Strong first-order phase transitions: Vorticity, droplets and gravitational waves

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Contents

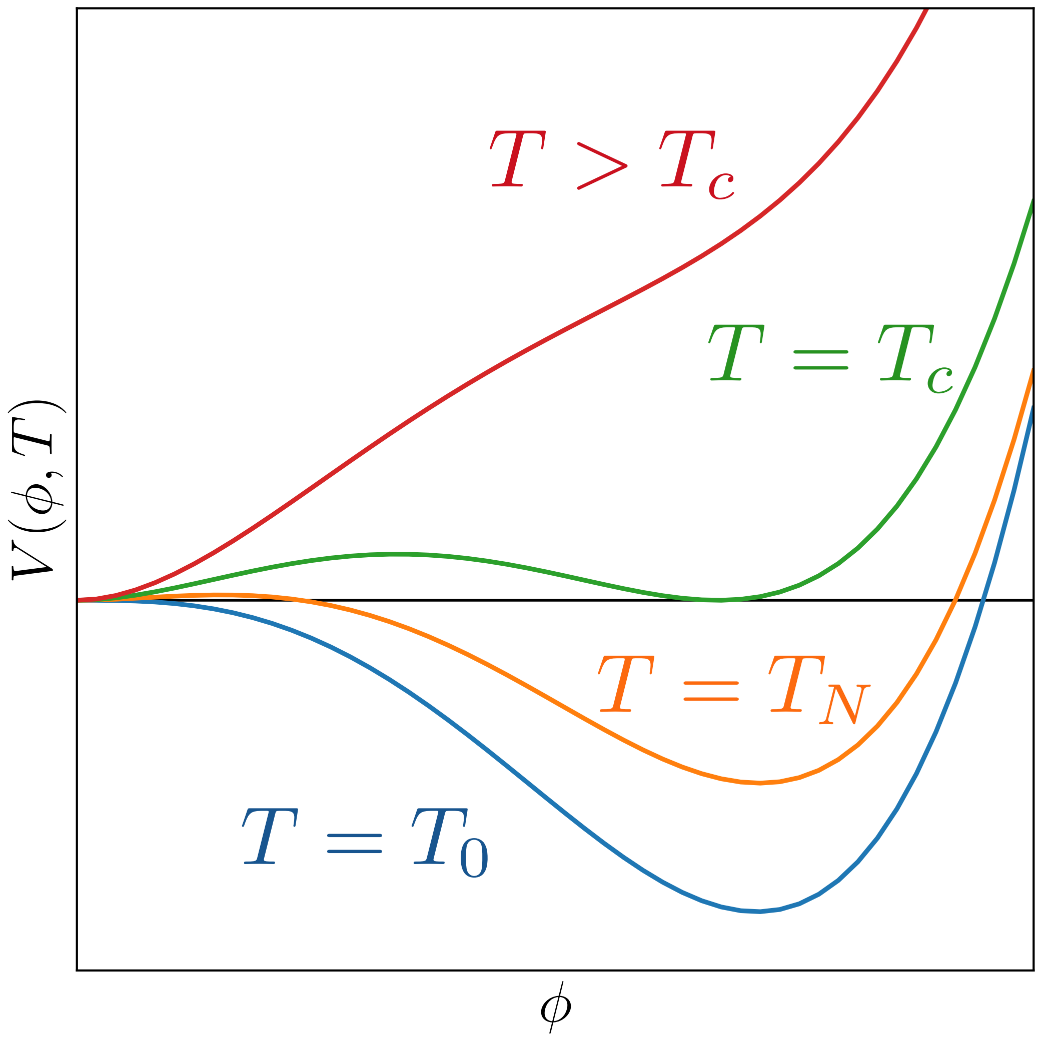
- Background and model
- Simulation details
- Results:
  - Vorticity
  - Heated droplets
  - Kinetic deficit
  - Gravitational waves

Cosmological phase transitions

- In cosmological phase transitions (PTs) an (effective) scalar field transitions from false vacuum to true vacuum.

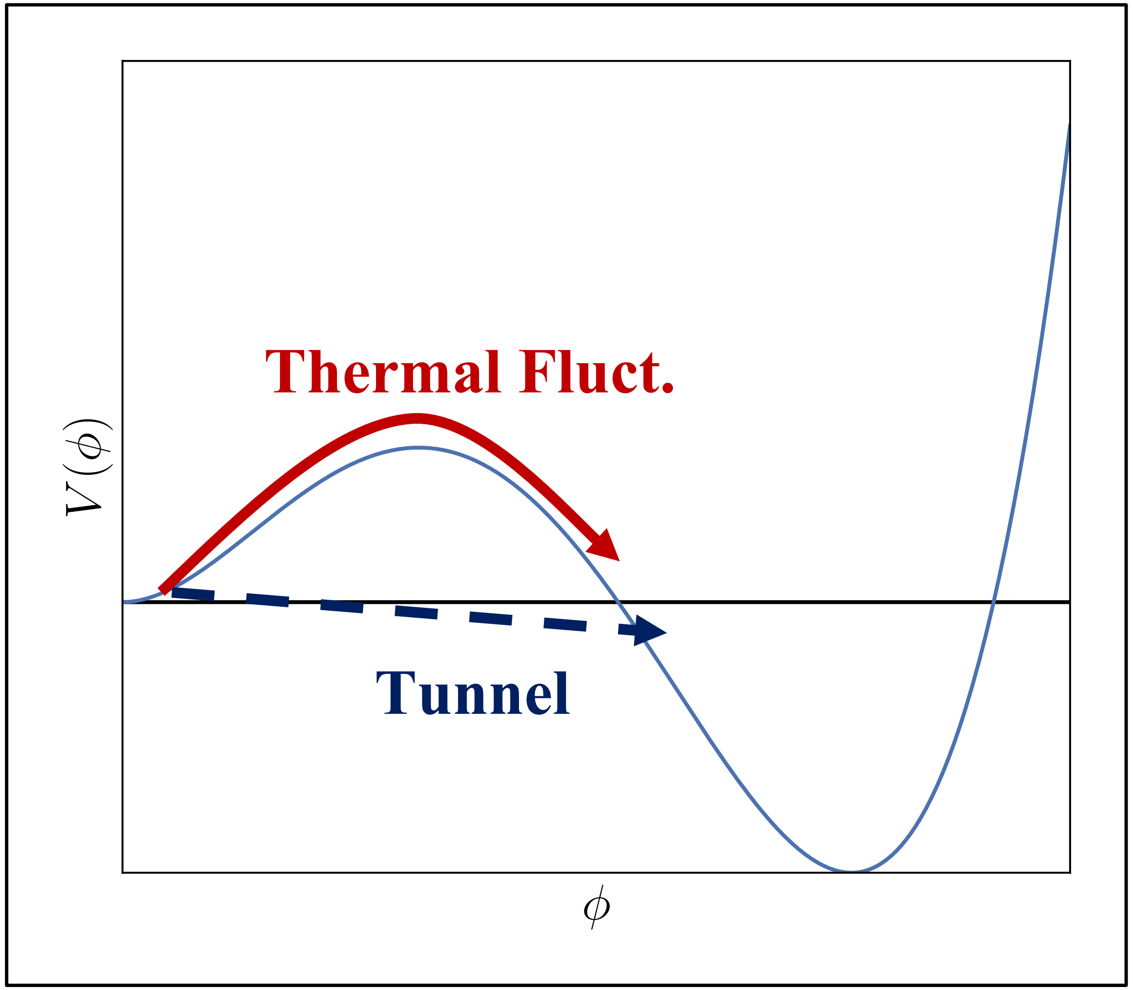
  [Coleman 1977][Linde 1982][Kirzhnits, Linde 1975]

- Phases separated by potential barrier.
- Bubbles nucleate, expand, merge.
- Extensions to the Standard Model can give a first-order PT around electroweak scale.



Thermal vs vacuum

- Thermal PTs:
  - Thermally fluctuate over barrier.
  - Fluid shell around bubble wall, exerts friction.
  - Terminal wall velocity.
  - Free energy difference mostly shared between bulk motion of fluid and thermal radiation.
- Vacuum PTs:
  - Quantum tunnel through barrier.
  - Fluid effects negligible.
  - Bubble wall accelerates until collision.
  - Free energy difference deposited into motion of the bubble wall.



Gravitational waves from phase transitions

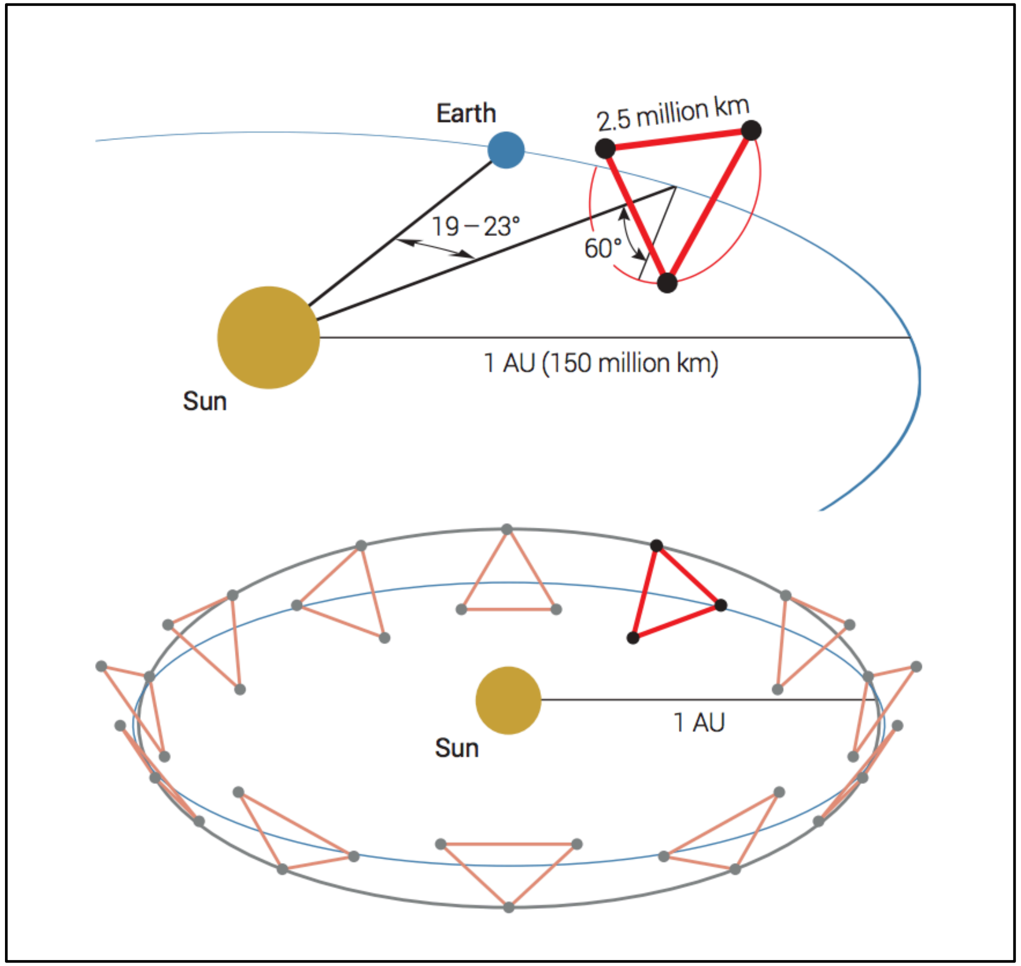
- Early studies used envelope approximation.
  - Energy momentum in infinitesimally thin shell at wall.
  - Energy momentum disappears upon collision.
  - Broken power law: k^3 in IR, k^{-1} in UV.

  [Kosowsky, Turner, Watkins, 1992] [Kosowsky, Turner, 1993][Kamionkowski, Kosowsky, Turner, 1994] [Huber, Konstandin, 2008]
- Sound waves/fluid motion found to be dominant source in thermal transitions.

  [Hindmarsh, Huber, Rummukainen, Weir, 2014, 2015, 2017][Giblin, Mertins 2014][Weir 2016]
- Lattice simulations give a steeper power law in UV than envelope for vacuum transitions. *[Cutting, Hindmarsh, Weir, 2018]*

Gravitational waves and LISA

- Laser interferometer space antenna (LISA) due to fly in 2030s.
- Sensitive to frequencies of $10^{-4} Hz$ to $10^{-1} Hz$. *[arXiv:1702.00786]*
- Signal from PT at electroweak scale peaks in this window!
- Can probe BSM physics if we can characterise the gravitational wave (GW) signal.



