Perturbative construction of gravity action via BRST ghosts

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Many promising results

Examples of solutions in classical gravitational theories from double copy piling up [Bahjat-Abbas,Bern,Bjerrum-Bohr,Carrillo Gonzalez,Cheung,Damgaard,Festuccia,Goldberger,Kosower,Li,Luna,Maybee,Monteiro,Nicholson,Ochirov,O'Connell,Penna Penco,Prabhu, Ridgway,Rothstein,Shen,Solon,Thompson,Trodden,Vanhove,Westerberg,White]

How general is the double copy?

- Can we construct the full space of the gravity theory in a *gauge-independent* way ?
- Could we *derive* the gravity action from YM ?

How general is the double copy?

- Can we construct the full space of the gravity theory in a *gauge-independent* (and respectively *coordinate independent*) way ?
- Could we *derive* the gravity action from YM ?
- First and second order in perturbation theory for pure gravitational theory.

Minimal YM^2

- Schematically:

$$A_\mu * \tilde{A}_\nu \equiv h_{\mu\nu} + B_{\mu\nu} + \eta_{\mu\nu}\phi$$

- Want to construct the full theory with arbitrary boundary conditions of graviton, dilaton and two-form from the double copy.
- If this is achievable, it should be possible to extract (pure) gravitational solutions by requiring $B_{\mu\nu} = \phi = 0$
- Start by constructing Lorenz-covariant dictionaries for all the fields compatible with symmetries and equations of motion.

The set-up

- We build gravitational fields as convolutions of gauge fields with the bi-adjoint scalar [Anastasiou, Borsten, Duff, Hughes, SN 2014]:

$$Z_{\mu\nu} = A_\mu{}^i \star [\Phi_{ii'}]^{-1} \star \tilde{A}_\nu{}^{i'} \equiv A_\mu \star \tilde{A}_\nu$$

- The convolution is defined as

$$[f \star g](x) = \int d^4y f(y) g(x-y).$$

- Importantly, is **doesn't** obey the Leibnitz rule when the functions f, g have appropriate fall-off:

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

Symmetry requirement

- Reproduces the correct local transformations at linear level:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$
$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$
$$\delta \phi = 0$$

from

$$\delta A^a_\mu = \partial_\mu \alpha^a + f^a{}_{bc} A^b_\mu \theta^c$$
$$\delta \phi_{a\tilde{a}} = -f^b{}_{ac} \phi_{b\tilde{a}} \theta^c - f^{\tilde{b}}{}_{\tilde{a}\tilde{c}} \phi_{a\tilde{b}} \theta^{\tilde{c}}$$

Dynamics requirement

- Reproduce gravtational eom

$$R_{\mu\nu} = j^{(h)}_{\mu\nu}$$
$$\partial^\rho H_{\rho\mu\nu} = j^{(B)}_{\mu\nu}$$
$$\Box\phi = j^{(\phi)}$$

from

- Yang-Mills

$$\partial^\mu F^i_{\mu\nu} = j^i_\nu, \quad \partial^\mu({}^*F^i_{\mu\nu}) = 0, \quad \partial^\mu j^i_\mu = 0$$

Sources have a dual role : they ensure proper fall-off for the fields to be convoluted and they will illuminate a fundamental issue of the double copy.

Field dictionary[Cardoso,Inverso,SN,Nampuri]

$$h_{\mu\nu} = A_\mu \star \tilde{A}_\nu + A_\nu \star \tilde{A}_\mu + q\eta_{\mu\nu}\left[A_\rho \star \tilde{A}^\rho - \frac{1}{\Box}(\partial \cdot A) \star (\partial \cdot \tilde{A})\right]$$

$$B_{\mu\nu} = A_\mu \star \tilde{A}_\nu - A_\nu \star \tilde{A}_\mu$$

$$\phi = A_\rho \star \tilde{A}^\rho - \frac{1}{\Box}(\partial \cdot A) \star (\partial \cdot \tilde{A})$$

Source dictionary

$$j^{(h)}_{\mu\nu} = -j_{(\mu} \star \tilde{j}_{\nu)} + (q+1)\frac{\partial_\mu \partial_\nu}{\Box} j_\rho \star \tilde{j}^\rho$$
$$j^{(B)}_{\mu\nu} = 2 j_{[\mu} \star \tilde{j}_{\nu]}$$
$$j^{(\phi)} = j_\rho \star \tilde{j}^\rho$$

Source dictionary and constraint

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Note that out theory is constrained:

$$\boxed{j^\phi \propto -T^\rho{}_\rho}$$

Obstruction to obtaining pure gravity - even for the most general dictionary !!

