Five Loops in $\mathcal{N} = 8$ Supergravity

Alex Edison

with

Zvi Bern, John Joseph Carrasco, Wei Ming Chen, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, and Mao Zeng

1804.09311, ongoing

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

Outline

Setup

Prediction from Symmetries

Why are we interested in five loops?

Prediction from Symmetries

Why are we interested in five loops?

- Supersymmetry arguments predict critical dimension for $\mathcal{N} = 8$ SUGRA at five loops to be $D_c = \frac{24}{5}$.
  (Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove; Green and Björnsson; Bossard, Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger ; Bossard, Howe, Stelle, Vanhove)

Prediction from Symmetries

Why are we interested in five loops?

- Supersymmetry arguments predict critical dimension for $\mathcal{N} = 8$ SUGRA at five loops to be $D_c = \frac{24}{5}$.
  (Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove; Green and Björnsson; Bossard, Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger ; Bossard, Howe, Stelle, Vanhove)
- Predicted divergence in $D_c = 4$ comes at seven loops from same operator.

Prediction from Symmetries

Why are we interested in five loops?

- Supersymmetry arguments predict critical dimension for $\mathcal{N} = 8$ SUGRA at five loops to be $D_c = \frac{24}{5}$.
  (Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove; Green and Björnsson; Bossard, Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger ; Bossard, Howe, Stelle, Vanhove)
- Predicted divergence in $D_c = 4$ comes at seven loops from same operator.
- Explicit calculations show "enhanced cancellations" for lower SUSY and lower loops. (Bern, Davies, Dennen; Bern, Davies, Dennen, Huang)
  Enhanced Cancellations: Cancellations of divergences when combining diagrams into amplitude, that are not consequences of standard symmetry arguments.

Generalized Double Copy

"Traditional" BCJ double-copy:

1. Impose color jacobi relations on corresponding numerators
2. Match unitarity cuts
3. Numerators can be "squared" to get gravity

Generalized Double Copy

"Traditional" BCJ double-copy:

1. Impose color jacobi relations on corresponding numerators
2. Match unitarity cuts
3. Numerators can be "squared" to get gravity

Generalized Double Copy:

1. ~~Impose color jacobi relations on corresponding numerators~~
2. Build some local,diagrammatic sYM representation. Better power counting still helps on the gravity side.
3. "Square numerators". Integrand is not a gravity integrand, but close.
4. Correct integrand with contact terms.

See Radu Roiban's talk from earlier, and John Joseph Carrasco's talks from QCD Meets Gravity 2017 and Amplitudes 2018.

Large ℓ Expansion

Large loop momentum implemented by small external momenta expansion:

$$p_i \to \epsilon p_i$$

Large ℓ Expansion

Large loop momentum implemented by small external momenta expansion:

$$p_i \to \epsilon p_i$$

Each diagram can be expanded to target order in ϵ to extract log divergent terms in a given dimension.

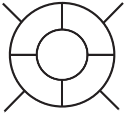
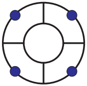
Large ℓ Expansion

Large loop momentum implemented by small external momenta expansion:

$$p_i \to \epsilon p_i$$

Each diagram can be expanded to target order in ϵ to extract log divergent terms in a given dimension.

$$D = \frac{22}{5}: \ \mathcal{I}_k \to \epsilon^6 \mathcal{I}_k^{(6)} \qquad\qquad D = \frac{24}{5}: \ \mathcal{I}_k \to \epsilon^8 \mathcal{I}_k^{(8)}$$

$\mathcal{I}_k = N$  $\to (s^2 + t^2 + u^2)^2 \epsilon^8 \left(N^{(8)} + \frac{1}{\ell^2} N^{(6)} + \dots \right)$ 

 $=$ dots $\leftrightarrow$ doubled propagators $= \frac{1}{(\ell^2)^2}$

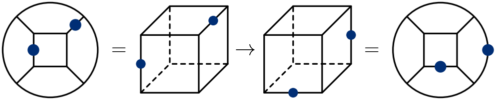
UV Diagram Symmetries and IBP Relations

Two Classes of Symmetries

1. Isomorphisms: changing location of doubled propagators
2. Automorphisms: fixing doubled propagators in place

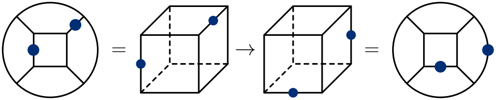
Isomorphisms

- Graph/propagator relabelings that change the location of dots



Isomorphisms

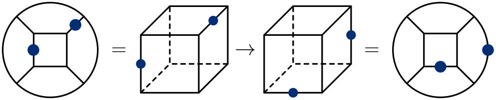
- Graph/propagator relabelings that change the location of dots



