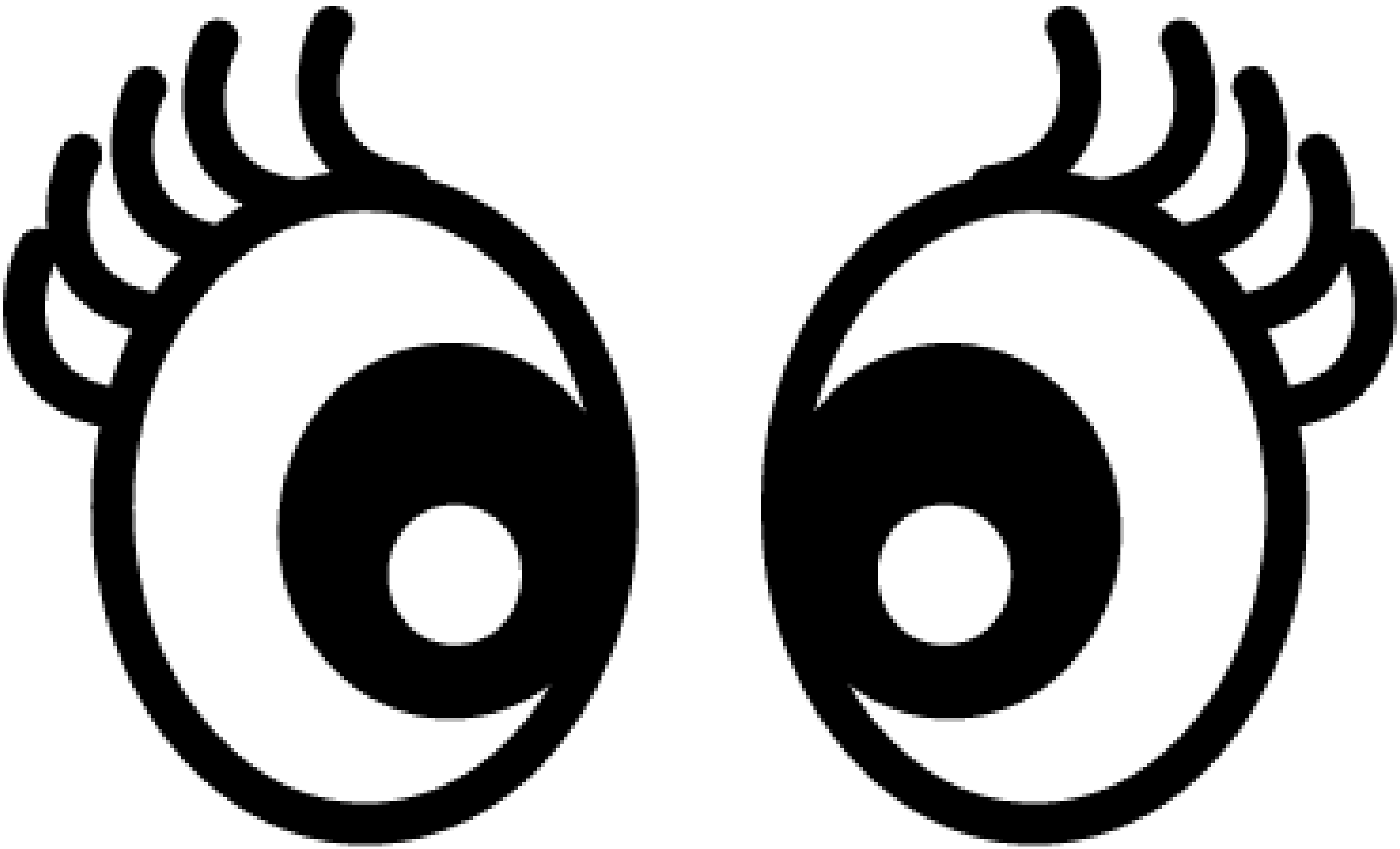
Gravitational waves as a probe of the history of the universe

Jun'ichi Yokoyama

RESearch Cnter for the Early Universe (RESCEU)
The University of Tokyo

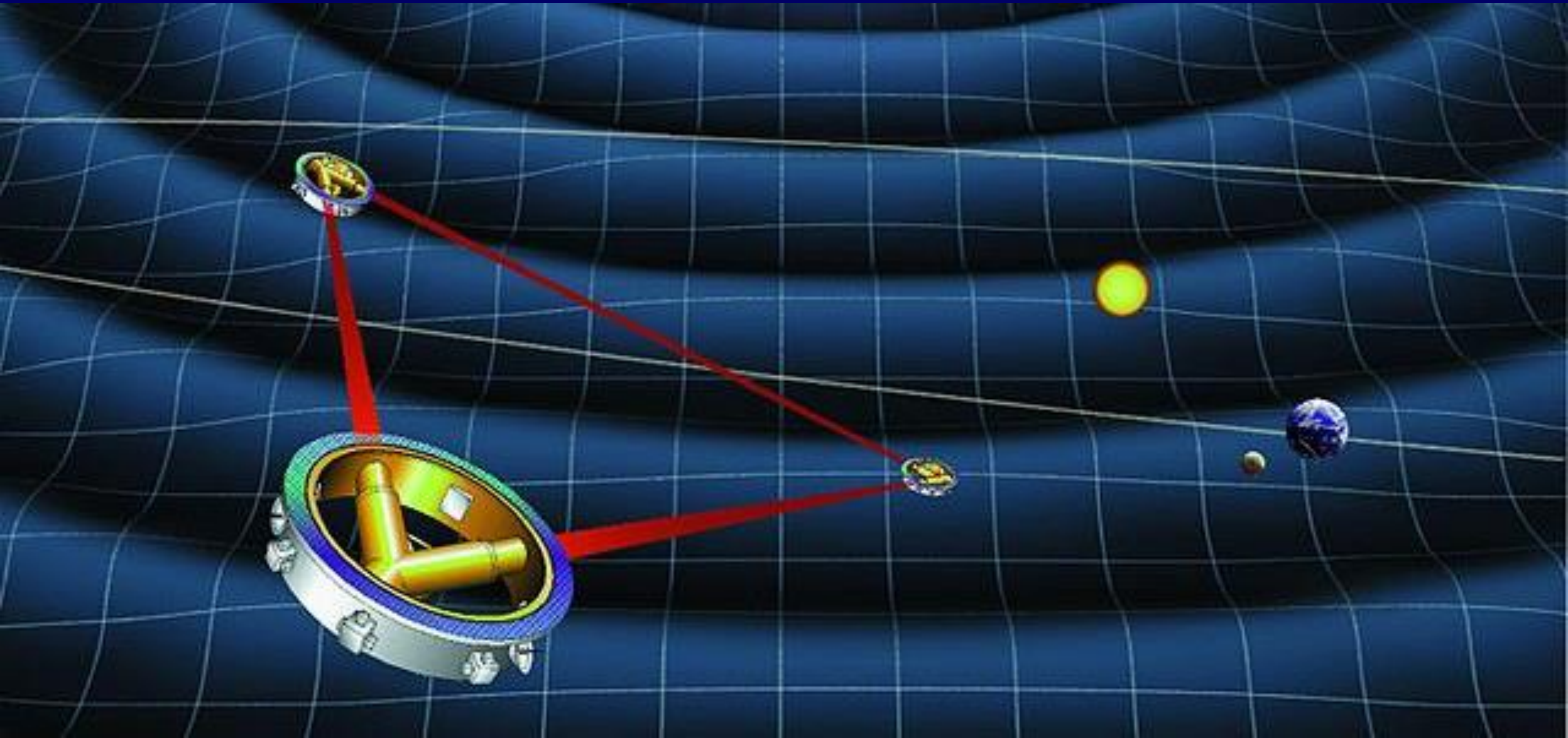
Gravitational wave Physics and Astronomy
New Eyes to Observe the Universe



Gravitational wave Physics and Astronomy Science Goals

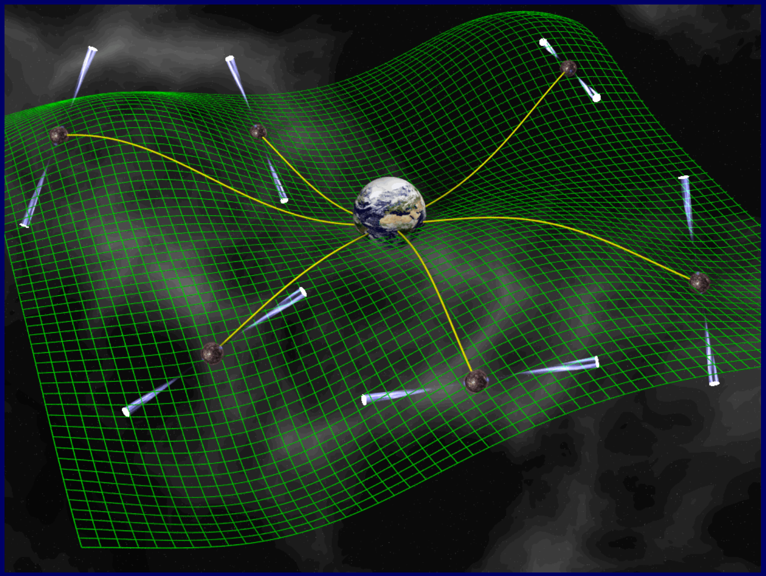
- Cosmology
  - Can we directly see before the CMB era?

Direct detection of cosmological Gravitational waves can be achieved by space-based laser interferometers Such as LISA, DECIGO, TianQin, Taiji…



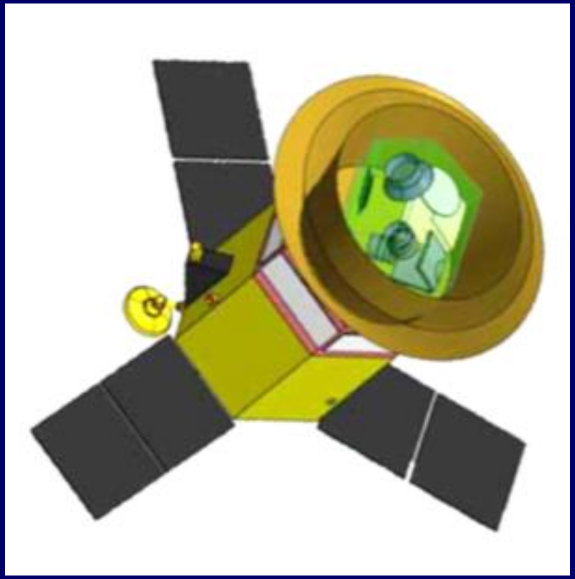
Other means of detection of Cosmological gravitational waves

Pulsar timing



©David J. Champion

B-mode polarization of Cosmic Microwave Background (CMB)