Taken from: arXiv:1702.00786

Model

- Perfect fluid coupled to a scalar field:

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \frac{1}{2} g^{\mu\nu} (\partial\phi)^2 + (\epsilon + p) U^\mu U^\nu + g^{\mu\nu} p \, .$$

- Bag-like equation of state with pressure and energy in symmetric (+) and broken phase (-):

$$p_+ = a_+ T^4 - \Delta V, \qquad \epsilon_+ = 3a_+ T^4 + \Delta V,$$
$$p_- = a_- T^4, \qquad \epsilon_- = 3a_- T^4,$$

with $a = \frac{\pi^2}{90} g_*$ and g_* the degrees of freedom.

- The enthalpy is given by $w = p + \epsilon$.

[Ignatius, Kajantie, Kurki-Suonio,Laine 1994]

Model

- Friction term couples field and fluid

$$\partial_\mu T^{\mu\nu}_\phi = -\partial_\mu T^{\mu\nu}_{\mathrm{f}} = -\eta U^\mu \partial_\mu \phi \partial^\nu \phi.$$

- Define the transition strength as *[Espinosa, Konstandin, No, Servant, 2010]*

$$\alpha = \left. \frac{\frac{1}{4}\Delta\left(\epsilon - 3p\right)}{\epsilon_r} \right|_{T_n} = \frac{\Delta V}{3a_+ T_n^4}\,.$$

- Gravitational waves from field negligible unless $\alpha \gg 1$ or bubbles grow to Hubble length.

Strong transitions

- Previously up to $\alpha \sim 0.1$ studied, what changes with $\alpha \sim 1$?

  [Hindmarsh, Huber, Rummukainen, Weir, 2014, 2015, 2017] [Giblin, Mertins 2014]

- Fluid kinetic energy (and GW signal) depends strongly on v_{w} and α.

- For transitions with $\alpha \sim 1$ fluid velocity $v \sim \mathcal{O}(1)$.

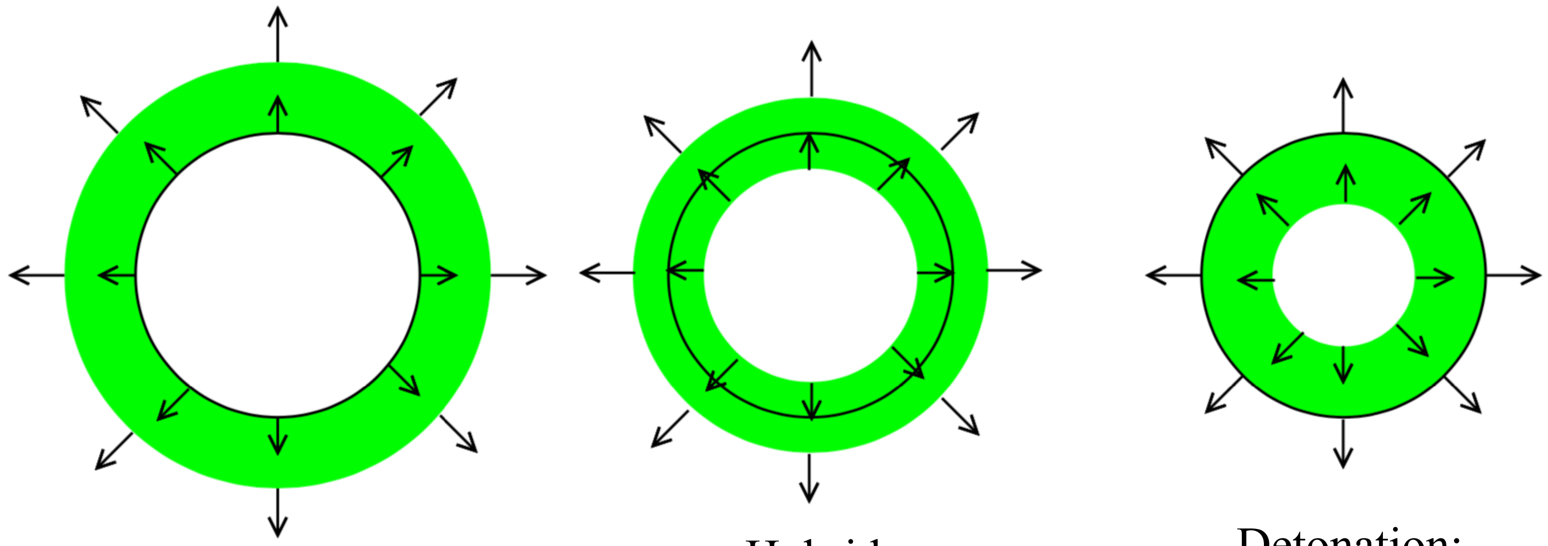
- Large $v \rightarrow$ non linearities become important.

- Vorticity important: different GW signal for turbulence and sound waves.

  [Gogoberidze, Kashniavili, Kosowsky 2007][Caprini, Durrer, Servant 2009]
  [Caprini, Durrer, Konstandin, Servant 2009][Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky 2019]

Fluid profiles

- Fluid shell develops around expanding bubbles.
- Different types of fluid shell depending on wall velocity v_{w}, speed of sound c_{s}, and Jouget detonation speed c_{J}:



Deflagration: $v_{\mathrm{w}} < c_s$

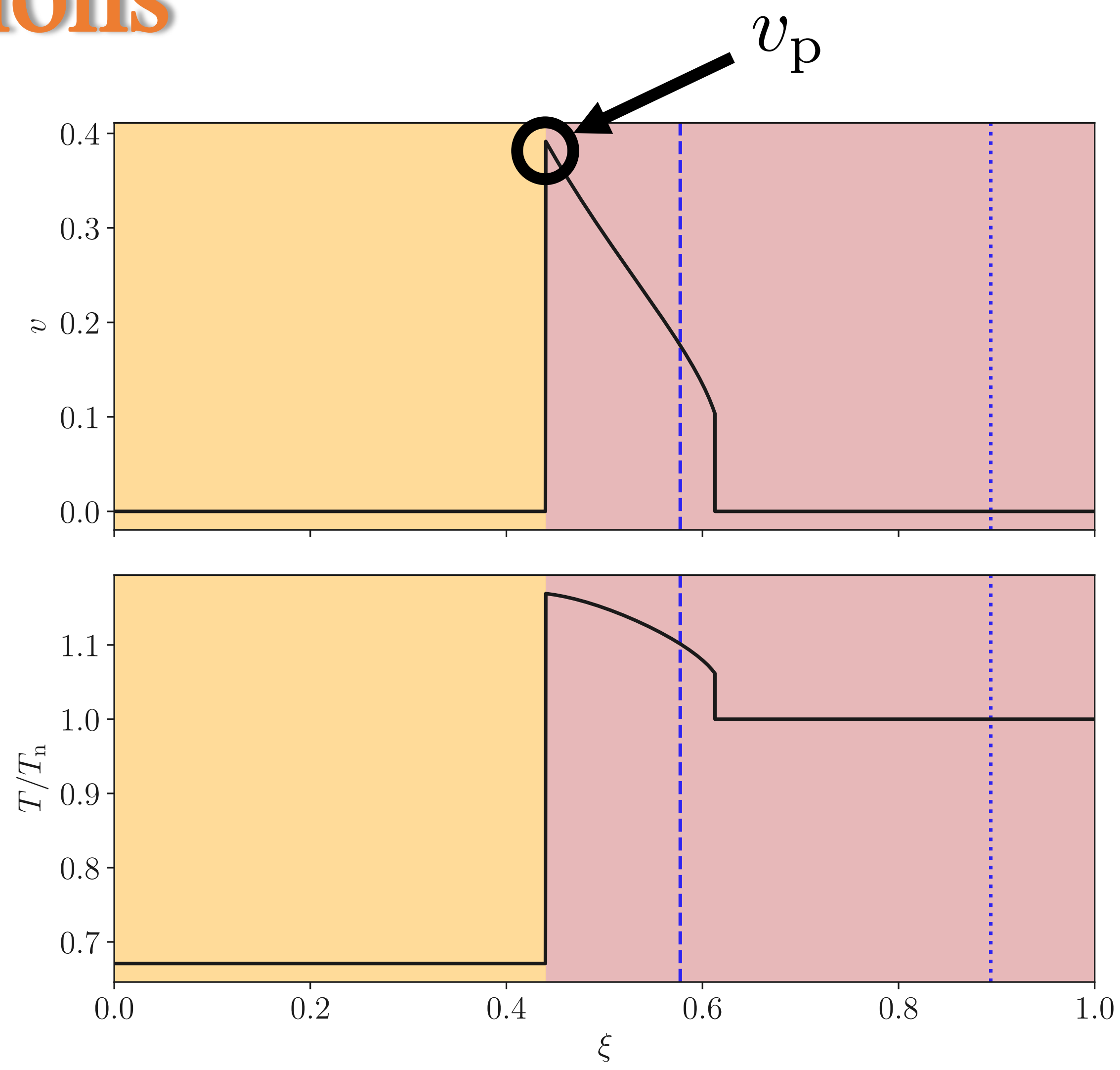
Hybrid: $c_{\mathrm{s}} \leq v_{\mathrm{w}} < c_J$

Detonation: $c_{\mathrm{s}} < c_{\mathrm{J}} \leq v_{\mathrm{w}}$

Fig: [Espinosa, Konstandin, No, Servant, 2010]

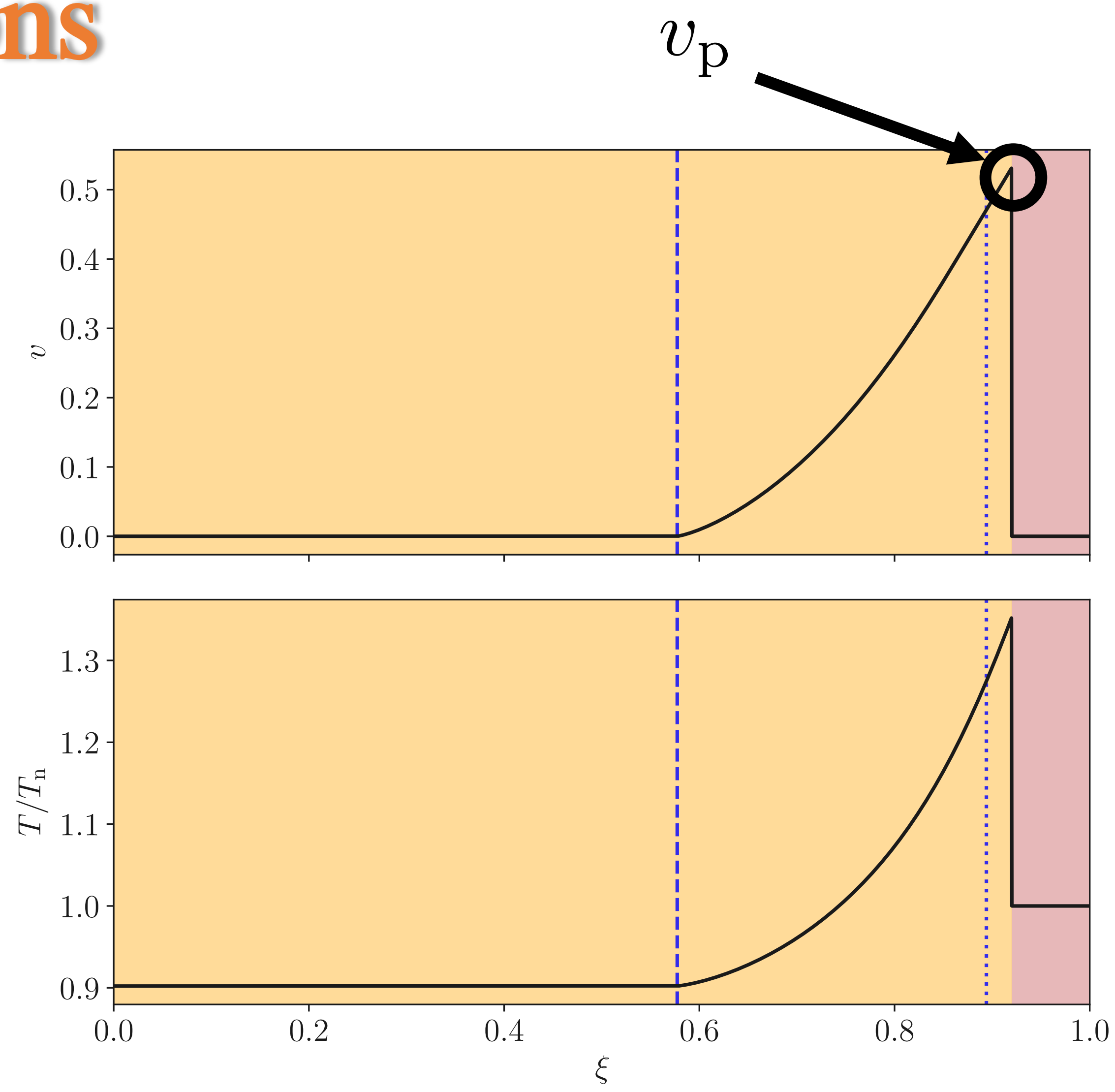
Fluid profiles: deflagrations

- Self-similar profile.
- Fluid velocity v, temperature T, and ϕ all functions of $\xi \equiv r/t$.
- Fluid velocity peaks by the wall.
- Slower than speed of sound (blue dashed line)
  $v_\mathrm{w} < c_s$.
- Fluid at rest inside of the bubble.
- Heated in front of the bubble.



