Decoding the Cosmological Polytopes

Cosmological Observables and the Flat-Space Physics

Paolo Benincasa

Niels Bohr International Academy

QCD meets Gravity 2018 – Nordita

based on: N. Arkani-Hamed, P.B., A. Postnikov – 1709.02813;
N. Arkani-Hamed, P.B. – 1811.01125;
P.B. – 1811.02515;
N. Arkani-Hamed, P.B. – work in progress

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Amplitudes meet Cosmology

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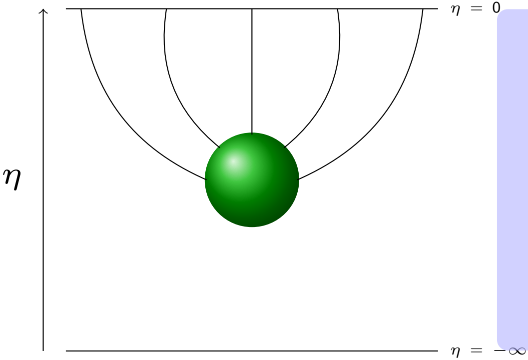
Late-time correlators



Observations at present time

Spatial averages

Accelerated expansion

Universe ∞ large

Observables and boundary data

- Limited control on the correlators:

  Interest for: non-gaussianities

  prediction of existence of particles during inflation..

- Consistency conditions not well understood
- Not general rules for extracting fundamental physics
- Which are the invariant properties they ought to satisfy

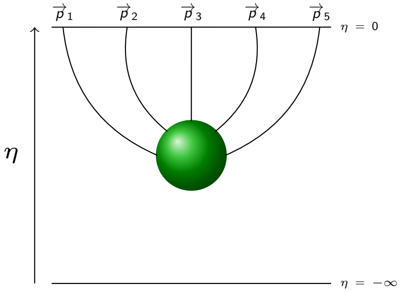
  in order to be consistent with a unitary evolution

  in cosmological space-times?

What can we say?

Observables and boundary data

The wavefunction of the universe $\Psi[\phi]$ encodes the correlations



$$\Psi[\phi_0] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \text{free solution} \\ \varphi(\eta = 0) = 0 \\ \varphi(\eta \longrightarrow -\infty) = 0 \end{cases}$$

$$\langle \prod_{i=1}^{n} \phi(p_i) \rangle = \mathcal{N} \int D\phi \prod_{i=1}^{n} \phi(p_i) \, |\Psi[\phi]|^2$$

Observables and boundary data

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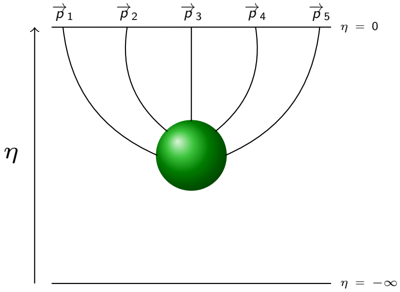

$$\Psi[\phi_0] = \int D\varphi \, e^{iS[\phi_0+\varphi]}$$

$$\begin{cases} \phi_0 = \phi(p) e^{iE\eta}, \; E \equiv |\vec{p}| \\ \varphi(\eta = 0) = 0 \\ \varphi(\eta \longrightarrow -\infty) = 0 \end{cases}$$

$$S[\phi] = \int_{-\infty}^{0} d\eta \int d^d x \left[\frac{1}{2}(\partial\phi)^2 - \sum_{k\geq 3} \frac{\lambda(\eta)}{k!}\phi^k\right], \quad \lambda(\eta) = \int_0^{\infty} d\varepsilon \, e^{i\epsilon\eta} \tilde{\lambda}_k(\varepsilon)$$

Observables and boundary data

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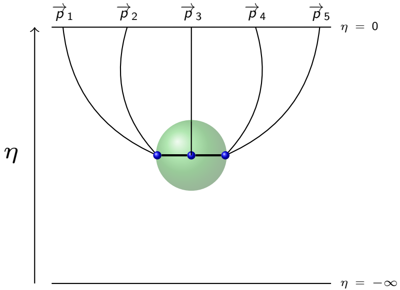

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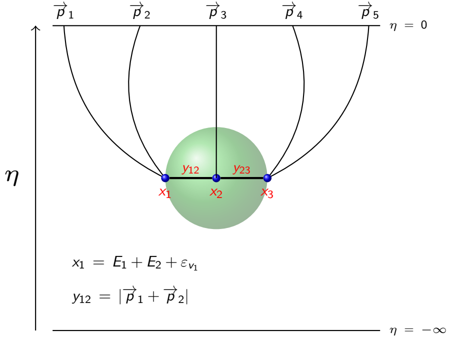
Observables and boundary data in cosmology