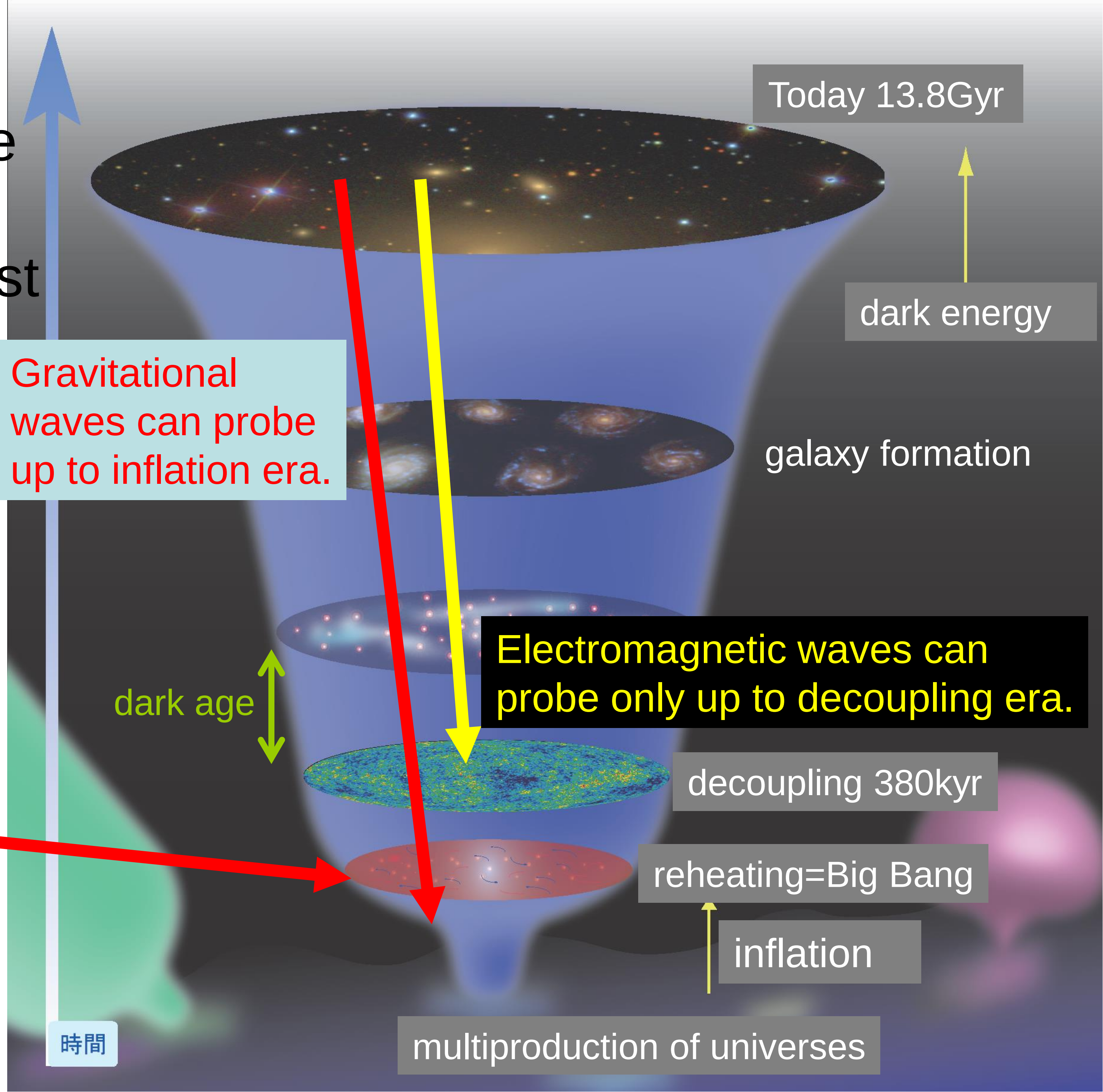


LiteBIRD: approved by JAXA in May 2019!

Gravitational waves provide new eyes to see the earliest Universe.

We can probe another tiny dark age between inflation and Big Bang Nucleosynthesis

Shedding new “lignt” on this epoch



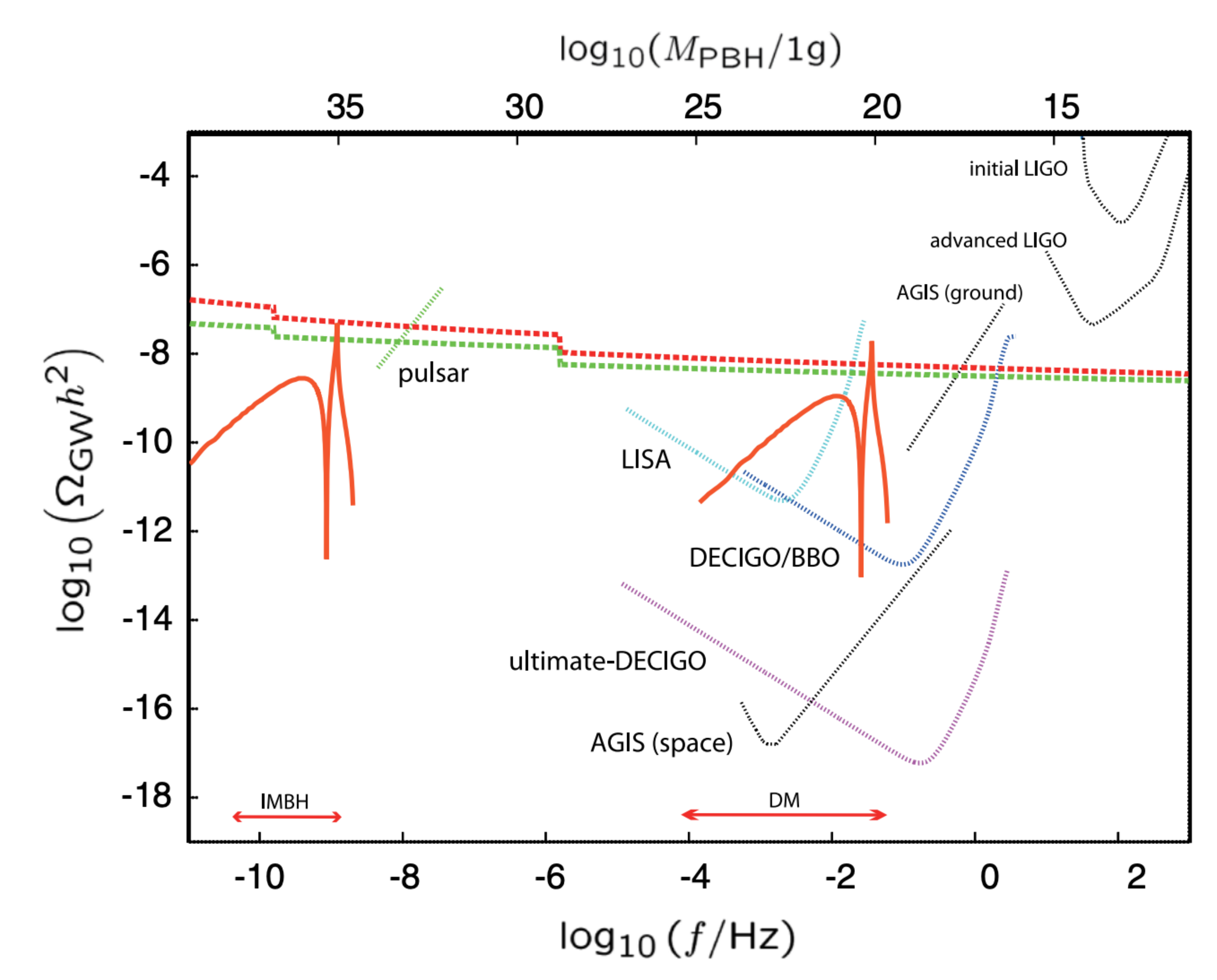
Sources of gravitational waves from the early universe

① Quantum tensor perturbations from inflation

② GWs from second-order scalar perturbations

PBH DM scenario can be tested by LISA & DECIGO/BBO.

Saito & JY PRL107(2011)069901



Sources of gravitational waves from the early universe

① Quantum tensor perturbations from inflation

② GWs from second-order scalar perturbations

③ GWs from bubble collisions after phase transition

④ GWs from self-ordering scalar fields

⑤ GWs from topological defects esp. cosmic strings

⑥ Gravitational particle production of gravitons

⑦ 。。。

Tensor Perturbations (Quantum Gravitational Waves)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h_+ \varepsilon_{ij}^+ + h_\times \varepsilon_{ij}^\times$$

transverse
-traceless

They are equivalent with two massless scalar fields.

$$h_{ij}(t, \boldsymbol{x}) = \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \epsilon_{ij}^\lambda(\boldsymbol{k}) h_{\boldsymbol{k}}^\lambda(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

satisfies massless Klein-Gordon eqn

Quantization in De Sitter background yields nearly scale-invariant long-wave perturbations during inflation.

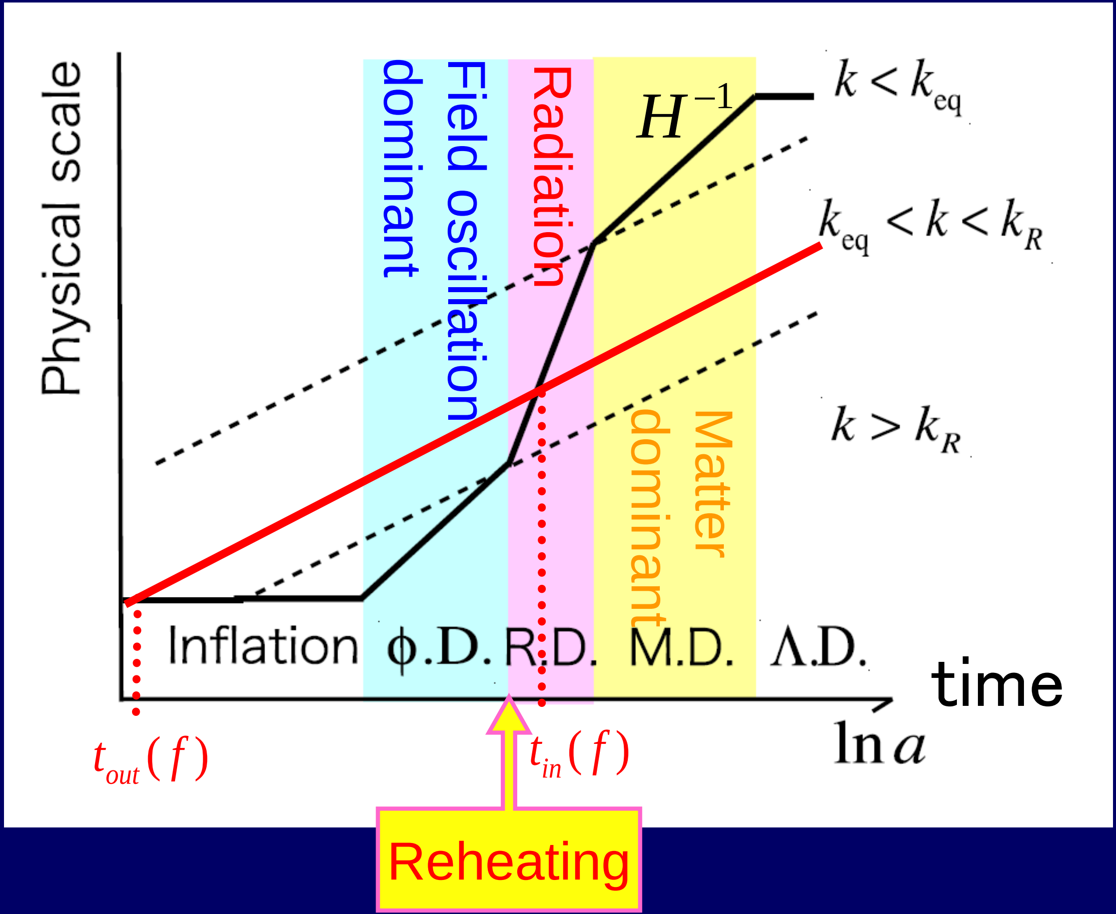
$$\Delta_h^2(k) = \langle h_{ij} h^{ij}(k) \rangle = 64\pi G \left(\frac{H(t_k)}{2\pi} \right)^2$$

Starobinsky (1979)

Evolution of gravitational waves in the standard inflationary Universe

- Amplitude of GW is constant when its wavelength is longer than the Hubble radius between $t_{out}(f)$ and $t_{in}(f)$.

