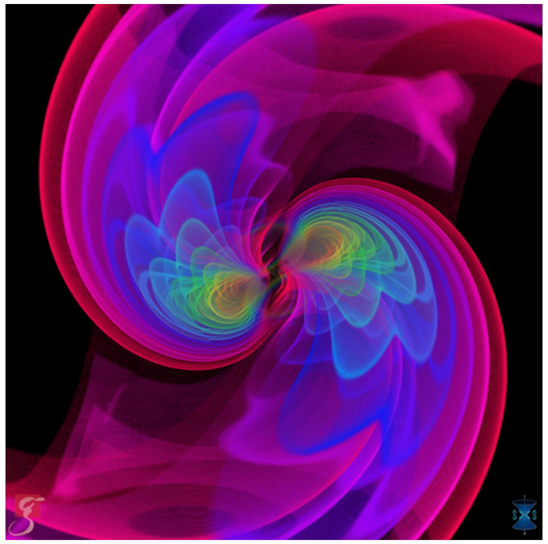
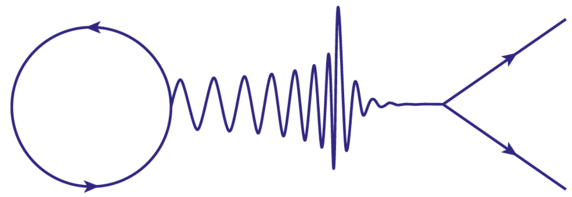
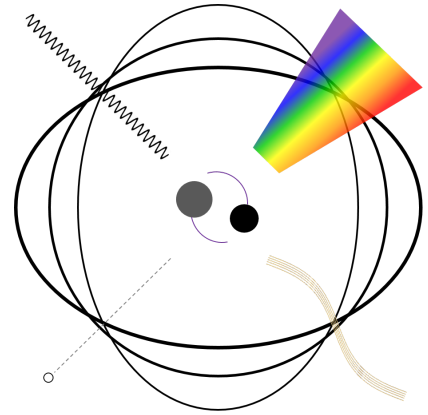
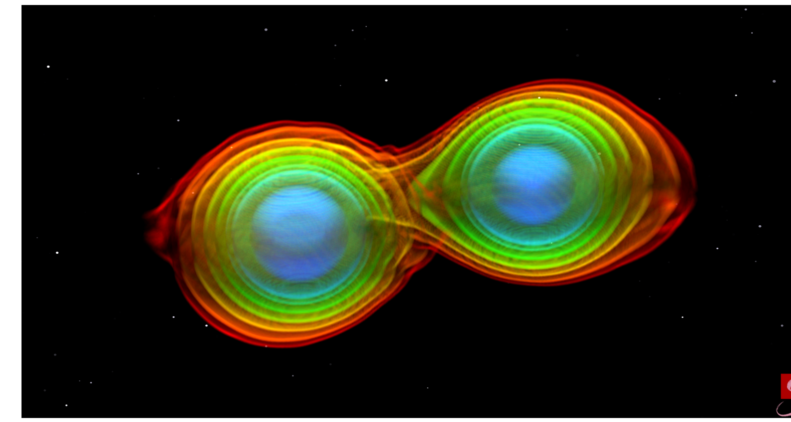

The Need for High-Precision Gravitational Waveforms

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"QCD Meets Gravity IV", NORDITA, Stockholm **Dec 10-14, 2018**

Outline

- Observing gravitational waves and inferring astrophysical/physical information stem on our ability to make precise predictions of 2-body dynamics and gravitational radiation.

- Several perturbative methods available: post-Newtonian theory, post-Minkowskian theory, gravitational self-force (small mass-ratio expansion).

- How do we assess accuracy of perturbative waveform models? Why perturbative calculations alone are not sufficient to predict high-precise waveform models?

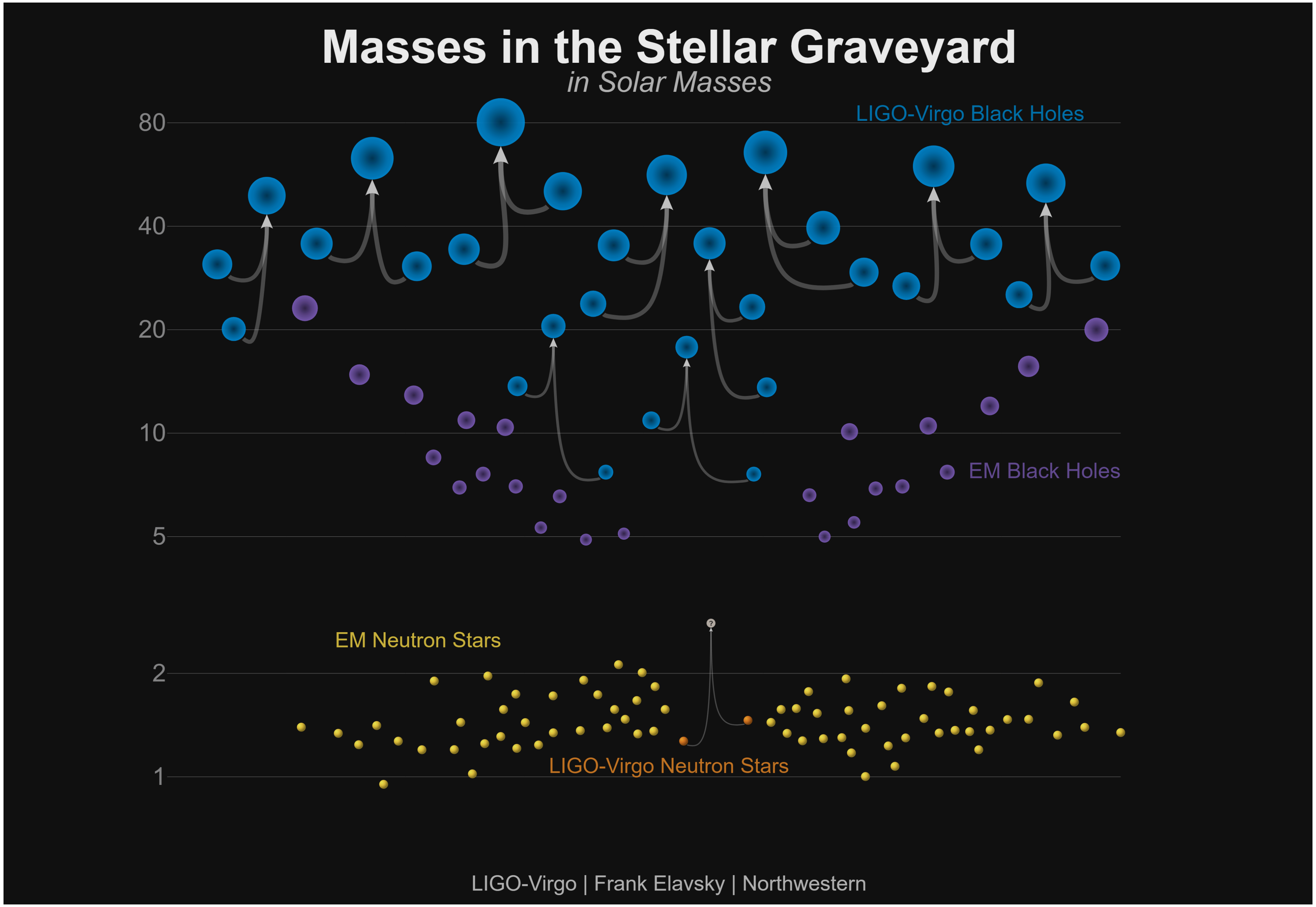
- Need to re-sum and re-organize perturbative results to improve accuracy and include strong-field effects close to merger: effective-one-body theory. Need also interface with numerical relativity.

- Gravitational-wave experiments on ground and in space require more accurate waveform models: new theoretical challenges and opportunities.

Black holes & neutron stars in GWTC-1

- **4 new GWs** from binary black holes observed by LIGO & Virgo

(Abbott et al. arXiv:1811.12907)

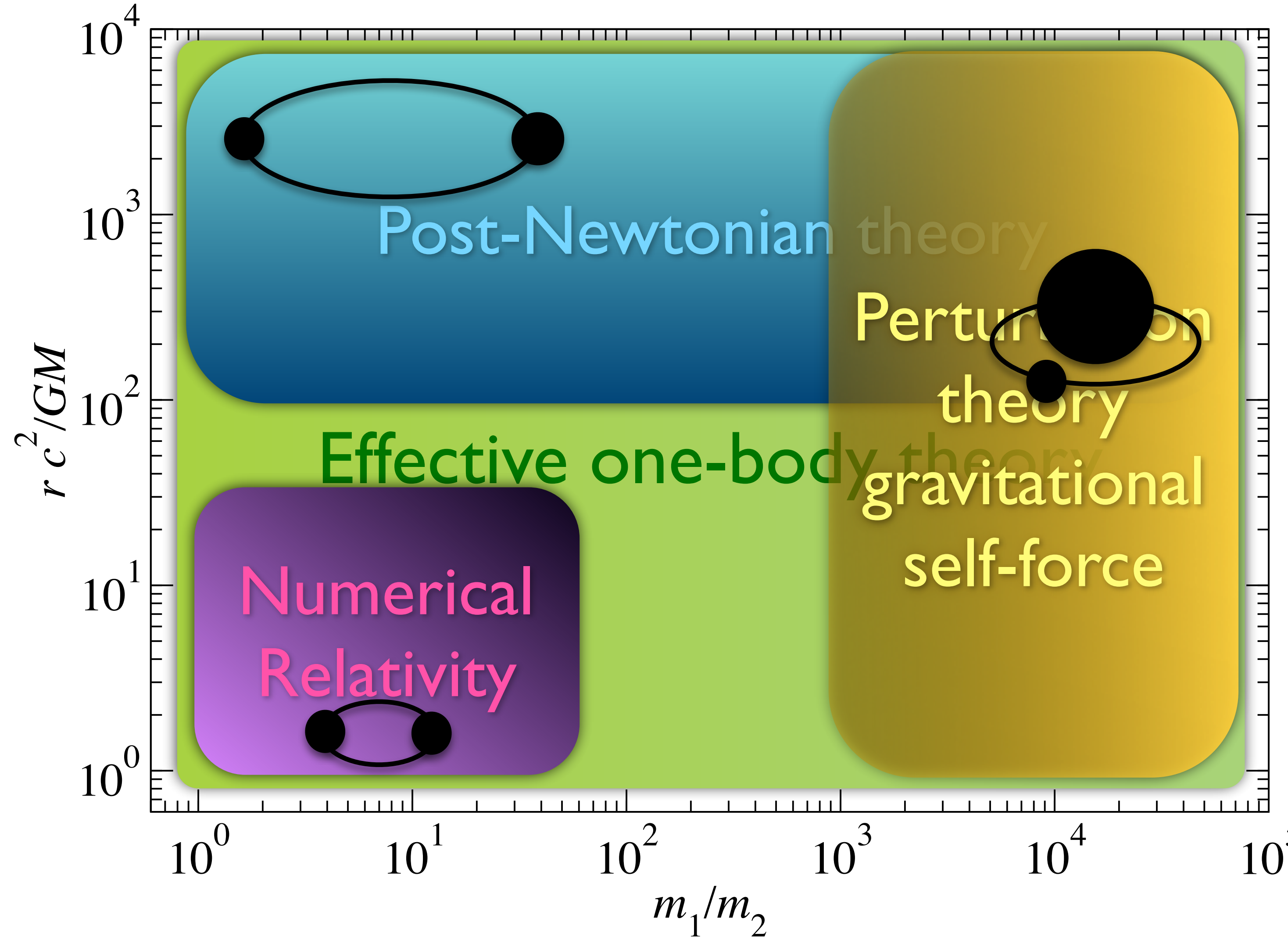


Solving two-body problem in General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

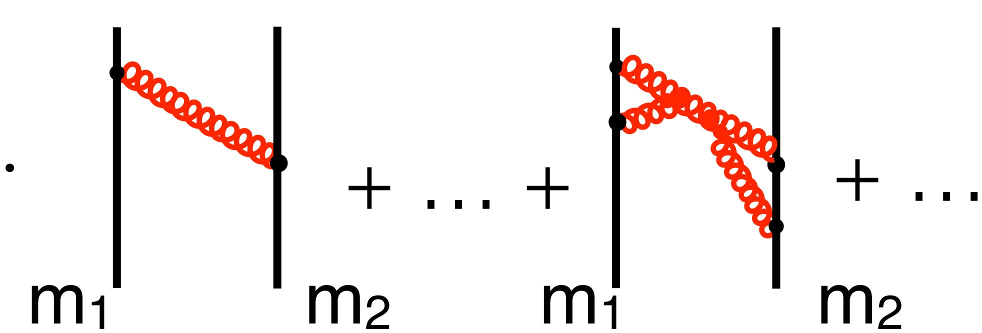
- **GR** is **non-linear theory**. Complexity similar to QCD.
- Einstein's field equations can be solved:
  - **approximately,** but **analytically** (**fast** way)
  - **"exactly"**, but **numerically** on supercomputers (**slow** way)

$v^2/c^2 \sim GM/rc^2$



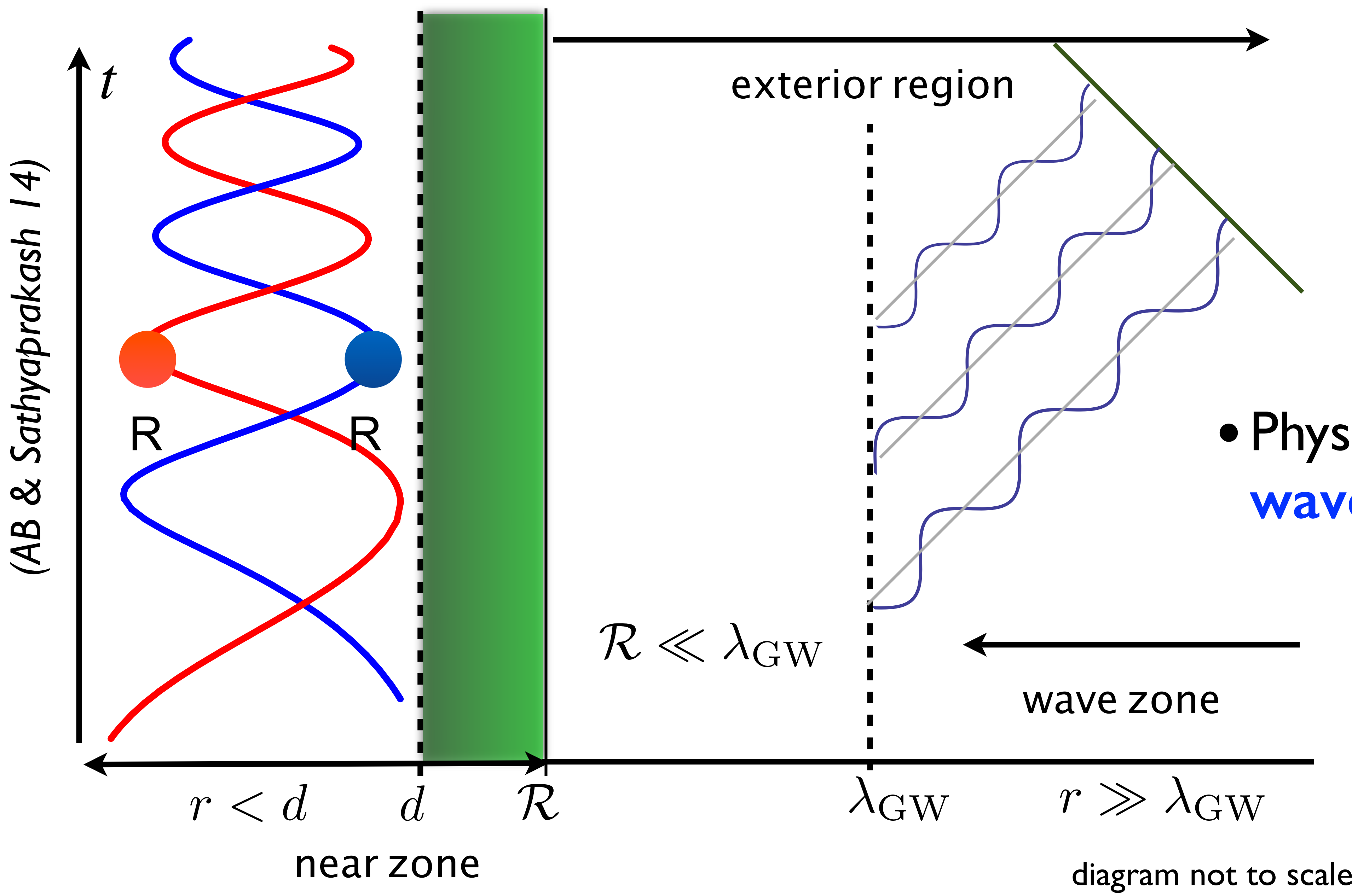
(AB & Sathyaprakash 14)

- **Analytical methods**: post-Newtonian/post-Minkowskian/small mass-ratio expansion, effective-one-body theory
  - **effective field-theory**, **dimensional regularization**, etc.
  - **diagrammatic** approach to organize expansions



Post-Newtonian/post-Minkowskian formalism/effective field theory

diverse scales: R, d, $\lambda_{\rm GW}$



- Physical **observables are waveforms** at null infinity

- **Multi-chart** approach to describe motion of **strong self-gravity bodies**, such as NS & BH.
- **Radiation-reaction** problem.

- **Matched asymptotic** expansions.
- **Generation** problem.

Post-Newtonian approximation

- **First introduced in 1917** *(Droste & Lorentz 1917, … Einstein, Infeld & Hoffmann 1938)*

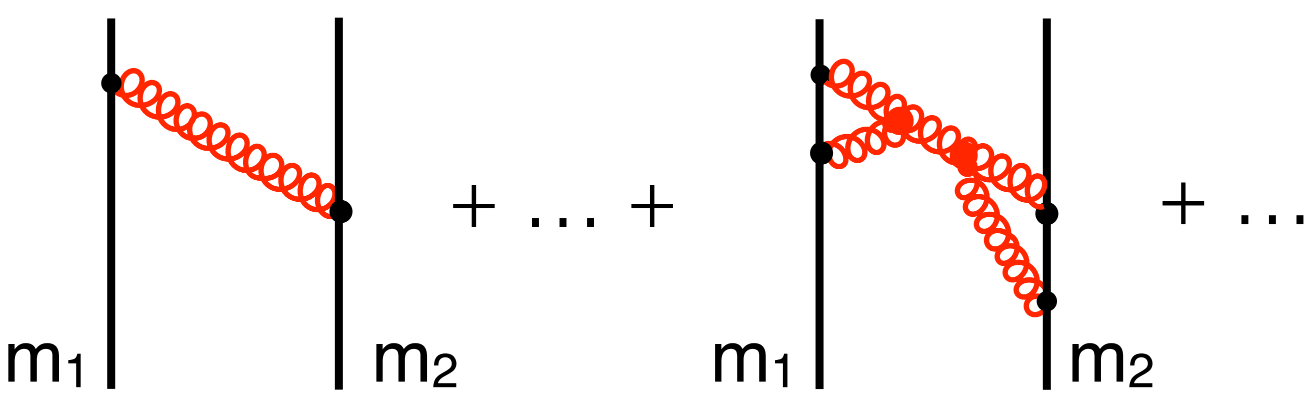
(Blanchet, Damour, Iyer, Faye, Bernard, Bohe', AB, Marsat; Jaranowski, Schaefer, Steinhoff; Will, Wiseman; Flanagan, Hinderer, Vines; Goldberger, Porto, Rothstein; Kol, Levi, Smolkin; Foffa, Sturani; …)

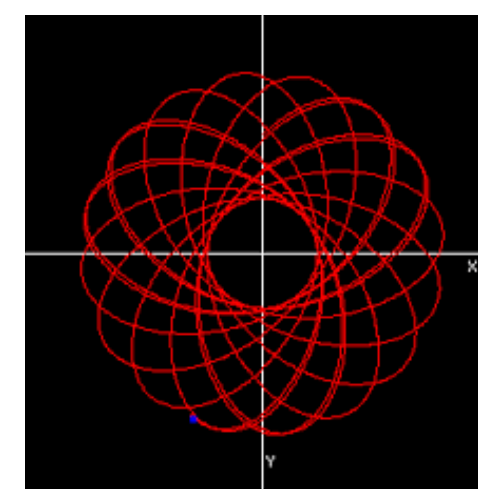
$$\widehat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r}$$

Small parameter is **v/c << 1, $v^2/c^2 \sim GM/rc^2$**

$$\widehat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{r} + \frac{1}{2r^2}$$

$$\widehat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\Big\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 \quad + \ldots$$





- Compact object is **point-like body endowed** with time-dependent **multipole moments** (skeletonization).

