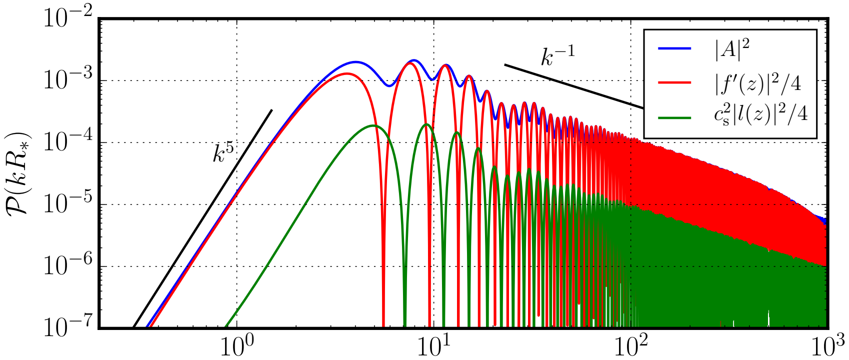
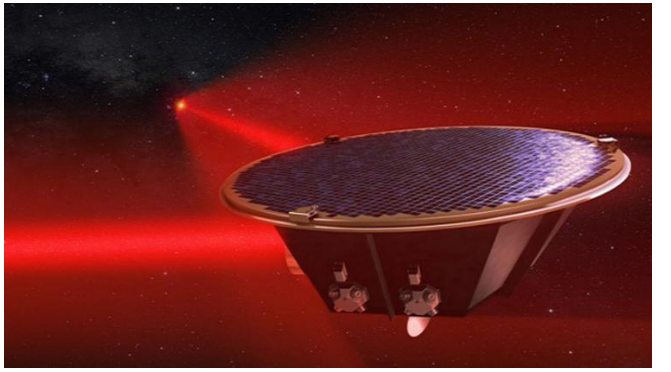




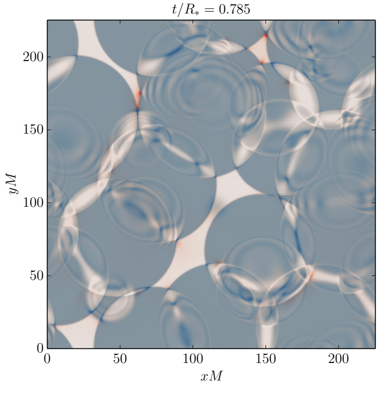
Gravitational waves from the sound of a phase transition

Mark Hindmarsh

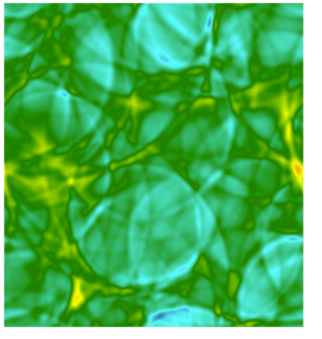
Helsinki Institute of Physics & Dept of Physics, University of Helsinki

and

Dept of Physics and Astronomy, University of Sussex

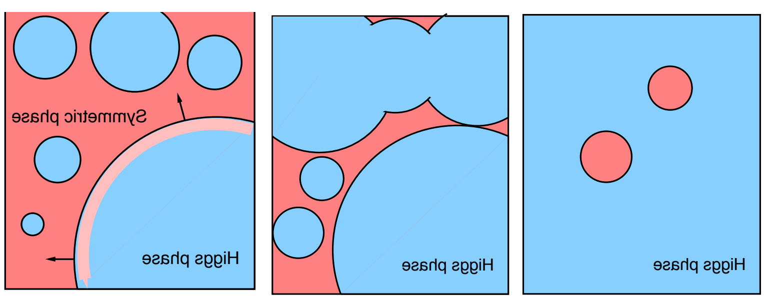


Nordita
18. syyskuuta 2019



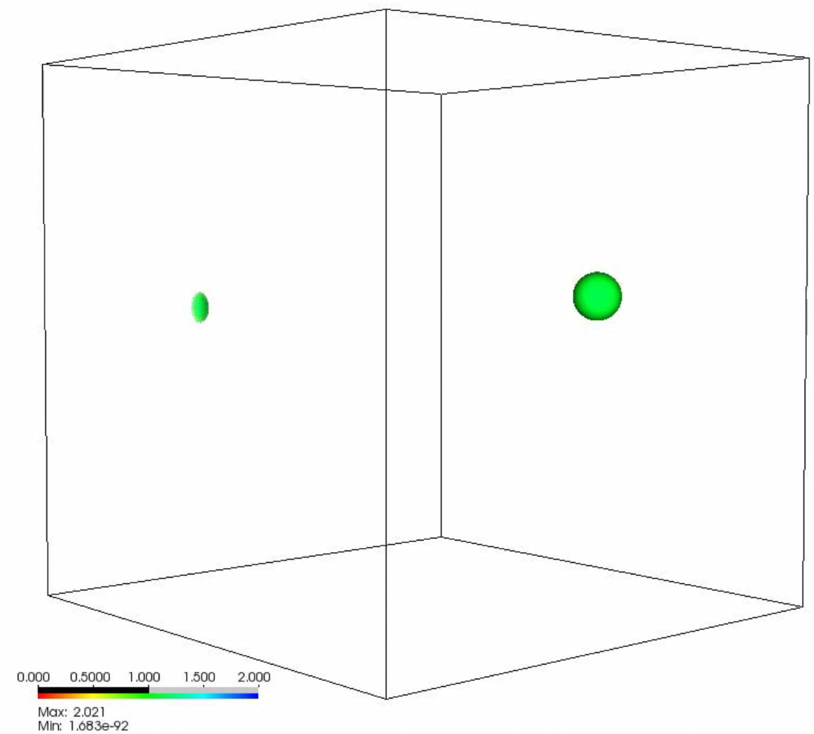
Little bangs in the Big Bang

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Detectable gravitational waves



Steinhardt (1982); Gyulassy et al (1984); Witten (1984); Enqvist et al (1992);

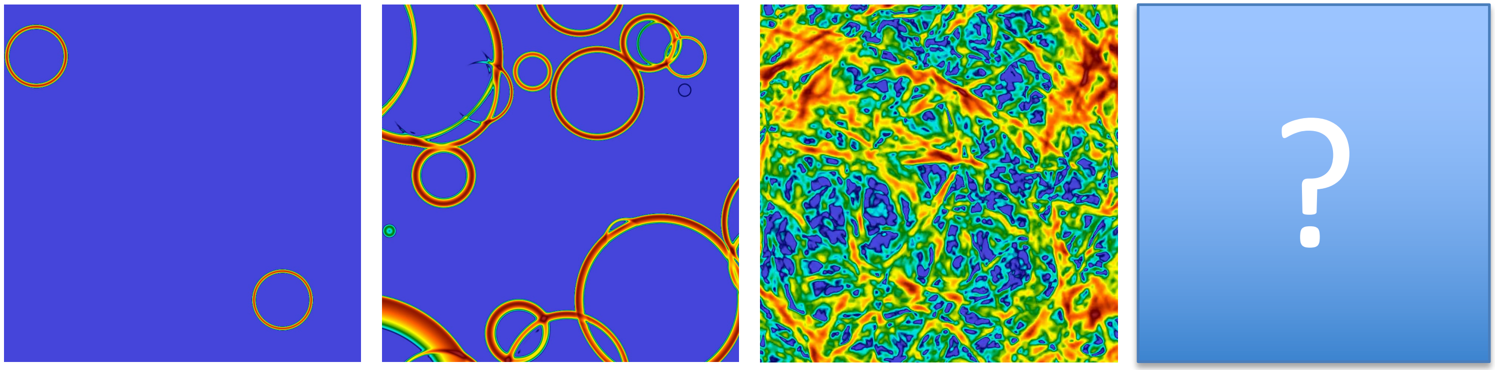
Scalar field $\phi(\mathbf{x}, t)$



David Weir, Helsinki University

MH, Huber, Rummukainen, Weir (2013,5,7)
Scalar only: Child, Giblin (2012);
Cutting, MH, Weir (2018)