- After entering the Hubble radius, the amplitude decreases as $\propto a^{-1}(t)$ and the energy density as $\propto a^{-4}(t)$.



When $a(t) \propto t^p \quad (p < 1)$, the tensor perturbation evolves as

$$h(f, a) \propto a(t)^{\frac{1-3p}{2p}} J_{\frac{3p-1}{2(1-p)}}\left(\frac{p}{1-p}\frac{k}{a(t)H(t)}\right), \quad k = 2\pi f a(t_0)$$

Density parameter in GW per logarithmic frequency interval

$$\Omega_{GW}(f,t) = \frac{1}{\rho_{cr}(t)} \frac{d\rho_{GW}(f,t)}{d\ln f}$$

When the mode reentered the Hubble horizon at $t \equiv t_{in}(f)$,
the angular frequency is equal to $\omega = H\left(t_{in}(f)\right)$, so we find

$$\frac{d\rho_{GW}(f,t_{in}(f))}{d\ln f} = \frac{\omega^2}{32\pi G} h_{\inf}^2(f) = \frac{H^2\left(t_{in}(f)\right)}{32\pi G} h_{\inf}^2(f) = \frac{1}{24}\rho_{cr}(t_{in}(f))\Delta_h^2(f)$$

$$\boxed{\Omega_{GW}(f,t_{in}(f)) = \frac{1}{24}\Delta_h^2(f)}$$

at horizon reentry

After entering the Hubble horizon at $t_{in}(f)$,

$$\Omega_{GW}(f,t) = \frac{\rho_{GW,\ln f}(f,t)}{\rho_{tot}(t)} \quad \begin{matrix} \propto a^{-4}(t) \\ \propto a^{-3(1+w)}(t) \end{matrix}$$

$w \equiv \dfrac{p}{\rho_{tot}}$: equation of state parameter

$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w}$$

Radiation dominated era: $w = \dfrac{1}{3}$ $\quad \Omega_{GW}(f,t) = \text{const.}$

Matter dominated era: $w = 0$ $\quad \Omega_{GW}(f,t) \square \quad \propto a^{-1}(t)$

$t_{in}(f)$ as a function of frequency

$$2\pi f = aH \propto t^{\frac{2}{3(1+w)}} \frac{2}{3(1+w)t} \propto t^{-\frac{3w+1}{3w+3}} \quad \text{at} \quad t = t_{in}(f)$$

$$\therefore t_{in}(f) \propto f^{-\frac{3w+3}{3w+1}}$$

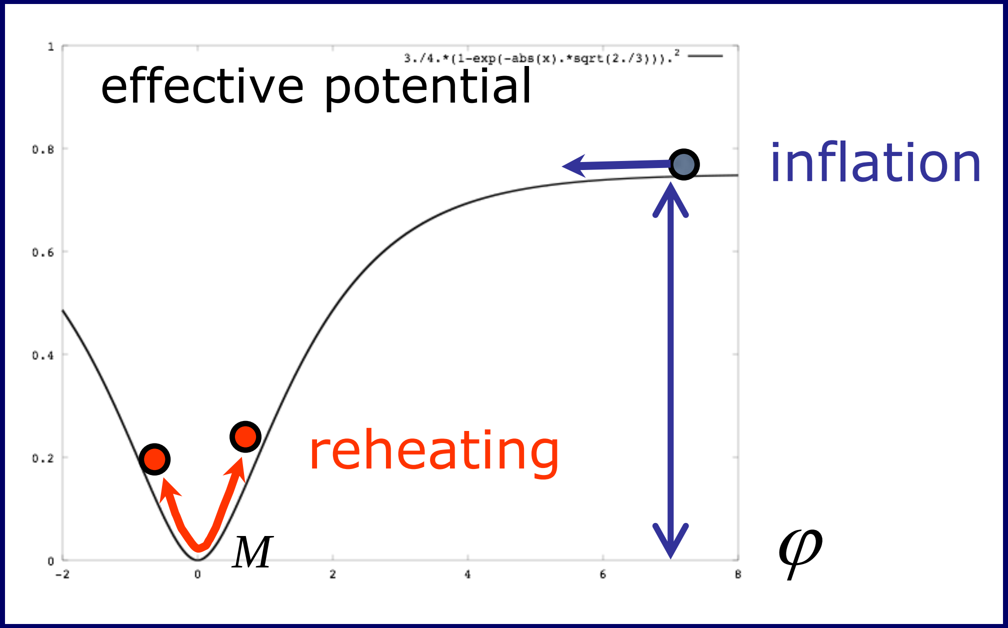
$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w} \propto \Delta_h^2(f)\, f^{\frac{6w-2}{3w+1}}$$

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), Boyle & Steinhardt (08), Nakayama, Saito, Suwa, JY (08), Kuroyanagi et al (11)…

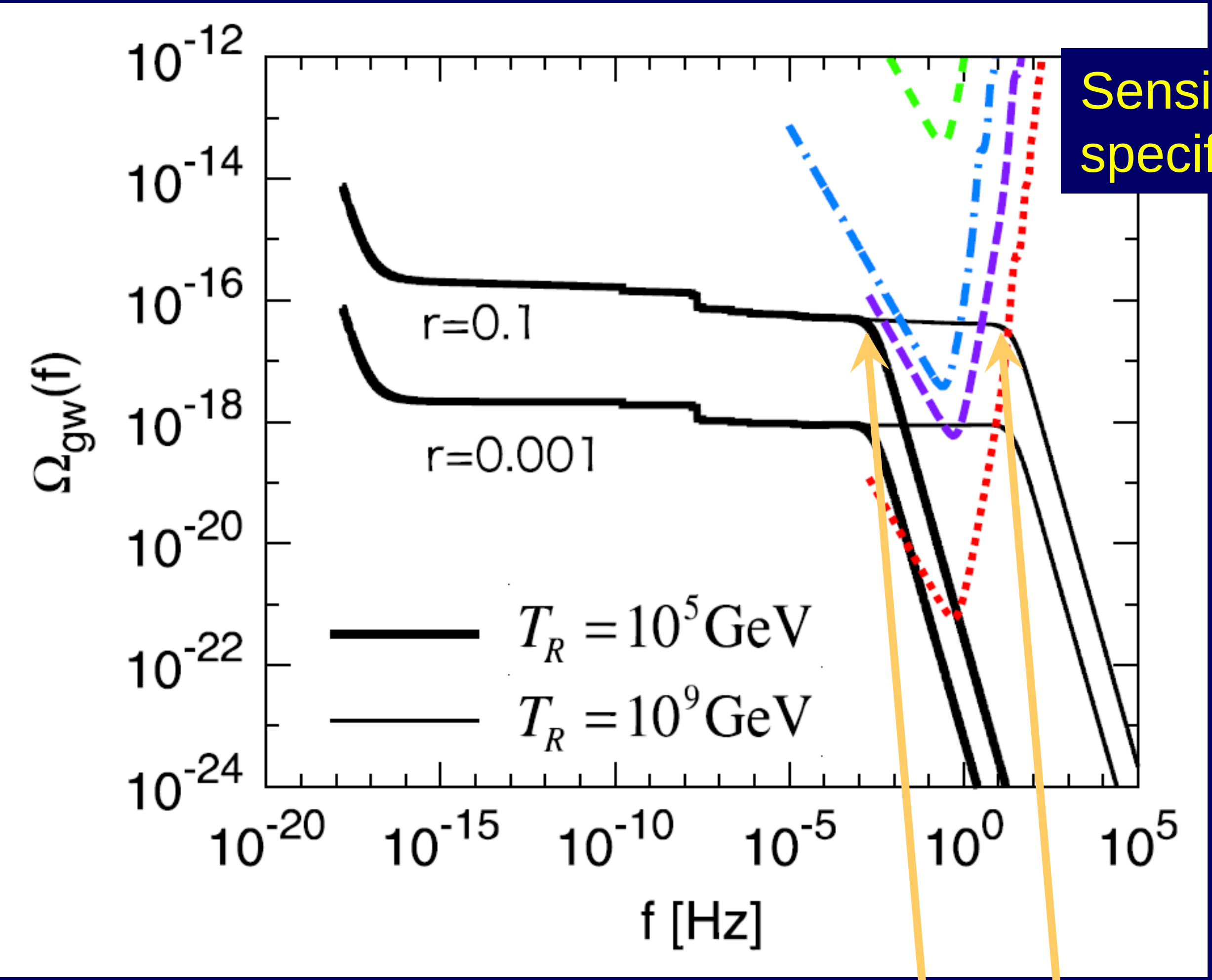
Standard potential-driven inflation models



Inflation is followed by a coherent oscillation of the inflaton which behaves like non-relativistic matter with EOS parameter $w=0$ if the mass term dominates the potential.

$$\Omega_{GW}(f) \propto \Delta_h^2(f)\, f^{\frac{6w-2}{3w+1}} \propto f^{-2}$$ in high frequency region

Thermal History is inprinted on the spectrum of GWs.



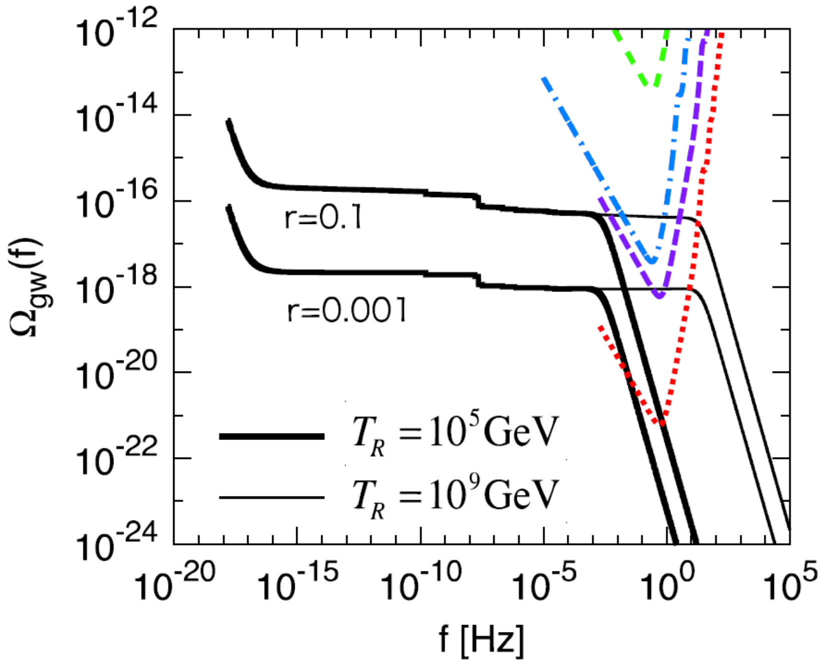
Sensitivity curves of various specifications of DECIGO