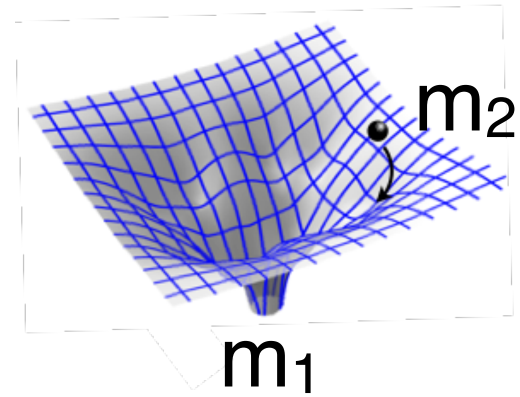
Small mass-ratio expansion/gravitational self-force formalism

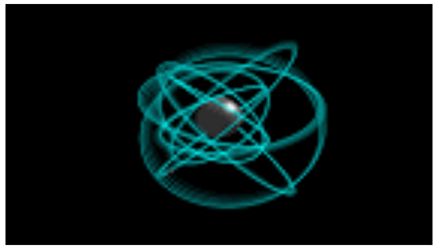
- **First works in 50-70s** *(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)*

Small parameter is $m_2/m_1 \ll 1$, $v^2/c^2 \sim GM/rc^2 \sim 1$, $M = m_1 + m_2$

Equation of gravitational perturbations in black-hole spacetime:

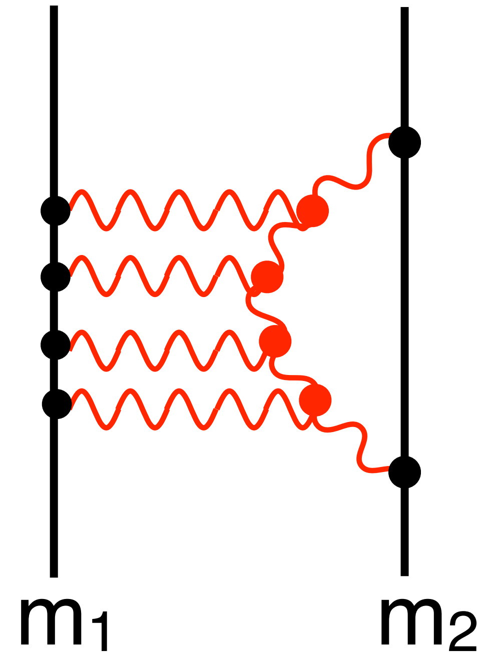
$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_\star^2} + V_{\ell m}\Psi = S_{\ell m}$$





Green functions in Schwarzschild/Kerr spacetimes.

(Fujita, Poisson, Sasaki, Shibata, Khanna, Hughes, Bernuzzi, Harms, Nagar…)



- Accurate modeling of **relativistic dynamics of large mass-ratio** inspirals **requires** to include **back-reaction effects** due to interaction of small object with its own gravitational perturbation field.

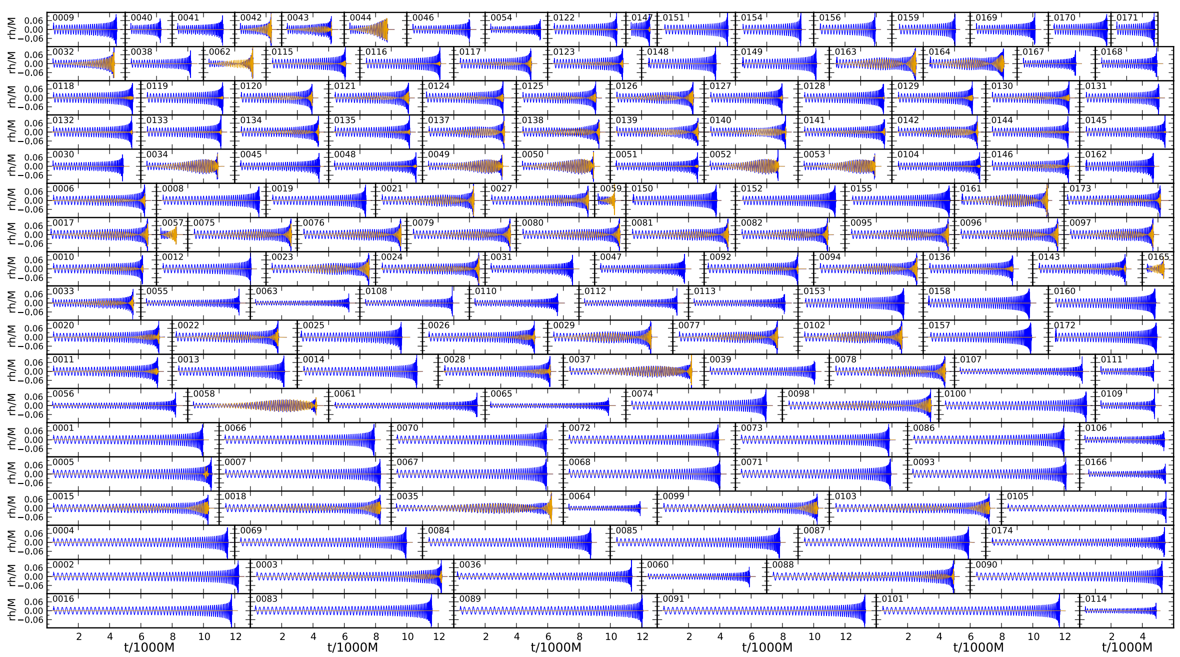
(Deitweiler, Whiting, Mino, Poisson, Quinn, Wald, Sasaki, Tanaka, Barack, Ori, Pound, van de Meent, …)

Numerical Relativity: binary black holes

- **Breakthrough** in 2005 *(Pretorius 05, Campanelli et al. 06, Baker et al. 06)*

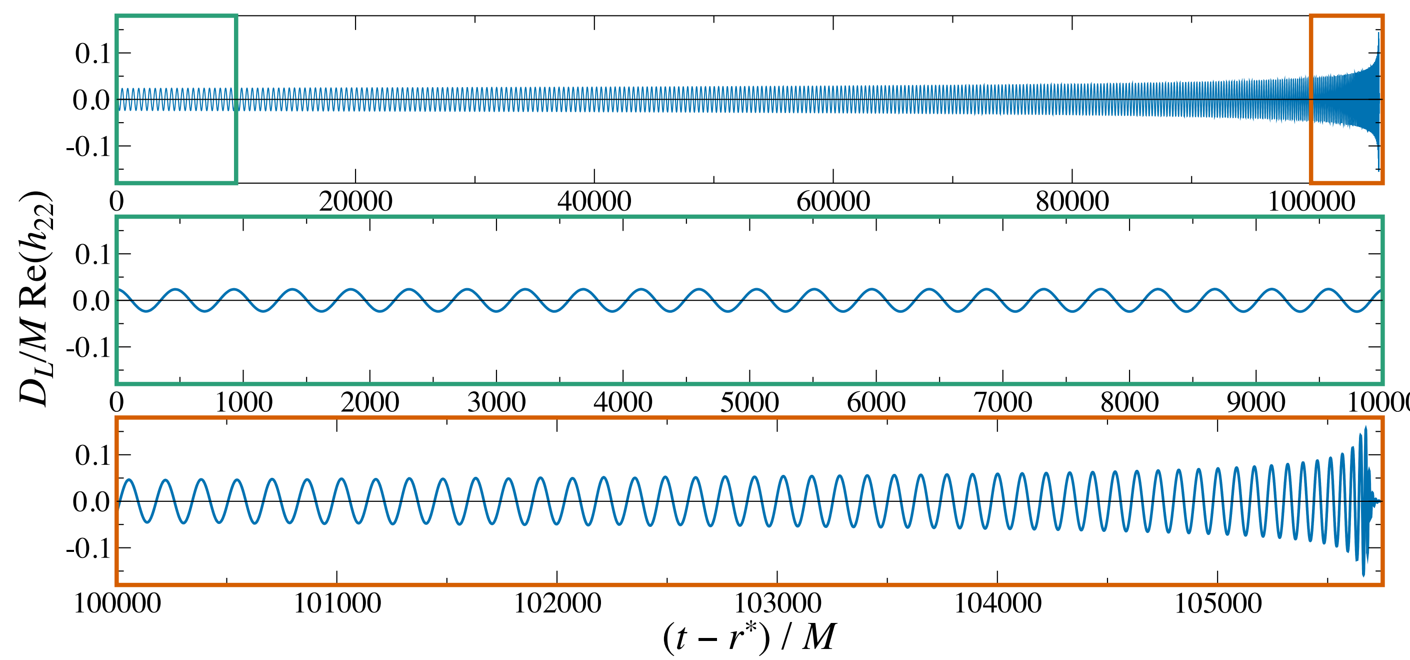
(Kidder, Pfeiffer, Scheel, Lindblom, Szilagyi; Bruegmann; Hannam, Husa, Tichy; Laguna, Shoemaker; …)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



- **376 GW cycles**, zero spins & mass-ratio 7 (8 months, few millions CPU-h)

(Szilagyi, Blackman, AB, Taracchini et al. 15)



- **Simulating eXtreme Spacetimes (SXS)** collaboration *(Mroue et al. 13)*

- **Numerical-Relativity & Analytical-Relativity** collaboration *(Hinder et al. 13)*

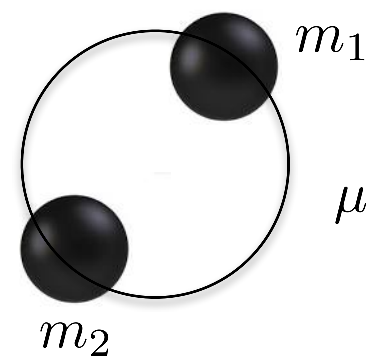
Gravitational waveforms built from conservative & dissipative dynamics

- GW from time-dependent **quadrupole moment**: $$h_{ij} \sim \frac{G}{c^4}\frac{\ddot{Q}_{ij}}{\mathrm{R}}$$

$$h = \nu \left(\frac{GM}{c^2\,\mathrm{R}}\right)\frac{v^2}{c^2}\cos 2\Phi$$

$$\frac{v}{c} = \left(\frac{GM\omega}{c^3}\right)^{1/3}$$

$$\nu = \mu/M$$



$$\mu = m_1 m_2/M$$

- Center-of-mass energy: $E(\omega)$

$$E(v) = -\frac{\mu}{2}\,v^2 + \cdots$$

- GW luminosity: $\mathcal{L}_{\mathrm{GW}}(\omega) \equiv F(\omega)$

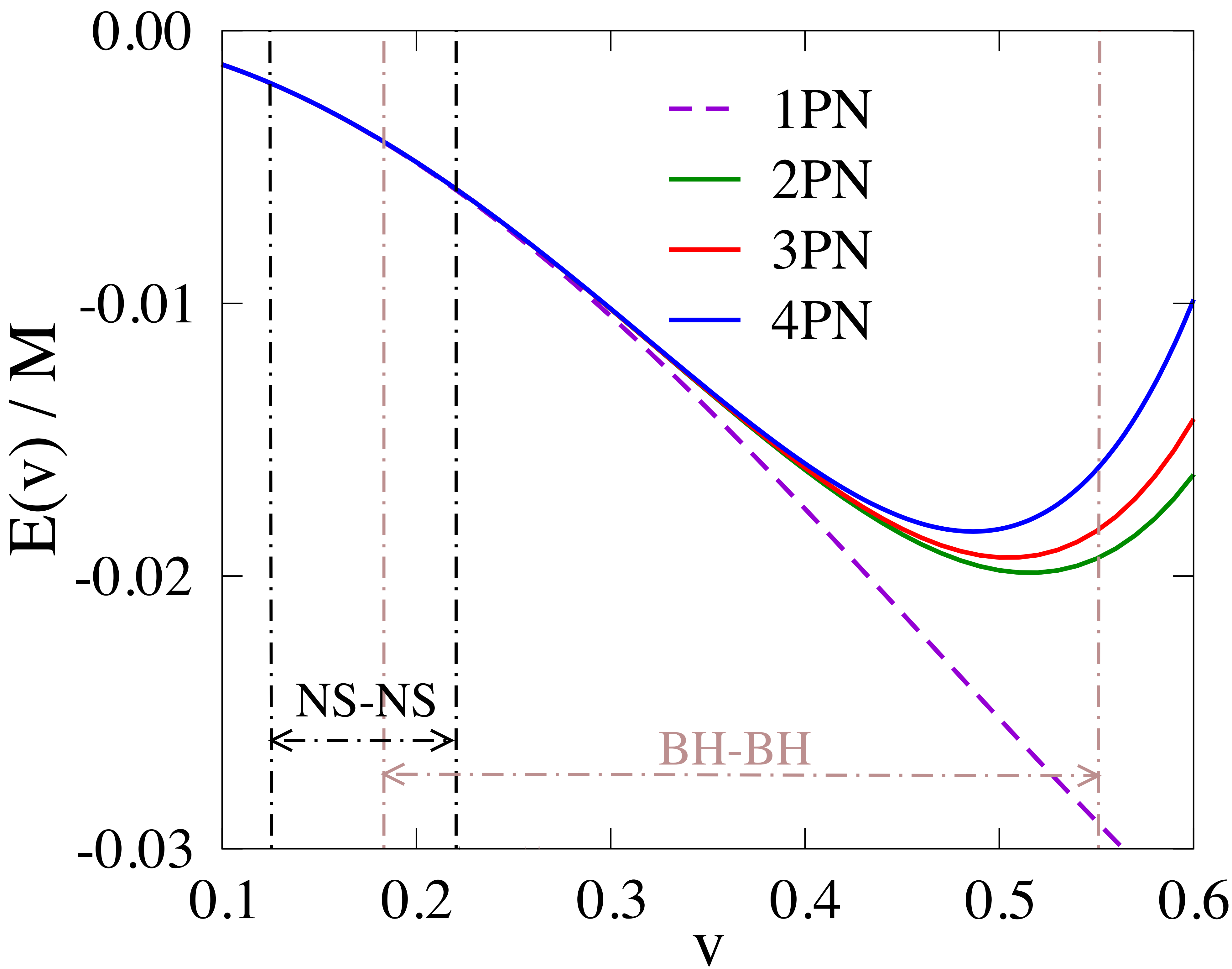
$$F(v) = \frac{32}{5}\,\nu^2\,\frac{c^5}{G}\left(\frac{v}{c}\right)^{10} + \cdots$$

- Balance equation: $$\frac{dE(\omega)}{dt} = -F(\omega) \quad \longrightarrow \quad \dot{\omega}(t) = -\frac{F(\omega)}{dE(\omega)/d\omega}$$

- Gravitational-wave **phase**: $$\Phi_{\mathrm{GW}}(t) = 2\Phi(t) = \frac{1}{\pi}\int^{t}\omega(t')dt'$$

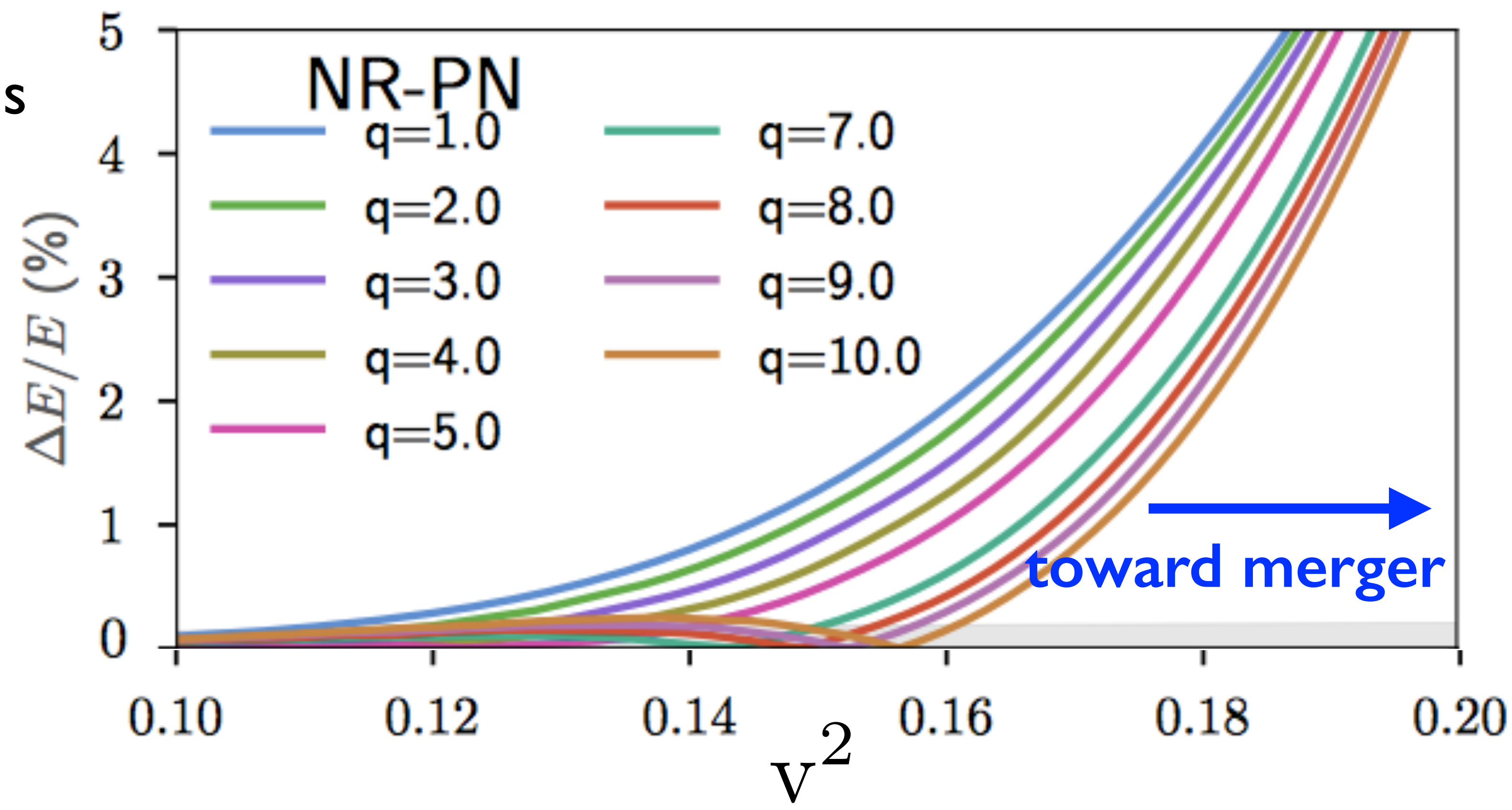
PN (binding) energy versus velocity

- Equal-mass, non-spinning binary



Comparing the energetics of NR against PN

non-spinning BHs

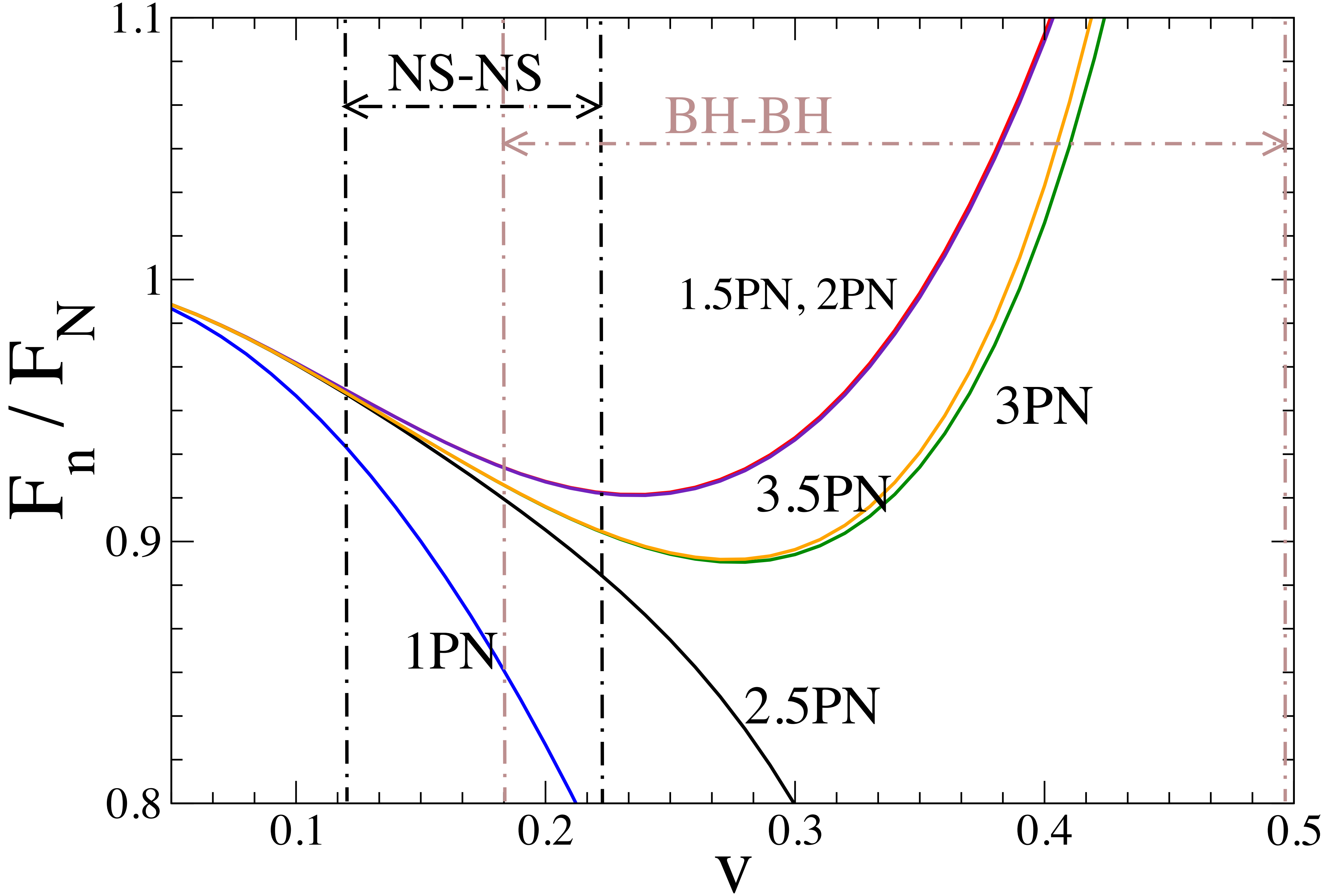


(Ossokine, Dietrich et al. 17)

$$\mathrm{v}^2 = (M\Omega)^{2/3}$$

PN gravitational-wave flux versus velocity

- Equal-mass, non-spinning binary



Number of GW cycles predicted by PN theory

$$N_{\rm tot} = \frac{1}{\pi}(\Phi_{\rm max} - \Phi_{\rm min}) = \frac{1}{\pi}\int_{f_{\rm min}}^{f_{\rm max}} df\, \frac{d\Phi(f)}{df} = \int_{f_{\rm min}}^{f_{\rm max}} \frac{df}{f}\, N_{\rm inst}(f)$$

$$N_{\rm inst}(f) = \frac{f^2}{df/dt}, \qquad f = \frac{1}{\pi}\frac{d\Phi}{dt}$$