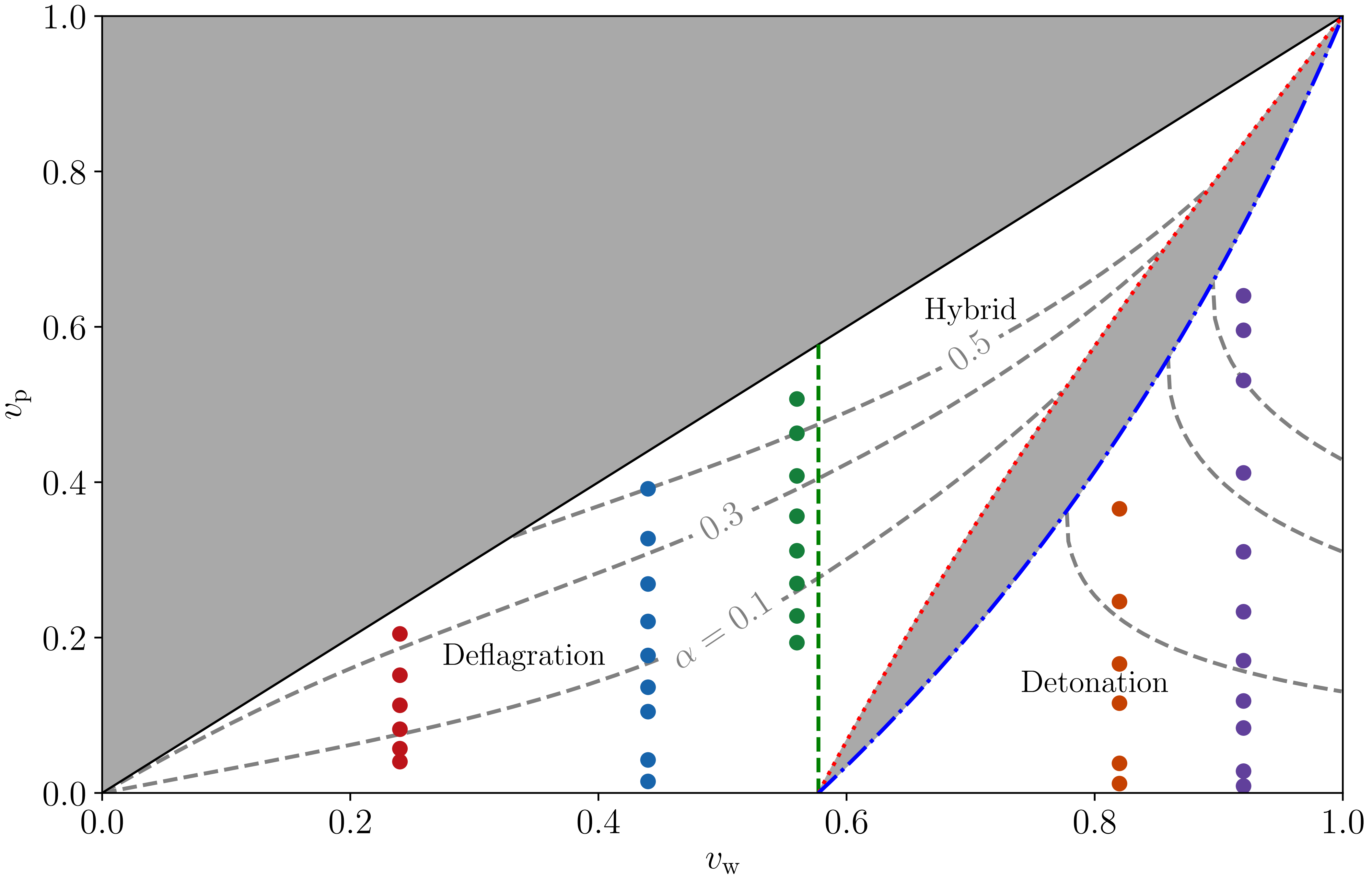
Fluid profiles: detonations

- Self-similar profile.
- Fluid velocity v, temperature T, and ϕ all functions of $\xi \equiv r/t$.
- Fluid velocity peaks by the wall.
- Faster than Jouget detonation speed (blue dotted line) $v_w \geq c_J > c_s$.
- Fluid at rest outside of the bubble.
- Heated inside the bubble.



Simulations

- Classical lattice simulations in 3+1 dimensions.
- Periodic boundary conditions.
- Nucleate 8 bubbles simultaneously.
- Scan across v_w and α, keeping the same bubble positions.
  - Deflagrations: $v_\mathrm{w} = \{0.24, 0.44, 0.56\}$
  - Detonations: $v_\mathrm{w} = \{0.82, 0.92\}$
- Keep same potential parameters, increase α by dropping T_n.
- Average bubble separation $R_* = (V/N_\mathrm{b})^{1/3}$.



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
v_{p}
v_{w}
Hybrid
Deflagration
Detonation
0.5
0.3
$\alpha = 0.1$

Simulations

- Track the enthalpy-weighted RMS four-velocity,

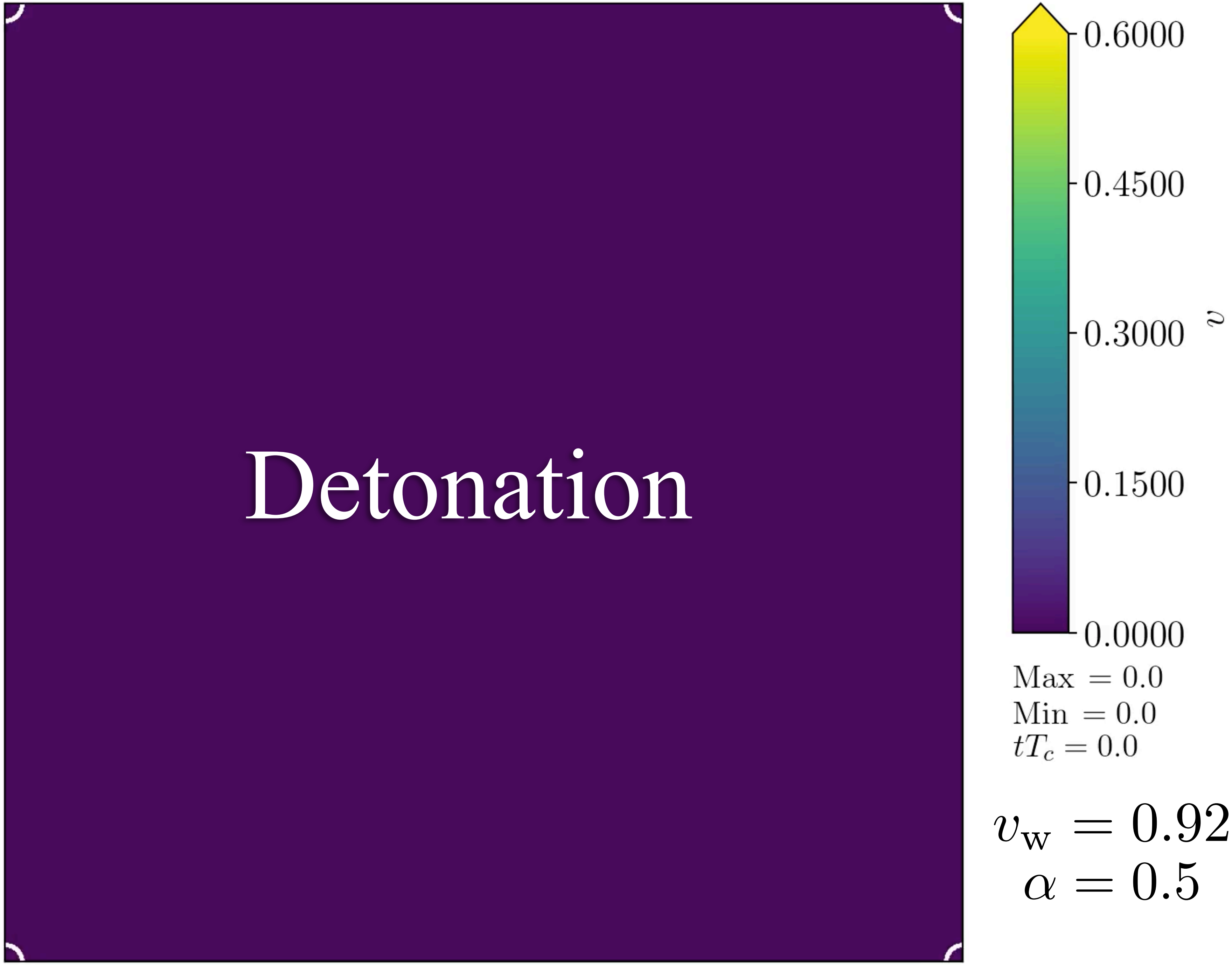
$$\overline{U}_{\mathrm{f}}^{2} = \frac{1}{\overline{w}\mathcal{V}} \int_{\mathcal{V}} \mathrm{d}^3x \, w\gamma^2 v^2 .$$