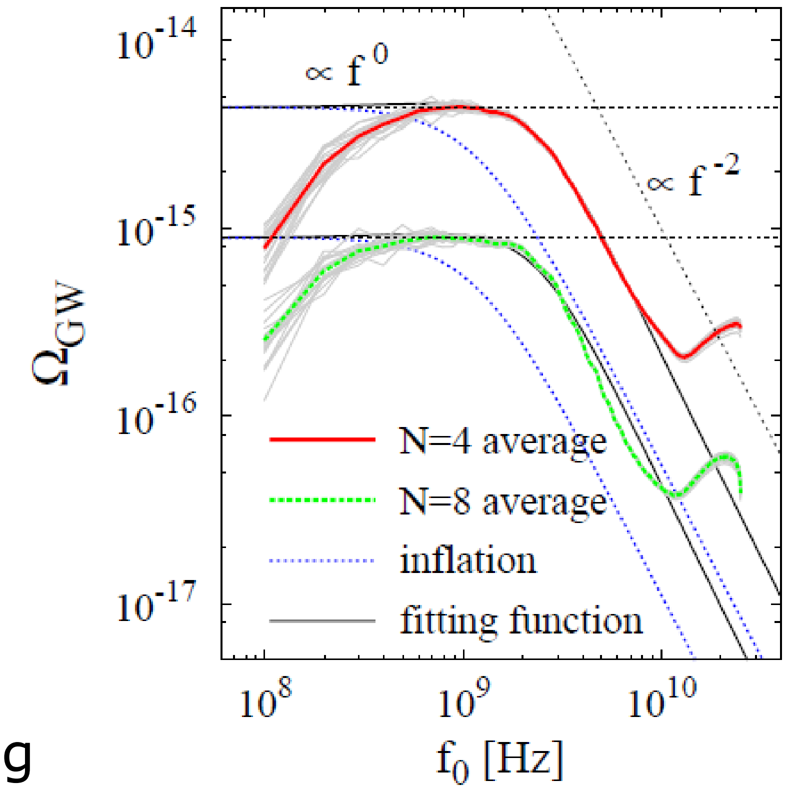
(Nakayama, Saito, Suwa & JY 08)

$$f_{\rm R} = \frac{k_{\rm R}}{2\pi a_0} \simeq 0.26{\rm Hz} \left(\frac{g_{*s}(T_{\rm R})}{106.75}\right)^{1/6} \left(\frac{T_{\rm R}}{10^7{\rm GeV}}\right)$$

characteristic frequency at reheating time

Comparison between inflationary tensor perturbations and GWs from self-ordering N-component scalar fields





Transfer functions due to reheating

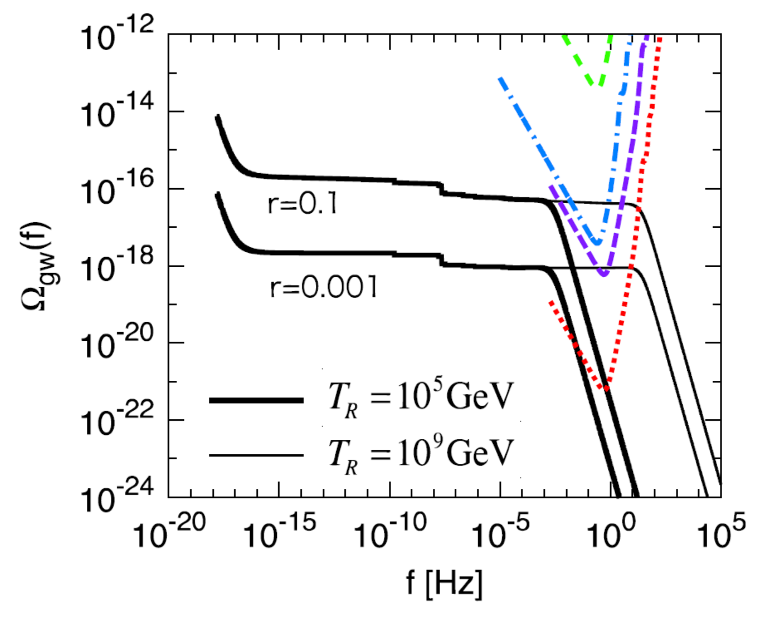
$$T^2_{\rm inflation}(x_R) = (1 - 0.22 x_R^{1.5} + 0.65 x_R^2)^{-1} \qquad T^2_{O(N)}(x_{R'}) = (1 - 0.6 x_{R'}^{1.5} + 0.65 x_{R'}^2)^{-1}$$

$$x_R = \frac{f}{f_R} \qquad f_R = 0.26 \left(\frac{g_*(T_R)}{106.75}\right)^{\frac{1}{6}} \left(\frac{T_R}{10^7\,\mathrm{GeV}}\right) \mathrm{Hz}$$

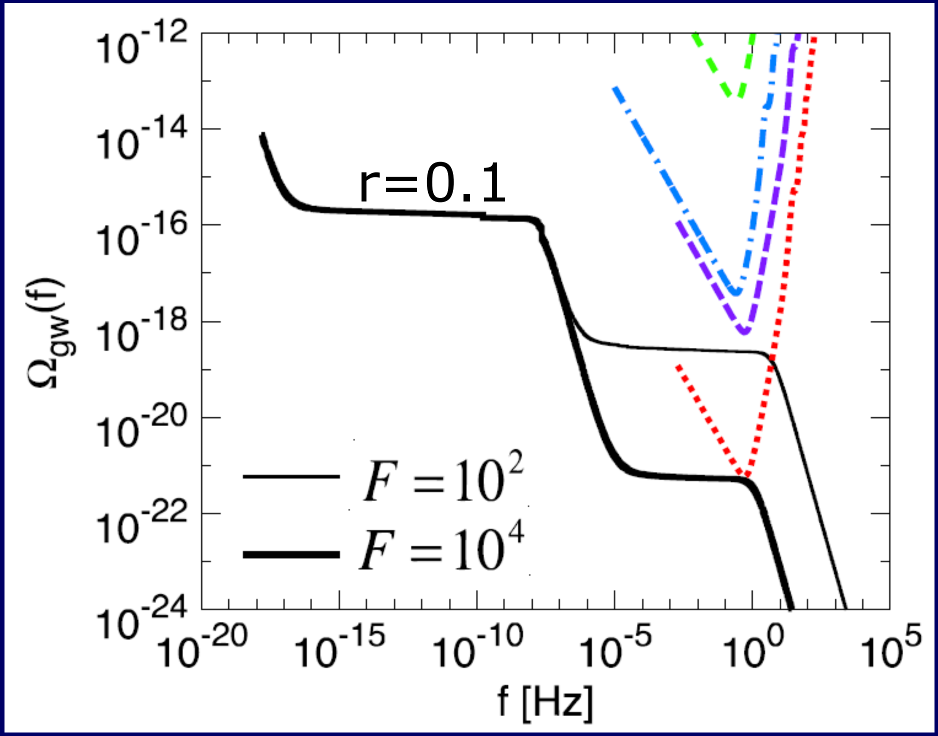
can be distinguished

(Kuroyanagi, Hiramatsu & JY 16)

If we could measure the GWs at two different frequencies, we could probe entropy production between two regimes, too.



No entropy production after reheating



Entropy production w/ dilution factor

$$F \equiv \frac{s(T_d)a^3(T_d)}{s(T_R)a^3(T_R)}$$

T_d: temperature after entropy production

High frequency part is modified as

$$\Omega_{GW}(f,t) \rightarrow F^{-4/3}\Omega_{GW}(f,t) \quad \text{and} \quad f_R \rightarrow F^{-1/3} f_R$$

If we could measure the GWs at two different frequencies, we could probe entropy production between two regimes, too.

We do not consider entropy production after reheating hereafter.

No

aft

ctor

$s(T_R)a^3(T_R)$

e after entropy production

High frequency part is modified as

$$\Omega_{GW}(f,t) \to F^{-4/3}\Omega_{GW}(f,t) \text{ and } f_R \to F^{-1/3} f_R$$

Broadband Approach to Inflationary Cosmology

Generalized G-inflation

The most general single-field inflation with second order field equations

$$S = \sum_{i=2}^{5} \int \mathcal{L}_i \sqrt{-g}\, d^4x$$

Generalized Galileon = Horndeski Theory

(Kobayashi, Yamaguchi & JY 2011)

4 arbitrary functions of ϕ and $X \equiv -\frac{1}{2}(\partial\phi)^2$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Generalized G-inflation

The most general single-field inflation with second order field equations

$$S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$$

Generalized Galileon = Horndeski Theory

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$G_4 \supset M_{Pl}^2/2$ gives the Einstein action

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

Generalized G-inflation is a framework to study the most general single-field inflation model with second-order field equations.

G-inflation model

$$K(\phi, X) - G(\phi, X)\Box\phi$$

Inflation driven by kinetic energy

Ex. G-inflation (k-inflation is its special case w/ G=0)

(Kobayashi, Yamaguchi & JY 10) (Armendariz-Picon, Damour & Mukhanov 99)

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \qquad X \equiv -\frac{1}{2}(\partial\phi)^2$$

canonical kinetic function

Energy momentum tensor

$$T_{\mu\nu} = (K_X - G_X)\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\left(K + \nabla_\lambda G \nabla^\lambda \phi\right) - 2\nabla_{(\nu} G \nabla_{\nu)}\phi$$

Flat RW background $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

$$T^\mu_\nu = \mathrm{diag}\left(-\rho, p, p, p\right) \text{ with } \rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X$$

$$p = K - 2\left(G_\phi + G_X\ddot{\phi}\right)X$$

leading to

$$3M_{Pl}^2 H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + 2\dot{H}\right) = -p$$

Simplest de Sitter k-inflation Solution

- We seek for a solution with $H = \text{const.}$ and $\dot{\phi} = \text{const.}$ in the simplest k-inflation model with

$$K(\phi, X) \equiv K(X), \quad G(\phi, X) \equiv 0.$$

No scalar potential

shift symmetry $\phi \to \phi + c$

- $3M_{Pl}^2 H^2 = \rho = 2K_X X - K, \quad M_{Pl}^2\left(3H^2 + 2\dot{H}\right) = -p = -K$

A simple choice $K(X) \equiv -X + \dfrac{X^2}{2M^4}$ ⇒ $K_X = 0$ at $X = M^4$

We find de Sitter solution $H^2 = \dfrac{M^4}{6M_{Pl}^2}$ with $p = -\rho = -\dfrac{M^4}{2}$.

(Armendariz-Picon, Damour & Mukhanov 99)

G-de Sitter solution can also be found with $G(\phi, X) \neq 0$.