Phases of a phase transition



1 2 3 4

1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} \sim L_f/\bar{U}_f$$

$L_{\rm f}$ – fluid flow length scale
$U_{\rm f}$ – RMS fluid velocity

'exponential' nucleation

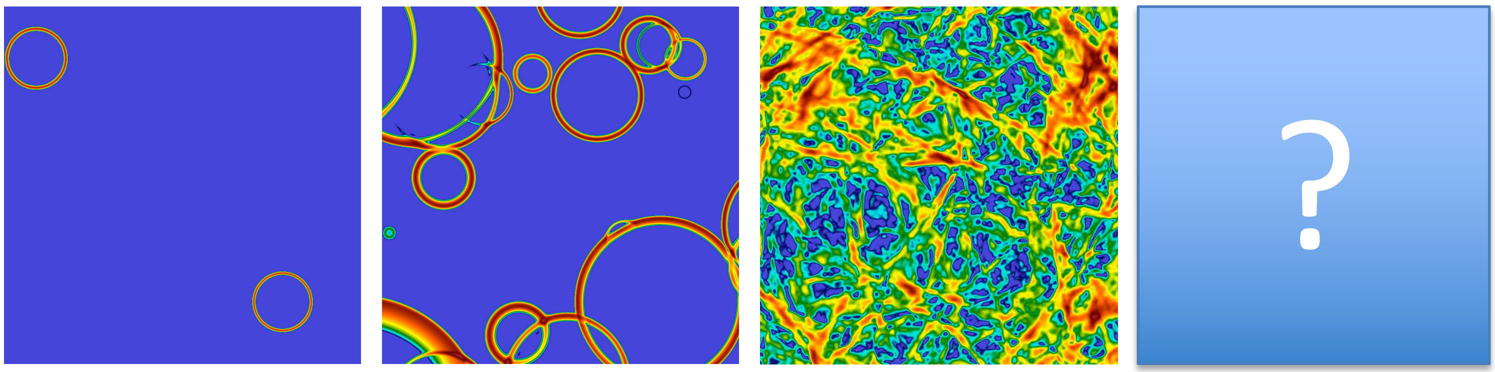
$$p(t) = p_n e^{\beta(t-t_n)}$$

$$\tau_{\rm co} = \beta^{-1}$$

Guth, Weinberg 1983; Enqvist et al 1992; Turner, Weinberg, Widrow 1992;

p – nucleation rate/volume
β – transition rate parameter

Phases of a phase transition



1 2 3 4

Not discussed here

- Generation of magnetic field?

  Baym, Bodeker, McLerran, 1996

  Vachaspati 2001

  Ferrer, Pogosian, Vachaspati 2019

- MHD turbulence

  Brandenburg et al 2017

  Roper Pol et al 2019

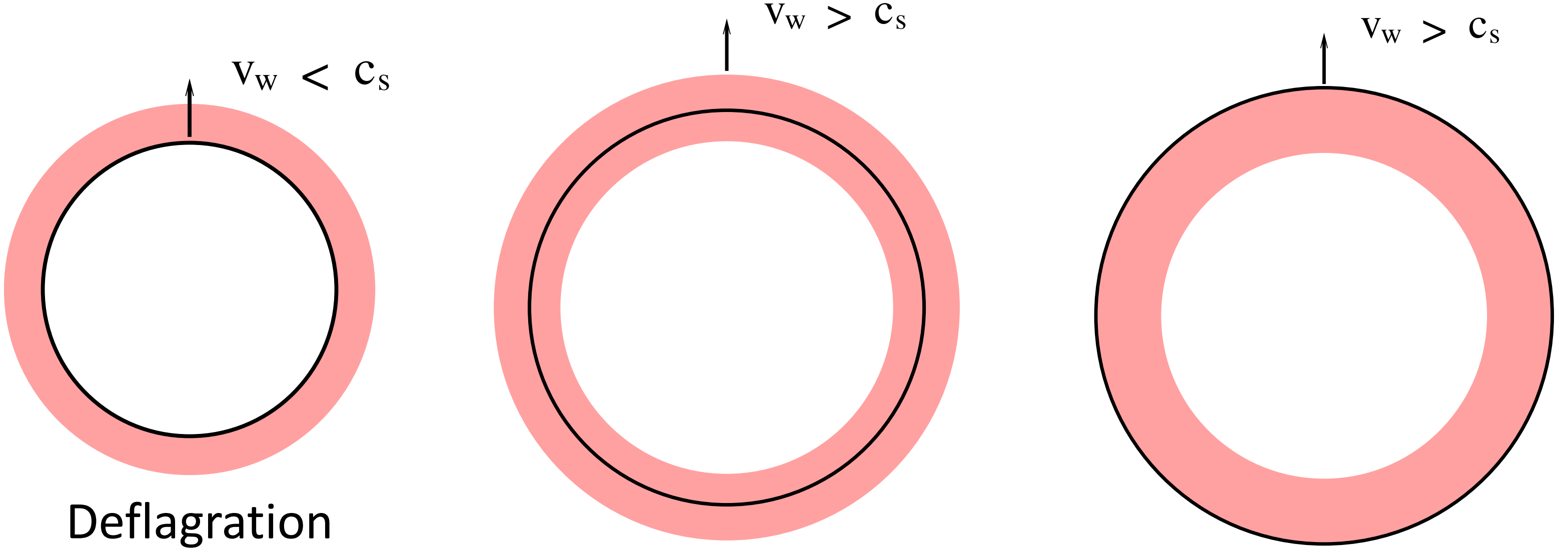
1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm dy} \sim L_f/\bar{v}_\perp$$

$L_{\rm f}$ – fluid flow length scale

$\bar{V}_\perp$ – RMS rotational velocity

Relativistic combustion



Deflagration

Supersonic deflagration ("hybrid")

Detonation

Landau & Lifshitz
Steinhardt (1984)
Kurki-Suonio, Laine (1991)
Espinosa et al (2010)

- Scalar potential energy (trace anomaly θ) to kinetic energy, heat energy
- Wall velocity v_w from pressure difference and Higgs-plasma coupling $\eta(\phi)$
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$ from v_w and $\Delta\theta$
- Similarity solution $v(r/t)$, $w(r/t)$
- Runaway ($v_w \to 1$) (near-vacuum transition)

Bodeker Moore 2010, 2017

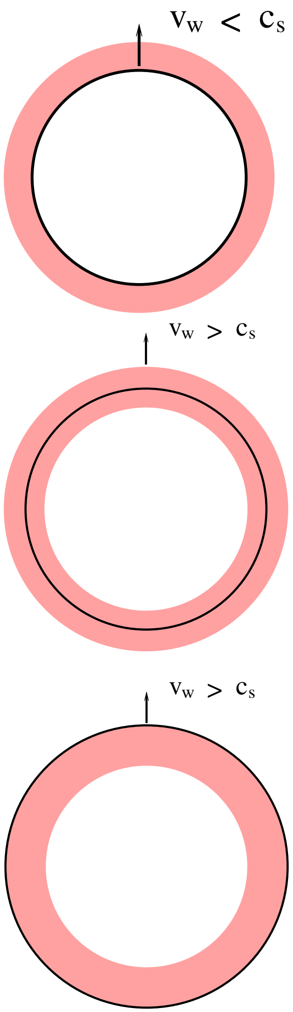
GWs from first order phase transitions

- Parameters of transition:
  - α = (“Potential energy”)/(“Heat energy”)
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - H_n = Hubble rate at nucleation
- Derived parameters:
  - R_* = mean bubble separation $= (8\pi)^{1/3} v_w/\beta$
  - K = fluid kinetic energy fraction (depends on α, v_w)

Steinhardt ’84
Espinosa et al 2010

- Aim: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$



GWs from phase transitions

- Gravitational waves generated by shear stress fluctuations

$$\Omega_{\rm GW} \sim \frac{1}{G\rho} \left\langle \left| \dot{h}_{ij}(t) \right|^2 \right\rangle$$

$$\dot{h}_{ij} \sim G \int dt' \cos[k(t-t')] T^{TT}_{ij}(k,t')$$

- Shear stress ~ kinetic energy
- Kinetic energy from potential energy
  - *$K(\alpha, v_w)$* = fluid kinetic energy fraction

$$T_{ij} \sim \rho U_i U_j$$

$$\dot{h} \sim \tau (G\rho) K$$

- Timescales τ_v and τ_c
  - τ_v duration of stresses from fluid velocity
  - τ_c coherence time of stress fluctuations

