A Real-Time Semiclassical Picture of Vacuum Decay

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w/ Matt Johnson, Hiranya Peiris, Andrew Pontzen, and Silke Weinfurtner
1712.02356, 1806.06069, 1904.07873 and in progress
2018 Buchalter Cosmology Prize

GWs from the Early Universe, Nordita, Aug. 27, 2019

How Quantum is QFT?

$$|\Psi\rangle$$

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$$

$$P[\phi, \Pi]$$

$$\frac{\partial \phi}{\partial t} = \frac{\delta H}{\delta \Pi}$$

$$\frac{\partial \Pi}{\partial t} = -\frac{\delta H}{\delta \phi}$$

Nonlinear, Nonperturbative, Nonequilibrium Phenomena

Sourced GWs

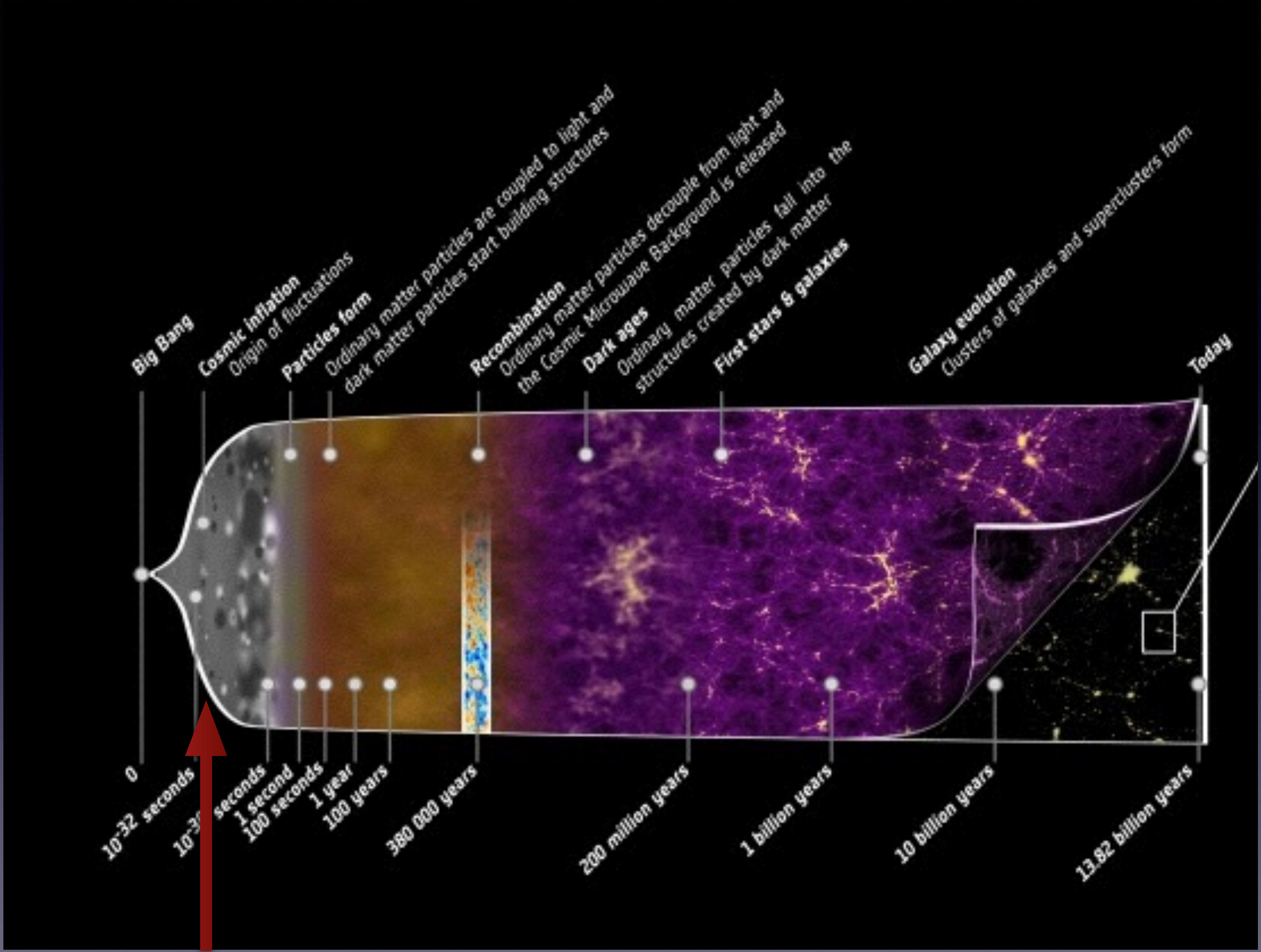
$$(ah_{ij}^{\mathrm{TT}})'' - \left(\nabla^2 + \frac{a''}{a}\right)(ah_{ij}^{\mathrm{TT}}) = \frac{2}{M_P^2} a^3 \Pi_{ij}^{\mathrm{TT}}$$

$$a^2 \Pi_{ij} = T_{ij} - \langle P \rangle g_{ij}$$

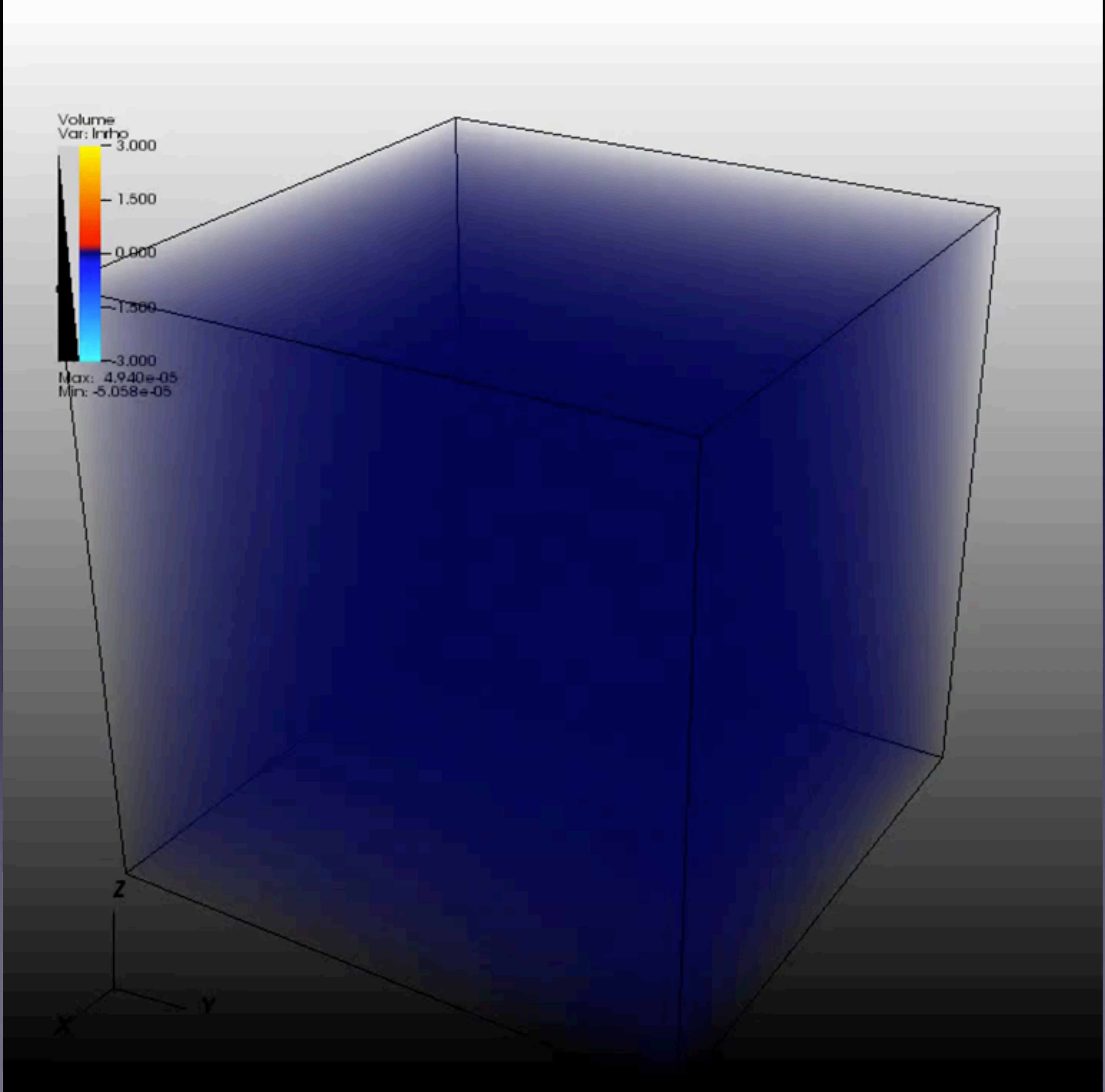
$$\rho_{\mathrm{GW}} = \frac{M_P^2}{4} \left\langle \dot{h}_{ij} \dot{h}^{ij} \right\rangle_{\mathrm{V,T}}$$

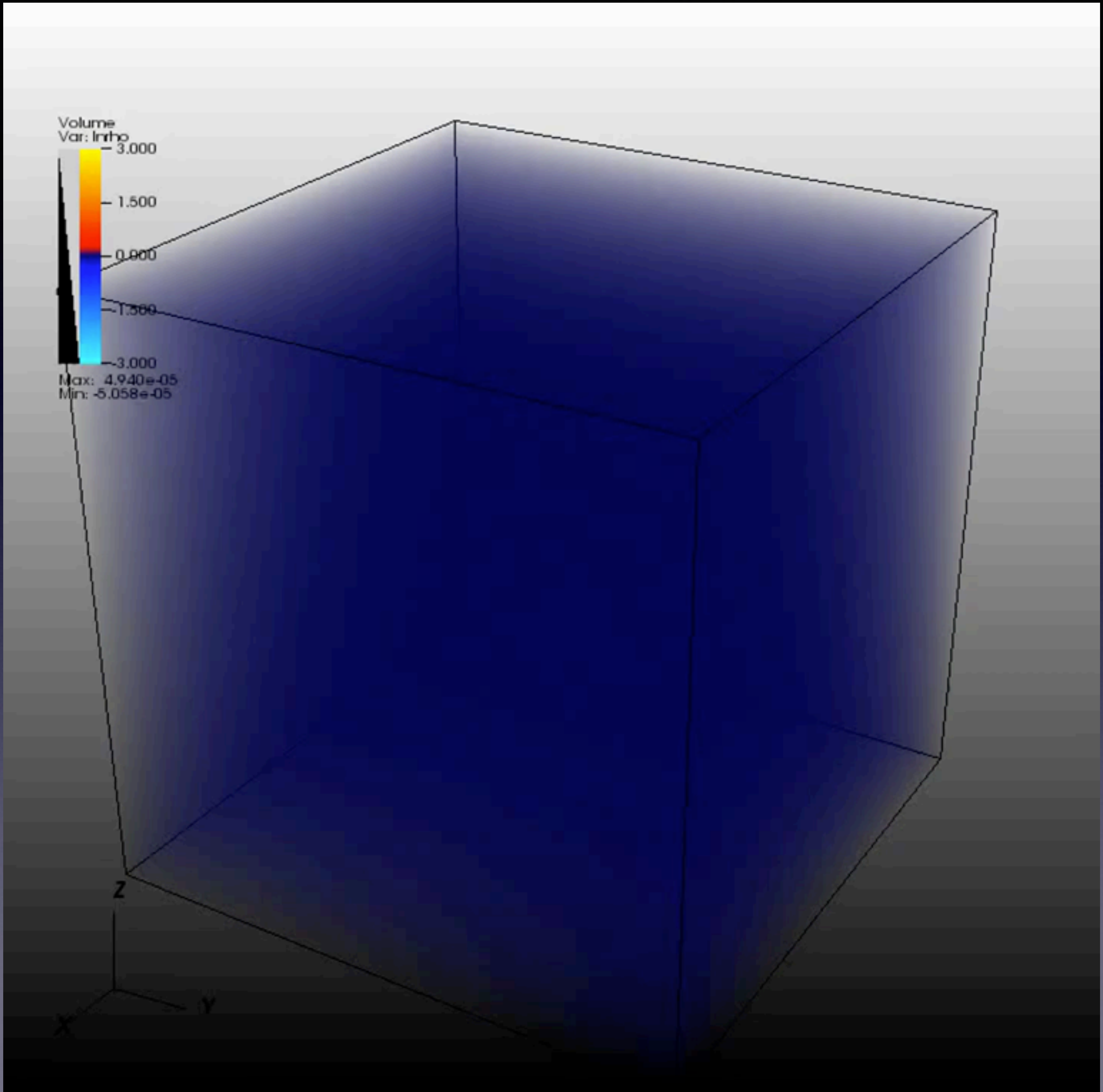
Scalar Fields

$$\Pi_{ij}^{\mathrm{TT}} = \mathcal{O}_{ij,lm}^{\mathrm{TT}} \partial_i \phi \partial_j \phi$$



End-of-Inflation


Volume
Var: lnrho
3.000
1.500
0.000
-1.500
-3.000
Max: 4.940e-05
Min: -5.058e-05
Z
Y
X



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-1.500
-3.000
Max: 4.940e-05
Min: -5.058e-05
Z
Y
X

[JB (to appear)]

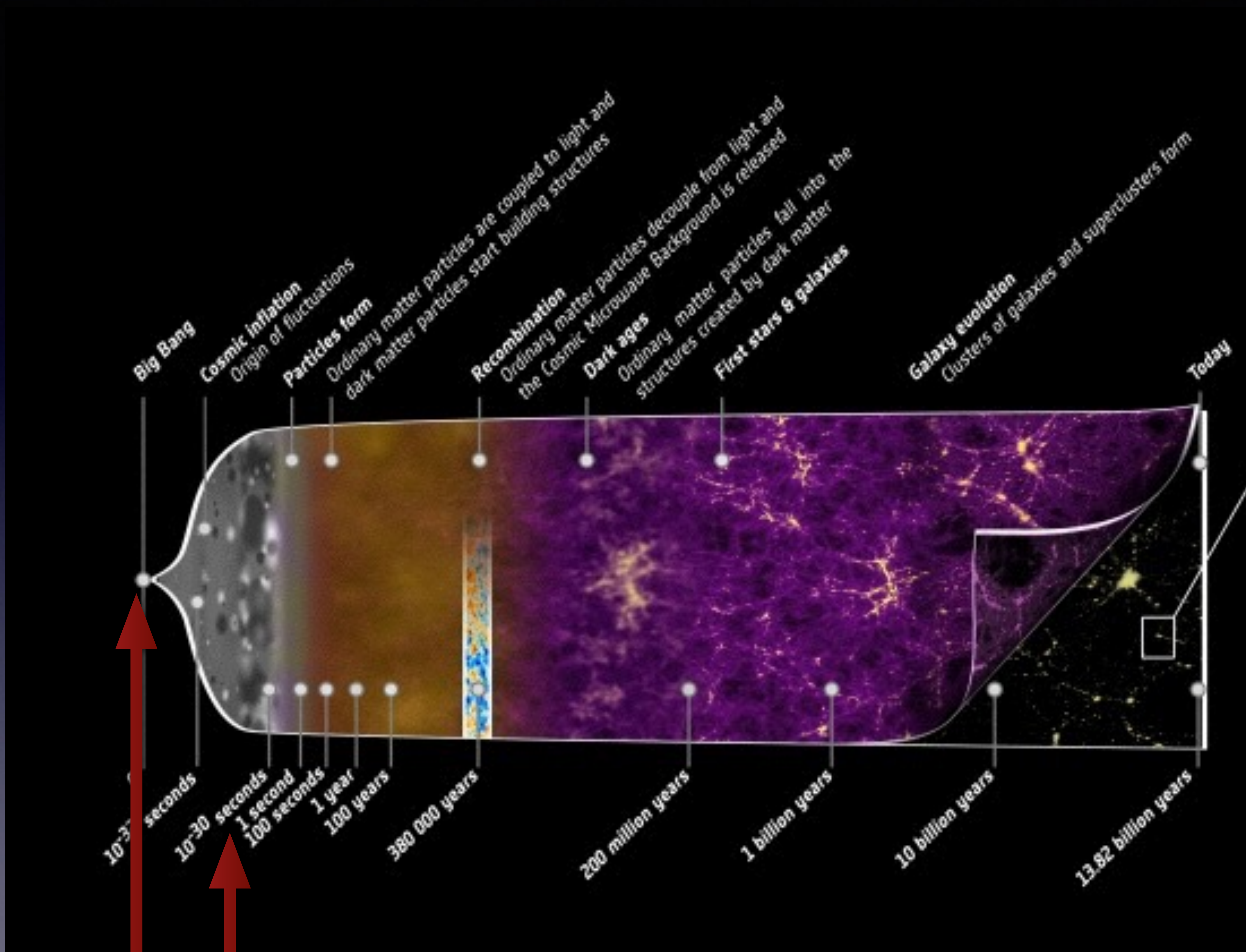
Long-Lived Single Frequency Scaling Source

$$\phi(\mathbf{x}, t) = a^{-\alpha} A_0(a^{\beta}\mathbf{x}) \cos(\omega t)$$

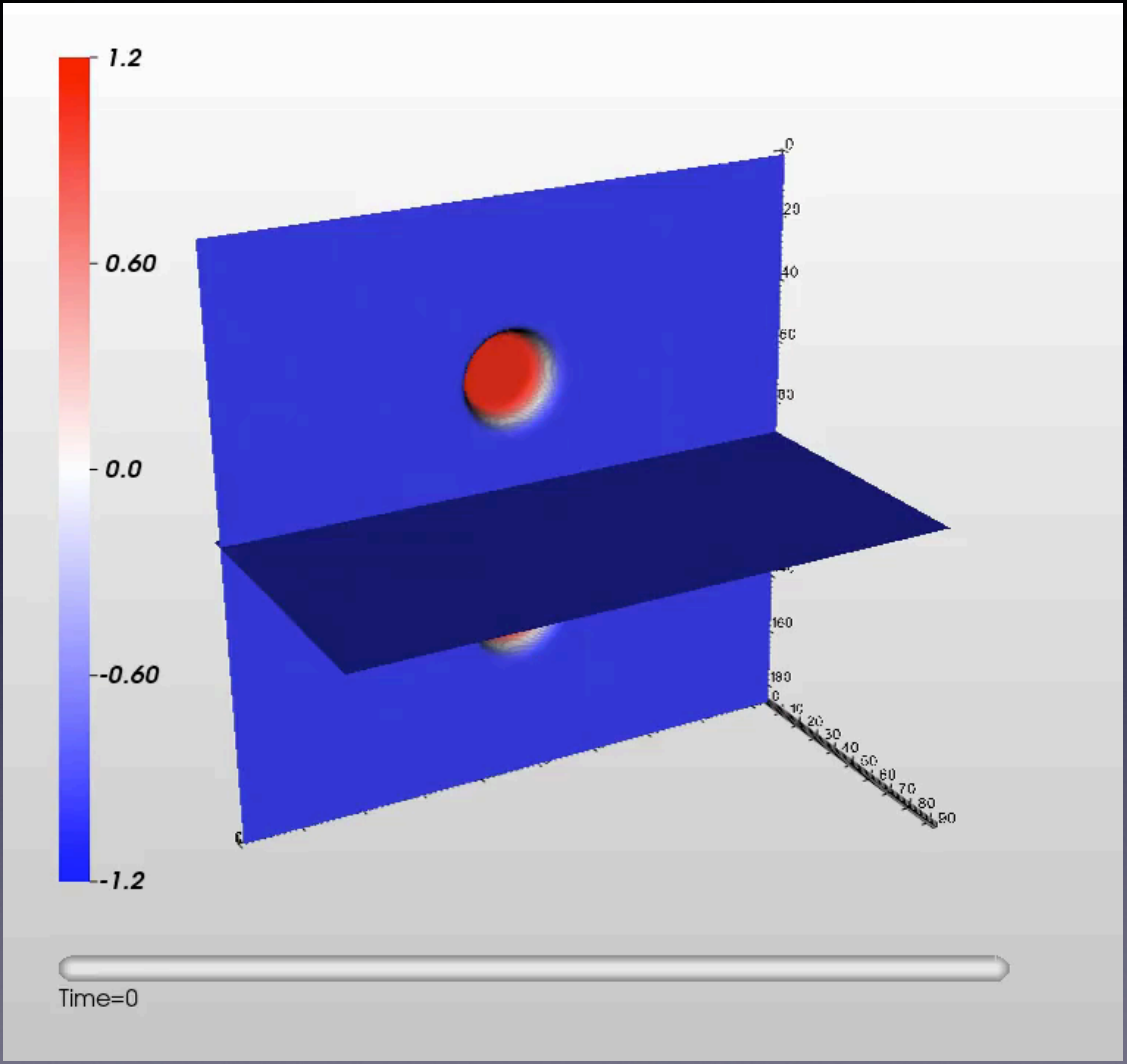
$$\Omega_{\mathrm{GW}} \approx A_{\mathrm{GW}} \left(\frac{k}{k_{\mathrm{pivot}}}\right)^{n_{\mathrm{GW}}} \Theta(k - 2\omega a_i)\Theta(2\omega a - k)$$

$$A_{\mathrm{GW}} \approx \frac{\pi}{12}\left(\frac{\phi_0}{M_P}\right)^4 \frac{1}{H_0^3 V}\sigma^2\omega^2 \left(\frac{k_{\mathrm{pivot}}}{2\omega a}\right)^{n_{\mathrm{GW}}} a^{n_{\mathrm{GW}}-1+3w}\tilde{F}_{\mathrm{N}}$$

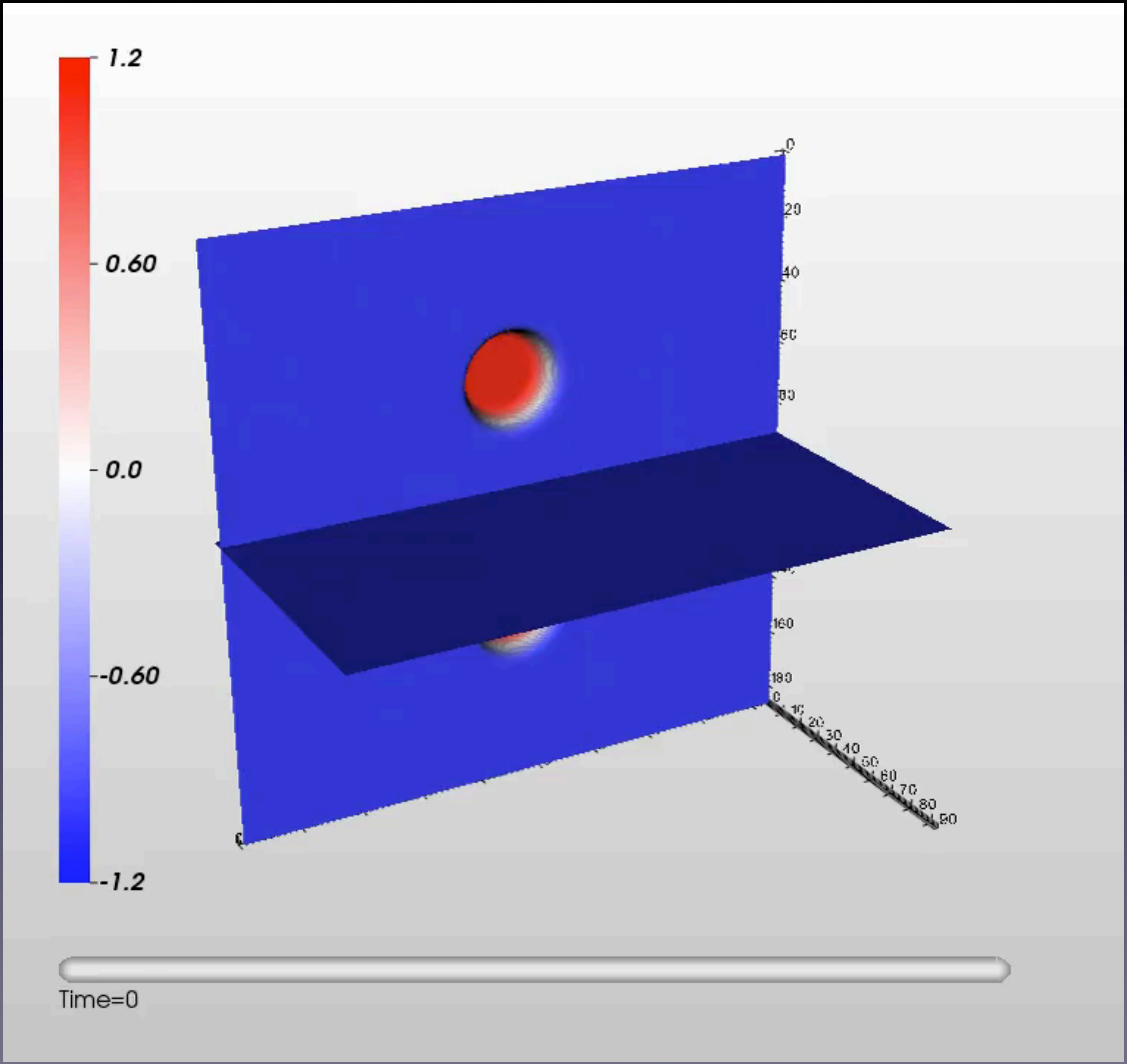
$$n_{\mathrm{GW}} = \frac{5}{2} + \frac{3}{2}w - 2(\beta - 1) - 4\alpha$$



