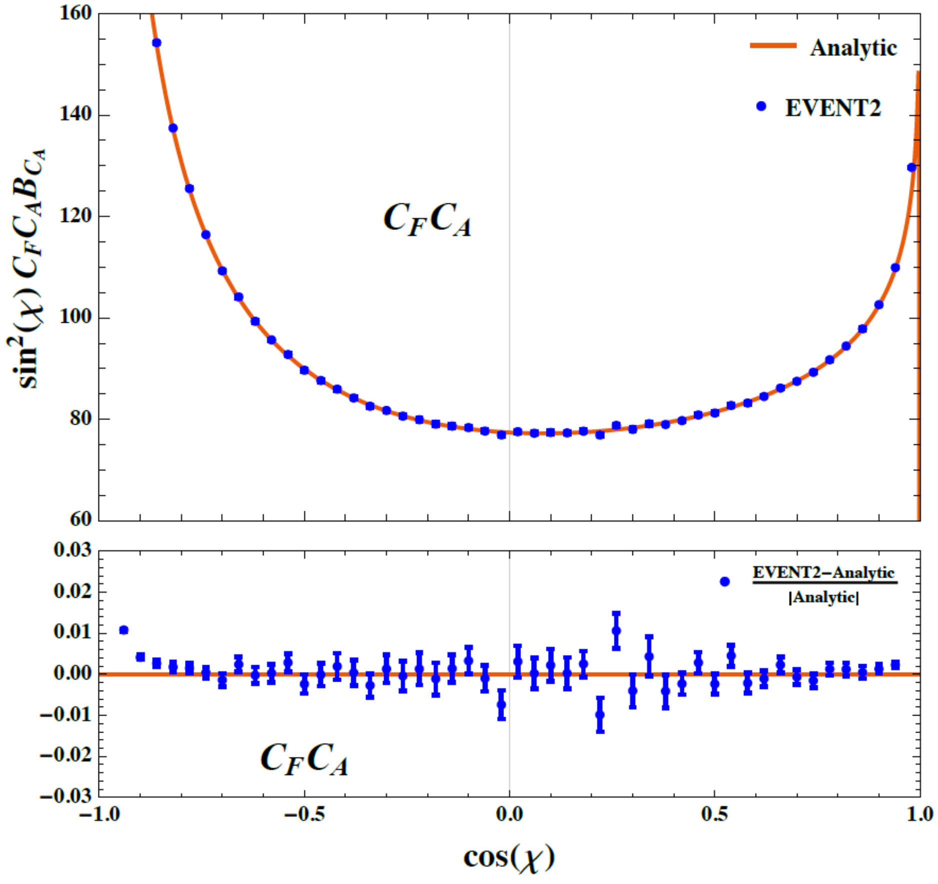
$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle_q = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle 0|O^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)O(0)|0\rangle$$

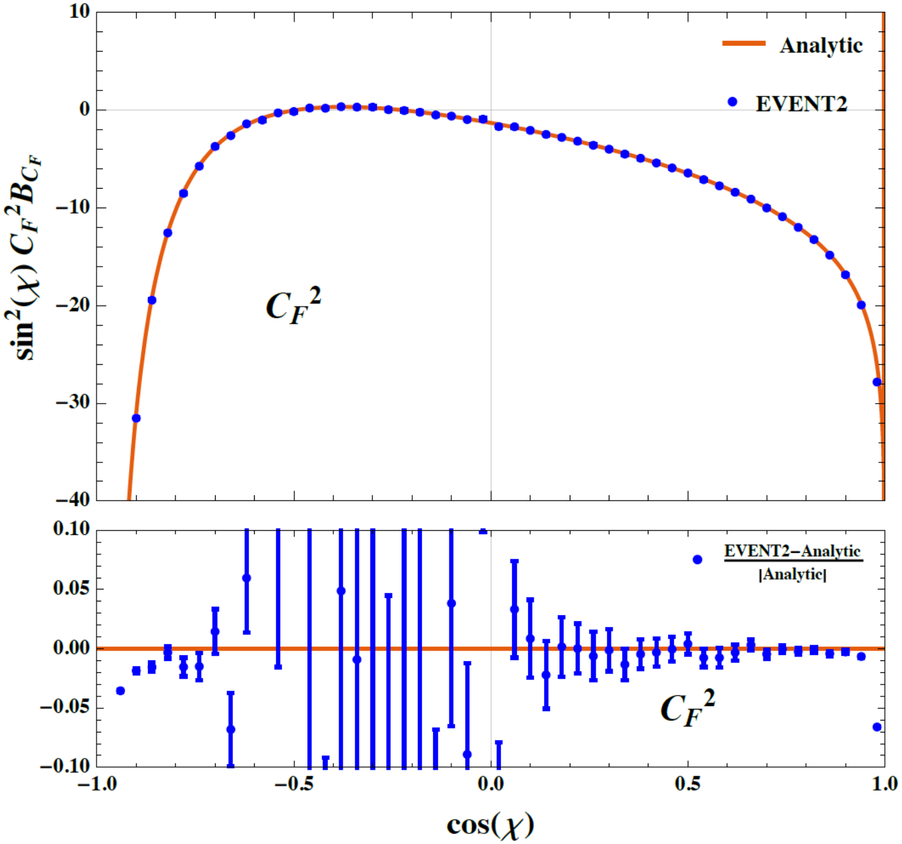
$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} d\tau \lim_{r\to\infty} r^2 n^i T_{0i}(t=\tau+r, r\vec{n})$$