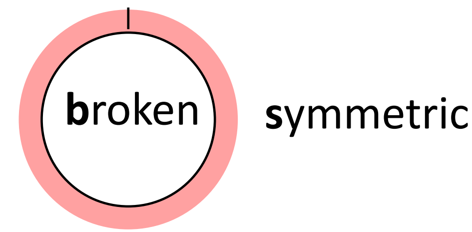
$$\Omega_{\rm gw} \sim \frac{\tau_v \tau_c}{G\rho} (G\rho)^2 K^2$$

$$\Omega_{\rm gw} \sim (H_n \tau_v)(H_n \tau_c) K^2$$

$$\Omega_{\rm gw,0} \sim \Omega_{\rm rad,0} (H_n \tau_v)(H_* \tau_c) K^2$$

Energy budget of a phase transition

Espinosa et al 2010



- Conservation of energy around expanding bubble

$$T^{00} = w\gamma^2 - p = w\gamma^2 v^2 + e$$

$w = e + p$ enthalpy

$\theta = \frac{1}{4}(e - 3p)$ trace anomaly

- Internal energy = "heat" + "potential" energy

$$e = \frac{3}{4} w + \theta$$

$$e_K = 4\pi \int_0^{\xi_{\rm max}} d\xi \xi^2 w \gamma^2 v^2,$$

$$\Delta e_Q = 4\pi \int_0^{\xi_{\rm max}} d\xi \xi^2 \frac{3}{4}(w - w_{\rm n}),$$

$$\Delta e_\theta = 4\pi \int_0^{\xi_{\rm max}} d\xi \xi^2 (\theta_{\rm n} - \theta).$$

- Initial: v = 0, $e = e_{\rm n}$, $w = w_{\rm n}$, $\theta = \theta_{\rm n}$ (uniform)
- Around bubble: $v(\xi)$, $w(\xi)$, $\theta(\xi)$, with $\xi = r/t$

$$e_K + \Delta e_Q = -\Delta e_\theta$$

- Kinetic energy fraction K: strength parameter $\alpha_{\rm n}$

$$e_K = K \cdot e_{\rm n} \frac{4\pi}{3} v_w^3 \qquad \Delta e_\theta \simeq \alpha_{\rm n} \cdot e_{Q,{\rm s}}(T_{\rm n}) \frac{4\pi}{3} v_w^3$$

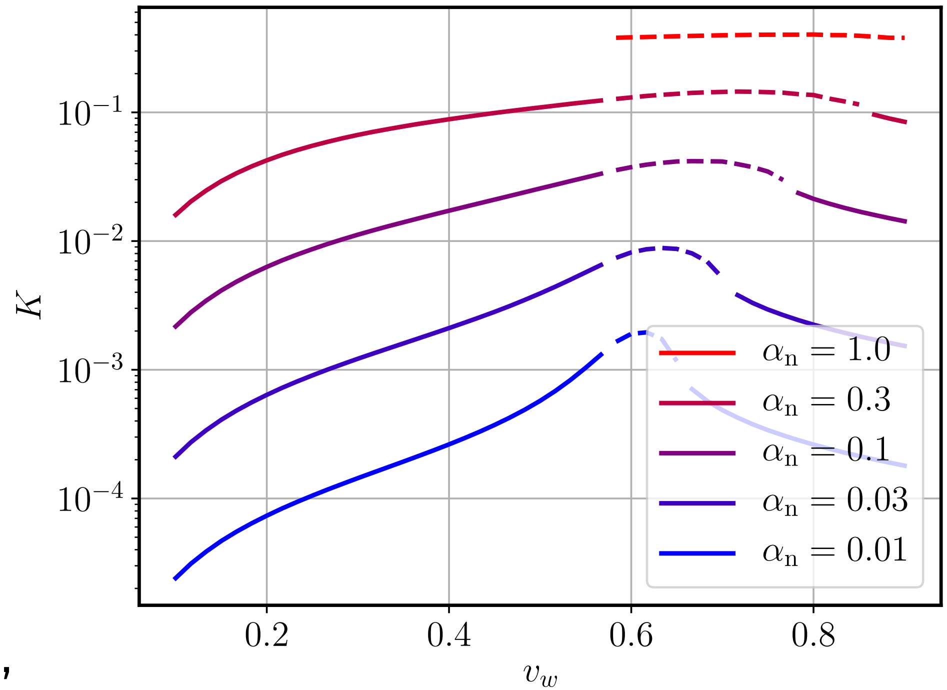
$$\alpha_{\rm n} = \left. \frac{\theta_{\rm s} - \theta_{\rm b}}{e_{Q,{\rm s}}} \right|_{T_{\rm n}}$$

$\alpha_{\rm n} = \frac{\epsilon}{a_n T_n^4}$ Espinosa et al 2010

Estimating GW power

- Recall GW energy fraction:
  - τ_v duration of stresses
  - τ_c coherence time
- Numerical simulations:
  - $\tau_c \sim R_*$ (bubble separation)
- Analytical estimate:
  - $\tau_v = \min(H_n^{-1}, R_*/U_f)$
  - N.B. $K = (4/3)U_f^2$
  - U_f RMS velocity (slow flows)
- Pure acoustic ($H_n R_* >> U_f$)

$$\Omega_{\rm gw} \sim (H_{\rm n}\tau_v)(H_{\rm n}\tau_c)K^2$$



$$\Omega_{\rm gw} \simeq (H_{\rm n}R_*)K^2\tilde{\Omega}_{\rm gw},$$
$$\tilde{\Omega}_{\rm gw} \sim 10^{-2}$$

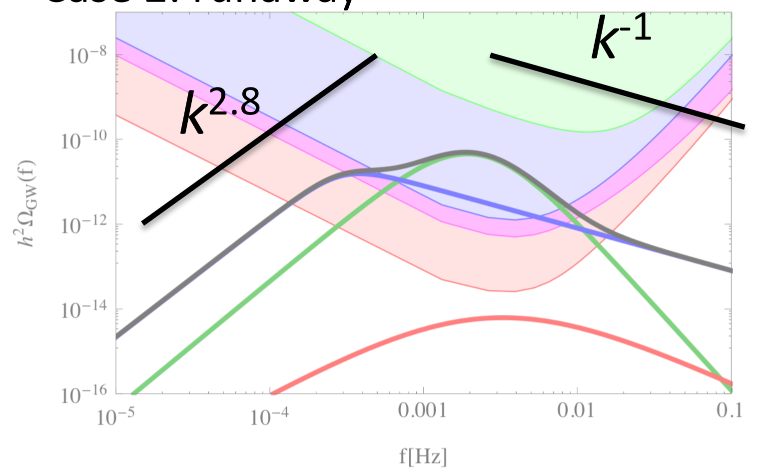
LISA CWG party line 2016

- Three contributions to total power:
  - Scalar field ϕ
  - Acoustic ac
  - Turbulent tu

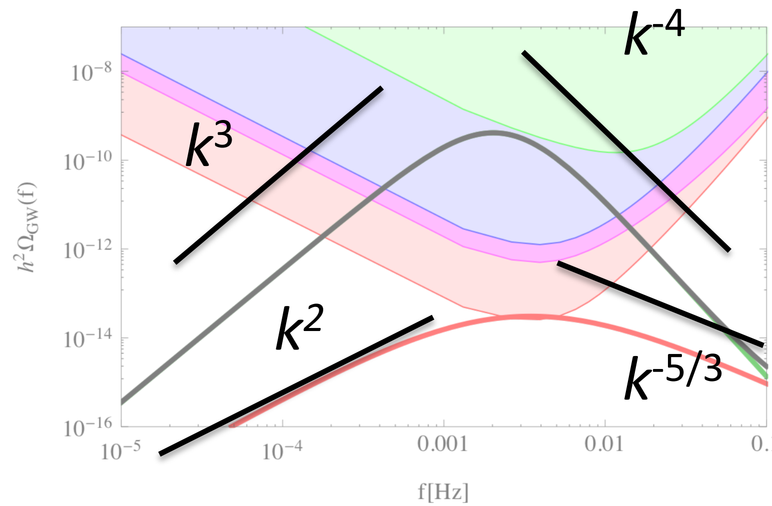
$$\Omega_{\text{gw}} = \Omega_{\text{gw}}^{\phi} + \Omega_{\text{gw}}^{\text{ac}} + \Omega_{\text{gw}}^{\text{tu}}$$

- Scalar field: bubble wall collisions
  - relevant only for runaway walls
  - “envelope approximation”
  - Kosowsky, Turner 1992
  - Huber, Konstandin 2008
- Acoustic production:
  - M.H. et al 2013, 2015
- Turbulent production:
  - Caprini, Durrer, Servant 2009