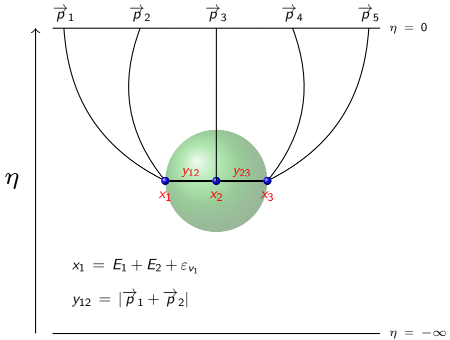
General properties of the wavefunction



$$x_1 = E_1 + E_2 + \varepsilon_{v_1}$$

$$y_{12} = |\vec{p}_1 + \vec{p}_2|$$

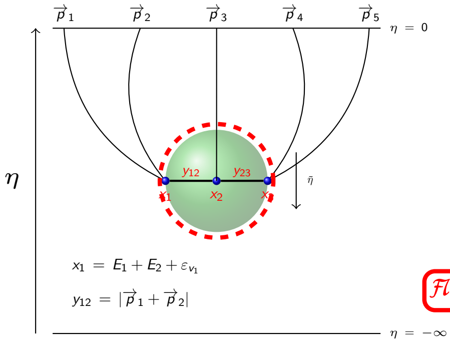
General properties of the wavefunction



- Function of sum of energies
- Singularities ≡ sum of energies corresponding to each subgraph

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1+x_2+x_3)(x_1+y_{12})(x_1+x_2+y_{23})(y_{12}+x_2+y_{23})(y_{12}+x_2+x_3)(y_{23}+x_3)}$$

General properties of the wavefunction



- Function of sum of energies
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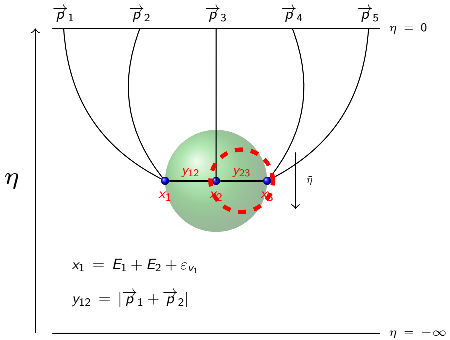
$$\sim \frac{\mathcal{A}_3}{x_1 + x_2 + x_3}$$

Flat-space scattering amplitude!

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1 + x_2 + x_3)(x_1 + y_{12})(x_1 + x_2 + y_{23})(y_{12} + x_2 + y_{23})(y_{12} + x_2 + x_3)(y_{23} + x_3)}$$

Observables and boundary data in cosmology

General properties of the wavefunction



- Function of sum of energies
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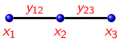
$$\sim \frac{\hat{\psi} \times \mathcal{A}_2}{y_{12} + x_2 + x_3}$$

Factorization: $\hat{\psi} \otimes A$

$$\psi_3 = \frac{x_1 + y_{12} + 2x_2 + y_{23} + x_3}{(x_1 + x_2 + x_3)(x_1 + y_{12})(x_1 + x_2 + y_{23})(y_{12} + x_2 + y_{23})(y_{12} + x_2 + x_3)(y_{23} + x_3)}$$

Observables and boundary data in cosmology

General properties of the wavefunction



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Observables and boundary data in cosmology

General properties of the wavefunction

A boundary recursive formula

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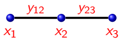
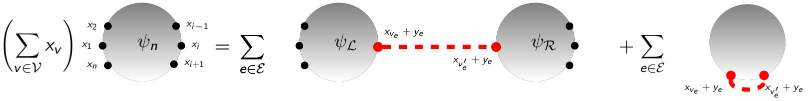
- Singularities $\equiv$ sum of energies corresponding to each subgraph





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General properties of the wavefunction

A boundary recursive formula

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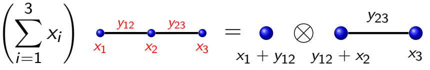
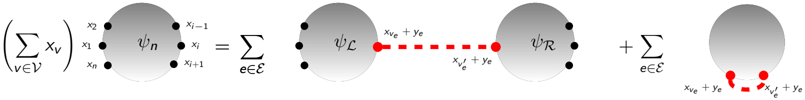


$$\frac{1}{x_1 + y_{12}} \otimes \frac{1}{(y_{12} + x_2 + x_3)(y_{12} + x_2 + y_{23})(y_{23} + x_3)}$$