(Kobayashi, Yamaguchi & JY 10)

Essential difference between k-inflation & G-inflation

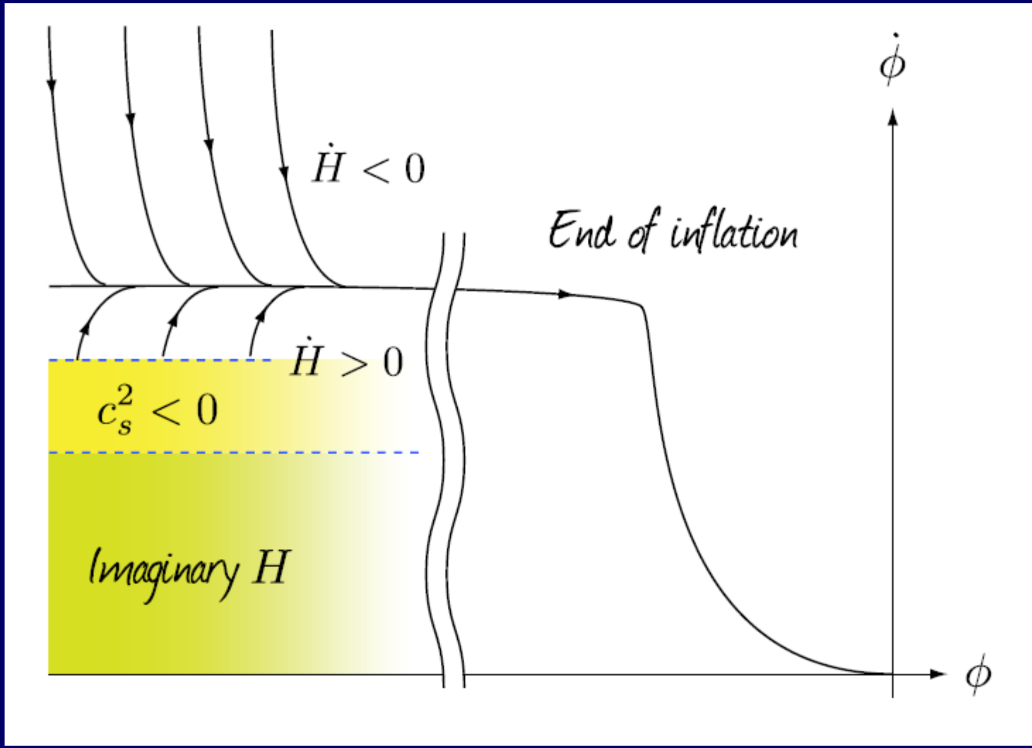
In G-inflation, the null energy condition may be violated, $2M_{Pl}^2\dot{H} = -(\rho + p) > 0.$

It can be violated without instabilities, keeping $c_s^2 > 0.$

The tensor spectral index can be positive,

$$n_T = -2\varepsilon = 2\frac{\dot{H}}{H^2} > 0.$$

Short wave tensor fluctuations may have a larger amplitude at formation.

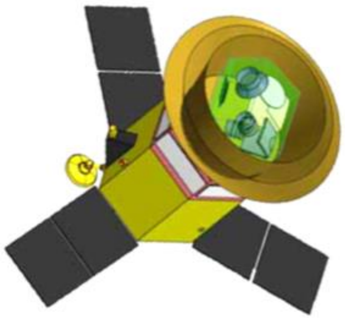


NB. In k-inflation, null energy condition cannot be violated, since it would cause gradient instability.

Many people say
positive tensor
spectral in…
wou… …tely
falsi… …nflation.

This is not true.

We hope LiteBIRD will measure $n_t>0$!!



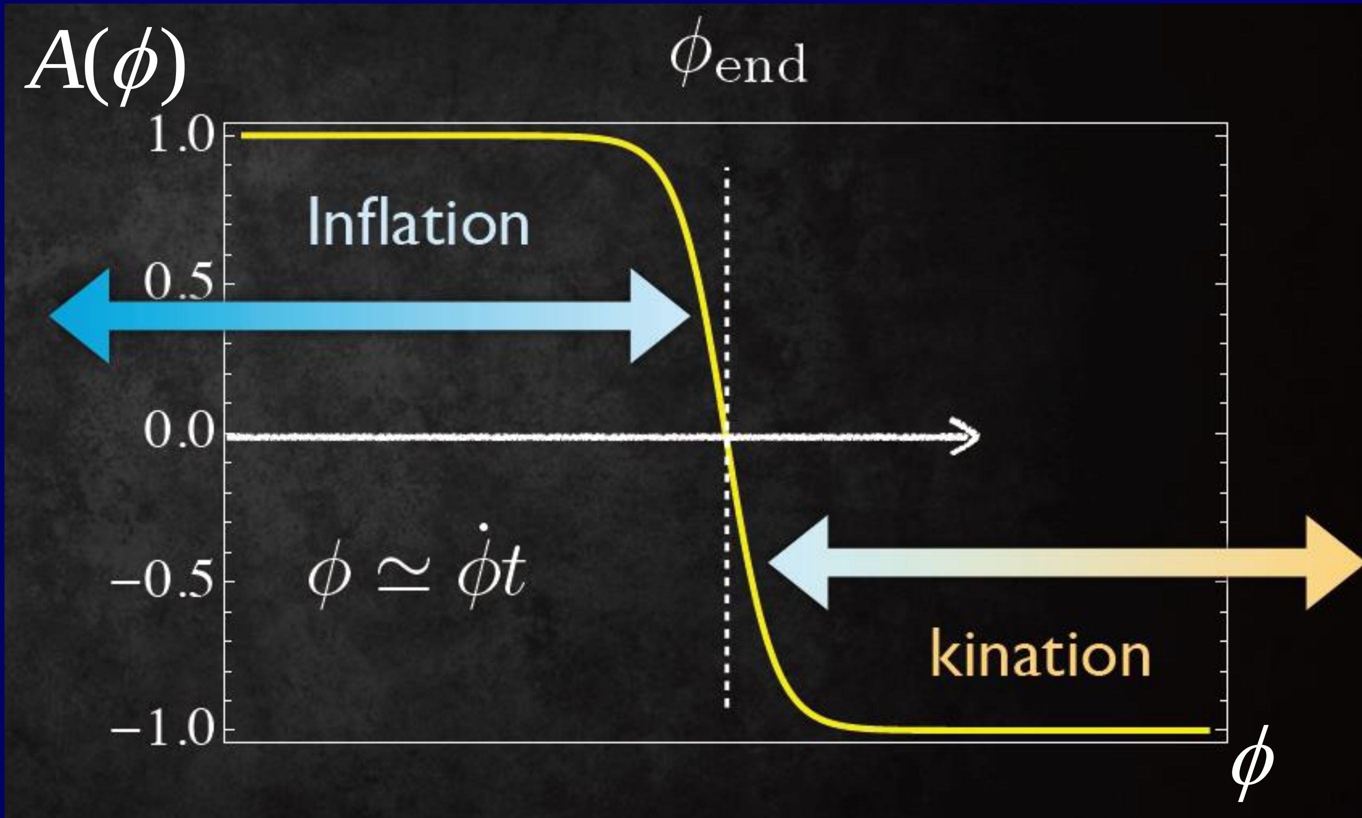
★ Inflation can be terminated by flipping the sign of $-X$.

$$K(\phi, X) \equiv -A(\phi)X + \frac{X^2}{2M^3\mu}$$

($K \cong -\rho < 0$ during inflation.)

A simple choice: $A(\phi) \equiv \tanh\left[\lambda\left(\phi_{end} - \phi\right)/M_{Pl}\right]$ with $\lambda = O(1)$.

Numerical solutions indicate ϕ stalls within one e-fold after crossing ϕ_{end} and all higher derivative terms become negligible.



The Universe after G-inflation

- After inflation the Universe is dominated by the kinetic energy of ϕ, which now behaves as a free massless field,

$$\rho = \frac{\dot{\phi}^2}{2} \propto a^{-6}(t). \qquad w = 1$$

$$\Omega_{GW}(f) \propto \Delta_h^2(f)\, f^{\frac{6w-2}{3w+1}} \propto f \, □$$ in high frequency region

(Chiba, Tashiro & Sasaki 04)

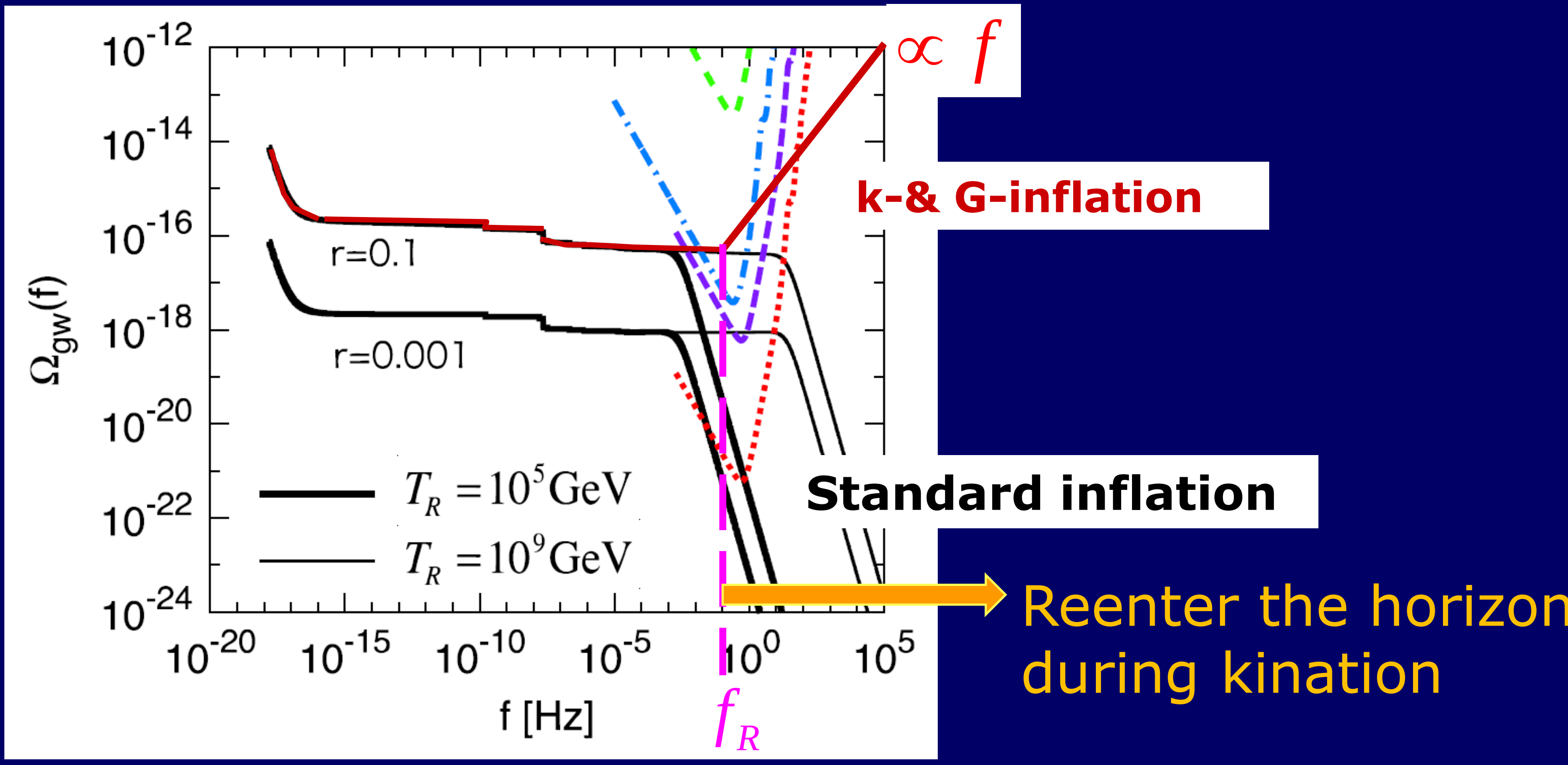
- Shift symmetry of the Lagrangian prevents direct inter-action between ϕ and standard particles.