- Analytically continue from Euclidean to physical region using double Mellin transform
- No infrared divergences at any step!
- In NLO QCD computation divergences cancel between virtual and real

NLO program EVENT2 validated

M. Seymour





Analytic properties of moments

- With analytic formulae, compute the integrals

$$B_N = \int_0^1 dz\ z^N B(z)$$

numerically to high accuracy, for each color coefficient

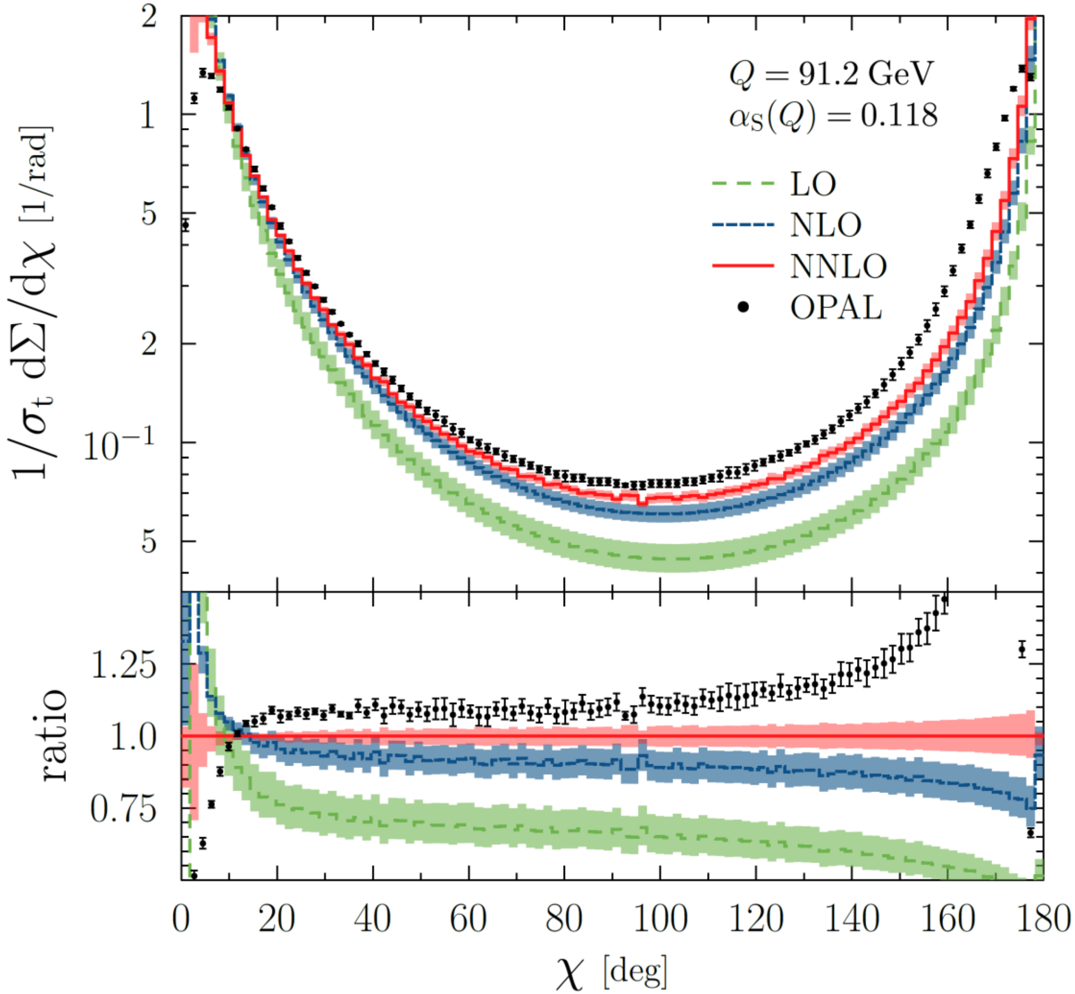
- Using PSLQ, it is always of the form

$$B_N = r_N^{(4)}\zeta(4)\ +\ r_N^{(3)}\zeta(3)\ +\ r_N^{(2)}\zeta(2)\ +\ r_N^{(0)}$$

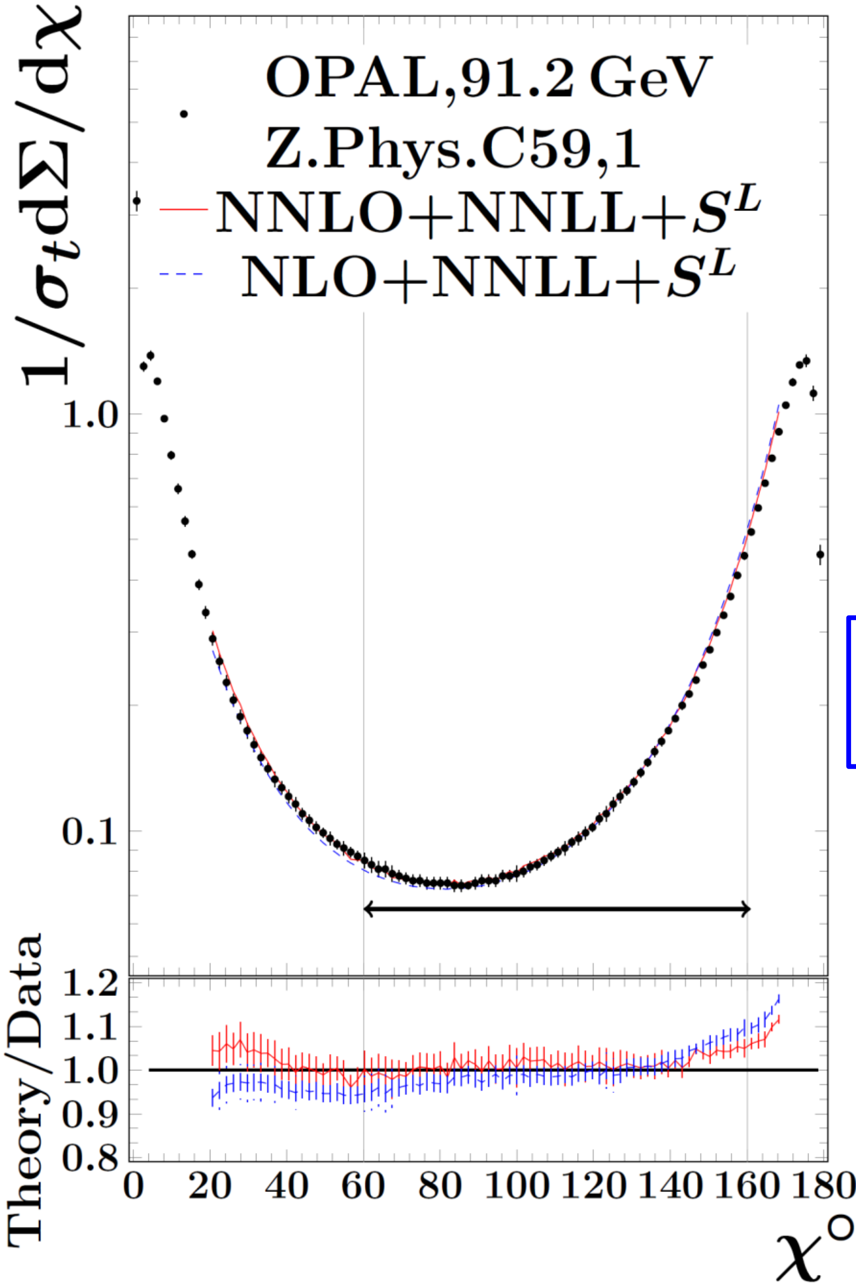
where the $r_N^{(w)}$ are rational numbers.