General properties of the wavefunction

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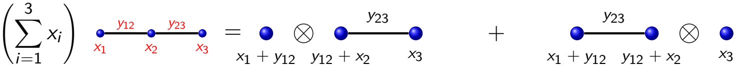
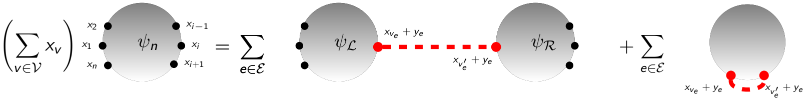


$$\frac{1}{x_1+y_{12}} \otimes \frac{1}{(y_{12}+x_2+x_3)(y_{12}+x_2+y_{23})(y_{23}+x_3)} + \frac{1}{(x_1+x_2+y_{12})(x_1+y_{12})(y_{12}+x_2+y_{23})} \otimes \frac{1}{(y_{23}+x_3)}$$



In this talk

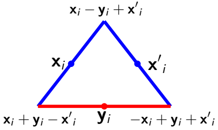
The wavefunction of the universe from first principles:
Cosmological Polytopes

Form cosmology to flat-space physics and back

Taking averages:
Late-time correlators from the Cosmological Polytopes

Decoding the Cosmological Polytopes:

The Wavefunction of the Universe



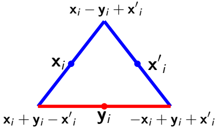
n_e triangles

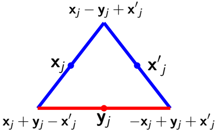
$$\mathbb{P}^{3n_e-1}$$

$$\mathcal{Y} = \sum_{v\in\mathcal{S}} x_v \mathbf{X}_v + \sum_{e\in\mathcal{S}} y_e \mathbf{Y}_e$$

$$\left\{\mathbf{x}_i - \mathbf{y}_i + \mathbf{x'}_i,\ \mathbf{x}_i + \mathbf{y}_i - \mathbf{x'}_i, -\mathbf{x}_i + \mathbf{y}_i + \mathbf{x'}_i\right\}_{i=1}^{n_e}$$

Cosmological Polytopes





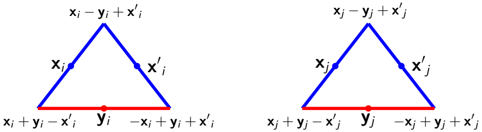
2 triangles

$$\mathbb{P}^5$$

$$\mathcal{Y} = \sum_{v \in \mathcal{S}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{S}} y_e \mathbf{Y}_e$$

$$\{\mathbf{x}_i - \mathbf{y}_i + \mathbf{x}'_i,\ \mathbf{x}_i + \mathbf{y}_i - \mathbf{x}'_i,\ -\mathbf{x}_i + \mathbf{y}_i + \mathbf{x}'_i,\ \mathbf{x}_j - \mathbf{y}_j + \mathbf{x}'_j,\ \mathbf{x}_j + \mathbf{y}_j - \mathbf{x}'_j,\ -\mathbf{x}_j + \mathbf{y}_j + \mathbf{x}'_j\}$$

Cosmological Polytopes



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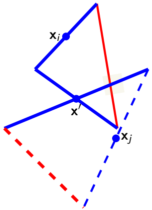
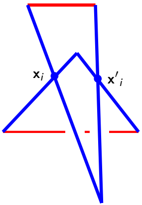


$\mathbf{x'}_i = \mathbf{x'}_j \equiv \mathbf{x}$

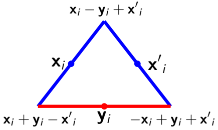
$\mathbb{P}^4$

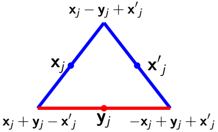


$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x'}_i = \mathbf{x'}_j$

$\mathbb{P}^3$

Cosmological Polytopes



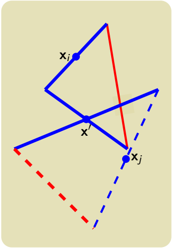


2 triangles

$$x'_i = x'_j \equiv x' \Rightarrow \mathbb{P}^4$$

$$\mathcal{Y} = (x_1, y_1, x', x_2, y_2)$$

$$\{x_i - y_i + x',\ x_i + y_i - x',\ -x_i + y_i + x',\ x_j - y_j + x',\ x_j + y_j - x',\ -x_j + y_j + x'\}$$



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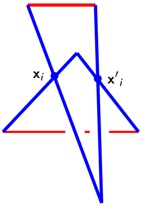

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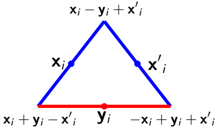

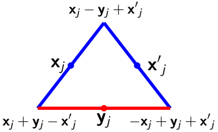

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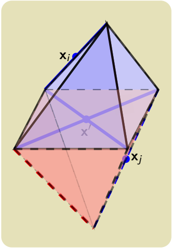
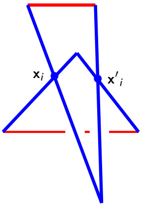


$x'_i x_{\mathcal{I}} = x'_j x_{\mathcal{J}} = x$

$\mathbb{P}^4$



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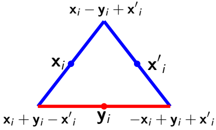