Phase Transitions



1.2
0.60
0.0
-0.60
-1.2
Time=0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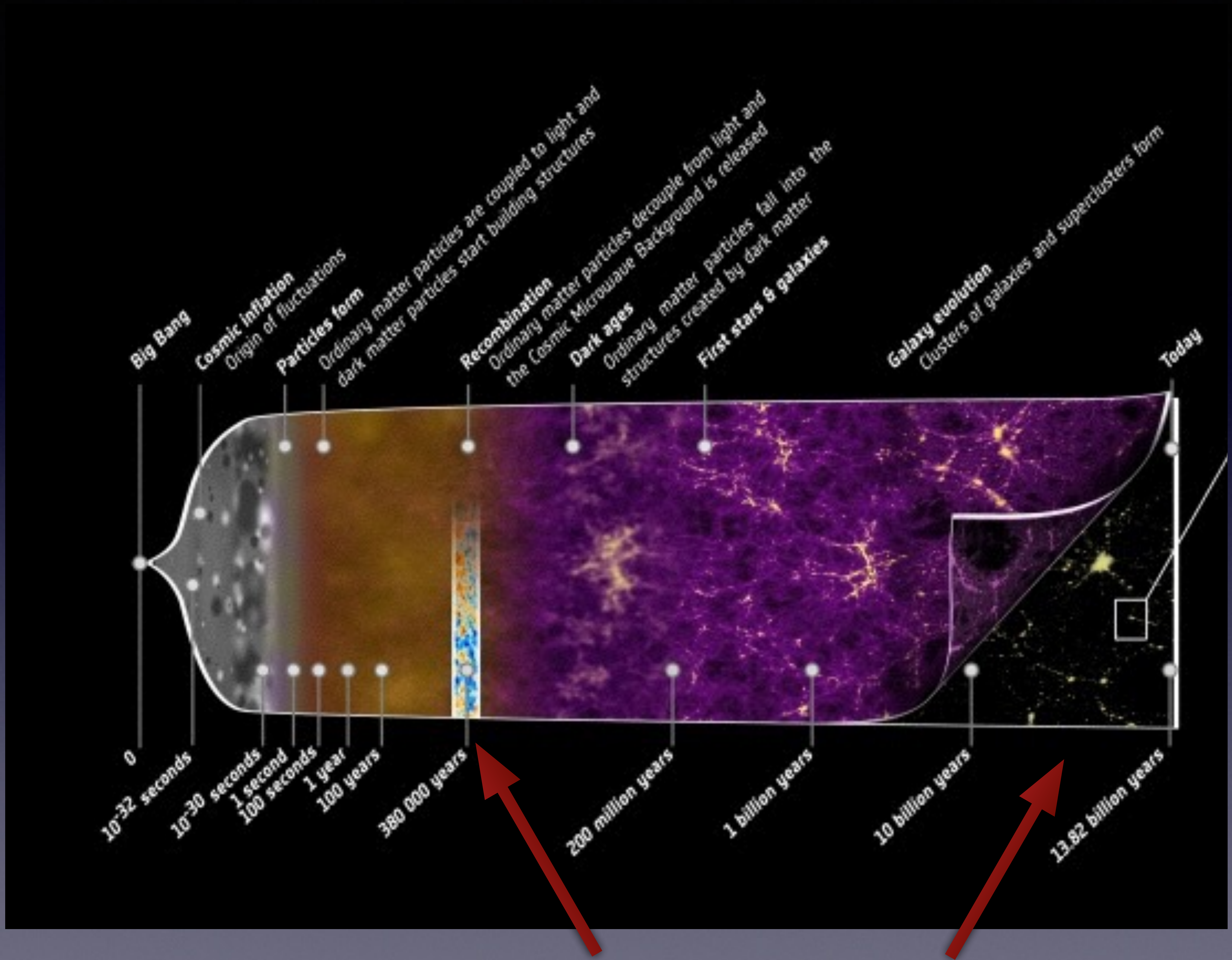
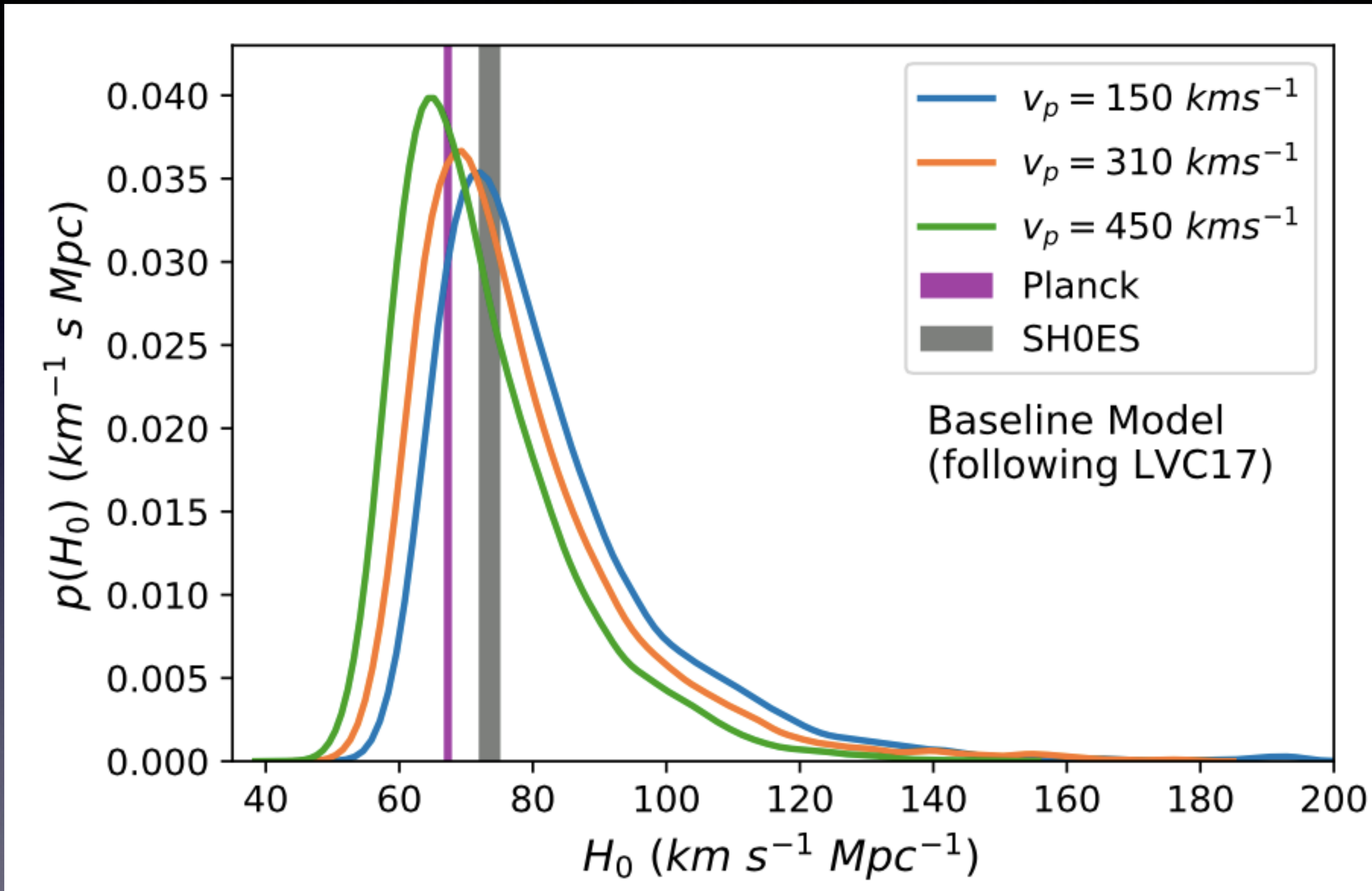


Expansion Rate of Universe



$v_p = 150\ kms^{-1}$
$v_p = 310\ kms^{-1}$
$v_p = 450\ kms^{-1}$
Planck
SH0ES
Baseline Model
(following LVC17)
$p(H_0)\ (km^{-1}\ s\ Mpc)$
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
40
60
80
100
120
140
160
180
200
$H_0\ (km\ s^{-1}\ Mpc^{-1})$

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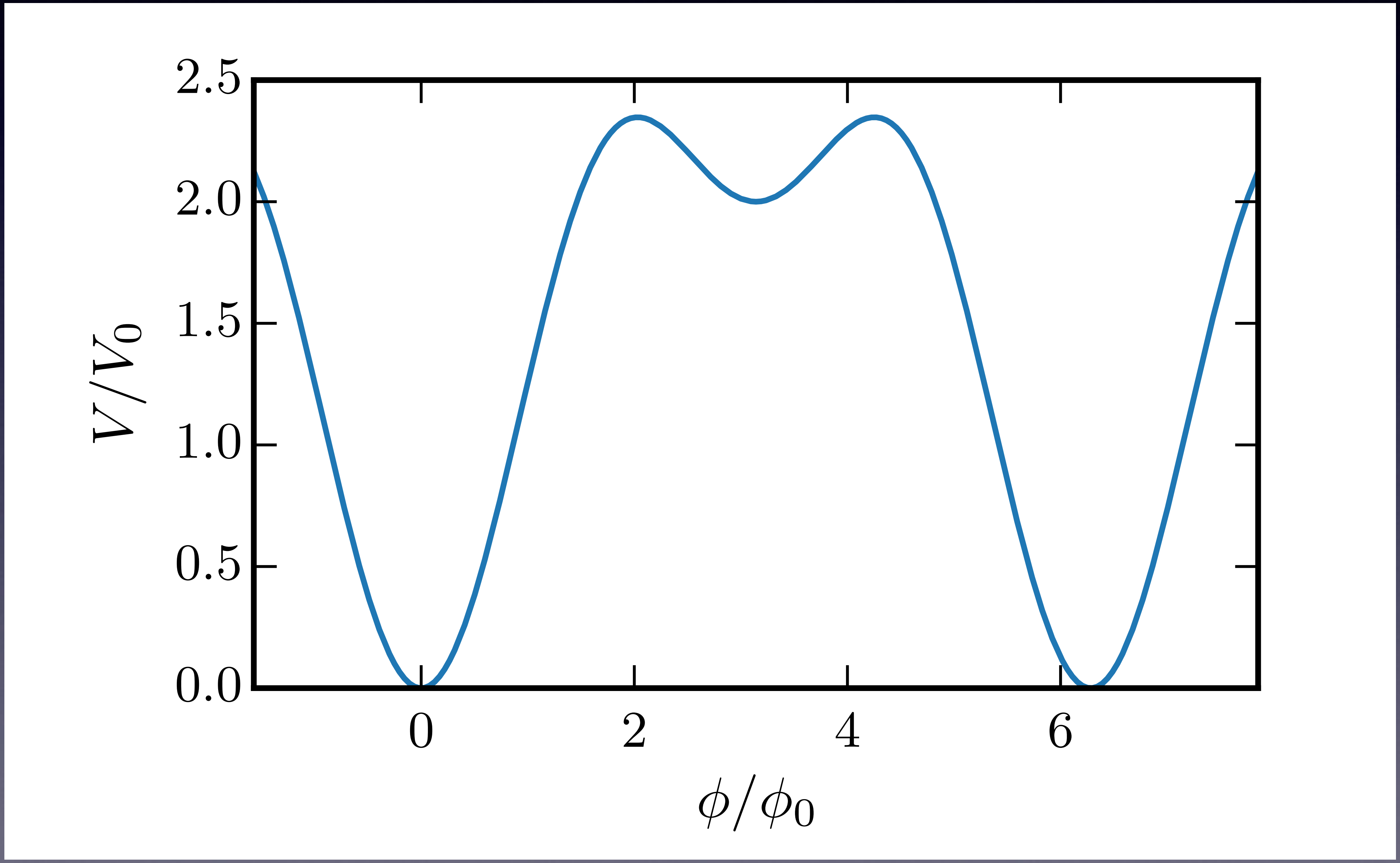
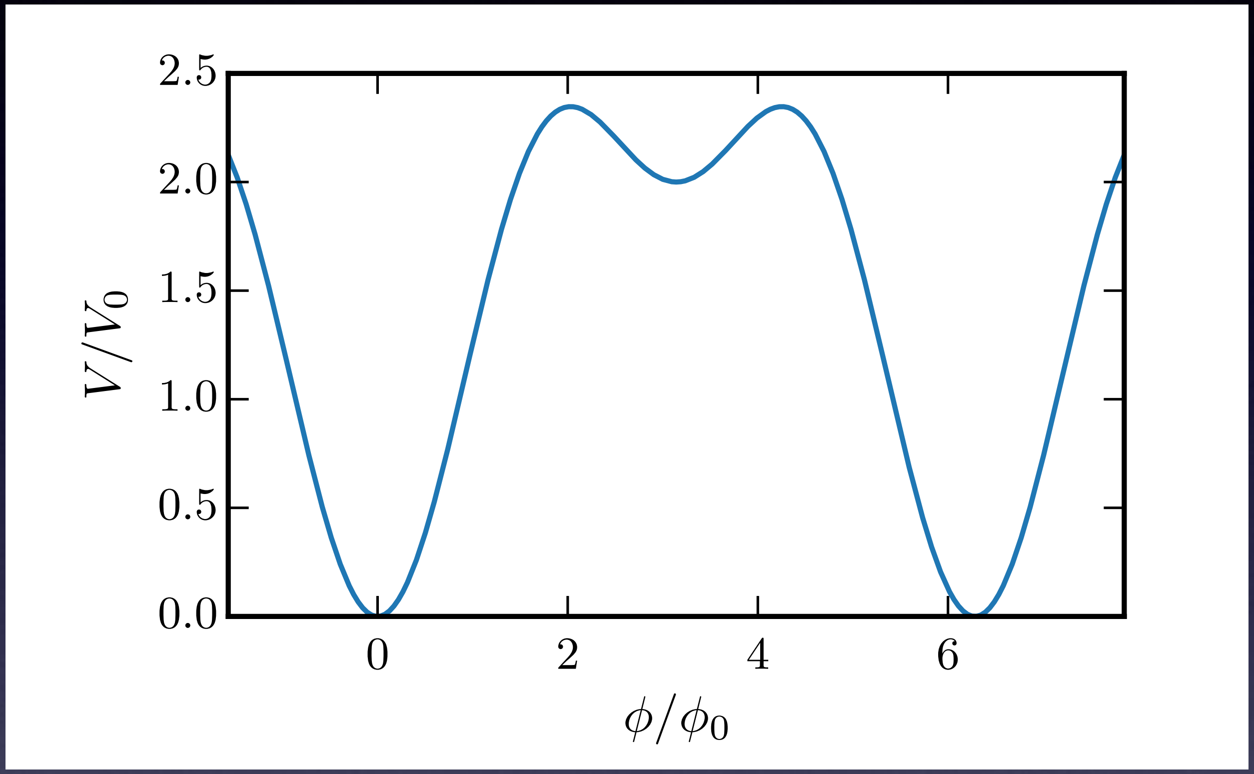
Outline

- Review of Vacuum Decay and 1st Order Phase Transitions
- Euclidean Description (including new computational method)
- Real-Time Description of Decay
- Novel Future Applications
- Connection to BECs (time permitting)

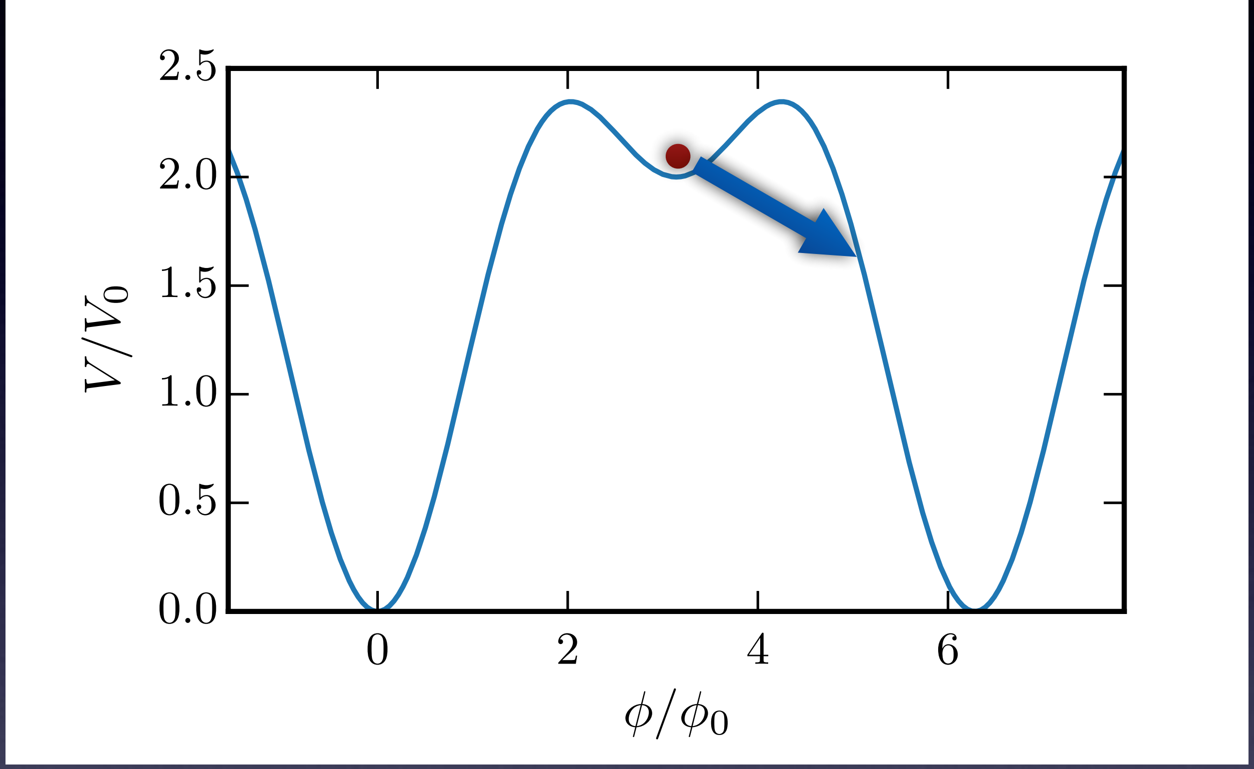
First Order Phase Transitions



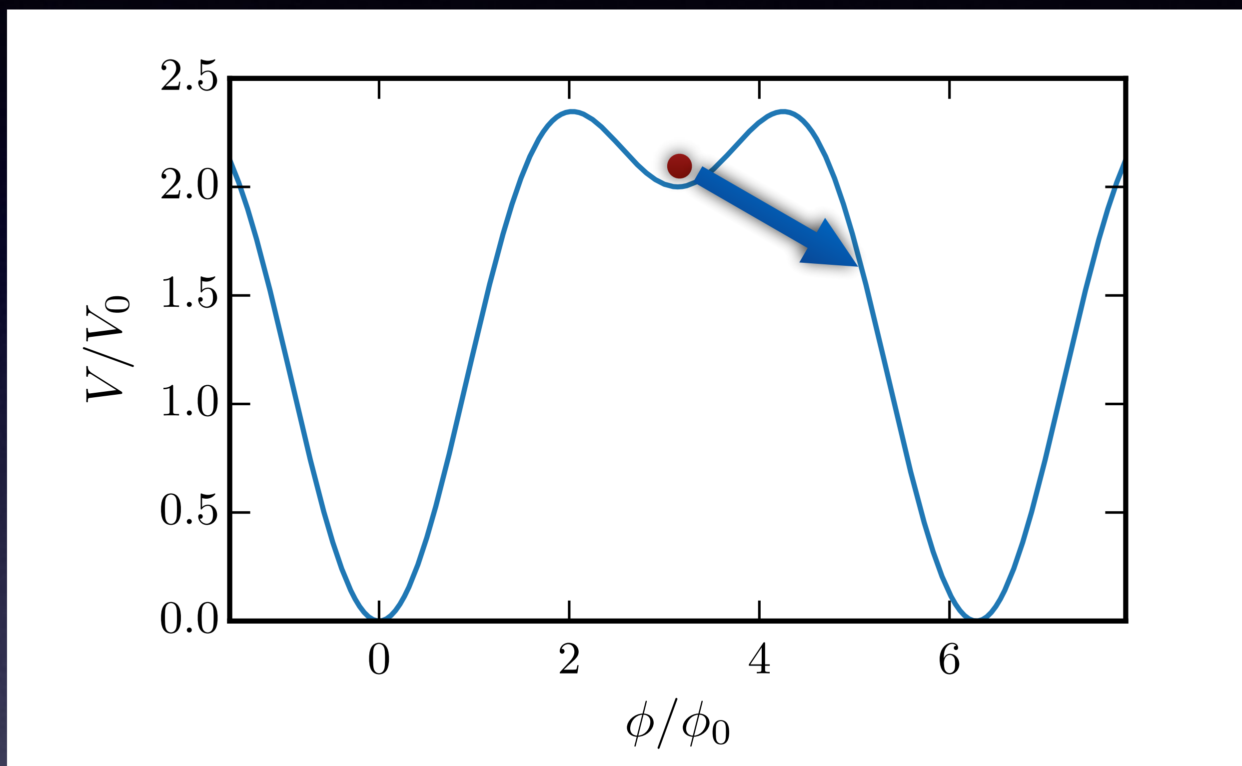
First Order Phase Transitions

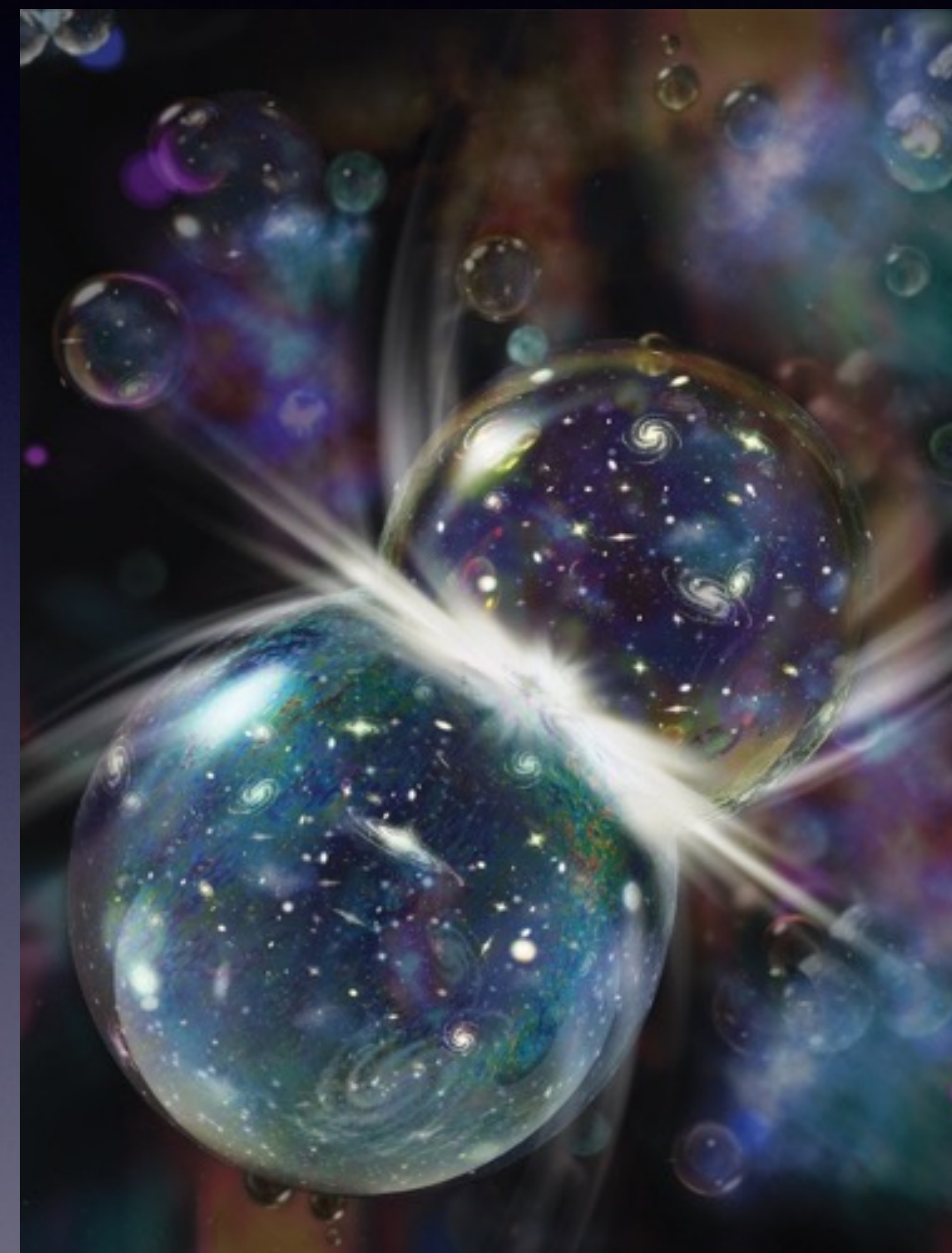


First Order Phase Transitions

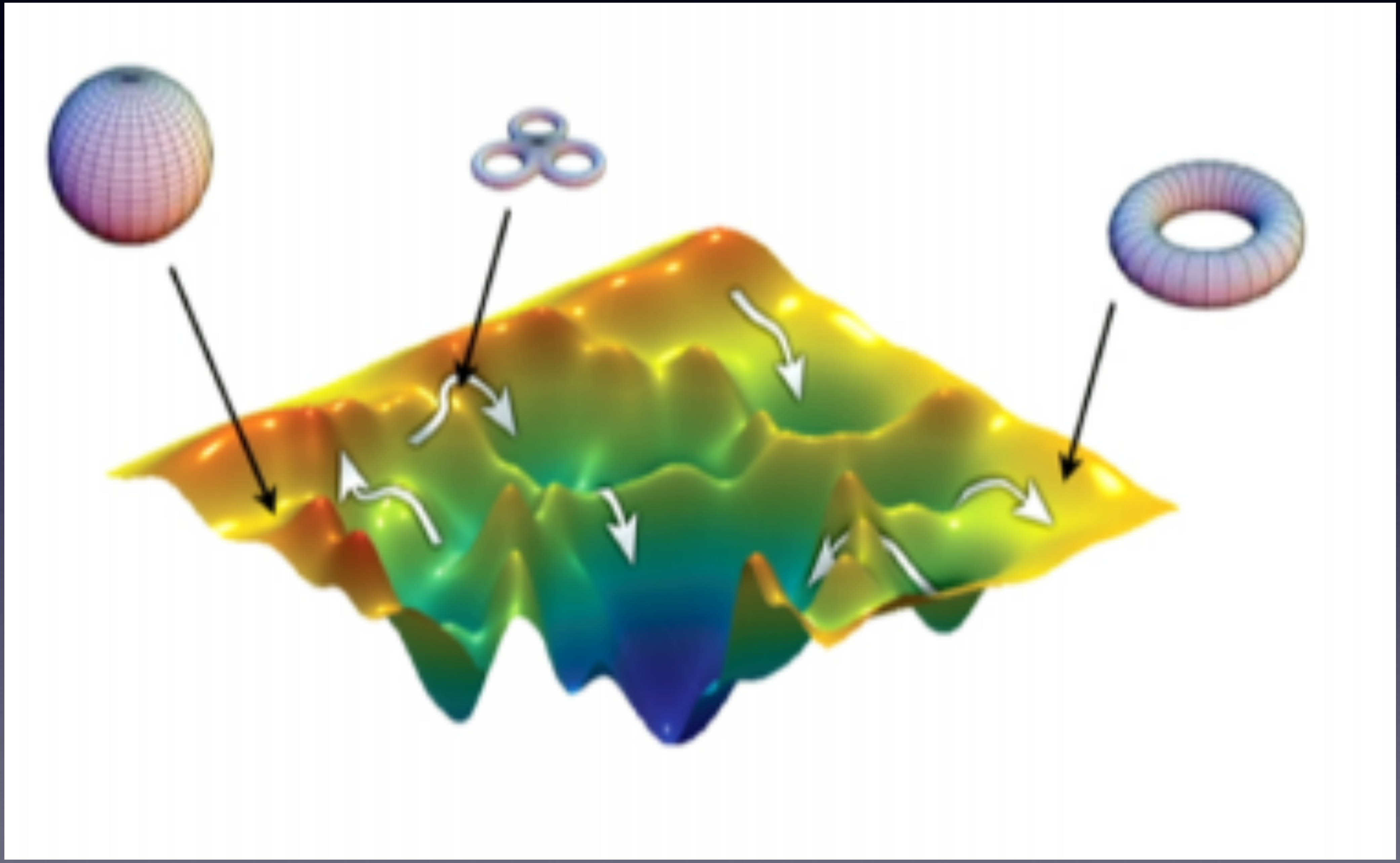


First Order Phase Transitions



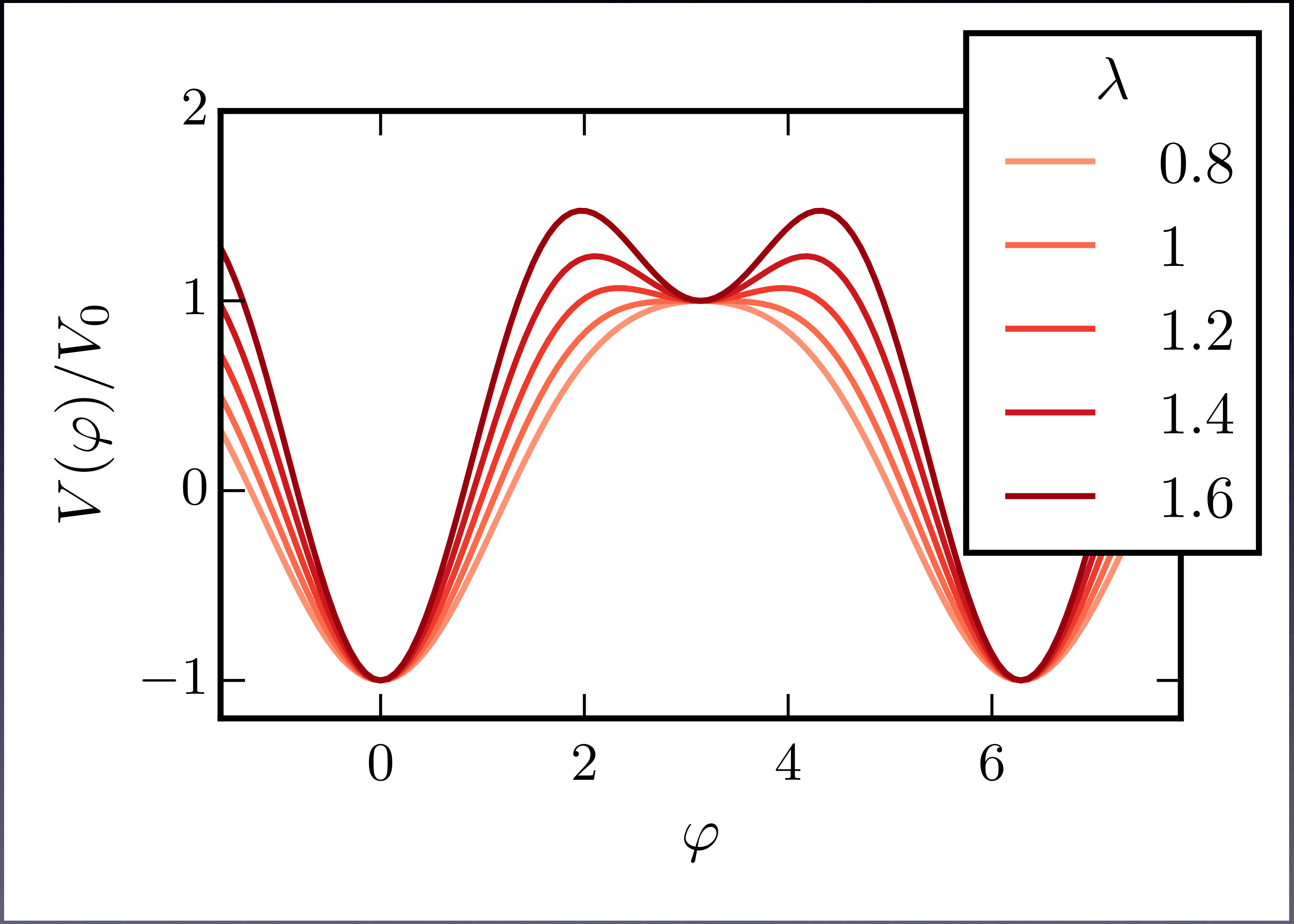


First Order Phase Transitions




INSTA

Model



$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2} \sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

0th Order Questions

- How fast does the vacuum decay?
- Do bubbles form?
- What do the bubbles look like?

