Nuclear Reaction Networks

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The physics of macronova
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HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES

SFB 1245

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Outline

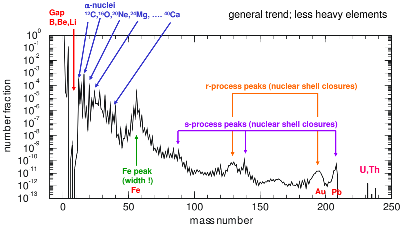
- Lecture 1: Basic concepts of nuclear reaction rates
- Lecture 2: Solving reaction networks
- Lecture 3: Applications for the r process

Lecture 1: Astrophysical reaction rates

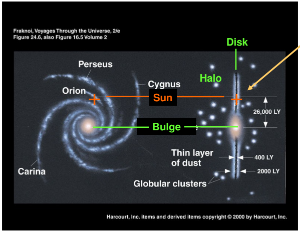
What is Nuclear Astrophysics?

- Nuclear astrophysics aims at understanding the nuclear processes that take place in the universe.
- These nuclear processes generate energy in stars and contribute to the nucleosynthesis of the elements and the evolution of the galaxy.

Hydrogen mass fraction	X = 0.71
Helium mass fraction	Y = 0.28
Metallicity (mass fraction of everything else)	Z = 0.019
Heavy Elements (beyond Nickel) mass fraction	4E-6



3. The solar abundance distribution

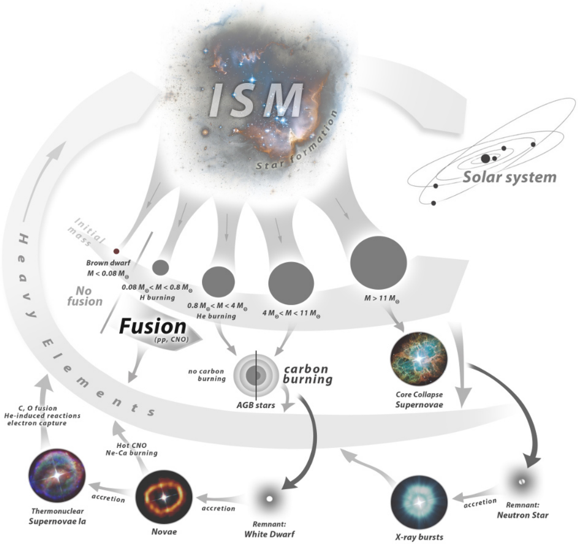


solar abundances:

Elemental (and isotopic) composition of Galaxy at location of solar system at the time of it's formation

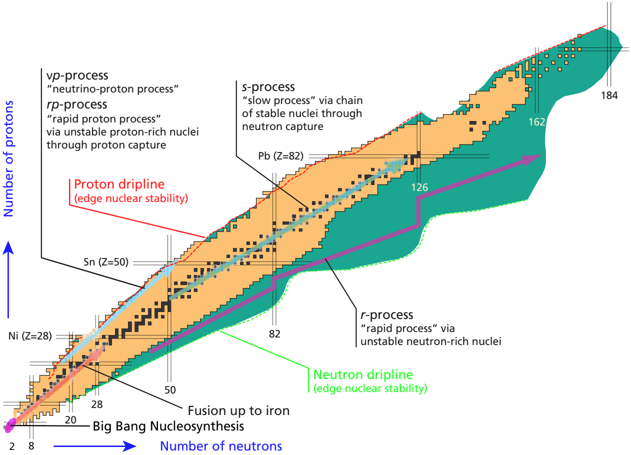
K. Lodders, Astrophys. J. **591**, 1220-1247 (2003)

Cosmic Cycle



Nucleosynthesis processes

In 1957 Burbidge, Burbidge, Fowler and Hoyle and independently Cameron, suggested several nucleosynthesis processes to explain the origin of the elements.



The r-process abundance

The r-process abundance is obtained by subtracting the calculated s-process abundance from the observed solar abundance of heavy elements, $N_r = N_\odot - N_s$. Thus, uncertainties in s-process models reflect in r-process abundances.

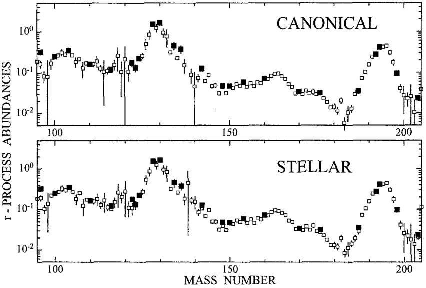
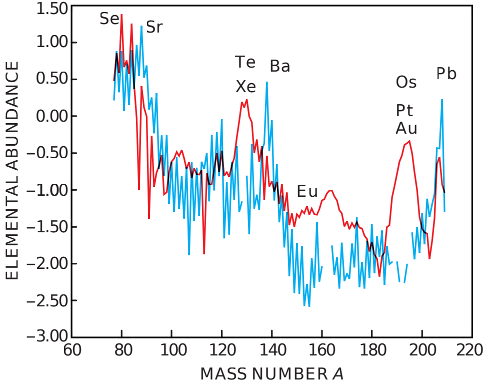


Fig. 24. The r-process abundances obtained as the differences between the solar and the s abundances, $N_\odot$ - N_s (open circles) compared to the r-only nuclei (full squares). Top: s abundances from canonical model. Bottom: s abundances from stellar model, represented by the average yields from thermally pulsing AGB stars of 1.5 and $3M_\odot$.

C. Arlandini, et al, Astrophys. J. **525**, 886 (1999).

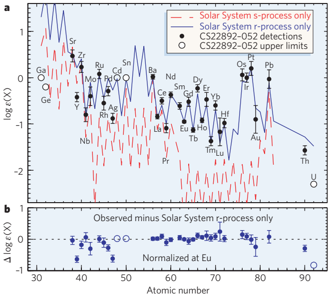
S and r-process abundances



- Notice the different behaviour with respect to odd-even staggering.
- Can we “directly” observe the r-process abundances?