$M = (1.4 + 1.4)M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 1570$ Hz

$\chi = |\mathbf{S}|/m^2$

	Number of cycles
Newtonian:	16034
1PN:	$+441$
1.5PN	-211
Spin-orbit:	$+65.7\chi_1 + 65.7\chi_2$
2PN	$+9.9$
2.5PN	$-11.7 + 9.2\chi_1 + 9.2\chi_2$
3PN:	$+2.6$
3.5PN:	-0.9

Number of GW cycles predicted by PN theory

$M = (1.4 + 1.4) M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 1570$ Hz

$\chi = |\mathbf{S}|/m^2$

$$N_{\rm useful} = \frac{\int_{f_{\rm min}}^{f_{\rm max}} w(f)\, N_{\rm inst}(f)\, df/f}{\int_{f_{\rm min}}^{f_{\rm max}} w(f)\, df/f}$$

$$w(f) = a^2(f)/[f S_n(f)]\,, \quad h(t) = 2a(t)\, \cos 2\Phi(t)$$

(Damour, Iyer & Sathyaprakash 03)

	Number of cycles	Number of *useful* cycles:
Newtonian:	16034	247.8
1PN:	$+441$	$+24.0$
1.5PN	-211	-20.0
Spin-orbit:	$+65.7\chi_1 + 65.7\chi_2$	$6.2\chi_1 + 6.2\chi_2$
2PN	$+9.9$	$+1.5$
2.5PN	$-11.7 + 9.2\chi_1 + 9.2\chi_2$	$-2.3 + 0.8\chi_1 + 0.8\chi_2$
3PN:	$+2.6$	$+0.6$
3.5PN:	-0.9	-0.2

Number of GW cycles predicted by PN theory

$$N_{\rm useful} = \frac{\int_{f_{\rm min}}^{f_{\rm max}} w(f)\, N_{\rm inst}(f)\, df/f}{\int_{f_{\rm min}}^{f_{\rm max}} w(f)\, df/f}$$

$M = (15+15)M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 147$ Hz

$\chi = |\mathbf{S}|/m^2$

$$w(f) = a^2(f)/[f S_n(f)]\,, \quad h(t) = 2a(t)\,\cos 2\Phi(t)$$

	Number of cycles	Number of *useful* cycles:
Newtonian:	302	10.7
1PN:	$+39$	$+4.0$
1.5PN	-37	-6.2
Spin-orbit:	$+11.7\chi_1 + 11.7\chi_2$	$1.9\chi_1 + 1.9\chi_2$
2PN	$+3.3$	$+0.8$
Spin-spin:	$-1.7\chi_1\chi_2$	$-0.4\chi_1\chi_2$
2.5PN	$-6.2 + 3.6\chi_1 + 3.6\chi_2$	$-2.3\ +0.8\chi_1 + 0.8\chi_2$
3PN:	$+2$	$+1.2$
3.5PN:	-0.8	-0.5

PN approximants for inspiraling waveforms

$$E(\omega) = E_0 v^2 \left[1 + E_{1\mathrm{PN}} v^2 + E_{2\mathrm{PN}} v^4 + \cdots\right]$$

$$F(\omega) = F_0 v^{10} \left[1 + F_{1\mathrm{PN}} v^2 + F_{1.5\mathrm{PN}} v^3 + F_{2\mathrm{PN}} v^4 + \cdots\right]$$

$$\Rightarrow \dot{\omega}(t) = -\frac{F(\omega)}{dE(\omega)/d\omega}$$

- **T4-PN-approximant at 2PN order:**

$$[\dot{\omega}(t)]_{2\mathrm{PN}} = -\mathcal{T}_{2\mathrm{PN}} \left\{ \frac{F(\omega)}{dE(\omega)/d\omega} \right\} \qquad \mathcal{T}_{\mathrm{nPN}} \Rightarrow \text{Taylor exp. up to nPN}$$

- **T1–PN-approximant at 2PN order:**

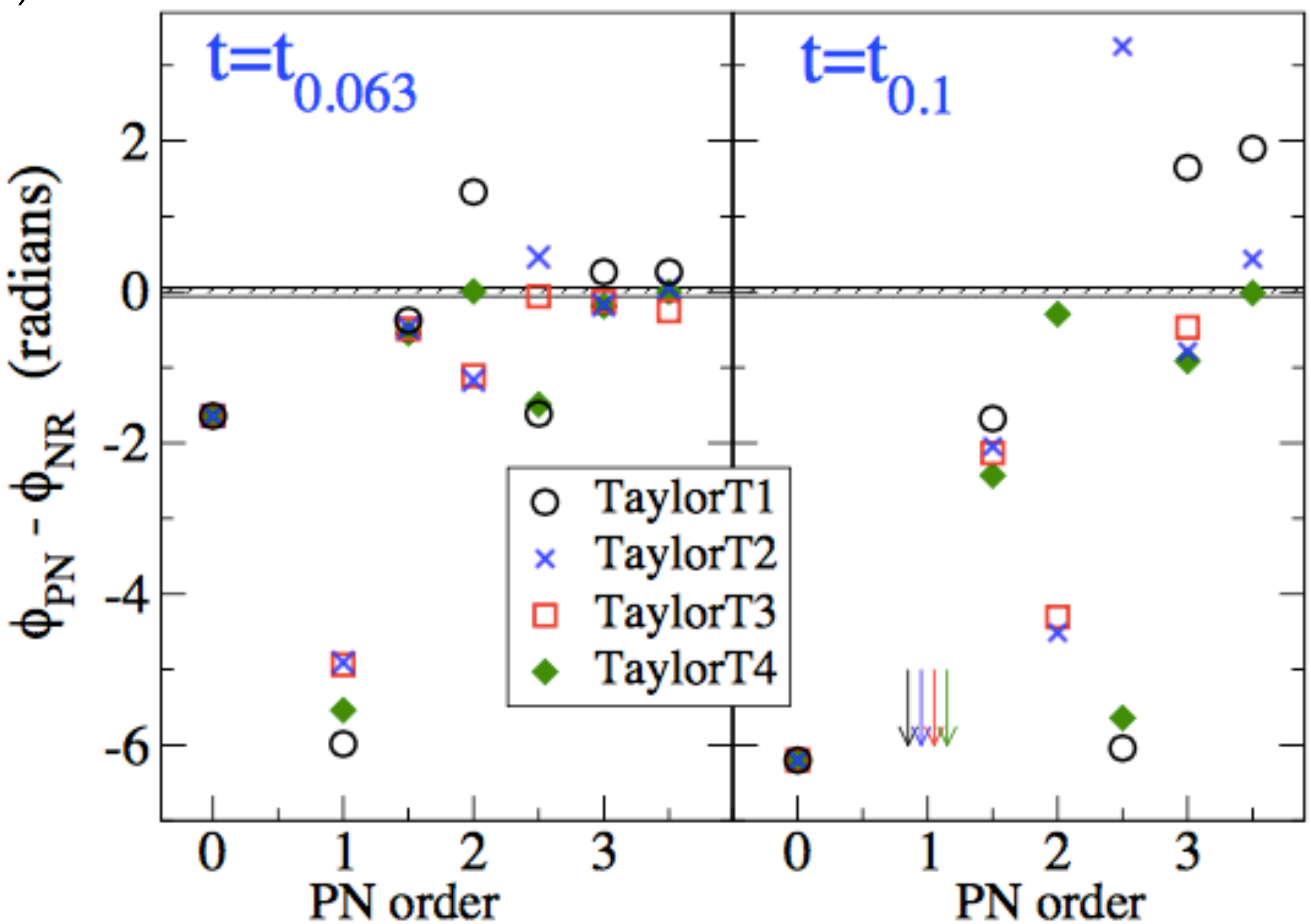
$$[\dot{\omega}(t)]_{2\mathrm{PN}} = -\frac{\mathcal{T}_{2\mathrm{PN}}\{F(\omega)\}}{\mathcal{T}_{2\mathrm{PN}}\{dE(\omega)/d\omega\}}$$

(see, e.g., AB, Iyer, Ochsner, Pan & Sathyaprakash 2011)

Comparing GW phase in NR and PN

(Boyle et al. 2007)

- Equal-mass, non-spinning binary

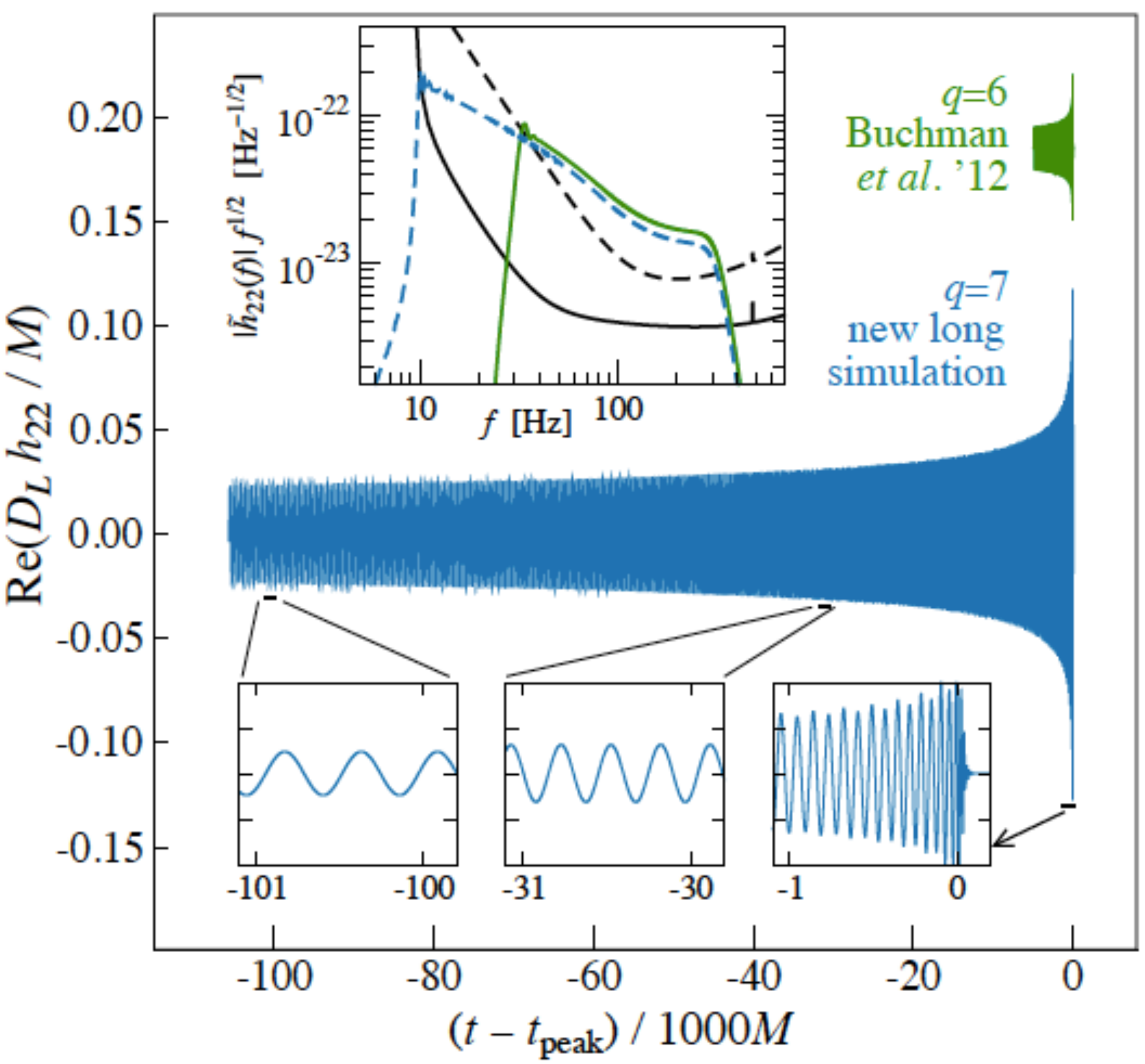


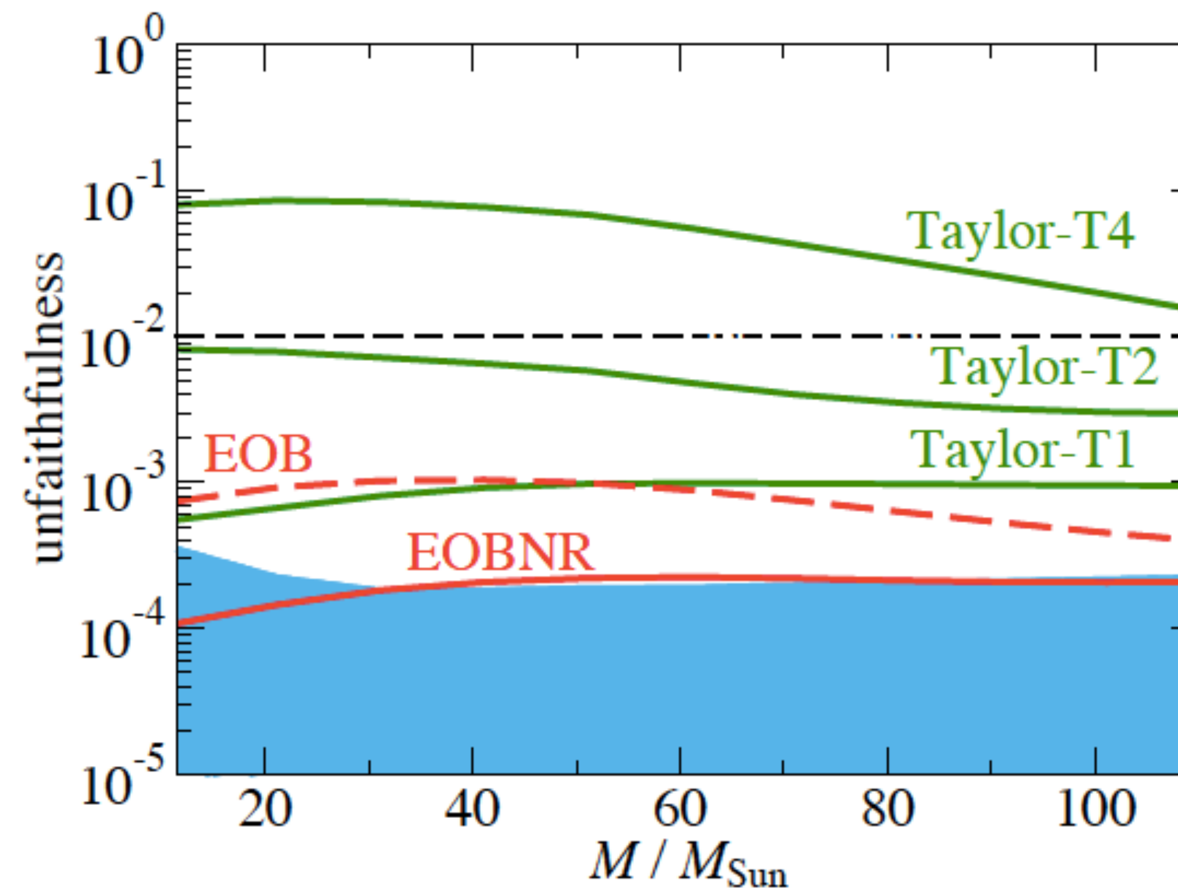
$$M = 20M_\odot \Rightarrow f^{\rm GW}_{0.063} = 98\,{\rm Hz},\ f^{\rm GW}_{0.1} = 161\,{\rm Hz},\ f^{\rm GW}_{\rm ISCO} = 220\,{\rm Hz}$$

$$M = 2.8M_\odot \Rightarrow f^{\rm GW}_{0.063} = 807\,{\rm Hz},\ f^{\rm GW}_{0.1} = 1350\,{\rm Hz},\ f^{\rm GW}_{\rm ISCO} = 1570\,{\rm Hz}$$

Comparing waveform models to ultralong NR waveform

- **376 GW cycles: 20 times longer than previously achieved**





- **NR waveform covers** *entire* **band for** $M > 45 M_\odot$

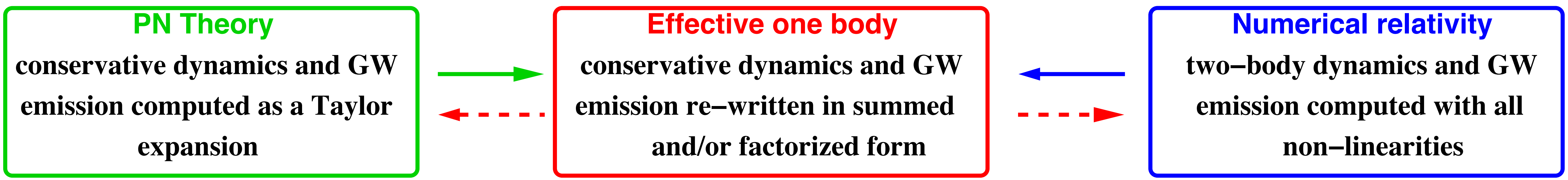
- **"EOBNR" means calibrated to much shorter NR waveforms with mass ratio** $\neq 7$

(Szilagyi, Blackman, AB, Taracchini et al. 15)

The effective-one-body (EOB) approach

- **EOB** approach **introduced before NR** breakthrough

(AB, Pan, Taracchini, Barausse, Bohe', Cotesta, Shao, Hinderer, Steinhoff, Vines; Damour, Nagar, Bernuzzi, Bini, Balmelli, Messina; Iyer, Sathyaprakash; Jaranowski, Schaefer)



- **EOB** model uses best information available in PN theory, but **resums PN terms** in suitable way to describe accurately dynamics and radiation during inspiral and plunge (going beyond quasi-circular adiabatic motion).

- **EOB** assumes **comparable-mass** description is **smooth deformation of test-particle limit**. It employs non-perturbative ingredients and **models analytically merger-ringdown** signal.