- Reheating proceeds through gravitational particle production due to the change of the cosmic expansion law: $a(t) \propto e^{H_{\inf} t} \to a(t) \propto t^{\frac{1}{3}}$.

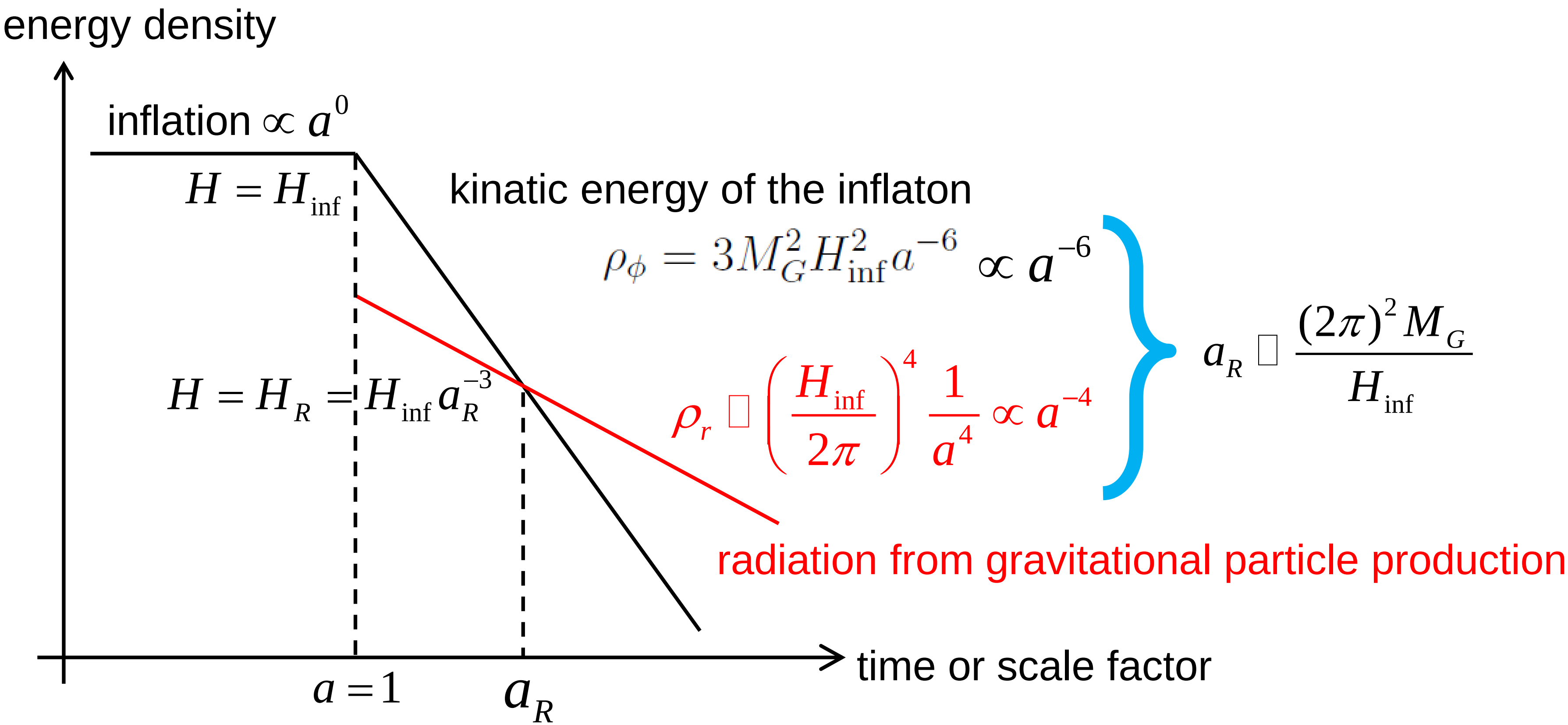
  This process may create radiation energy density of order of (the Hawking temperature)^4, namely, $\rho_r \, □ \, T_H{}^4 = \left(H_{\inf}/2\pi\right)^4$.
  (We return to this issue more in detail later.)

(Ford 87, Kunimitsu & JY 12)

High frequency tensor perturbation is enhanced.

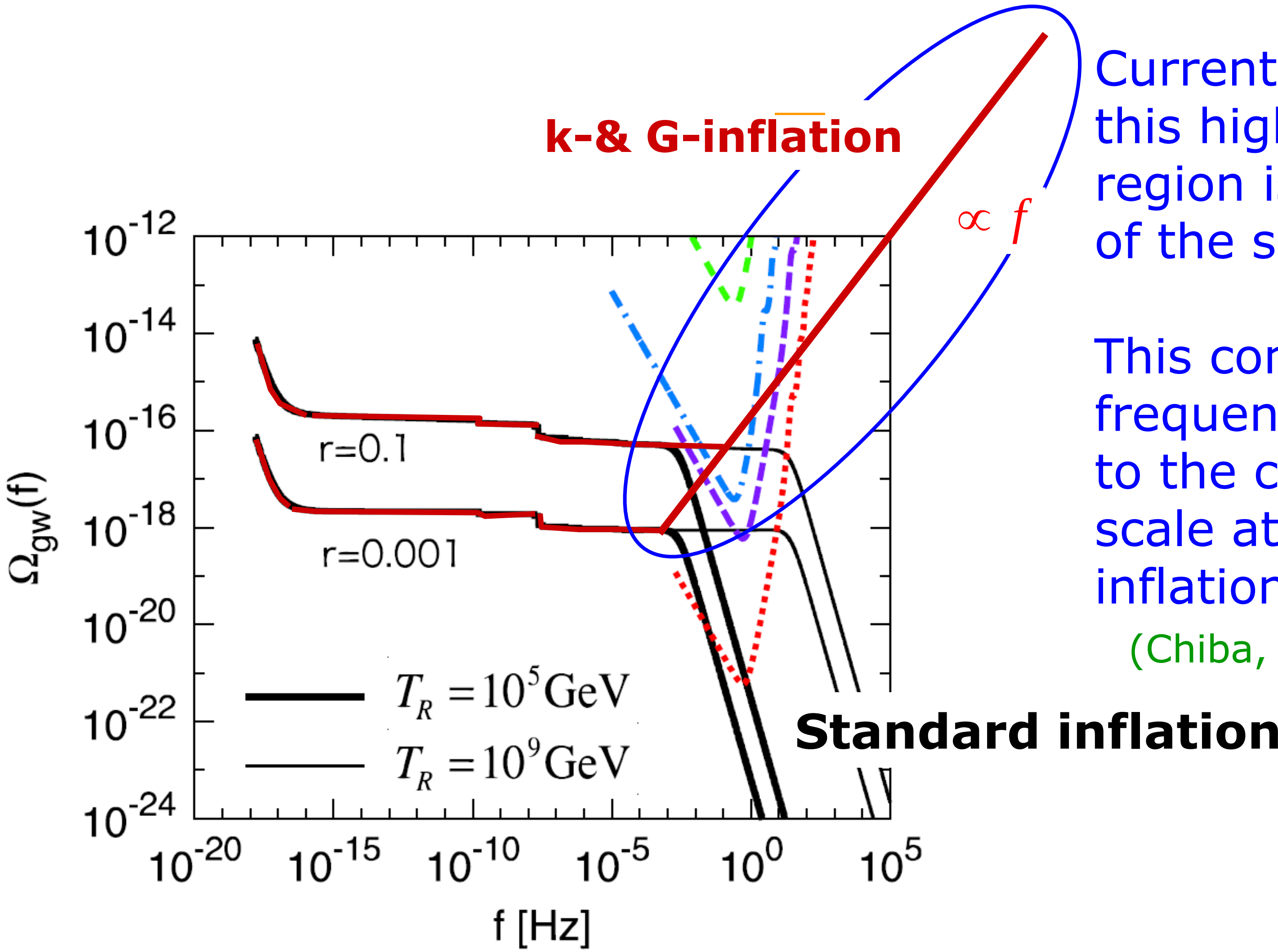


f_R : Frequency that reentered the horizon at reheating



$$f_R = \frac{a_R H_R}{2\pi a_0} \propto \frac{T_0}{T_R} H_R \propto \frac{T_0}{(H_R M_G)^{1/2}} H_R \propto H_R^{1/2} \propto H_{\inf}^2$$

$f > f_R$ $\quad \Omega_{GW}(f) \simeq \Delta_h^2(f) \frac{f}{f_R} \propto H_{\inf}^2 \frac{f}{H_{\inf}^2}$ is independent of $H_{\inf}$



Current amplitude in this high frequency region is independent of the scale of inflation.

This continues to the frequency corresponding to the comoving horizon scale at the end of inflation.

(Chiba, Tashiro & Sasaki 04)

Hence we study creation of gravitons around the horizon scale at the end of inflation more in detail.

Gravitational Particle Production from Inflation to Kination

A scalar field χ with $\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$ in $ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$.

Its mode function satisfies $\frac{d^2\chi_k}{d\eta^2} + \left[k^2 - V(\eta)\right]\chi_k = 0$

$$V(\eta) = -a^2(\eta)\left[m_\chi^2 + \left(\xi - \tfrac{1}{6}\right)R(\eta)\right] \xrightarrow{\xi = 0, m_\chi = 0} V(\eta) = \tfrac{1}{6}a^2(\eta)R(\eta)$$

$$\chi_k(\eta) = \chi_k^{(in)}(\eta) + \frac{1}{\omega}\int_{-\infty}^{\eta} V(\eta')\sin\omega(\eta - \eta')\chi_k(\eta')d\eta'$$

$$\chi_k^{(in)}(\eta) = \frac{e^{-i\omega\eta}}{\sqrt{2\omega}} \quad (\eta \to -\infty) \qquad \chi_k(\eta) = \frac{1}{\sqrt{2\omega}}\left(\alpha_k e^{-i\omega\eta} + \beta_k e^{i\omega\eta}\right) \quad (\eta \to \infty)$$

Bogoliubov coefficient $\beta_\omega = \frac{i}{2\omega}\int_{-\infty}^{\infty} e^{-2i\omega\eta}V(\eta)d\eta$

"Radiation" energy density $\rho_r = \frac{1}{2\pi^2 a^4}\int_0^\infty |\beta_\omega|^2\omega^3 d\omega.$

Gravitational Particle Production from Inflation to Kination

$$\rho_r = -\frac{1}{32\pi^2 a^4}\int_{-\infty}^{\eta_0} d\eta_1 \int_{-\infty}^{\eta_0} d\eta_2 \ln(|\eta_1 - \eta_2|\mu) V'(\eta_1) V'(\eta_2)$$

Energy density of created massless minimally coupled field

$$\rho_r = \frac{H_{\inf}^4}{128\pi^2 a^4} I, \qquad I = -\int_{-\infty}^{x} dx_1 \int_{-\infty}^{x} dx_2 \ln(|x_1 - x_2|) \tilde{V}'(x_1) \tilde{V}'(x_2)$$

where $x \equiv H_{\inf}\eta$

$$\tilde{V}(x) = \frac{f'' f - \frac{1}{2}(f')^2}{f^2} \qquad f(x) \equiv f(H_{\inf}\eta) \equiv a^2(\eta)$$

We model the transition from inflation to kination as

$$f(x) = \begin{cases} 1/x^2 & \text{De Sitter} \quad (x < -1) \\ a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 & (-1 < x < -1 + x_0) \\ b_0 (x + b_1) & \text{Kination} \quad (-1 + x_0 < x) \end{cases}$$

continuous transition up to $f^{(3)}$

We find $I \simeq 50 x_0^{-0.262}$ $\qquad x_0 \cong H_{\inf}\Delta t \approx 1$

$$\rho_r \simeq \frac{9H_{\inf}^4}{64\pi^2 a^4}$$

(Nakama & JY 19)

Since each polarization mode of the graviton satisfies the same field equation as a minimally coupled massless scalar field.

$$\rho_{graviton} = \rho_{GW} \simeq \frac{9H_{\rm inf}^4}{32\pi^2 a^4} \qquad a = 1 \text{ at the end of inflation}$$

It behaves as dark radiation similar to additional sterile massless neutrinos only to speed up cosmic expansion.