- E.g. $B_3(C_A) = -\frac{207}{2}\zeta(4) + \frac{14902}{35}\zeta(3) - \frac{553}{450}\zeta(2) - \frac{2369041}{5040}$
- Could they be MZV's at higher loop orders too?
- General expression in terms of ψ functions?

Fixed order vs. Z pole data



Tulipant, Kardos, Somogyi, 1708.04093



To measure strong coupling α_s :
Add NNLL $z \to 1$ resummation
+ MC estimate of
nonperturbative contributions

Kardos, Kluth, Somogyi, Tulipant, Verbytskyi, 1804.09146

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.)$$
$$\pm 0.00257(ren.) \pm 0.00078(res.)$$

Competitive measurement of α_s

Still room for theory improvement:
→ NNNLO (approx.?)
+ NNNLL $z \to 1$ resummation
+ (N?)NLL $z \to 0$ resummation

Back-to-back limit, $z \rightarrow 1$

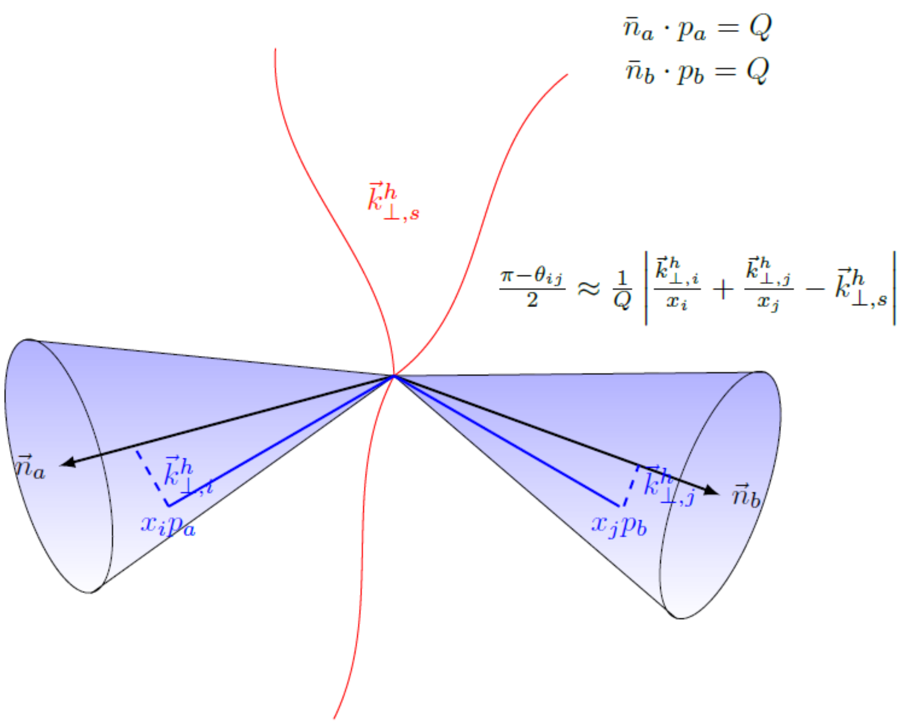
$$
\begin{aligned}
B(z) = C_F \Bigg\{ & \frac{1}{1-z} \Bigg[\frac{1}{2} C_F \ln^3(1-z) + \ln^2(1-z) \left(\frac{11 C_A}{12} + \frac{9 C_F}{4} - \frac{N_f T_f}{3} \right) \\
& + \ln(1-z) \left(C_A \left(\frac{\zeta_2}{2} - \frac{35}{72} \right) + C_F \left(\zeta_2 + \frac{17}{4} \right) + \frac{N_f T_f}{18} \right) \\
& + C_A \left(\frac{11\zeta_2}{4} + \frac{3\zeta_3}{2} - \frac{35}{16} \right) + C_F \left(3\zeta_2 - \zeta_3 + \frac{45}{16} \right) + N_f T_f \left(\frac{3}{4} - \zeta_2 \right) \Bigg] \\
& + \left(\frac{C_A}{2} + C_F \right) \ln^3(1-z) + \ln^2(1-z) \left(\frac{27 C_A}{8} + \frac{13 C_F}{2} - \frac{N_f T_f}{2} \right) \\
& + \ln(1-z) \left[C_A \left(22\zeta_2 - \frac{2011}{72} \right) + C_F (47 - 19\zeta_2) + N_f T_f \left(\frac{361}{36} - 4\zeta_2 \right) \right] \\
& + C_A \left(\frac{6347\zeta_2}{80} - 21\zeta_2 \ln 2 - \frac{137\zeta_3}{4} - \frac{3305}{72} \right) \\
& + C_F \left(-\frac{1727\zeta_2}{20} + 42\zeta_2 \ln 2 + \frac{121\zeta_3}{2} + \frac{3437}{96} \right) \\
& + N_f T_f \left(-\frac{1747\zeta_2}{120} + 12\zeta_3 + \frac{2099}{144} \right) \Bigg\} + \mathcal{O}(1-z)
\end{aligned}
$$