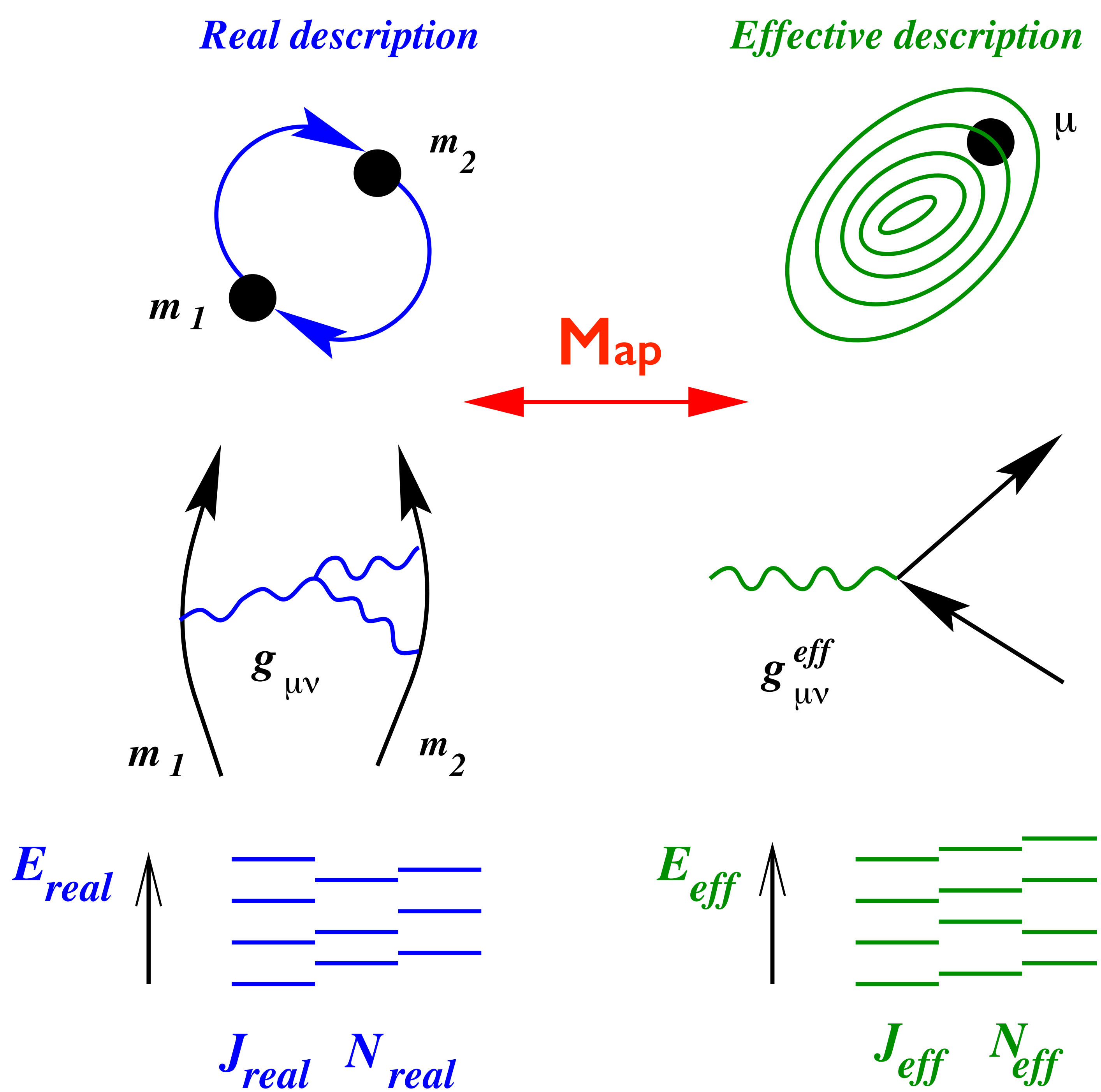
The effective-one-body approach in a nutshell

$$\nu = \frac{\mu}{M} \qquad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB model were **inspired by quantum** field **theory** when describing energy of comparable-mass charged bodies.



(AB & Damour 1998)

Energy for comparable-mass bodies

- Classical gravity: *(AB & Damour 98)*

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \left(\frac{E_{\rm eff}}{\mu}\right)$$

- Quantum electrodynamics: *(Brezin, Itzykson & Zinn-Justin 1970)*

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \frac{1}{\sqrt{1 + Z^2\,\alpha^2/(n-\epsilon_j)^2}}$$

- Considering scattering states:

$$\varphi(s) \equiv \frac{s - m_1^2 - m_2^2}{2m_2 m_2} = \frac{-(p_1 + p_2)^2 - m_1^2 - m_2^2}{2m_2 m_2} = -\frac{p_1 \cdot p_2}{m_1 m_2}$$

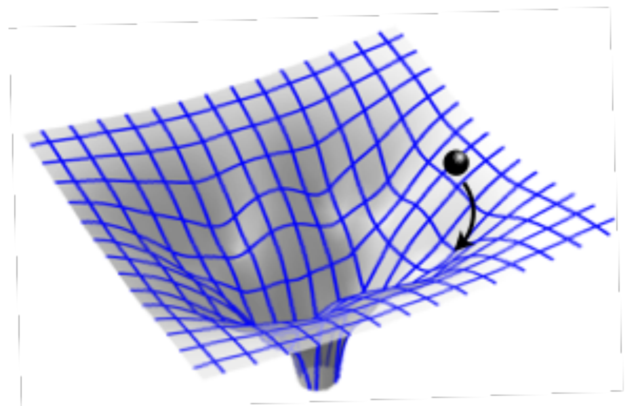
EOB Hamiltonian: resummed conservative dynamics (@2PN)

- Real Hamiltonian → - Effective Hamiltonian

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

$$H_{\rm eff}^{\nu} = \mu \sqrt{A_\nu(r)\left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\frac{p_r^2}{\mu^2}\right]}$$

$$ds^2_{\rm eff} = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$



(credit: Hinderer)

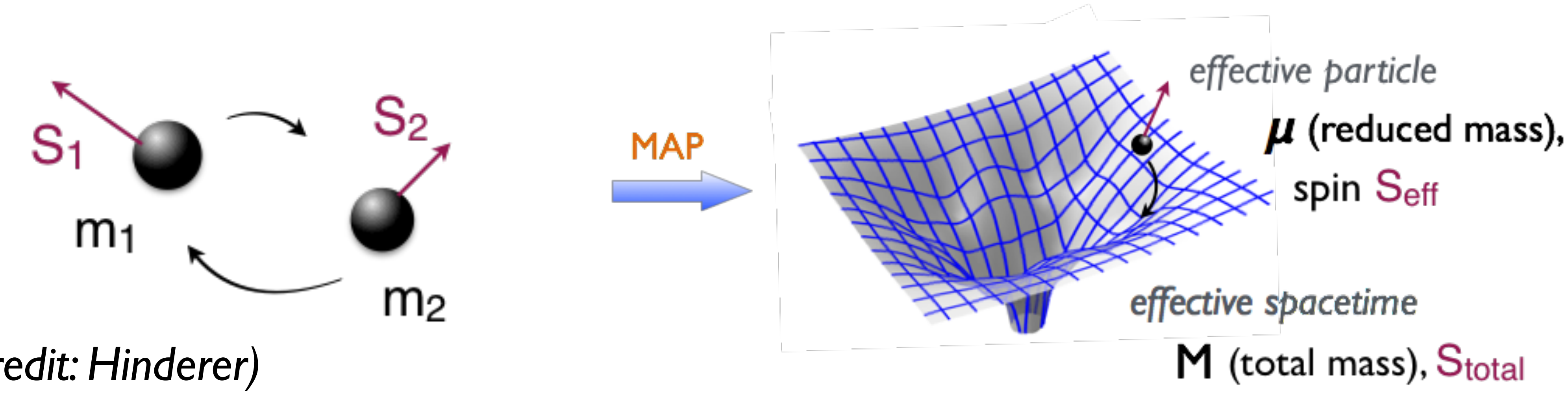
- EOB Hamiltonian: $H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$

- Dynamics condensed in $A_\nu(r)$ and $B_\nu(r)$

- $A_\nu(r)$, which encodes the energetics of circular orbits, is quite simple:

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

EOB resummed spin dynamics & waveforms



(credit: Hinderer)

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$

- $H_{\rm eff}{}^{\nu}$ with spins, two EOB resummations:

  (Barausse, Racine & AB 09; Barausse & AB 10, 11)

  (Damour 01, Damour, Jaranowski & Schäfer 08; Damour & Nagar 14)

- EOB equations of motion *(AB et al. 00, 05; Damour et al. 09)*:

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{p}} \qquad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{r}} + \mathbf{F} \qquad \dot{\mathbf{S}} = \{\mathbf{S}, H_{\rm real}^{\rm EOB}\}$$

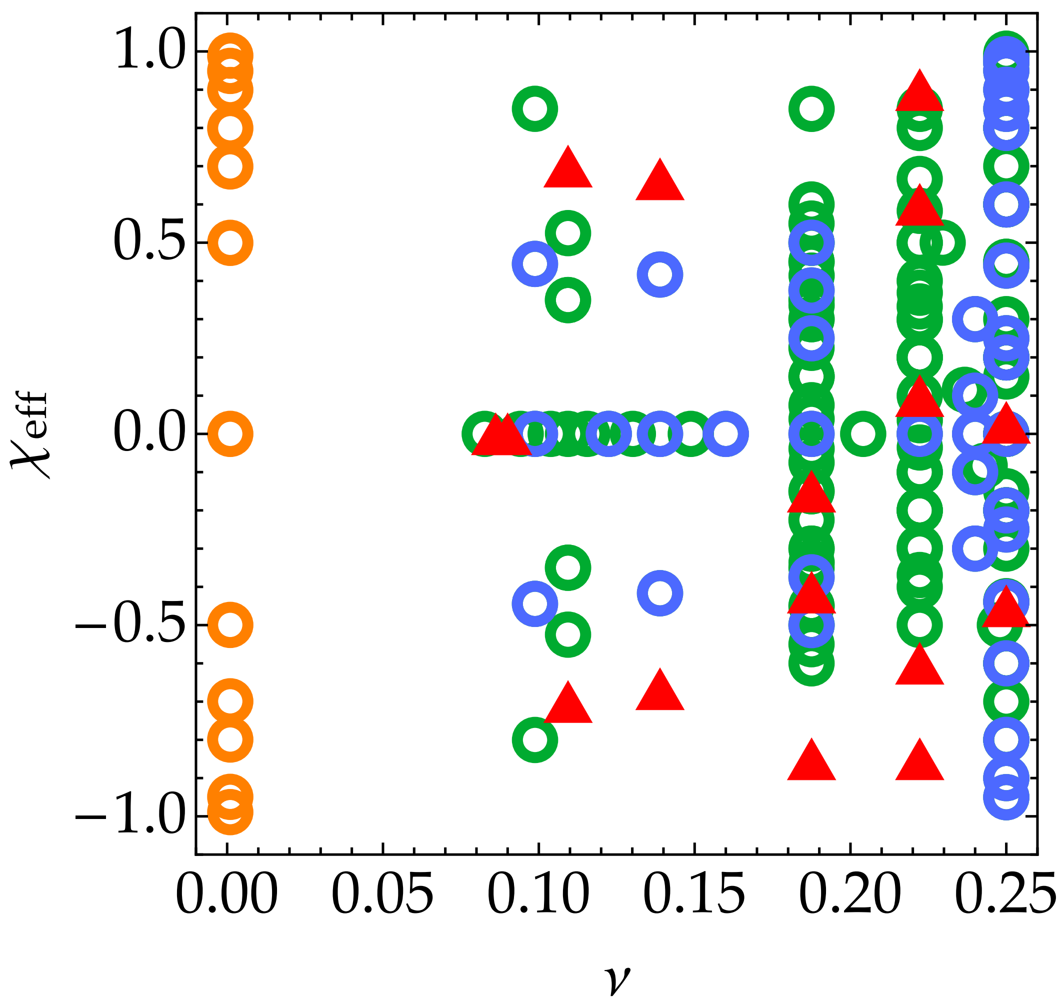
- EOB waveforms *(AB et al. 00; Damour et al. 09; Pan, AB et al. 11)*:

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

Waveforms combining analytical & numerical relativity

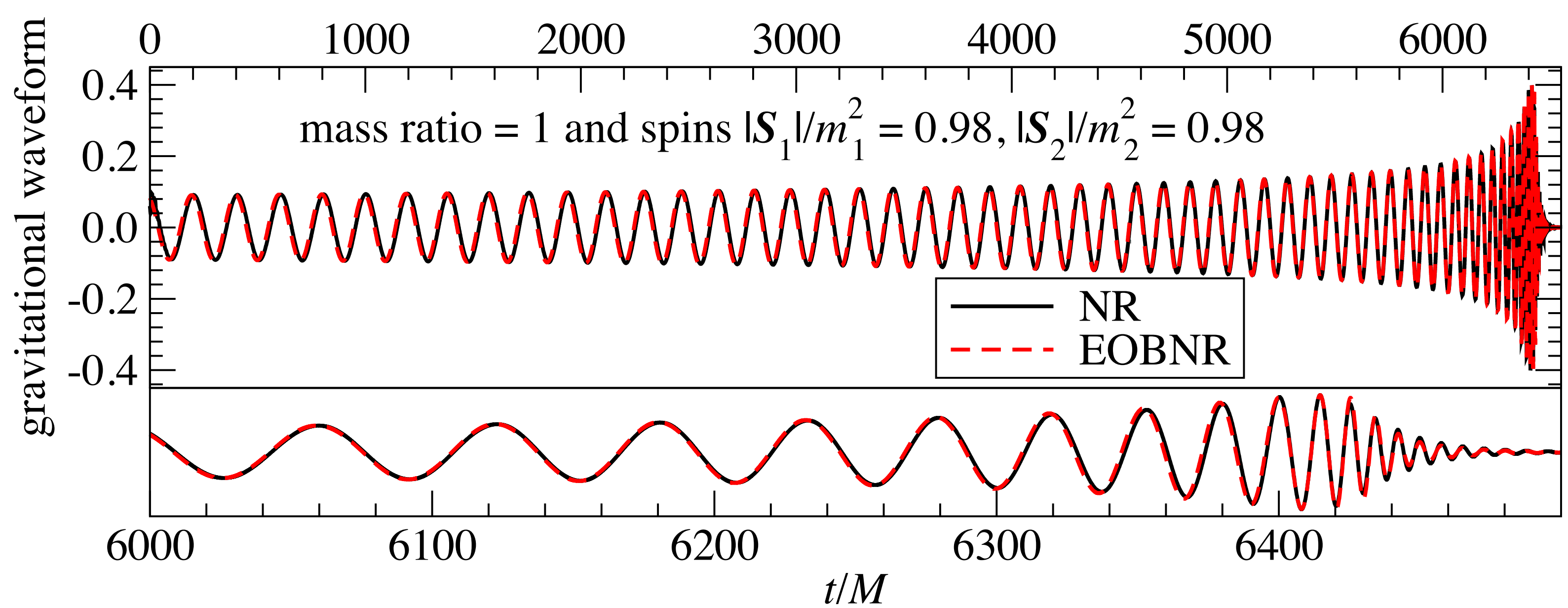
- **Effective-one-body** (EOB) theory & NR (EOBNR)

141 SXS simulations



(Pan, AB et al. 13, Taracchini, AB, Pan, Hinderer & SXS 14, Puerrer 15)

(Bohe', Shao, Taracchini, AB & SXS 16, Babak et al. 16)



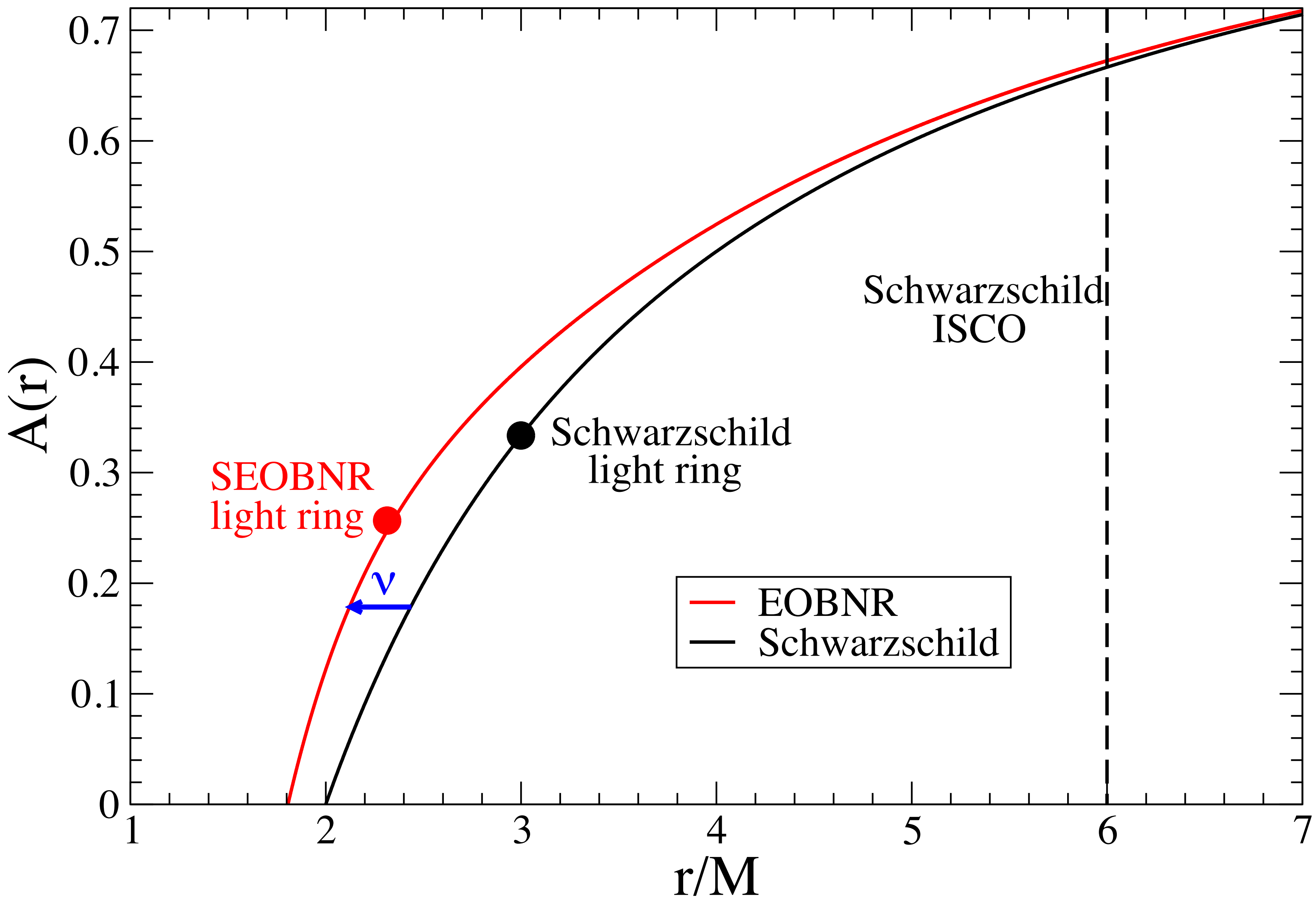
- **Inspiral-merger-ringdown phenomenological** waveforms fitting **EOB & NR** (IMRPhenom) *(Khan et al. 16, Hannam et al 16)*

(If PN were used instead, accuracy will degrade, because of "gap" between PN and NR)

Strong-field effects in binary black holes included in EOB

Finite mass-ratio effects make **gravitational** interaction **less attractive**

(Taracchini, AB, Pan, Hinderer & SXS 14)