Decay Rate

$$P_{\text{undecayed}} = |\langle \Omega_{\text{FV}}(t)|\Omega_{\text{FV}}(t=0)\rangle|^2 \sim e^{-\Gamma t}$$

Schematically

$$\langle \Omega_{\text{FV}}|\Omega_{\text{FV}}(t)\rangle = \langle \Omega_{\text{FV}}|e^{-iHt}|\Omega_{\text{FV}}\rangle$$

Work in Euclidean Time

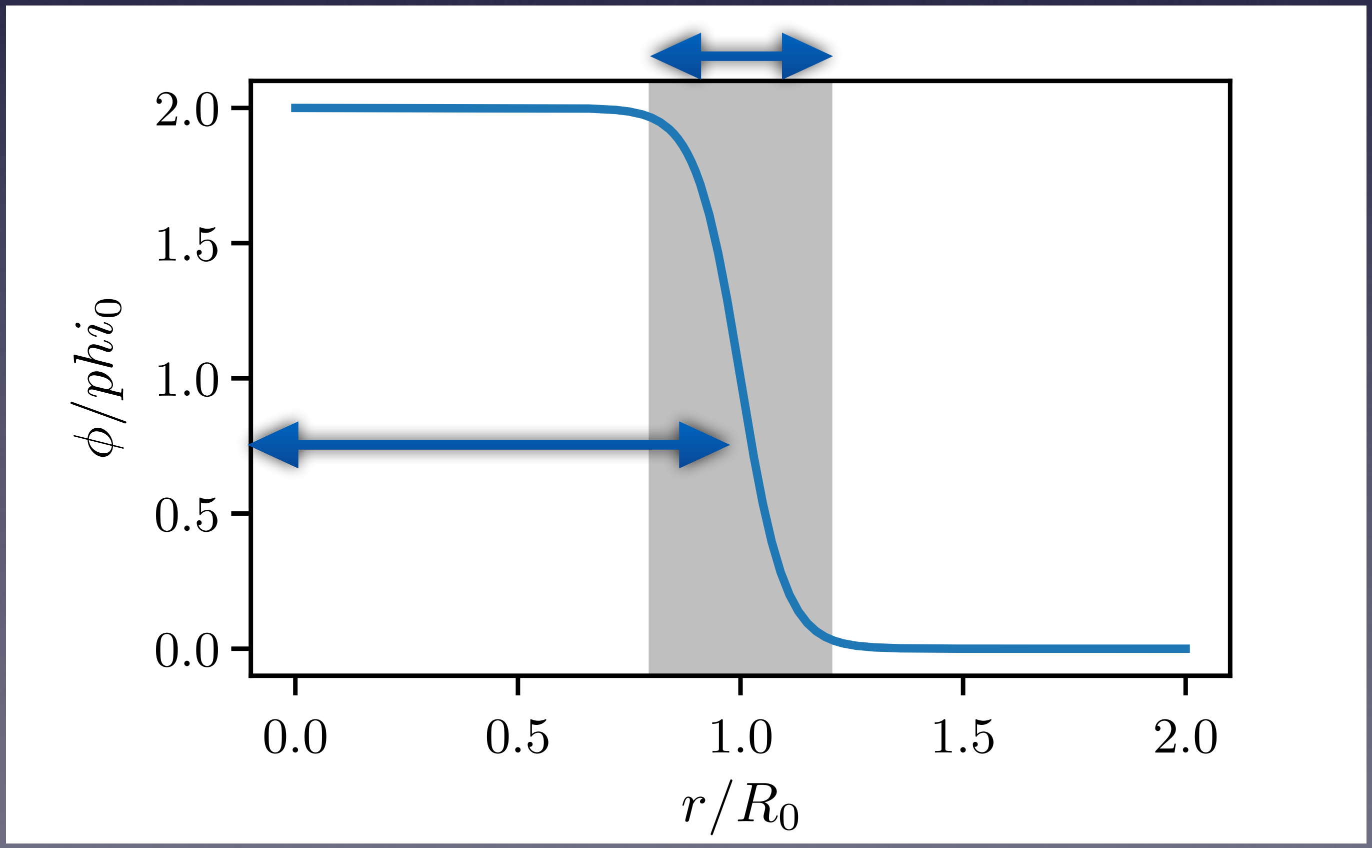
$$\langle \Omega_{\text{FV}}|e^{-HT}|\Omega_{\text{FV}}\rangle \sim e^{-E_0 T}$$

Imaginary Part of Energy Gives Decay in Real Time

Standard Description

$$r_{\mathrm{E}}^2 = \tau^2 + \mathbf{r}^2 \qquad \tau = it$$

$$\frac{\partial^2 \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}^2} + \frac{d}{r_{\mathrm{E}}}\frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}} - \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}}(0) = 0 \qquad \phi(\infty) = \phi_{\mathrm{fv}}$$

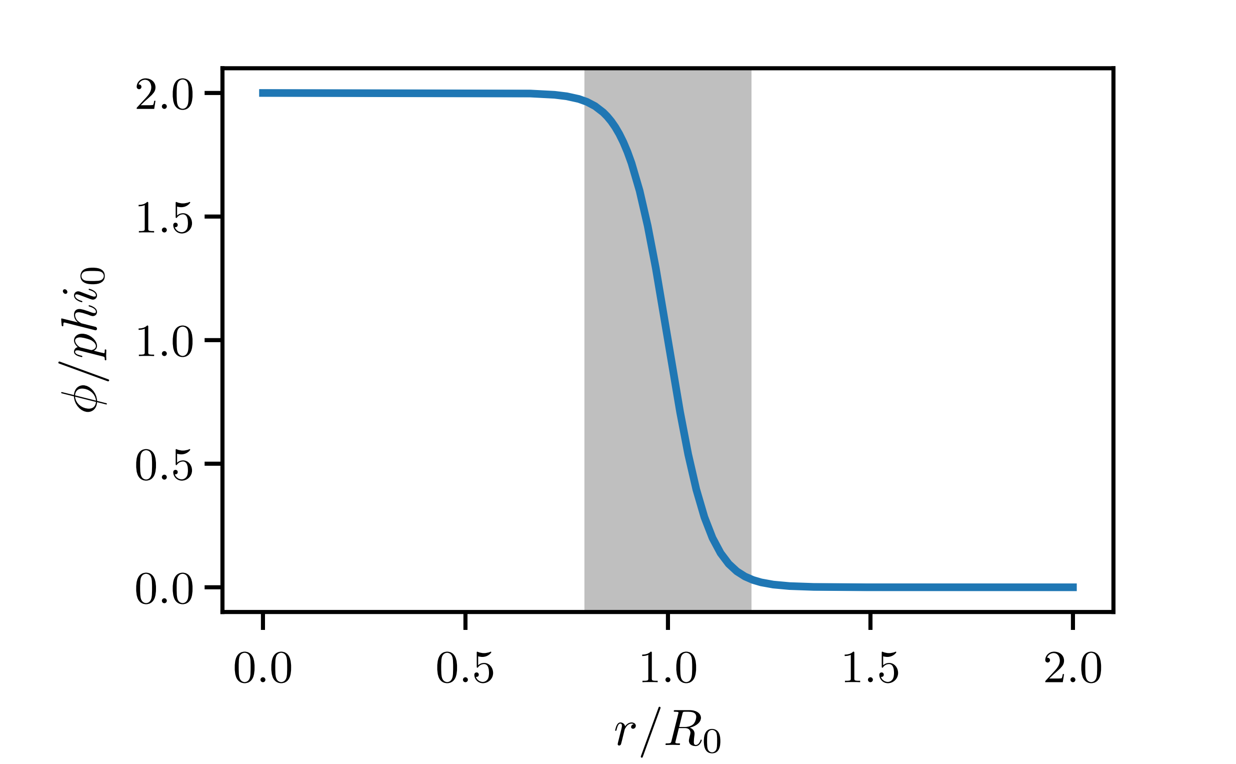


Typical Solution

Pseudospec Solution

$$\phi_{\mathrm{I}}(r) = \sum_n c_n B_{2n}\left(y\left(\frac{r}{\sqrt{r^2+L^2}}\right)\right)$$

$$y(x) = \frac{1}{\pi}\tan^{-1}\left(d^{-1}\tan\left(\pi\left[x-\frac{1}{2}\right]\right)\right)+\frac{1}{2}$$



Chebyshev Polynomials

$$B_n(x) = \cos(n\cos^{-1}(x))$$

Pseudospec Solution

Zero deriv.
at origin

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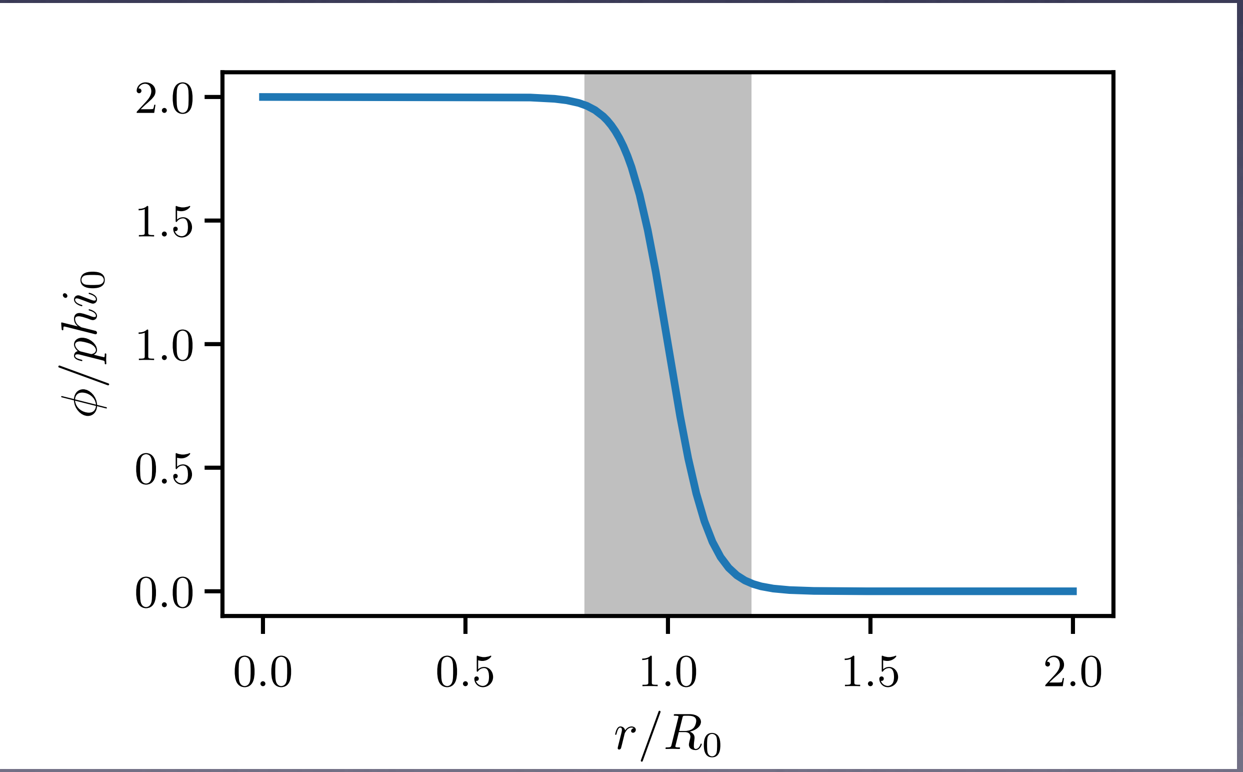

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Infinite Domain, feature at L

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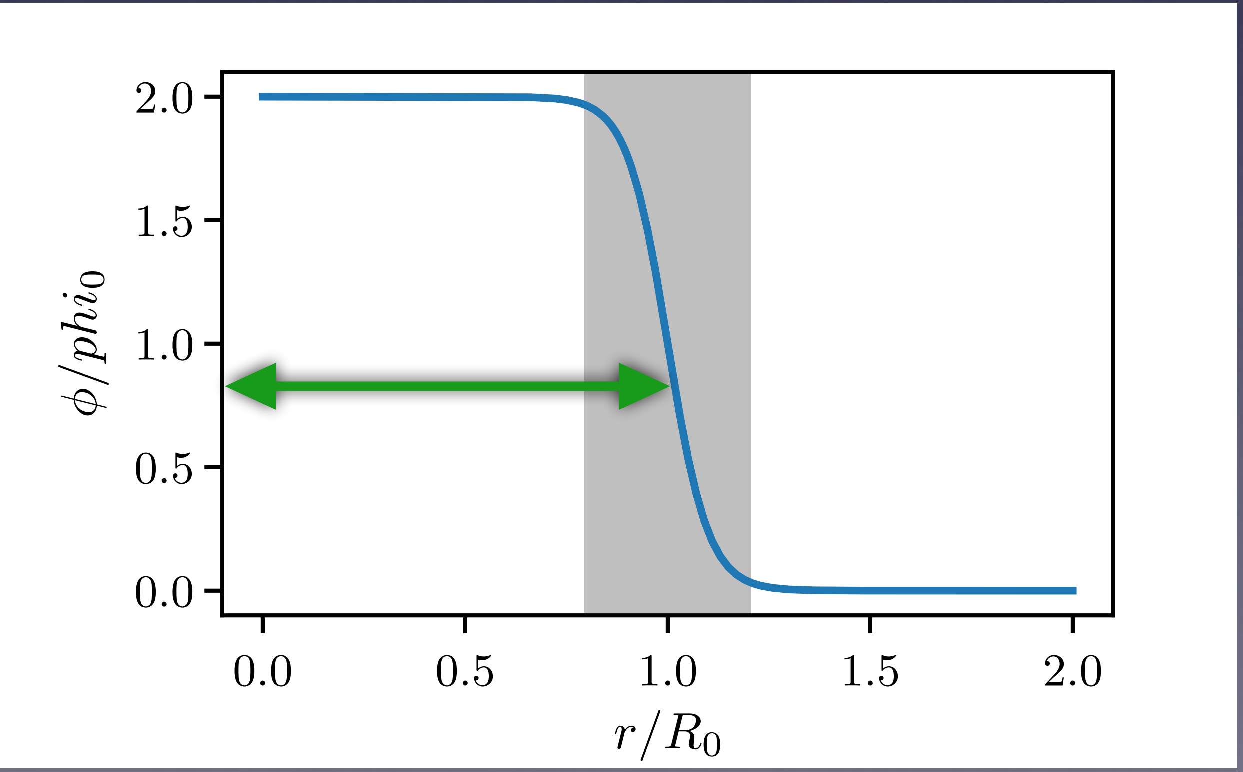

Chebyshev Polynomials

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Pseudospec Solution

Zero deriv. at origin

Width of Feature

Infinite Domain, feature at L

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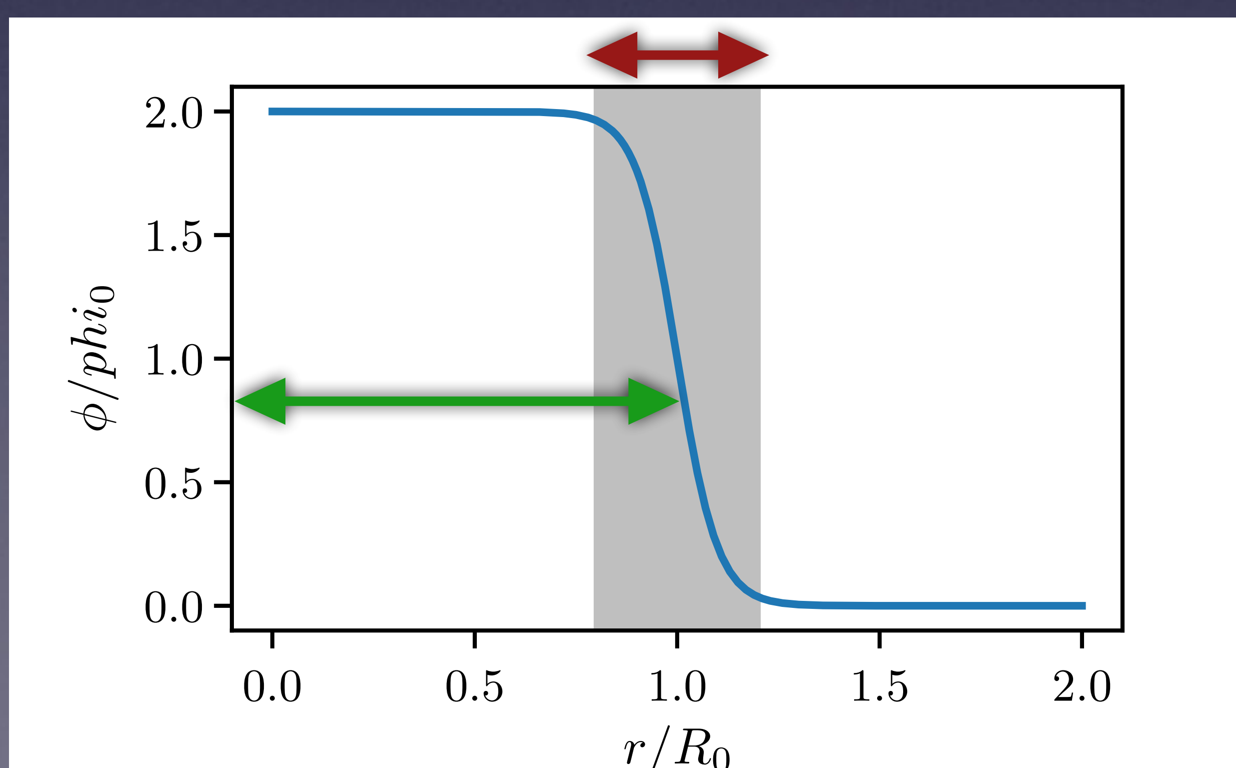

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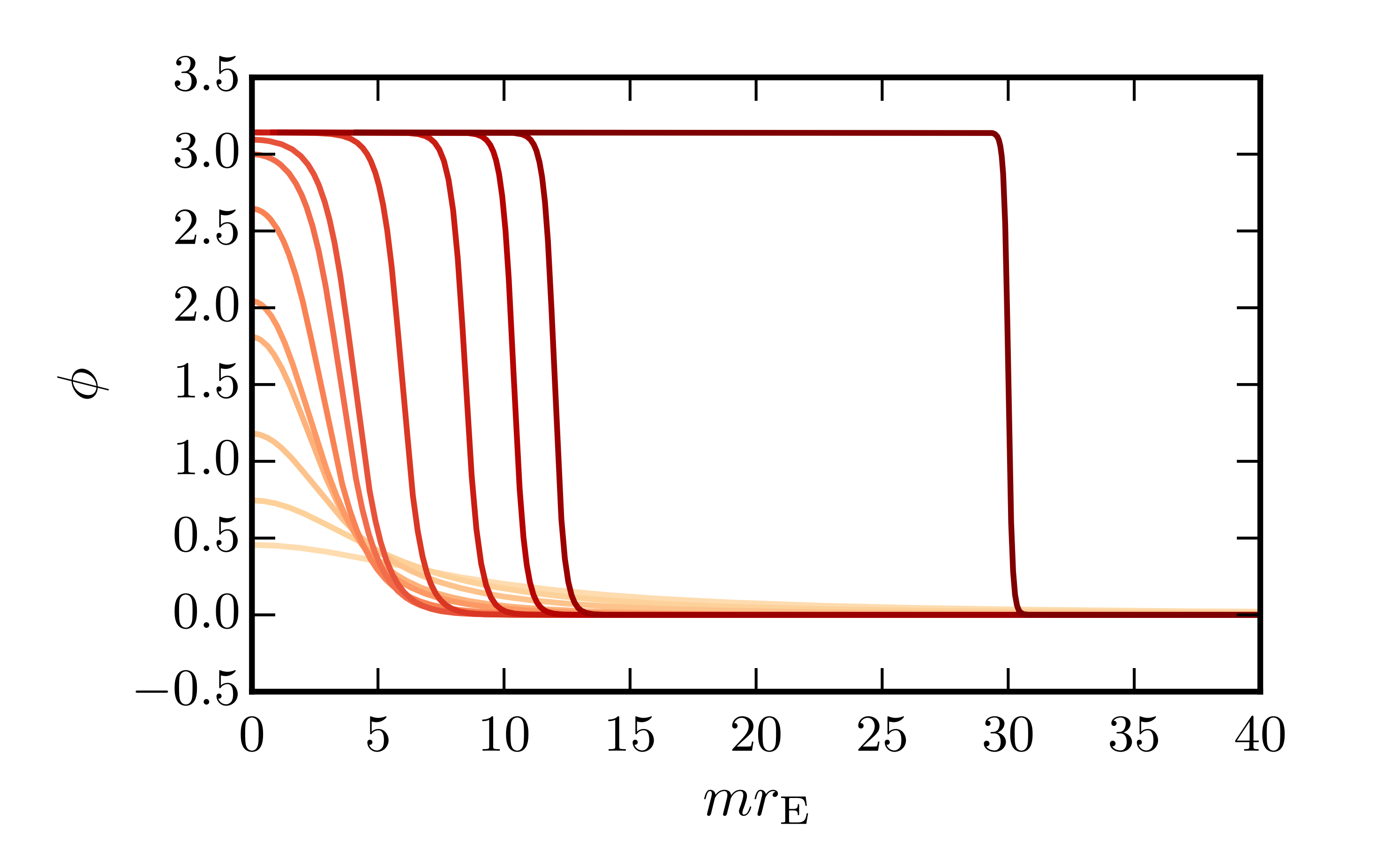
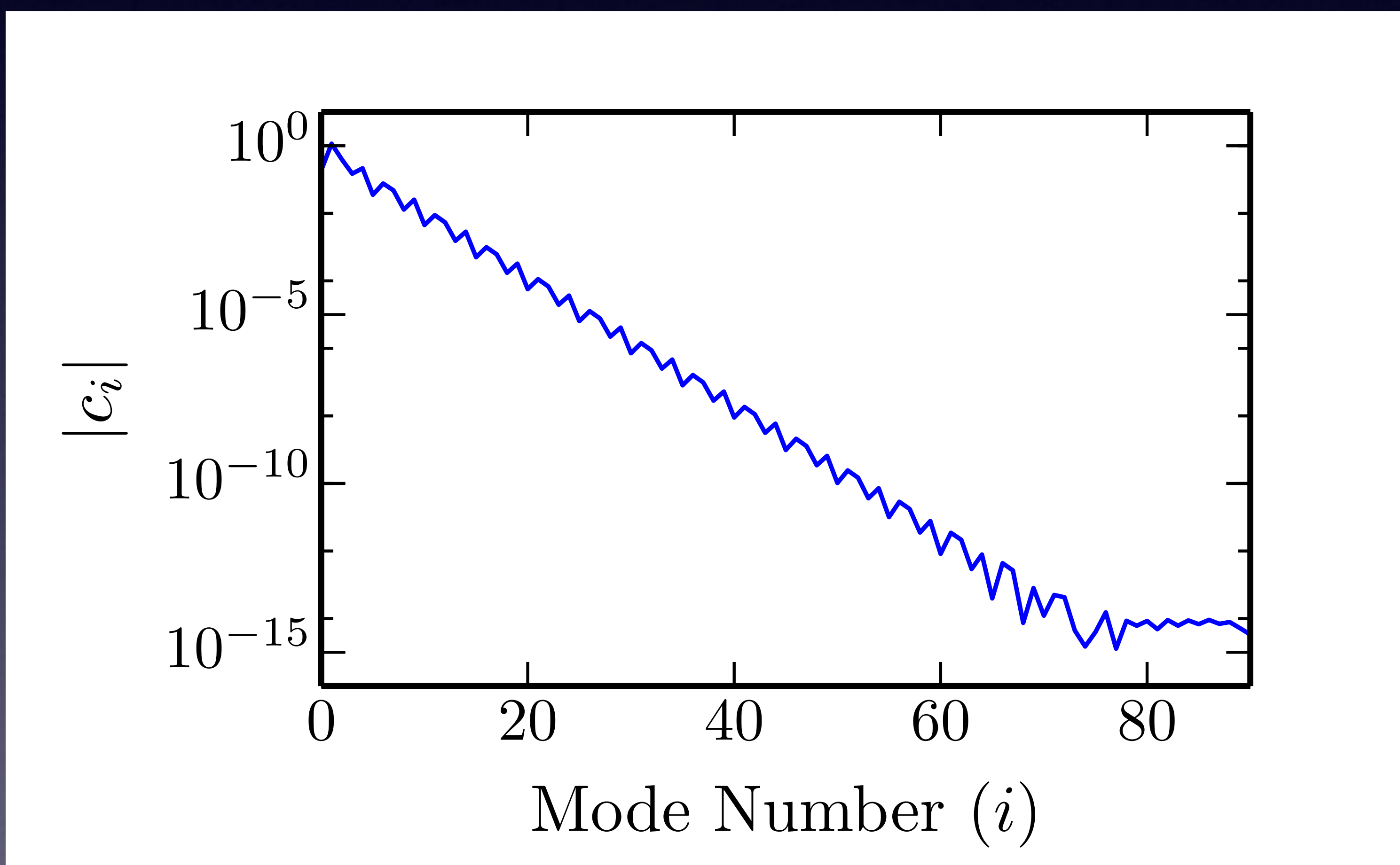
Bounce Profiles



- Outer boundary at ∞
- $\mathcal{O}(10^{-15})$: ~100 modes
- $N^3_{\mathrm{fields}}\mathcal{O}(10^{-3})\mathrm{s}$
- ~~Arbitrary precision arithmetic~~

Bounce Profiles



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Decay Rates

$$S_{\rm E} = A_{\rm d+1} \int dr_{\rm E} r_{\rm E}^d \left(\frac{\phi'^2}{2} + V(\phi) \right)$$

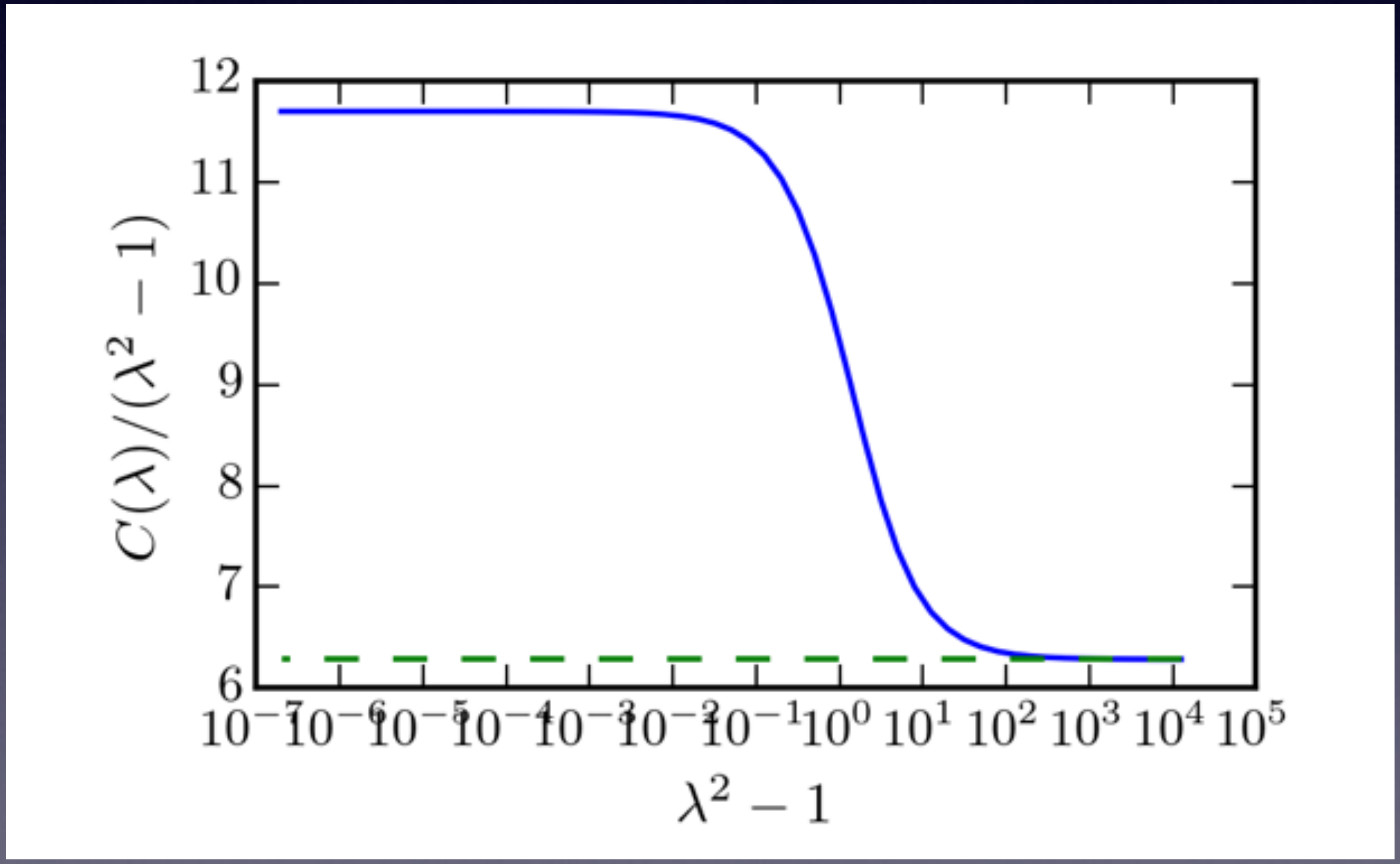
$$S_{\rm I} = S_{\rm E}[\phi_{\rm B}] - S_{\rm E}[\phi_{\rm fv}]$$