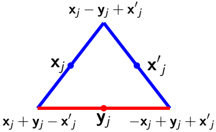

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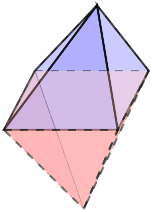
$$\left.\begin{array}{l} \mathbf{x}_i = \mathbf{x}_j \equiv \mathbf{x} \\ \mathbf{x}'_i = \mathbf{x}'_j \equiv \mathbf{x}' \end{array}\right\} \Rightarrow \mathbb{P}^3$$

$$\mathcal{Y} = (x, y_1, x', y_2)$$

$$\{\mathbf{x} - \mathbf{y}_i + \mathbf{x}', \mathbf{x} + \mathbf{y}_i - \mathbf{x}', -\mathbf{x} + \mathbf{y}_i + \mathbf{x}', \mathbf{x} - \mathbf{y}_j + \mathbf{x}', \mathbf{x} + \mathbf{y}_j - \mathbf{x}', -\mathbf{x} + \mathbf{y}_j + \mathbf{x}'\}$$



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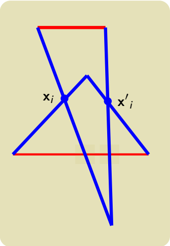

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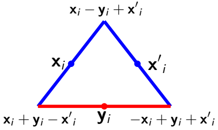
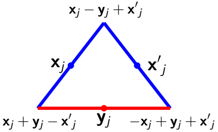
$\mathbb{P}^3$

Cosmological Polytopes





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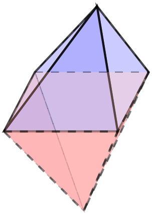

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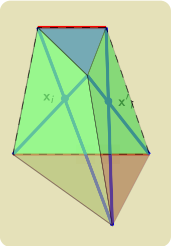
$\mathbb{P}^4$



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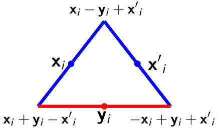
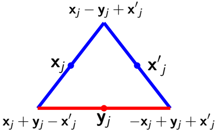
Cosmological Polytopes



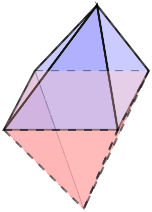


2 triangles

$$\mathbb{P}^5$$

$$\mathcal{Y} = \sum_{v \in \mathcal{S}} x_v \mathbf{X}_v + \sum_{e \in \mathcal{S}} y_e \mathbf{Y}_e$$

$$\Omega = \int \prod_{k=1}^{\nu} \frac{dc_k}{c_k - i\varepsilon_k} \delta^{(N)}\left(\mathcal{Y} - \sum_{k=1}^{\nu} c_k \mathbf{V}^{(k)}\right)$$



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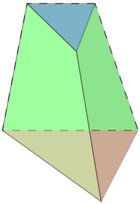

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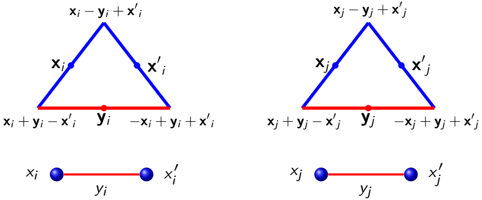
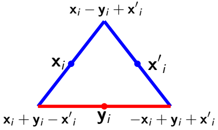
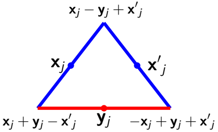


2 triangles

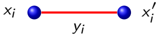
$$\mathbb{P}^5 \equiv \mathbb{P}^{n_v + n_e - 1}$$

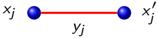
$$\mathcal{Y} = \sum_{v \in n_v} x_v \mathbf{X}_v + \sum_{e \in n_e} y_e \mathbf{Y}_e$$





2 triangles





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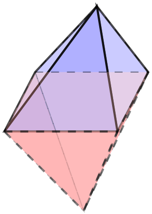
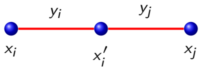
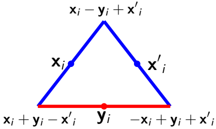


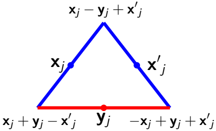


$$\mathbb{P}^4 \equiv \mathbb{P}^{n_v + n_e - 1}$$

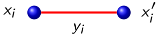
$$\mathbf{x'}_i = \mathbf{x'}_j$$

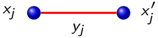
$\mathbb{P}^4$





2 triangles





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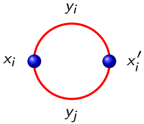