Metal-poor star observations

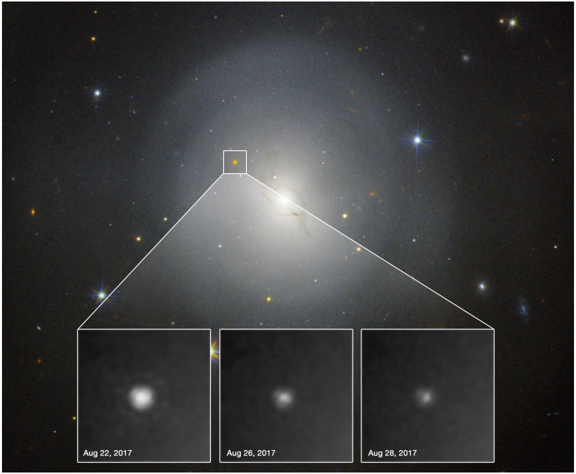
- The r-process is known to occur already at low metalicities, early times, while the s-process occurs later in galactic history.
- If we observe old enough stars we may be able to see a star that contains only r-process and no s-process.
- This may also allow to observe the contribution of a single r-process event to the solar composition.



$$\log \varepsilon(X) = \log_{10}(N_X/N_H) + 12.0$$

from Cowan and Sneden, Nature **440**, 1151 (2006)

Kilonova/Macronova luminosity



Abundance and mass fraction

Low energy astrophysical processes conserve number of nucleons:

$$n = \sum_i n_i A_i$$

n number nucleons per volume, $n \approx \dfrac{\rho}{m_u} = \rho N_A$ (CGS)

n_i number nuclei species i

Abundance: $Y_i = \dfrac{n_i}{n} \Rightarrow n_i = \dfrac{\rho}{m_u} Y_i$ (changes in density are factored out)

Mass fraction: $X_i = \dfrac{n_i m_i}{\rho} = \dfrac{n_i A_i m_u}{\rho} = Y_i A_i$

From conservation of number of nucleons: $\sum_i Y_i A_i = \sum_i X_i = 1$

Electron Abundance

From charge neutrality:

$$n_e = \sum_i n_i Z_i = n \sum_i Y_i Z_i$$

Introducing: $Y_e = \dfrac{n_e}{n}$

$$Y_e = \sum Y_i Z_i$$

In general one can not define a lepton abundance. Lepton number is not locally conserved (neutrinos leave the system).

Type of processes

Transfer (strong interaction)

$$^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}, \qquad \sigma \simeq 0.5\ \mathrm{b\ at}\ E_p = 2.0\ \mathrm{MeV}$$

Capture (electromagnetic interaction)

$$^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}, \qquad \sigma \simeq 10^{-6}\ \mathrm{b\ at}\ E_p = 2.0\ \mathrm{MeV}$$

Weak (weak interaction)

$$p(p,e^{+}\nu)d, \qquad \sigma \simeq 10^{-20}\ \mathrm{b\ at}\ E_p = 2.0\ \mathrm{MeV}$$

$$\mathrm{b\ (barn)} = 100\ \mathrm{fm}^2 = 10^{-24}\ \mathrm{cm}^2$$

Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

- Decay

$$^{56}\text{Ni} \rightarrow {}^{56}\text{Co} + e^{+} + \nu_e$$
$$^{15}\text{O} + \gamma \rightarrow {}^{14}\text{N} + p$$

$$\frac{dn_a}{dt} = -\lambda_a n_a$$

In order to dissentangle changes in the density (hydrodynamics) from changes in the composition (nuclear dynamics), the abundance is introduced:

$$Y_a = \frac{n_a}{n}, \quad n \approx \frac{\rho}{m_u} = \text{Number density of nucleons (constant)}$$

$$\frac{dY_a}{dt} = -\lambda_a Y_a$$

Rate can depend on temperature and density

Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

- Capture processes

$$a + b \rightarrow c + \gamma$$

$$\frac{dn_a}{dt} = -n_a n_b \langle \sigma v \rangle$$

$$\frac{dY_a}{dt} = -\frac{\rho}{m_u} Y_a Y_b \langle \sigma v \rangle$$

destruction rate particle a by reaction with b: $\lambda_a(b) = \rho Y_b \langle \sigma v \rangle / m_u$

- photodissociation rates

$$\gamma + c \rightarrow a + b$$

$$\frac{dY_c}{dt} = -Y_c \lambda_c = -Y_c n_\gamma \langle \sigma c \rangle$$

$\langle \sigma c \rangle$ photodissociation cross section averaged over thermal photon spectrum.
The balance between capture and photodissociation is determined by the photon-to-baryon ratio.

Three body reactions

Due to the fact that there is no stable nuclei with $A = 5$ and 8, nuclei heavier than ^{4}He have to be build by 3-body reactions. The most relevant reactions are:

3-α Dominant in proton-rich environments

$$3\,{}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$

$$\frac{dY_\alpha}{dt} = -\frac{3}{3!} Y_\alpha^3 \left(\frac{\rho}{m_u}\right)^2 \langle \alpha\alpha\alpha \rangle$$

$\alpha\alpha n$ Dominant in neutron-rich environments

$$2\,{}^4\mathrm{He} + n \rightarrow {}^{9}\mathrm{Be} + \gamma$$

$$\frac{dY_\alpha}{dt} = -\frac{2}{2} Y_\alpha^2 Y_n \left(\frac{\rho}{m_u}\right)^2 \langle \alpha\alpha n \rangle$$

These reactions are key for the build-up of heavy nuclei.

Reaction rates

Consider n_a and n_b particles per volume of species a and b. The number of nuclear reactions per unit of time and volume

$$a + A \rightarrow B + b$$

is given by:

$$r_{aA} = \frac{n_a(v_a) n_A(v_A)}{(1+\delta_{aA})} \sigma(v) v, \quad v = |\boldsymbol{v}_a - \boldsymbol{v}_A| \text{ (relative velocity)}$$

In stellar environment the velocity (energy) of particles follows a thermal distribution that depends of the type of particles.