- Define similar quantity with gradients of ϕ,

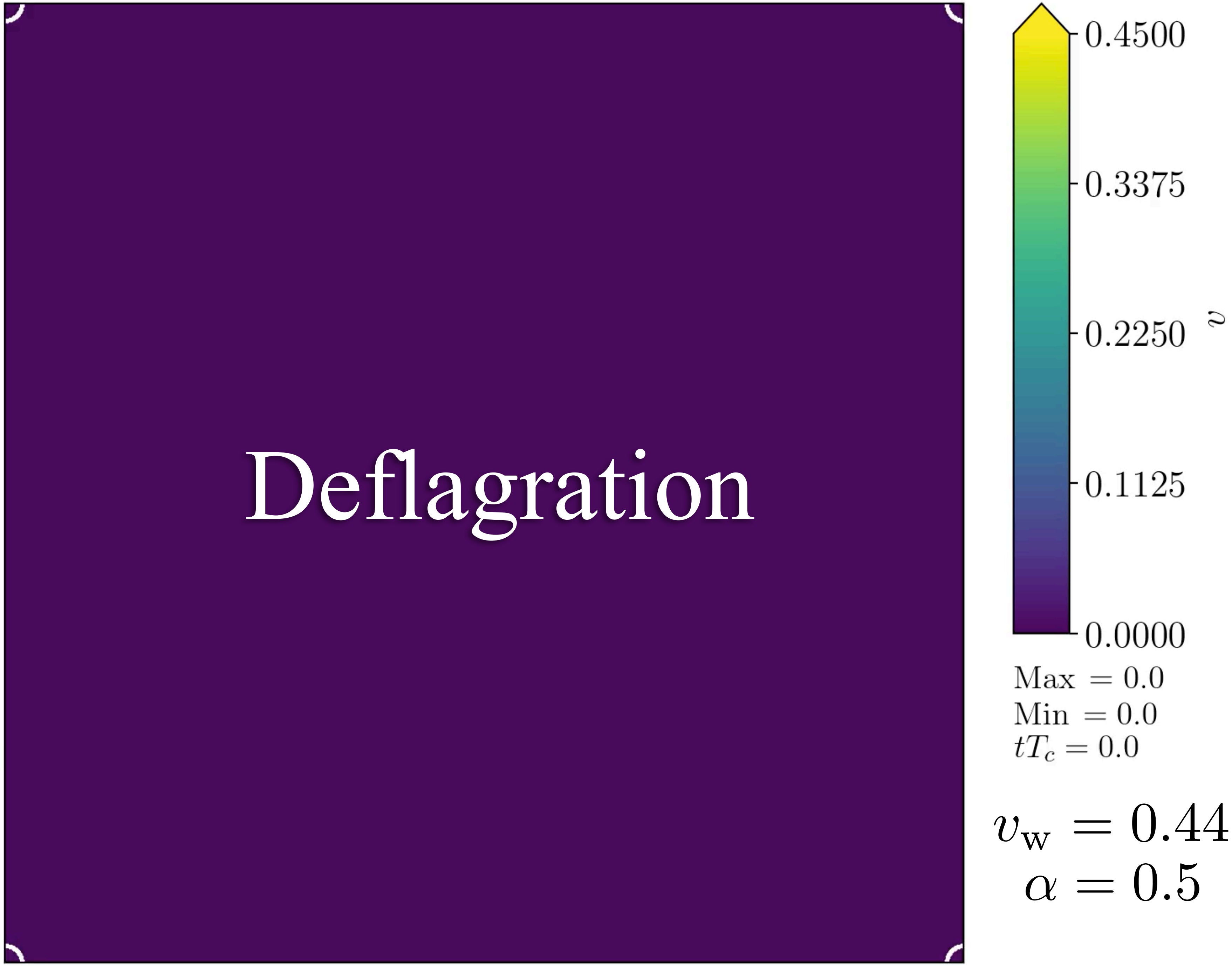
$$\overline{U}_{\phi}^{2} = \frac{1}{\overline{w}\mathcal{V}} \int_{\mathcal{V}} \mathrm{d}^3x \, \partial_i\phi\partial_i\phi .$$

- Output RMS fluid velocity $\bar{v}$, and rotational/longitudinal components $\bar{v}_{\perp}$, $\bar{v}_{\parallel}$.
- Track gravitational wave energy density parameter $\Omega_{\mathrm{gw}} = \rho_{\mathrm{gw}}/\rho_{\mathrm{c}}$.
- Write out slices of fluid velocity v, vorticity $|\nabla \times v|$, and temperature T.