- Single negative eigenmode

$$\frac{\Gamma}{V} = \left(\frac{S_{\rm I}}{2\pi} \right)^{D/2} \sqrt{\frac{\det \delta^2 S_{\rm E}[\phi_{\rm fv}]}{\det' \delta^2 S_{\rm E}[\phi_{\rm B}]}} e^{-S_{\rm I}} \left(1 + \mathcal{O}(\hbar)\right)$$

Nucleation Rates

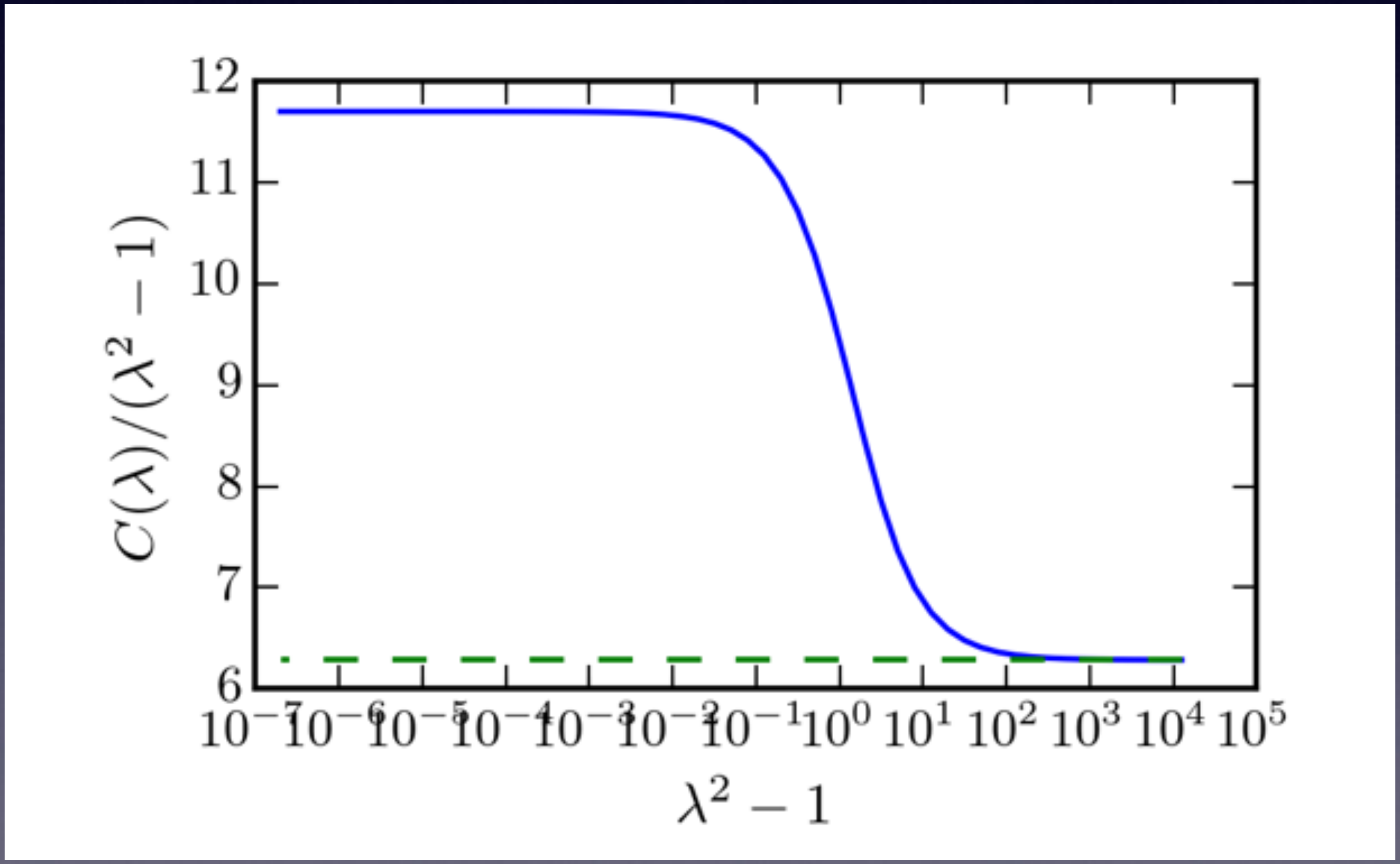
$$\frac{\Gamma}{V} \approx g(\lambda)\left[m_{\rm eff}^2\right]^{\frac{D}{2}}\left(\frac{S_{\rm I}}{2\pi}\right)^{\frac{D}{2}} e^{-S_{\rm I}}$$



$$V(\phi) = V_0\left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1\right)$$

Nucleation Rates

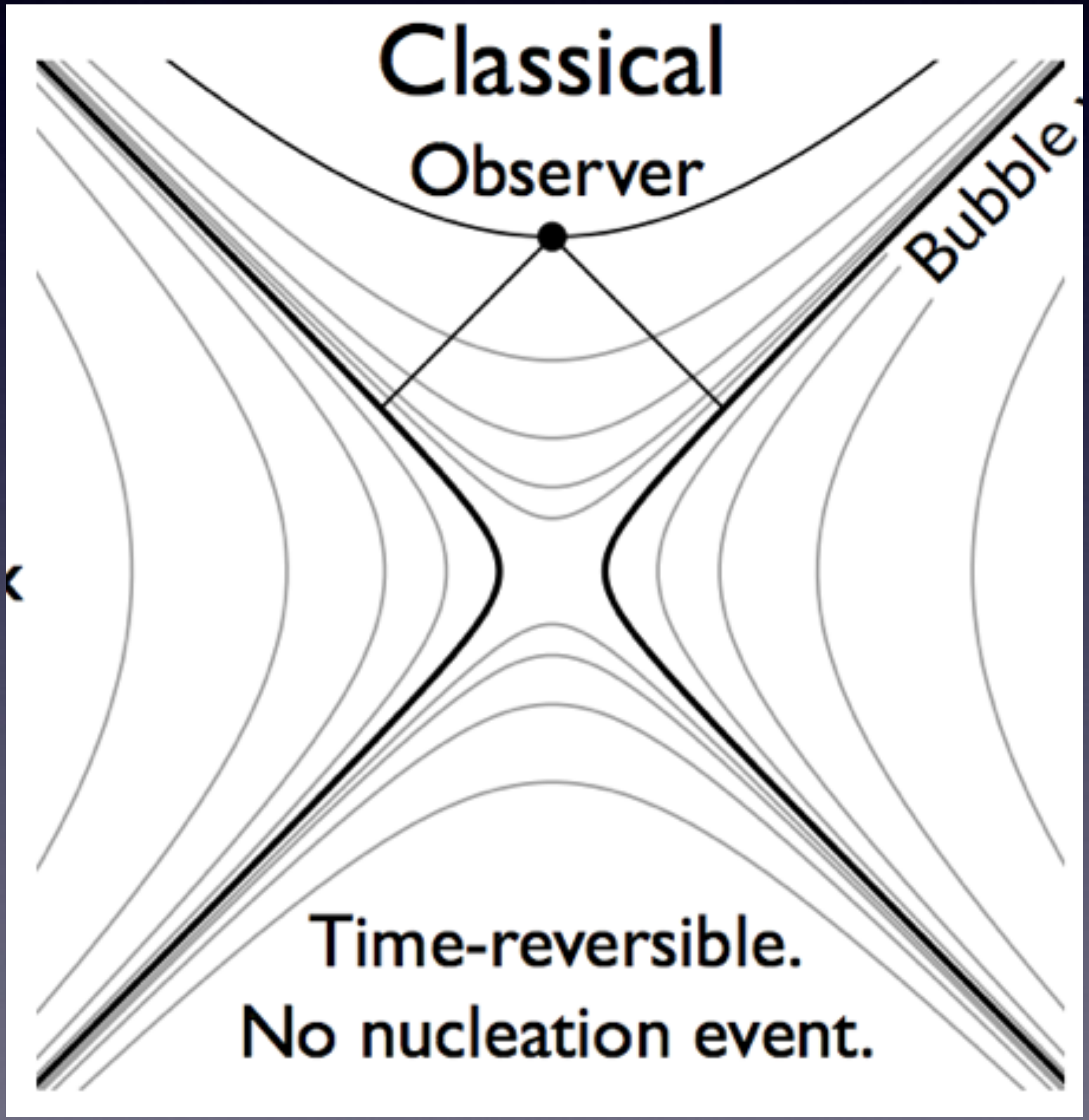
$$S_{\mathrm{I}} = 2\pi\phi_0^2 C(\lambda)$$



$$V(\phi) = V_0\left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1\right)$$

Real-Time Interpretation

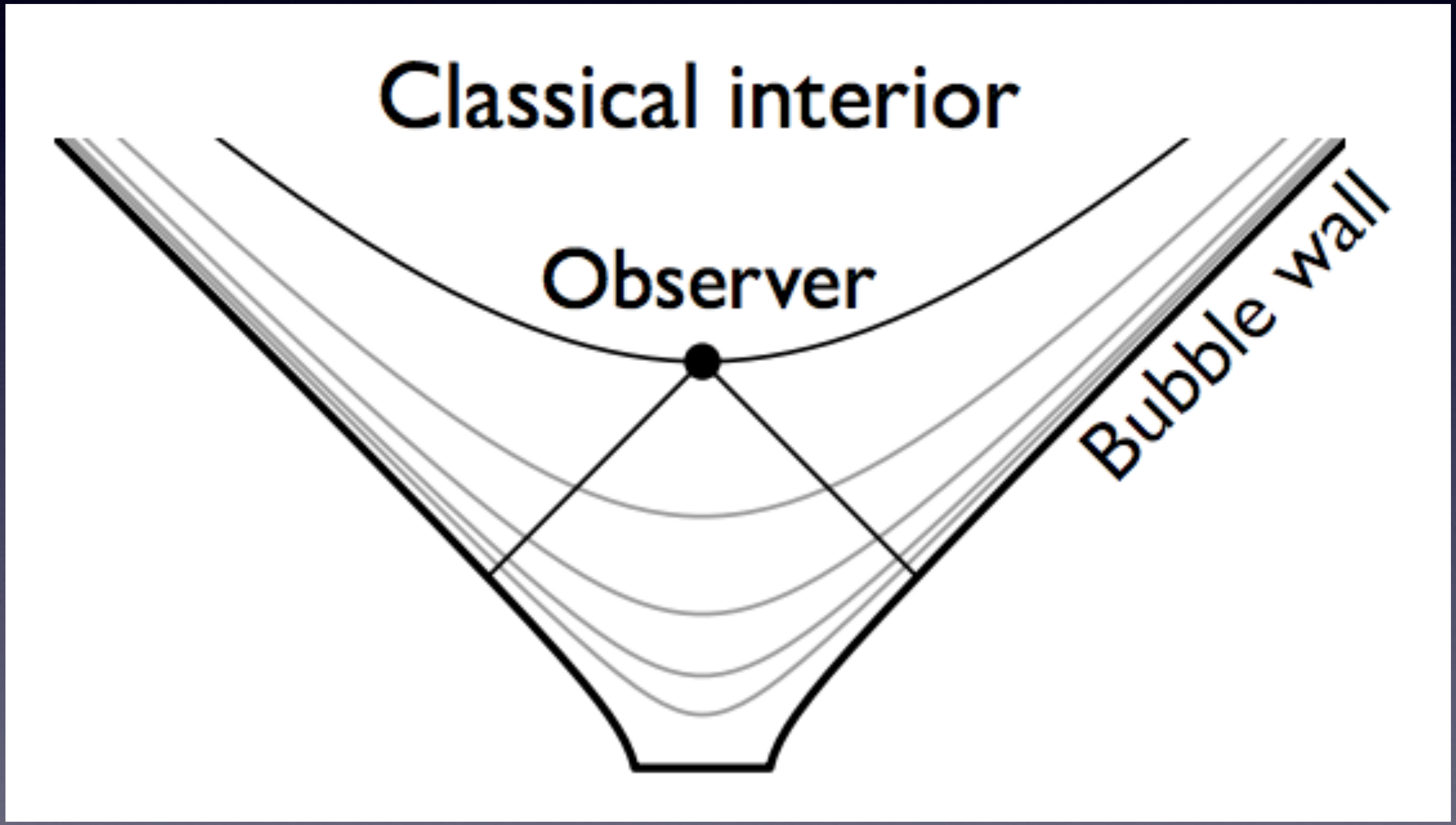
$$\phi(x,t) = \phi_{\mathrm{I}}(\sqrt{x^2 - t^2})$$



[Figure courtesy of Andrew Pontzen]

Ad-Hoc Nucleation

$$\phi(\mathbf{x}, t=0) = \phi_{\rm I}(|\mathbf{x}|)$$



No real-time classical description

[Figure courtesy of Andrew Pontzen]

Some Questions

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- Time-dependent description of nucleation

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  - Bubble precursor? Init. cond. at nucleation

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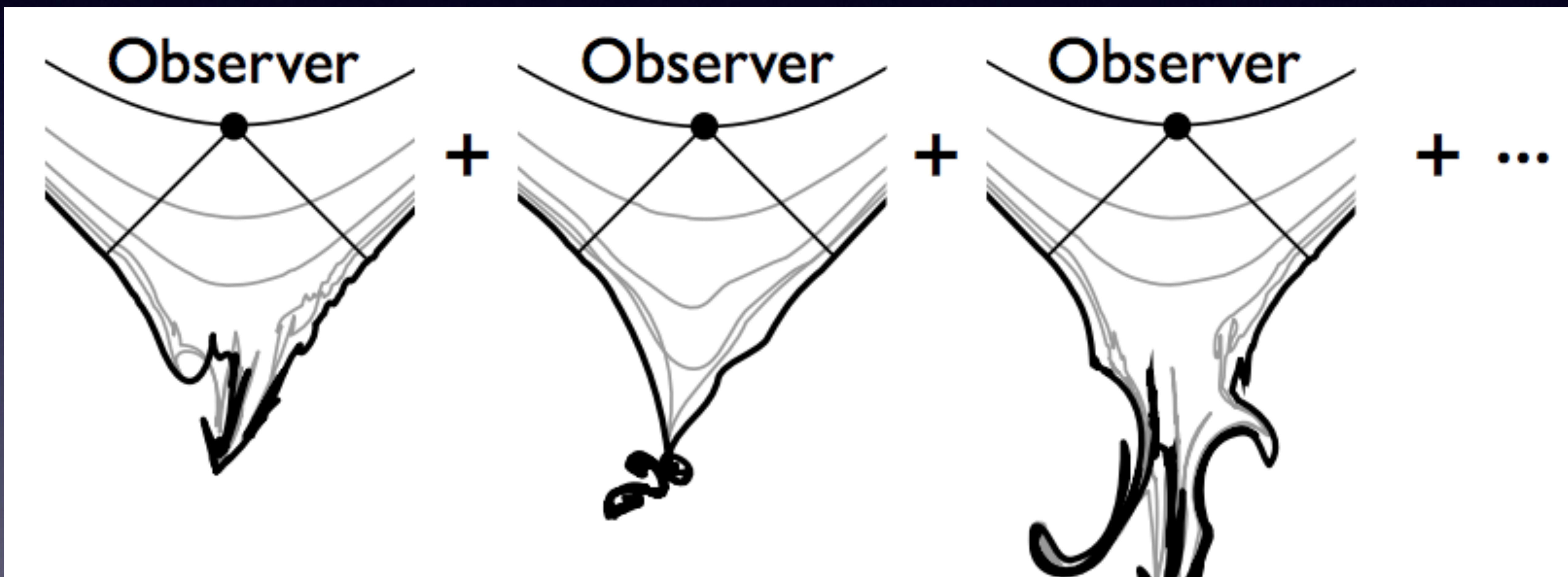
QFT exponentially complex.
Need Approximations!

Full Evolution?



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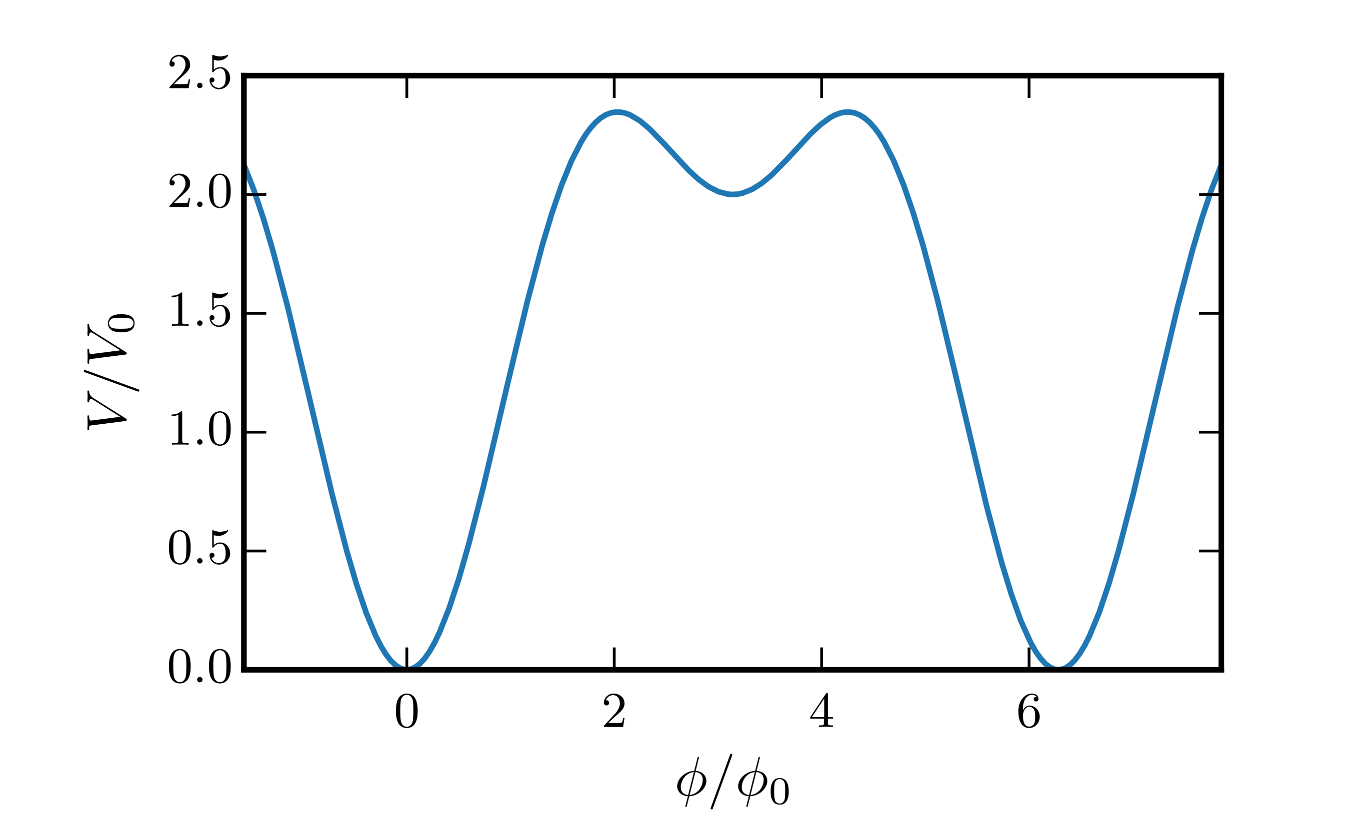
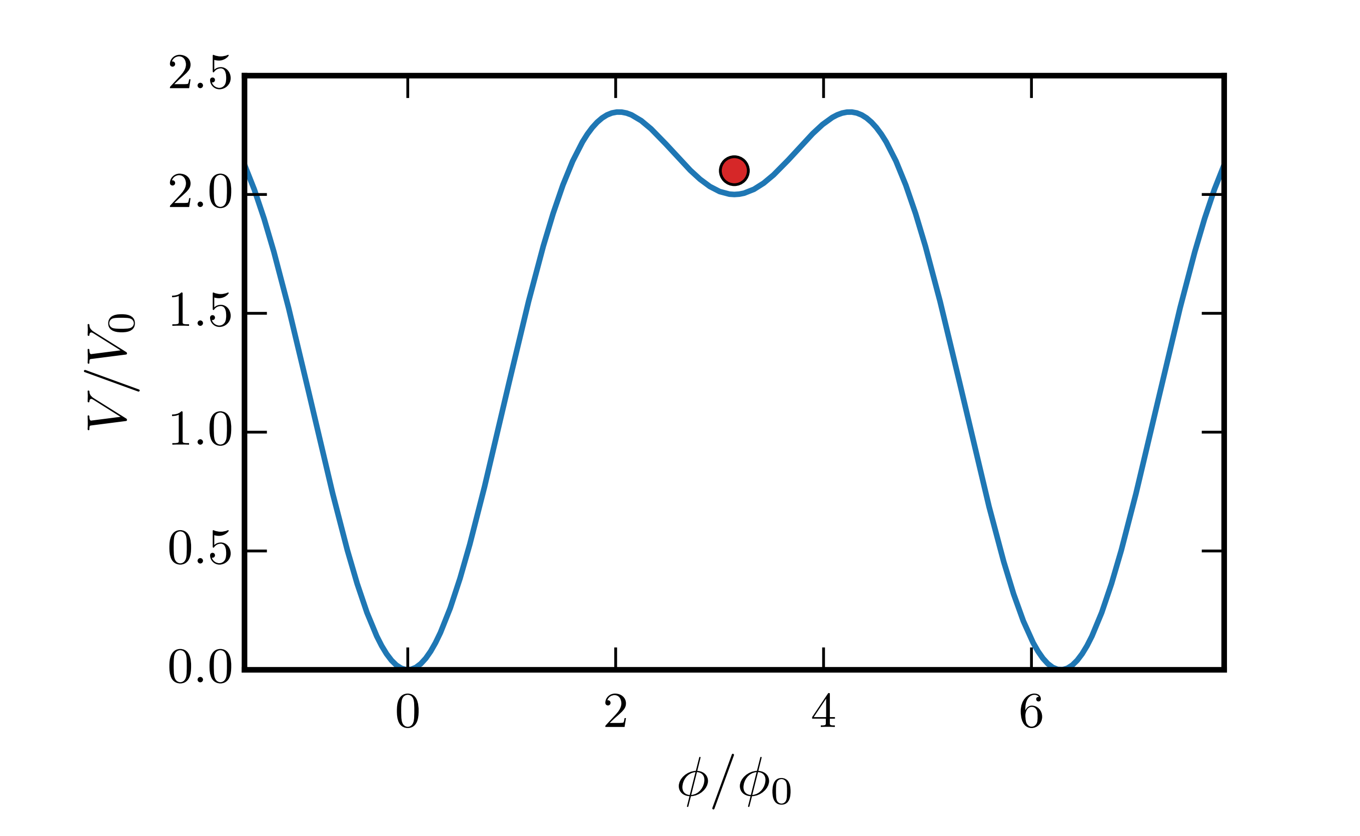