$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

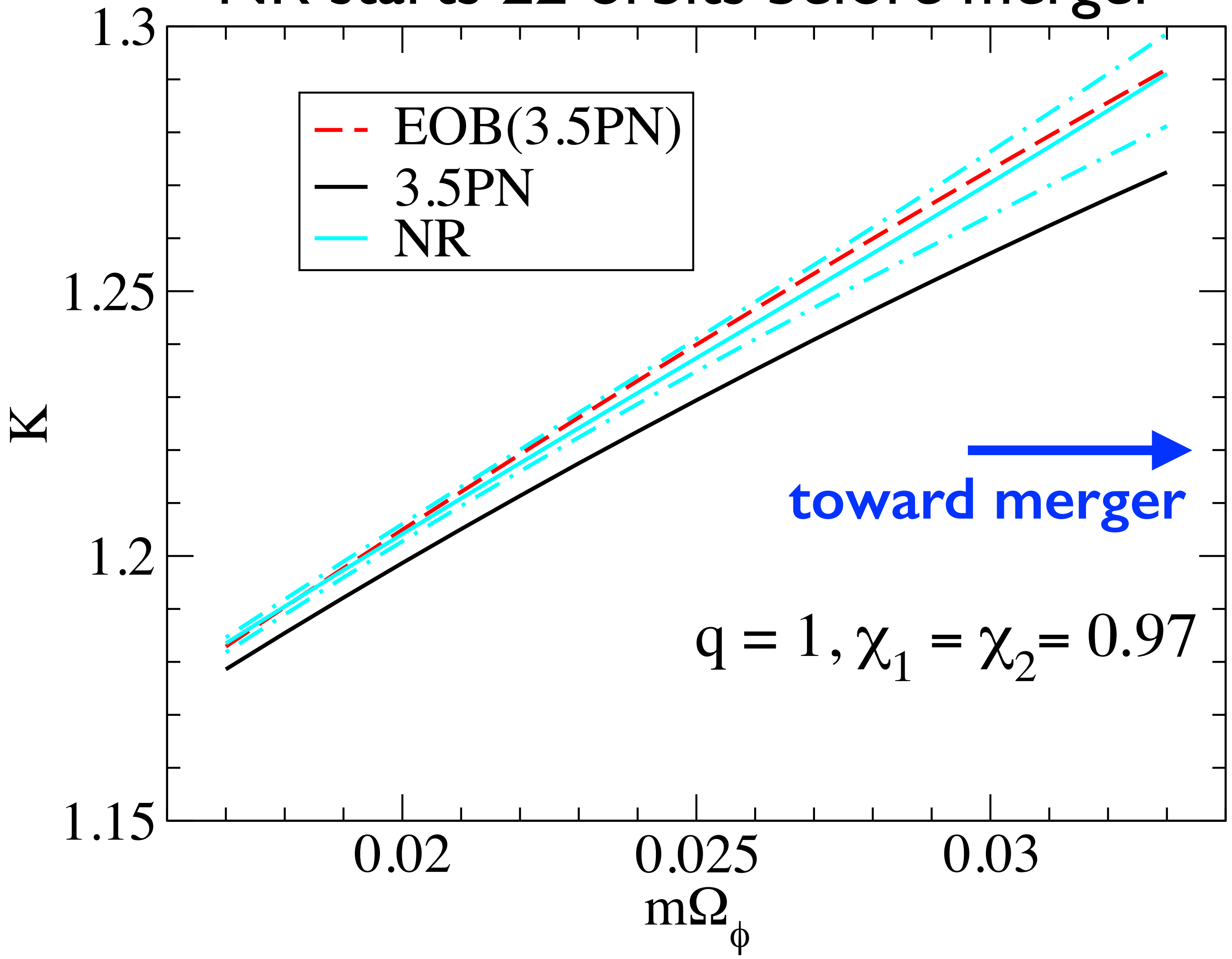
Comparing NR, PN & EOB beyond waveforms

- **Periastron advance**

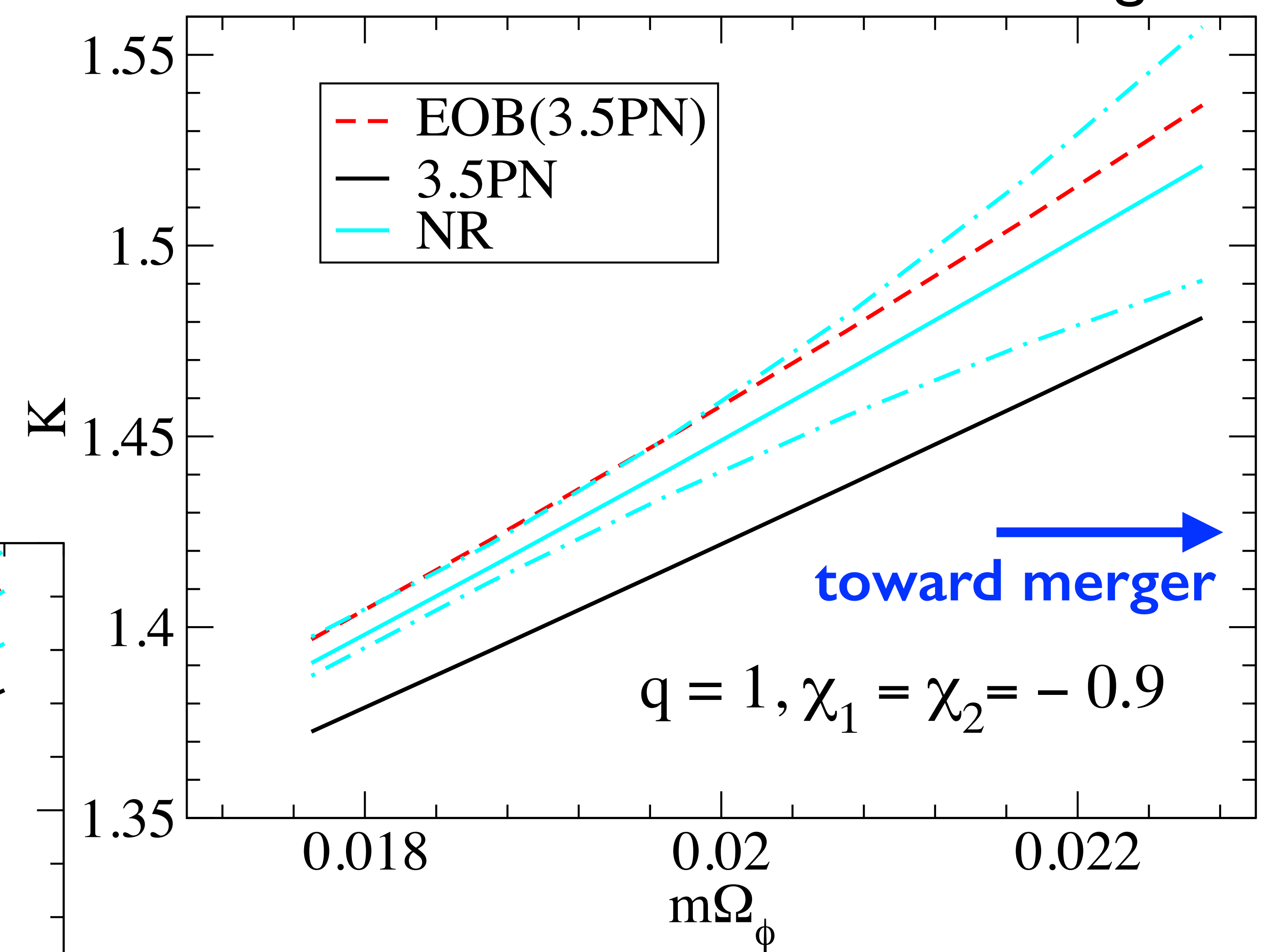
$$K = 1 + \frac{\Delta\Phi}{2\pi} = \frac{\Omega_\phi}{\Omega_r}$$

(Hinderer et al. 13, Le Tiec et al. 11, 13)

NR starts 22 orbits before merger

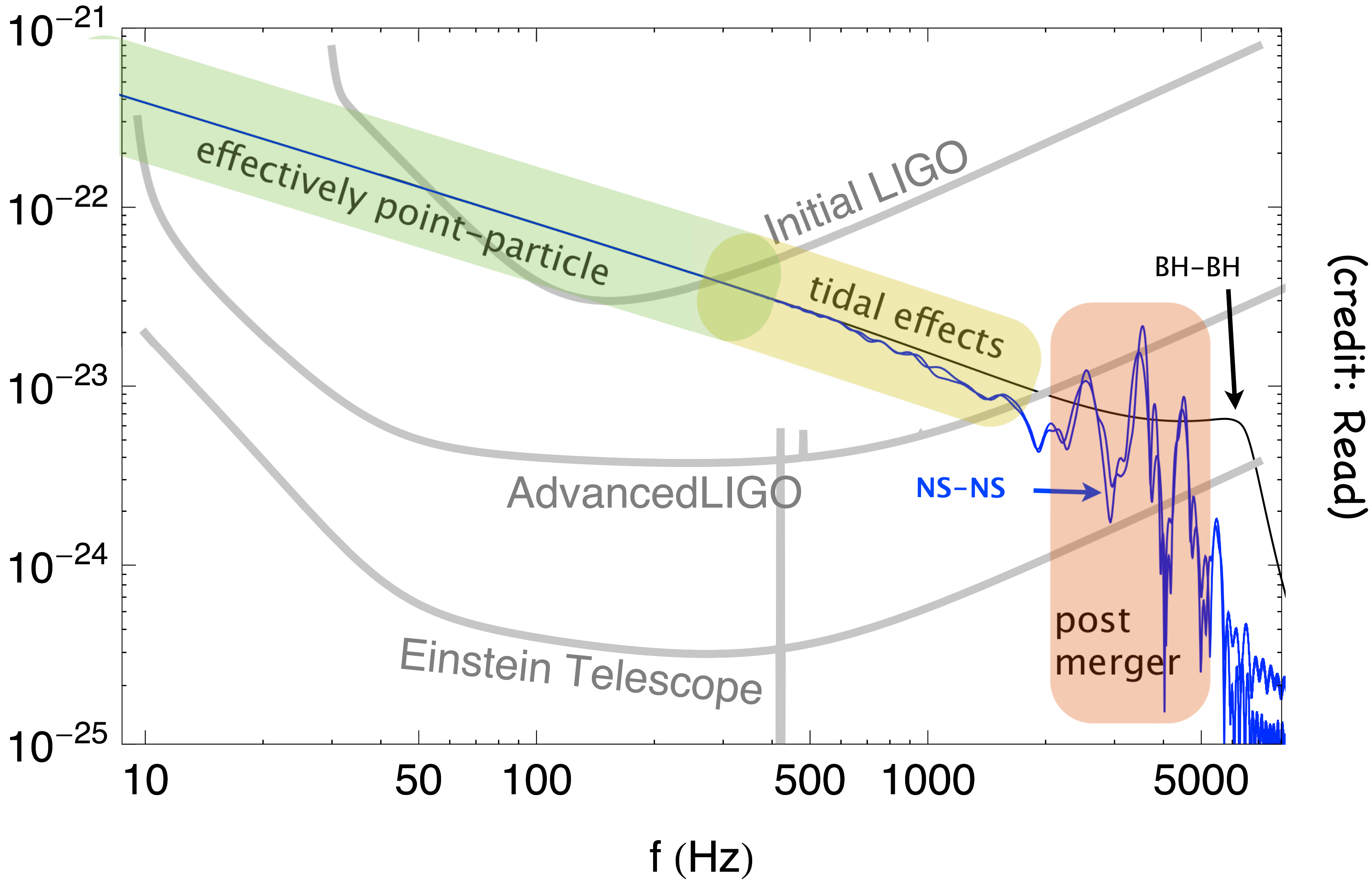


NR starts 12 orbits before merger



- **Energy/angular momentum** *(Damour et al. 11, Le Tiec et al. 11, Ossokine, Dietrich et al. 17)*
- **Scattering angle** of hyperbolic encounters *(Damour et al. 14)*

Probing equation of state of neutron stars



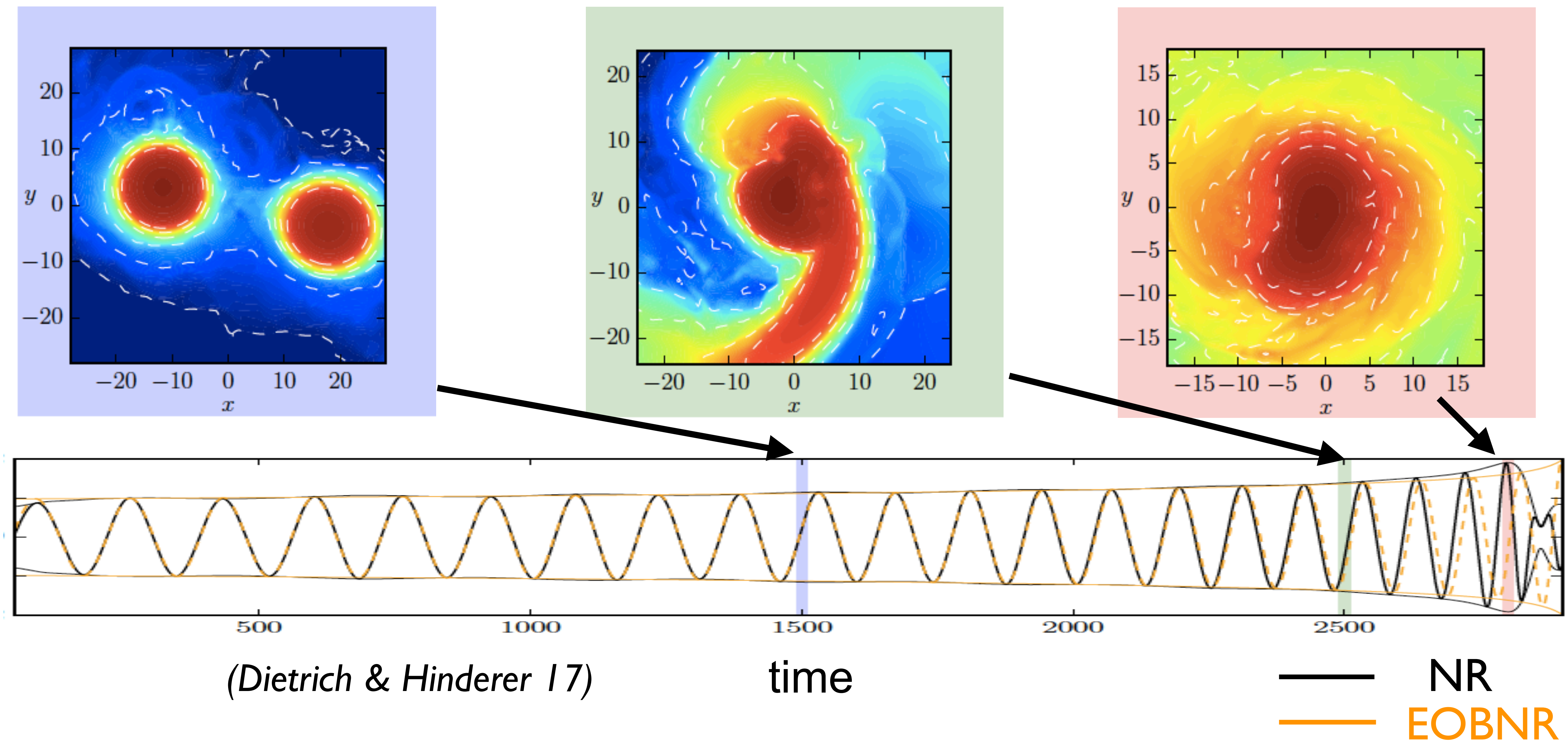
- **Tidal effects imprinted** on gravitational waveform during inspiral through **parameter** λ

- λ **measures** star's **quadrupole deformation** in response to companion **perturbing tidal field:**

$$Q_{ij} = -\lambda\, \mathcal{E}_{ij}$$

State-of-art waveform models for binary neutron stars

- Synergy between **analytical** and **numerical work** is **crucial.**

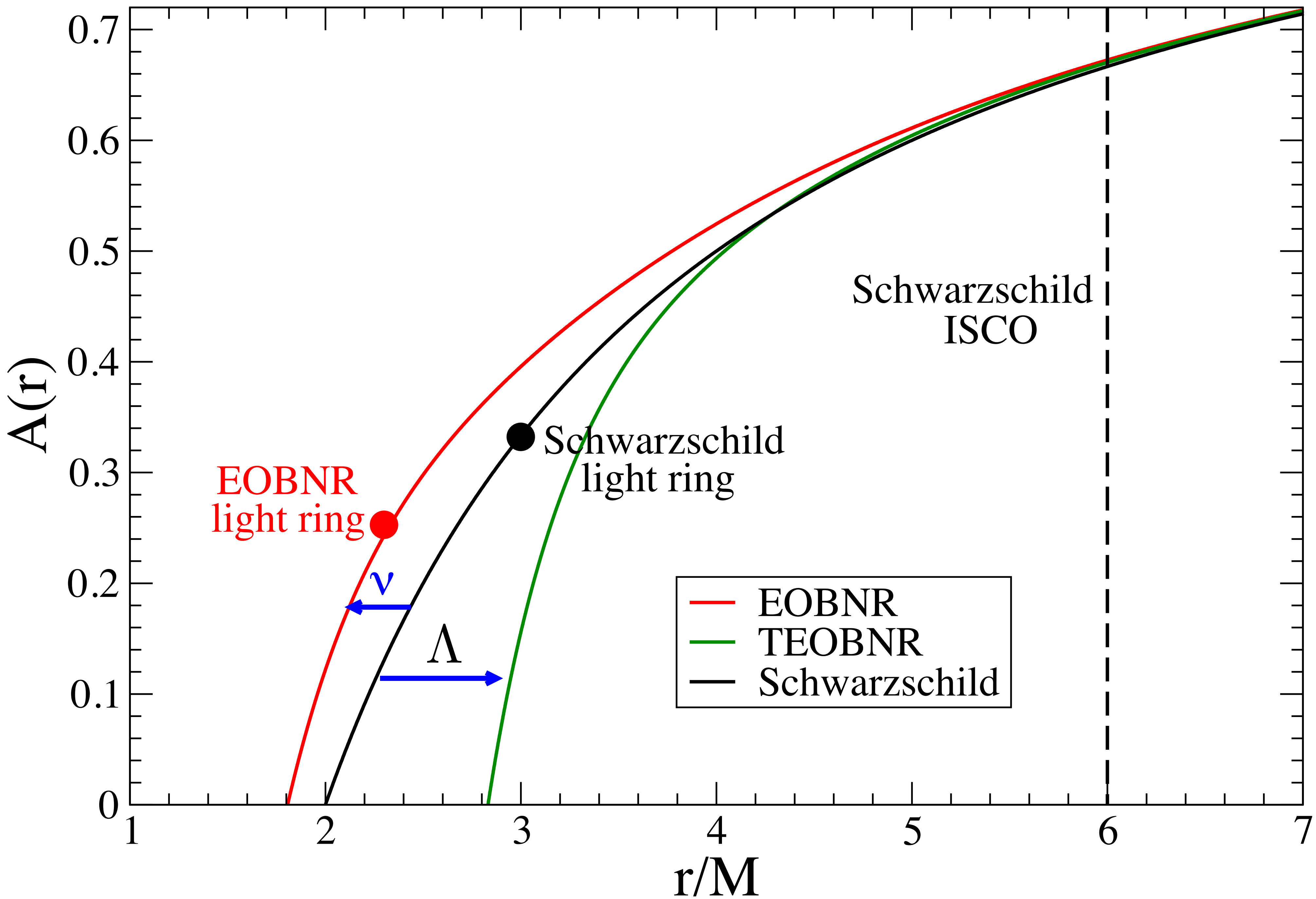


(Dietrich & Hinderer 17)

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer et al. 16, Steinhoff et al. 16, Dietrich et al. 17-18, Nagar et al. 18)

Strong-field effects in presence of matter in EOB theory

$$A(r) = A_\nu(r) + A_{\rm tides}(r)$$



(Hinderer et al. 2016, Steinhoff et al. 2016, see also Bernuzzi et al. 15)

Tides make **gravitational** interaction **more attractive**

PN templates for compact-object binary inspirals

$$\tilde{h}(f) = \mathcal{A}_{\rm SPA}(f)\, e^{i\psi_{\rm SPA}(f)}$$

$$\mathcal{M} = \nu^{3/5}\, M$$

$$\psi_{\rm SPA}(f) = 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\{1+$$
$$-\frac{5\hat{\alpha}^2}{336\omega_{\rm BD}}\nu^{2/5}(\pi\mathcal{M}f)^{-2/3} - \frac{128}{3}\frac{\pi^2 D\,\mathcal{M}}{\lambda_g^2(1+z)}(\pi\mathcal{M}f)^{2/3}$$
$$+\left(\frac{3715}{756}+\frac{55}{9}\nu\right)\nu^{-2/5}(\pi\mathcal{M}f)^{2/3} - 16\pi\,\nu^{-3/5}(\pi\mathcal{M}f) + 4\beta\,\nu^{-3/5}(\pi\mathcal{M}f)$$
$$+\left(\frac{15293365}{508032}+\frac{27145}{504}\nu+\frac{3085}{72}\nu^2\right)\nu^{-4/5}(\pi\mathcal{M}f)^{4/3} - 10\sigma\,\nu^{-4/5}(\pi\mathcal{M}f)^{4/3}$$
$$\cdots - \frac{39}{2}\nu^{-2}\tilde{\Lambda}(\pi\mathcal{M}f)^{10/3}\Big\}$$

0PN

dipole radiation → -1PN

graviton with non zero mass → 1PN

1PN

1.5PN

spin-orbit → 1.5PN

2PN

spin-spin → 2PN

5PN

tidal

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1 + (m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \frac{\lambda}{m_{\rm NS}^5} = \frac{2}{3}k_2\left(\frac{R_{\rm NS}c^2}{Gm_{\rm NS}}\right)^5$$