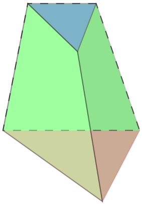

$$\mathbb{P}^3 \equiv \mathbb{P}^{n_v + n_e - 1}$$

$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$

$\mathbb{P}^3$

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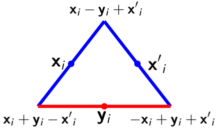
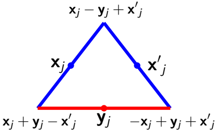
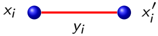
Cosmological Polytopes

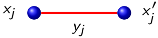




2 triangles

$$\mathbb{P}^{n_v+n_e-1}$$





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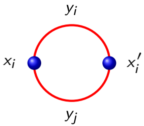
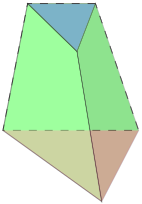




$$\Omega = \frac{\prod_{v,e} dx_v dy_e}{\text{Vol}\{GL(1)\}} \Psi_{\mathcal{G}}(x_v, y_e)$$

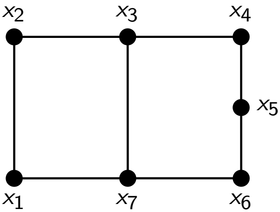
$\mathbf{x}_i = \mathbf{x}_j, \quad \mathbf{x}'_i = \mathbf{x}'_j$

$\mathbb{P}^3$

Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow$ 0

$$\mathcal{W} \cdot \mathbf{V} = 0$$



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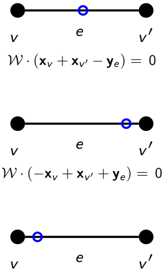
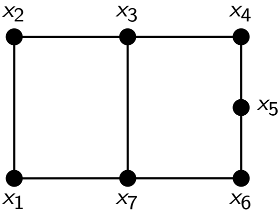


$\mathcal{W} \cdot (\mathbf{x}_v + \mathbf{x}_{v'} - \mathbf{y}_e) = 0$

$\mathcal{W} \cdot (-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$

$\mathcal{W} \cdot (\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$



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Cosmological Polytopes: The Face Structure

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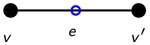
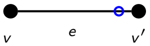
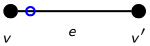


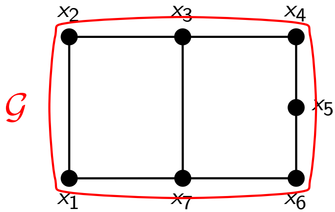
$\mathcal{W}\cdot(\mathbf{x}_v+\mathbf{x}_{v'}-\mathbf{y}_e) = 0$



$\mathcal{W}\cdot(-\mathbf{x}_v+\mathbf{x}_{v'}+\mathbf{y}_e) = 0$



$\mathcal{W}\cdot(\mathbf{x}_v-\mathbf{x}_{v'}-\mathbf{y}_e) = 0$



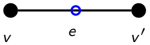
$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

$$\mathcal{W}\cdot\mathbf{V} = 0$$



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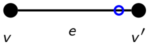
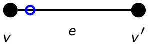
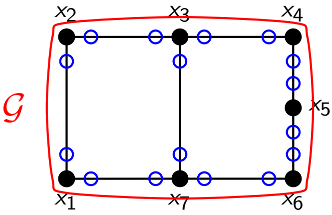


$\mathcal{W}\cdot(-\mathbf{x}_v + \mathbf{x}_{v'} + \mathbf{y}_e) = 0$



$\mathcal{W}\cdot(\mathbf{x}_v - \mathbf{x}_{v'} - \mathbf{y}_e) = 0$



$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Vertices $(2n_e)$

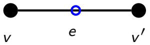
$$\{\mathbf{x}_i - \mathbf{x}_j + \mathbf{y}_{ij},$$

$$-\mathbf{x}_i + \mathbf{x}_j + \mathbf{y}_{ij}\}$$

Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

$$\mathcal{W}\cdot\mathbf{V} = 0$$



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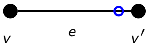
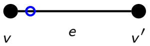


$\mathcal{W}\cdot(-\mathbf{x}_v+\mathbf{x}_{v'}+\mathbf{y}_e) = 0$



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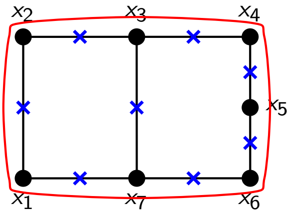
$\mathcal{G}$



$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

Vertices $(2n_e)$

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Cosmological Polytopes: The Face Structure

Faces (boundaries) $\Longleftrightarrow$ Subgraphs total energy $\longrightarrow 0$

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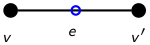

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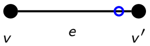

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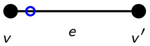