- Nuclei (Maxwell-Boltzmann)

$$n(v)dv = n 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) dv = n\phi(v)dv$$

- Electrons, Neutrinos (if thermal) (Fermi-Dirac)

$$n(p)dp = \frac{g}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{(E(p)-\mu)/kT} + 1} dp$$

- photons (Bose-Einstein)

$$n(p)dp = \frac{2}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{pc/kT} - 1} dp$$

Stellar reaction rate

The product σv has to be averaged over the velocity distribution $\phi(v)$ (Maxwell-Boltzmann)

$$\langle \sigma v \rangle = \int_0^\infty \phi(v)\sigma(v) v dv$$

that gives:

$$\langle \sigma v \rangle = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 \sigma(v) \exp\left(-\frac{mv^2}{2kT}\right) dv, \quad m = \frac{m_a m_b}{m_a + m_b}$$

or using $E = mv^2/2$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE \qquad (1)$$

Cross section determination

The calculation of the cross section requires the determination of the wave function for the system projectile (a) and target (A) for a particular value of energy E. This requires solutions of the Schrodinger equation for a potential

$$V(r) = V_{\text{nuclear}}(r) + V_{\text{coulomb}}(r) + V_{\text{centrifugal}}(r)$$

- Nuclear potential: complicated form with strong dependence on energy, E, angular momentum, J and parity, π (due to the internal structure of the target and projectile). It is of very short range: $R = 1.2(A_a^{1/3} + A_A^{1/3})$ fm.
- Coulomb potential (only for charged particles):

$$V(r) = \frac{Z_a Z_A e^2}{r}$$

- Centrifugal barrier:

$$V(r) = \frac{\hbar^2 l(l+1)}{2mr^2}$$

  cross section suppressed for high l values. Normally s-wave ($l = 0$) and p-wave ($l = 1$) dominate.

Cross section is mainly determined by long range behaviour of the potential

Cross section

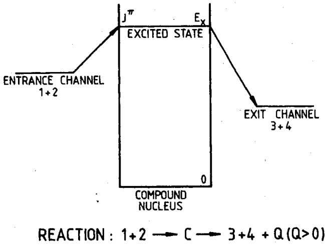
The general form of the total cross section for the formation of a nucleus with $A_C = A_a + A_A$ and $Z_C = Z_a + Z_A$

$$a + A \rightarrow C \rightarrow B + b$$

$$\sigma(E) = \pi \lambda\!\!\!^{-2} \sum_l (2l+1) T_l, \quad \lambda\!\!\!^{-} = \frac{\hbar}{mv} = \frac{\hbar}{\sqrt{2mE}}$$

T_l transmission coefficient through the potential barrier.
The problem reduces to a calculation of the tunneling probability through a barrier.

Neutron capture



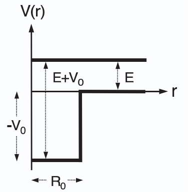


Fig. 2.7 Three-dimensional square-well potential of radius R_0 and potential depth V_0. The horizontal line indicates the total particle energy E. For the calculation of the transmission coefficient, it is necessary to consider a one-dimensional potential step that extends from $-\infty$ to $+\infty$. See the text.

$$A + n \rightarrow B + \gamma$$

$$\sigma_n(E) = \pi \lambda\!\!\!^{-2} \sum (2l+1) T_{l,n}(E) P_\gamma(E+Q)$$

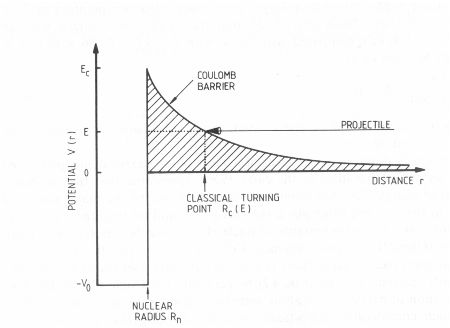
$T_{l,n}$ transmision coefficient, P_γ probability of gamma emission, E neutron energy ($\sim$ keV), $Q = m_A + m_n - m_B = S_n$, $Q \gg E_n$. Normally s-wave dominates and we have

$$\sigma_n(E) \propto \lambda\!\!\!^{-2} T_{0,n}(E), \quad T_n(E) \propto v$$

$$\sigma_n(E) \propto \frac{1}{v^2} v = \frac{1}{v}, \quad \langle \sigma_n v \rangle = \text{constant}$$

Charged-particle reactions

Stars' interior is a neutral plasma made of charged particles (nuclei and electrons). Nuclear reactions proceed by tunnel effect. For the $p + p$ reaction the Coulomb barrier is 550 keV, but the typical proton energy in the Sun is only 1.35 keV.



Assuming s-wave dominates:

$$\sigma(E) = \pi \lambda\!\!\!^{-2} T_0(E), \quad T_0 = \exp\left\{-\frac{2}{\hbar}\int_{R_n}^{R_c} \sqrt{2m[V(r)-E]}dr\right\}$$

S-factor

For the coulomb potential and assuming that $R_n \approx 0 \ll R_c$ the integral gives:

$$T_0 = e^{-2\pi\eta} = e^{b/E^{1/2}}, \quad \eta = \frac{Z_a Z_A e^2}{\hbar}\sqrt{\frac{m}{2E}} = \frac{b}{E^{1/2}}$$

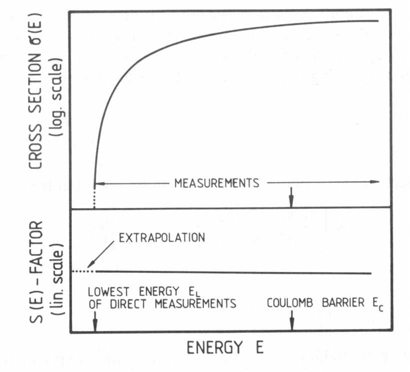
η is the Sommerfeld parameter that accounts for tunneling through a coulomb barrier.
We can rewrite the cross section as:

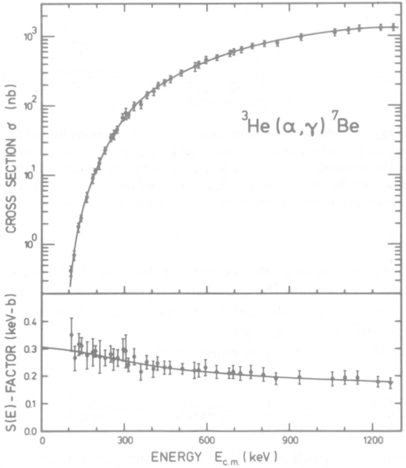
$$\sigma(E) = \frac{1}{E} S(E) e^{-2\pi\eta}$$

S is the so-called S-factor and accounts for the short distance dependence of the cross section on the nuclear potential. It is expected to be only mildly dependent on Energy.

S-factor

S factor makes possible accurate extrapolations to low energy.