Source dictionary and constraint

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\end{aligned}
$$

Note that out theory is constrained:

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\boxed{j^{\phi} \propto -T^{\rho}{}_{\rho}}
$$

Obstruction to obtaining pure gravity - even for the most general dictionary !!

- issue with off-shell d.o.f. counting: $A_\mu \star \tilde{A}_\nu$ has 9, whereas $h_{\mu\nu}$ has 6, $B_{\mu\nu}$ has 3 and ϕ has 1.

Is there a more natural home for the double copy?

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- Siegel('95) conjectured in that a string-inspired double-copy would be manifested at the level of BRST systems (physical fields+ghosts)

	$\widetilde{\text{physical}}$	$\widetilde{\text{ghost}}$
physical	physical	ghost
ghost	ghost	physical + gh-gh

- Originally considered because it resolves the apparent conflict between on-shell and off-shell d.o.f (in a way that extends naturally to supergravity theories).

BRST

- introduced to avoid issues caused by gauge symmetry in path integrals
- perform a gauge-fixing

$$\mathcal{L}^{BRST} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + b\mathcal{G}(A^i_\mu, b, \xi_{(A)})$$

- replace the gauge symmetry by a fermionic global symmetry by introducing a ghost-antighost pair for each original gauge parameter. The symmetries have the same form as the original ones, but with gauge parameters replaced by ghosts:

$$Q_{BRST}A^i_\mu = \partial_\mu c^i$$
$$Q_{BRST}c = 0$$
$$Q_{BRST}\bar{c} = b$$
$$Q_{BRST}b = 0$$

BRST

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$$\mathcal{L}^{BRST} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - b\mathcal{G}(A^i_\mu, b, \xi_{(A)})$$

- replace the gauge symmetry by a fermionic global symmetry by introducing a ghost-antighost pair for each original gauge parameter and write a new Lagrangian which manisfests this symmetry by the prescription:

$$\mathcal{L}^{BRST} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - b\mathcal{G}(A^i_\mu, b, \xi_{(A)}) - \bar{c}\int d^4y \frac{\delta\mathcal{G}}{\delta A_\mu}\partial^\mu c$$

- At classical level this is equivalent with the original one!

BRST for linearised YM

- Pick a Lorenz covariant ξ-gauge for YM field:

$$\mathcal{G}(A_\mu, b, \xi_{(A)}) = \partial^\mu A_\mu + \frac{\xi_{(A)}}{2} b$$

- and corresponding one for graviton

$$\mathcal{G}(h_{\mu\nu}, b_\mu, \xi_{(h)}) = h_{\mu\nu} - \frac{1}{2}\partial_\mu h + \frac{\xi_{(h)}}{2} b_\mu$$

with BRST transformations

$$\begin{aligned} Q_{BRST} h_{\mu\nu} &= \partial_\mu c_\nu + \partial_\nu c_\mu \\ Q_{BRST} c_\mu &= 0 \\ Q_{BRST} \bar{c}_\mu &= b_\mu \\ Q_{BRST} b_\mu &= 0 \end{aligned}$$

- Similarly for the two-form $B_{\mu\nu}$, though more complicated: we have ghosts $d_\mu, \bar{d}_\mu$ ghosts-for-ghosts $d, \bar{d}$ and and a physical spin-0 field η in the gauge-fixing term[Kimura '80].

BRST dictionary

- *symmetry requirement*: reproduce BRST symmetries of gravitational fields, ghosts, ghosts-for-ghosts from those if YM.
- *dynamics requirement*: reproduce gravitational e.o.m. in the BRST forrm

$$\Box h_{\mu\nu} - \frac{\xi_{(h)}+2}{\xi_{(h)}}\left(2\partial^{\rho}\partial_{(\mu}h_{\nu)\rho} - \partial_{\mu}\partial_{\nu}\right)h = j^{(h)}_{\mu\nu}$$

$$\Box B_{\mu\nu} + \frac{\xi_{(B)}+2}{\xi_{(B)}}2\partial^{\rho}\partial_{[\mu}B_{\nu]\rho} = j^{(B)}_{\mu\nu}$$

$$\Box\varphi = j^{(\varphi)}$$

from YM equations in BRST form

$$\Box A_{\mu} - \frac{\xi+1}{\xi}\partial_{\mu}\partial A = j_{\mu}, \quad \Box c^{\alpha} = j^{\alpha}$$