Detonation

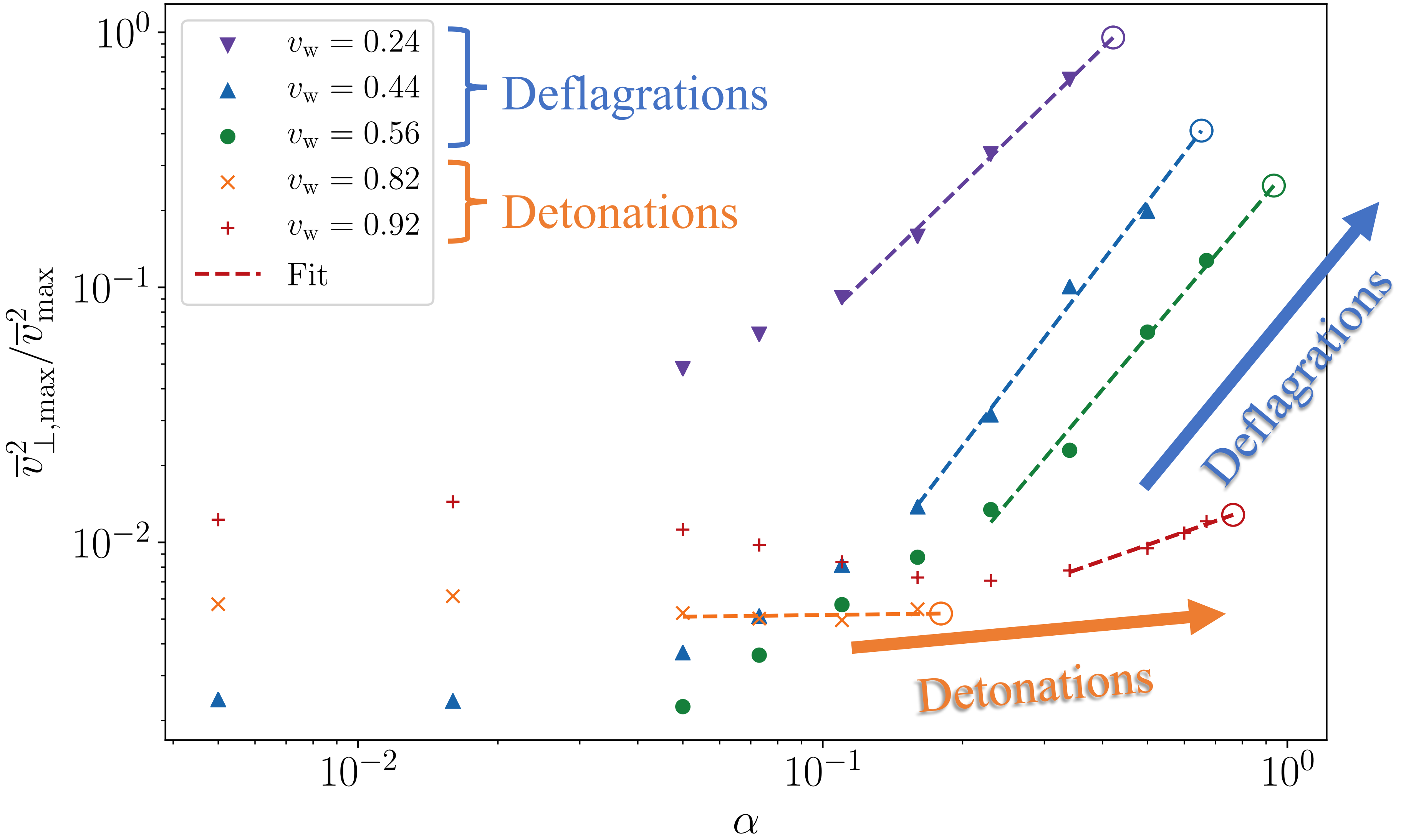


$$v_{\mathrm{w}} = 0.92$$
$$\alpha = 0.5$$

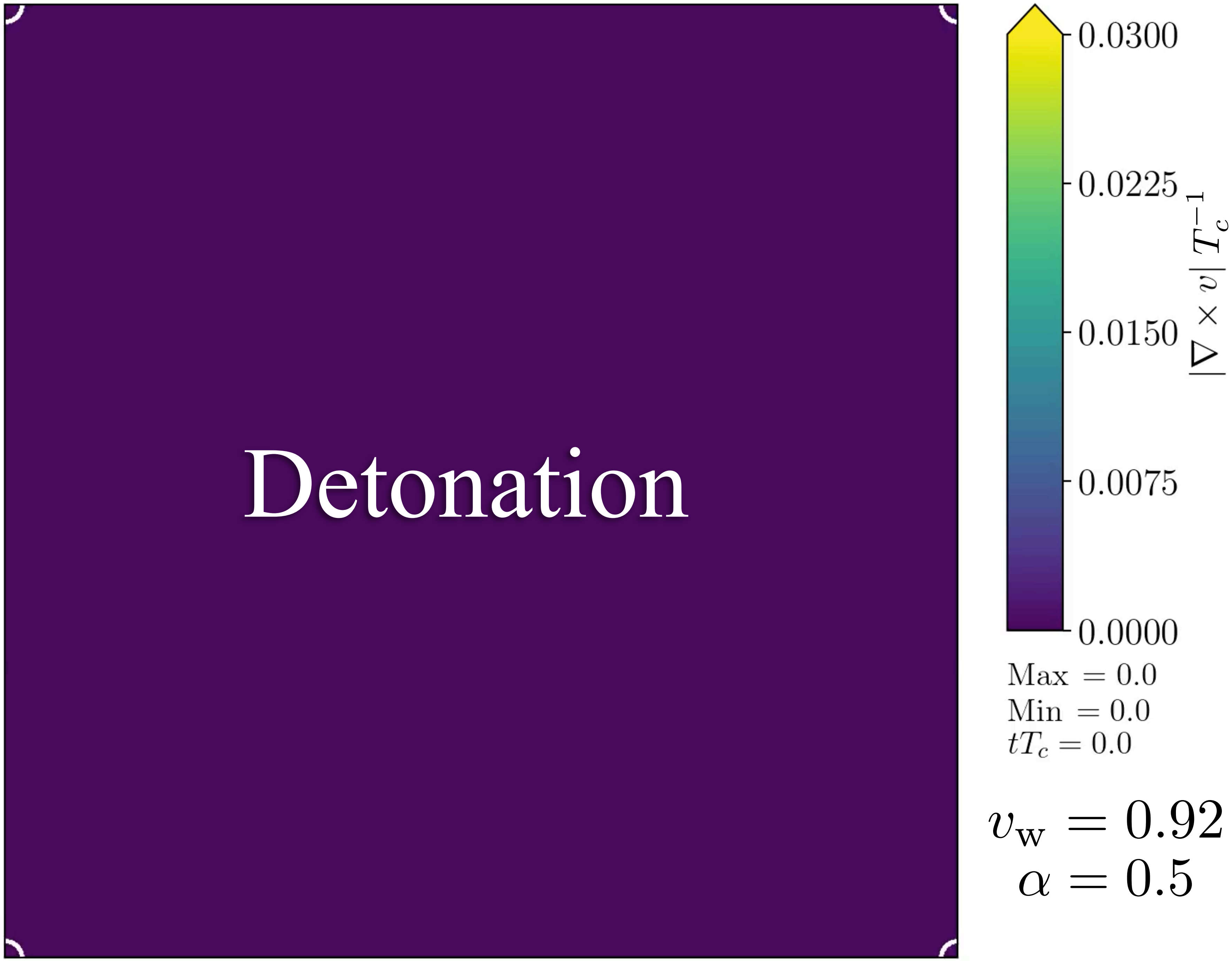


$v_{\mathrm{w}} = 0.44$
$\alpha = 0.5$

Vorticity

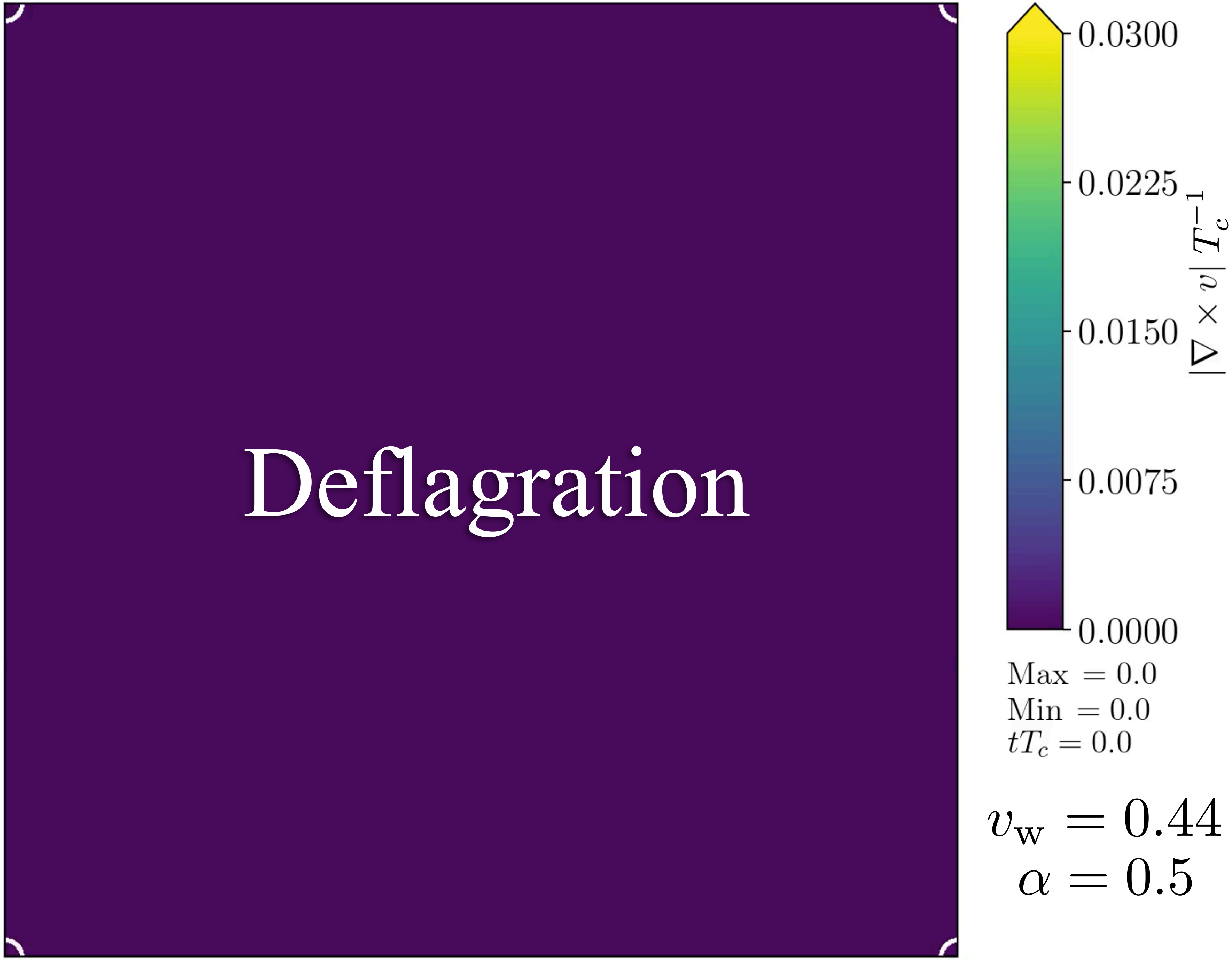


$v_w = 0.24$
$v_w = 0.44$
$v_w = 0.56$
$v_w = 0.82$
$v_w = 0.92$
Fit
Deflagrations
Detonations
Deflagrations
Detonations
$\overline{v}^2_{\perp,\max}/\overline{v}^2_{\max}$
α
10^0
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^0



$v_{\mathrm{w}} = 0.92$

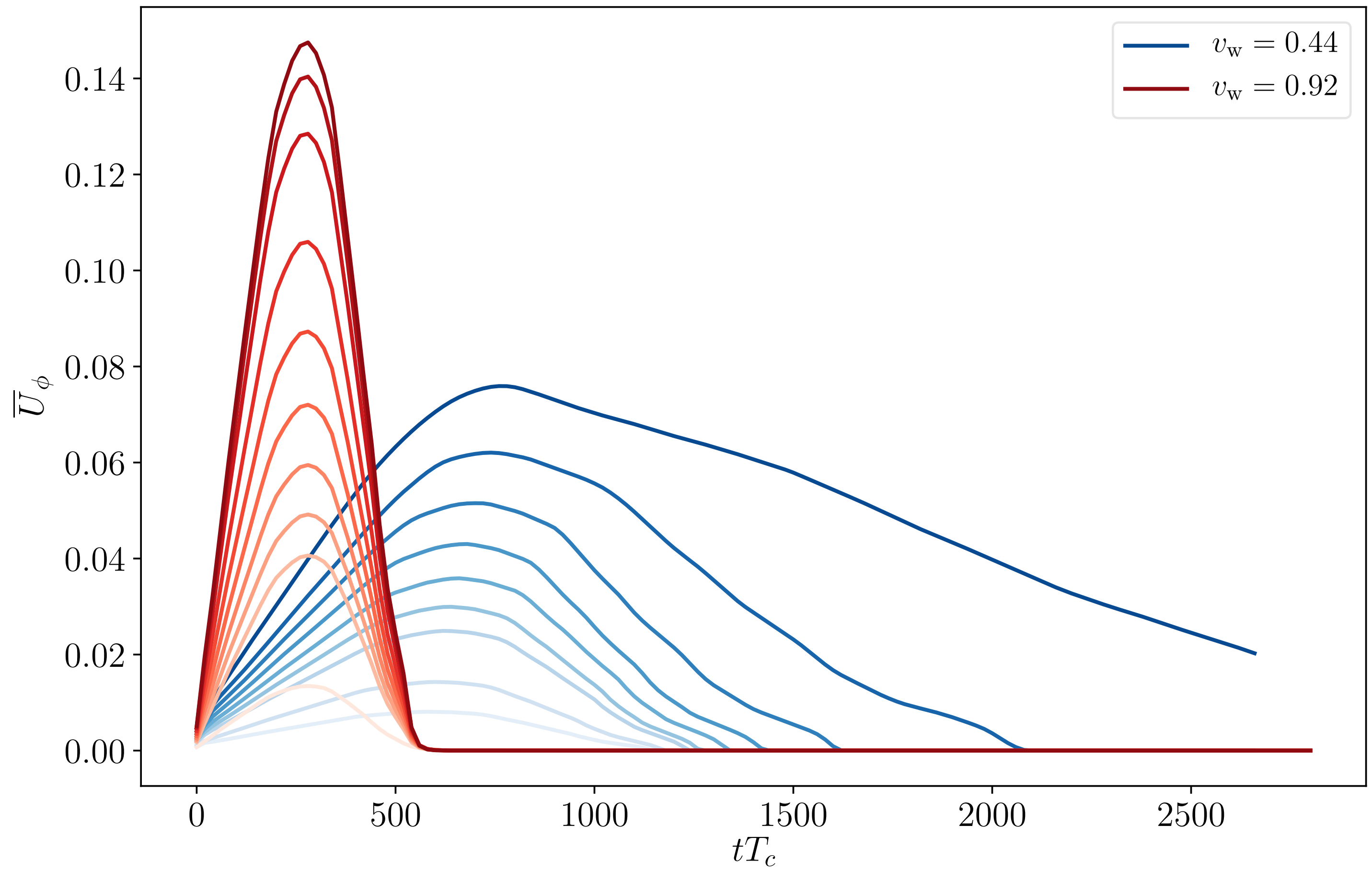
$\alpha = 0.5$

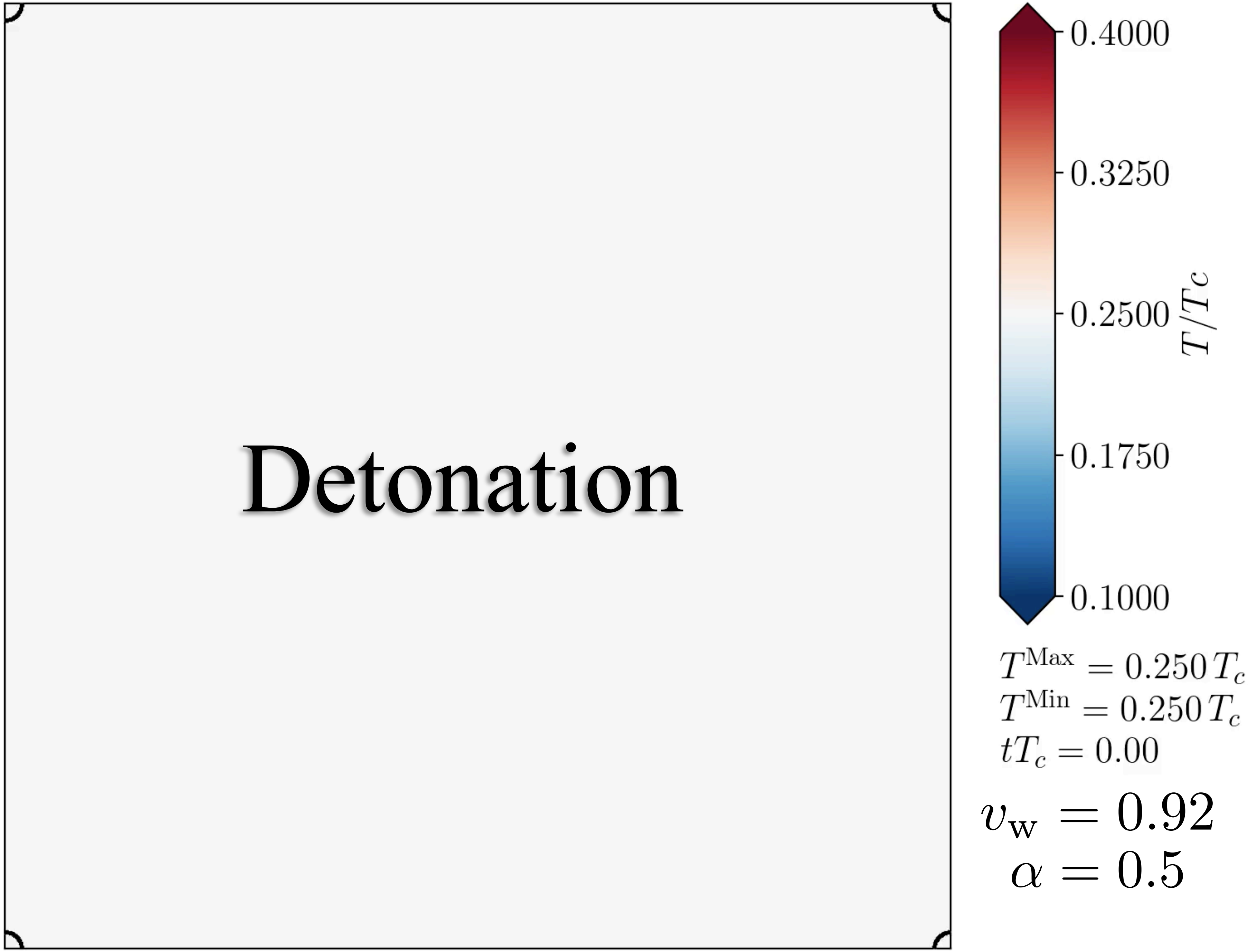


$$v_{\rm w} = 0.44$$
$$\alpha = 0.5$$

Heated droplets

- Shape of $\overline{U}_\phi$ tells us about transition duration and wall velocity.
- Duration of transition increases with α for deflagrations.
- Hot fluid pushed ahead of deflagrations.
- Form hot, high pressure droplets.
- Predicted in old QCD studies and more recent work. *[Kurki-Suonio, 1985]* *[Konstandin, No, 2011][Mégevand, Ramírez, 2018]*




Detonation
0.4000
0.3250
0.2500
0.1750
0.1000
T/T_c
$T^{\mathrm{Max}} = 0.250\,T_c$
$T^{\mathrm{Min}} = 0.250\,T_c$
$tT_c = 0.00$
$v_{\mathrm{w}} = 0.92$
$\alpha = 0.5$