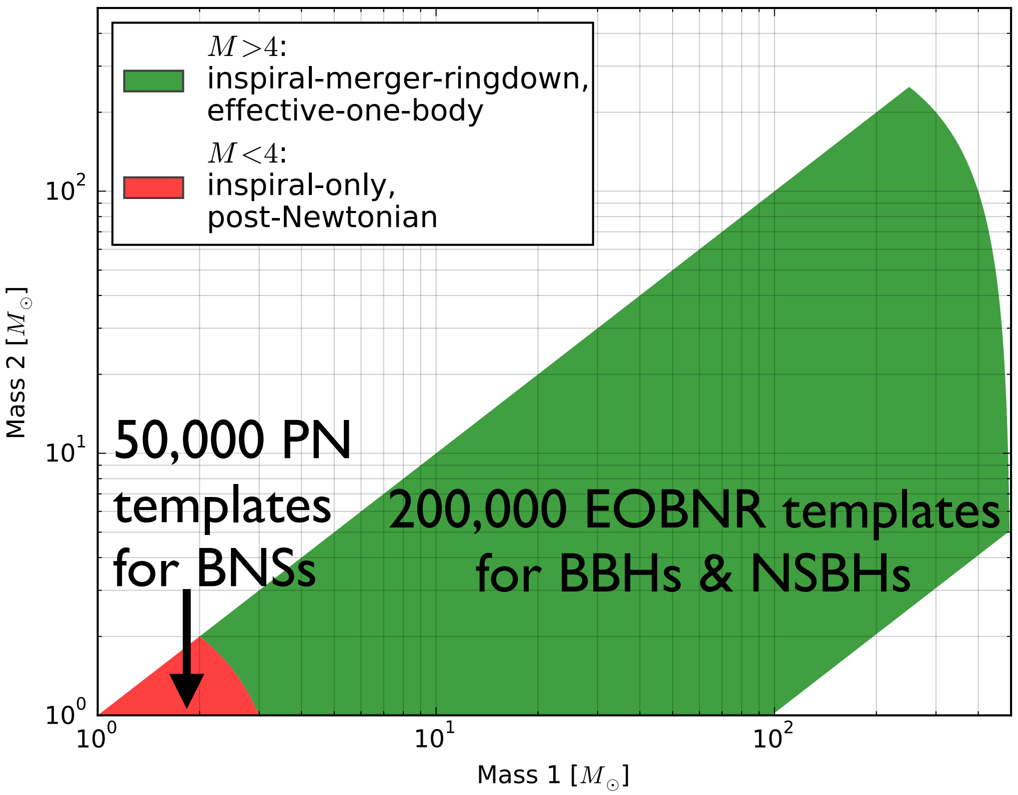
Depends on EOS & compactness → k_2

it can be large → $\frac{R_{\rm NS}c^2}{Gm_{\rm NS}}$

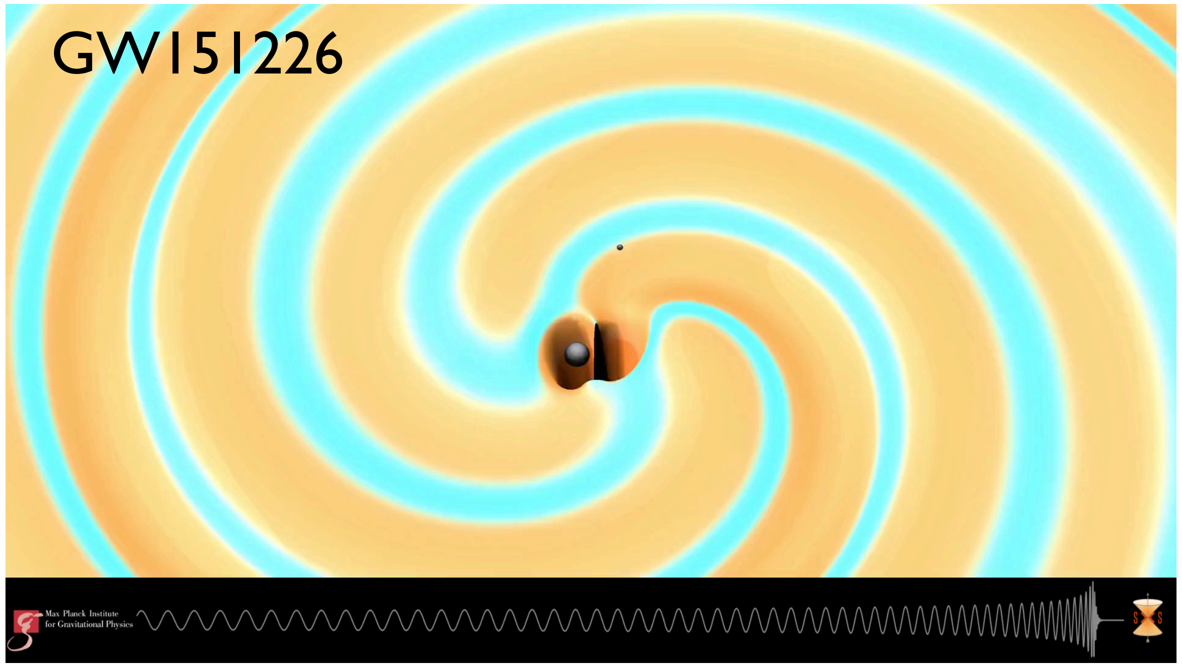
Template bank of modeled LIGO search, follow up studies

- **Matched filtering** employed

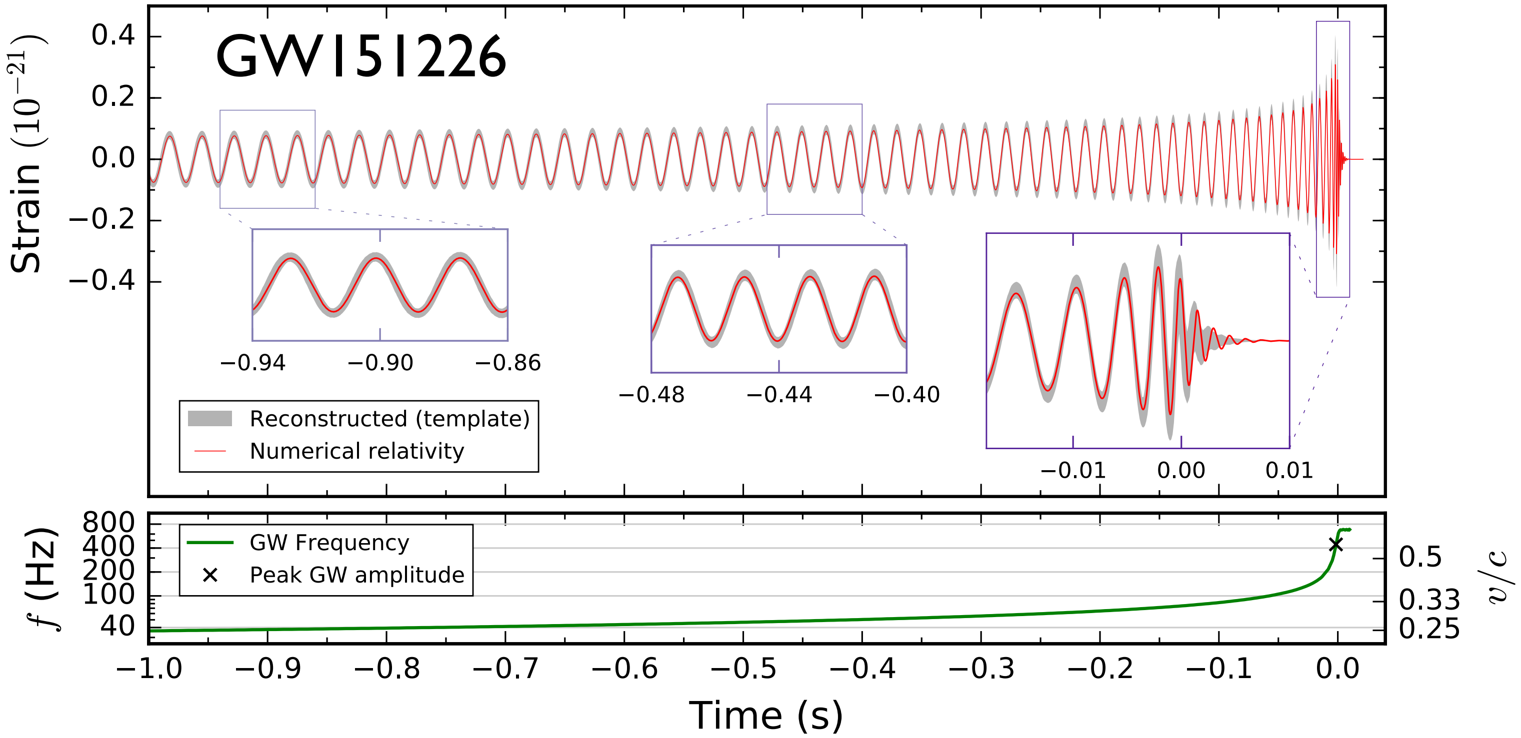
(Dal Canton & Harry 17)



(visualization credit: Dietrich, Haas @AEI)
(Ossokine, AB & SXS project)

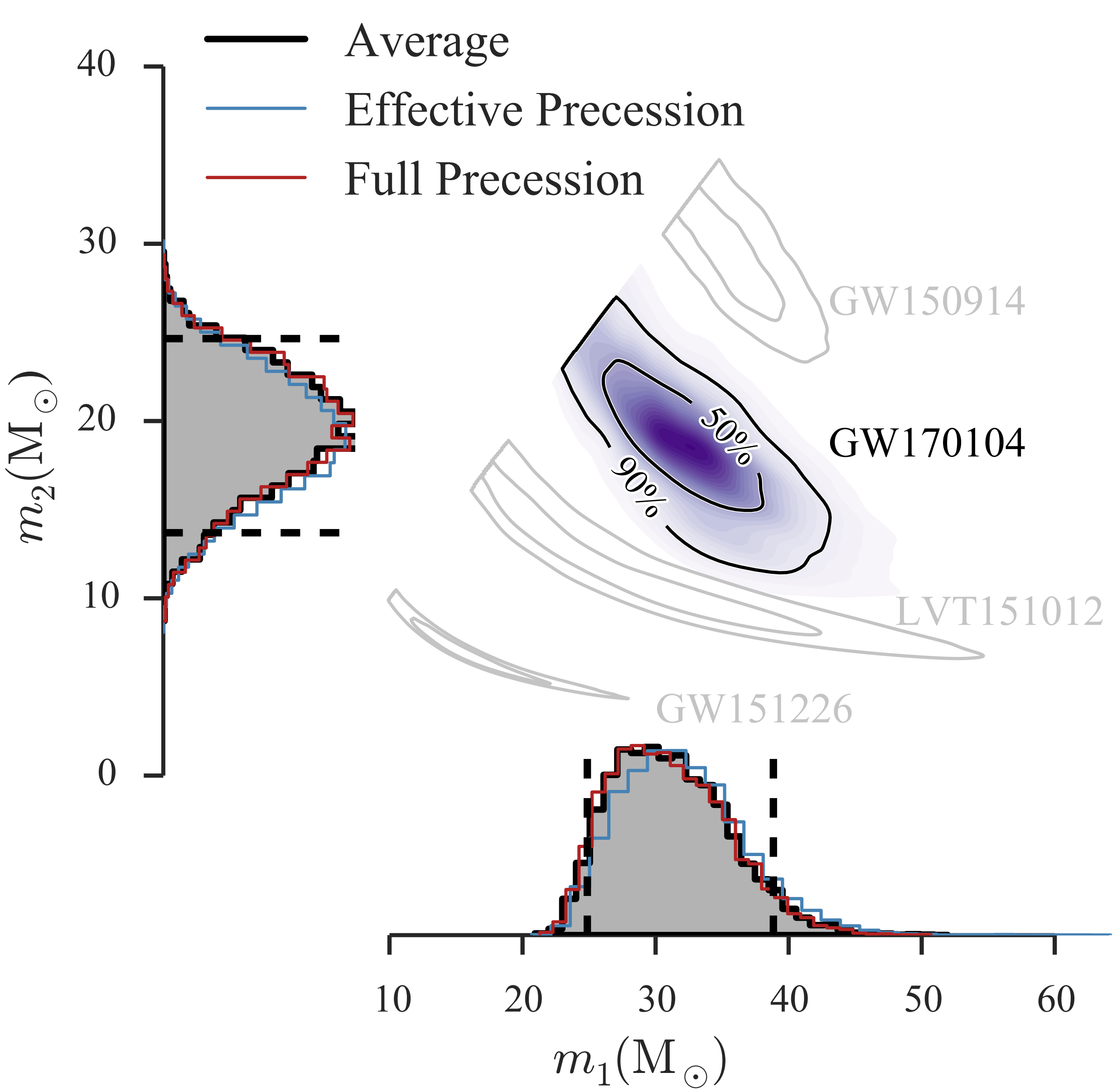


(Abbott et al. PRL 116 (2016) 241103)

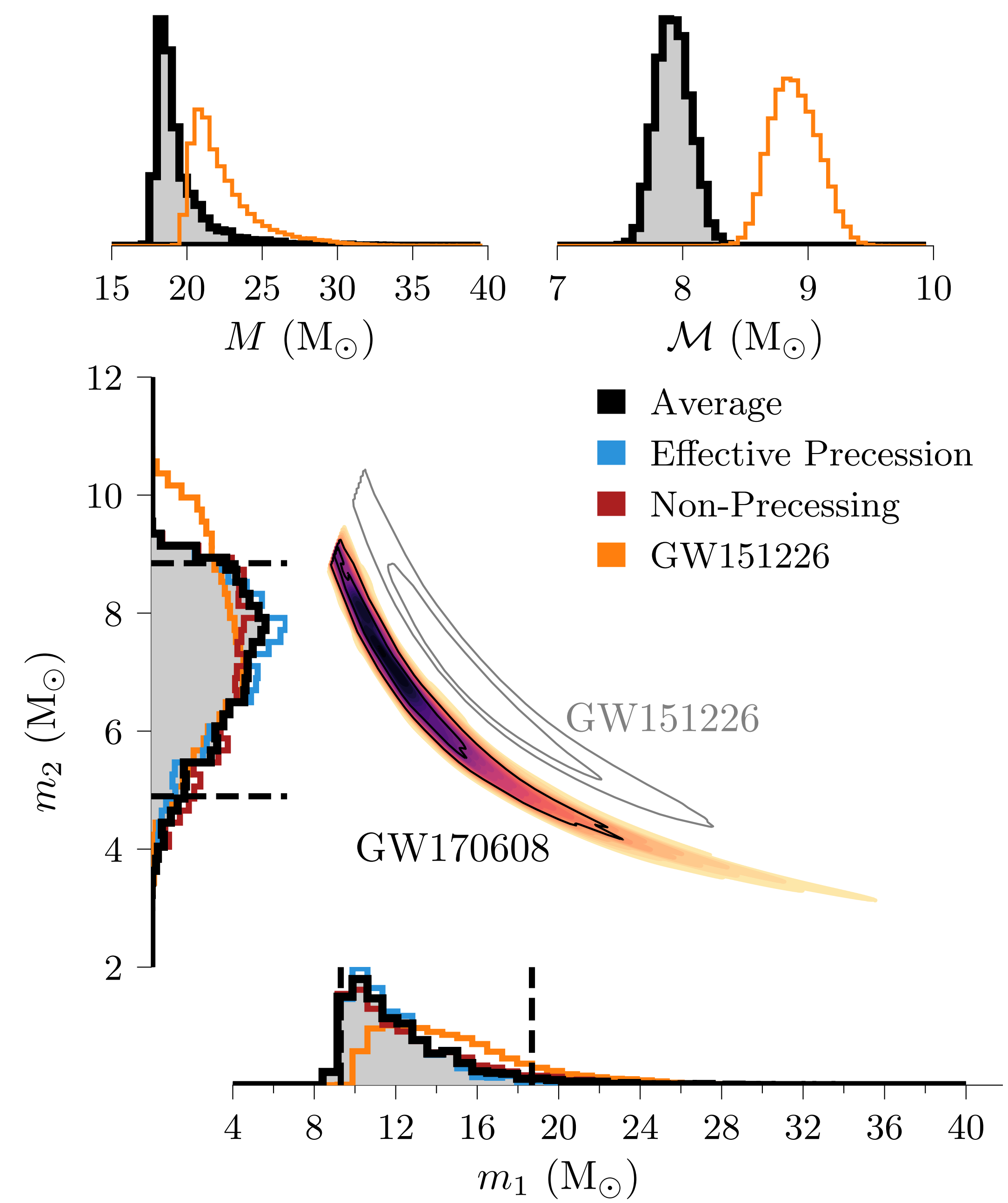


Unveiling binary black-hole properties: masses

(Abbott et al. PRL 118 (2017) 221101)



(Abbott et al. ApJ 851 (2017) L35)



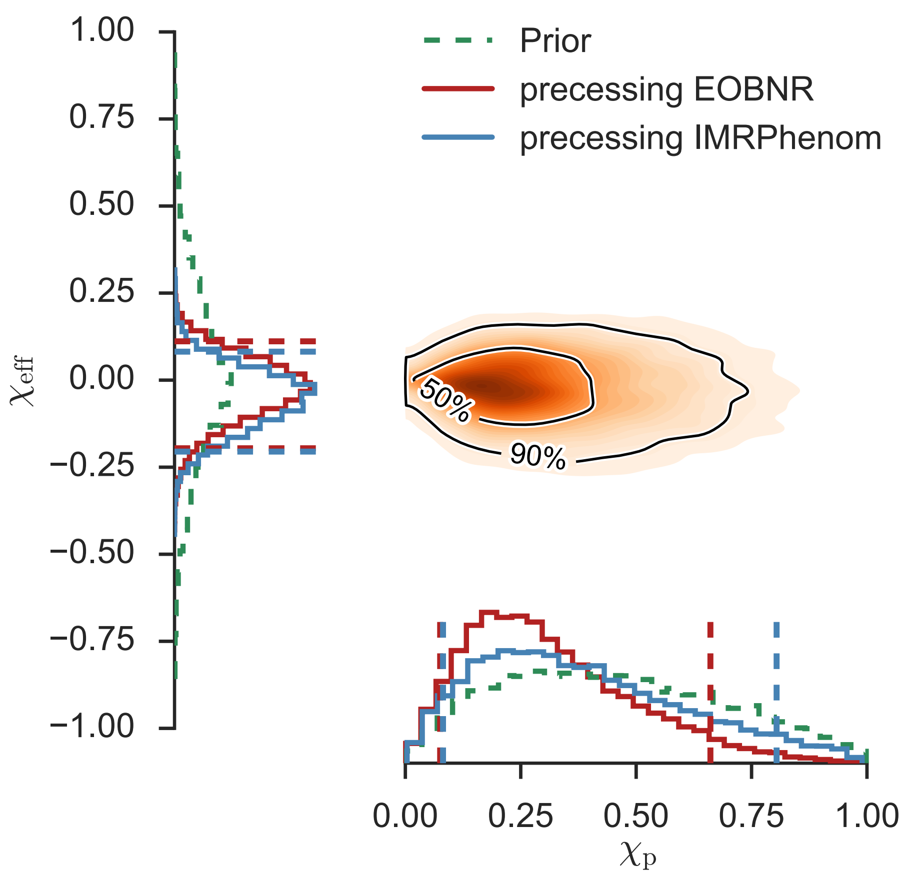
- **Current measurements of masses dominated by statistical error.**

Unveiling binary black-hole properties: spins

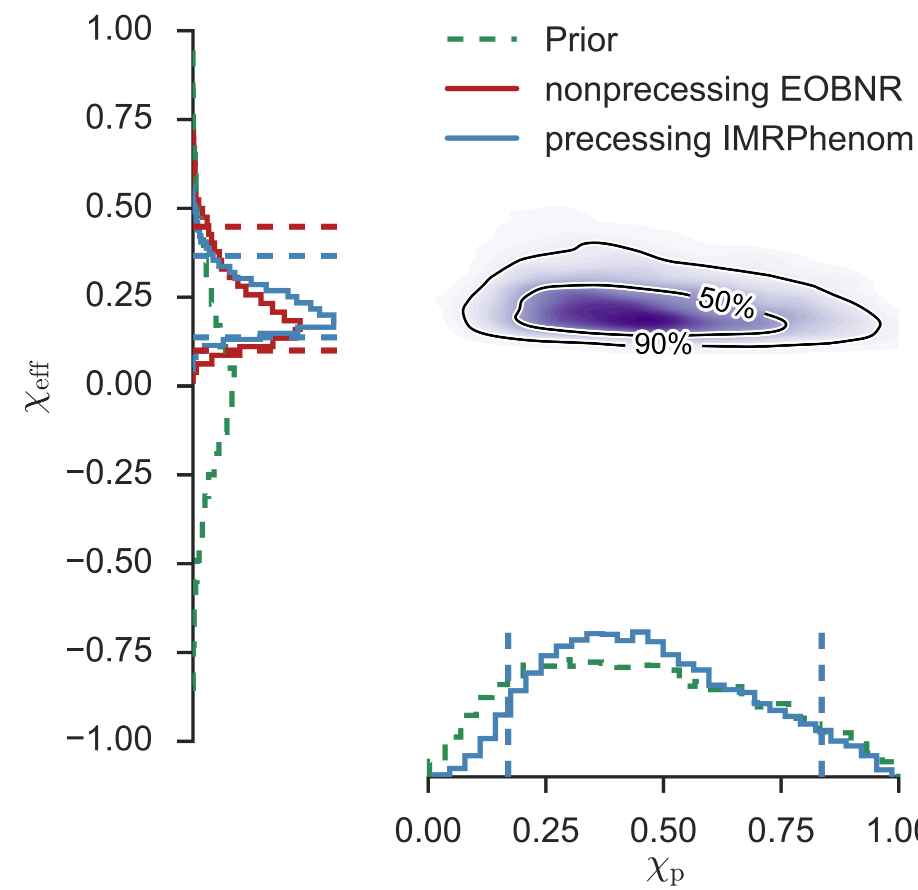
(measurements @ 25Hz)

(Abbott et al. PRX 6 (2016) 041014)

