Gravitational wave energy budget in strongly supercooled phase transitions

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Stockholm, 26 Aug 2019

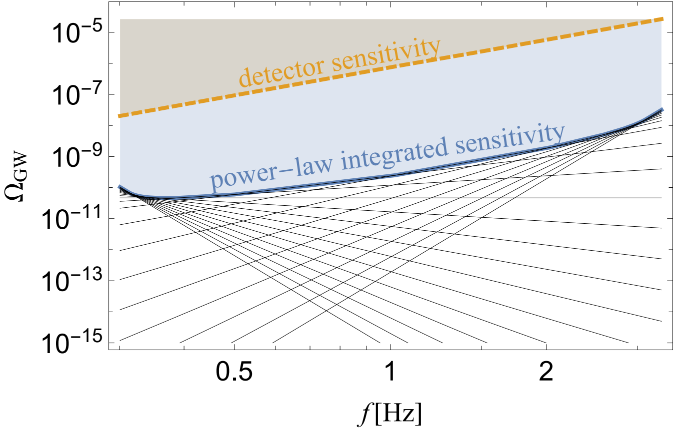
Based on:
J. Ellis, ML, J. M. No arXiv:1809.08242
J. Ellis, ML, J. M. No, V. Vaskonen arXiv:1903.09642
J. Ellis, M. Fairbairn, ML, J. M. No, V. Vaskonen, A Wickens arXiv:1907.04315
ML, V. Vaskonen arXiv:1909.XXXXX

Plan

- Experimental prospects
- Introduction to first order phase transitions
- Energy stored in the bubble walls
- Bubble wall and plasma sourced GWs
- Other observable consequences
    - Primordial Magnetic Fields
    - Primordial Black Holes
- Conclusions

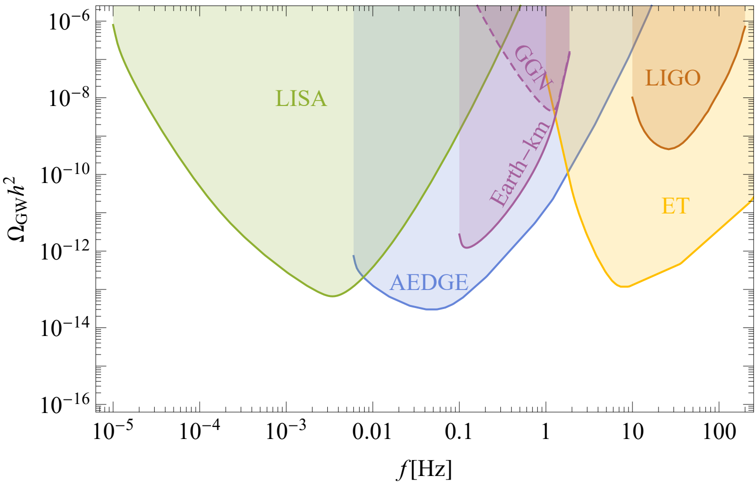
Power-law integrated sensitivity

$$\Omega_{\rm GW}^{\rm noise} = \frac{2\pi}{3}\frac{f^3 S_h}{H_0^2}, \qquad \text{SNR} = \sqrt{\mathcal{T}\int df \left(\frac{\Omega_{\rm GW}^{\rm signal}}{\Omega_{\rm GW}^{\rm noise}}\right)^2}$$



Thrane, Romano '13

Experimental outlook



phase transition dynamics

Bubble: static field configuration passing the barrier (excited through thermal fluctuations)

- decay rate

$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right),$$

- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- EOM $\to$ bubble profile

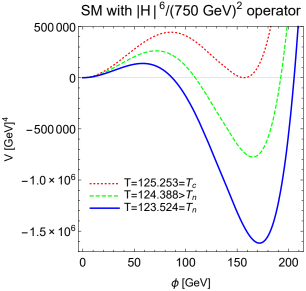
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi,T)}{\partial \phi} = 0,$$

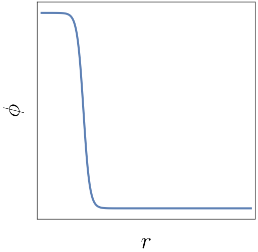
$$\phi(r \to \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

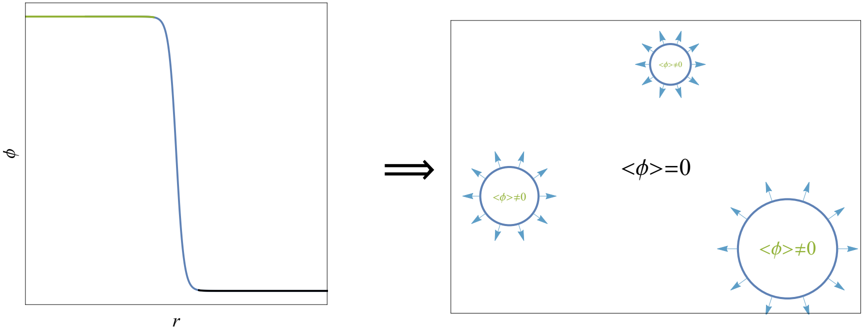
- nucleation temperature

$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T}\frac{\Gamma(T)}{H(T)^4} = 1$$