$v_{\mathrm{w}} = 0.44$

$\alpha = 0.5$

Estimating gravitational waves

- Enthalpy weighted RMS four-velocity,

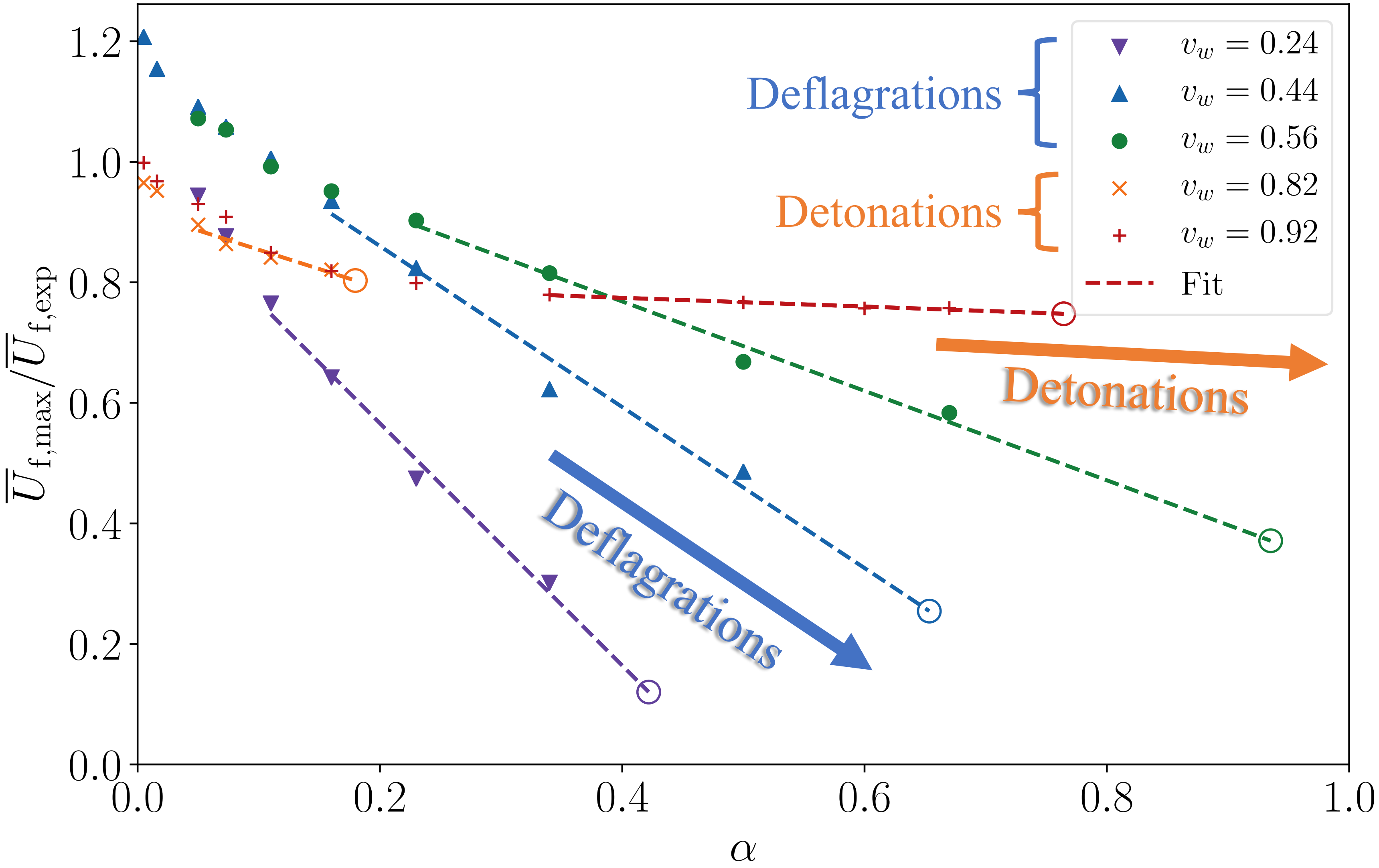
$$\overline{U}_{\rm f}^2 = \frac{1}{\overline{w}\mathcal{V}} \int_{\mathcal{V}} \mathrm{d}^3x\, w\gamma^2 v^2.$$

- Estimate $\overline{U}_{\rm f,exp}^2$ from fluid shell around single bubble with diameter R_*.
  - Works well for up to about $\alpha \sim 0.1$. [Hindmarsh, 2016]
- Gravitational wave energy density parameter $\Omega_{\rm gw} = \rho_{\rm gw}/\rho_{\rm c}$.
- $\Omega_{\rm gw}$ for sound waves with $\overline{U}_{\rm f,exp}^2$ given by

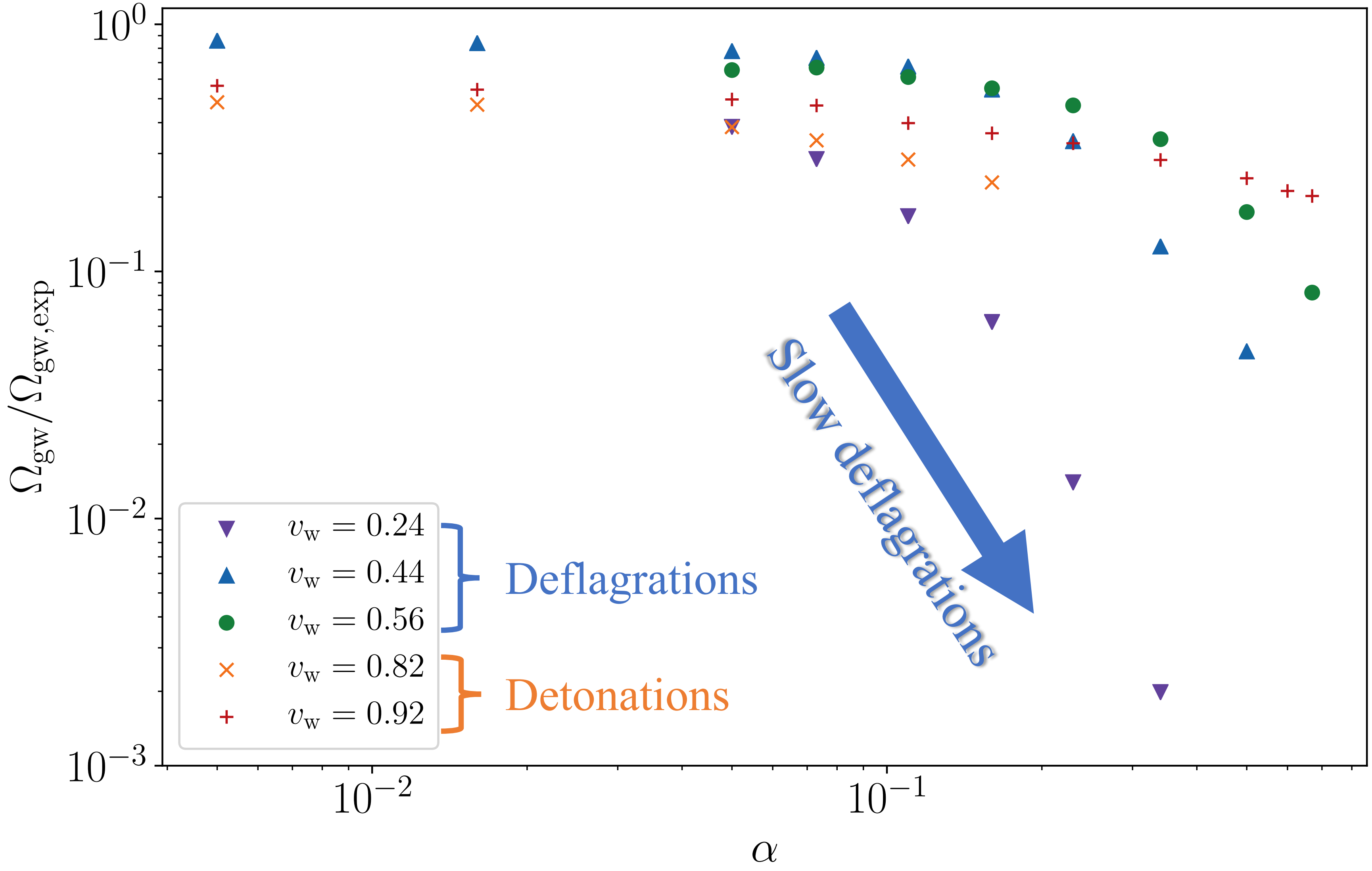
$$\Omega_{\rm gw,exp} = 3\,\tilde{\Omega}_{\rm gw} \left(\frac{\overline{w}}{\overline{\epsilon}}\right)^2 \overline{U}_{\rm f,exp}^4 (H_{\rm n}t)(H_{\rm n}R_*)\,,$$

with $\tilde{\Omega}_{\rm gw}$ a constant of $O(10^{-2})$. [Hindmarsh, Huber, Rummukainen, Weir, 2017]

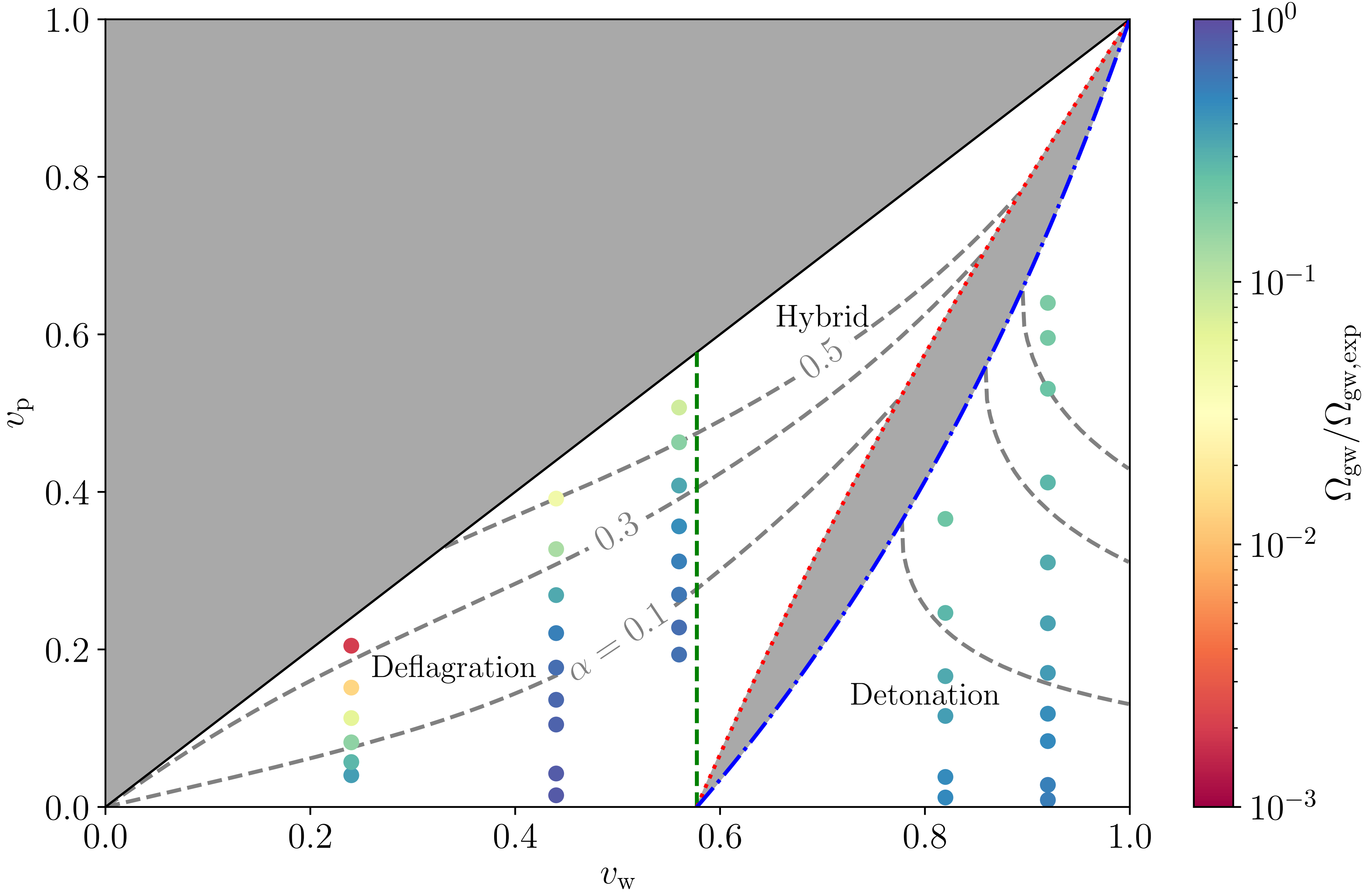
Kinetic Deficit



Gravitational waves



$\Omega_{gw,exp}$ from modelling in *[Hindmarsh, Huber, Rummukainen, Weir, 2017]*



1.0
0.8
0.6
0.4
0.2
0.0
$v_{\rm p}$
0.0
0.2
0.4
0.6
0.8
1.0
$v_{\rm w}$
Hybrid
0.5
0.3
$\alpha = 0.1$
Deflagration
Detonation
10^0
10^{-1}
10^{-2}
10^{-3}
$\Omega_{\rm gw}/\Omega_{\rm gw,exp}$

Summary

- Detonations and deflagrations behave differently in strong transitions.
- Deflagrations have substantial vorticity generation *during* transition.
- Hot droplets of the metastable state form in deflagrations.
- Droplets resist collapse → $\bar{U}_\mathrm{f}$ is suppressed.
- Reduced $\bar{U}_\mathrm{f}$ suppresses gravitational wave signal.
- Detonations behave similar to weak and intermediate transitions.

Caveats and further research:

- Does not apply to $\alpha \gg 1$.
- We assume that $HR_* \ll 1$.
- Small simulations:
  - Subject to finite volume effects.
  - Do not quite reach asymptotic flows.
- Find power spectra for strong transitions.
- Explore $\Delta a/a$ parameter space.
- Investigate development and decay of flows. ($\tau_{\mathrm{nl}} \sim R_*/\bar{U}_{\mathrm{f}}$)
- Behaviour of hybrids?
- More realistic equation of state?