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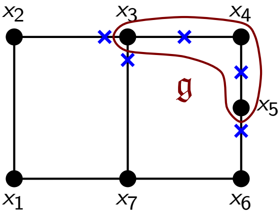


$$y_{23}+y_{37}+y_{56}+\sum_{j=3}^{5}x_j \longrightarrow 0$$

Cosmological Polytopes & Wavefunction: A dictionary

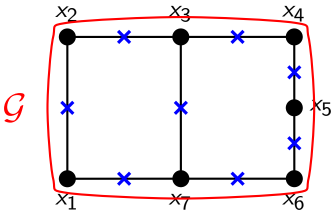
Cosmological Polytope $\mathcal{P}$	Universe Wavefunction Ψ
Canonical form	Ψ
Triangulations	Representations for Ψ
Boundaries	Residues of Ψ
Volume preserving transformations	Symmetries of Ψ

Decoding the Cosmological Polytopes:

The Flat-Space Physics

Scattering Facet: Emergent Unitarity

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



Scattering Facet: Emergent Unitarity

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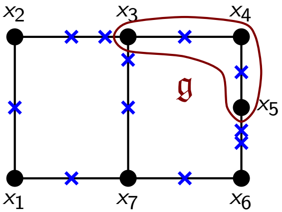
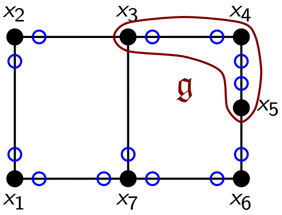


$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

Scattering Facet: Emergent Unitarity

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



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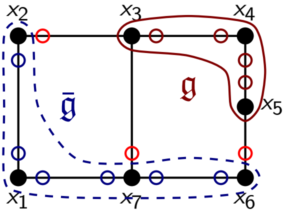

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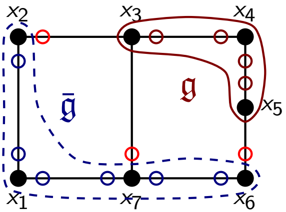

$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

$\circ \implies$ *Energy flow!*

$$\Omega = \left(\prod_{e \in \mathcal{E}} \frac{1}{2 y_e} \right) \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}]$$

cutting rules!

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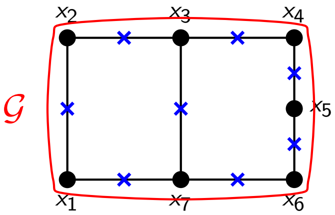
Scattering Facet: Emergent Lorentz Invariance

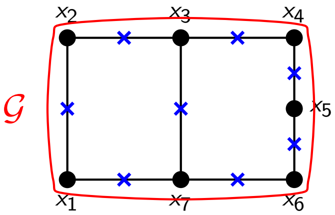
$$\sum_{j=1}^{7} x_j \longrightarrow 0$$



Scattering Facet: Emergent Lorentz Invariance

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$\Omega \sim \int_{\mathbb{R}^N} \prod_{j=1}^{\nu} \frac{dc_j}{c_j - i\varepsilon_j} \delta^{(N)}\left(\mathcal{Y} - \sum_{j=1}^{\nu} c_j \mathbf{V}^{(j)}\right)$$



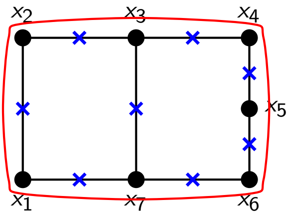
$$\mathbb{P}^{N-1}, \qquad N \equiv n_e + n_v - 1$$

$$\nu \equiv n_e + n_v - 1 + L$$

Scattering Facet: Emergent Lorentz Invariance

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$\mathcal{G}$



$$\Omega \sim \int_{\mathbb{R}^N} \prod_{j=1}^{\nu} \frac{dc_j}{c_j - i\varepsilon_j} \delta^{(N)} \left(\mathcal{Y} - \sum_{j=1}^{\nu} c_j \mathbf{V}^{(j)} \right)$$

$$\sim \int \prod_{j=1}^{L} \frac{dc_j}{\left(c_j - \frac{y_{e_j}}{2}\right)^2 - \left(\frac{y_{e_j}}{2} - i\varepsilon_j\right)^2} \times$$

$$\times \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} c_r - \frac{\mathfrak{y}_s}{2}\right)^2 - \left(\frac{y_s}{2} - i\varepsilon_s\right)^2}$$

$$\mathbb{P}^{N-1}, \qquad N \equiv n_e + n_v - 1$$

$$\nu \equiv n_e + n_v - 1 + L$$

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Scattering Facet: Emergent Lorentz Invariance

$$\Omega \sim \int \prod_{j=1}^{L} \frac{dc_j}{\left(c_j - \frac{y_{e_j}}{2}\right)^2 - \left(\frac{y_{e_j}}{2} - i\varepsilon_j\right)^2} \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} c_r - \frac{\mathfrak{y}_s}{2}\right)^2 - \left(\frac{y_s}{2} - i\varepsilon_s\right)^2}$$

$$\mathcal{I} \sim \int \prod_{j=1}^{L} d\vec{l}^{(j)} dl_0^{(j)} \frac{1}{(l_0^{(j)})^2 - (|\vec{l}^{(j)}| - i\varepsilon_j)^2} \prod_{s=1}^{n_e - L} \frac{1}{\left(\sum_r \sigma_{r_s} l_0^{(j)} - \mathfrak{p}_s\right)^2 - \left(|\vec{P}_s| - i\varepsilon_s\right)^2}$$