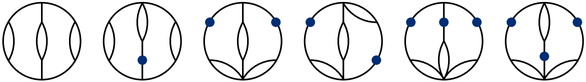
  - Orbit of automorphism group of undotted graph w.r.t. dots.
  - Induces non-trivial mapping of numerators

Isomorphisms

- Graph/propagator relabelings that change the location of dots

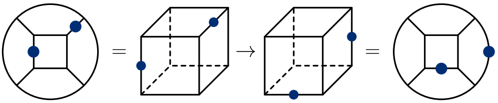


  - Orbit of automorphism group of undotted graph w.r.t. dots.
  - Induces non-trivial mapping of numerators
- Integrals are invariant under exchanging adjacent bubbles and propagators



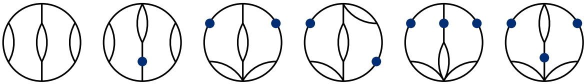
Isomorphisms

- Graph/propagator relabelings that change the location of dots



  - Orbit of automorphism group of undotted graph w.r.t. dots.
  - Induces non-trivial mapping of numerators

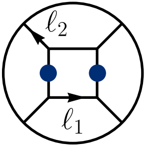
- Integrals are invariant under exchanging adjacent bubbles and propagators

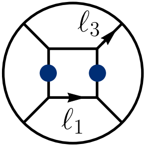


Pick canonical configuration under both of these actions

Automorphisms

- Relabelings that leave dots fixed, but relabel other propagators

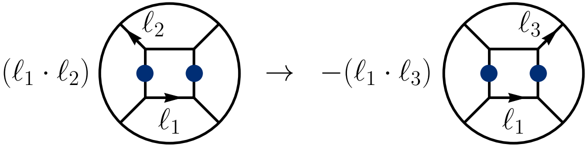
$(\ell_1 \cdot \ell_2)$ 

$\rightarrow \quad -(\ell_1 \cdot \ell_3)$ 

- Stabilizer group w.r.t. dots

Automorphisms

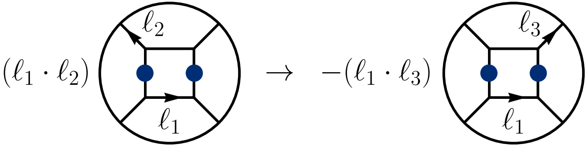
- Relabelings that leave dots fixed, but relabel other propagators



- Stabilizer group w.r.t. dots
- Almost always leads to equalities between irreducible numerators, or relations between contacts

Automorphisms

- Relabelings that leave dots fixed, but relabel other propagators



- Stabilizer group w.r.t. dots
- Almost always leads to equalities between irreducible numerators, or relations between contacts
- Solve these as equations in conjunction with integration by parts relations.

$\mathfrak{sl}(L)$ Relations

Use two loop example so that it actually fits on slide:

$\mathfrak{sl}(L)$ Relations

Use two loop example so that it actually fits on slide:

- Generic vacuum integral

$$V_{A,B,C} = \int \frac{d^D\ell_1}{(2\pi)^2}\frac{d^D\ell_2}{(2\pi)^2}\frac{1}{[(\ell_1)^2 - m^2]^A[(\ell_2)^2 - m^2]^B[(\ell_1 - \ell_2)^2 - m^2]^C}$$

- Only interested in logarithmically divergent integrals, $A + B + C = D$.

$\mathfrak{sl}(L)$ Relations

Use two loop example so that it actually fits on slide:

- Generic vacuum integral

$$V_{A,B,C} = \int \frac{d^D\ell_1}{(2\pi)^2}\frac{d^D\ell_2}{(2\pi)^2}\frac{1}{[(\ell_1)^2 - m^2]^A[(\ell_2)^2 - m^2]^B[(\ell_1-\ell_2)^2 - m^2]^C}$$

- Only interested in logarithmically divergent integrals, $A + B + C = D$.
- Want to generate relations between UV poles of the integrals [1]. Start by looking at

$$0 = \int \frac{d^D\ell_1}{(2\pi)^2}\frac{d^D\ell_2}{(2\pi)^2}\frac{\partial}{\partial \ell_i^\mu}\frac{\Omega_{ij}\ell_j^\mu}{\prod_j P_j}$$

where $\Omega \in \mathfrak{gl}(L) \sim$ infinitesimal relabeling of loop variables.