Linde '81 '83






ϕ
r
<ϕ>=0
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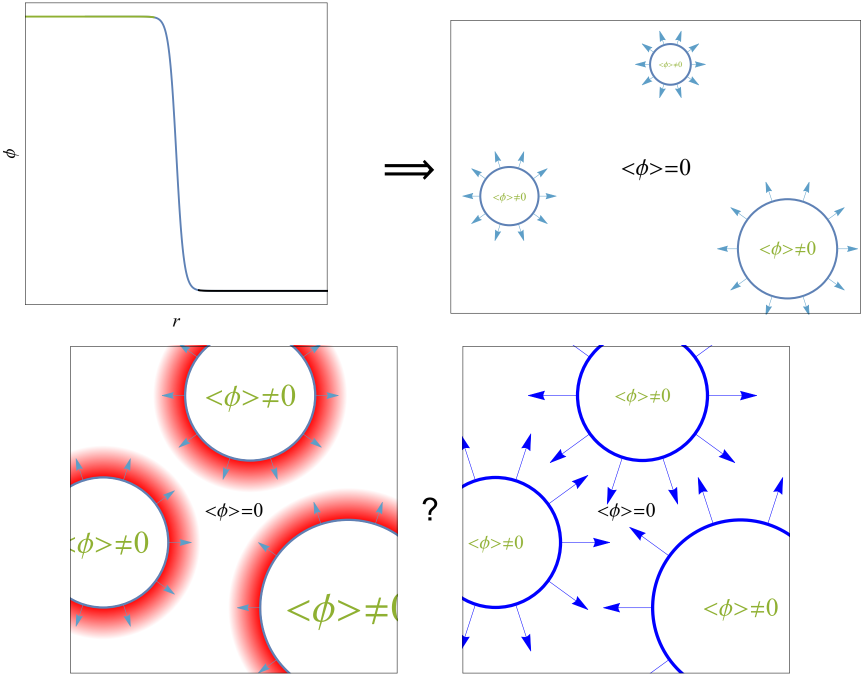
<ϕ>≠0
<ϕ>≠0



ϕ
r
$\Longrightarrow$
$<\phi>\neq 0$
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?
$<\phi>\neq 0$
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- Energy of the bubble

$$\mathcal{E} = 4\pi R^2 \sigma \gamma - \frac{4\pi}{3} R^3 p \,, \qquad \gamma = \frac{1}{\sqrt{1 - \dot{R}^2}}$$

- Vacuum pressure on the wall
Coleman '73

$$p_0 = \Delta V$$

- Energy of the bubble

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- Vacuum pressure on the wall
  Coleman '73

$$p_0 = \Delta V$$

- Leading order plasma contribution
  Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\mathrm{LO}} \approx \Delta V - \frac{\Delta m^2 T^2}{24} \,,$$

- Energy of the bubble

$$\mathcal{E} = 4\pi R^2 \sigma \gamma - \frac{4\pi}{3} R^3 p \,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

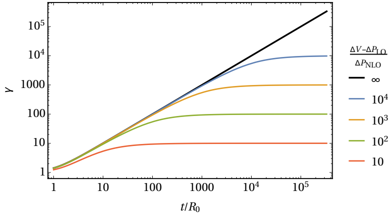
$$p_0 = \Delta V$$

- Leading order plasma contribution
  Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\rm LO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} \,,$$

- Next-To-Leading order plasma contribution
  Bodeker '17

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma\, g^2 \Delta m_V T^3 \,.$$



- γ factor at which the bubble stops accelerating and the value it would reach if we neglected $\Delta P_{\rm NLO}$

$$\gamma_{\rm eq} \equiv \frac{\Delta V - \Delta P_{\rm LO}}{\Delta P_{\rm NLO}}\,, \qquad \gamma_* \equiv \frac{2}{3}\frac{R_*}{R_0}\,,$$

- Finally the efficiency factors read

$$\kappa_{\rm col} = \frac{E_{\rm wall}}{E_V} = \begin{cases} \frac{\gamma_{\rm eq}}{\gamma_*}\left[1 - \frac{\Delta P_{\rm LO}}{\Delta V}\left(\frac{\gamma_{\rm eq}}{\gamma_*}\right)^2\right], & \gamma_* > \gamma_{\rm eq} \\ 1 - \frac{\Delta P_{\rm LO}}{\Delta V}\,, & \gamma_* \le \gamma_{\rm eq}\,, \end{cases}$$

$$\kappa_{\rm sw} = \frac{\alpha_{\rm eff}}{\alpha}\frac{\alpha_{\rm eff}}{0.73 + 0.083\sqrt{\alpha_{\rm eff}} + \alpha_{\rm eff}}\,, \quad \text{with} \quad \alpha_{\rm eff} = \alpha(1 - \kappa_{\rm col})\,.$$

Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left.\frac{\Delta V}{\rho_R}\right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

$$HR_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H}\right)^{-1}$$

- Signals are produced by three main mechanisms:
    - collisions of bubble walls: $\Omega_{\rm col} \propto \left(\kappa_{\rm col}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)^2$

      Kamionkowski ‘93, Huber ‘08, Hindmarsh ‘18,
    - sound waves: $\Omega_{\rm sw} \propto \left(\kappa_{\rm sw}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)(H\tau_{sw})$

      Hindmarsh ‘13 ‘15 ‘17, Ellis ‘18
    - turbulence $\Omega_{\rm turb} \propto \left(\kappa_{\rm sw}\frac{\alpha}{\alpha+1}\right)^{\frac{3}{2}} (HR_*)(1 - H\tau_{sw})$

      Caprini ‘09, Ellis ‘19
- Sound wave period lasts $H\tau_{\rm sw} \equiv \min\left[1, \frac{HR_*}{U_f}\right]$

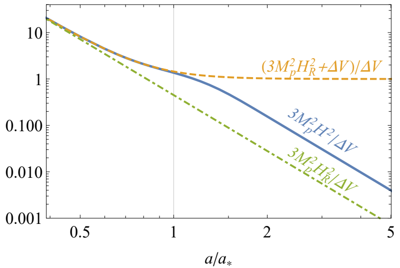
Particle physics examples

- Two models with very different potential cosmological evolution

$$H = \frac{1}{3M_{\rm pl}^2}\left(\rho_R + \Delta V\right) = H_R + \frac{\Delta V}{3M_{\rm pl}^2}$$

Standard Model $+|H|^6$ operator

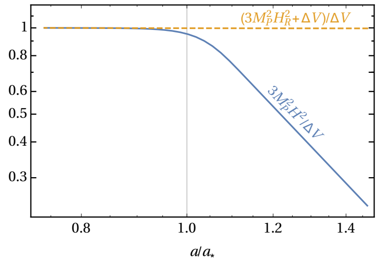
$$V(\phi) \simeq m^2\phi^2 + \lambda\phi^4 + \frac{\phi^6}{\Lambda^2}$$



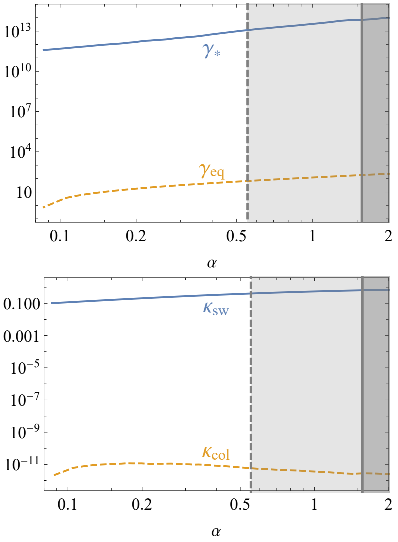
$U(1)_{B-L}$ extension of SM

$$V(\varphi) \simeq \frac{3g_{\rm B-L}^4\varphi^4}{4\pi^2}\left[\log\left(\frac{\varphi^2}{v_\varphi^2}\right) - \frac{1}{2}\right] + g_{\rm B-L}^2 T^2\varphi^2$$