Dictionary

We find

$$\xi_{(h)} = \xi_{(B)} = \xi$$

and

$$\begin{aligned}
h_{\mu\nu} =& A_{(\mu} * A_{\nu)} + \tfrac{1}{1-\xi}\frac{\partial_\mu \partial_\nu}{\Box} A * A + \tfrac{1+\xi}{2(1-\xi)}\frac{\partial_\mu \partial_\nu}{\Box} c * c \\
& - \tfrac{1}{2}\frac{1}{\Box}\left(\partial A * \partial_{(\mu} A_{\nu)} + \partial_{(\mu} A_{\nu)} * \partial A\right) \\
& + \tfrac{\xi}{(2-D)(\xi-1)}\eta_{\mu\nu}\left(A * A + \tfrac{1}{\xi} c * c + \left(\tfrac{1}{\xi^2} - 1\right)\frac{1}{\Box}\partial A * \partial A\right) \\
B_{\mu\nu} =& A_{[\mu} * A_{\nu]} - \tfrac{1}{2}\frac{1}{\Box}\left(\partial A * \partial_{[\mu} A_{\nu]} - \partial_{[\mu} A_{\nu]} * \partial A\right) \\
\phi =& A * A + \tfrac{1}{\xi} c * c + \left(\tfrac{1}{\xi^2} - 1\right)\frac{1}{\Box}\partial A * \partial A
\end{aligned}$$

Gravity dictionary

Kill $B_{\mu\nu}$ by setting $A_\mu = \tilde{A}_\mu$ and kill dilaton by setting

$$\tfrac{1}{\xi} c * c = -A * A \left(1 - \tfrac{1}{\xi^2}\right) \frac{1}{\Box} \partial A * \partial A$$

so graviton dictionary reduces to

$$h_{\mu\nu} = A_{(\mu} * A_{\nu)} + \tfrac{1}{1-\xi} \frac{\partial_\mu \partial_\nu}{\Box} A * A + \tfrac{1+\xi}{2(1-\xi)} \frac{\partial_\mu \partial_\nu}{\Box} c * c$$
$$- \tfrac{1}{2} \frac{1}{\Box} \left(\partial A * \partial_{(\mu} A_{\nu)} + \partial_{(\mu} A_{\nu)} * \partial A\right)$$

E.o.m. dictionary

Source dictionary for graviton-dilaton

$$j^{(h)}_{\mu\nu} = a_6 \frac{1}{\Box} j_{(\mu} * \tilde{j}_{\nu)} + a_7 \frac{\partial_\mu \partial_\nu}{\Box^2} j^\rho * \tilde{j}_\rho + a_8 \frac{\partial_\mu \partial_\nu}{\Box^2} j^a * \tilde{j}_a + \eta_{\mu\nu} \frac{1}{\Box} \left[b_4 j^\rho * \tilde{j}_\rho + b_5 j^a * \tilde{j}_a \right]$$

$$j^\phi = \frac{1}{\Box} \left[j^\rho * \tilde{j}_\rho + \frac{1}{\xi} j^a * \tilde{j}_a \right]$$

with

$$a_6 = \frac{1}{1-\xi}, \; a_7 = \frac{(1+\xi)^2}{\xi(1-\xi)}, \; a_8 = \frac{(1+\xi)^2}{\xi^2(1-\xi)}, \quad b_4 = \frac{\xi}{2(1-\xi)}, \; b_5 = \frac{1}{2(1-\xi)}$$

E.o.m. dictionary

Kill dilaton source this reduces to

$$j^{(h)}_{\mu\nu} = \frac{1}{1-\xi}\frac{1}{\Box} j_{(\mu} \star \tilde{j}_{\nu)}$$

of

$$\boxed{e.o.m.(h_{\mu\nu}) = \frac{1}{1-\xi}\frac{1}{\Box}\left[e.o.m.(A_\mu) \star e.o.m.(A_\nu)\right]}$$

Can construct gravity Lagrangian directly from YM fields !

Higher order

- First obstacle: how are the gravity and YM gauge fixing terms $\mathcal{G}$ related?
- At linear level, we made a guess that generalised Lorenz corresponds to generalised De Donder, which turned out to work, because of the simplity of the action.
- At second order, could be more complicated, even restricting to Lorenz covariant gauges
- Writing most general $\mathcal{G}$ can be very ugly!