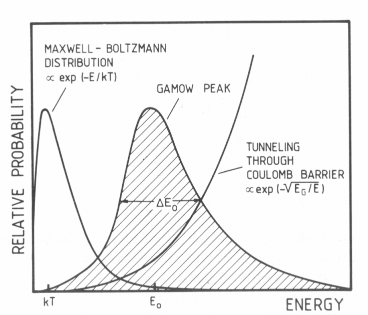




Gamow window

Using definition S factor:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}} \right] dE$$



$$\exp\left[-\frac{E}{k_B T} - \frac{b}{E^{1/2}} \right] \approx \exp\left[-\frac{3E_0}{k_B T} \right] \exp\left[-\left(\frac{E - E_0}{\Delta/2} \right)^2 \right]$$

Gamow window

Assuming the S factor is constant over the gamow window and approximating the integrand by a Gaussian one gets:

$$\langle \sigma v \rangle = \left(\frac{2}{m}\right)^{1/2} \frac{\Delta}{(k_B T)^{3/2}} S(E_0) \exp\left(-\frac{3E_0}{k_B T}\right)$$

with

$$E_0 = \left(\frac{b k_B T}{2}\right)^{2/3} = 1.22 (Z_a^2 Z_A^2 A T_6^2)^{1/3} \text{ keV}$$

$$\Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 k_B T} = 0.749\ (Z_a^2 Z_A^2 A T_6^5)^{1/6} \text{ keV}$$

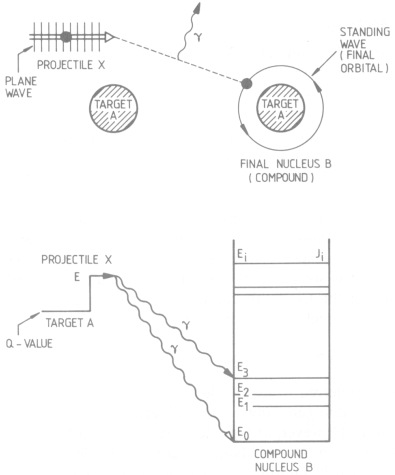
($A = m/m_u$ and $T_6 = T/10^6$ K)
Examples for solar conditions ($T = 15 \times 10^6$ K):

reaction	E_0 (keV)	$\Delta/2$ (keV)	$\exp(-3E_0/k_B T)$	T dependence
$p + p$	5.9	3.2	1.1×10^{-6}	$T^{3.6}$
$^{14}N + p$	26.5	6.8	1.8×10^{-27}	T^{20}
$^{12}C + \alpha$	56.0	9.8	3.0×10^{-57}	T^{42}
$^{16}O + ^{16}O$	237.0	20.2	6.2×10^{-239}	T^{182}

Reaction rate depends very sensitively on temperature

Direct reactions

So far we have discussed the so-called “direct reactions” in which the reaction proceeds directly to a bound nuclear state:



S-factor $^3He(^3He, 2p)^4He$

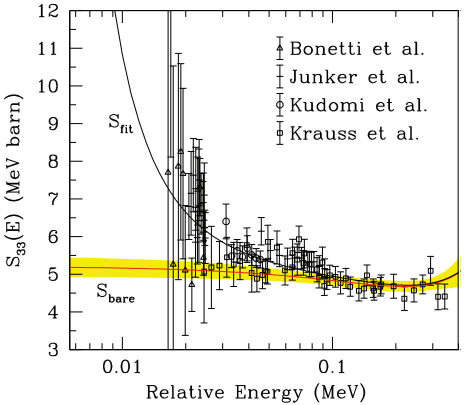
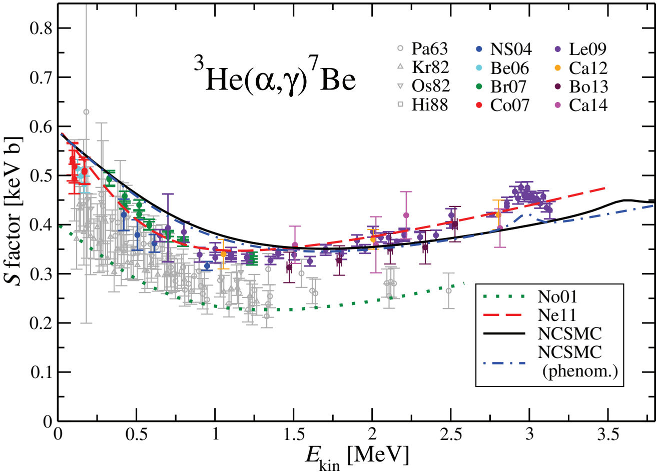


FIG. 4 (color online). The data, the best quadratic + screening result for $S_{33}(E)$, and the deduced best quadratic fit (line) and allowed range (band) for $S_{33}^{\rm bare}$. See text for references.

Adelberger *et al*, Rev. Mod. Phys. **83**, 195 (2011)

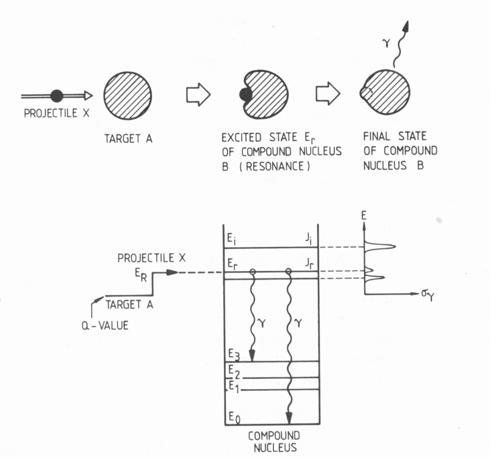
$^3He(\alpha,\gamma)^7Be$



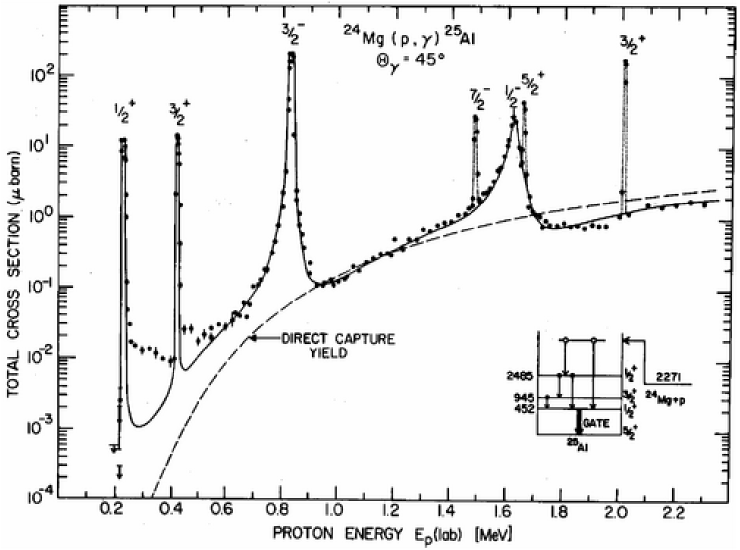
Dohet-Eraly, et al, PLB **757**, 430 (2016)

Resonant reactions

The cross section can also have contributions from resonances that can be seen like quasi-bound states. During the reaction a quasi-bound, compound, state forms that decays by particle or gamma emission.