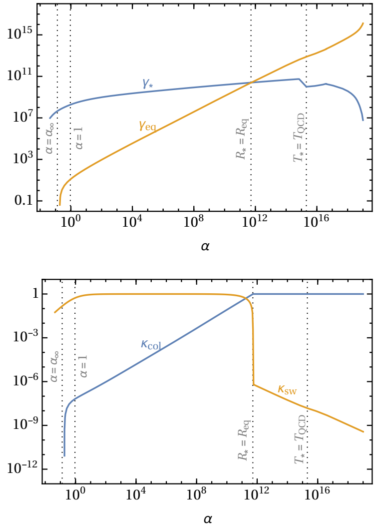
$$m_{\rm Z'} = 2g_{\rm B-L}v_\varphi$$

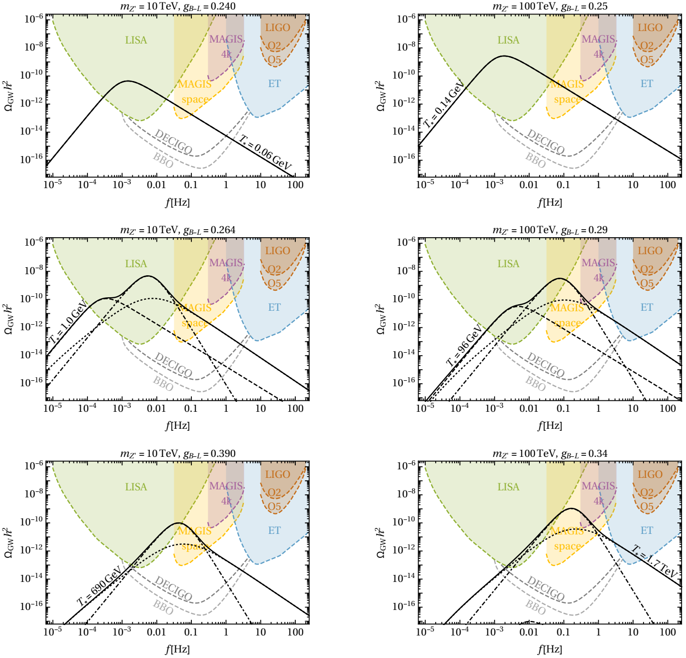


Standard Model $+|H|^6$ operator

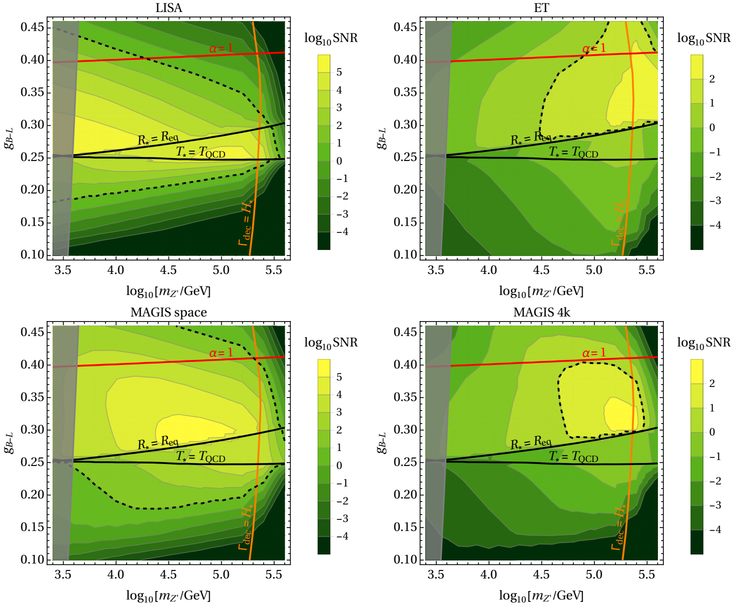


$U(1)_{B-L}$ extension of SM





$m_{Z'} = 10\,\text{TeV},\ g_{B-L} = 0.240$
$m_{Z'} = 100\,\text{TeV},\ g_{B-L} = 0.25$
$m_{Z'} = 10\,\text{TeV},\ g_{B-L} = 0.264$
$m_{Z'} = 100\,\text{TeV},\ g_{B-L} = 0.29$
$m_{Z'} = 10\,\text{TeV},\ g_{B-L} = 0.390$
$m_{Z'} = 100\,\text{TeV},\ g_{B-L} = 0.34$
LISA
MAGIS 4k
MAGIS space
LIGO
O2
O5
ET
DECIGO
BBO
$T_* = 0.06\,\text{GeV}$
$T_* = 0.14\,\text{GeV}$
$T_* = 1.0\,\text{GeV}$
$T_* = 96\,\text{GeV}$
$T_* = 690\,\text{GeV}$
$T_* = 1.7\,\text{TeV}$
$\Omega_{\rm GW} h^2$
$f\,[\text{Hz}]$



LISA
ET
MAGIS space
MAGIS 4k
$\log_{10}$SNR
g_{B-L}
$\log_{10}[m_{Z'}/\mathrm{GeV}]$
$\alpha = 1$
$R_* = R_{eq}$
$T_* = T_{QCD}$
$\Gamma_{dec} = H_*$

Plasma related GW sources

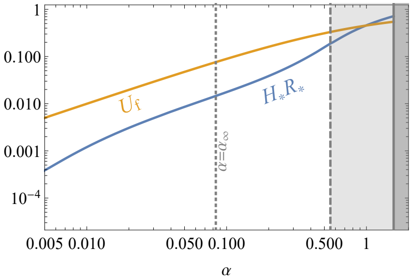
- Sound wave period last a fraction of the Hubble time

$$H\tau_{\rm sw} \equiv \min\left[1, \frac{HR_*}{U_f}\right]$$

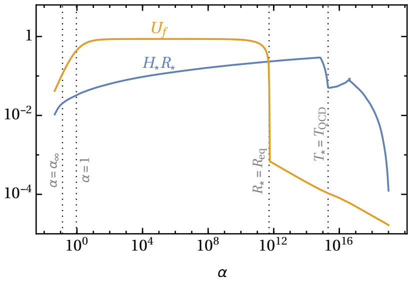
- Root-mean-square four-velocity of the plasma