Potential - more details

- Energy and pressure:

$$\epsilon = 3a(\phi)T^4 + V_0(\phi)\,, \qquad p = a(\phi)T^4 - V_0(\phi)\,.$$

- Potential given by:

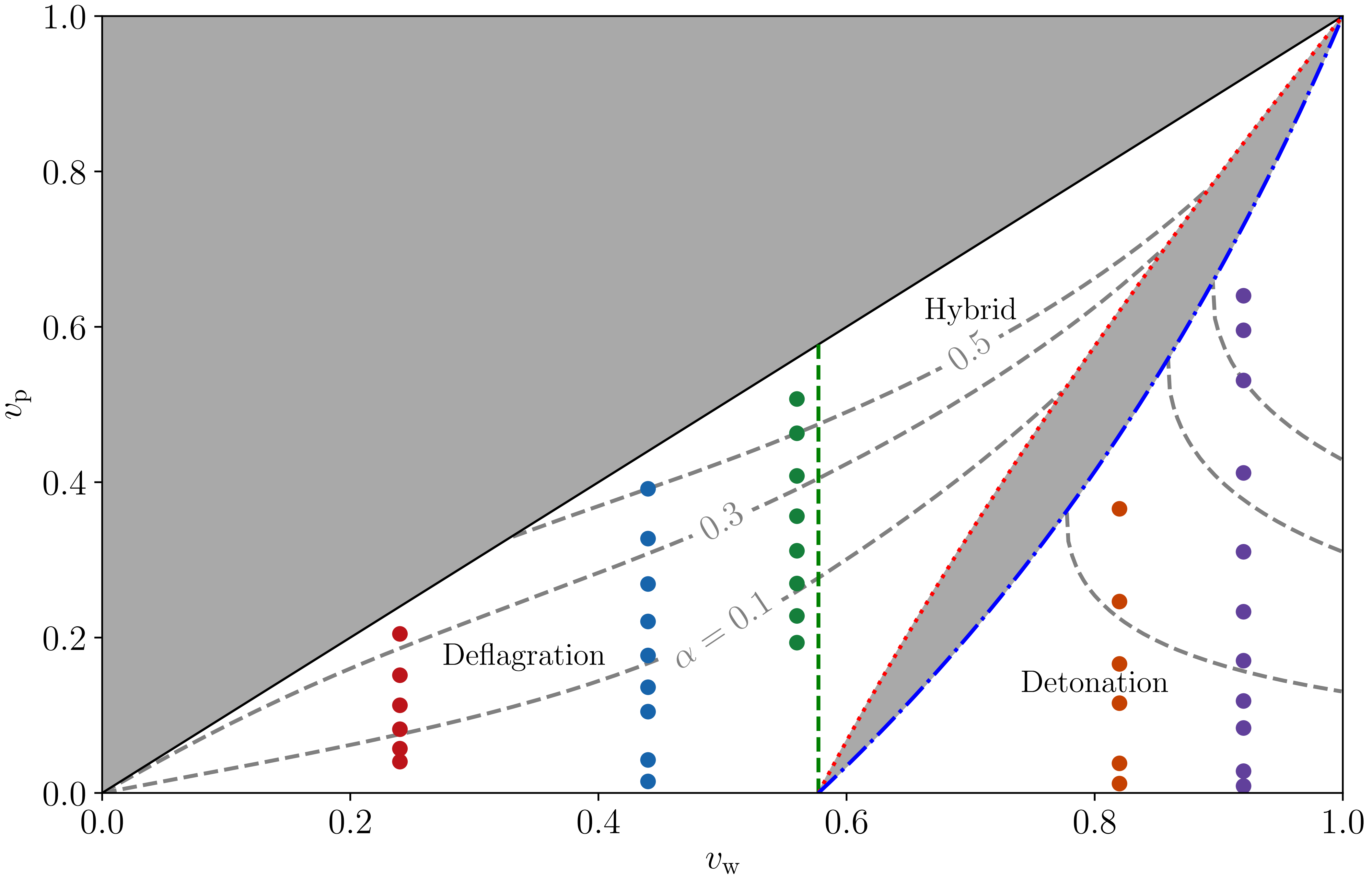
$$V_0(\phi) = \frac{1}{2}M^2\phi^2 - \frac{1}{3}\mu\phi^3 + \frac{1}{4}\lambda\phi^4 - V_\mathrm{c}\,,$$

with V_c chosen such that $V(\phi_b) = 0$, and ϕ_b the value of ϕ in the broken phase

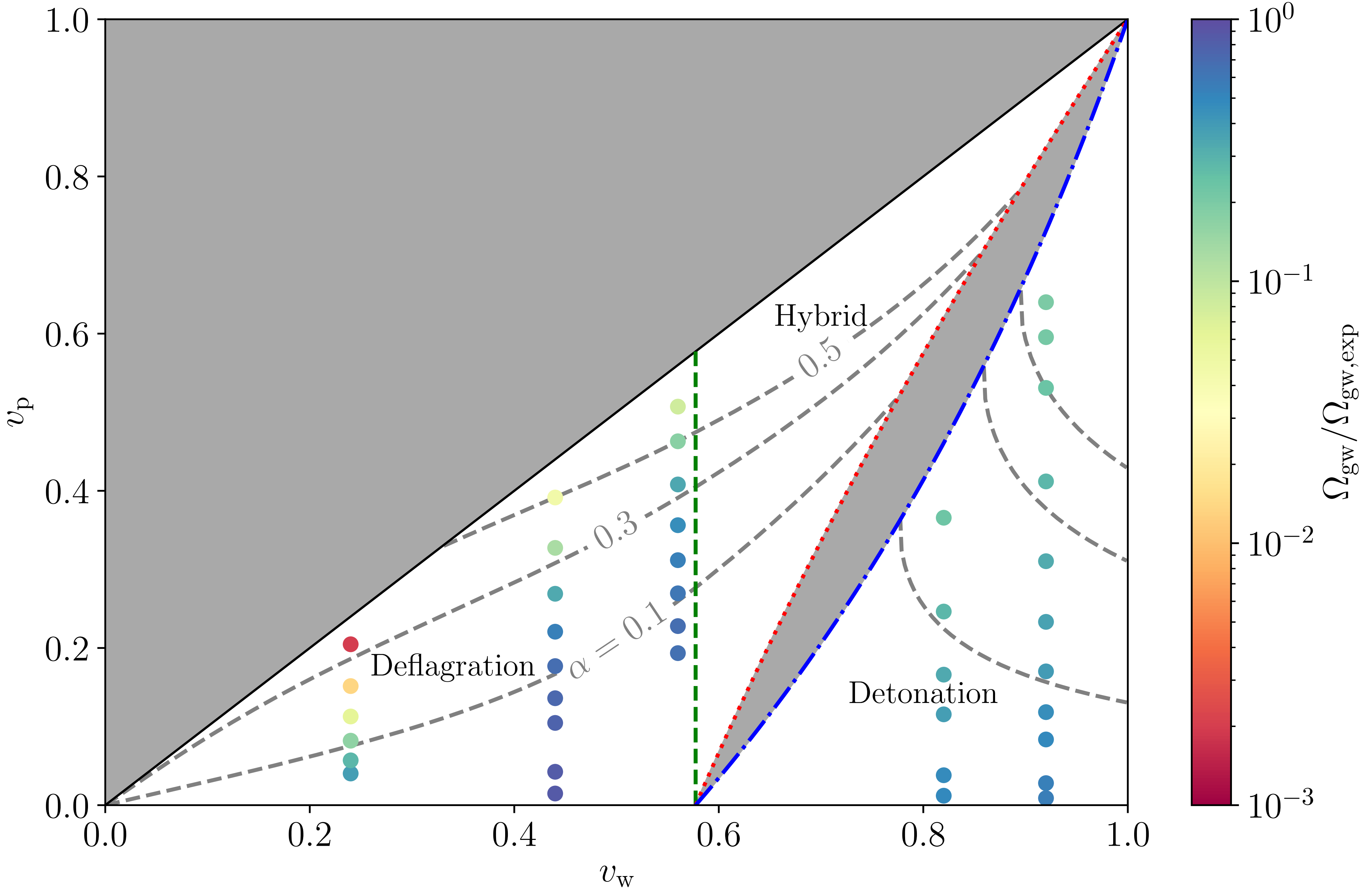
$$\Delta V = V_0(0) - V_0(\phi_b)\,.$$

- Finally, choose how degrees of freedom varies with ϕ,

$$a(\phi) = a_0 - \frac{\Delta V}{T_\mathrm{c}^4}\left[3\left(\frac{\phi}{\phi_\mathrm{b}}\right)^2 - 2\left(\frac{\phi}{\phi_\mathrm{b}}\right)^3\right].$$

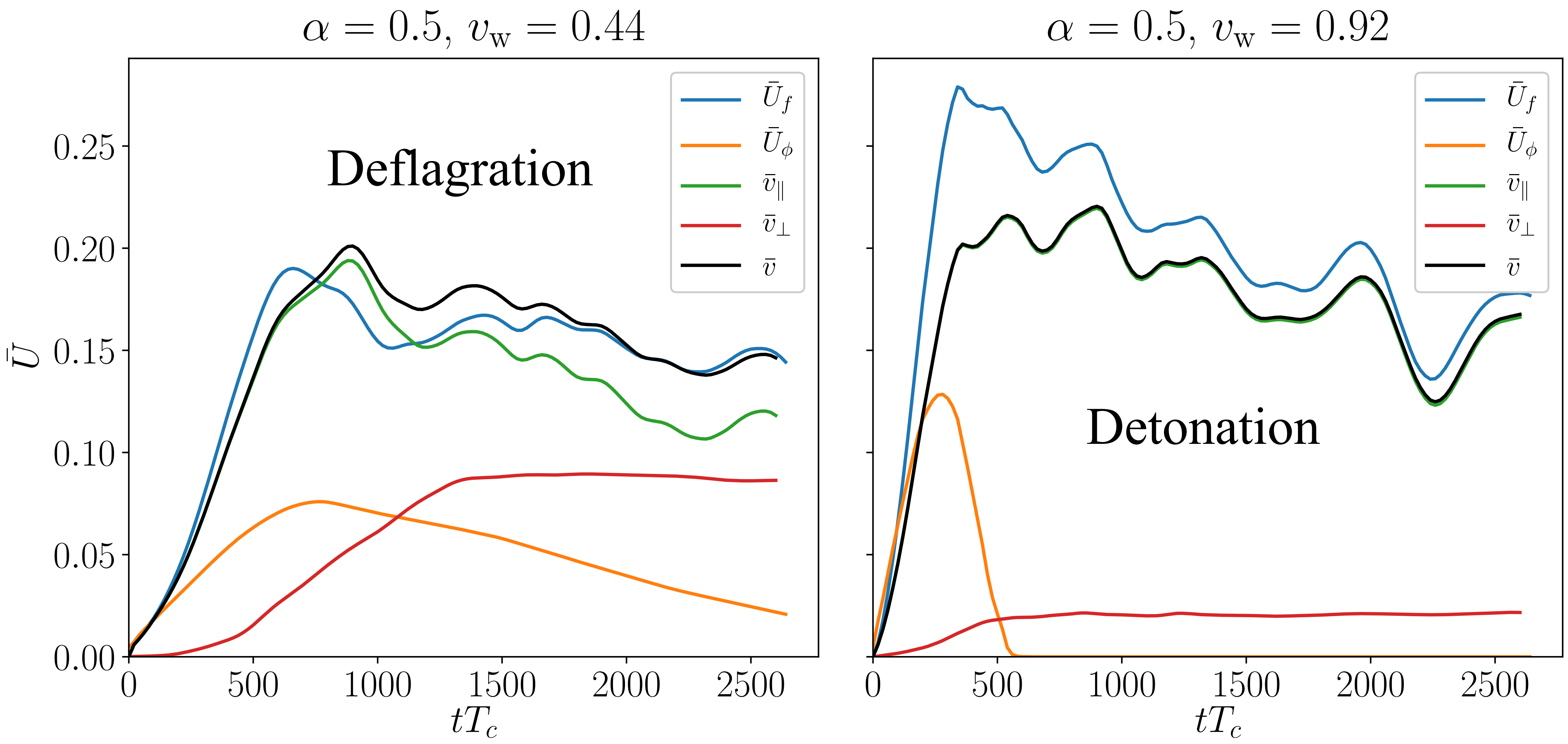


Hybrid
0.5
0.3
$\alpha = 0.1$
Deflagration
Detonation
v_p
v_w
0.0
0.2
0.4
0.6
0.8
1.0