Case 2: runaway



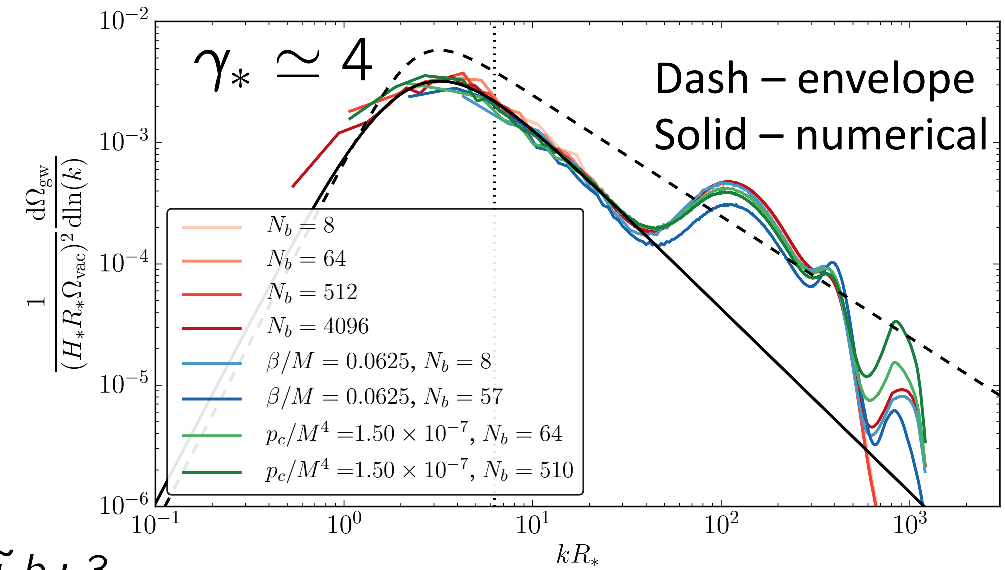
Case 1: constant v_w



Developments 1: scalar field

- Numerical simulations show differences from envelope approximation

Cutting, MH, Weir 2018

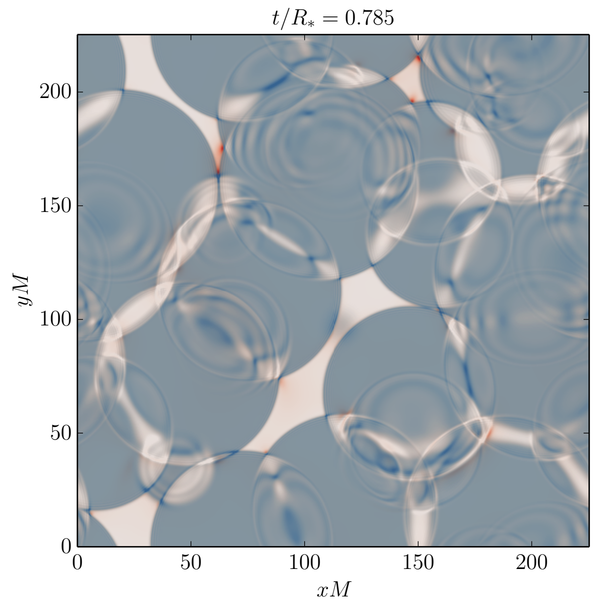


$$\frac{d\Omega_{\rm gw}^{\rm fit}}{d\ln k} = \Omega_{\rm p}^{\rm fit} \frac{(3+b)^c \tilde{k}^b k^3}{(b\tilde{k}^{(3+b)/c} + 3k^{(3+b)/c})^c}$$

$$\Omega_{\rm p}^{\rm fit} = (3.22 \pm 0.04) \times 10^{-3} (H_{\rm n} R_*)^2 \Omega_\phi^2,$$

$$\tilde{k} R_* = 3.20 \pm 0.04,$$

$$b = 1.51 \pm 0.04, \qquad c = 2.18 \pm 0.15$$

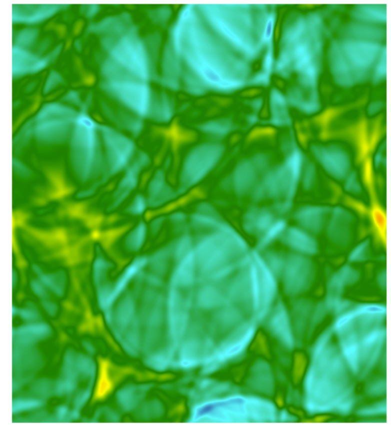


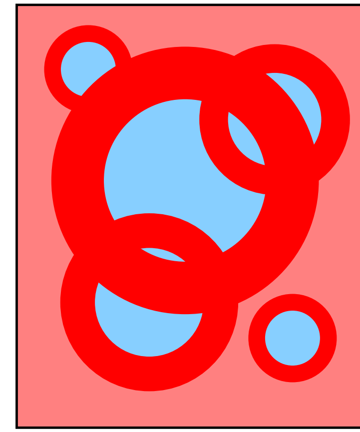
Developments 2: sound shell model

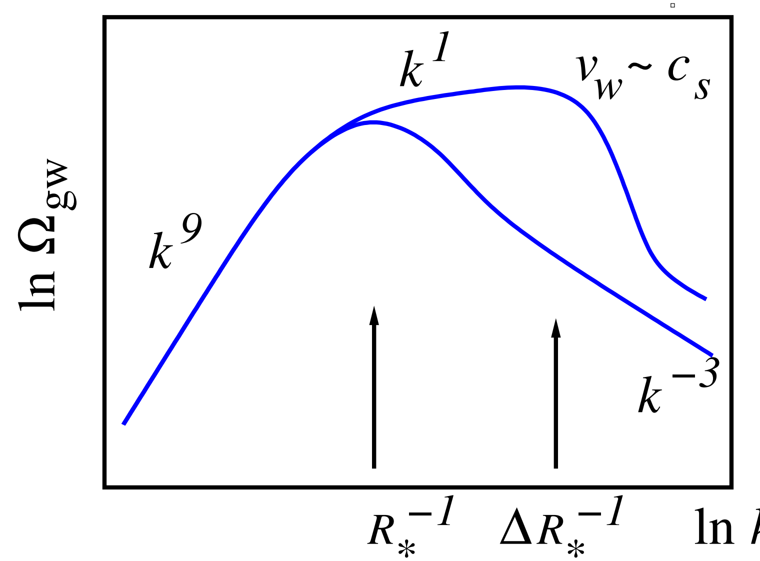
- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_{\mathbf{q}}(t_i)$

  MH 2017, MH, Hijazi (in prep 2019)

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \frac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude:
  - Detonations: good (< 10%)
  - Deflagrations: overestimated