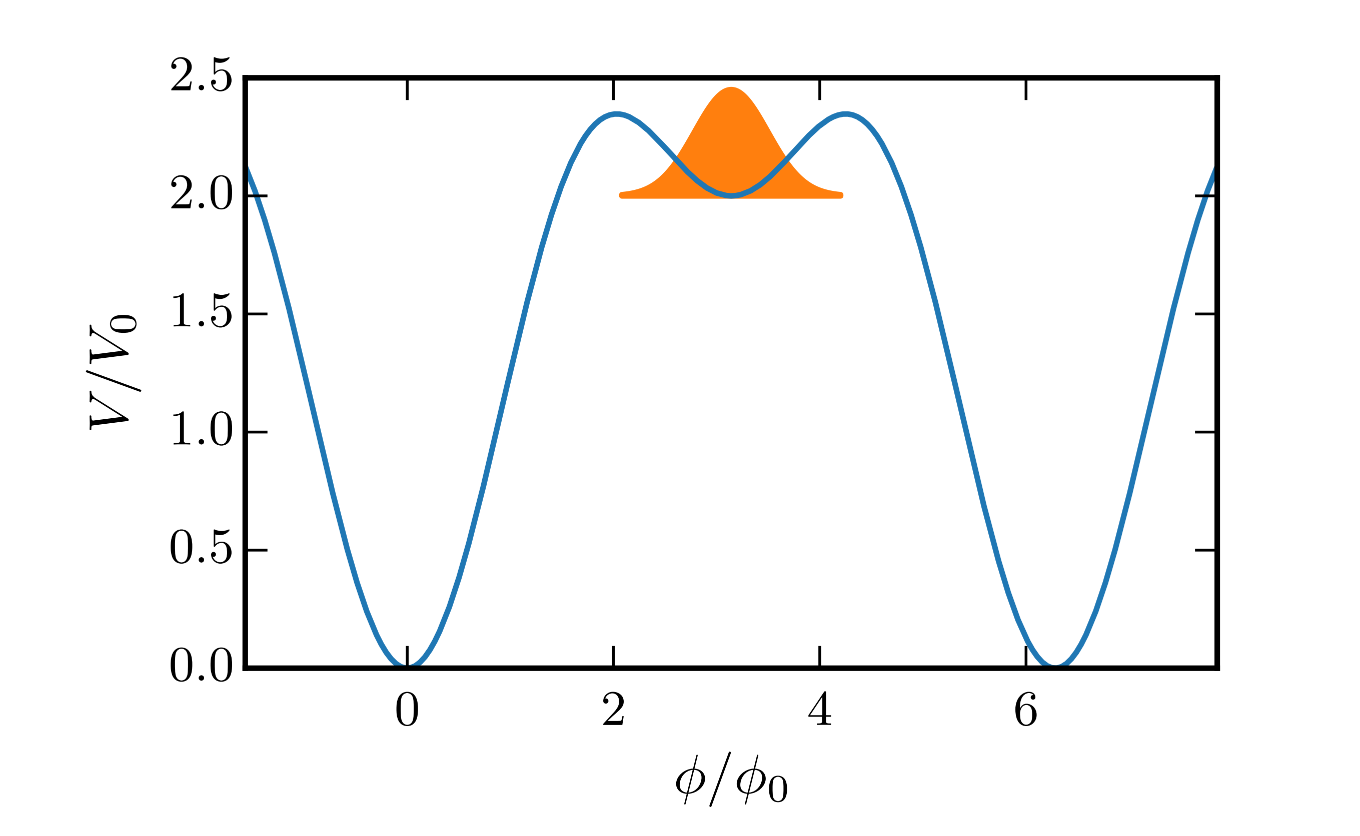
$$\phi =$$

$$\Pi =$$



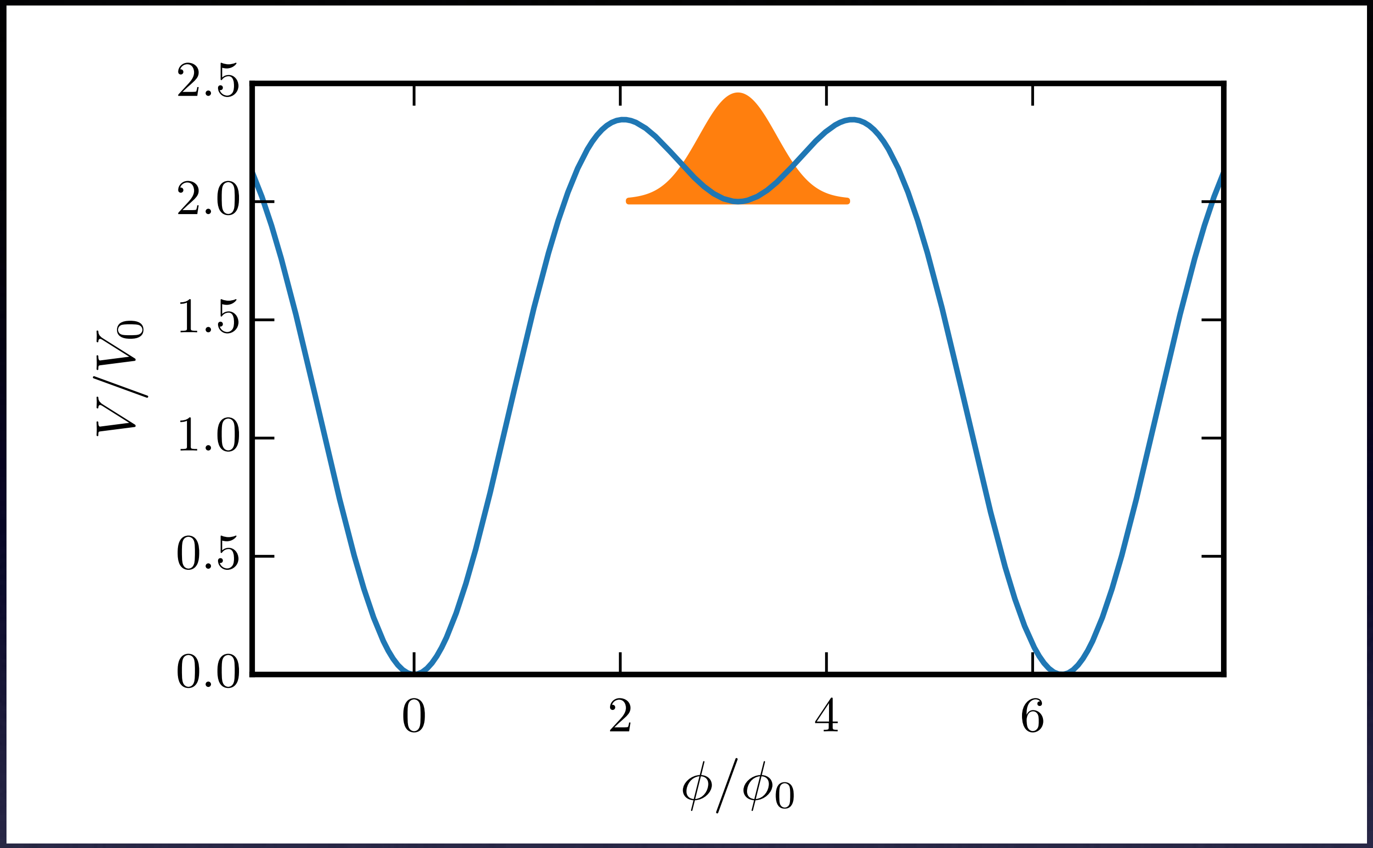
$$\phi = \phi_{\mathrm{fv}}$$

$$\Pi = 0$$



$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

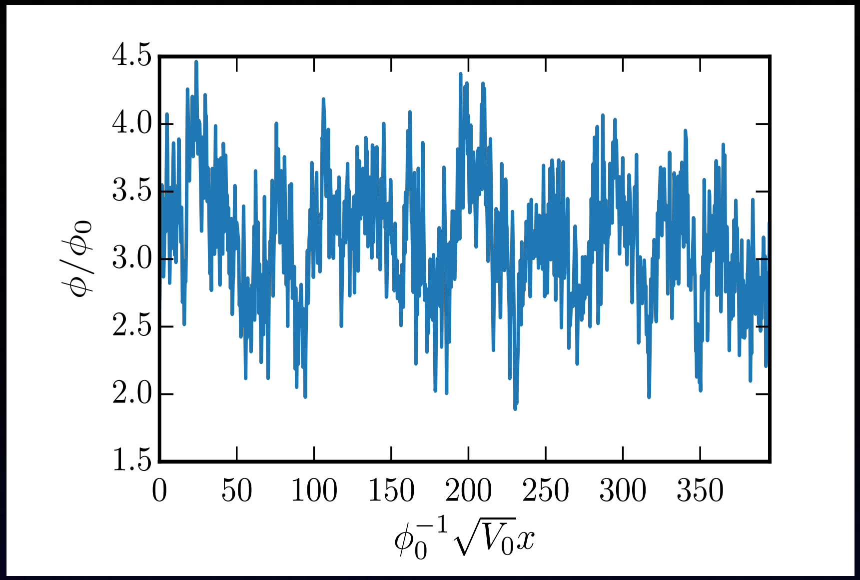


$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

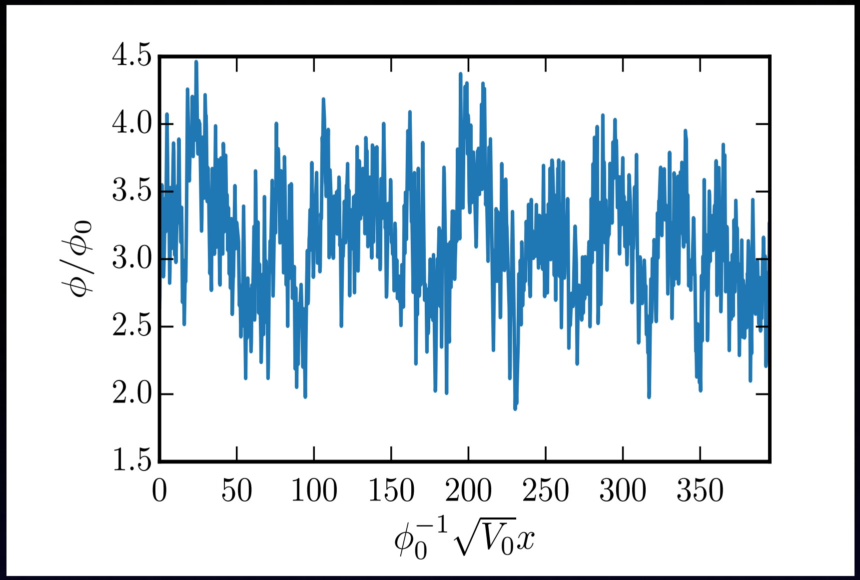
$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

$$\langle \delta\tilde{\phi}_k \delta\tilde{\phi}^*_p \rangle = \frac{1}{2\omega_k}\delta(k-p) \qquad \langle \delta\tilde{\Pi}_k \delta\tilde{\Pi}^*_p \rangle = \frac{\omega_k}{2}\delta(k-p)$$

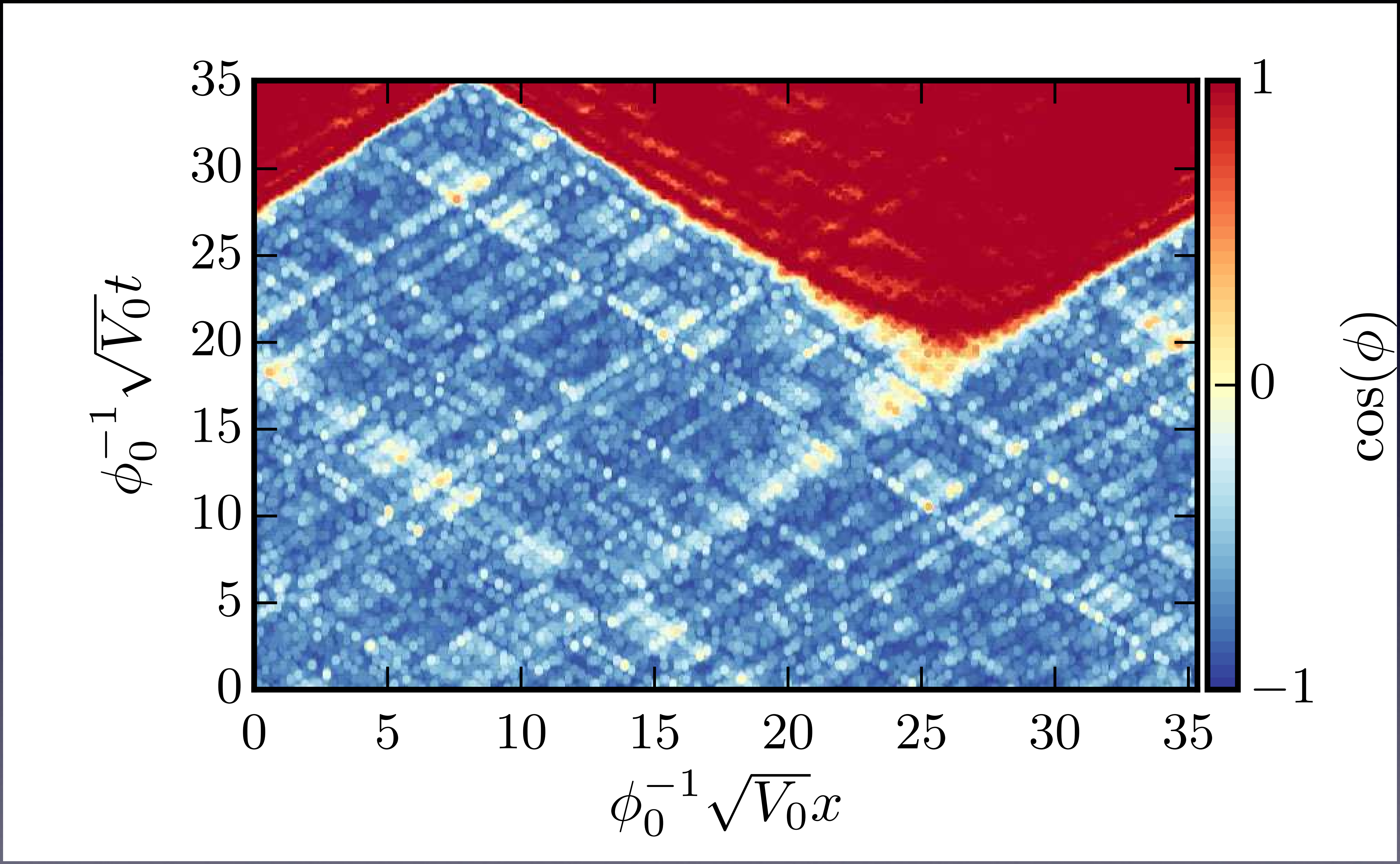
Quantum Commutators



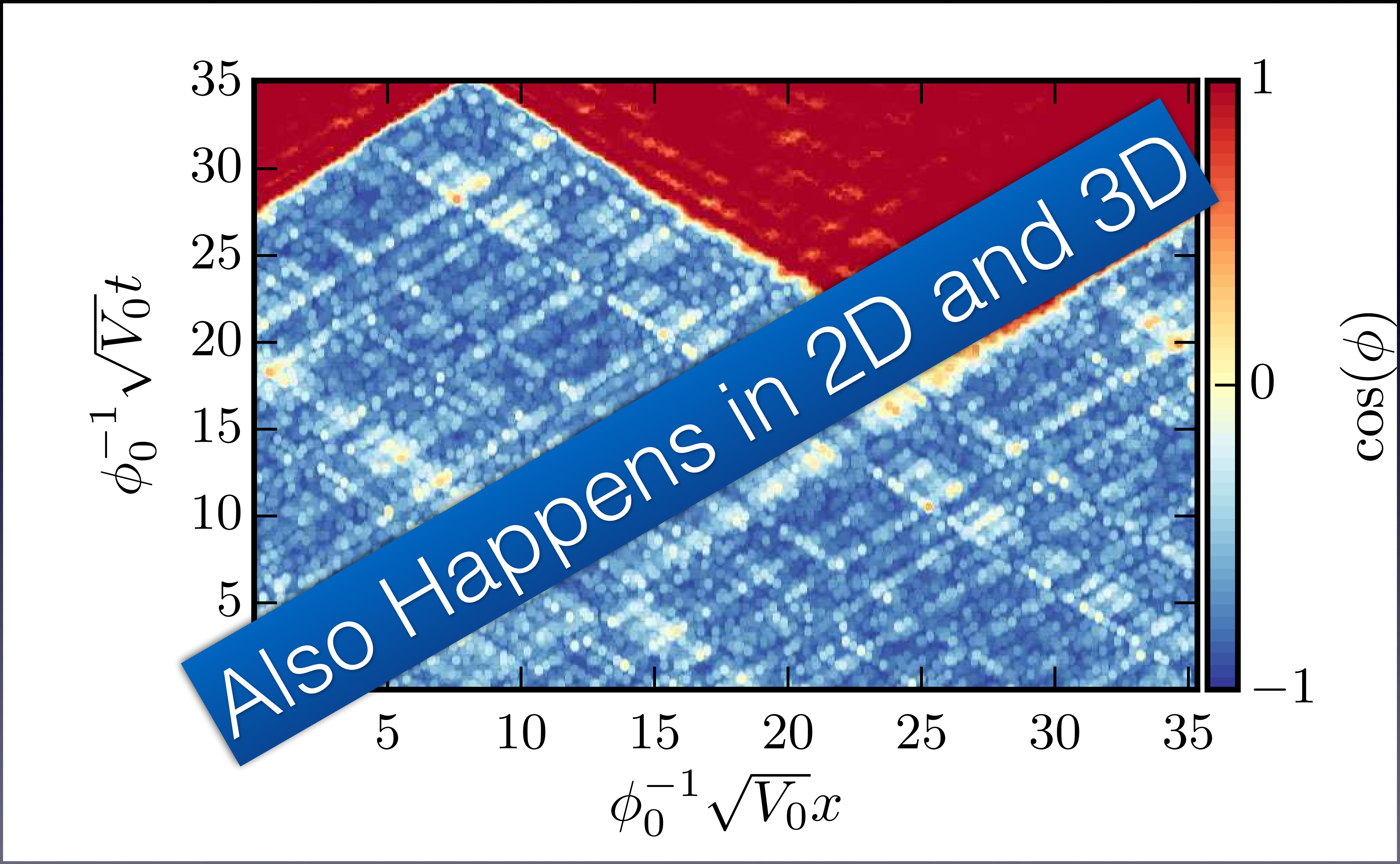
Quantum Commutators



$$\ddot{\phi} - \nabla^2 \phi + V'(\phi) = 0$$



Classically-Allowed Vacuum Decay



Classically-Allowed Vacuum Decay

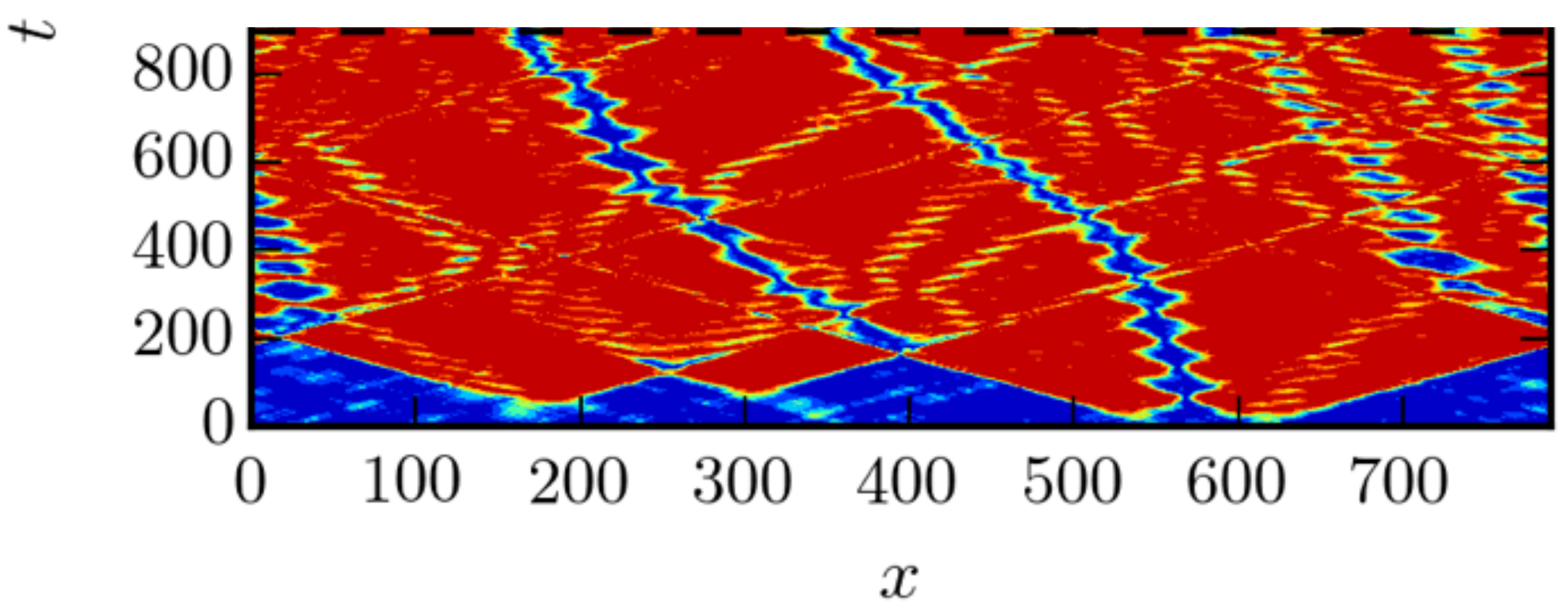
Numerical Artifact?

- Spatial Discretization: Fourier pseudospectral (exponential convergence)
  Temporal Discretization: Gauss-Legendre (10th order in dt, symplectic)

- Energy conservation: $\mathcal{O}(10^{-15})$
  Momentum conservation: $\mathcal{O}(10^{-15})$
  Pointwise convergence with dt step: $\mathcal{O}(10^{-15})$
  Pointwise convergence with dx step: $\mathcal{O}(10^{-15})$

NO

Numerical Reversibility



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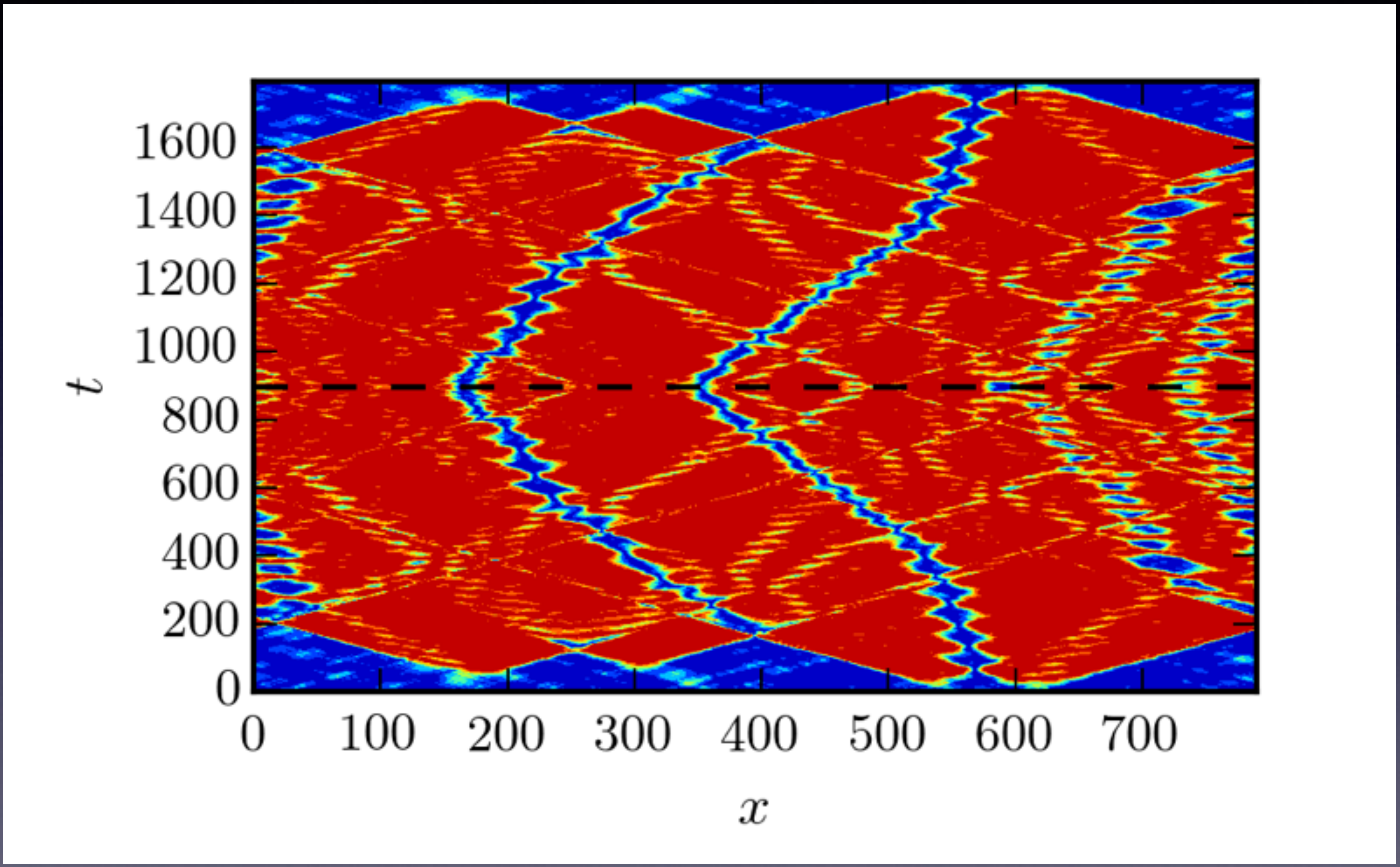
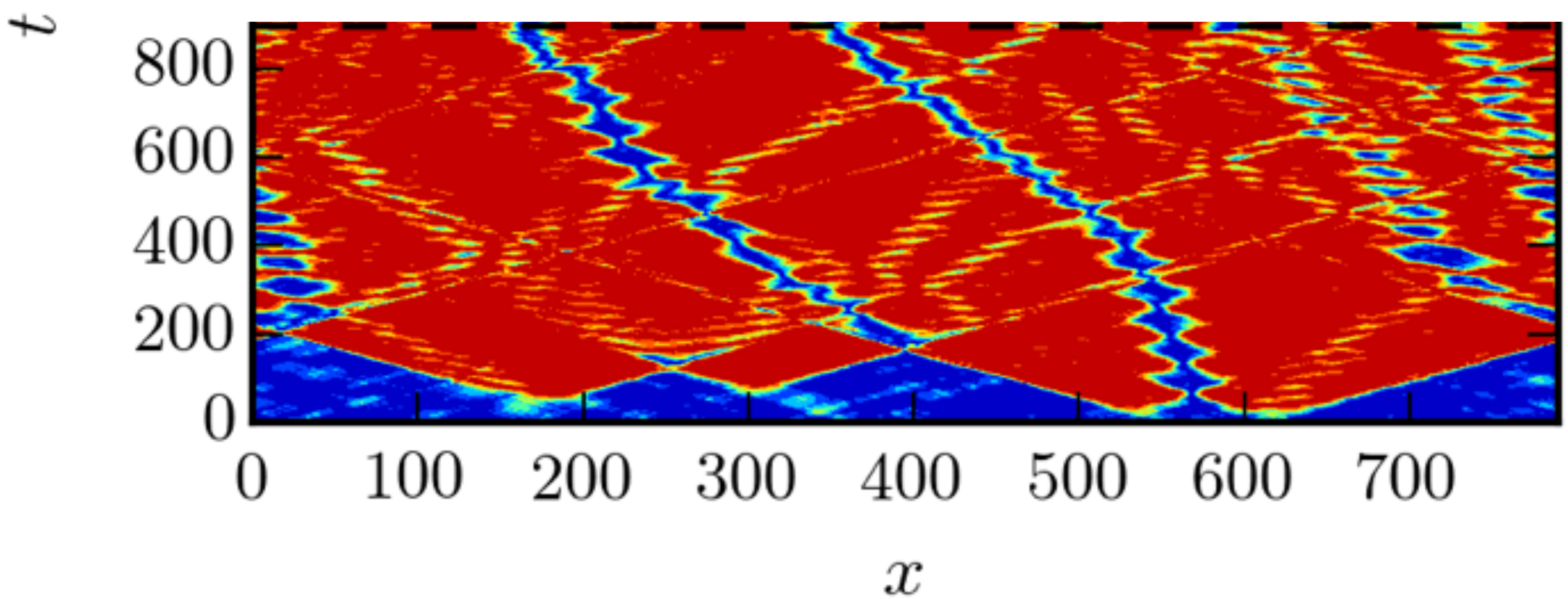


Destroyed by Addition of Noise



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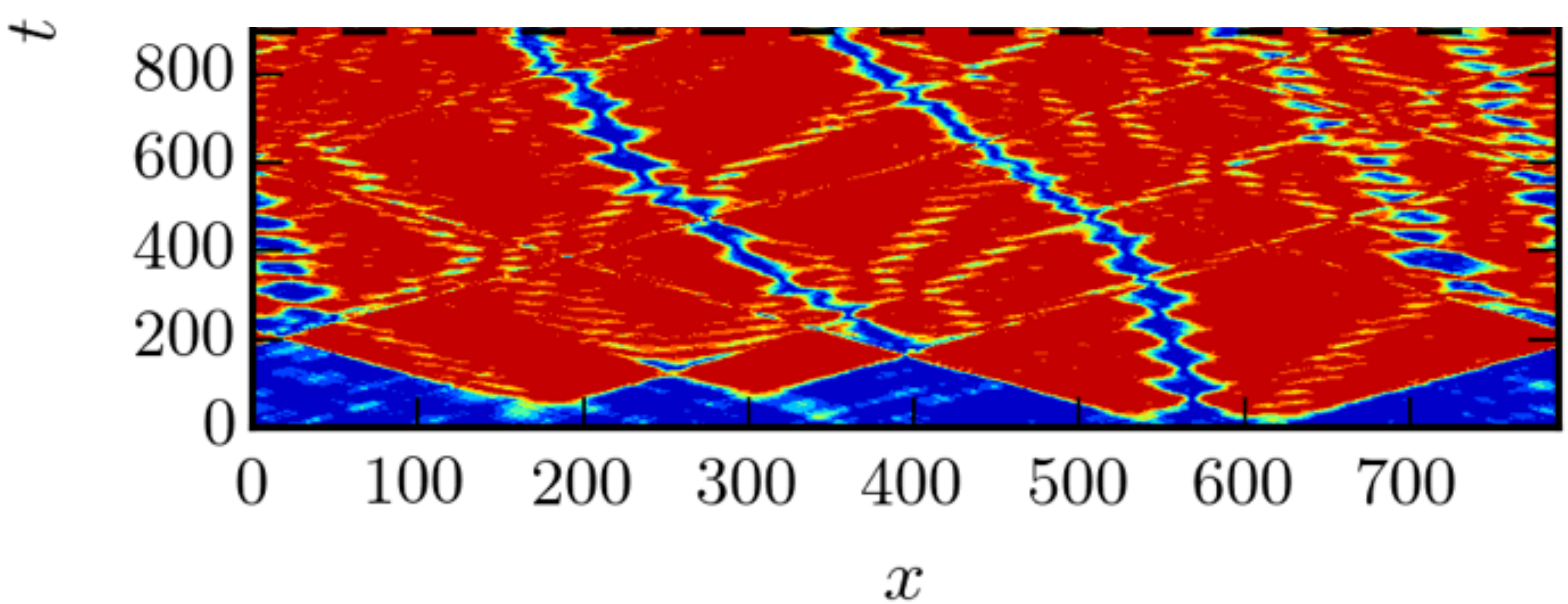
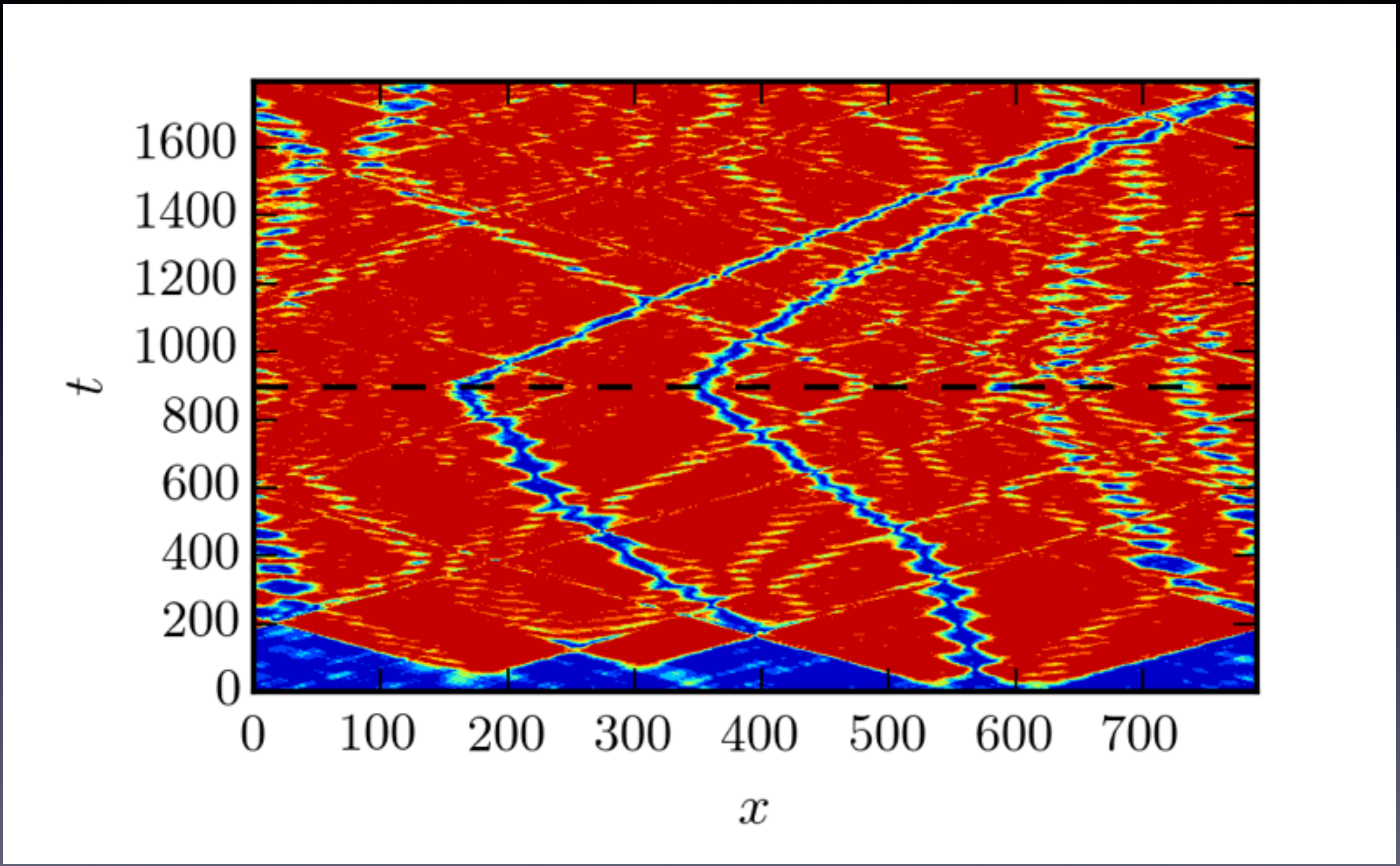