GW150914



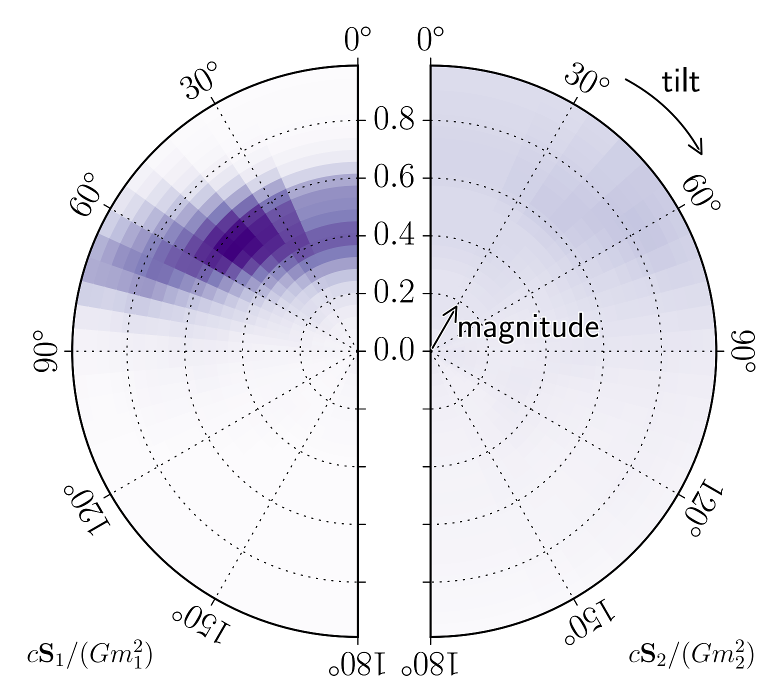
GW151226



(Abbott et al. PRL 116 (2016) 241103)

$$\chi_{\rm eff} = \frac{c}{GM}\left(\frac{\boldsymbol{S}_1}{m_1}+\frac{\boldsymbol{S}_2}{m_2}\right)\cdot\frac{\boldsymbol{L}}{|\mathbf{L}|}$$

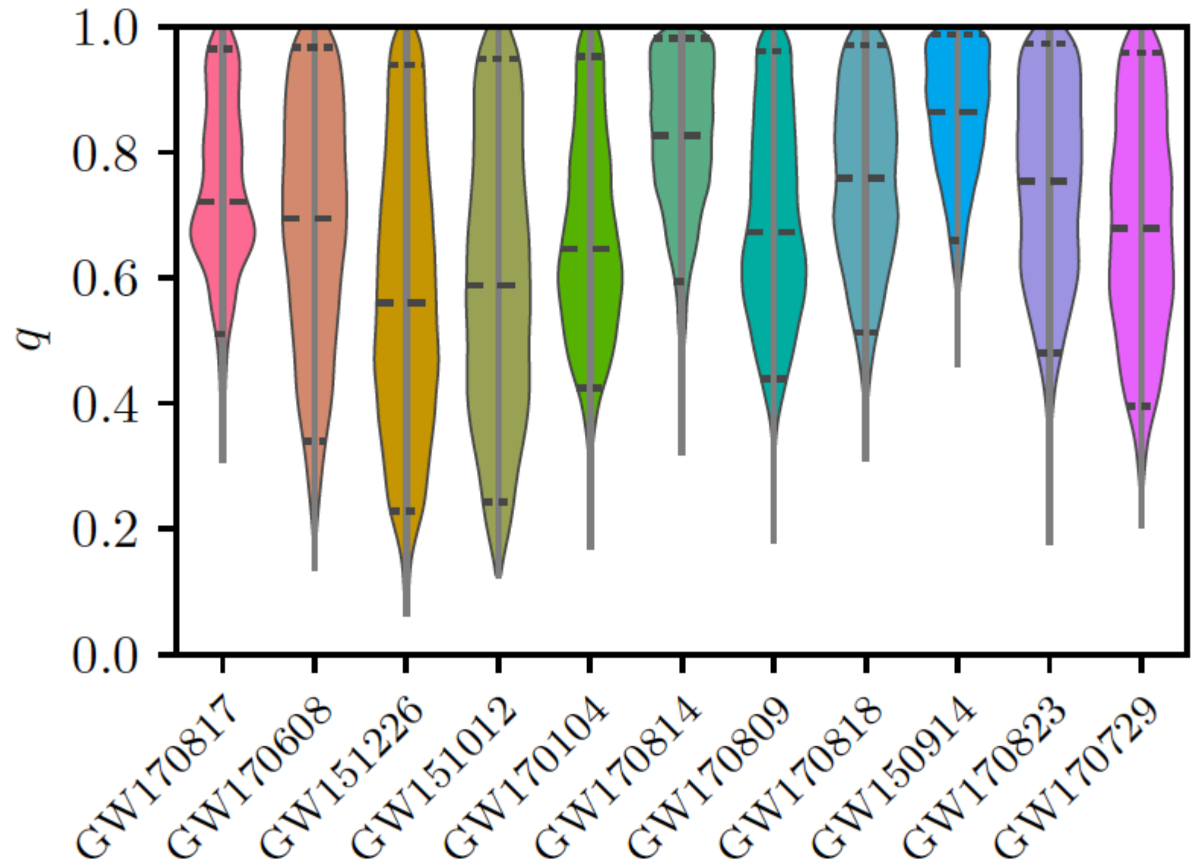
(constant through 2PN order)

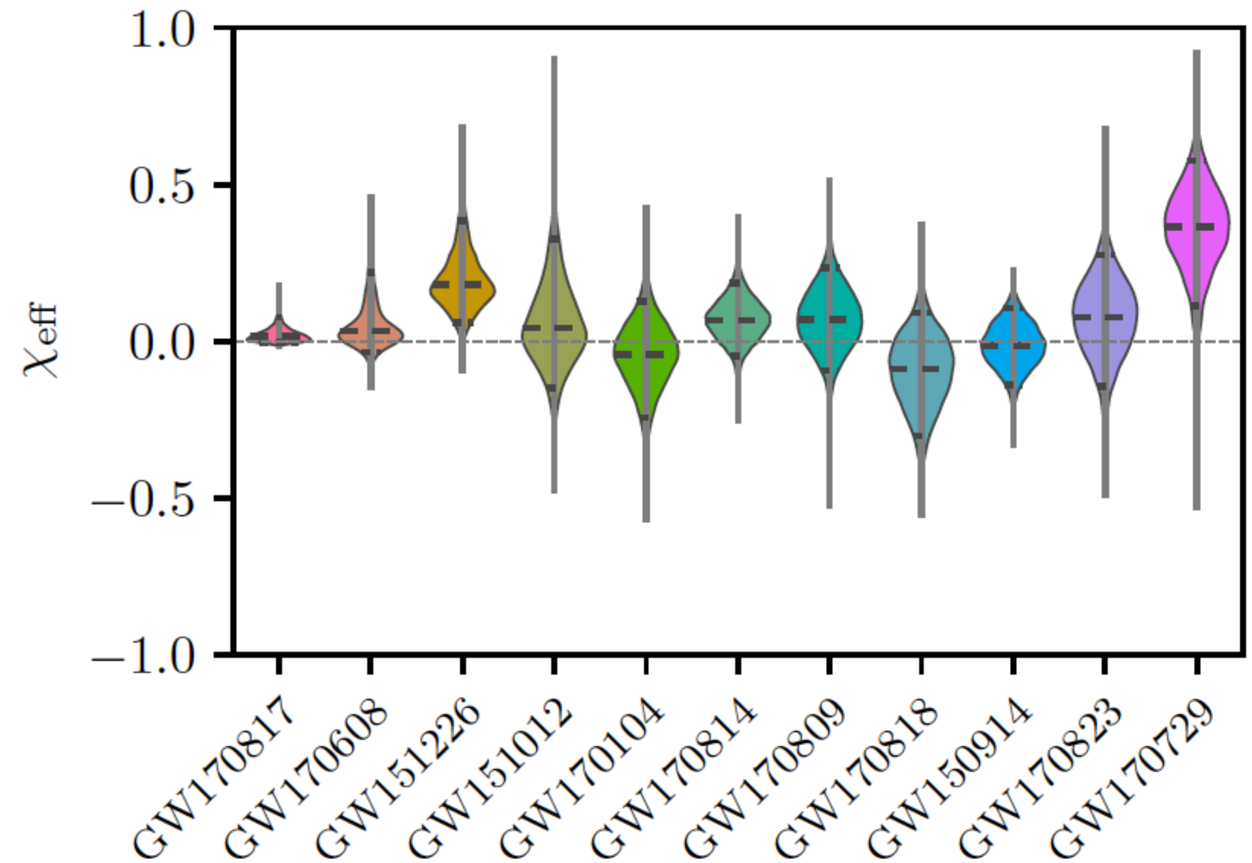


- **Current measurements of masses dominated by statistical error.**

Unveiling binary black-hole properties: results GWTC-1

(Abbott et al. arXiv:1811.12907)





- Current **measurements** of masses and spins for GWTC-1 **dominated by statistical instead of modeling** error.
- **Inferences** obtained combining the **effective (IMRPhenom)** and **full (SEOBNR) precessing** waveform models.

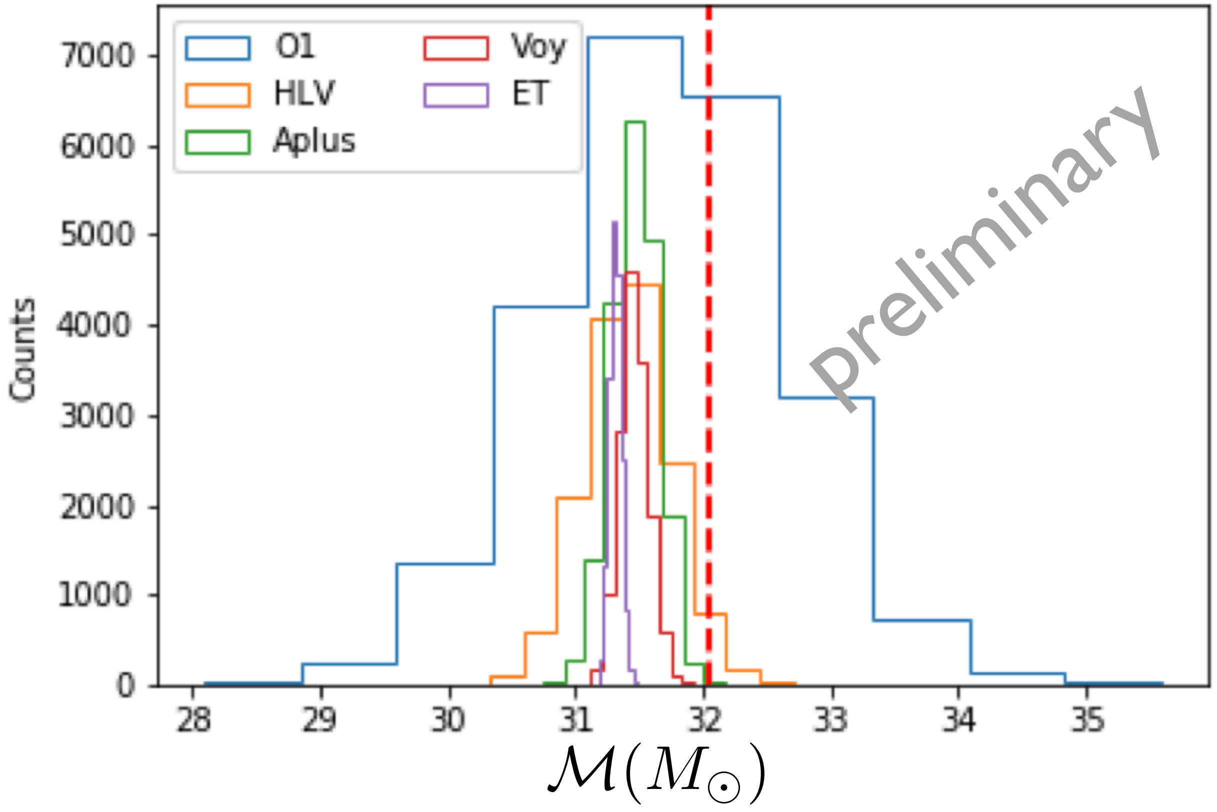
Systematics due to modeling for GW150914-like @ aLIGO

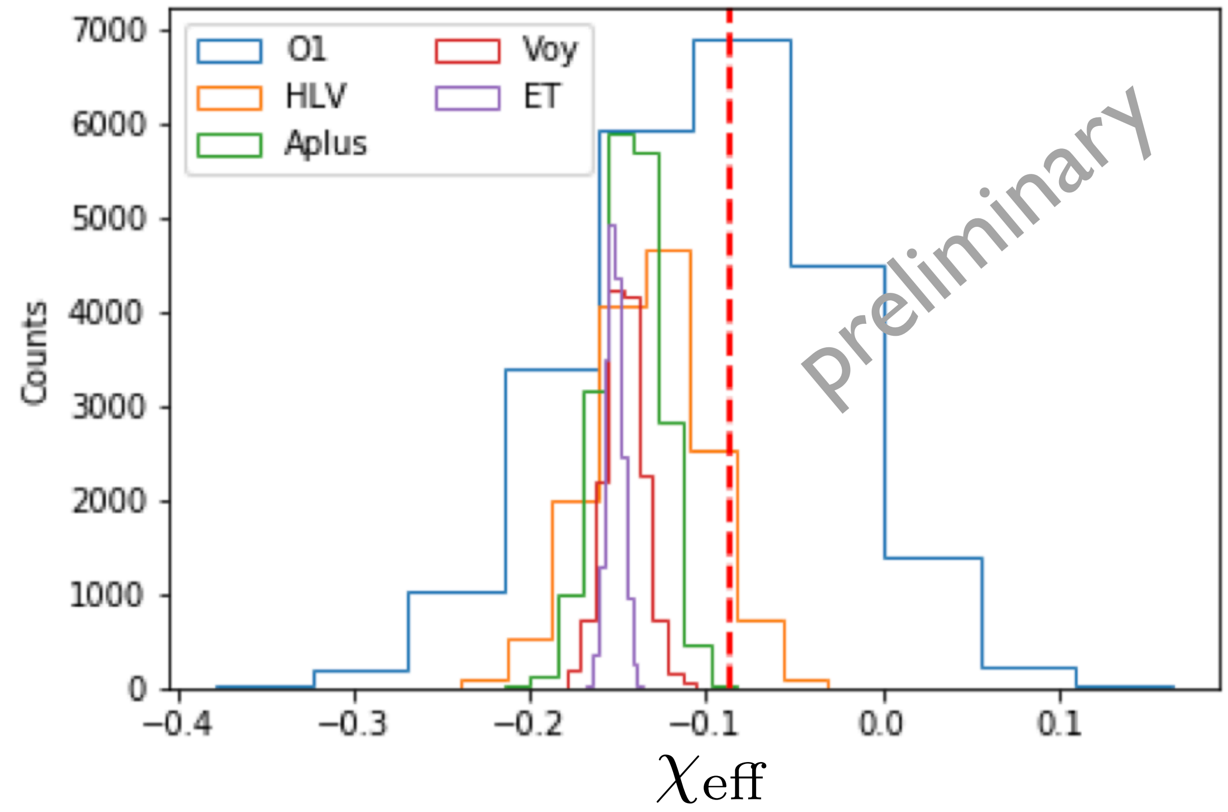
- Synthetic GW signal of a **binary black hole** at **400 Mpc** is **injected** in Gaussian noise with **aLIGO design-sensitivity** noise-spectral density **(SNR ~ 70)**.

- **Inference** with one of currently used waveform models (IMRPhenom).

(Pürrer & Haster in prep)

- **GW150914-like NR signal** is injected

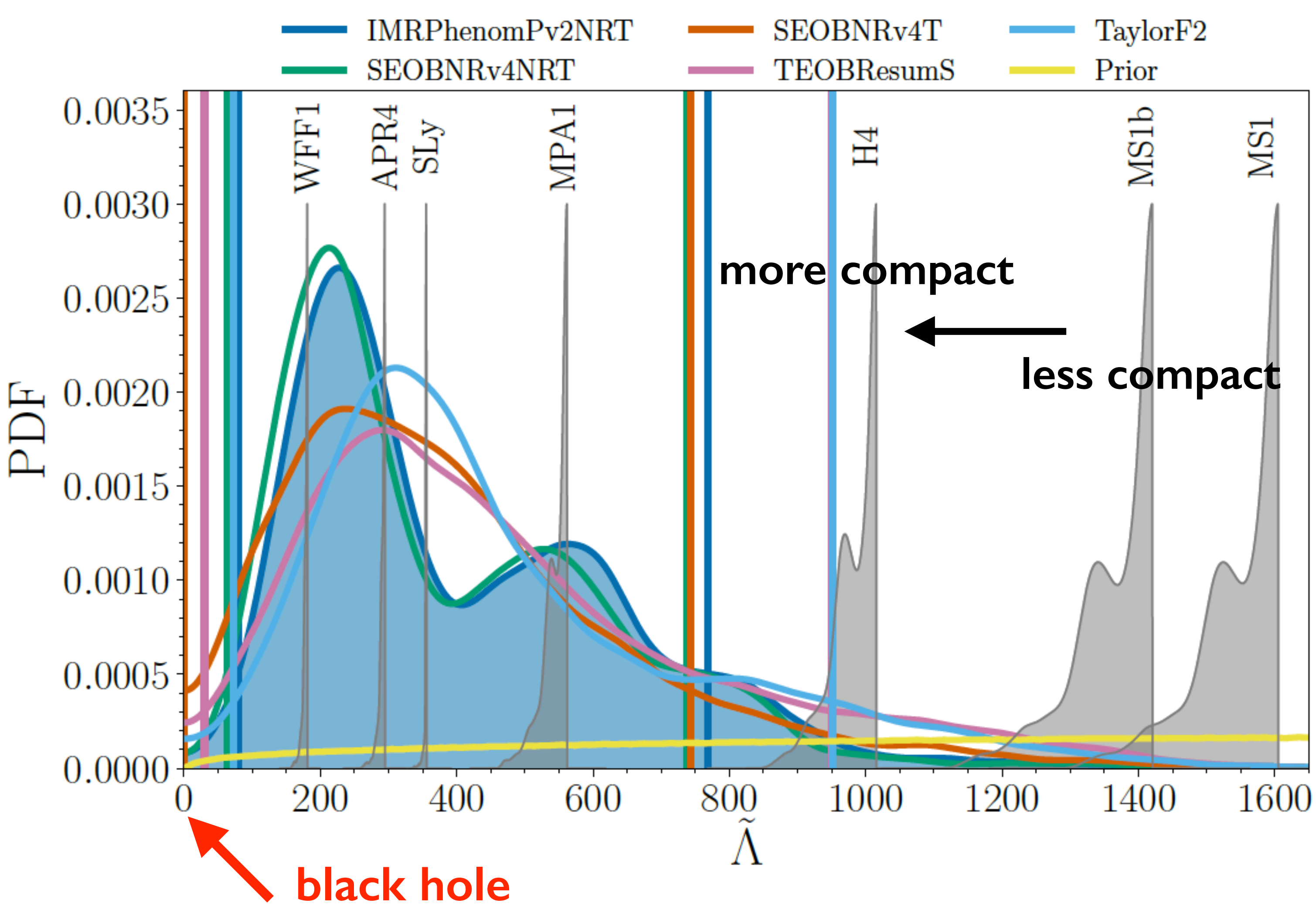




Constraining NS equation of state with GW170817

(Abbott et al. arXiv:1811.12907)

$$|\chi| \leq 0.05$$



Depends on EOS & compactness

$$\Lambda = \frac{\lambda}{m_{\rm NS}^5} = \frac{2}{3} k_2 \left(\frac{R_{\rm NS} c^2}{G m_{\rm NS}} \right)^5$$

- **Effective tidal deformability** enters **GW phase at 5PN order:**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

$$\tilde{\Lambda} : 330^{+438}_{-251}$$ @ 90% CL

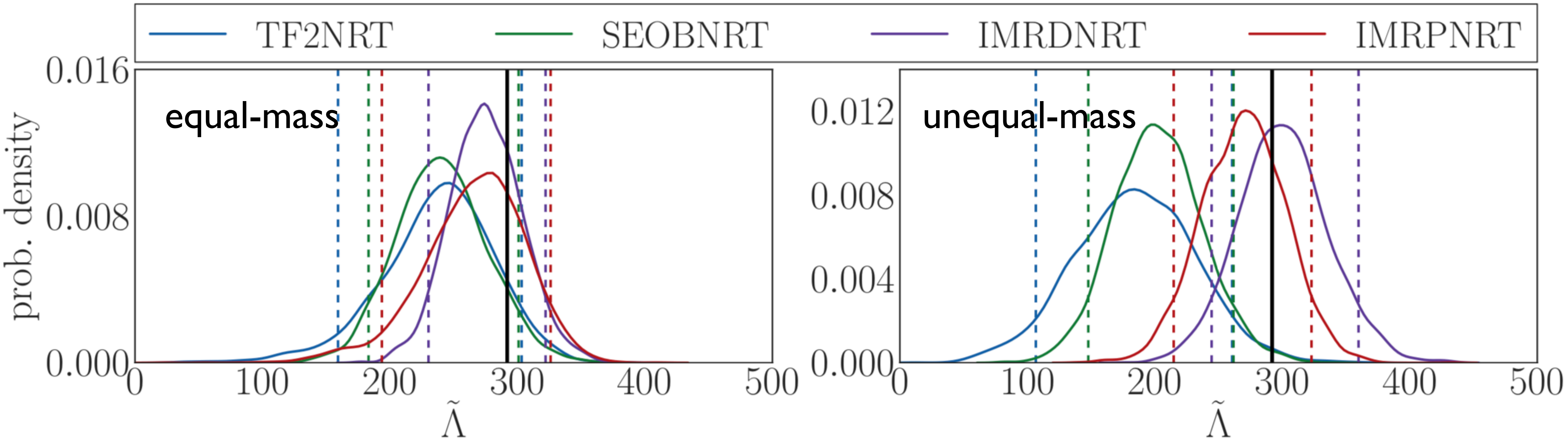
- Current **measurements** of tidal effects **dominated by statistical** error, but **inference** with **PN inspiral-only** waveform somewhat **stands out**.