Cross section example



There could be important interferences between resonances if these are broad

Resonance cross section

The cross section for capture through an isolated resonance is given by the Breit-Wigner formula:

$$a + A \rightarrow C \rightarrow B + b$$

$$\sigma(E) = \pi \lambda\!\!\!^{-2} \frac{(2J_C + 1)(1 + \delta_{aA})}{(2J_a + 1)(2J_A + 1)} \frac{\Gamma_{aA}\Gamma_{bB}}{(E - E_r)^2 + (\Gamma/2)^2}, \qquad \lambda\!\!\!^{-} = \frac{1}{k} = \frac{\hbar}{p}$$

with $\Gamma = \Gamma_{aA} + \Gamma_{bB} + \ldots$ (sum over all partial widths for all possible decay channels). They depend on energy.

Particle width

$$\Gamma^{(l)}(E) = \frac{2\hbar}{R} v P_l(E) \theta_l^2$$

Photon width

$$\Gamma_\gamma^{(l)}(E) = \frac{8\pi}{l[(2l+1)!!]^2} B(\omega l) E_\gamma^{(2l+1)}, \omega = \text{Electric or Magnetic}$$

Incoming particle, $E = E_{aA}$

Outgoing particle, $E = E_{aA} + Q \approx Q$ ($Q \gg E_{aA}$, independent of energy)

Reaction rate for a narrow resonance

If we assume a narrow resonance ($\Gamma \ll E_r$ and $\Gamma \ll k_B T$) the astrophysical reaction rate [see eq. (1)] is given by:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} \frac{1}{(k_B T)^{3/2}} E_r \exp\left(-\frac{E_r}{k_B T}\right) \pi \lambda_r^2 \omega \frac{\Gamma_{aA}\Gamma_{bB}}{\Gamma} \int_0^\infty \frac{\Gamma}{(E - E_r)^2 + (\Gamma/2)^2} dE$$

to give:

$$\langle \sigma v \rangle = \left(\frac{2\pi}{m k_B T}\right)^{3/2} \hbar^2 (\omega\gamma)_r \exp\left(-\frac{E_r}{k_B T}\right)$$

$\omega\gamma$ is denoted the resonant strength

$$\omega = \frac{2J_C + 1}{(2J_a + 1)(2J_A + 1)}, \quad \gamma = \frac{\Gamma_{aA}\Gamma_{bB}}{\Gamma}$$

Typically $\Gamma = \Gamma_{aA} + \Gamma_{bB}$

- if $\Gamma_{aA} \ll \Gamma_{bB}$ then $\gamma = \Gamma_{aA}$
- if $\Gamma_{aA} \gg \Gamma_{bB}$ then $\gamma = \Gamma_{bB}$

reaction rate determined by smaller width

Inverse reactions

Let's have the reaction

$$a + A \to B + \gamma \quad Q = m_a + m_A - m_B$$

We are interested in the inverse reaction. One can use detailed-balance to determine the inverse rate. Simpler using the concept of chemical equilibrium.

$$\frac{dn_a}{dt} = -n_a n_A \langle \sigma v \rangle_{aA} + (1 + \delta_{aA}) n_B \lambda_\gamma = 0$$

$$\left(\frac{n_a n_A}{n_B} \right)_{\text{eq}} = (1 + \delta_{aA}) \frac{\lambda_\gamma}{\langle \sigma v \rangle_{aA}}$$

Using equilibrium condition for chemical potentials: $\mu_a + \mu_A = \mu_B$

$$\mu(Z, A) = m(Z, A)c^2 + kT \ln \left[\frac{n(Z, A)}{G_{(Z,A)}(T)} \left(\frac{2\pi\hbar^2}{m(Z, A)k_B T} \right)^{3/2} \right], \quad G_{(Z,A)}(T) = \sum_i (2J_i + 1) e^{-E_i/(kT)}$$

Inverse reactions

One obtains:

$$\left(\frac{n_a n_A}{n_B}\right)_{\text{eq}} = \frac{G_a G_A}{G_B}\left(\frac{m_a m_A}{m_B}\right)^{3/2}\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-Q/k_B T}$$

Finally, we obtain:

$$\lambda_\gamma = \frac{G_a G_A}{(1+\delta_{aA})G_B}\left(\frac{m_a m_A}{m_B}\right)^{3/2}\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-Q/k_B T}\langle\sigma v\rangle$$

For a reaction $a + A \rightarrow B + b$ ($Q = m_a + m_A - m_B - m_b$):

$$\langle\sigma v\rangle_{bB} = \frac{(1+\delta_{bB})}{(1+\delta_{aA})}\frac{G_a G_A}{G_b G_B}\left(\frac{m_a m_A}{m_b m_B}\right)^{3/2} e^{-Q/k_B T}\langle\sigma v\rangle_{aA}$$

Hauser-Feshbach cross section and reaction rate

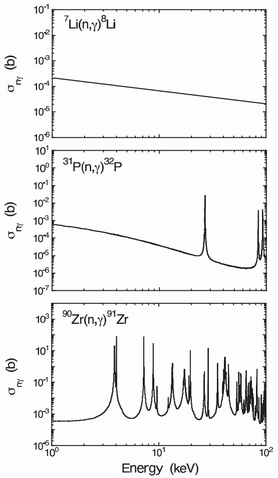


Fig. 3.30 Cross sections for neutron capture on ^{7}Li, ^{31}P, and ^{90}Zr versus energy. The curve in the upper panel shows a $1/v$ behavior, while resonances are visible in the middle and lower panels.