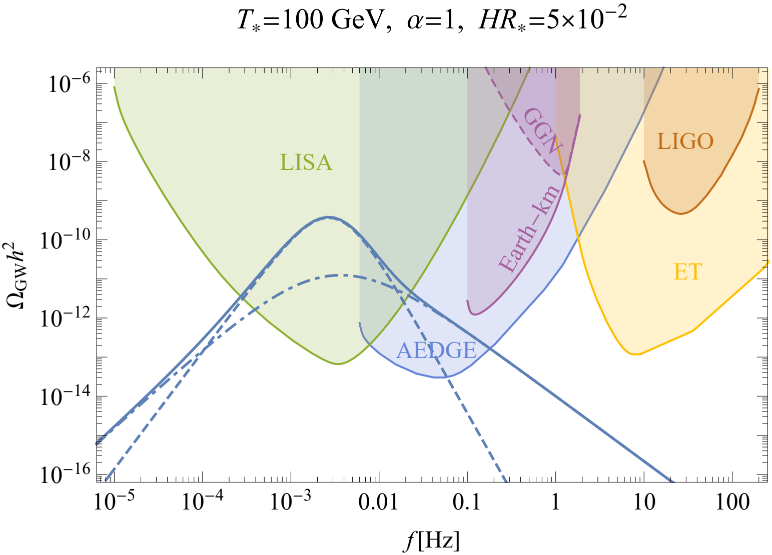
$$U_f \approx \sqrt{\frac{3}{4}\frac{\kappa_{\rm sw}\alpha}{1+\alpha}} \xRightarrow{v_w \approx 1} \frac{\sqrt{3}\alpha}{2(1+\alpha)\sqrt{0.73+0.083\sqrt{\alpha}+\alpha}}\,.$$

Standard Model $+|H|^6$ operator



$U(1)_{B-L}$ extension of SM

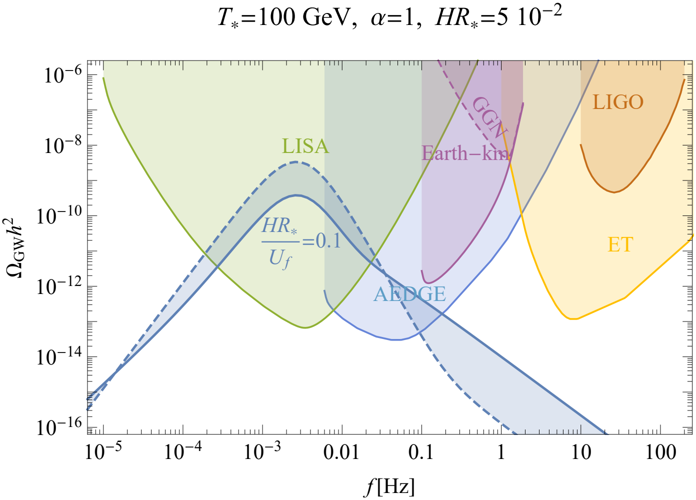




T_*=100 GeV, α=1, HR_*=5×10^{-2}
$\Omega_{\rm GW}h^2$
f[Hz]
LISA
AEDGE
GGN
Earth–km
LIGO
ET
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-5}
10^{-4}
10^{-3}
0.01
0.1
1
10
100

- Sound wave spectrum reduction and earlier onset of turbulence

$$\Omega_{\rm sw} \propto H\tau_{\rm sw} = \frac{HR_*}{U_f}\,, \quad \Omega_{\rm turb} \propto 1 - H\tau_{\rm sw} = 1 - \frac{HR_*}{U_f}$$