leading power

first subleading power

- Double log behavior, $ln^{2L+1}(1-z)/(1-z)$ characteristic of Sudakov suppression from soft/collinear gluon emission. Collins, Soper,…
- Coefficients of leading-power terms agree precisely with NNLL resummation DeFlorian, Grazzini,hep-ph/0407241

$z \rightarrow 1$ (cont.)



Moult, Zhu, 1801.02627

Soft gluons contribute, but only via recoil, by deflecting the hard quark jet

- Factorization theorem recently proved: Relate EEC to back-to-back production of identified hadrons Collins, Soper 1981-1982
- Should allow NNNLL resummation soon

Intra-jet limit, $z \to 0$

$$
\begin{aligned}
B(z) = C_F \Bigg\{ & \frac{1}{z}\Bigg[\ln z \left(-\frac{107C_A}{120} + \frac{25C_F}{32} + \frac{53N_fT_f}{240}\right) + C_A\left(-\frac{25\zeta_2}{12} + \frac{\zeta_3}{2} + \frac{17683}{2700}\right) \\
& + C_F\left(\frac{43\zeta_2}{12} - \zeta_3 - \frac{8263}{1728}\right) - \frac{4913N_fT_f}{3600}\Bigg] \\
& + \ln z \left[C_A\left(\frac{33\zeta_2}{2} - \frac{703439}{25200}\right) + C_F\left(\frac{42109}{1200} - 21\zeta_2\right) + N_fT_f\left(\frac{86501}{12600} - 4\zeta_2\right)\right] \\
& + C_A\left(\frac{213\zeta_2}{5} - \frac{101\zeta_3}{2} - \frac{26986007}{5292000}\right) + C_F\left(-\frac{1541\zeta_2}{30} + 65\zeta_3 + \frac{18563}{2700}\right) \\
& + N_fT_f\left(-\frac{46\zeta_2}{3} + 12\zeta_3 + \frac{2987627}{330750}\right)\Bigg\} + \mathcal{O}(z)
\end{aligned}
$$

leading power

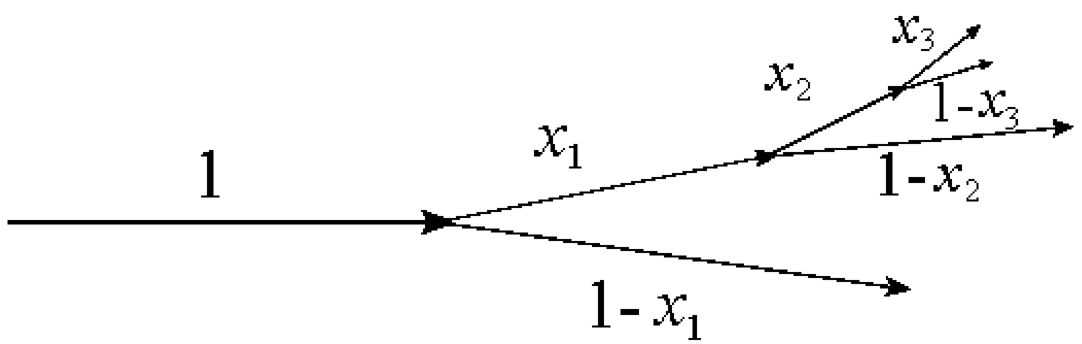
first subleading power

- Single log behavior, $ln^L z/z$ characteristic of pure collinear observable.
- Leading log (LL) resummation first performed in "jet calculus" approach   Konishi, Ukawa, Veneziano, Phys.Lett.1978,1979
- Coefficients of leading-power terms agree precisely with LL result   Richards, Stirling, Ellis, NPB229, 317, 1983

$z \to 0$ (cont.)