Christian Iliadis, *Nuclear Physics of Stars*

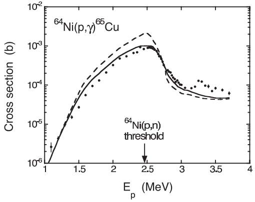


Fig. 2.29 Cross section versus bombarding energy for the ^{64}Ni(p,γ)^{65}Cu reaction. Beyond an energy of $\approx$ 2.5 MeV the endothermic ^{64}Ni(p,n)^{64}Cu reaction is energetically allowed. The sharp drop in the cross section at the neutron threshold reflects the decrease of the flux in all other decay channels of the compound nucleus ^{65}Cu. The curves show the results of Hauser–Feshbach statistical model calculations with (solid line) and without (dashed line) width fluctuation corrections. Reprinted from F. M. Mann et al., Phys. Lett. B, Vol. 58, p. 420 (1975). Copyright (1975), with permission from Elsevier.

- With increasing mass number reactions are determined by a larger number of resonances
- Often it is not possible to experimentally resolve resonances. Astrophysical reaction rate is an energy average over many resonances.
- Hauser-Feshbach provides and statistically averaged cross section from the contribution of many resonances in an energy interval.

Hauser-Feshbach cross section

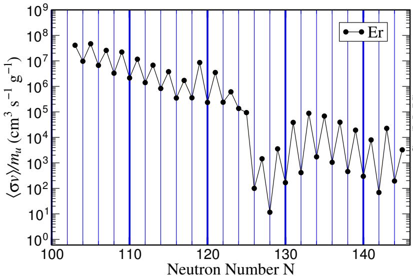
The Hauser-Feshbach expression for the cross section of an (n, γ) reaction proceeding from the target nucleus i in the state μ with spin J_i^μ and parity π_i^μ to a final state ν with spin J_m^ν and parity π_m^ν in the residual nucleus m via a compound state with excitation energy E, spin J, and parity π is given by

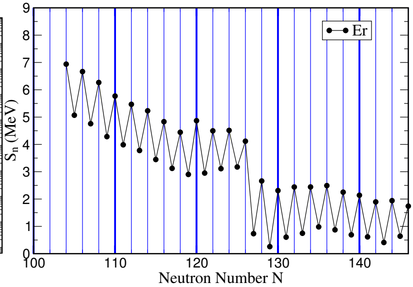
$$\sigma_{(n,\gamma)}^{\mu\nu}(E_{i,n}) = \frac{\pi\hbar^2}{2M_{i,n}E_{i,n}} \frac{1}{(2J_i^\mu + 1)(2J_n + 1)} \sum_{J,\pi} (2J + 1) \frac{T_n^\mu T_\gamma^\nu}{T_n^\mu + T_\gamma^\nu}$$

where $E_{i,n}$ and $M_{i,n}$ are the center-of-mass energy and the reduced mass for the initial system. $J_n = 1/2$ is the neutron spin. Normally we have situations in which $T_n \gg T_\gamma$. The transmission coefficients are related to the average decay width and level density (ρ)

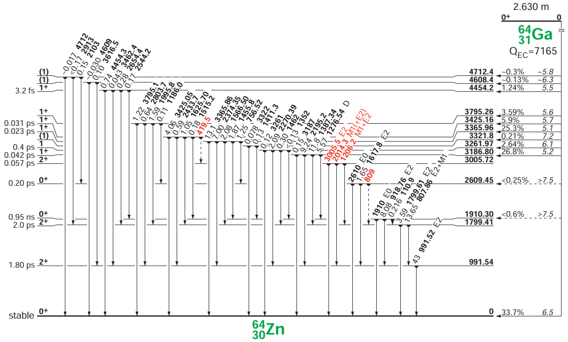
$$T = 2\pi\rho\langle\Gamma\rangle$$

Systematics $\langle\sigma v\rangle$ and neutron separation energies





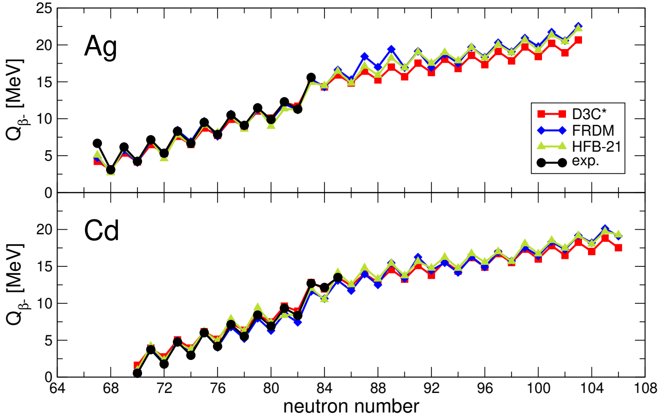
Beta-decay



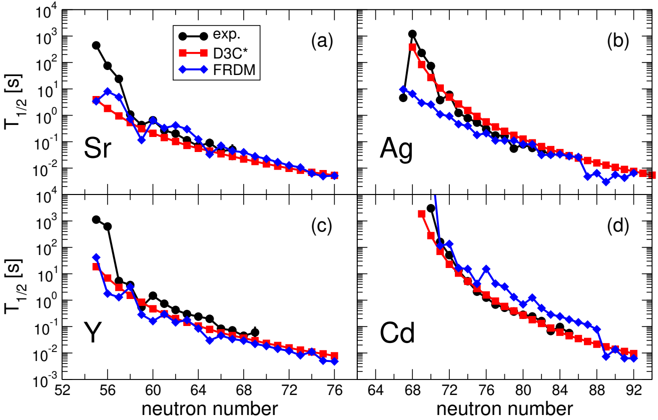
$$\lambda_\beta = \frac{\ln 2}{K} f(Q)[B(F) + B(GT)], \quad K = \frac{2\pi^3 (\ln 2)\hbar^7}{G_F^2 V_{ud}^2 g_V^2 m_e^5 c^4} = 6147.0 \pm 2.4\ s$$

- $f(Q)$ phase space function ($\sim Q^5$).
- $B(F)$ Fermi matrix element ($\sim \langle f | \sum_k t_+^k | i \rangle$).
- $B(GT)$ Gamow-Teller matrix element ($\sim \langle f | \sum_k \sigma t_+^k | i \rangle$).