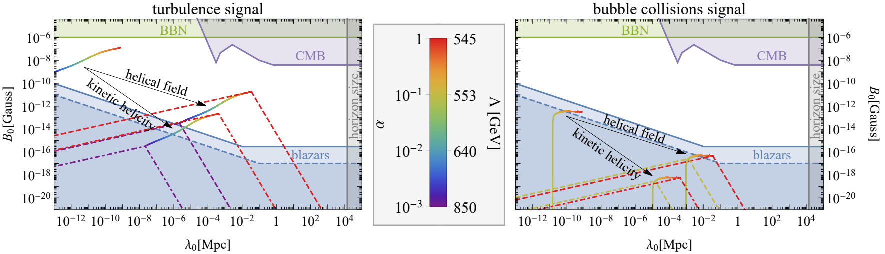
Primordial Magnetic Fields

- Energy density and correlation length of produced Magnetic Field

  Durrer ‘13 Brandenburg ‘17 Vachaspati ‘19

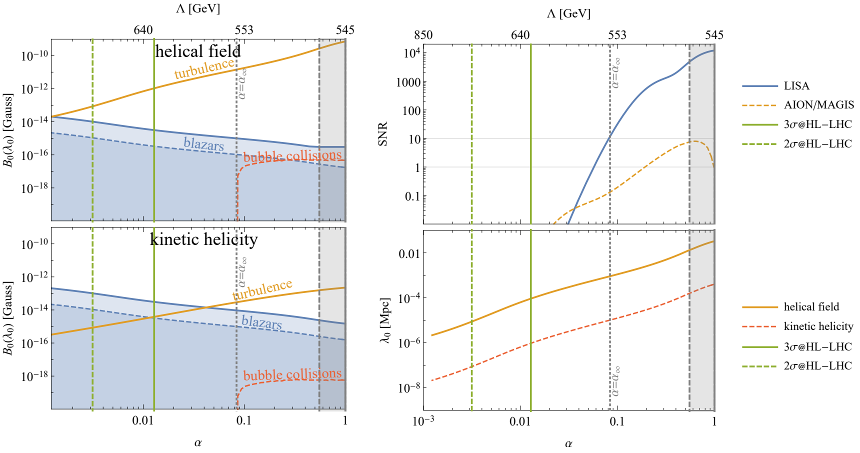
$$\rho_{B,*} = 0.1\kappa_{\mathrm{col/sw}}\frac{\alpha}{1+\alpha}\rho_* \qquad \lambda_* = R_*$$

- In the $SM + H^6$ model



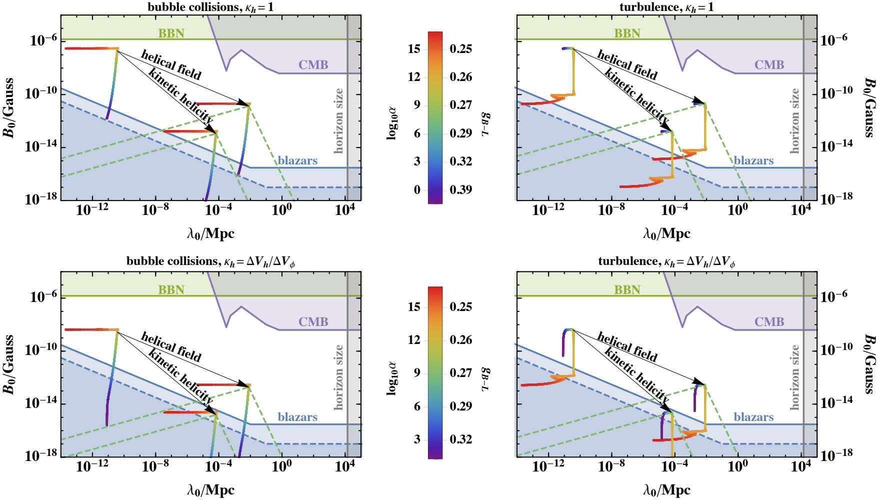
Primordial Magnetic Fields

- In the $SM + H^6$ model



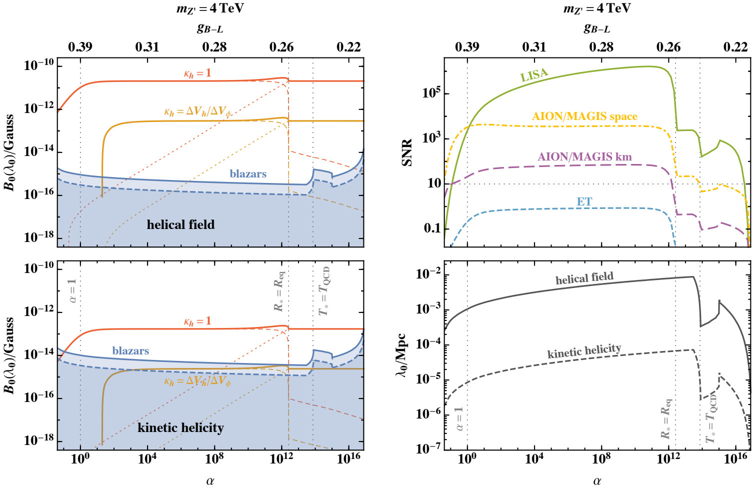
Primordial Magnetic Fields

- In the $U(1)_{\mathrm{B-L}}$ model with $m_{Z'} = 4\mathrm{TeV}$



Primordial Magnetic Fields

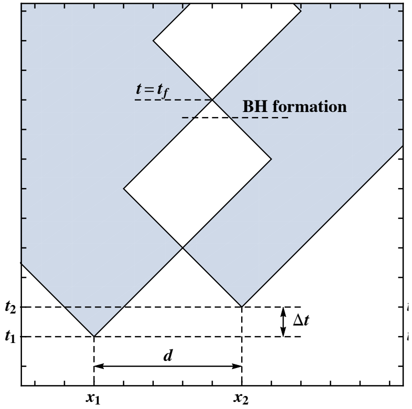
- In the $U(1)_{\mathrm{B-L}}$ model with $m_{Z'} = 4\mathrm{TeV}$