- Limit dominated by collinear emission. At leading log, only a single moment N=3 of time-like splitting function dominates

Konishi, Ukawa, Veneziano, Richards, Stirling, Ellis, Hofman, Maldacena, 0803.1467



Energy weighting → $$\int_0^1 dx\, x(1-x)\, P_{ij}(x) \;\to\; -\int_0^1 dx\, x^2\, P_{ij}(x) \;\equiv\; \gamma_{ij}^{(N=3)}$$

Momentum sum rule controls x^1 term,
→ can drop it.

$$\int_0^1 dx\, x\, P_{ij}(x) \;\equiv\; -\gamma_{ij}^{(N=2)}$$

LL resummed formula

Richards, Stirling, Ellis, NPB229, 317, 1983

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\sqrt{z}Q)}{16\pi\ z} \sum_{i,j=q,g} \Gamma_{ij}^{(0)} \left[\frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(Q)}\right]_{jq}^{-\Gamma^{(0)}/b_0}$$

$$\Gamma_{ij}^{(0)} = \begin{bmatrix} \frac{25}{6}C_F & -\frac{7}{15}N_f \\ -\frac{7}{6}C_F & \frac{14}{5}C_A + \frac{2}{3}N_f \end{bmatrix} \qquad b_0 = \frac{11}{3}C_A - \frac{2}{3}N_f$$

One-loop (LO) N=3 time-like moments

To expand back into fixed order:

$$\frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(Q)} = \left[1 + b_0\frac{\alpha_s(Q)}{4\pi}\ln z\right]^{-1}$$

Beyond LL as $z \to 0$

LD, Moult, Zhu, to appear

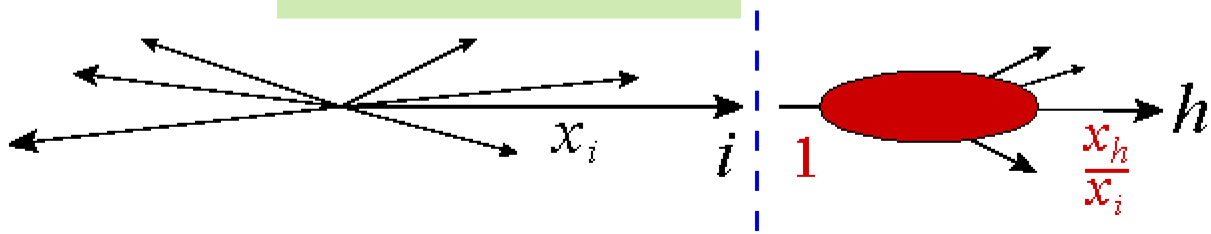
- Factorize on single parton states, similar to production of identified hadrons h with momentum $p_h = x \times Q/2$

$$\frac{d\sigma(e^+e^- \to h + X)}{dx} = \sum_{i=q,g} \int_0^1 dx_i \underbrace{\frac{d\sigma(e^+e^- \to i + X)}{dx_i}} \underbrace{D_{i\to h}(x/x_i)}$$