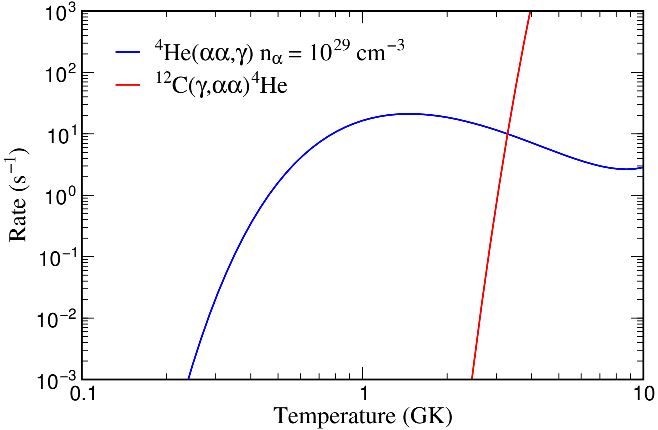
Systematics beta-decay



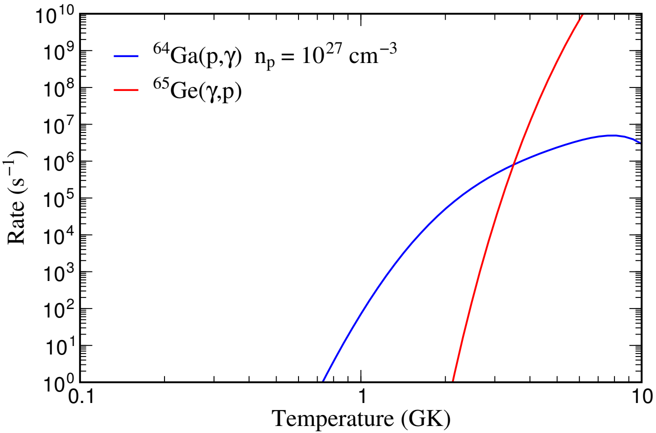
Systematics half-lives



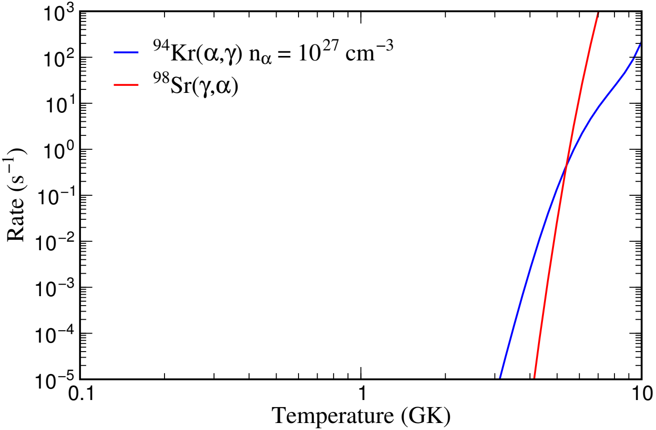
Rate Examples: $^4\mathrm{He}(\alpha\alpha,\gamma)$



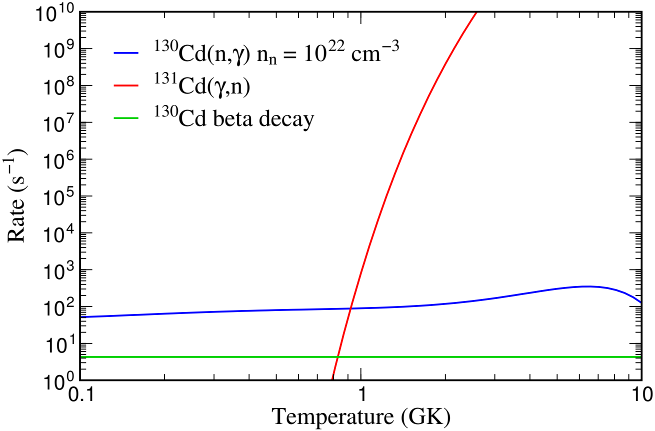
Rate Examples: (p, γ)



Rate Examples: (α, γ)



Rate examples: (n, γ)



r process calculations

r process calculations require to solve the system of differential equations:

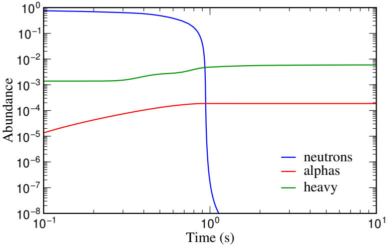
$$\begin{aligned}\frac{dY(Z,A)}{dt} &= \left(\frac{\rho}{m_u}\right)\langle\sigma v\rangle_{Z,A-1}Y_nY(Z,A-1) + \lambda_\gamma(Z,A+1)Y(Z,A+1)\\ &+ \sum_{j=0}^{J}\lambda_{\beta jn}(Z-1,A+j)Y(Z-1,A+j)\\ &-\left(\left(\frac{\rho}{m_u}\right)\langle\sigma v\rangle_{Z,A}Y_n + \lambda_\gamma(Z,A) + \sum_{j=0}^{J}\lambda_{\beta jn}(Z,A)\right)Y(Z,A)\end{aligned}$$

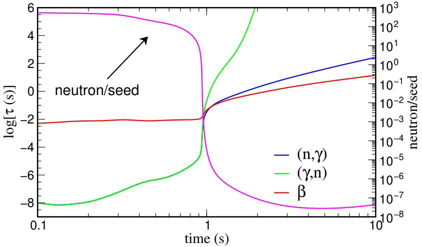
$$\begin{aligned}\frac{dY_n}{dt} = &- \sum_{Z,A}\left(\frac{\rho}{m_u}\right)\langle\sigma v\rangle_{Z,A}Y_nY(Z,A)\\ &+ \sum_{Z,A}\lambda_\gamma(Z,A)Y(Z,A)\\ &+ \sum_{Z,A}\left(\sum_{j=1}^{J}j\lambda_{\beta jn}(Z,A)\right)Y(Z,A)\end{aligned}$$

We are neglecting fission

Evolution during r process

Example of r process calculation for very neutron rich ejecta (Based on trajectory from merger simulation from A. Bauswein)





From Meng-Ru Wu

$(n,\gamma) \rightleftarrows (\gamma,n)$ equilibrium

Neutron capture reactions proceed much faster than beta-decays and an $(n,\gamma) \rightleftarrows (\gamma,n)$ equilibrium is achieved

$$\mu(Z,A+1) = \mu(Z,A) + \mu_n$$

$$\frac{Y(Z,A+1)}{Y(Z,A)} = n_n \left(\frac{2\pi\hbar^2}{m_u k_B T}\right)^{3/2} \left(\frac{A+1}{A}\right)^{3/2} \frac{G(Z,A+1)}{2G(Z,A)} \exp\left[\frac{S_n(Z,A+1)}{k_B T}\right]$$

Only even-even nuclei participate in the path so we can write:

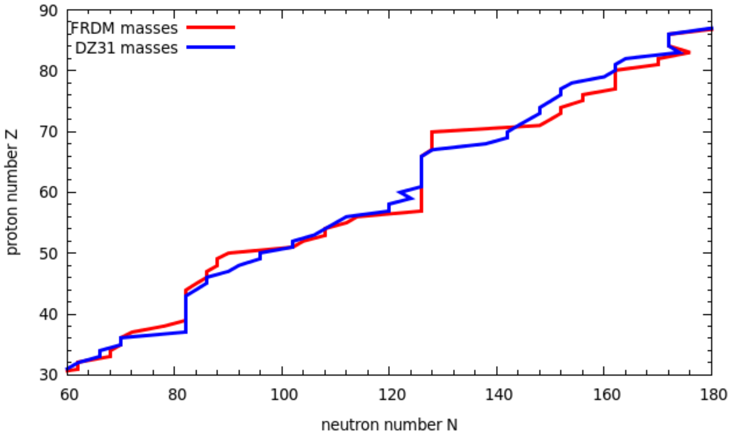
$$\frac{Y(Z,A+2)}{Y(Z,A)} = n_n^2 \left(\frac{2\pi\hbar^2}{m_u k_B T}\right)^{3} \left(\frac{A+2}{A}\right)^{3/2} \frac{G(Z,A+2)}{4G(Z,A)} \exp\left[\frac{S_{2n}(Z,A+1)}{k_B T}\right]$$

The maximum of the abundance defines the r-process path:

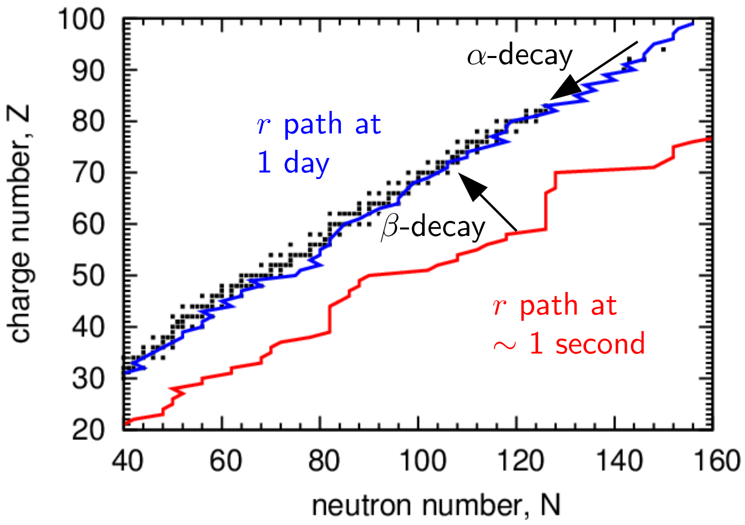
$$S_{2n}^0(\mathrm{MeV}) = \frac{2T_9}{5.04}\left(34.075 - \log n_n + \frac{3}{2}\log T_9\right)$$

For $n_n = 5\times 10^{21}$ cm^{-3} and $T = 1.3$ GK corresponds at $S_{2n}^0 = 6.46$ MeV, $S_n^0 = S_{2n}^0/2 = 3.23$ MeV,

r process path



path dependence on time



Beta-flow equilibrium

Assuming $(n,\gamma) \rightleftarrows (\gamma,n)$ equilibrium, it is sufficient to compute the time evolution of the total abundance for an isotopic chain

$$Y(Z) = \sum_A Y(Z,A)$$

The differential equation reduces to

$$\frac{dY(Z)}{dt} = \lambda_\beta(Z-1)Y(Z-1) - \lambda_\beta(Z)Y(Z)$$

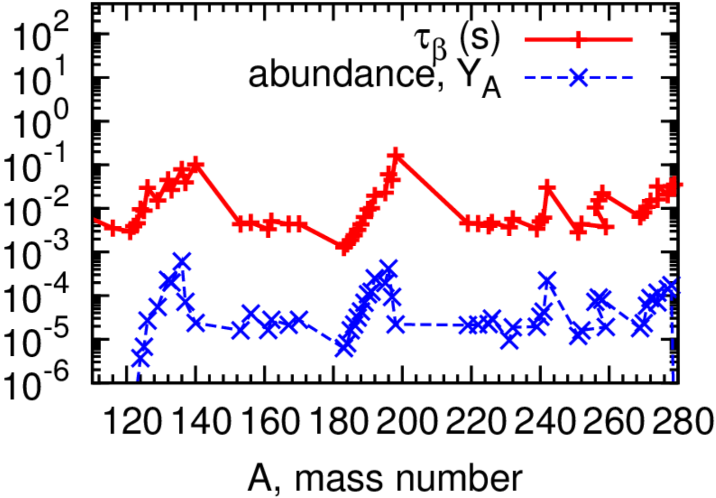
with

$$\lambda_\beta(Z) = \frac{1}{Y(Z)} \sum_A \lambda_\beta(Z,A)Y(Z,A)$$

Only beta-decays are necessary to determine the evolution. If the duration of the r process is larger than the beta-decay lifetimes an equilibrium is reaches denotes as steady β flow equilibrium that satisfies for each Z value

$$\lambda_\beta(Z-1)Y(Z-1) = \lambda_\beta(Z)Y(Z), \quad \text{i.e. } Y(Z) \propto \tau_\beta(Z)$$

Abundances vs beta lifetimes



Generic reaction network

The most general reaction we can consider is:

$$N_a A + N_b B + N_c C \rightarrow N_d D + N_e E + \ldots$$

N_i number of particles destroyed (negative) or produced (positive) by the reaction.
The change in abundance is given by

$$\frac{dY_i}{dt} = \sum_j N_i \lambda_j Y_j + \sum_{i,j} N^i_{jk} \frac{\rho}{m_u} \langle jk \rangle Y_j Y_k + \sum_{jkl} N^i_{jkl} \left(\frac{\rho}{m_u} \right)^2 \langle jkl \rangle Y_j Y_k Y_l$$

$$N^i_{jk} = \frac{N_i}{|N_j|!|N_k|!}, \quad N^i_{jkl} = \frac{N_i}{|N_j|!|N_k|!|N_l|!}$$

Exercise: write network for the reactions $3\alpha \rightleftarrows {}^{12}C + \gamma$ and ${}^{12}C + \alpha \rightleftarrows {}^{16}O + \gamma$