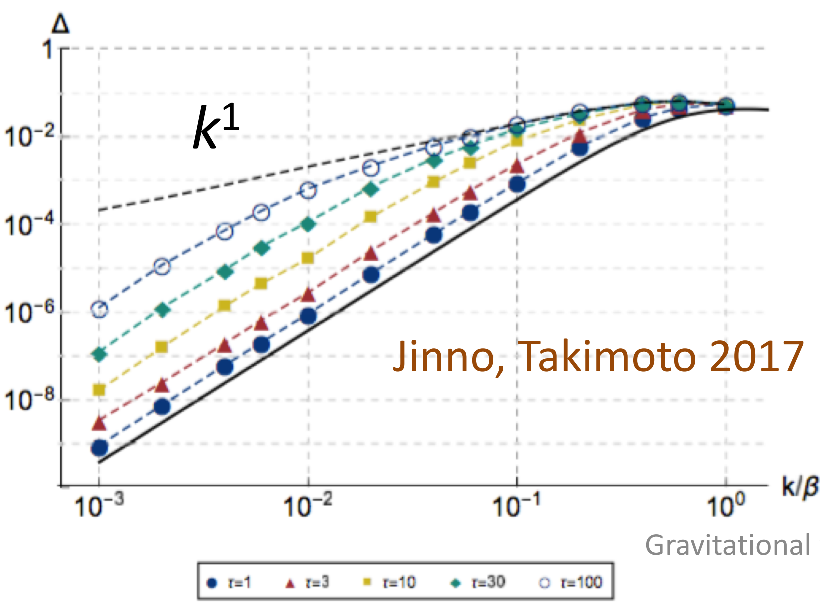




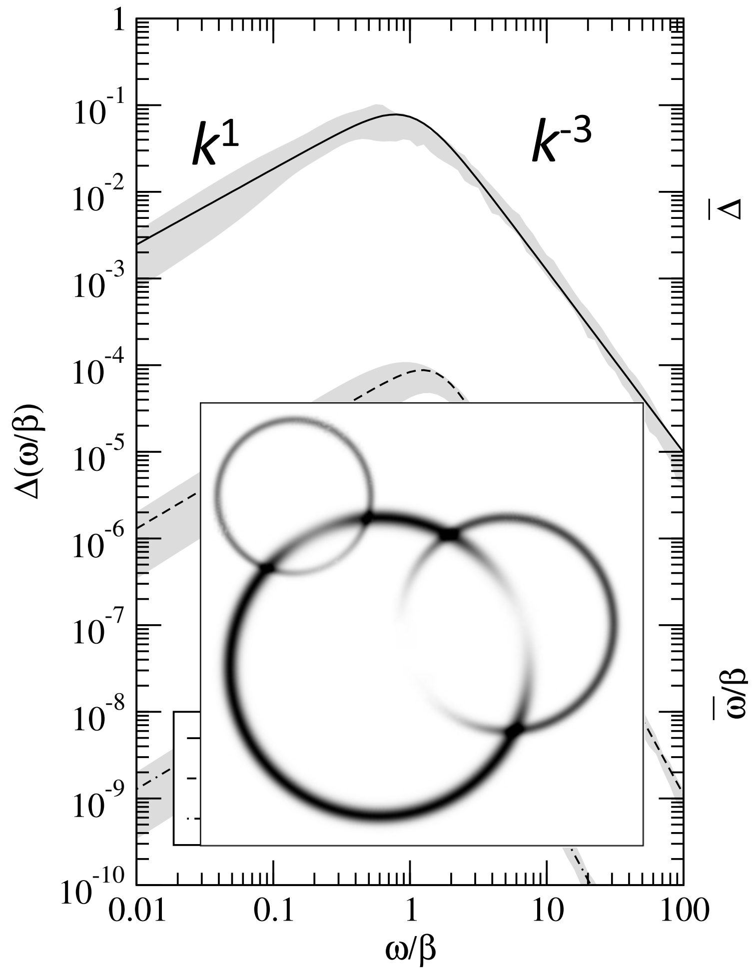


Developments 3: bulk flow models

- Based on envelope approximation
- Model overlapping energy shells in real space
- Disagrees with sound shell model at low k



Jinno, Takimoto 2017

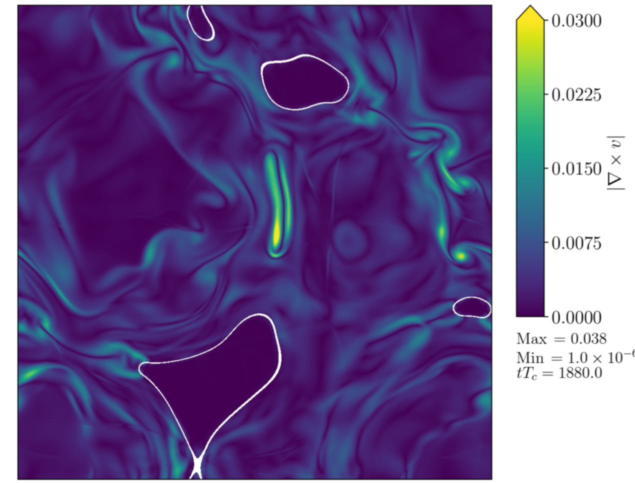


Konstandin 2017

Developments 4: non-linearities

- Longitudinal v
  - Kinetic energy suppression
- Transverse v
  - Vorticity production

  Cutting, MH, Weir 2019

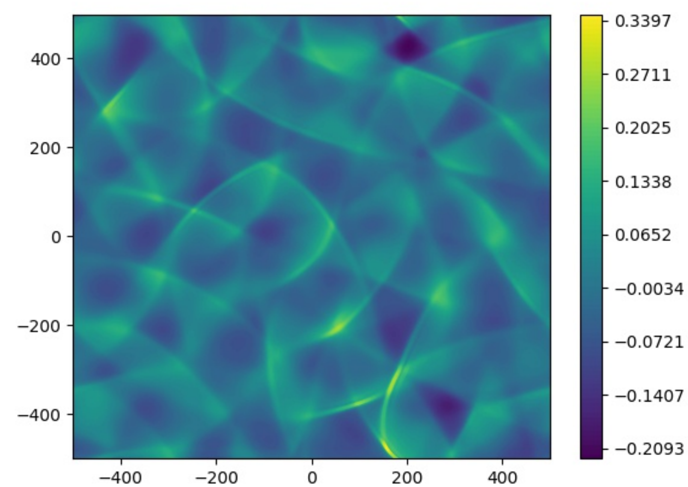


- Non-linearity timescale

  $$\tau_{\rm nl} \sim L_f / \bar{U}_f$$

  - Shock development
  - Further vorticity production
  - Decay of flows

T, $n = 1000$, $dx = dy = 1.0$, $dt = 0.01$, $\nu = 0.05$



J Dahl, U Helsinki

Developments 5: turbulence

- Modelling

Green: Gogoberidze, Kahniashvili, Kosowsky 2007
Black: Caprini, Durrer, Servant 2008

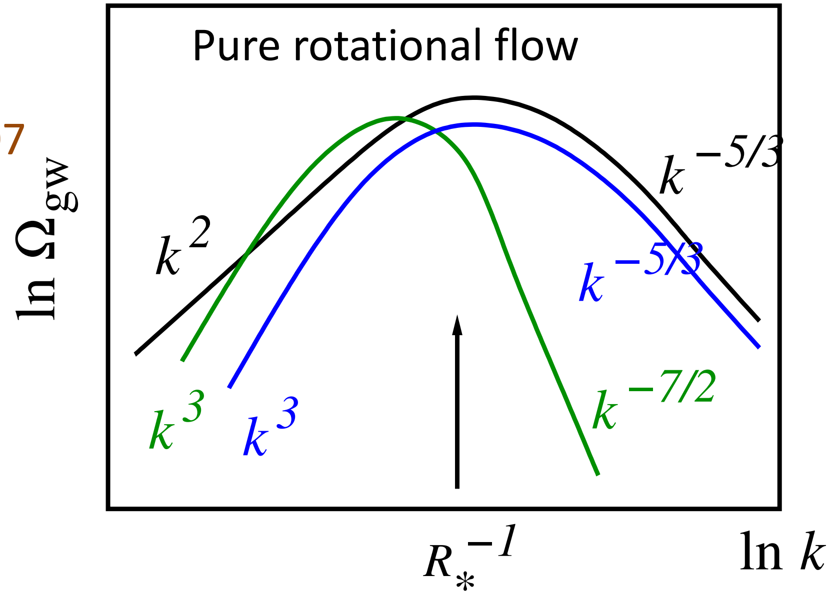
Blue: Niksa, Schlederer, Sigl 2018

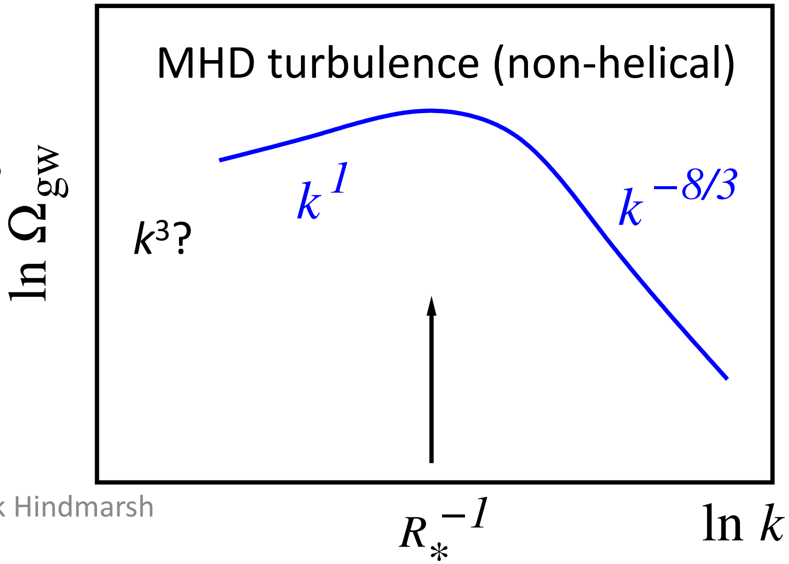
  - Kraichnan sweeping model: velocity autocorrelation time

$$\tau_k \sim 1/k\bar{v}_\perp$$

  - Pure rotational flow: high k GW power spectrum $k^{-5/3}$
  - Mixed acoustic-turbulent $k^{-8/3}$
- MHD simulation