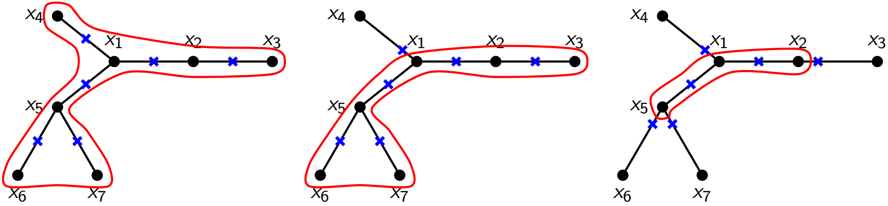
Lorentz invariant loop integrand!

$c_j \sim l_0,$

$\Omega - i\varepsilon \sim \text{Feynman} - i\varepsilon$

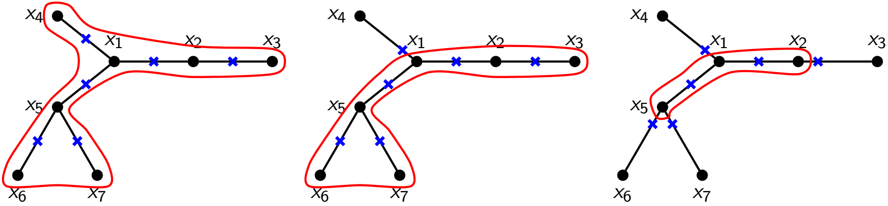
From the flat-space S-matrix to the wavefunction

Isomorphic facets Simplices



From the flat-space S-matrix to the wavefunction

Isomorphic facets | Simplices



$$\Omega = (-1)^{\dim\{\mathcal{E}^{\text{ext}}\}}\mathcal{A}$$

Flat-space scattering amplitude from other facets!

Flat-space scattering amplitude as residues of several poles!

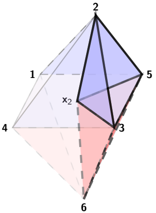
Combinatorial automorphisms map these facets into the scattering one

Symmetries in energy space $x_I \longleftrightarrow y_I$

From the flat-space S-matrix to the wavefunction

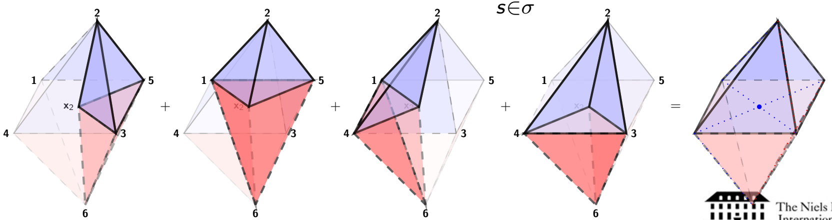
- Data: flat-space amplitude $\mathcal{A}^{\text{tree}}_{\mathcal{G}}$ + combinatorial automorphisms σ
- Representative of the wavefuntion:

$$\hat{\Psi}^{\text{tree}}_{\mathcal{A}_{\mathcal{G}}} = \frac{\mathcal{A}^{\text{tree}}}{\sum_k x_k}$$



Scattering facet
+ extra vertex $\mathbf{x}_2$

- Images of $\hat{\Psi}_{\mathcal{A}_{\mathcal{G}}}$ under σ. Then: $\Psi = \sum_{s\in\sigma} \mathfrak{Im}_{\sigma}\left\{\hat{\Psi}_{\mathcal{A}_{\mathcal{G}}}\right\}$



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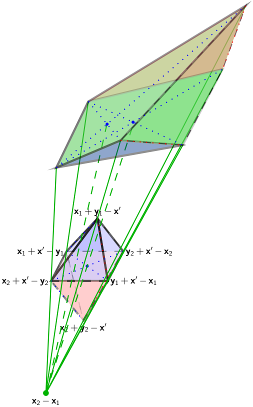
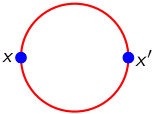
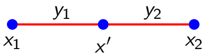
- Loops from trees: Projection through the cone $\mathbf{x}_i - \mathbf{x}_j$







$\mathbf{x}_2$

Decoding the Cosmological Polytopes:

The Late-Time Correlators

Taking averages

$$\Psi[\phi] \quad \Longrightarrow \quad \langle \prod_{i=1}^{n} \phi(p_i) \rangle = \mathcal{N} \int D\phi \prod_{i=1}^{n} \phi(p_i) \, |\Psi[\phi]|^2 \equiv \left(\prod_{k=1}^{n} \frac{1}{E_k} \right) \mathcal{C}_{\mathcal{G}}$$

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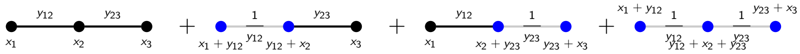

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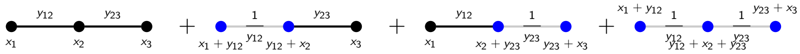
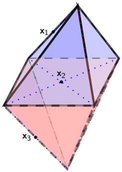
$\mathcal{C}_{\mathcal{G}} =$ 



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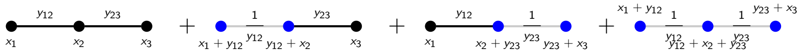

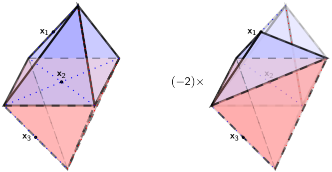

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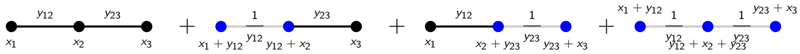

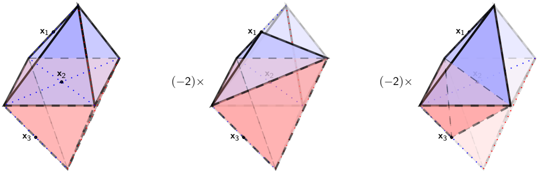

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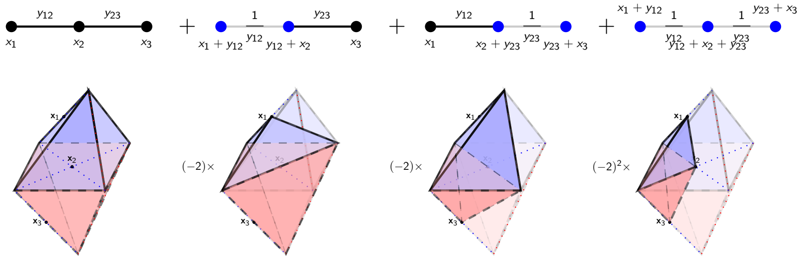
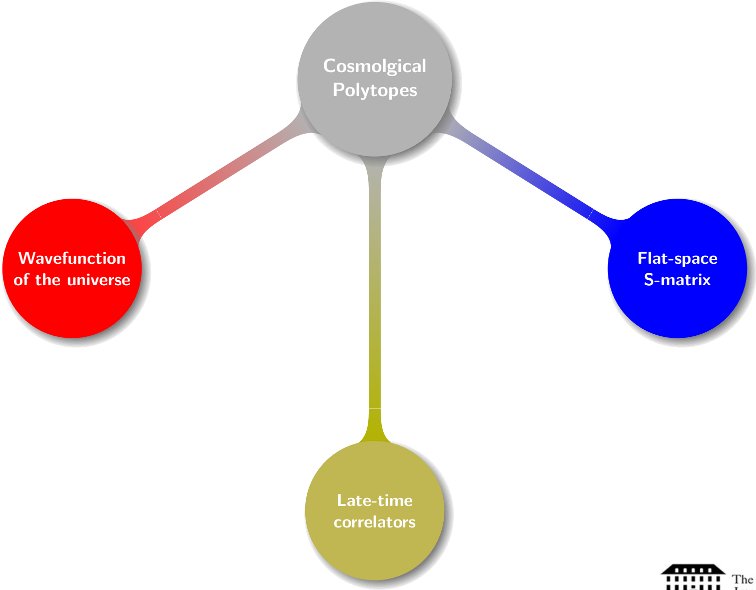


Conclusion

- Our knowledge of cosmological observables is quite primitive, both conceptually and computationally.
- Hints from amplitudes: Symmetries + Control on the analytic structure; New mathematical structures..
- Cosmological polytopes as a first combinatorial def of Ψ
- Triangulations $\Longleftrightarrow$ representations of Ψ (Feynman, OFPT..)
- Face structure $\Longleftrightarrow$ Singularity structure.
- It contains the flat-space S-matrix, with unitarity manifesting in the vertex structure of its facets
- Rules for Ψ which *do not* refer to Lorentz invariance & unitarity but contains a Lorentz-invariant & unitary object.
- Rules for extracting the late-time correlators from $\mathcal{P}$

The Niels Bohr International Academy

Conclusion



Cosmolgical Polytopes
Wavefunction of the universe
Flat-space S-matrix
Late-time correlators