[1] Bern, Enciso, Parra-Martinez, and Zeng 2017

$\mathfrak{sl}(L)$ Relations

- Trace part: $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$0 = -4\epsilon V_{A,B,C} - 10m^2(V_{A+1,B,C} + V_{A,B+1,C} + V_{A,B,C+1})$$

Probes scaling weight, mixes IR and UV, not what we are interested in.

$\mathfrak{sl}(L)$ Relations

- Trace part: $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$0 = -4\epsilon V_{A,B,C} - 10m^2(V_{A+1,B,C} + V_{A,B+1,C} + V_{A,B,C+1})$$

Probes scaling weight, mixes IR and UV, not what we are interested in.

- Traceless part, $\mathfrak{sl}(L)$ e.g. $\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{aligned} 0 =& (-2A + 2B)V_{A,B,C} - 2CV_{A-1,B,C+1} + 2CV_{A,B-1,C+1} \\ &+ m^2(-2AV_{A+1,B,C} + 2BV_{A,B+1,C}) \end{aligned}$$

Setting $m \to 0$ decouples UV and IR. Generates relations between UV log divergent integrals, exactly what we want.

Results

Problem Statistics

- Gravity Integrand Diagrams: (Recall Roiban's talk)

Contact Level	Max	N^1	N^2	N^3	N^4	N^5	N^6
Num. Diagrams	752	0	9007	17479	22931	20657	13071

Most of the contacts are actually zero. Many expand to zero.

Problem Statistics

- Gravity Integrand Diagrams: (Recall Roiban's talk)

Contact Level	Max	N^1	N^2	N^3	N^4	N^5	N^6
Num. Diagrams	752	0	9007	17479	22931	20657	13071

  Most of the contacts are actually zero. Many expand to zero.

- Integration System:

  3 million topology relations

Problem Statistics

- Gravity Integrand Diagrams: (Recall Roiban's talk)

Contact Level	Max	N^1	N^2	N^3	N^4	N^5	N^6
Num. Diagrams	752	0	9007	17479	22931	20657	13071

Most of the contacts are actually zero. Many expand to zero.

- Integration System:

3 million topology relations
4.5 million integral equations
850,000 integrals

Problem Statistics

- Gravity Integrand Diagrams: (Recall Roiban's talk)

Contact Level	Max	N^1	N^2	N^3	N^4	N^5	N^6
Num. Diagrams	752	0	9007	17479	22931	20657	13071

Most of the contacts are actually zero. Many expand to zero.

- Integration System:

3 million topology relations
4.5 million integral equations
850,000 integrals
$\sim$ QCD β-function

Problem Statistics

- Gravity Integrand Diagrams: (Recall Roiban's talk)

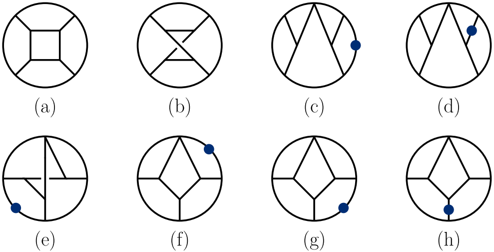
Contact Level	Max	N^1	N^2	N^3	N^4	N^5	N^6
Num. Diagrams	752	0	9007	17479	22931	20657	13071

Most of the contacts are actually zero. Many expand to zero.

- Integration System:

3 million topology relations
4.5 million integral equations
850,000 integrals
$\sim$ QCD β-function
8 relevant master integrals

SUGRA Master Integrals



The Result

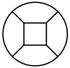
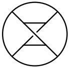
- Constructing a gravity integrand – Years

The Result

- Constructing a gravity integrand – Years
- Solving the IBP system – Seven days on the cluster

The Result

- Constructing a gravity integrand – Years
- Solving the IBP system – Seven days on the cluster
- Get a two-term result

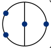
$$\mathcal{M}_4^{(5)}\Big|_{\text{leading UV}}^{D=24/5} \propto \frac{1}{48}$$  $$+ \frac{1}{16}$$ 

A Useful Pattern

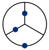
$\mathcal{N}=4$ super-Yang–Mills

 $=$ dots $\leftrightarrow$ doubled propagators $= \frac{1}{(\ell^2)^2}$

$\mathcal{A}_4^{(1)}\Big|_{\text{leading}} = g^4\mathcal{K}_{\text{YM}}\left(N_c(\tilde{f}^{a_1a_2b}\tilde{f}^{ba_3a_4}+\tilde{f}^{a_2a_3b}\tilde{f}^{ba_4a_1})-3B^{a_1a_2a_3a_4}\right)$  ,