Roper pol et al 2019



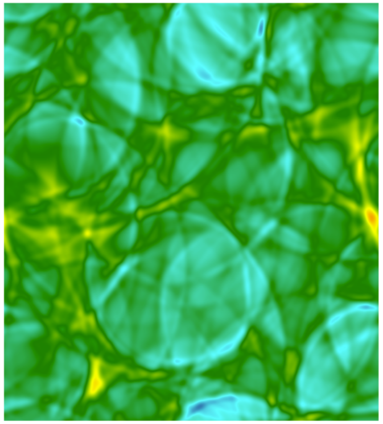


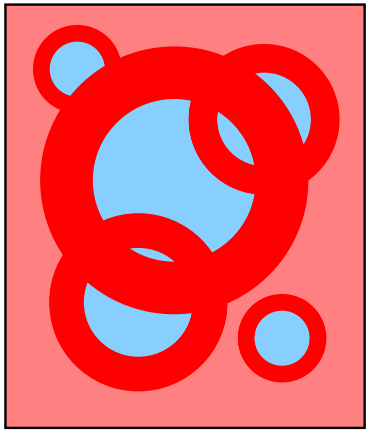
Sound shell model

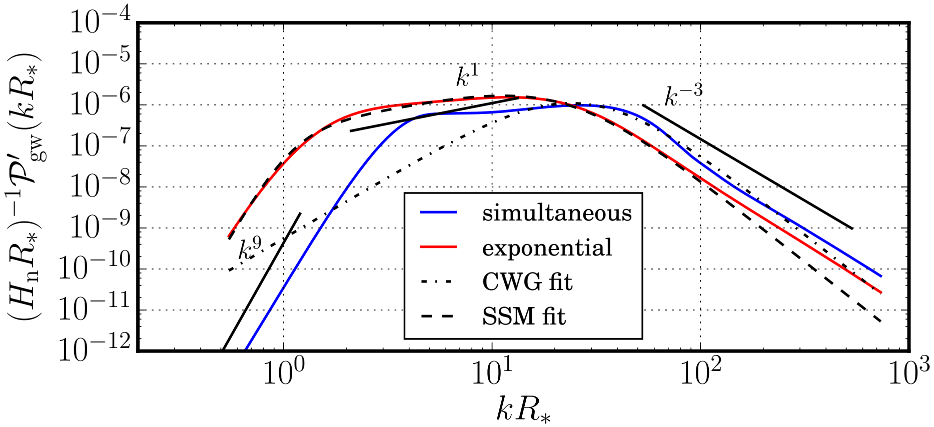
- Gaussian velocity field from weighted addition of sound shells $\mathbf{v_q}(t_\mathrm{i})$

  MH 2017, MH, Hijazi (in prep 2019)

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \dfrac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude:
  - Bubble separation
  - (Kinetic energy)^2



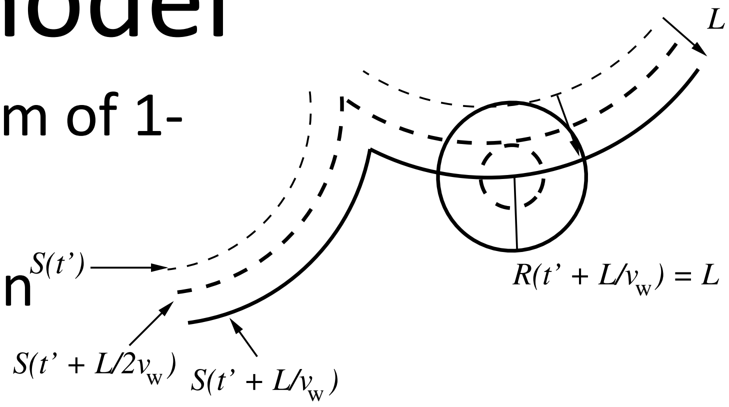




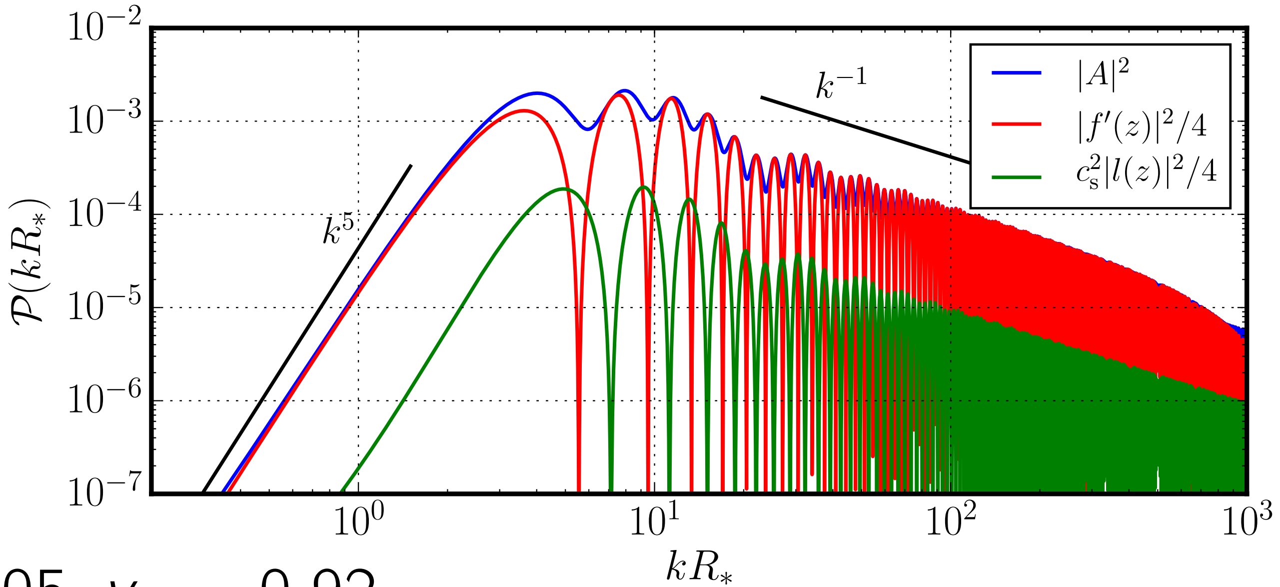
$\alpha_\mathrm{n} = 0.0046 \quad v_w = 0.56$

Sound shell model

- Velocity power spectrum is weighted sum of 1-bubble power spectra
- Weighting by bubble lifetime distribution



$$\nu(\beta T) = \begin{cases} \exp(-\beta T) & \text{exponential} \\ \frac{1}{2}(\beta T)^2 \exp(-\frac{1}{6}(\beta T)^3) & \text{simultaneous} \end{cases}$$