The energy density of GW is quantified in terms of an extra effective species of neutrinos, $N_{\rm eff,GW}$, and constrained by Big Bang Nucleosynthesis (BBN) and Cosmic Microwave Background (CMB) anisotropy.

The effective number appears in the expression of the total radiation energy density as follows.

$$\rho_{\mathrm{tot}} = \frac{\pi^2}{30}\left(2 + \frac{7}{8}\cdot 2\cdot\left(\frac{4}{11}\right)^{4\epsilon/3}\left[2(1-\epsilon) + N_\nu + N_{\mathrm{eff,GW}}\right]\right) T^4$$

$\varepsilon = 0$: at BBN (before neutrino decoupling)

$\varepsilon = 1$: at photon decoupling $N_\nu = 3 + 0.046\epsilon,$

$$\left.\frac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{rad}}}\right|_{t>t_{\mathrm{th}}}\left(=\left[\frac{g_*}{g_*(t_{\mathrm{th}})}\right]^{1/3}\left.\left(\frac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{rad}}}\right)\right|_{t=t_{\mathrm{th}}}\right) = g_*^{-1}\cdot\frac{7}{4}\left(\frac{4}{11}\right)^{4\epsilon/3} N_{\mathrm{eff,GW}}$$

At BBN $N_{\mathrm{eff,GW}} = 2.86\dfrac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{rad}}}$ $g_* = 10.75$

At photon decoupling $N_{\mathrm{eff,GW}} = 2.36\dfrac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{rad}}}$ $g_* = 3.38$

Big Bang Nucleosynthesis constraint $N_{\mathrm{eff,GW}} = 2.86 \frac{r_{GW}}{r_{\mathrm{rad}}} < 1.65$

CMB power spectrum (Planck) constraint $N_{\mathrm{eff,GW}} = 2.36 \frac{r_{GW}}{r_{\mathrm{rad}}} < 0.72$

In order to achieve reheating through minimally coupled massless scalar field χ, for which we find $\rho_\chi = \rho_{\mathrm{GW}}/2$, we need 7 or more species of them.

Since massless particles cannot decay, thermalization must proceed by scattering.
But χ field acquires a large vev during inflation due to quantum fluctuations, which means that particles coupled to χ are heavy if they are coupled to χ with sufficient strength.
Otherwise, they cannot thermalize and remain as a dark radiation.

Two working scenarios of gravitational reheating that can evade GW constraints with bonuses:

I. The universe is reheated by decay of long-lived massive scalar particles “A” and dark matter is explained by another heavy scalar particle “X”, both of which are created gravitationally.
This serves as an ideal realization mechanism of the PGDM (Purely Gravitational Dark Matter) scenario.

Garny, Sandora & Sloth (2016) Tang & Wu (2016) Ema, Nakayama & Tang (2018)

Hashiba & JY PRD99(2019)043008

II. Three species of massive right-handed neutrinos are created gravitationally, reheat the universe, generate lepton asymmetry and dark matter, evading the GW constraint.

Hashiba & JY 1905.12423

We consider reheating through production of massive (conformally coupled) scalar fields $\chi = A, X$.

A scalar field χ with $\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$ in $ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$.

Its mode function satisfies $\frac{d^2\chi_k}{d\eta^2} + \left[k^2 - V(\eta)\right]\chi_k = 0$

$$V(\eta) = -a^2(\eta)\left[m_\chi^2 + \left(\xi - \tfrac{1}{6}\right)R(\eta)\right] \xrightarrow{\xi = \frac{1}{6},\, m_\chi = m_{A,X}} V(\eta) = -a^2(\eta)m_{A,X}^2$$

Inflation

$$\frac{d^2\chi_k}{d\eta^2} + \left(k^2 + \frac{m^2}{H_{\rm inf}^2\eta^2}\right)\chi_k = 0$$

Bunch-Davies vacuum

$$\chi_k^{\rm BD}(\eta) = \frac{\sqrt{-\pi\eta}}{2}e^{-i\frac{2\nu+1}{4}\pi}H_\nu^{(1)}(-k\eta)$$

Kination

$$\frac{d^2\chi_k}{d\eta^2} + \left[k^2 + m^2(2H_{\rm inf}\eta + 3)\right]\chi_k = 0$$

Kination vacuum

$$\chi_k^{\rm K}(\eta) = \sqrt{\frac{\pi}{6}}\,(2m^2H_{\rm inf})^{-1/6}\exp\left[\left(\frac{k^3}{3m^2H_{\rm inf}} + \frac{3k}{2H_{\rm inf}} - \frac{5}{12}\pi\right)i\right]\sqrt{x}H_{1/3}^{(2)}\left(\frac{2}{3}x^{3/2}\right)$$

$$x(k,\eta) = \frac{k^2 + 3m^2 + 2m^2H_{\rm inf}\eta}{(2m^2H_{\rm inf})^{2/3}}$$

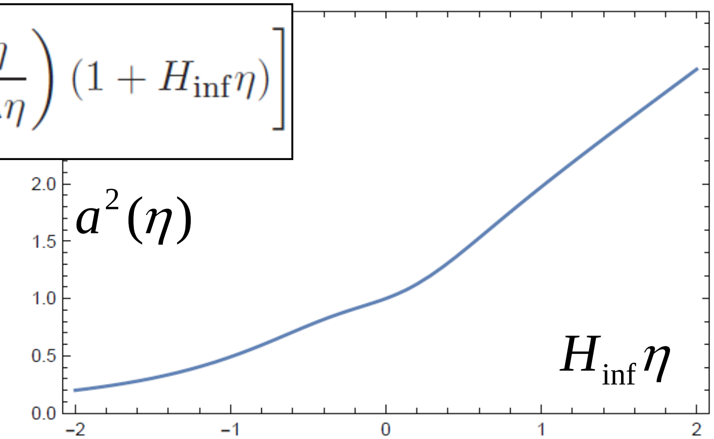
$$\chi_k(\eta) = \begin{cases} \chi_k^{\rm BD}(\eta) & \text{Inflation} \\ \alpha_k\chi_k^{\rm K}(\eta) + \beta_k\chi_k^{\rm K*}(\eta) & \text{Kination} \end{cases}$$

Bogoliubov coefficients

The mode function is numerically solved throughout a smooth transition from inflation to kination.

$$a^2(\eta) = \frac{1}{2}\left[\left(1 - \tanh\frac{\eta}{\Delta\eta}\right)\frac{1}{1 + H_{\rm inf}^2\eta^2} + \left(1 + \tanh\frac{\eta}{\Delta\eta}\right)(1 + H_{\rm inf}\eta)\right]$$

$\Delta\eta = 0.5 H_{\rm inf}^{-1}$ ⇨



to calculate b_k and obtain

$$\rho_\chi = \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3}\sqrt{m^2 + k^2}|\beta_k|^2$$

We have numerically calculated many different cases

A. vary $H_{\rm inf}$, $(\Delta\eta, m)$ while fixing $H_{\rm inf}\Delta\eta$ and $m/H_{\rm inf}$.

B. vary $H_{\rm inf}$ while fixing $\Delta\eta$ and m.

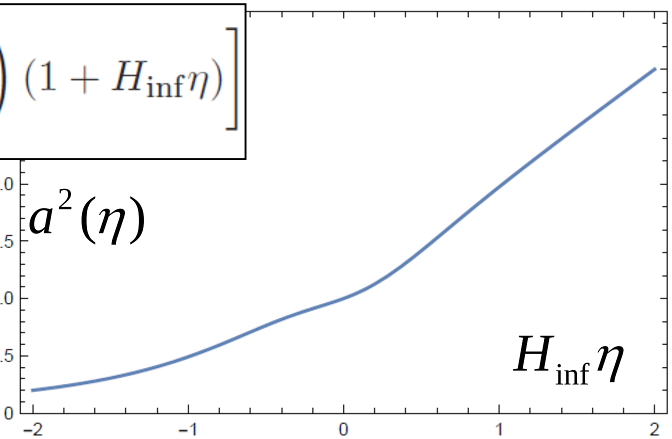
C. vary m while fixing $\Delta\eta$ and $H_{\rm inf}$.

D. vary $\Delta\eta$ while fixing m and $H_{\rm inf}$.

The mode function is numerically solved throughout a smooth transition from inflation to kination.

$$a^2(\eta) = \frac{1}{2}\left[\left(1 - \tanh\frac{\eta}{\Delta\eta}\right)\frac{1}{1 + H_{\rm inf}^2\eta^2} + \left(1 + \tanh\frac{\eta}{\Delta\eta}\right)(1 + H_{\rm inf}\eta)\right]$$

$\Delta\eta = 0.5 H_{\rm inf}^{-1}$ ⇨



to obtain

$$\rho_\chi = \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3}\sqrt{m^2 + k^2}|\beta_k|^2$$

$c = A, X$

We have numerically calculated many different cases and found that the results are well approximated as