…, Mitov, Moch, Vogt, 2006
Moch, Vogt, 0709.3899,
Almasy, Moch, Vogt, 1107.2263

perturbative hard function, computed to NNLO + evolution

nonperturbative fragmentation function

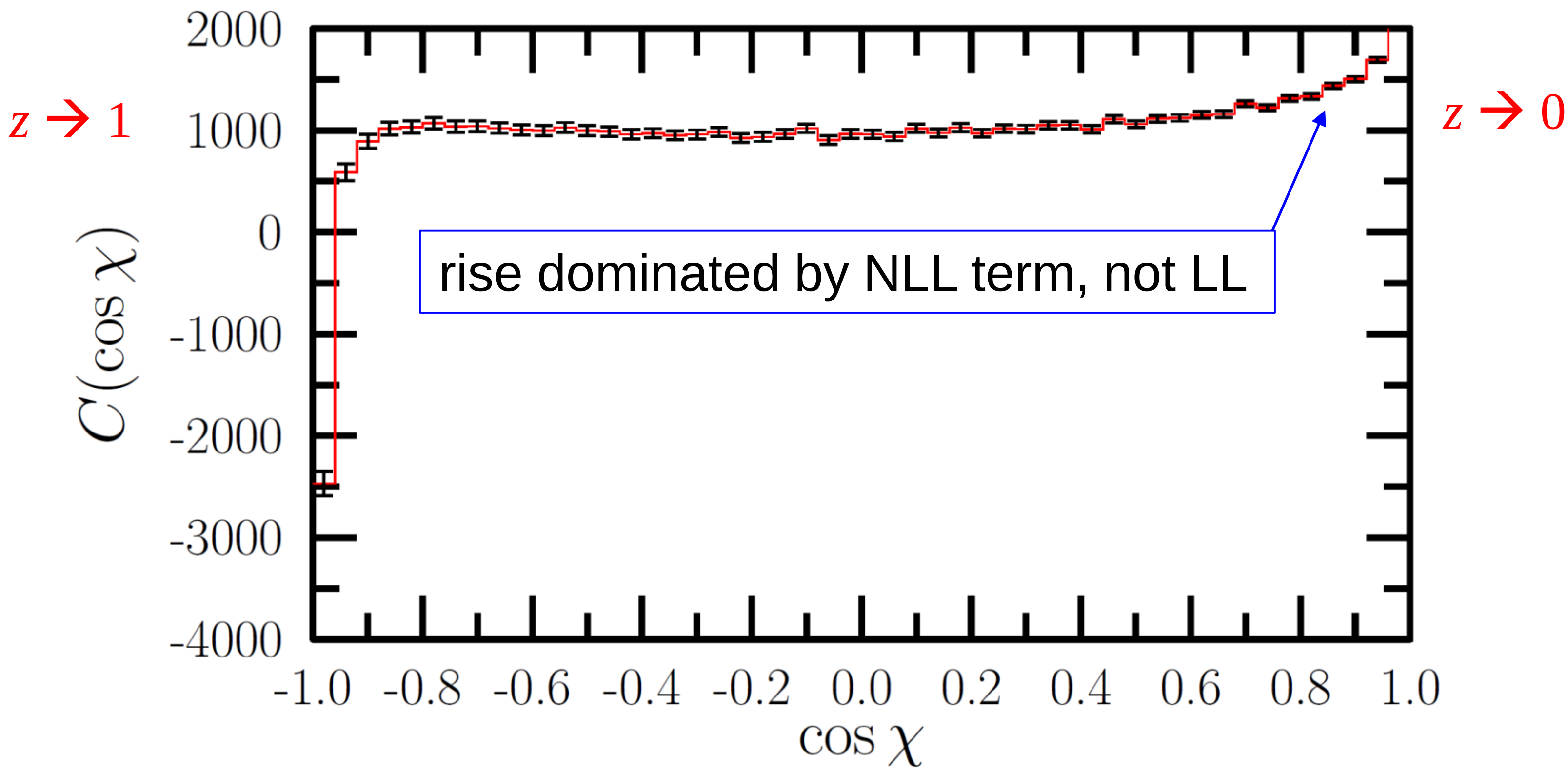


Beyond LL (cont.)

- **SCET factorization –** reuse hard function
- Replace nonperturbative fragmentation function with a perturbative jet function J which includes the small angle EEC measurement
- Computed J to $O(\alpha_s)$ so far → NLL accuracy
- Reproduce the coefficient of $(\ln z)^0/z$ in the fixed order NLO computation
- In progress: NLL resummation and eventually NNLL resummation

NNLO coefficient

Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, 1603.08927



Amazingly flat away from endpoints!

Conclusions

- Analytical results possible at NLO in QCD for at least one event shape in e^+e^- annihilation, the EEC.
- Transcendental structure no worse than for N=4 SYM, but rational functions considerably more complicated.
- Moments are linear combinations of zeta values.
- Limiting values of use in checking soft-gluon resummation for $z \to 1$ – and beyond leading power when available
  Moult, Stewart, Vita, Zhu, 1804.04665
  and collinear resummation beyond LL for $z \to 0$
- May eventually lead to more precise value of α_s, as well as more precise jet substructure understanding at LHC.
- Only meets **gravity** through holography, AdS/CFT (sorry!)

Extra Slides