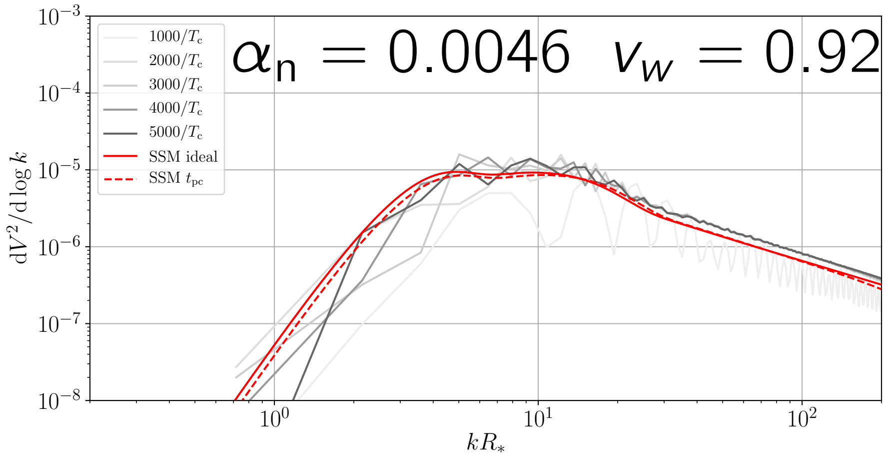


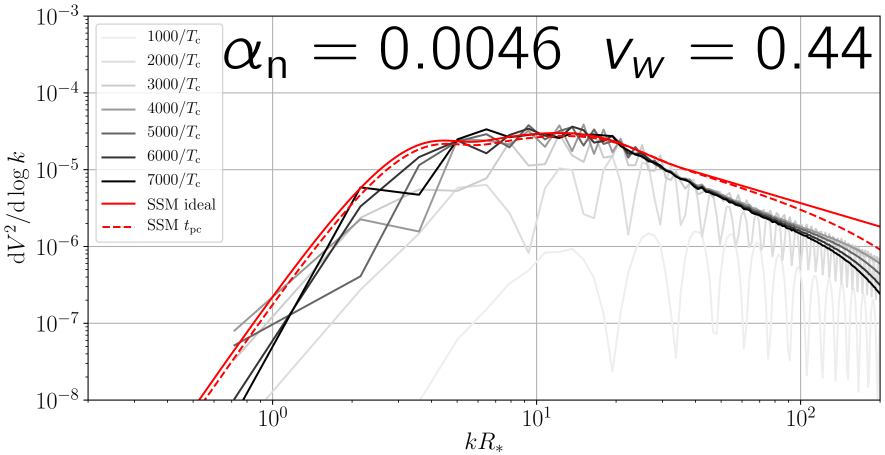
$\alpha_n = 0.05 \quad v_w = 0.92$

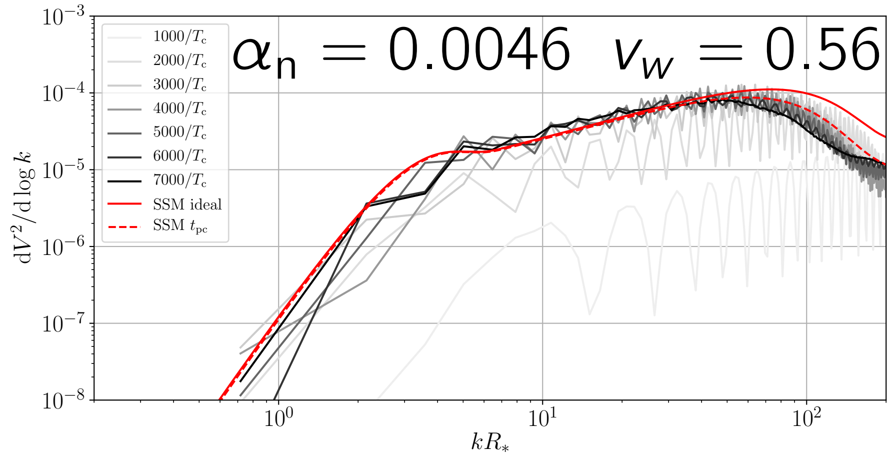
Sound shell model vs. simulations P_v

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019







Sound shell model vs. simulations P_{gw}

- Solid: self-similar sound shell
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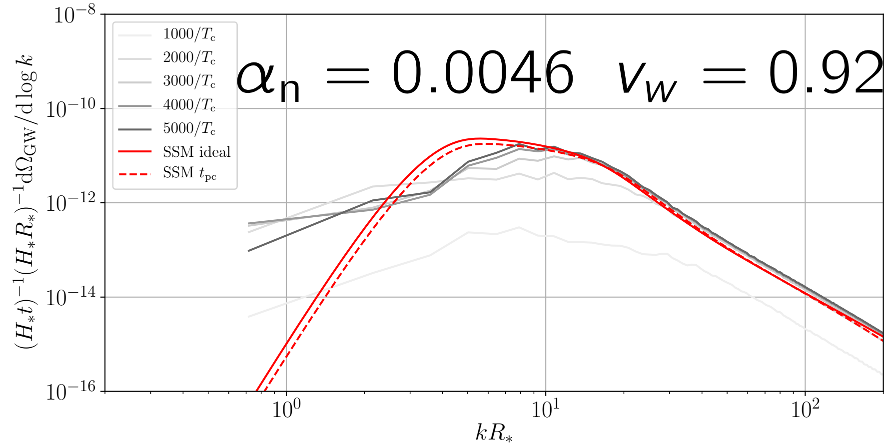
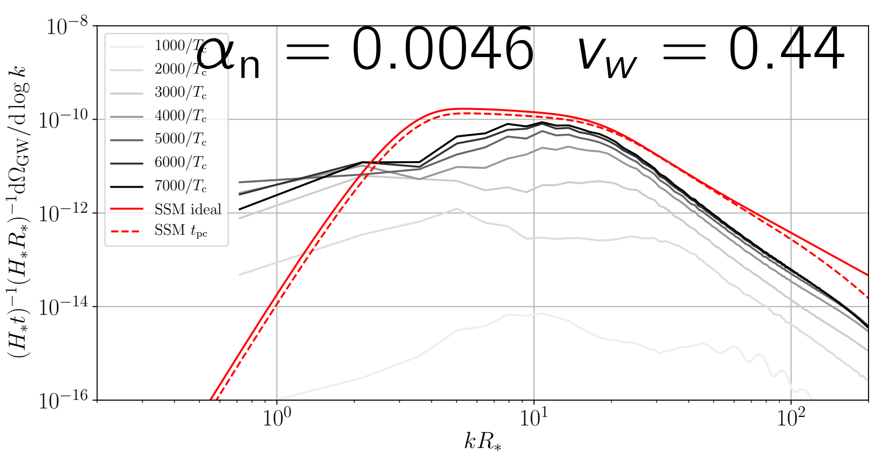
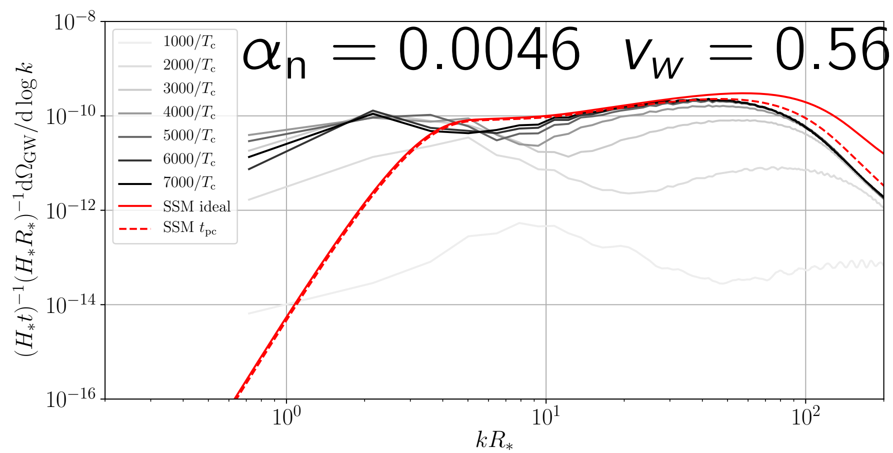
MH et al in prep 2019







Sound shell model fit function

MH, Hijazi 2019

$$\Omega_{\mathrm{gw}}^{\mathrm{ac}}(f) = F_{\mathrm{gw},0}(H_{\mathrm{n}}R_*)A_M\, M\left(\frac{f}{f_{p,0}}\right)$$

- Double broken power law $s = z_p(f/f_{p,0})$

$$M(s) = s^9\left(\frac{r_{\mathrm{p}}^4+1}{r_{\mathrm{p}}^4+s^4}\right)^2\left(\frac{5}{5-m+ms^2}\right)^{5/2}, \qquad A_M \simeq K^2\tilde{\Omega}_{\mathrm{gw}}$$

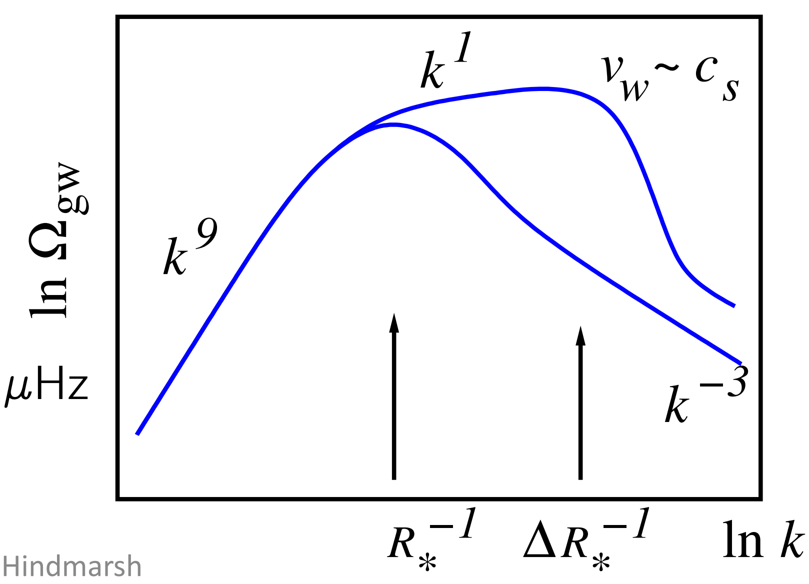
$$m = (9r_{\mathrm{p}}^4+1)/(r_{\mathrm{p}}^4+1)$$

- Radiation energy density redshift

$$F_{\mathrm{gw},0} \simeq 3.6\times 10^{-5}$$

- Frequency redshift

$$f_{p,0} \simeq 26\,(H_{\mathrm{n}}R_*)^{-1}\left(\frac{T_{\mathrm{n}}}{100\ \mathrm{GeV}}\right)\mu\mathrm{Hz}$$