Primordial Black Holes

- In a very strong transition, false vacuum region produced in the collision of two bubbles can collapse into a black hole

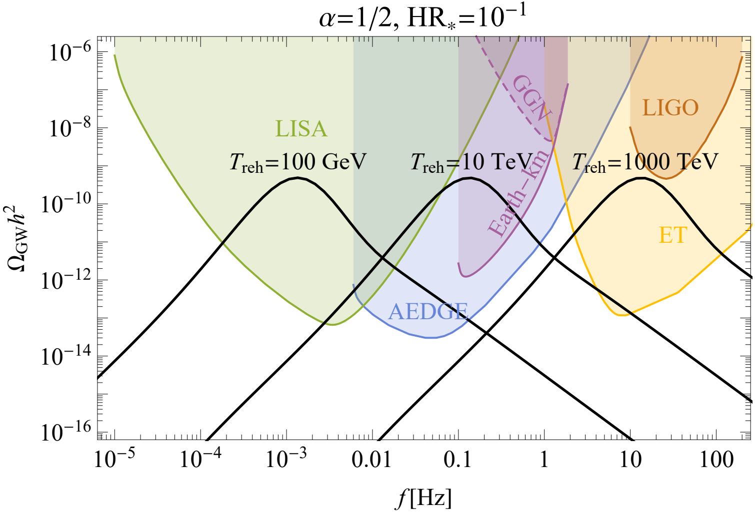
Hawking ‘82, Moss ‘94, Khlopov ‘99



Conclusions

- Observable bubble collision GW signal requires significant supercooling. We derived the efficiency factor quantifying this requirement precisely.
- Sound wave period generically last less than a Hubble time. This leads to a much weaker sound wave sourced GW signal and potentially a significant increase in the signal sourced by turbulence.
- PTs an also have leave other complimentary signals including production of primordial magnetic fields and primordial black holes.

- The frequency of the signal changes as $f \propto T_{\mathrm{reh}}$



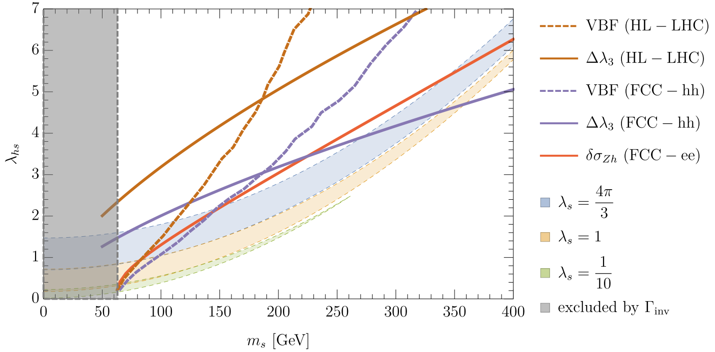
EWBG Model 2: Neutral singlet

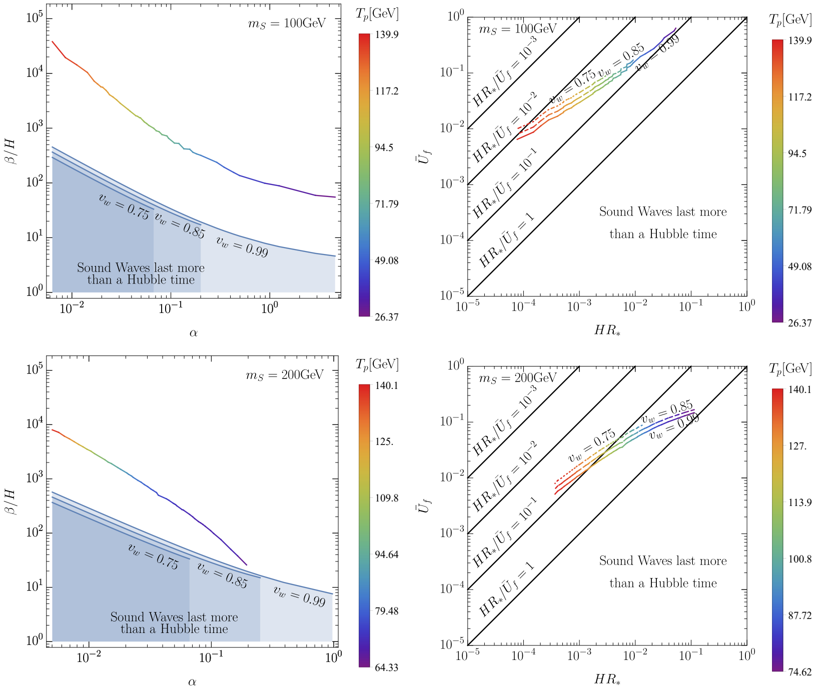
- We add an additional singlet scalar to SM

$$V^{\text{tree}}(H,s) = -\mu_h^2 |H|^2 + \lambda_h |H|^4 + \frac{\lambda_{hs}}{2} |H|^2 s^2 + \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4$$