Higher order

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- at second order, could be more complicated, even restricting to lorenz coariant gauges
- use BCJ double copy to work out relation between $\mathcal{G}(h)^{(2)}$ and $\mathcal{G}(A)^{(2)}$

Second order

Sticking with generalised Lorenz for YM, we get, at second order

$$\Box A_{\mu}^{(2)a} - \frac{\xi+1}{\xi}\partial_{\mu}\partial A^{(2)a} = -f^{abc}\Big[\partial A^{(1)b} A_{\mu}^{(1)c} + 2A_{\rho}^{(1)b}\partial^{\rho} A_{\mu}^{(1)c} + \partial_{\mu}(\bar{c}^{(1)b} c^{(1)c})\Big]$$

Second order

Pick $\xi = -1$ for simplicity

$$\Box A^{(2)a}_{\mu} = -f^{abc}\left[\partial A^{(1)b} A^{(1)c}_{\mu} + 2A^{(1)b}_{\rho}\partial^{\rho} A^{(1)c}_{\mu} + \partial_{\mu}(\bar{c}^{(1)b} c^{(1)c})\right]$$

Fourier transform and write in BCJ form

$$-p^2 A^{(2)\mu a}(-p) = if^{a}{}_{bc}\int \bar{d}k\bar{d}q\bar{\delta}\Big[n^{\mu\nu\sigma} A^{(1)b}_{\nu}(k) A^{(1)c}_{\sigma}(q) + n^{\mu} c^{(1)\alpha b}(k) c^{(1)}{}_{\alpha}{}^{c}(q)\Big]$$

with

$$\begin{aligned} n^{\mu\nu\sigma} &= \tfrac{1}{2}\left[(q-k)^{\mu}\eta^{\nu\sigma} + (k-p)^{\sigma}\eta^{\mu\nu} + (p-q)^{\nu}\eta^{\sigma\mu}\right], \\ n^{\mu} &= \tfrac{1}{2}\left(k^{\mu} + q^{\mu}\right). \end{aligned}$$

second order

we get

$$-p^2 H^{(2)\mu\tilde{\mu}} = i \int dkdq\delta\Big[n^{\tilde{\mu}}{}_{\tilde{\nu}\tilde{\sigma}} n^{\mu\nu\sigma} H^{(1)\tilde{\nu}}_{\nu} H^{(1)\tilde{\sigma}}_{\sigma} + n^{\tilde{\mu}}{}_{\tilde{\nu}\tilde{\sigma}} n^{\mu} D^{(1)\alpha\tilde{\nu}} D^{(1)}{}_{\alpha}{}^{\tilde{\sigma}} + n^{\tilde{\mu}} n^{\mu\nu\sigma} D^{(1)\tilde{\alpha}}_{\nu} D^{(1)}_{\sigma\tilde{\alpha}} + n^{\tilde{\mu}} n^{\mu} C^{(1)\alpha\tilde{\alpha}} C^{(1)}{}_{\alpha\tilde{\alpha}}\Big]$$

with

$$H^{(i)}_{\mu\tilde{\mu}} = A^{(i)}_{\mu} \star \tilde{A}^{(i)}_{\tilde{\mu}} \tag{1a}$$

$$D^{(i)}_{\mu\tilde{\alpha}} = A^{(i)}_{\mu} \star \tilde{c}^{(i)}_{\tilde{\alpha}} \tag{1b}$$

$$D^{(i)}_{\alpha\tilde{\mu}} = c^{(i)}_{\alpha} \star \tilde{A}^{(i)}_{\tilde{\mu}} \tag{1c}$$

$$C^{(i)}_{\alpha\tilde{\alpha}} = c^{(i)}_{\alpha} \star \tilde{c}^{(i)}_{\tilde{\alpha}} \tag{1d}$$

Second order

- Could stop at this point and study solutions
- We want general second order equation: invert linear dictionary to obtain expressions for $A \star A$ etc and use these to extract the gravitational gauge fixing term for our chosen YM $\mathcal{G}$:

$$\mathcal{G}(h)^{(2)} = \left[\partial_\mu \left(\sqrt{-g} g^{\mu\nu}\right)\right]^{(2)} + \frac{1}{\Box} f(h)$$

- Sanity check: the ghost terms in the e.o.m. as they come from the amplitude d.c. have to match the terms as they come from acting on $\mathcal{G}$ with Q_{BRST}

In progress

- Is it possible to give a direct fields dictionary at second order, now that we understand how to match e.o.m. ? (bi-adjoint scalar will play a role here)
- quantum applications (anomalies etc)
- more speculative: understanding non-perturbative double copy systematically will require Batalin Vilkovisky formalism.

Thank You !