Flow lifetime uncertainty

- Non-linearities important after

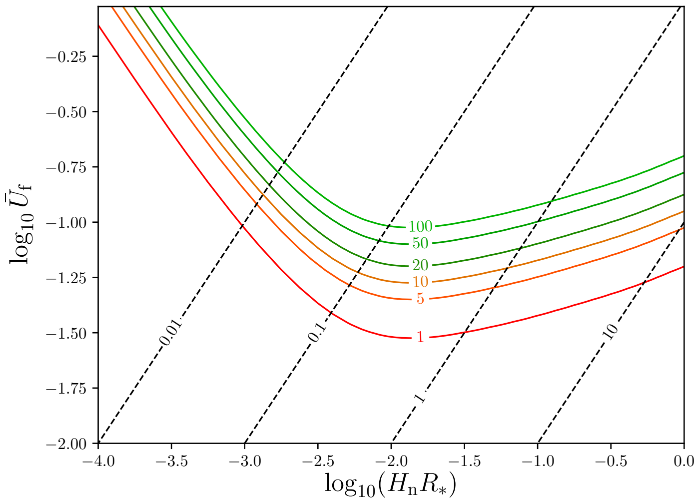
$$\tau_{\rm nl} \sim L_f / \bar{U}_f$$

- CWG 2016: Non-linear dissipation ignored
- source lifetime assumed to be $H_{\rm n}^{-1}$

- Estimate: multiply PS by

$$\min(1, H_{\rm n}\tau_{\rm nl})$$

- GW power $\Omega_{\rm gw} \sim (H_{\rm n} R_*)^2 K^{3/2}$



Outlook

- Non-linear evolution:
  - Longitudinal
    - Kinetic energy suppression
    - Shocks, wave turbulence
  - Transverse
    - Vorticity generation
    - Turbulence
  - Magnetic fields and MHD turbulence
- Connection to fundamental theory:
  - Scalar effective potential $V(\phi)$ → T_n, α, β, $g_{\rm eff}$ (equilibrium)
  - Field-fluid coupling $\eta(\phi)$ → v_w (non-equilibrium)

Parameters connecting to fundamental theory:

T_n = nucleation temperature

α = (scalar potential)/(thermal energy)

β = transition rate

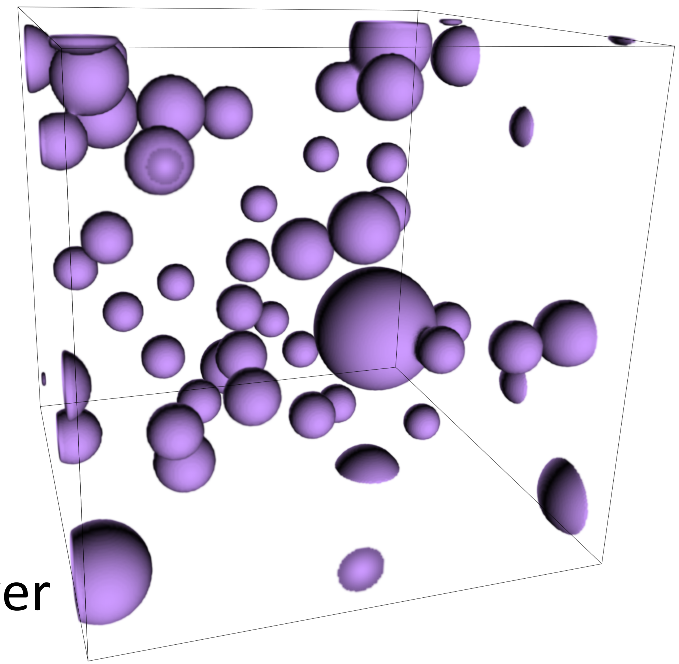
$v_{\rm w}$ = bubble wall speed

$g_{\rm eff}$ = effective d.o.f.

+ physics of magnetic field generation

Conclusions

- GWs probe of physics at very high energy
- LISA will probe physics of Higgs phase transition from 2034
- Measure/constrain phase transition parameters
  - T_n = nucleation temperature
  - α = (scalar potential)/(thermal energy)
  - R_* = mean bubble centre separation
  - v_w = bubble wall speed
  - g_{eff} = effective d.o.f.



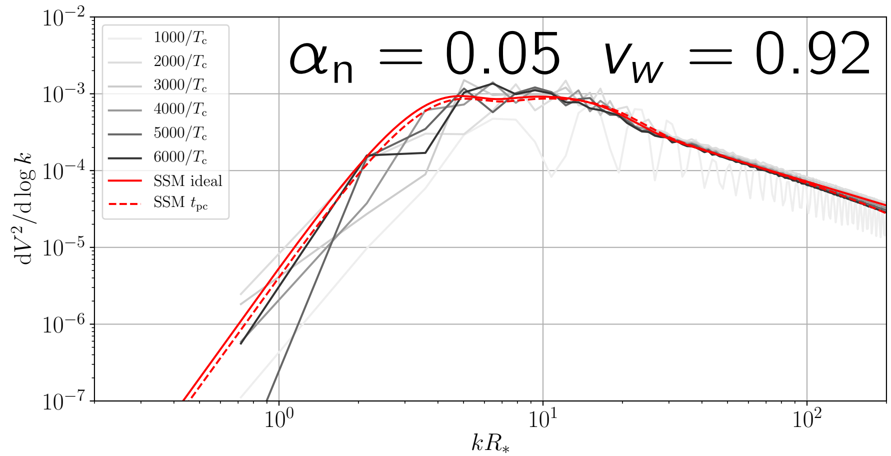
D Weir

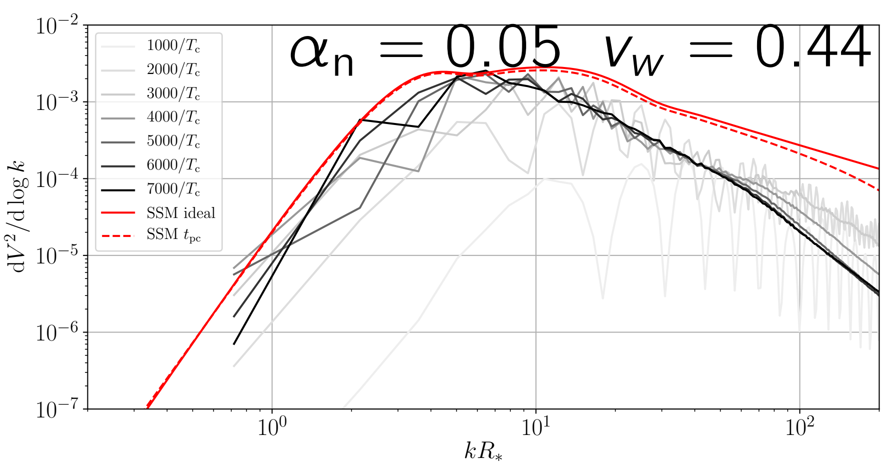
- Towards accurate calculations of GW power spectrum from parameters
  - Good understanding of acoustic production from numerical simulations & sound shell model
  - Non-linear evolution (turbulence, shocks) not well understood: likely to be important

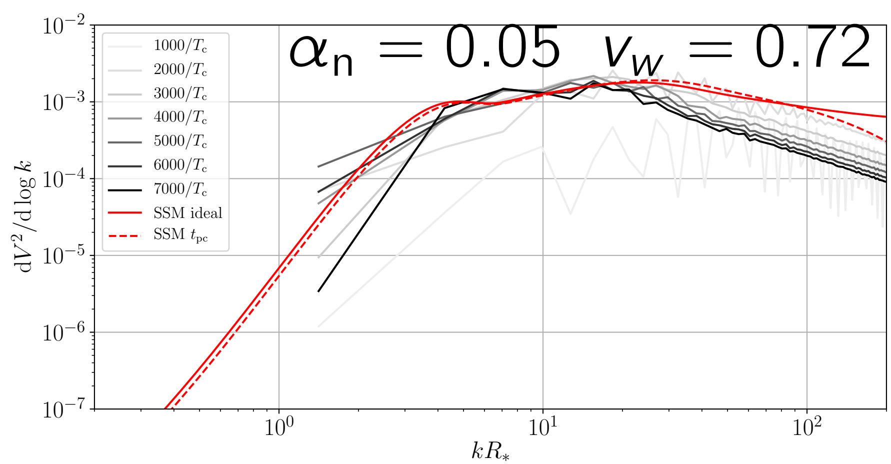
Sound shell model vs. simulations P_v

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019







Sound shell model vs. simulations P_{gw}

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019