$\mathcal{A}_4^{(2)}\Big|_{\text{leading}} = -\,g^6\,\mathcal{K}_{\text{YM}}\Big[F^{a_1a_2a_3a_4}\Big(N_c^2$  $+\,48\Big(\frac{1}{4}$  $+\frac{1}{4}$  $\Big)\Big)$

$+\,48\,N_c\,G^{a_1a_2a_3a_4}\Big(\frac{1}{4}$  $+\frac{1}{4}$  $\Big)\Big]$,

$\mathcal{A}_4^{(3)}\Big|_{\text{leading}} = 2\,g^8\,\mathcal{K}_{\text{YM}}\,N_cF^{a_1a_2a_3a_4}\Big(N_c^2$  $+\,72\Big(\frac{1}{6}$  $+\frac{1}{2}$  $\Big)\Big)$,

$\mathcal{A}_4^{(4)}\Big|_{\text{leading}} = -6\,g^{10}\,\mathcal{K}_{\text{YM}}\,N_c^2F^{a_1a_2a_3a_4}\Big(N_c^2$  $+\,48\Big(\frac{1}{4}$  $+\frac{1}{2}$  $+\frac{1}{4}$  $\Big)\Big)$,

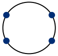
$\mathcal{A}_4^{(5)}\Big|_{\text{leading}} = \frac{144}{5}g^{12}\mathcal{K}_{\text{YM}}\,N_c^3\,F^{a_1a_2a_3a_4}\Big(N_c^2$  $+\,48\Big(\frac{1}{4}$  $+\frac{1}{2}$  $+\frac{1}{4}$  $\Big)\Big)$,

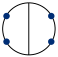
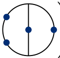
$A_4^{(6)}\Big|_{\text{leading}} = -120g^{14}\mathcal{K}_{\text{YM}}\,F^{a_1a_2a_3a_4}N_c^6\Big(\frac{1}{2}$  $+\frac{1}{4}(\ell_1+\ell_2)^2$  $-\frac{1}{20}$  $\Big)$

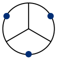
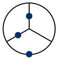
$+\,\mathcal{O}(N_c^4)$,

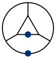
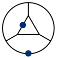
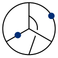
Green, Schwarz, Brink; Bern, Rozowsky, Yan; Bern, Dixon, Dunbar, Perelstein, Rozowsky; Carrasco, Dixon, Johansson, Roiban; Bern, Carrasco, Dixon, Douglas, von Hippel, Johansson

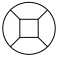
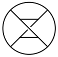
$\mathcal{N} = 8$ SUGRA

$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^4$  ,

$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2)\left(\frac{1}{4}\right.$  $+\frac{1}{4}$  $\left.\right)$,

$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^8 stu\left(\frac{1}{6}\right.$  $+\frac{1}{2}$  $\left.\right)$,

$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2\left(\frac{1}{4}\right.$  $+\frac{1}{2}$  $+\frac{1}{4}$  $\left.\right)$,

$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2\left(\frac{1}{48}\right.$  $+\frac{1}{16}$  $\left.\right)$,

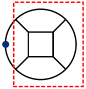
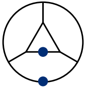
Green, Schwarz, Brink; Bern, Dixon, Dunbar, Perelstein , Rozowsky; Carrasco, Dixon, Johansson, Roiban; Bern, Carrasco, Chen, AE, Johansson, Roiban, Parra-Martinez, Zeng

Relations Between Loops in sYM and SUGRA

We noticed some interesting patterns between loops for the UV expressions.

In sYM, we see

- Planar diagrams are related between loops:

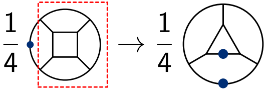
$\frac{1}{4}$  $\rightarrow$ $\frac{1}{4}$ 

Relations Between Loops in sYM and SUGRA

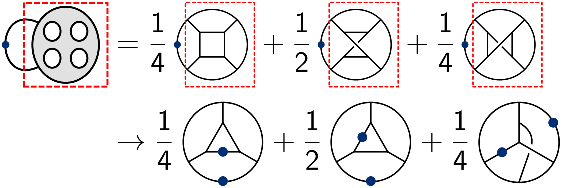
We noticed some interesting patterns between loops for the UV expressions.