Destroyed by Addition of Noise



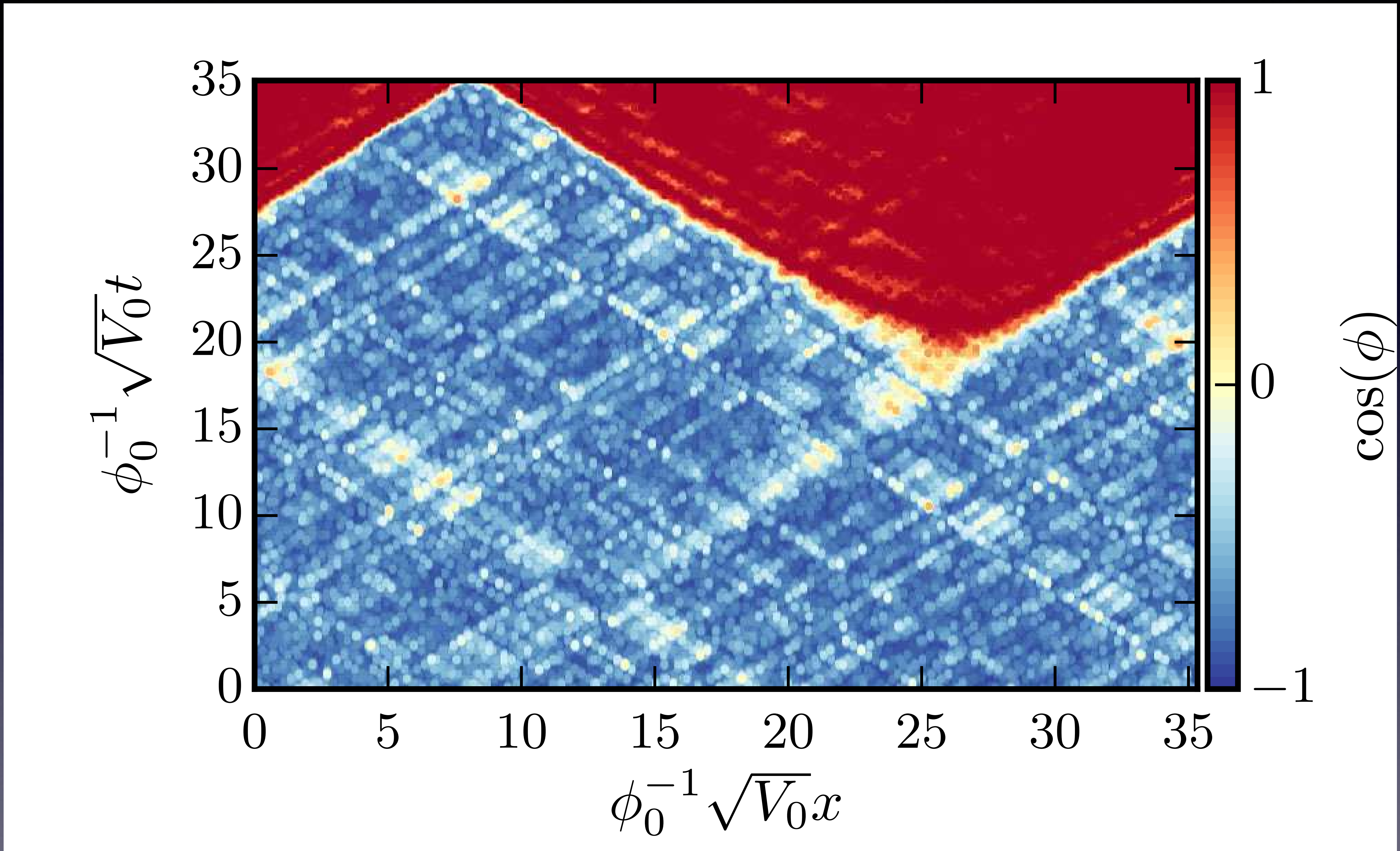
Decay Rates?

Prediction

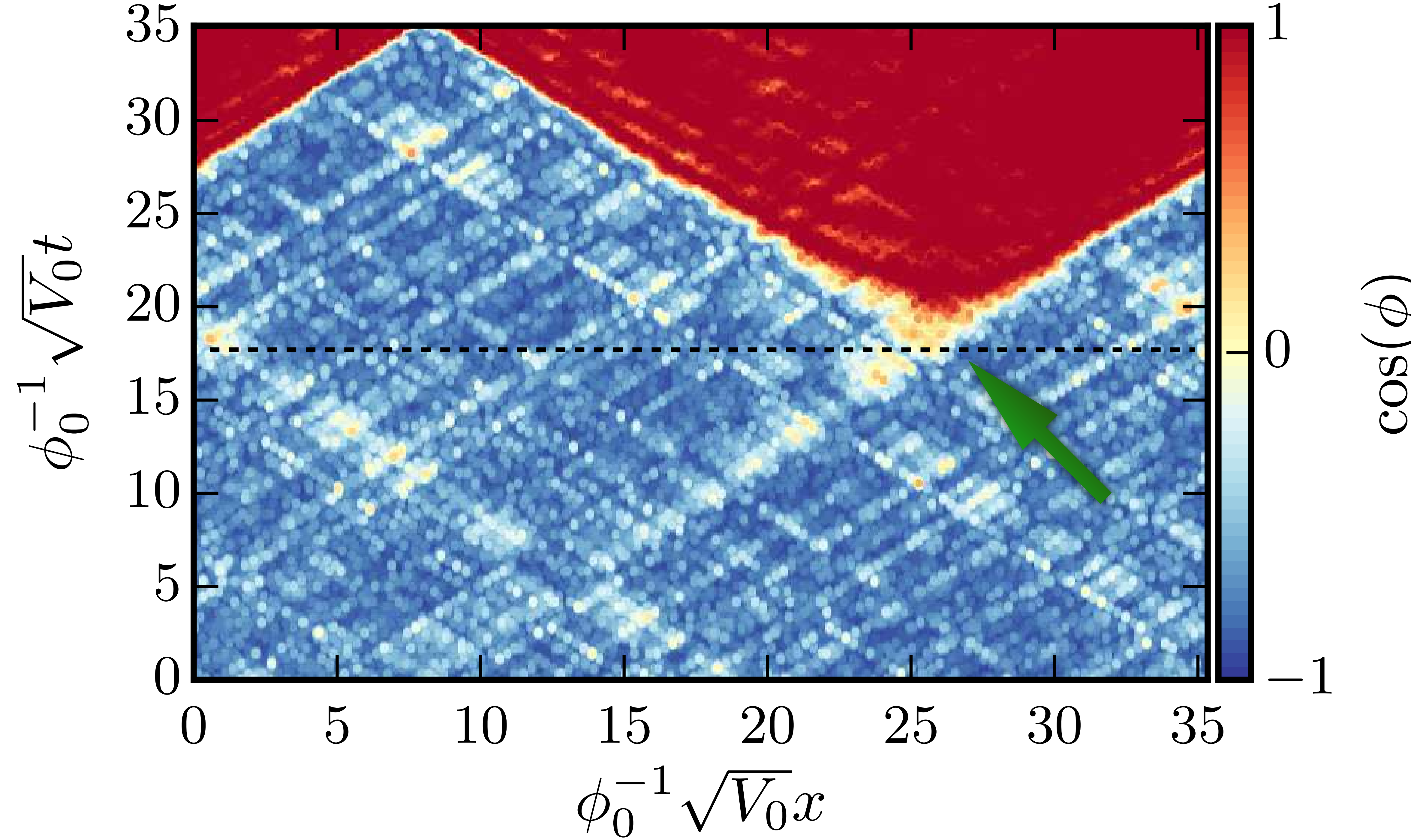
$$\frac{\Gamma_{\rm I}^{(1+1)}}{L} \approx g(\lambda, V_0, \phi_0) m_{\rm eff}^2 \phi_0^2 C(\lambda) e^{-2\pi\phi_0^2 C(\lambda)}$$

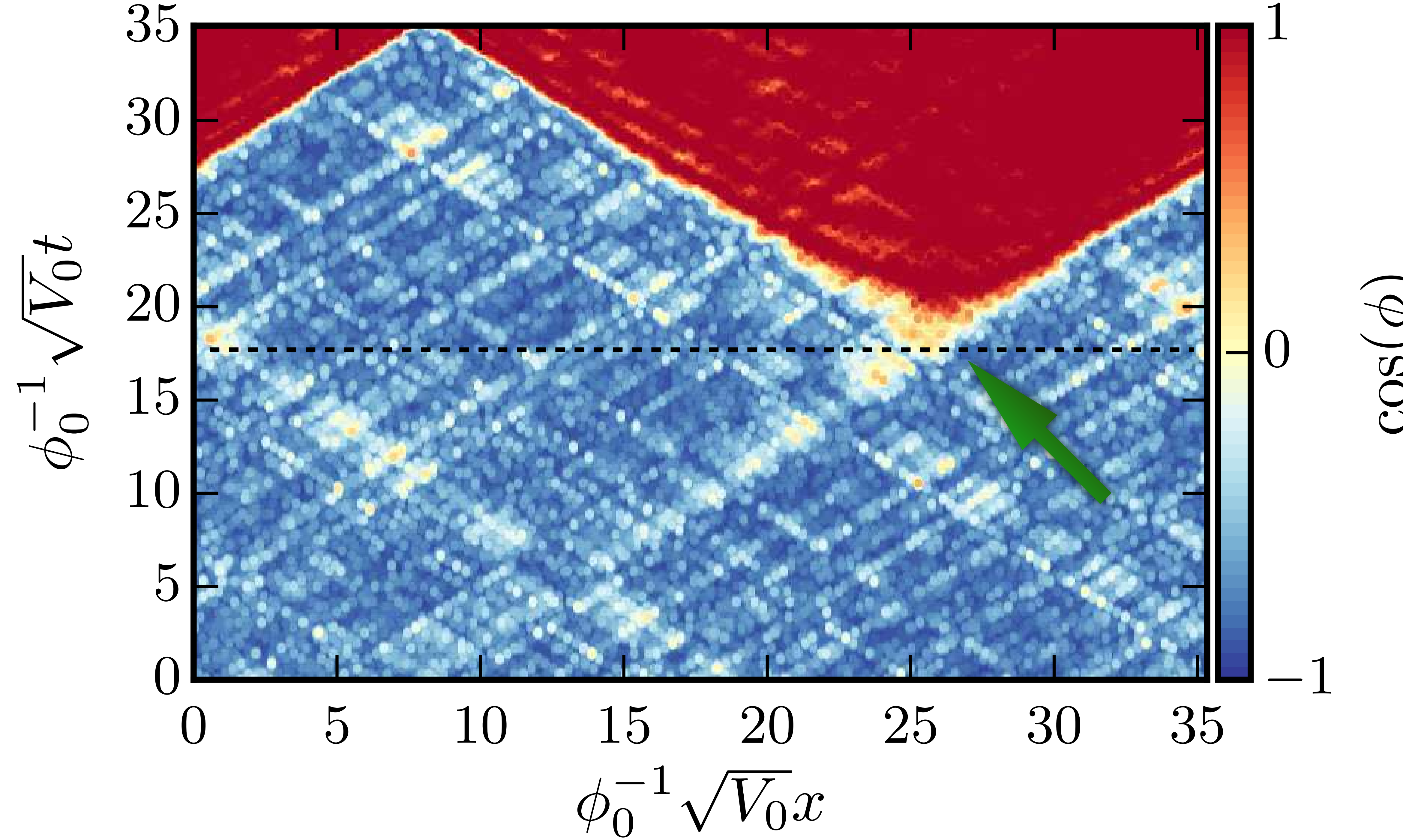
$\mathcal{O}(1)$ $\sim V''(\phi_{\rm fv})$ Instanton

$$V(\phi) = V_0\left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1\right)$$

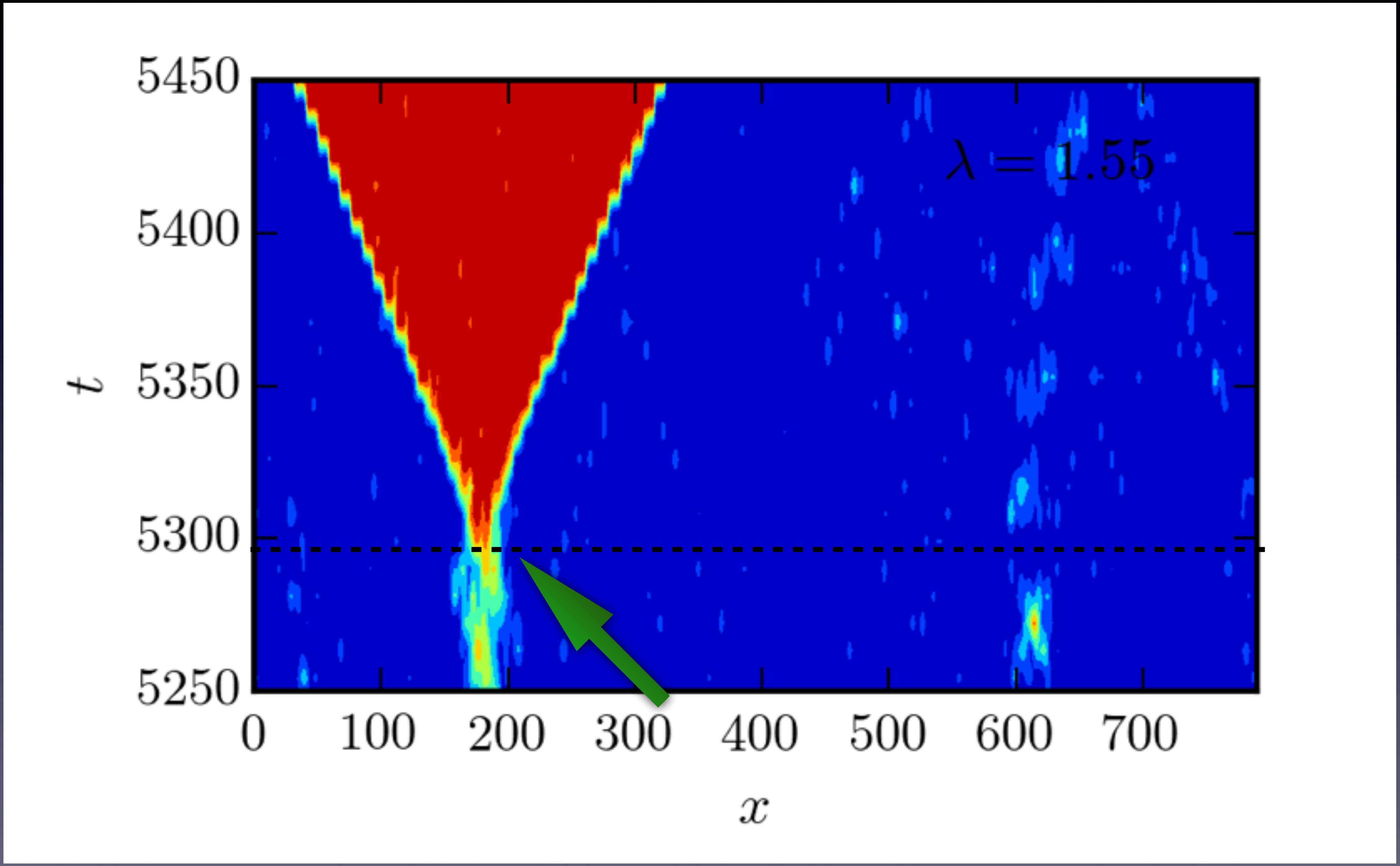


35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
$\phi_0^{-1}\sqrt{V_0}t$
$\phi_0^{-1}\sqrt{V_0}x$
1
0
−1
$\cos(\phi)$


35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
$\phi_0^{-1}\sqrt{V_0}t$
$\phi_0^{-1}\sqrt{V_0}x$
1
0
-1
$\cos(\phi)$

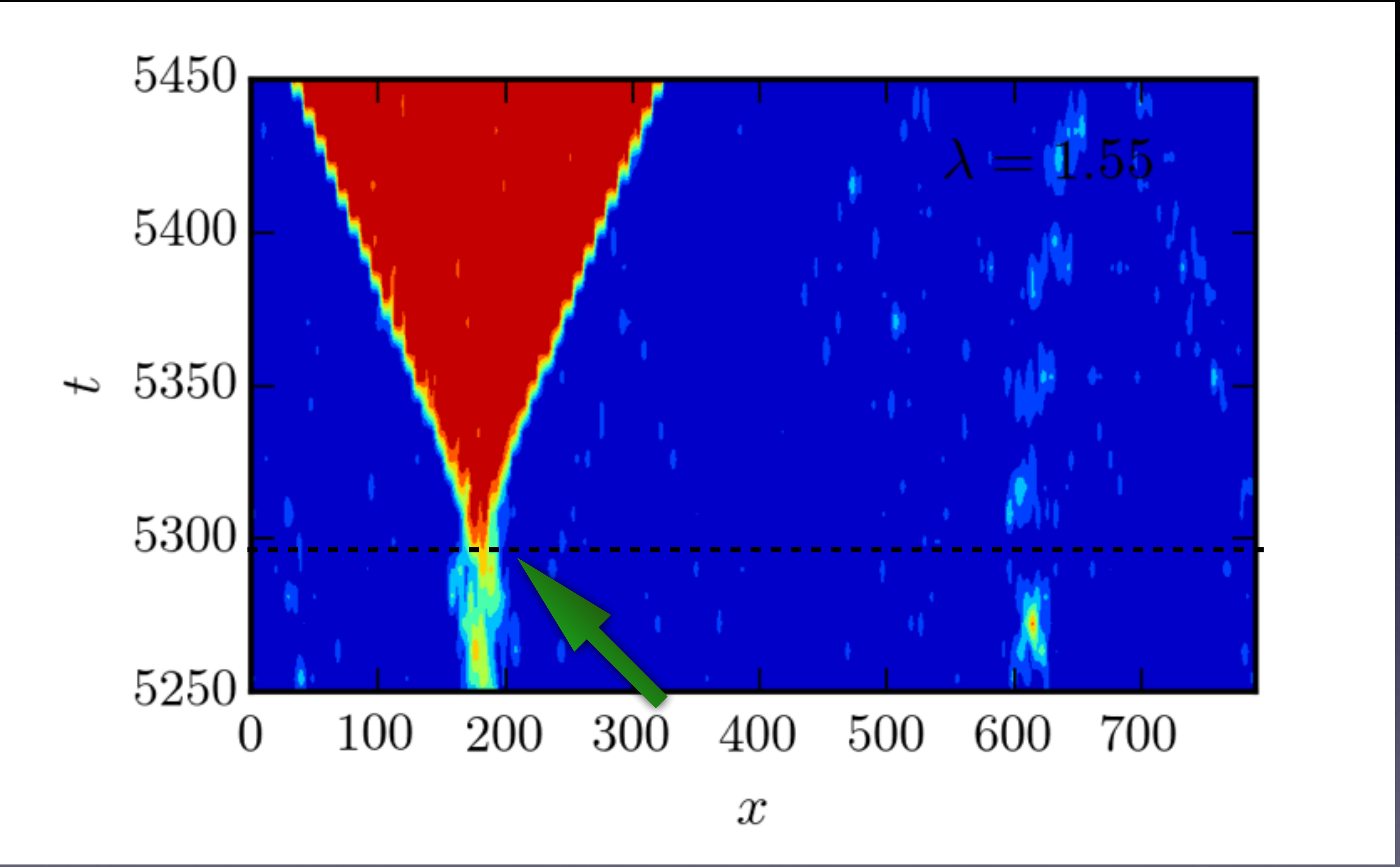


$t_{\mathrm{decay}}^{(i)}$



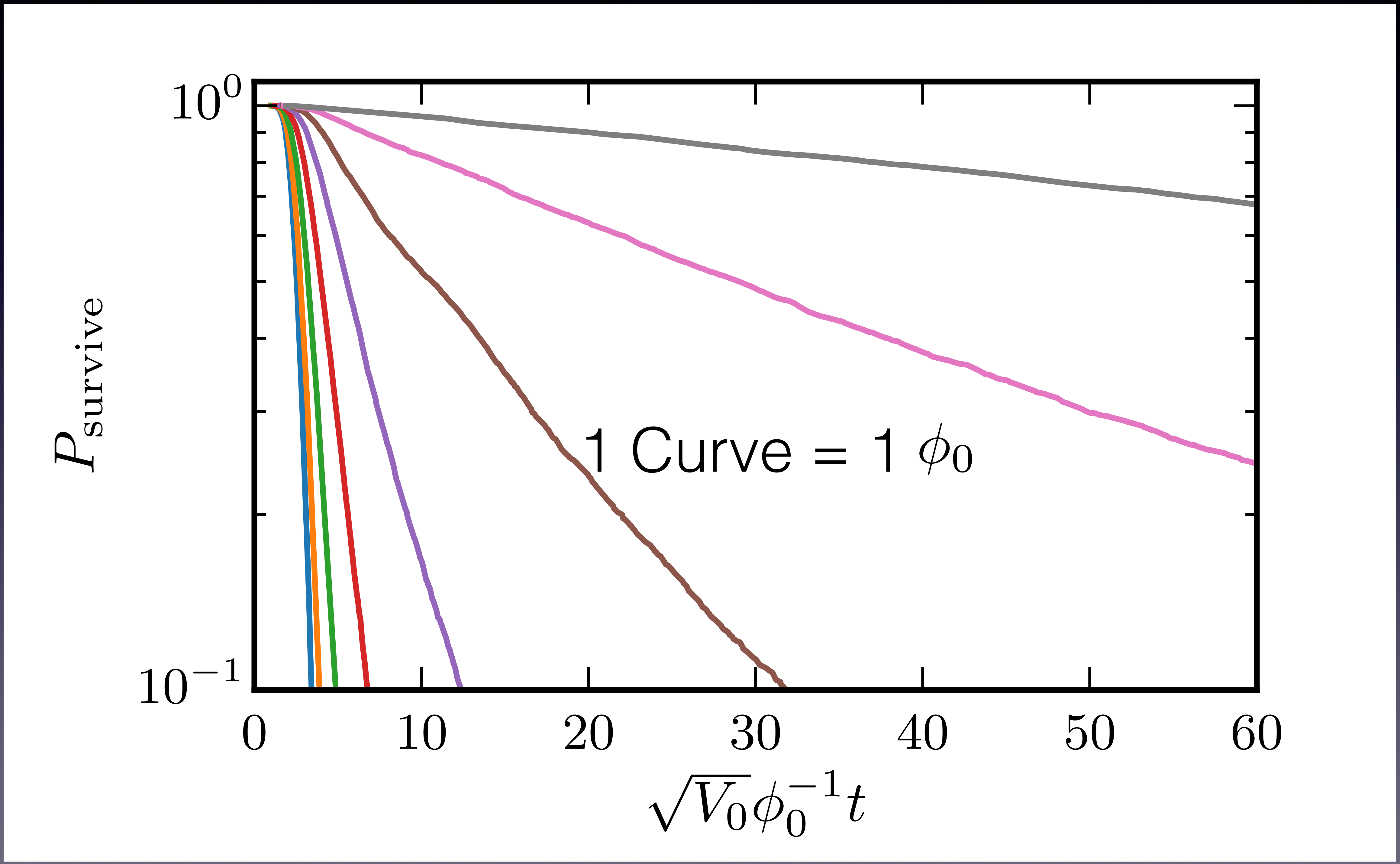
$t^{(i)}_{\mathrm{decay}}$

Not Just Peaks in Initial Field!



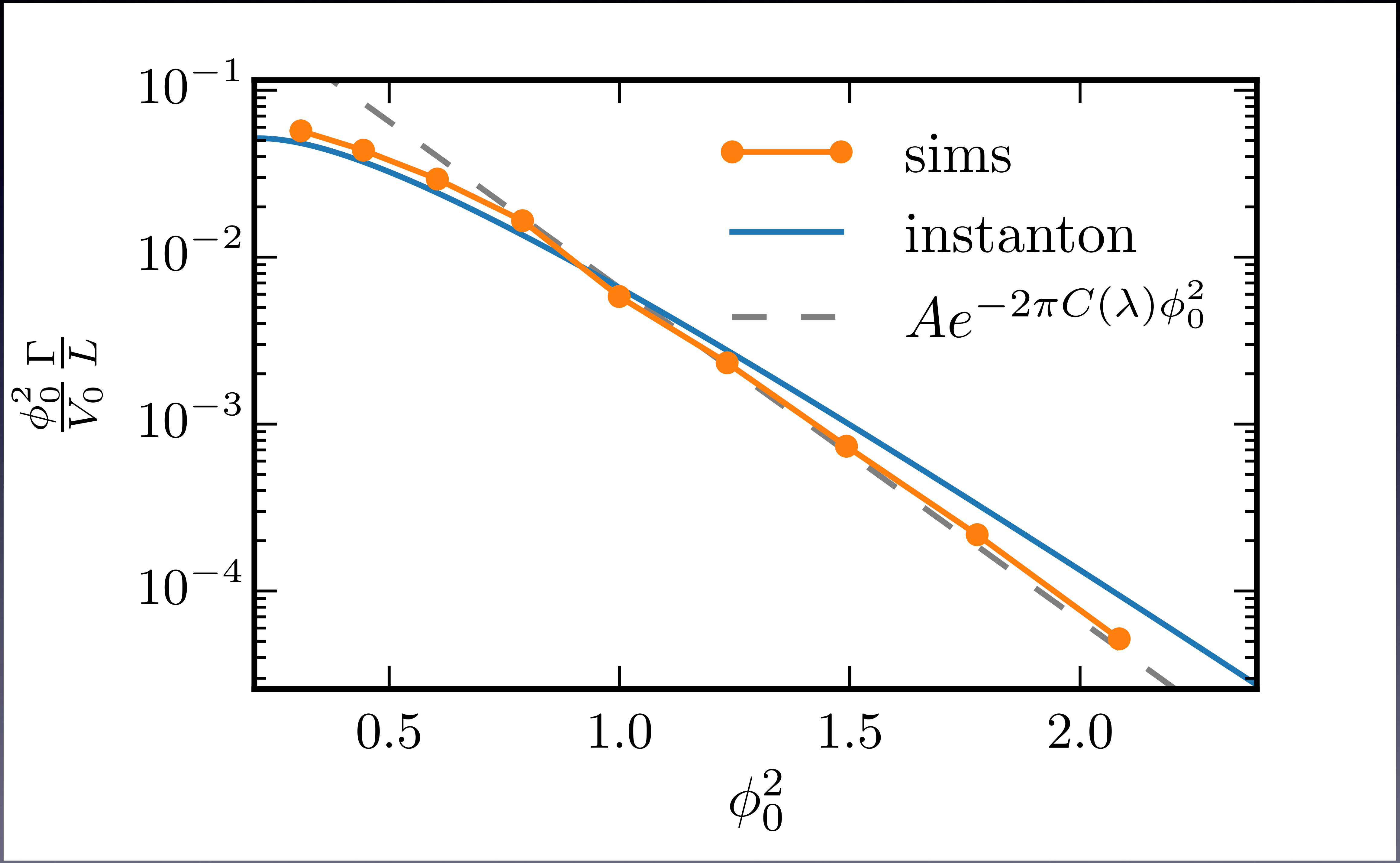
$t^{(i)}_{\text{decay}}$

$$P_{\text{survive}} \sim e^{-\Gamma(t-t_0)}$$



Sanity Check : $\Gamma \propto L$

$$\frac{\Gamma_{\mathrm{I}}^{(1+1)}}{L} = g(\lambda, \phi_0) m_{\mathrm{eff}}^2 \phi_0^2 e^{-2\pi\phi_0^2 C(\lambda)}$$



[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

First Principles Derivation of Approximation

Why Does This Work?