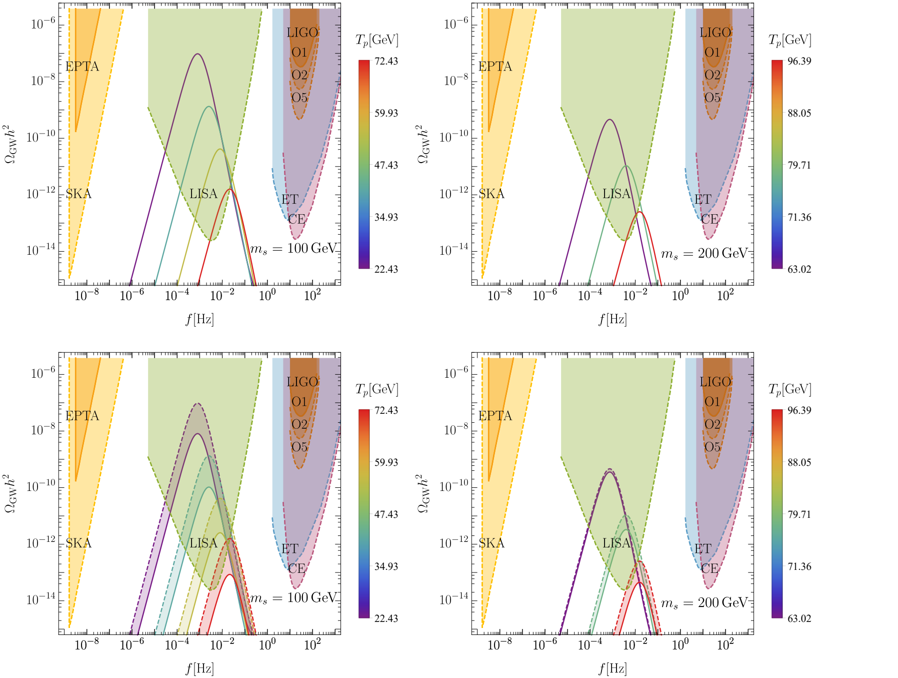
- Singlets physical mass

$$m_s{}^2 = \mu_s^2 + \lambda_{hs} v^2/2$$



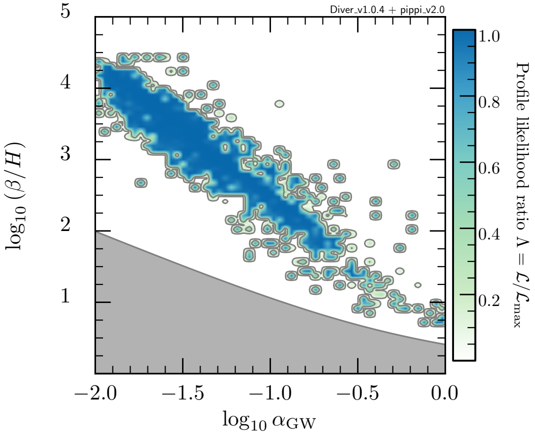


$m_S = 100\text{GeV}$
β/H
α
$v_w = 0.75$
$v_w = 0.85$
$v_w = 0.99$
Sound Waves last more than a Hubble time
T_p[GeV]
139.9
117.2
94.5
71.79
49.08
26.37
$\bar{U}_f$
HR_*
$HR_*/\bar{U}_f = 10^{-3}$
$HR_*/\bar{U}_f = 10^{-2}$
$HR_*/\bar{U}_f = 10^{-1}$
$HR_*/\bar{U}_f = 1$
$m_S = 200\text{GeV}$
140.1
125.
109.8
94.64
79.48
64.33
127.
113.9
100.8
87.72
74.62


EPTA
SKA
LISA
LIGO
O1
O2
O5
ET
CE
$m_s = 100\,\mathrm{GeV}$
$m_s = 200\,\mathrm{GeV}$
$T_p[\mathrm{GeV}]$
72.43
59.93
47.43
34.93
22.43
96.39
88.05
79.71
71.36
63.02
$\Omega_{\mathrm{GW}} h^2$
$f[\mathrm{Hz}]$

EWBG Model 2.5: Neutral singlet without $\mathbf{Z}_2$

$$V^{\text{tree}}(H,s) = -\mu_h^2 |H|^2 + \lambda_h |H|^4 + \frac{\lambda_{hs}}{2} |H|^2 s^2 + \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \frac{\mu_{hs}}{2} |H|^2 s + \frac{\mu_3}{2} s^3$$



See: Ankit Beniwal, ML, Martin White and Anthony G. Williams arXiv:1810.02380