In sYM, we see

- Planar diagrams are related between loops:



- As are the subleading color pieces:

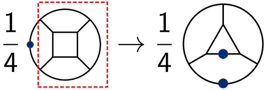


Relations Between Loops in sYM and SUGRA

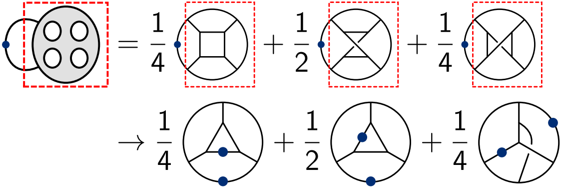
We noticed some interesting patterns between loops for the UV expressions.

In sYM, we see

- Planar diagrams are related between loops:



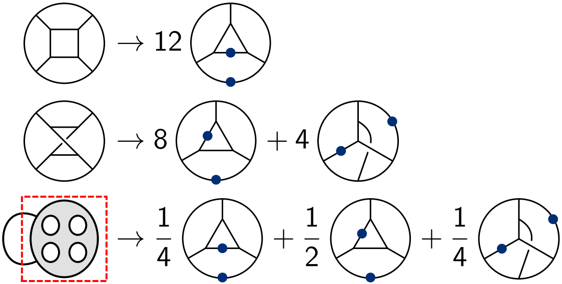
- As are the subleading color pieces:



Up to four loops, SUGRA diagrams are same as subleading color in sYM.

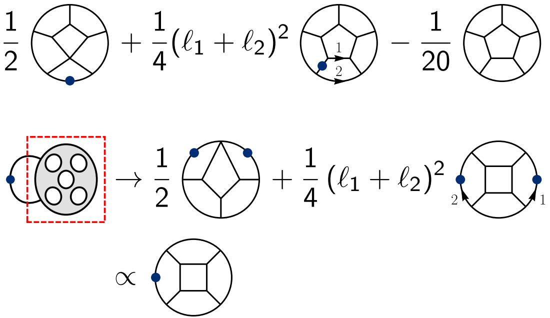
Relations Between Loops in sYM and SUGRA

Five Loop SUGRA first difference between sYM and SUGRA



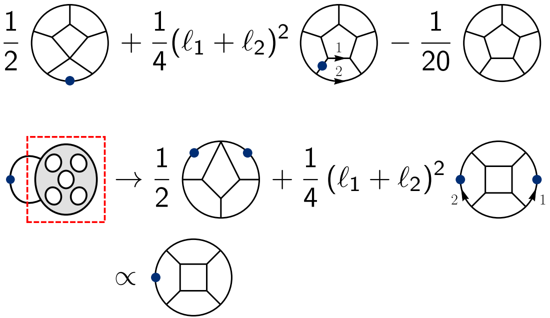
Relations Between Loops in sYM and SUGRA

- Works even when numerators present e.g. six → five loops, but need IBPs:



Relations Between Loops in sYM and SUGRA

- Works even when numerators present e.g. six → five loops, but need IBPs:

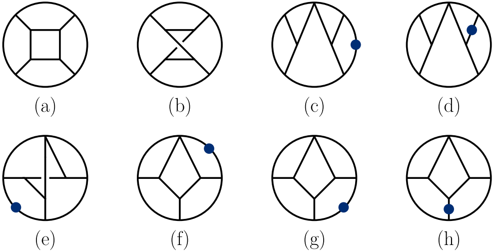


- Can be continued all the way to one loop → boxes or better

No One Loop Triangles

No one loop triangles occurs in both sYM and SUGRA for all data we have:

- SUGRA master integrals at five loops:



$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} \propto \frac{1}{48}$  $+ \frac{1}{16}$ 

Relative coefficients are just combinatoric factors.

Closing Comments

- Found some useful consistency relations between leading behavior at different loop levels

Closing Comments

- Found some useful consistency relations between leading behavior at different loop levels
- Can use to relations to help target unitarity cuts, skipping lots of work for six and seven loops

Closing Comments

- Found some useful consistency relations between leading behavior at different loop levels
- Can use to relations to help target unitarity cuts, skipping lots of work for six and seven loops
- Would be interesting to see if similar relations present in different theories, e.g. $\mathcal{N} = 4, 5$ SUGRA, $\mathcal{N} = 1, 2$ sYM, QCD

Closing Comments

- Found some useful consistency relations between leading behavior at different loop levels
- Can use to relations to help target unitarity cuts, skipping lots of work for six and seven loops
- Would be interesting to see if similar relations present in different theories, e.g. $\mathcal{N} = 4, 5$ SUGRA, $\mathcal{N} = 1, 2$ sYM, QCD
- Recent work gives evidence that enhanced cancellations happen specifically in $D = 4$ (Herrmann and Trnka 2018). It would be very interesting to check for these cancellations for $L = 7$, $D = 4$ where symmetry arguments predict a divergence

Thank You