My Original Question

$$|\Psi\rangle$$

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$$

$$P[\phi, \Pi]$$

$$\frac{\partial\phi}{\partial t} = \frac{\delta H}{\delta\Pi}$$

$$\frac{\partial\Pi}{\partial t} = -\frac{\delta H}{\delta\phi}$$

Nonlinear, Nonperturbative, Nonequilibrium Phenomena

QFT in Phase Space

Consider the Wigner functional

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

Important Properties

$$\int \mathcal{D}\phi \mathcal{D}\Pi \, W[\phi,\Pi] = 1$$

$$\langle \hat{\mathcal{O}}(\hat{\phi},\hat{\Pi}) \rangle = \int \mathcal{D}\phi \mathcal{D}\Pi \, W(\phi,\Pi) \mathcal{O}_{\mathrm{W}}(\phi,\Pi)$$

W ~ quantum probability distribution

(caveat: Not postive definite in general,
but is for Gaussian states)

Wigner Approach

$$W[\phi, \Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$$

Wigner Approach

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$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\Pi \frac{\delta}{\delta\phi} + \nabla^2\phi \frac{\delta}{\delta\Pi} - \frac{2}{i\hbar} V(\phi) \sin\left(\overleftarrow{\nabla}_\phi \frac{i\hbar}{2} \overrightarrow{\frac{\partial}{\partial\Pi}}\right)\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Wigner Approach

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Wigner Approach

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

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Initial State (t=0)
(Uncertainty Prin.)

Wigner Approach

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Classical Evolution

Initial State (t=0)
(Uncertainty Prin.)

Wigner Approach

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^dx \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \right| \Psi \Big\rangle \Big\langle \Psi \left| \phi - \frac{\eta}{2} \right\rangle$$

$$\left[\frac{\partial}{\partial t} + \int d^dx \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x),\Pi(x);t] = 0$$

Classical Evolution

Quantum "Noise"
(Interference)

Initial State (t=0)
(Uncertainty Prin.)

Wigner Approach

$$W[\phi,\Pi] \equiv \int \mathcal{D}\eta e^{-\frac{i}{\hbar}\int d^d x \Pi(\mathbf{x})\eta(\mathbf{x})} \left\langle \phi + \frac{\eta}{2} \middle| \Psi \right\rangle \left\langle \Psi \middle| \phi - \frac{\eta}{2} \right\rangle$$

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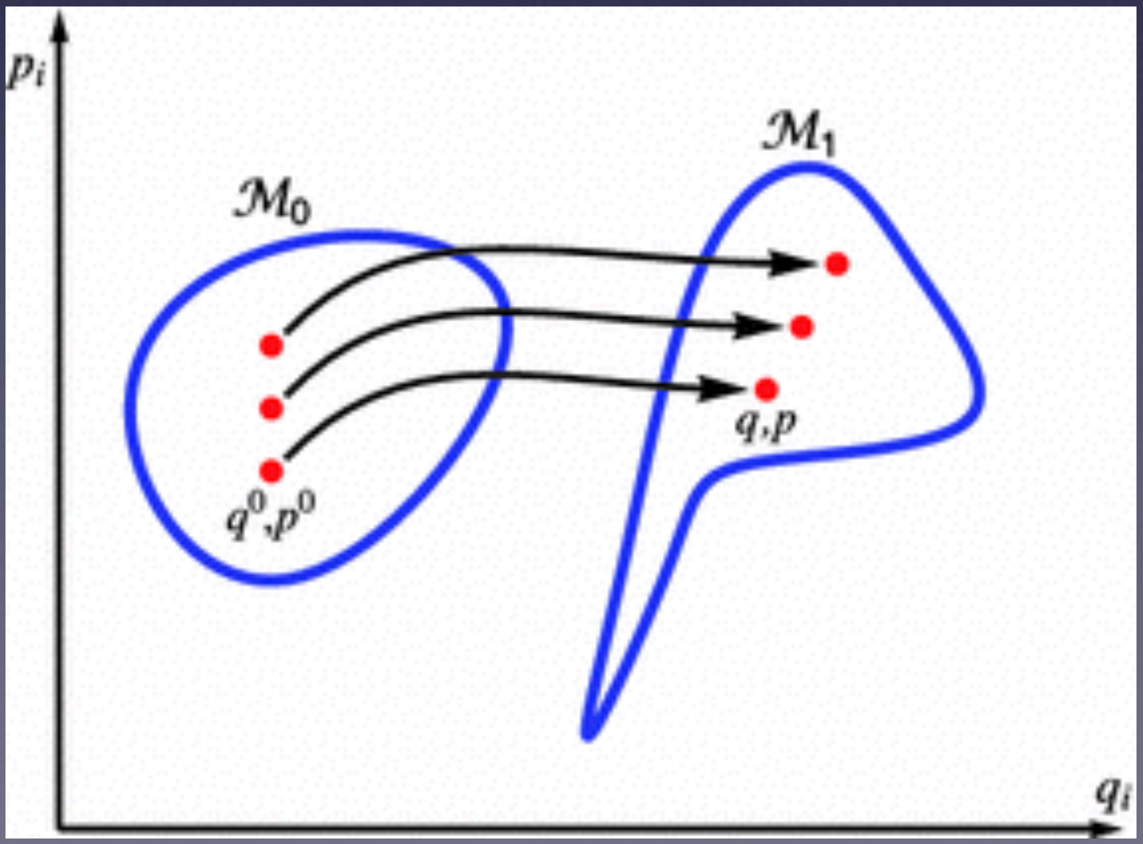
Classical Evolution

Quantum "Noise" (Interference)

Initial State (t=0) (Uncertainty Prin.)



Quantum Noise

$$(L_0 + \hbar^2 L_1)W = 0$$

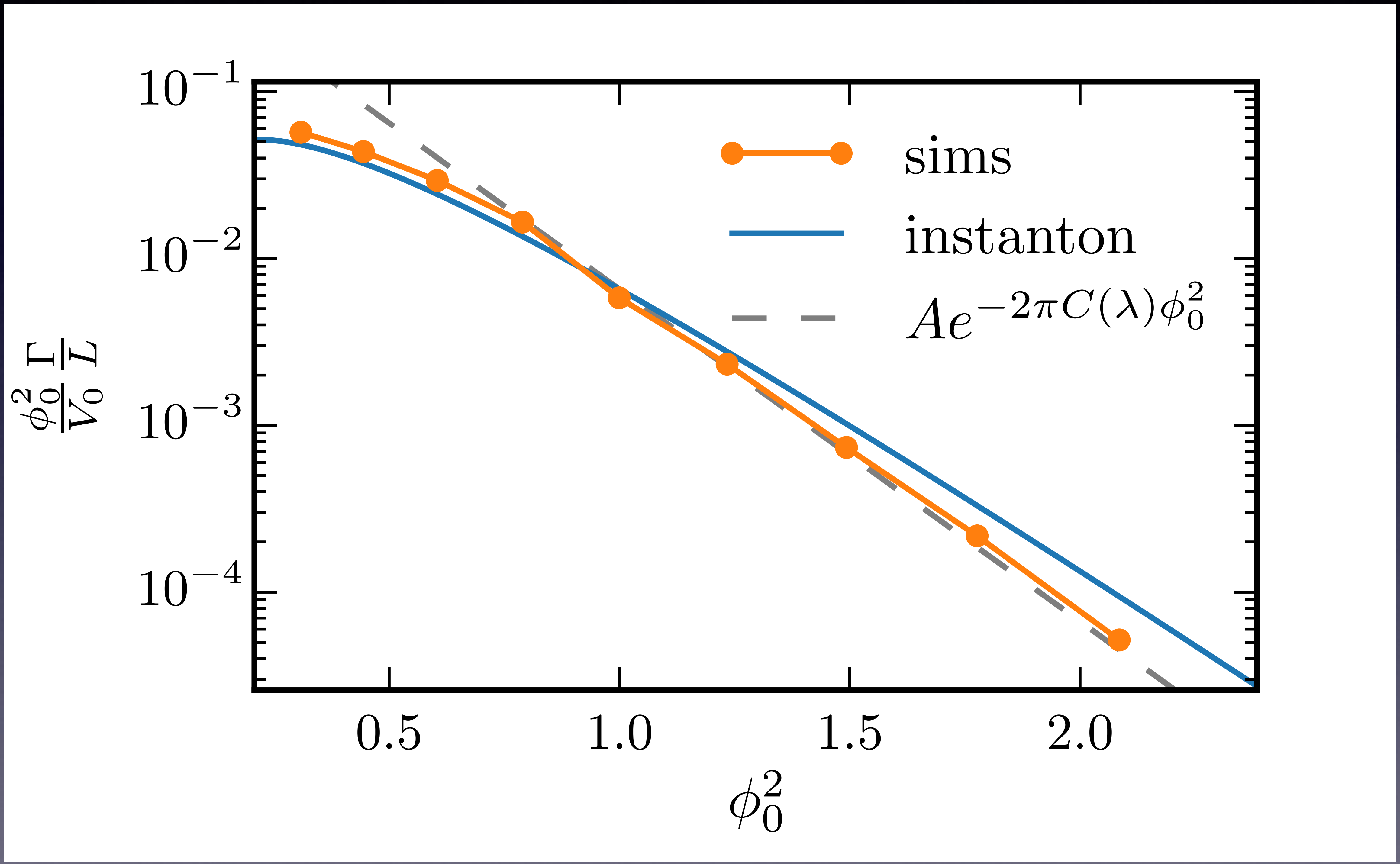
$$W = W_0 + \hbar^2 W_1$$

$$L_0 W_1 = L_1 W_0$$

Nonlinear Response

Stochastic Kick

Why The Discrepancy?



[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

$\hbar$ Expansions

$$\left[\frac{\partial}{\partial t} + \int d^d x \left(\dot{\phi}\frac{\delta}{\delta\phi} + \dot{\Pi}\frac{\delta}{\delta\Pi}\right) + \mathcal{O}\left(\hbar^2 V'''(\phi)\frac{\delta^3}{\delta\Pi^3}\right)\right] W[\phi(x), \Pi(x); t] = 0$$

Leading Order

First Correction

$$\frac{\Gamma}{V} = \left(\frac{S_{\rm I}}{2\pi}\right)^{D/2} \sqrt{\frac{\det\delta^2 S_{\rm E}[\phi_{\rm fv}]}{\det'\delta^2 S_{\rm E}[\phi_{\rm B}]}}\, e^{-S_{\rm I}} \left(1 + \mathcal{O}(\hbar)\right)$$

Fluctuation Determinant

$$\frac{\det{}'\delta^2 S(\phi_{\mathrm{B}})}{\det\delta^2 S(\phi_{\mathrm{fv}})}$$

$$\det(\delta^2 S(\phi_{\mathrm{B,fv}})) = \Pi_i \lambda_i^{B,fv}$$

$$\left[-\nabla_{\mathrm{E}}^2 + V''(\phi_{\mathrm{B,fv}})\right]\delta\phi = \lambda^{\mathrm{B,fv}}\delta\phi$$

Expand in Spherical Harmonics

$$\phi_{\mathrm{I}}(x, y, z, \tau) = \phi_{\mathrm{I}}(r_{\mathrm{E}})$$

$$\delta\phi = \sum_{\ell,\vec{m}} \delta\phi_{\ell,\vec{m}} R_\ell(r) Y_{\ell,\vec{m}}(\vec{\theta})$$

Fluctuation Determinant

$$\left[-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}\right)+\frac{\ell(\ell+d-2)}{r^2}+V''(\phi_{\mathrm{B,fv}})\right]R_\ell=\lambda R_\ell$$

$\ell = 0$ 1 negative mode (instability)

$\ell = 1$ d+1 zero modes (spacetime translations)

$$\ln\left(\frac{\delta^2 S(\phi_{\mathrm{B}})}{\delta^2 S(\phi_{\mathrm{fv}})}\right)=\Gamma_{(\ell=0)}+\Gamma_{(\ell=1)}+\sum_{\ell=2}^{\infty}g_\ell \ln\Gamma_{(\ell)}$$

Gelfand-Yaglom Theorem

$$\hat{L}f = \left[\frac{d}{dx}\left(P(x)\frac{d}{dx}\right) + Q(x)\right] f = \lambda f$$

$$f(0) = f(L) = 0$$

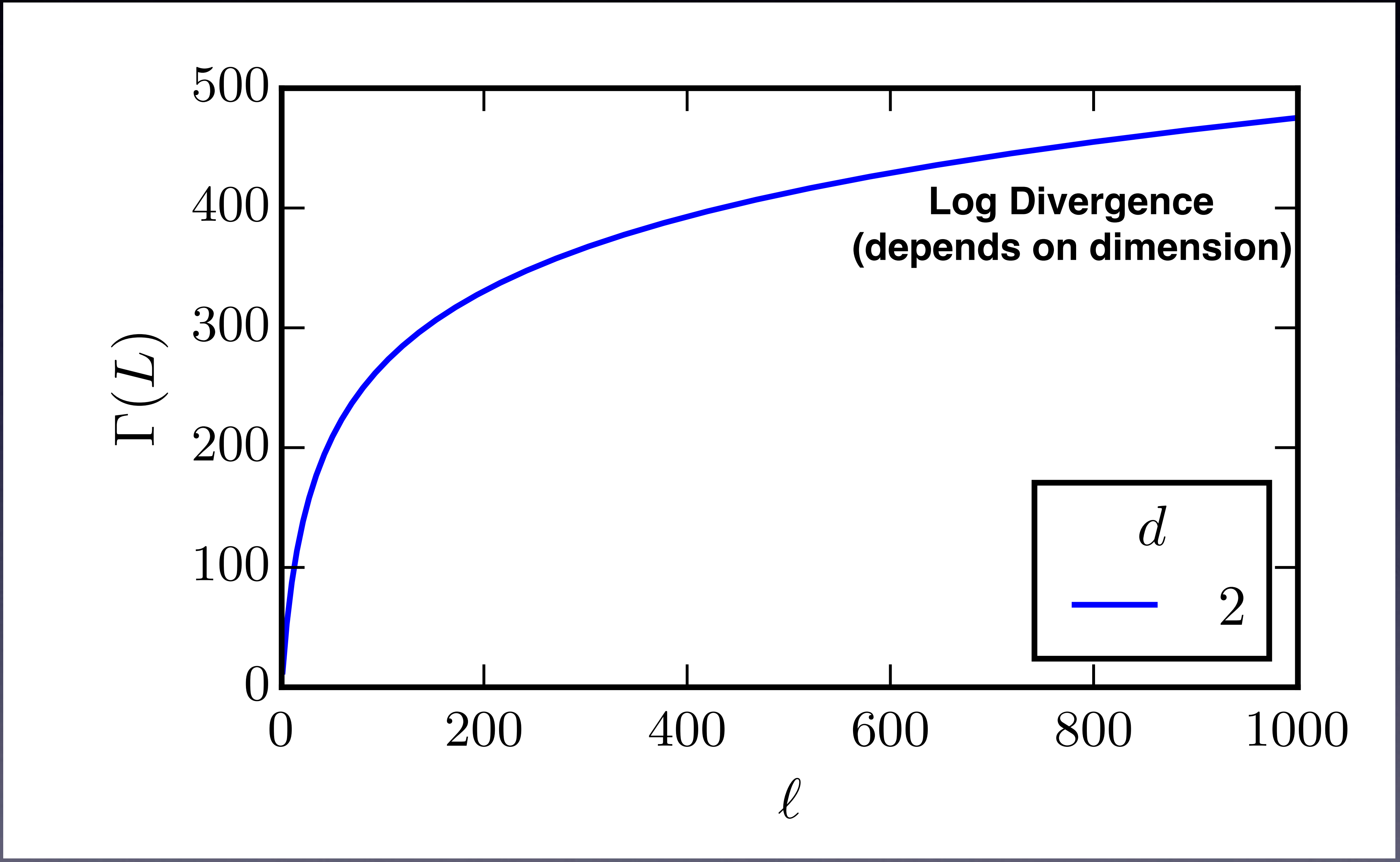
We can compute the determinant as

$$\det\left(\frac{\hat{L}}{\hat{L}_0}\right) = \frac{g(L)}{g_0(L)}$$

Where g satisfies the initial value problem

$$\hat{L}g = 0 \qquad g(0) = 0, \;\; g'(0) = 1$$

Fluctuations and Decay



Divergences appear that we must renormalise

Renormalization

Standard 1PI Effective Potential

$$V_{\mathrm{eff}}^{\mathrm{1PI}}(\bar{\phi}) = V(\bar{\phi}) + \frac{1}{2}\int \frac{\mathrm{d}^{d+1}k}{(2\pi)^{d+1}} \ln\left(\frac{V''(\bar{\phi}) + k_{\mathrm{E}}^2}{V''(\bar{\phi}_{\mathrm{fv}}) + k_{\mathrm{E}}^2}\right) + \dots$$

(Implicit) Assumptions

- Homogeneous background:
- Linear fluctuations
- Vacuum fluctuation statistics

Lattice Effective Potential

$$V_{\text{eff}}^{\text{lat}} \equiv \langle \rho \rangle = V(\bar{\phi}) + \frac{1}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^3} \sqrt{k^2 + V''(\bar{\phi})} + \mathcal{O}\langle \delta\phi^3 \rangle \,.$$

Lattice Effective Potential

$$V_{\rm eff}^{\rm lat} \equiv \langle \rho \rangle = V(\bar\phi) + \frac{1}{2}\int \frac{{\rm d}^d k}{(2\pi)^3}\sqrt{k^2 + V''(\bar\phi)} + \mathcal{O}\langle \delta\phi^3 \rangle \,.$$

$$\omega = \int d\omega^2 \frac{1}{2\omega} = \int_{-\infty}^{\infty} \frac{{\rm d}k_4}{2\pi} \int \frac{{\rm d}\omega^2}{\omega^2 + k_4^2} = \int \frac{dk_4}{2\pi} \ln(\omega_k^2 + k_4^2)$$

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$$V_{\text{eff}}^{\text{lat}} \equiv \langle \rho \rangle = V(\bar{\phi}) + \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^3}\sqrt{k^2 + V''(\bar{\phi})} + \mathcal{O}\langle \delta\phi^3 \rangle \,.$$

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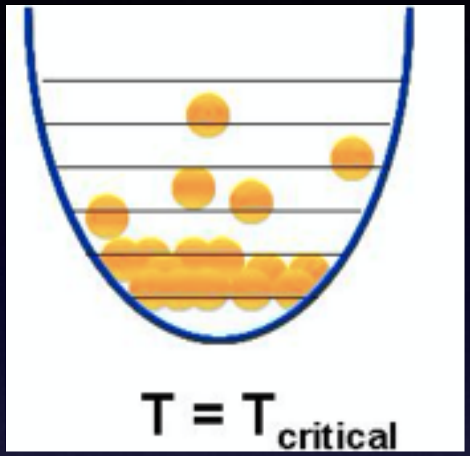
Also holds dynamically

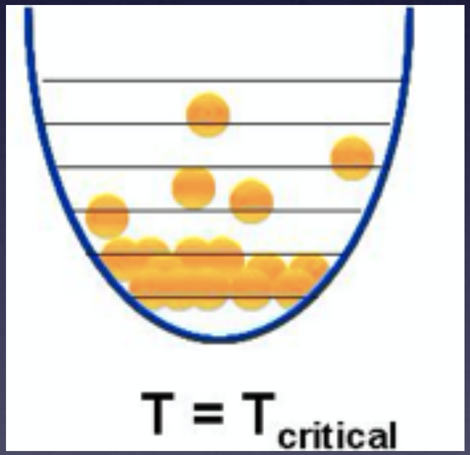
$$\ddot{\bar{\phi}} = -\langle V'(\bar{\phi} + \delta\phi)\rangle = -\frac{\partial V_{\text{eff}}(\bar{\phi})}{\partial\bar{\phi}}$$

Is This Testable?

Analog Cold Atom BEC

[JB, Johnson, Peiris, Weinfurtner, 1712.02356]





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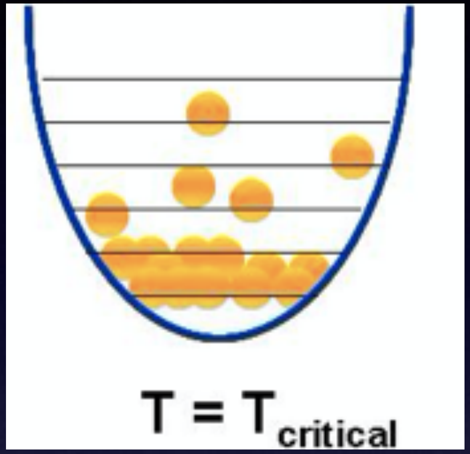

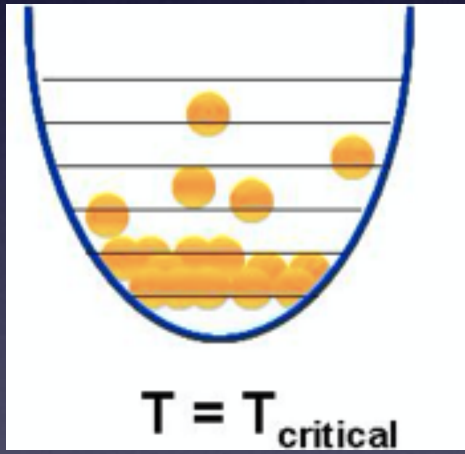

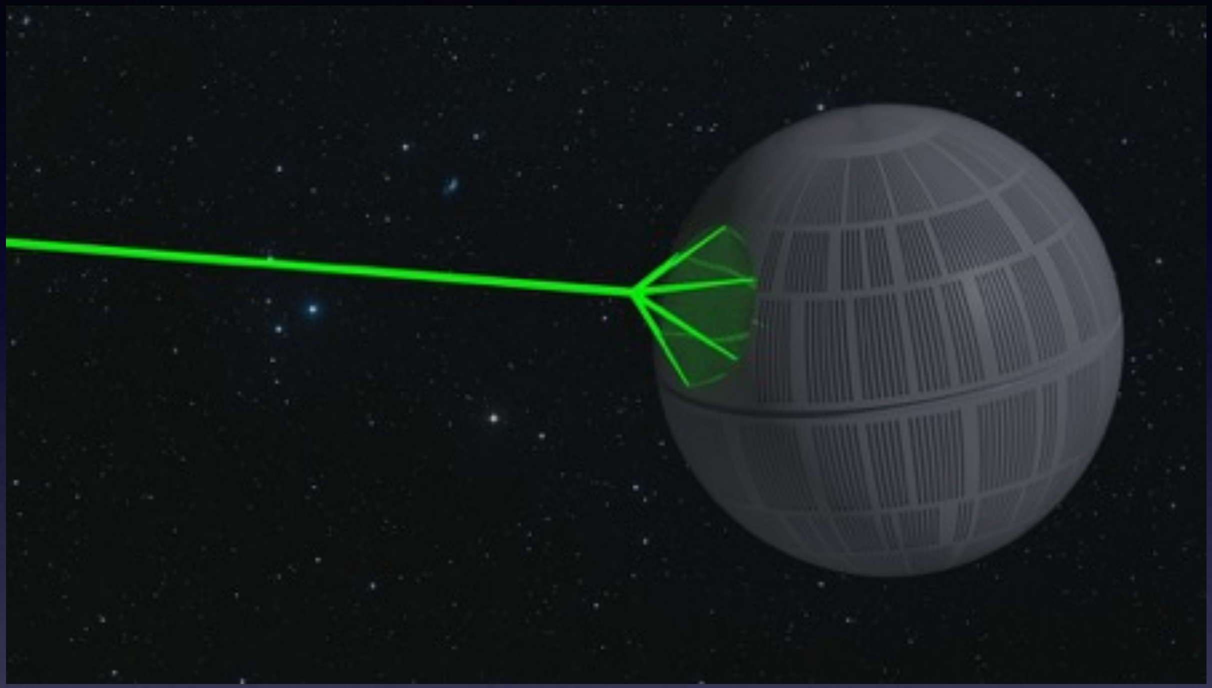

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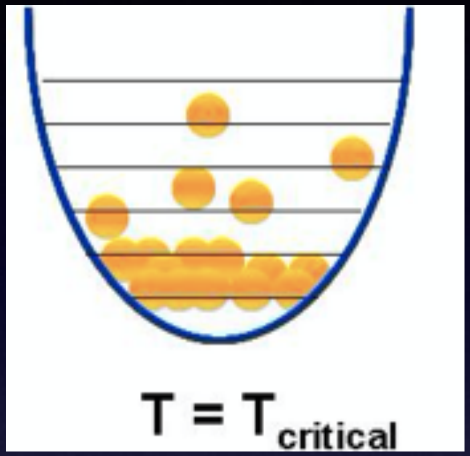
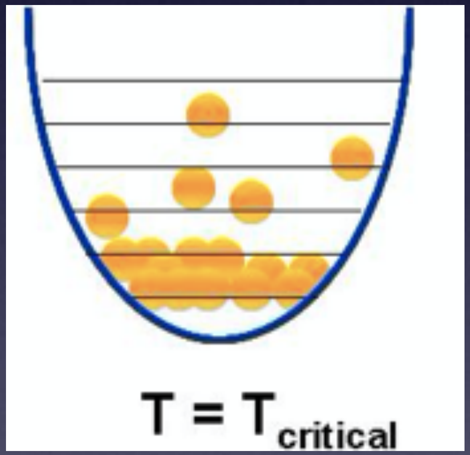
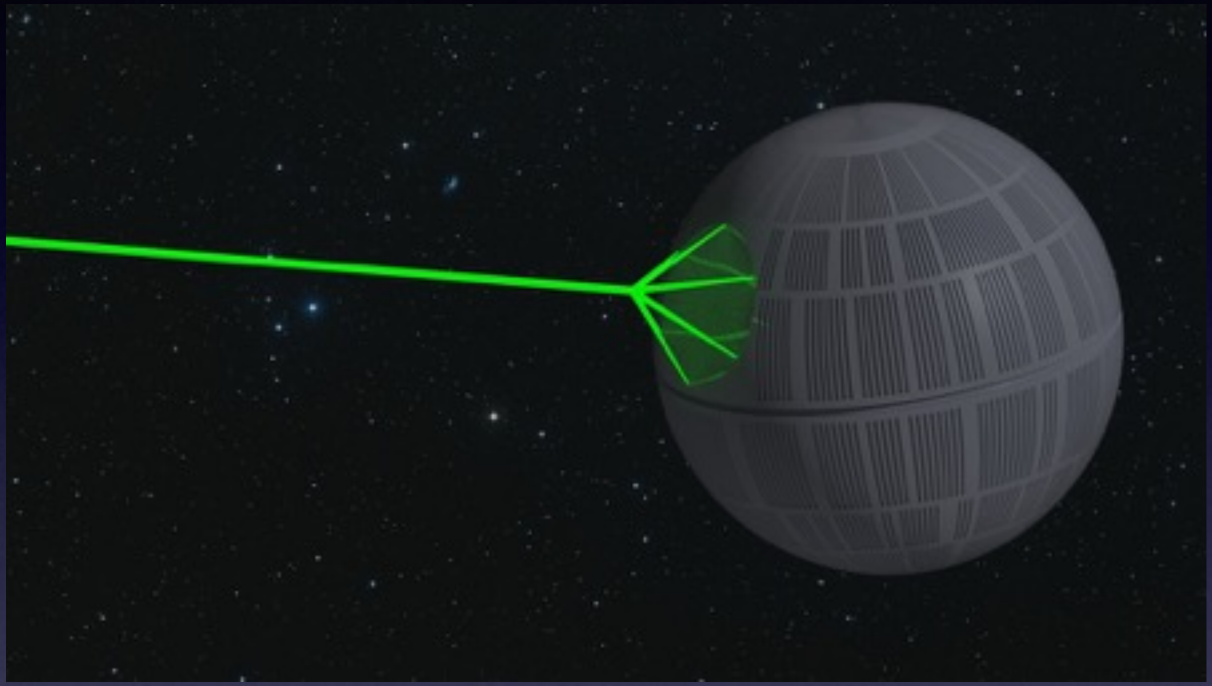
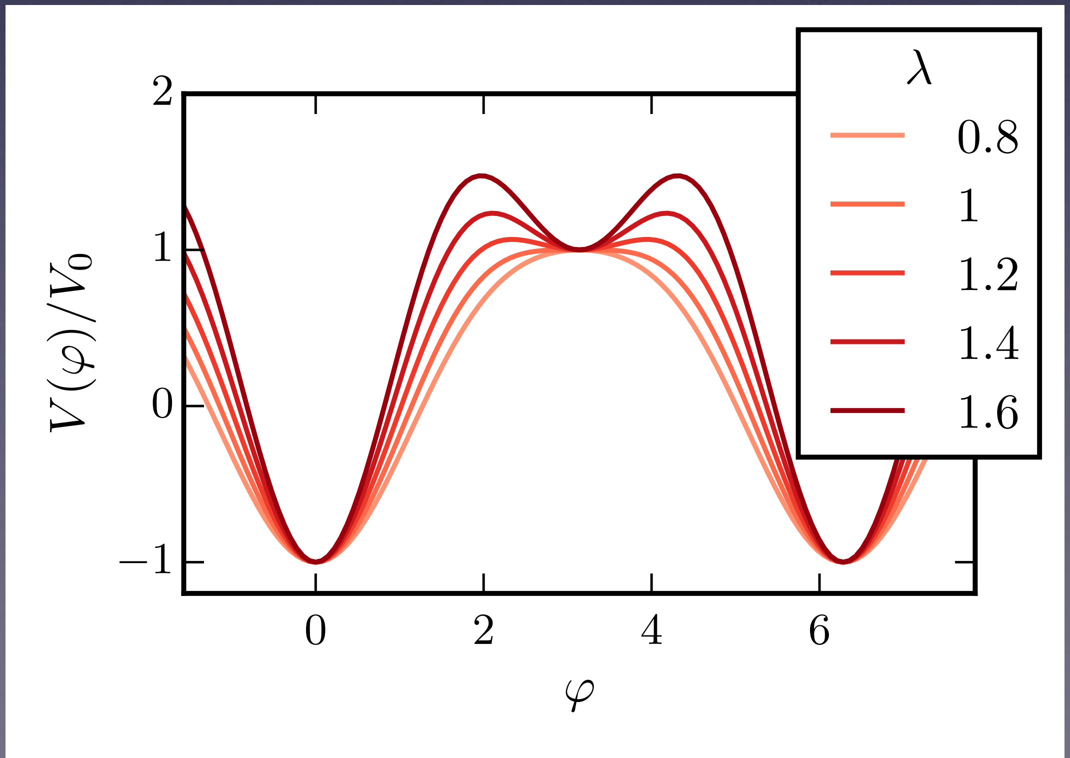