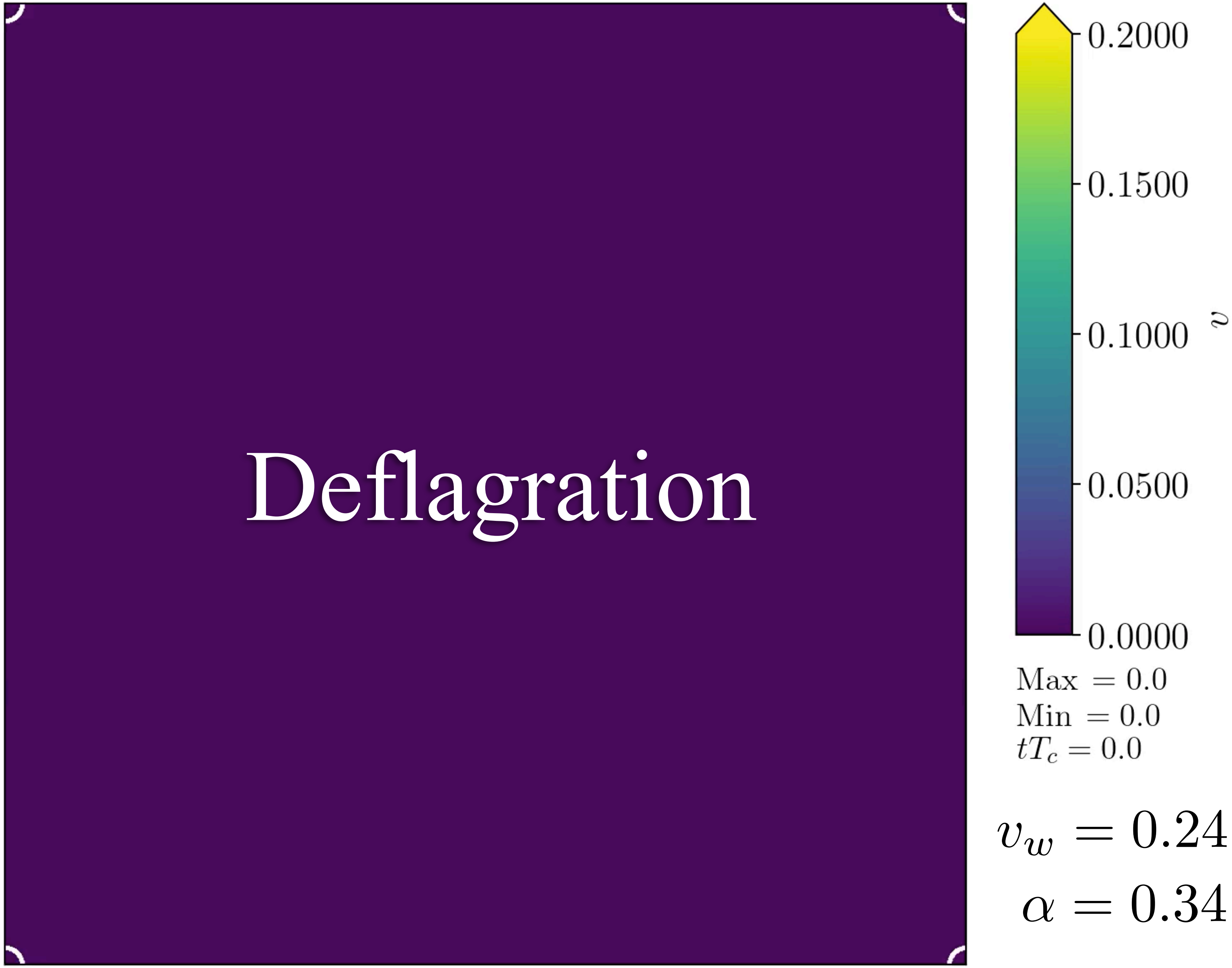


Hybrid
0.5
0.3
$\alpha = 0.1$
Deflagration
Detonation
v_w
v_p
$\Omega_{gw}/\Omega_{gw,exp}$

Evolution

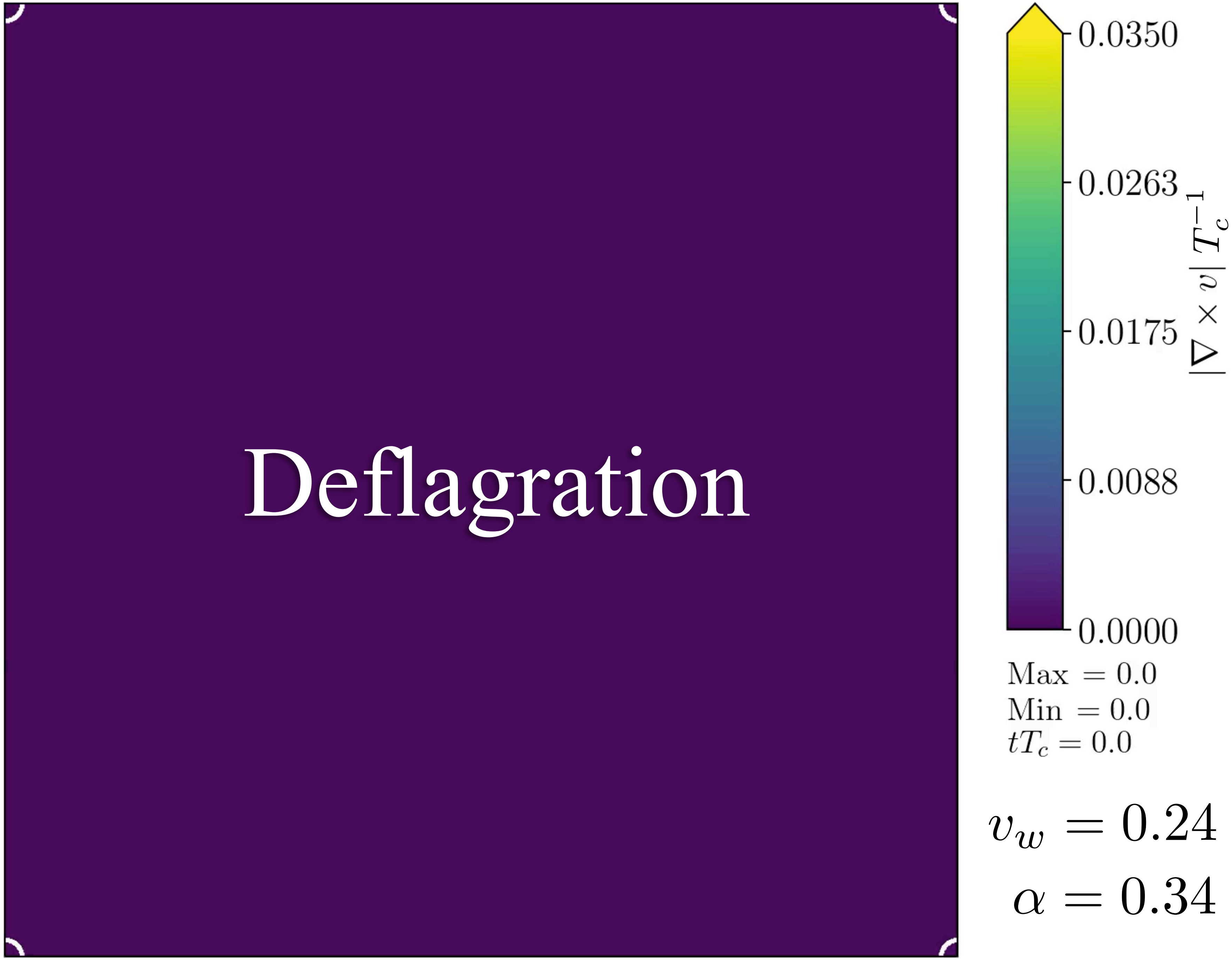


$\alpha = 0.5,\ v_w = 0.44$
Deflagration
$\bar{U}_f$
$\bar{U}_\phi$
$\bar{v}_\parallel$
$\bar{v}_\perp$
$\bar{v}$
$\bar{U}$
0.25
0.20
0.15
0.10
0.05
0.00
0
500
1000
1500
2000
2500
tT_c
$\alpha = 0.5,\ v_w = 0.92$
Detonation



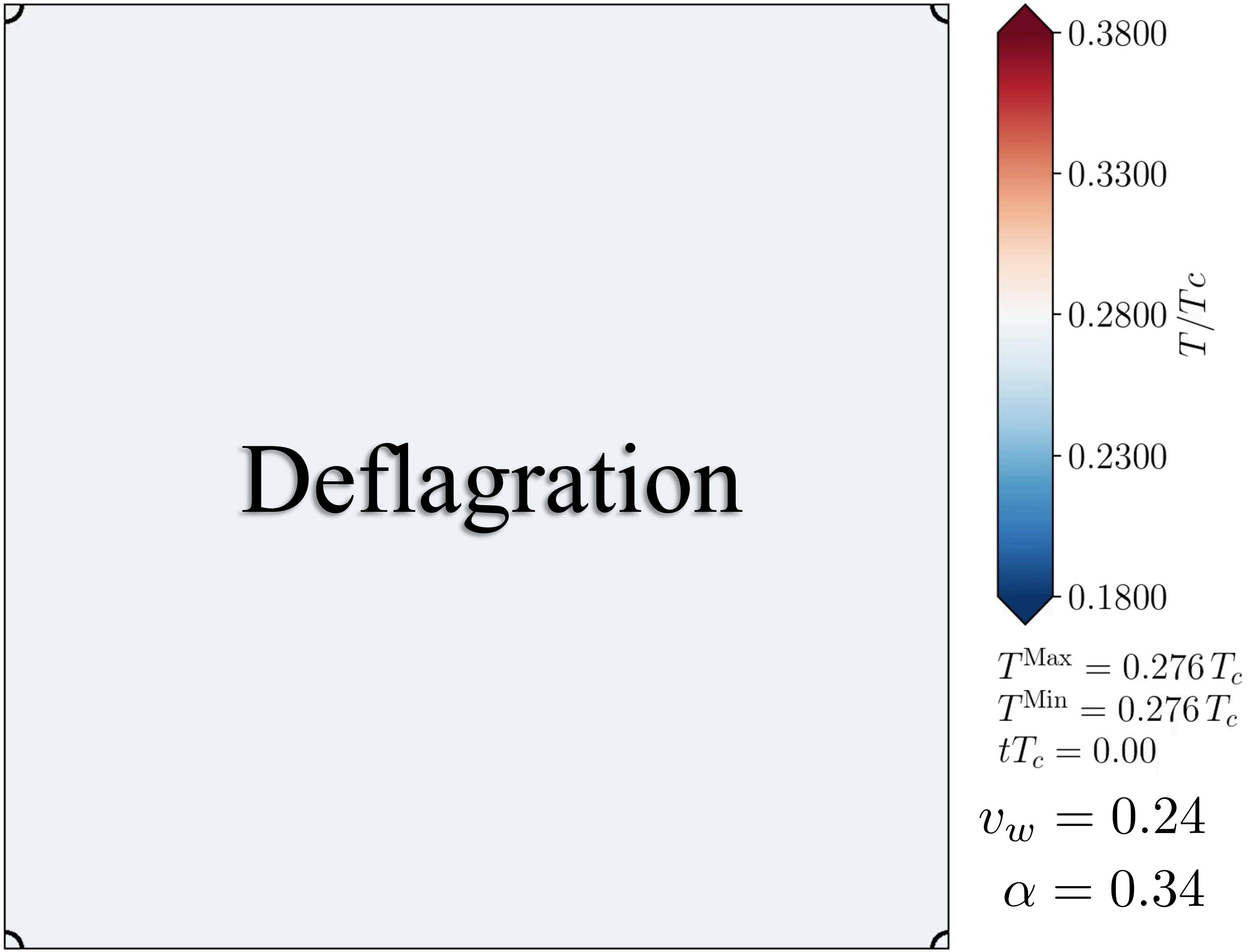
$$v_w = 0.24$$

$$\alpha = 0.34$$



$$v_w = 0.24$$

$$\alpha = 0.34$$



$v_w = 0.24$

$\alpha = 0.34$