Systematics due to modeling for GW170817-like @ aLIGO

- Synthetic GW signal of a **binary neutron star** at **50 Mpc** is **injected** in Gaussian noise with **aLIGO design-sensitivity** noise-spectral density **(SNR ~ 87)**.
- **Inference** with waveform models that have **same matter effects**, but **baseline point-mass model** is **different**.

(Samajdar & Dietrich 18)

- **IMRDNRT** is injected



Need more efficient ways to solve two-body problem, analytically

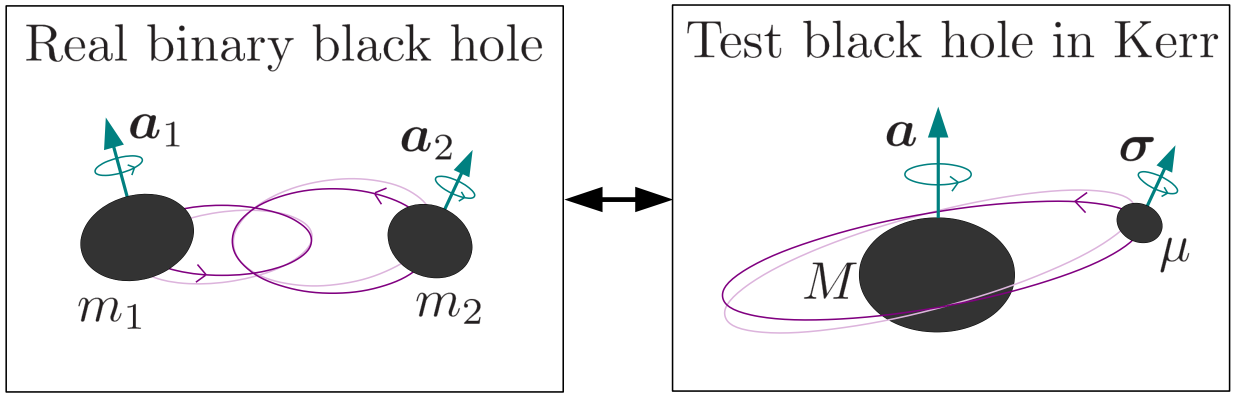
- In test-body limit, spinning EOB Hamiltonian includes **linear terms in spin of test body at all PN orders.**

(Barausse et al. 10, Barausse & AB 11, 12; Vines et al. 15)

$$(\sigma + \sigma^2)\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

- Is EOB **mapping unique** at all orders?

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$



exact mapping at the leading PN orders

Using **unbound orbits**, using **scattering angle** as adiabatic invariant, **at 1PM: mapping unique** & 2-body relativistic motion equivalent to 1-body motion in Kerr. *(Damour 16, Bini et al. 17-18, Vines 17)*

- Results at **leading PN order** but **all orders in spin.**

(Vines & Steinhoff 16; Vines & Harte 16, Siemonsen, Steinhoff & Vines 17)

see Steinhoff's & Damour's talks

$$\frac{Gm}{rc^2}\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

GM/rc² << v²/c² ~1

$$\frac{v^2}{c^2}\left(S_i + S_i^2 + \cdots\right)$$

PN versus PM expansion for conservative two-body dynamics

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\ldots\right] + \frac{1}{c^4}\left[\ldots\right] + \ldots$$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results

current known PM results

overlap between PN & PM results

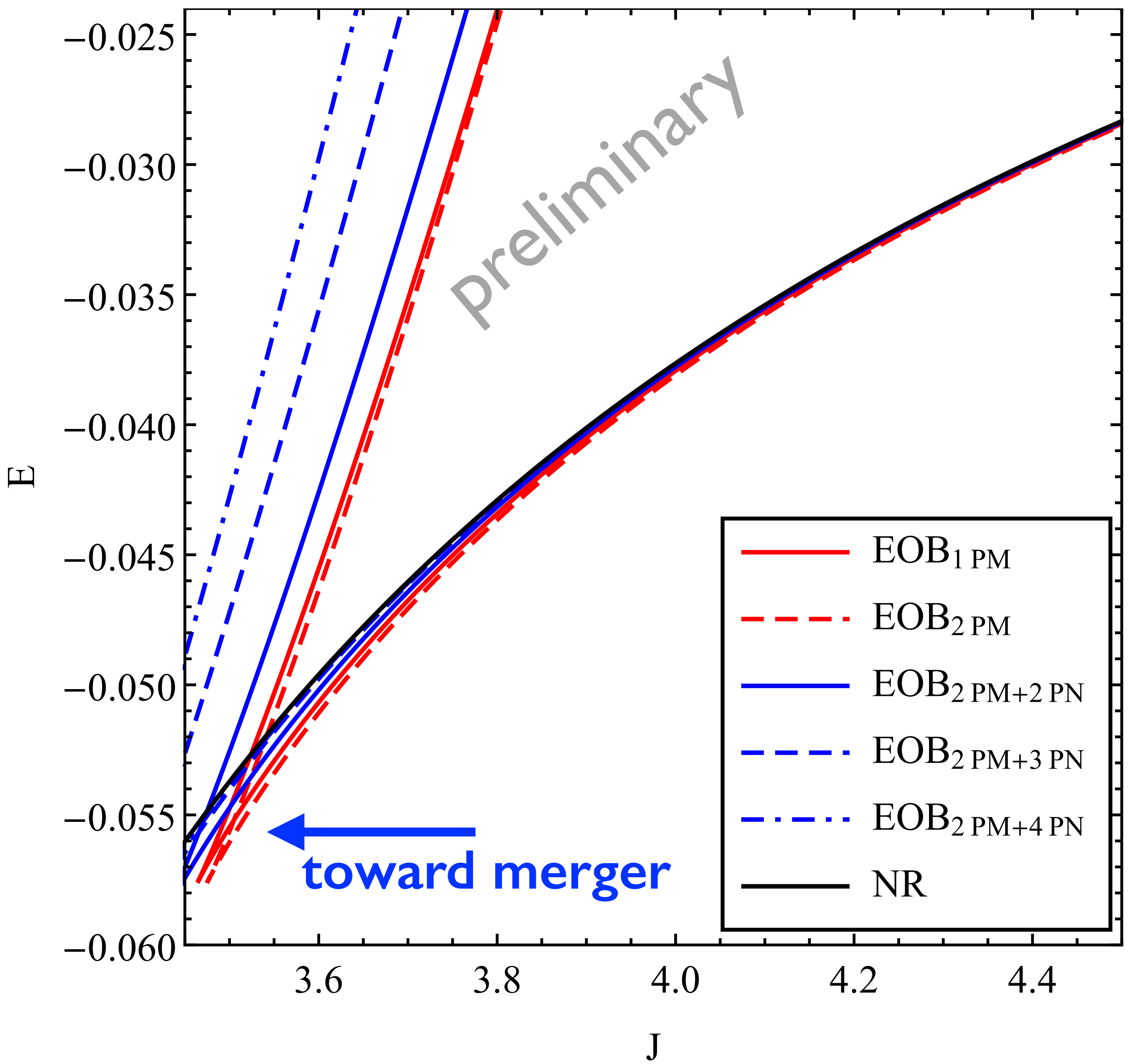
unknown

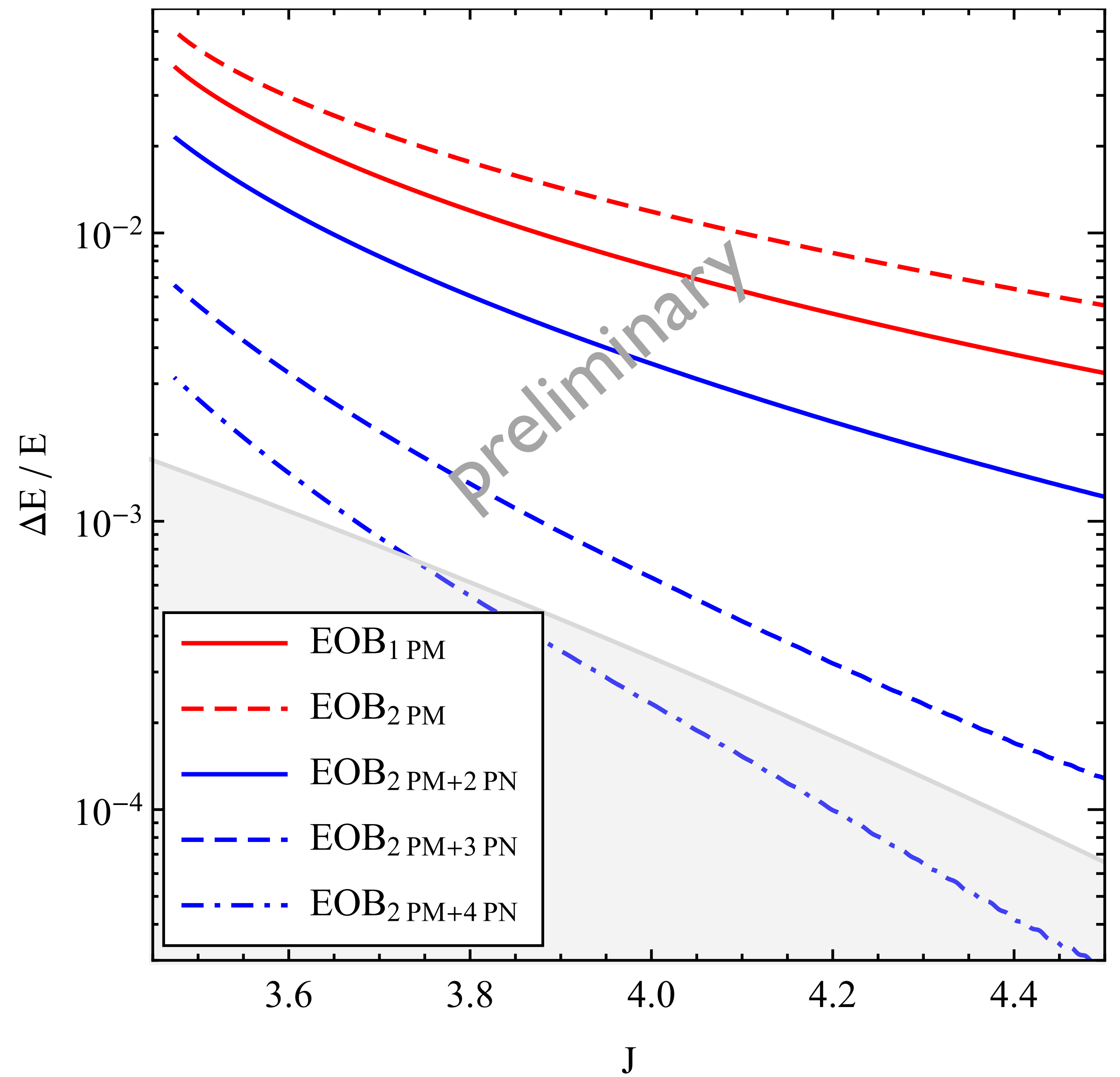
- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

Comparison between 2PM EOB and NR: binding energy

(Antonelli, van de Meent, … AB et al. in prep)

non-spinning, equal mass BBH



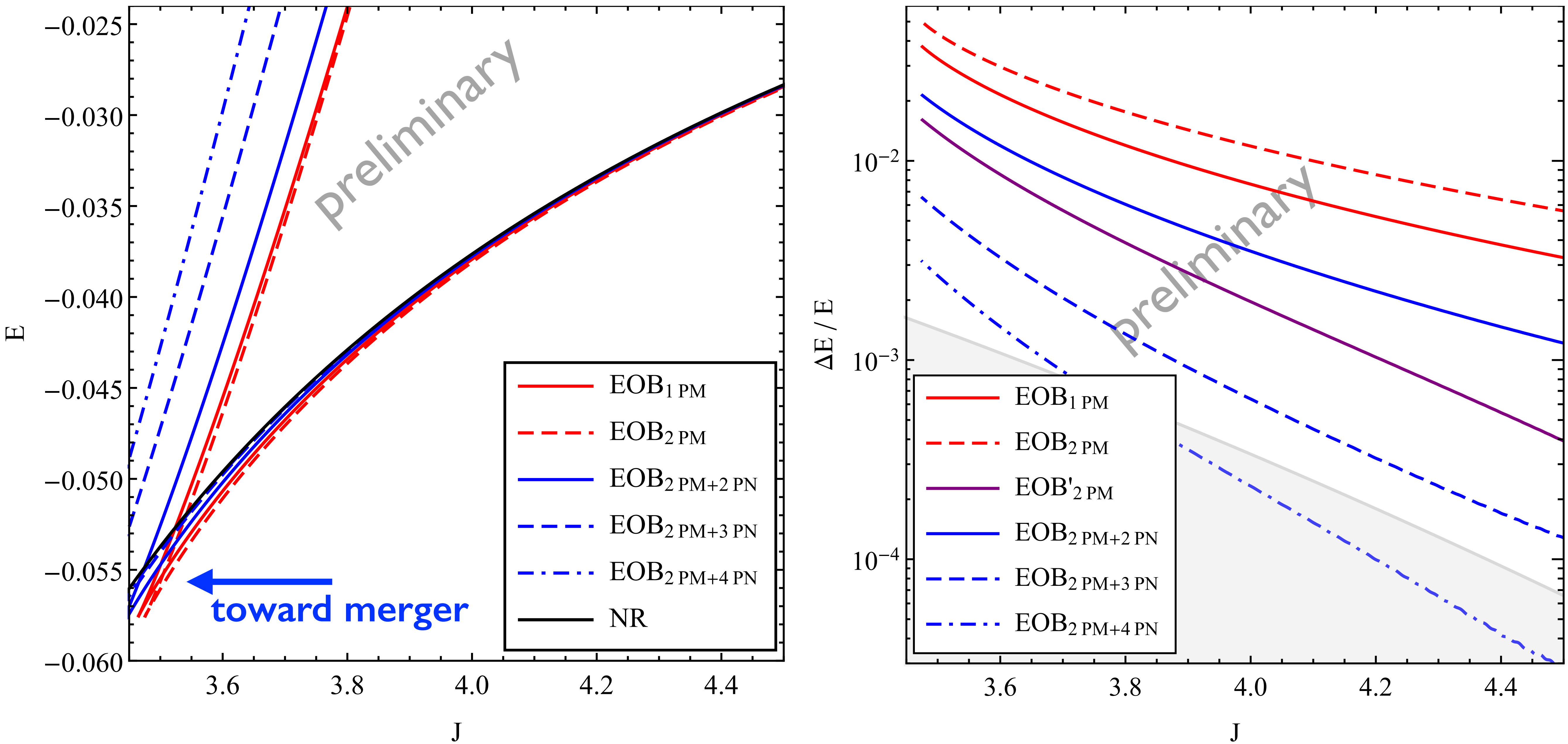


- Accurate **NR data** *(Ossokine & Dietrich 17)*
- Using **"a" 2PM EOB Hamiltonian.** *(Damour 17)*
- Crucial to **complete 2PM** results **with PN** terms for **bounded orbits** to improve accuracy.
- **Important** to compute **3PM**.

Comparison between 2PM EOB and NR: binding energy

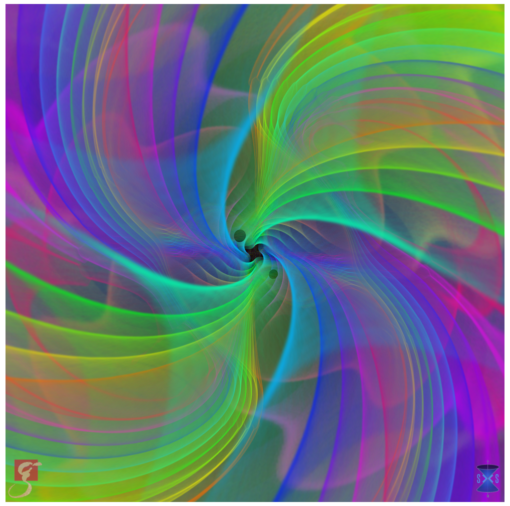
(Antonelli, van de Meent, … AB et al. in prep)

non-spinning, equal mass BBH



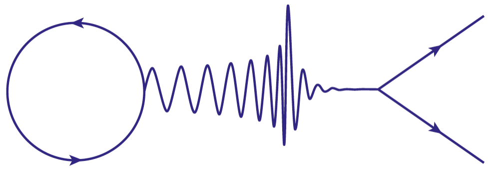
- Accurate **NR data** *(Ossokine & Dietrich 17)*
- Using **another 2PM EOB Hamiltonian.** (Antonelli et al. in prep)
- Crucial to **complete 2PM** results **with PN** terms for **bounded orbits** to improve accuracy.
- **Important** to compute **3PM**.

The new era of precision gravitational-wave astrophysics



- Theoretical groundwork in **analytical and numerical relativity** has allowed us to build **faithful waveform models** to **search** for signals, **infer properties** and **test GR**.

- So far, **inference from GW** observations is **dominated by statistical** instead of modeling **error**.

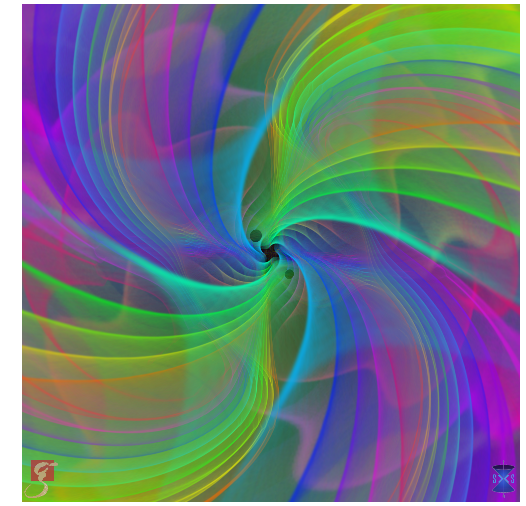


- **To extract best science** and **take full advantage** of **discovery potential** in next years and decades, we need to **develop highly-accurate waveforms** that **cover** the **entire parameter space** and **include all physical effects** (higher modes, matter, spin precession, eccentricity, etc.)

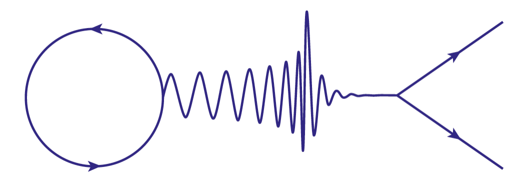
- **Post-Minkowskian results** through **modern scattering-amplitude** calculations **may help** improving accuracy.

- Could **2-body problem** be obtained **from 1-body problem**? Maybe **in part**, or maybe **fully.**

The new era of precision gravitational-wave astrophysics (cont.)

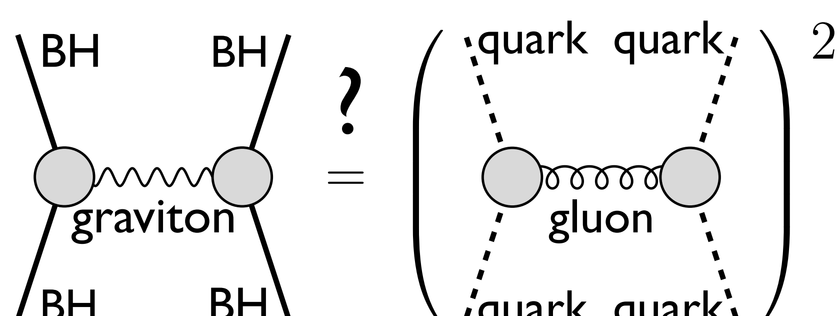


- Until we have **new results** (PN, PM, GSF, ...) to **check** against **current analytical waveform models** and against **numerical-relativity computations**, it is **not possible to assess** their **"real" phenomenological** (LIGO-Virgo) **impact.**



- To have **"real" phenomenological impact,** we **need conservative** and **dissipative results** (i.e., also **waveforms**).

Thank you!



(credit: Vines)

(postdoc openings @ AEI-Potsdam: http://www.aei.mpg.de/gwjobs2019)