Dynamics of relative phase
is a relativistic field
with periodic potential



Modelling BEC Dynamics

$$\hat{\Psi}_i = \psi_i + \Delta\hat{\psi}_i$$

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(\mathbf{x}) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

KE PE S-wave Propagator

$$\mathcal{H} = \frac{\hbar^2}{2m_i}|\nabla\psi_i|^2 + V(x)|\psi_i|^2 + \frac{g_{ij}}{2}|\psi_i|^2|\psi_j|^2 + \frac{\nu_{ij}}{2}\left(\psi_i\psi_j^* + \psi_j\psi_i^*\right)$$

BECs and Relativity

[JB,Johnson, Peiris, Weinfurtner, 1712.02356]

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum — Can. Position

$$\mathcal{H} = \frac{\hbar^2}{8m_i}\rho_i(\nabla \ln \rho_i)^2 + \frac{\hbar^2}{2m_i}\rho_i(\nabla\phi_i)^2 + \frac{g_{ij}}{2}\rho_i\rho_j - \nu_{ij}\sqrt{\rho_i\rho_j}\cos(\phi_j - \phi_i)$$

Assumptions

$$\rho_i(x,t) = n_i + \delta\rho_i(x,t)$$

1) Homogeneous

2) Small

Modelling BEC Dynamics

$$i\hbar\dot{\psi}_1 = \left(-\frac{\hbar^2}{2m_1}\nabla^2 + g_{11}|\psi_1|^2 + g_c|\psi_2|^2\right)\psi_1 - \nu\psi_2$$

$$i\hbar\dot{\psi}_2 = \left(-\frac{\hbar^2}{2m_2}\nabla^2 + g_{22}|\psi_2|^2 + g_c|\psi_1|^2\right)\psi_2 - \nu\psi_1$$

$$g_{11} = g + \frac{\delta g}{2} \qquad g_{22} = g - \frac{\delta g}{2}$$

Simplified Case

$$m_1 = m_1 = m \qquad \delta g = 0$$

Small Density Fluctuations

$$Z = \int d\psi_i d\psi_i^* e^{i \int \mathcal{L}}$$

$$\alpha = \frac{\phi_1 + \phi_2}{2} \qquad \phi = \phi_2 - \phi_1 \qquad \delta\rho = \delta\rho_1 + \delta\rho_2 \qquad \Delta\rho = \frac{\rho_2 - \rho_1}{2}$$

$$\mathcal{L} = -\frac{\hbar^2 n}{m}(\nabla\alpha)^2 - \frac{\hbar^2 n}{4m}(\nabla\phi)^2 + 2\nu n \cos\phi$$

$$+ (\dot{\alpha} + J_\delta)\,\delta\rho + \left(\frac{\dot{\phi}}{2} + J_\Delta\right)\Delta\rho$$

$$+ \frac{1}{2}\begin{pmatrix} \delta\rho & \Delta\rho \end{pmatrix} C_{\delta\rho}^{-1} \begin{pmatrix} \delta\rho & \Delta\rho \end{pmatrix}^T + \mathcal{O}(\delta\rho^3, \Delta\rho^3)$$

Limit of Small Number Fluctuations

A convenient variable is

$$\phi = \phi_2 - \phi_1$$

Integrate out fluctuations in number density

$$Z_{\text{eff}} \propto \int d\phi e^{i\mathcal{L}_{\text{eff}}}$$

$$\mathcal{L}_{\text{eff}} \sim G(\phi)\frac{\dot{\phi}^2}{2} - c_s^2\frac{(\nabla\phi)^2}{2} + \nu\Lambda\cos\phi + \ldots$$

Limit of Small Number Fluctuatio

Sine-Gordon Model

A convenient variable is

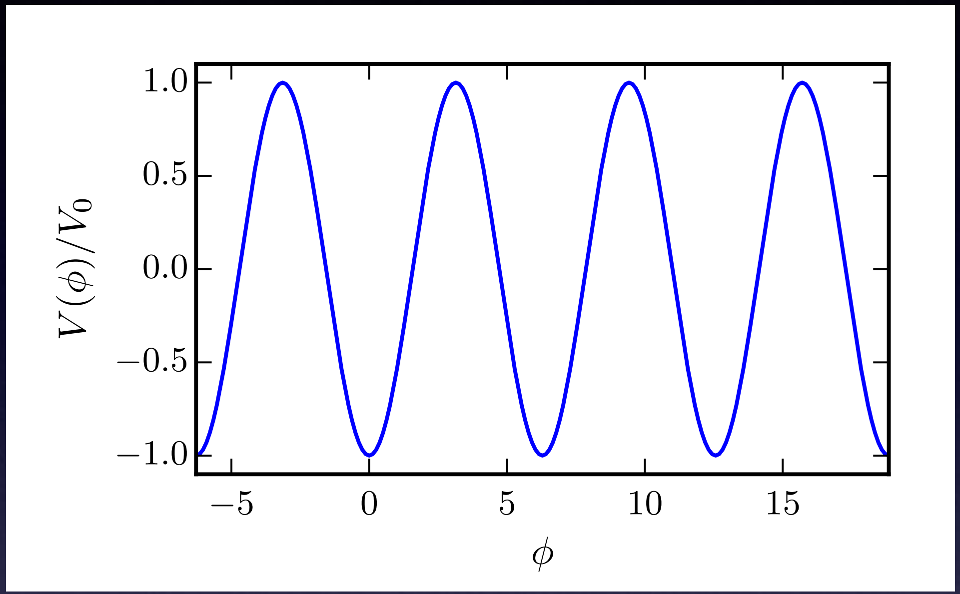
$$\phi = \quad \phi_1$$

Integrate out flu in number density

$$\propto \int d\phi e^{i\mathcal{L}_{\mathrm{eff}}}$$

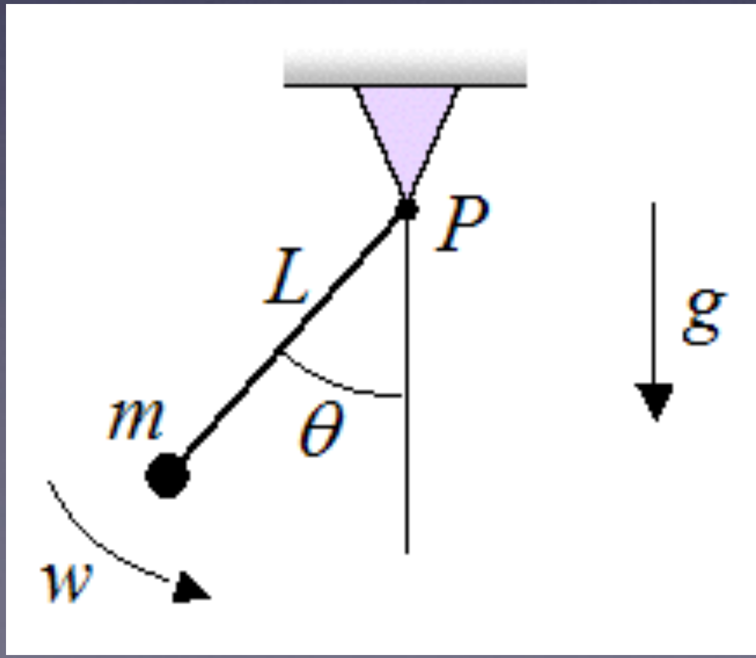
$$\mathcal{L}_{\mathrm{eff}} \quad G(\phi)\frac{\dot{\phi}^2}{2} - c_s^2\frac{(\nabla\phi)^2}{2} + \nu\Lambda\cos\phi + \dots$$

SG and the Pendulum



Where's My False Vacuum!

Homogeneous Limit:
Rigid Pendulum



Modulate Transition Rate

$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$

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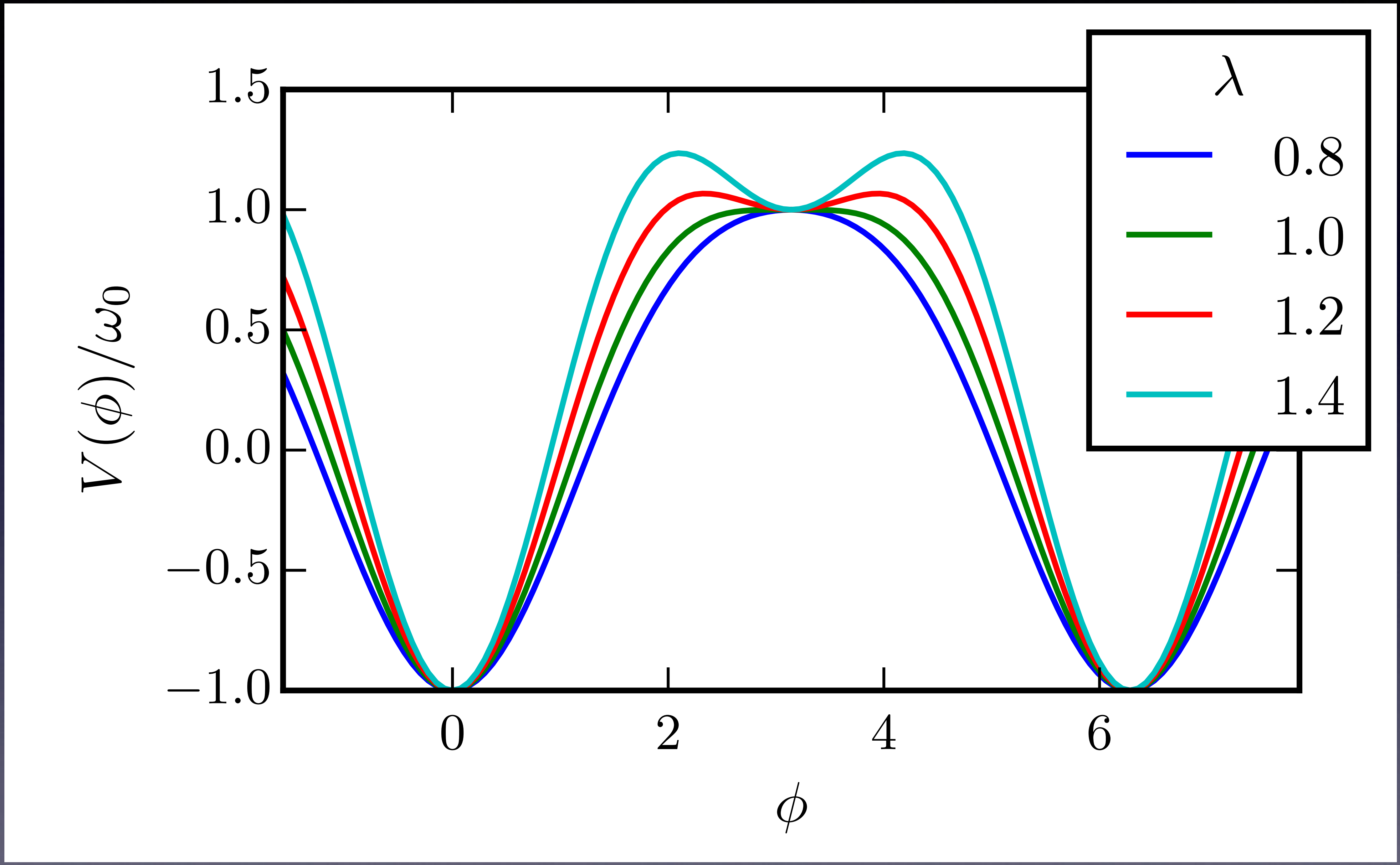
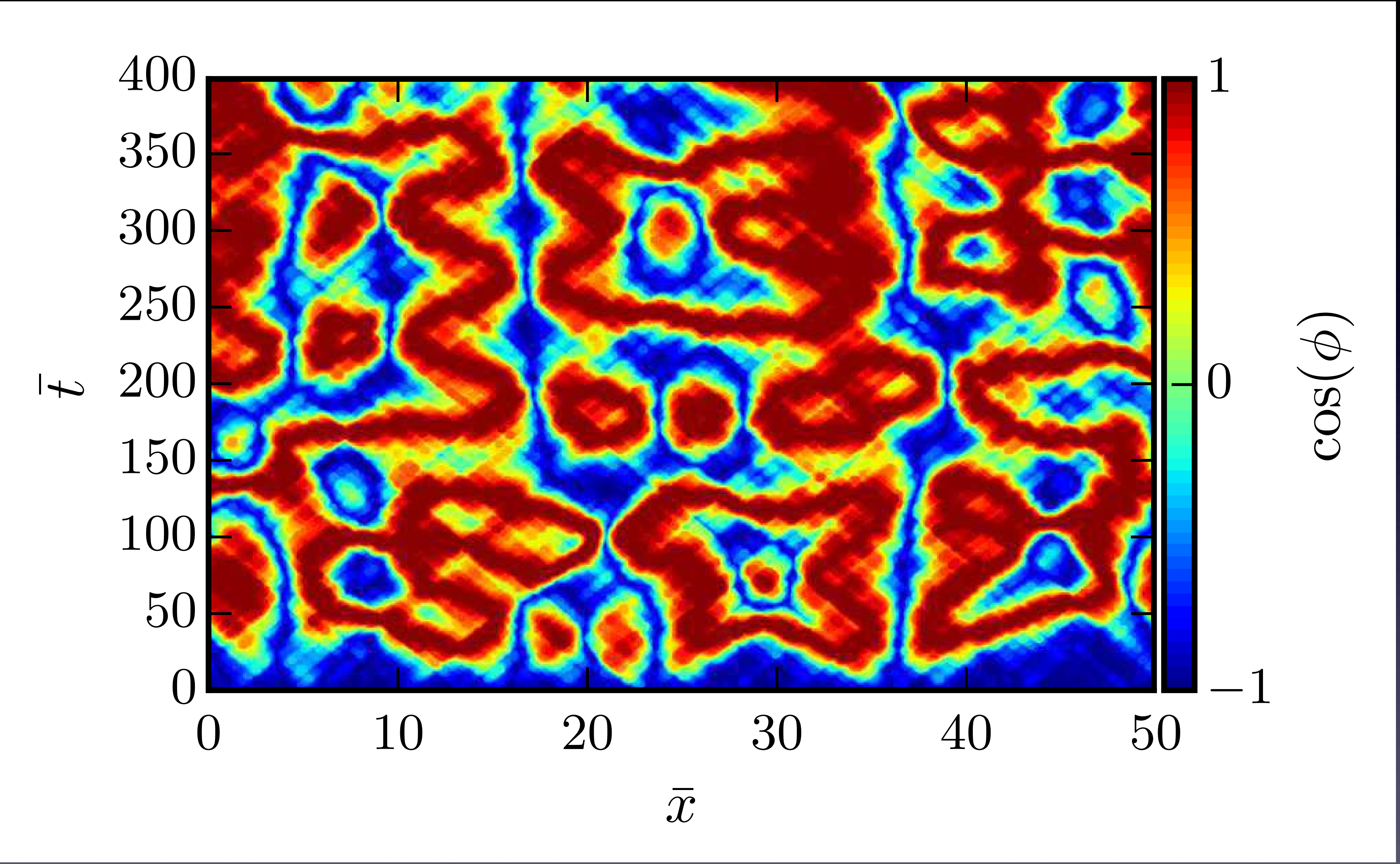
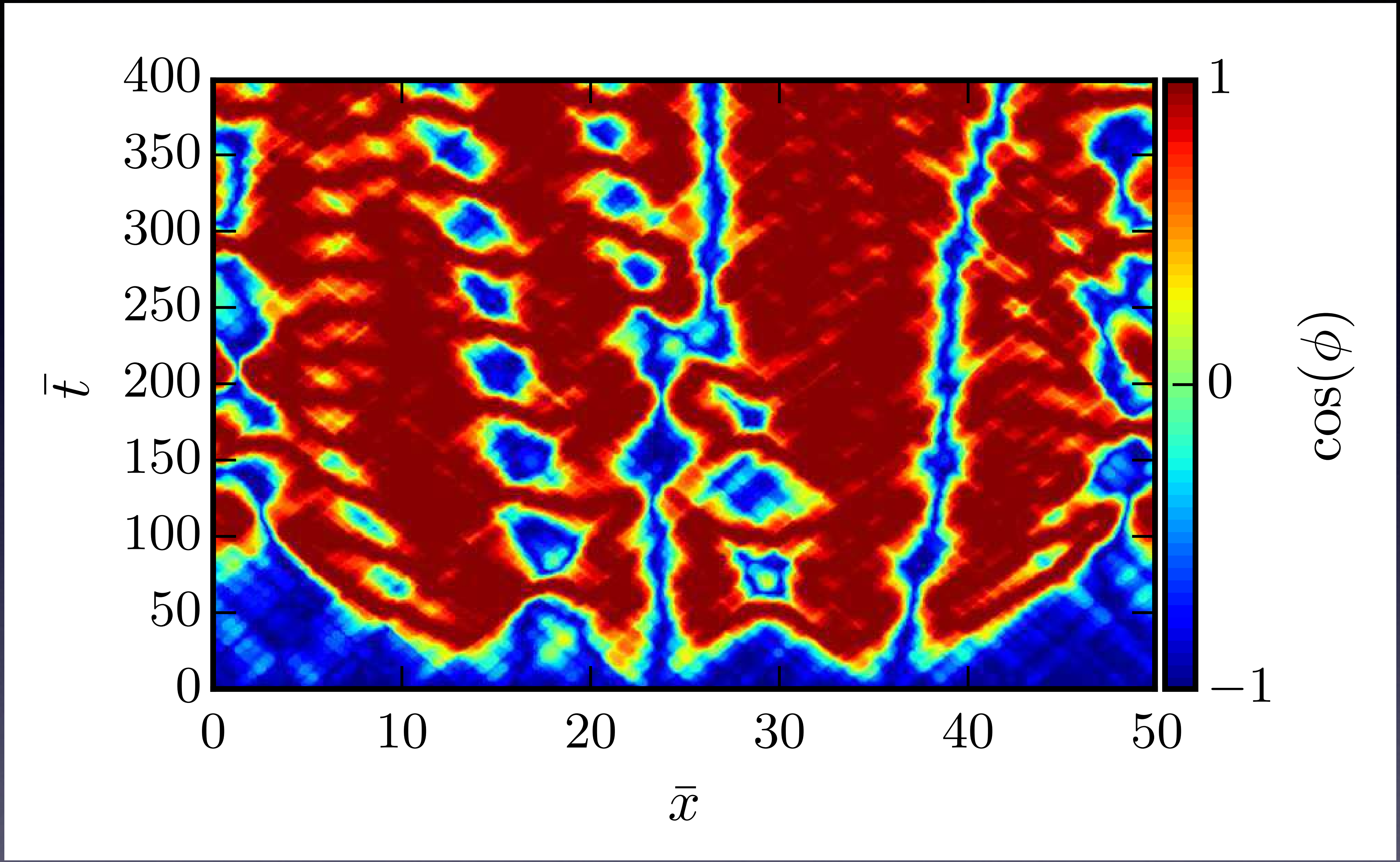
Time Averaged Potential



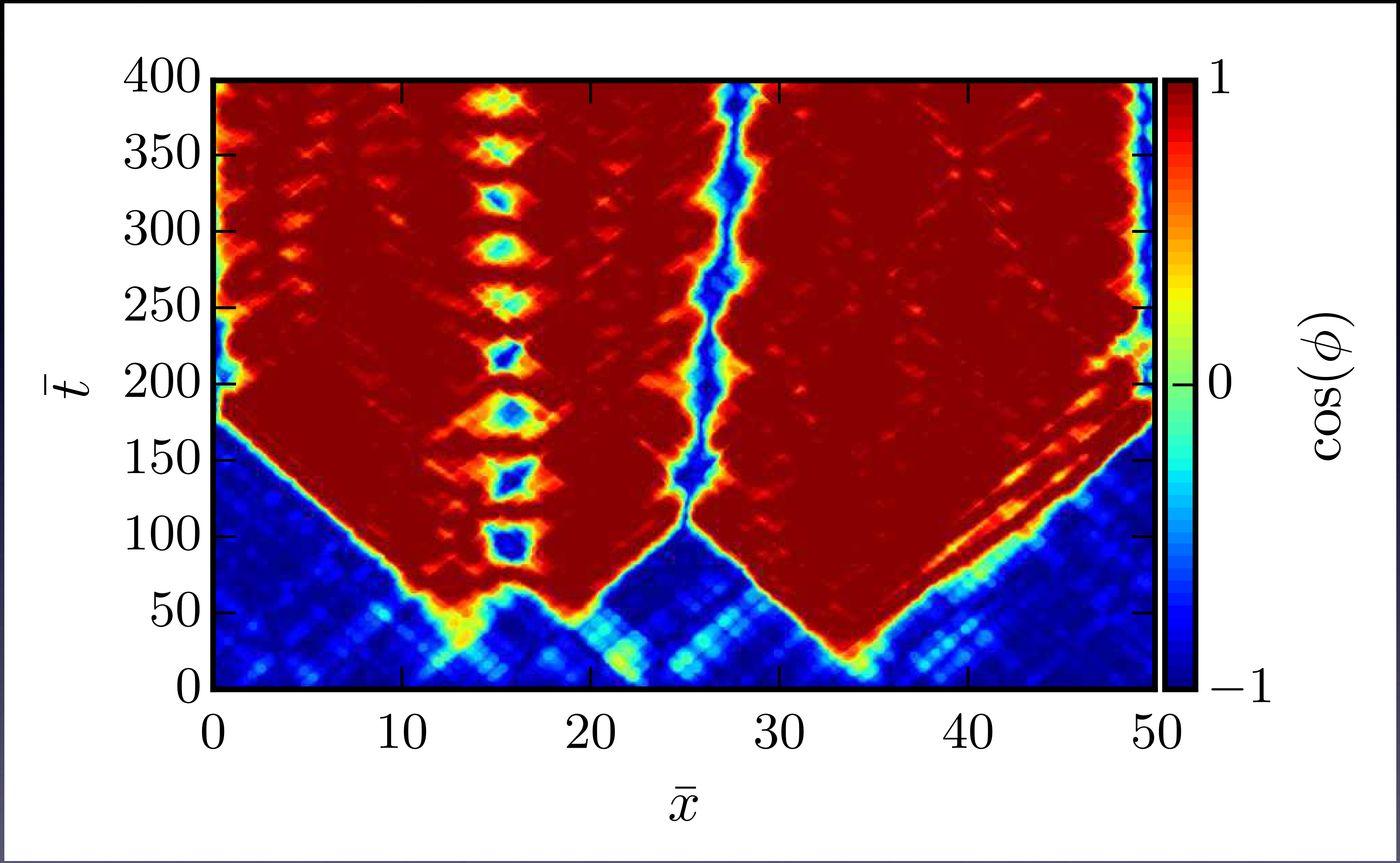
$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$



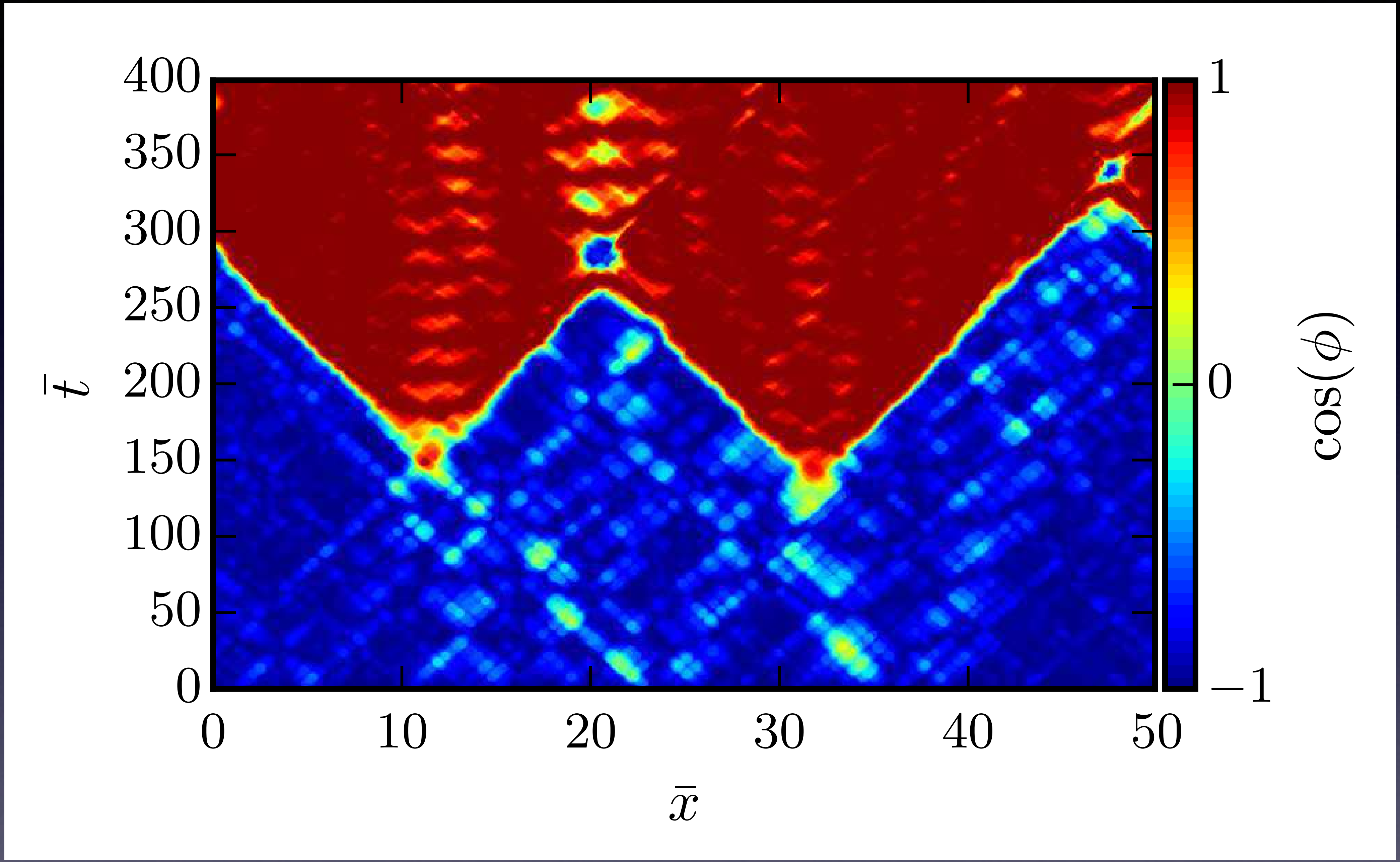
Spinodal Instability



Transition Regime



Rapid Nucleation



Slower Nucleation

Conclusions

Physical Process: False Vacuum Decay

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- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)

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Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
    - Alternative description of instanton (no tunnelling)
    - Complimentary to instanton (Euclidean rate wrong)

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
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  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)
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Current/Future Work

- Real-time ↔ Instanton
  - Renormalisation, Fluc. Determinant, Wigner
- Mean bubble profile = instanton?
- Bubble-bubble correlations?
- Time-dependent background or potential
- Non-vacuum initial states (pure or mixed)
- Application to many fields
- Testability in BEC experiments?

THANK YOU