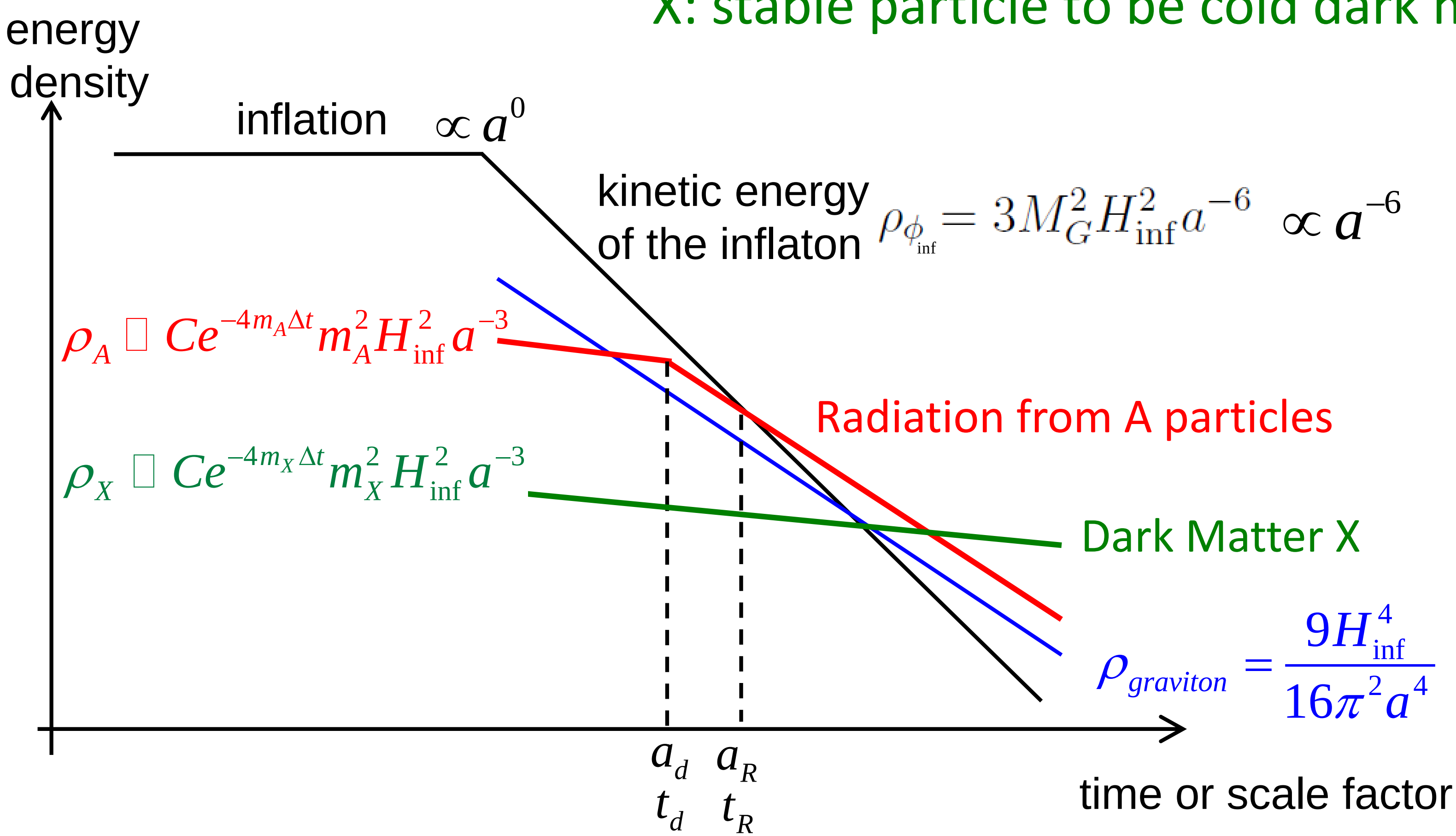
$$r_c = C e^{-4 m_c \Delta t} m_c^2 H_{\rm inf}^2 a^{-3}(t)$$

$$C \simeq 2\times 10^{-4}$$

$$\Delta t \simeq H_{\rm inf}^{-1}$$

★ We consider gravitational particle production of two conformally coupled massive scalar. A: decay into radiation to reheat the universe

X: stable particle to be cold dark matter today



A particles decay at $a = a_d$ to radiation with the decay rate $\Gamma_A \equiv \alpha m_A$.

The Universe became radiation dominant at $a = a_R$: reheating time.

★ The dominant part of the entropy is produced at t_d , when we find

$$s|_d = \frac{2\pi^2}{45} g_{*d} T_d^3 \quad \text{with} \quad T_d = 5 \times 10^{-2} \alpha^{1/4} e^{-m_A \Delta t} m_A^{3/4} H_{\rm inf}^{1/4},$$

$$g_{*d} = 106.75$$

$$\rho_X|_d = C\alpha e^{-4m_X \Delta t} m_A m_X^2 H_{\rm inf}.$$

$$\Rightarrow \quad \frac{\rho_X}{s} = 4 \times 10^{-2} \alpha^{1/4} e^{(3m_A - 4m_X)\Delta t} \frac{m_X^2 H_{\rm inf}^{1/4}}{m_A^{5/4}} \approx 4 \times 10^{-10}\ \mathrm{GeV}$$

should hold to explain CDM

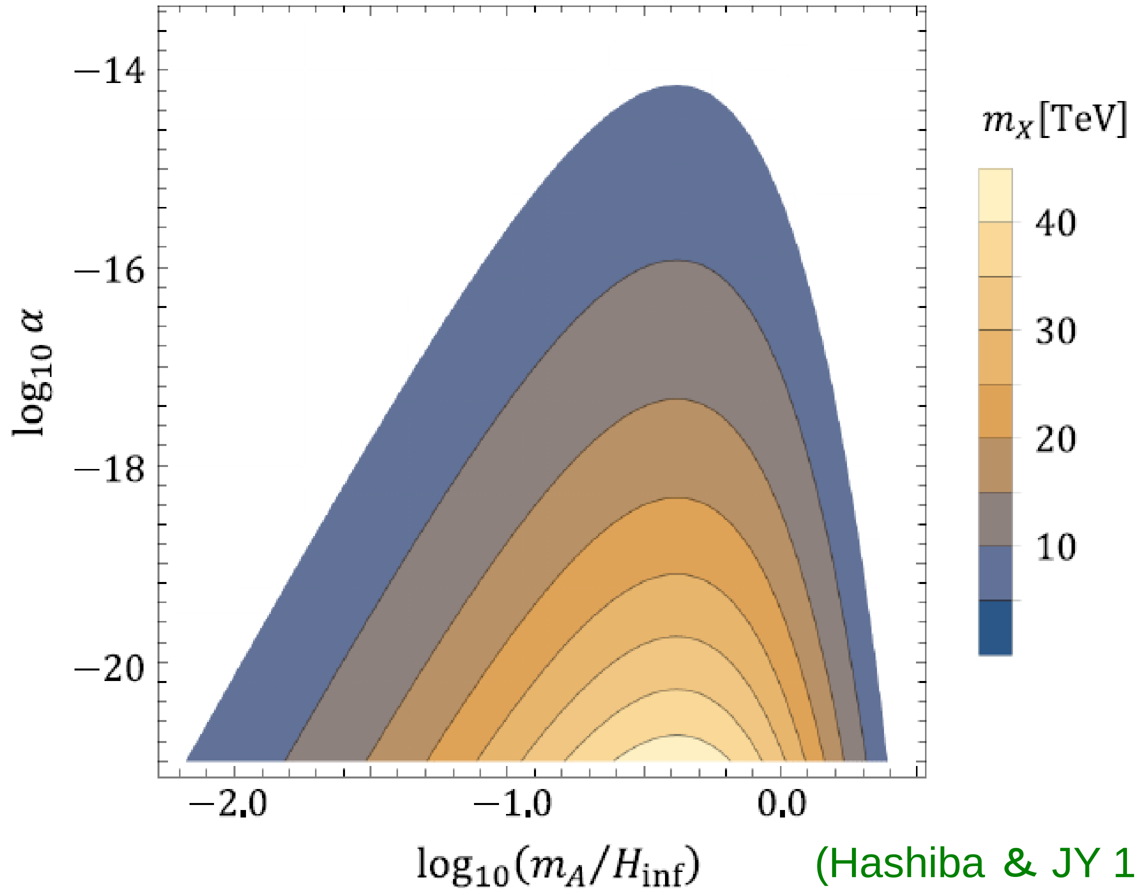
★ Gravitons (quantum GWs) must be sufficiently diluted

$$N_{\rm eff,GW} = 2.36 \frac{r_{GW}}{r_{\rm rad}} = 2.36 \left.\frac{r_{GW}}{r_A}\right|_d < 0.72$$

$$\Rightarrow \quad \alpha^{-1/3} e^{-4m_A \Delta t} \left(\frac{m_A}{H_{\rm inf}}\right)^{5/3} > 2.3 \times 10^3.$$

A severe constraint on a in $G_A = a m_A$

Allowed region to explain CDM and reheating



(Hashiba & JY 19)

$\Gamma_A = \alpha m_A$ with $\alpha \sim 10^{-14}$ can be easily realized if *A* is coupled to the standard model with a Planck-suppressed interaction $e^{-\lambda \frac{A}{M_{Pl}}} \mathscr{L}_{std}$ with $\lambda = O(1)$.

This model is a nightmare scenario for particle DM experiments as our DM interacts only gravitationally.

There is an astrophysical implication, though;
Comoving free streaming scale at equality time t_{eq} is very small

$$l_{fs} = \int_{t_*}^{t_{eq}} \frac{v(t)}{a(t)} \simeq \frac{c t_{eq}}{a_{eq} T_*} \ln\left(\frac{T_*}{T_{eq}}\right) \qquad T_* \geq T_R$$

and the minimum mass of DM halo is also very small.

$M_{\min} \simeq 10^{-15} M_{\odot}$ for $T_* = 1\mathrm{TeV}$ and even smaller for higher temperature.

This may be tested by pulsar timing measurements.

Kashiyama & Oguri (2018)

Key Question:

What are A and X, after all?

II Neutrinos produce everything!

Hashiba & JY 1905.12423

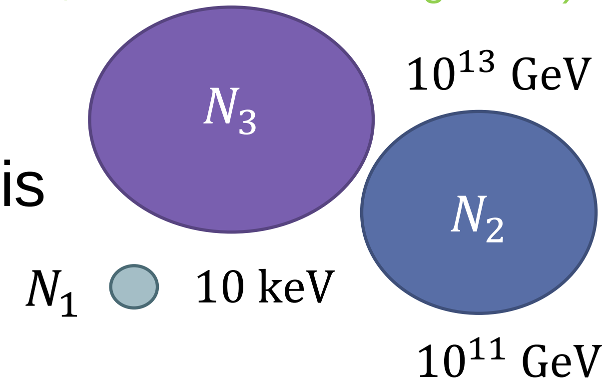
- We consider 3 hierarchical right-handed neutrinos

(Kusenko, Takahashi & Yanagida 10)

$N_3 : M_3 \sim 10^{13}$ GeV → Reheating

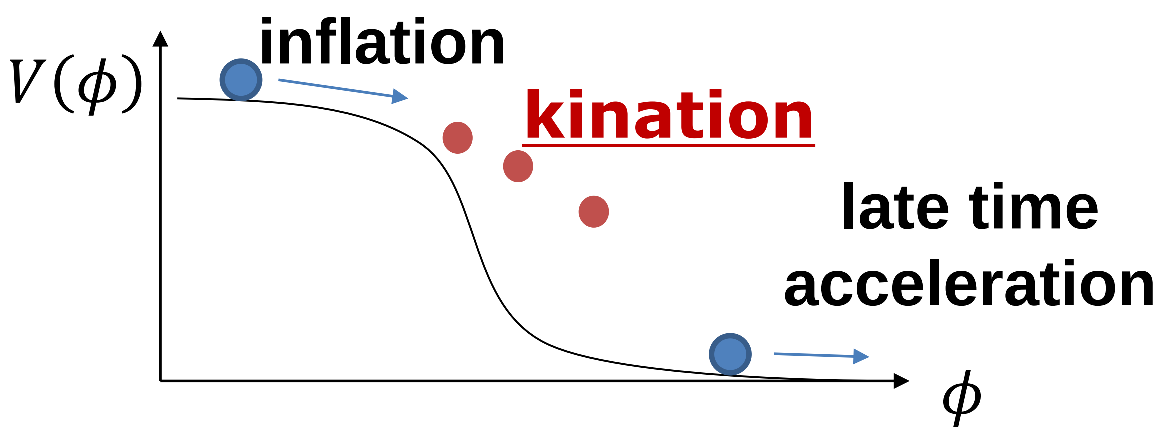
$N_2 : M_2 \sim 10^{11}$ GeV → Baryogenesis

$N_1 : M_1 \sim 10$ keV → Dark matter



- $$\mathcal{L}_N = M_i \bar{N}_i^c N_i + h_{i\alpha} N_i L_\alpha H^\dagger$$

- Quintessential inflation which is followed by kination



(Peebles & Vilenkin 99)

Gravitational particle creation of fermions

- Produced fermion energy density

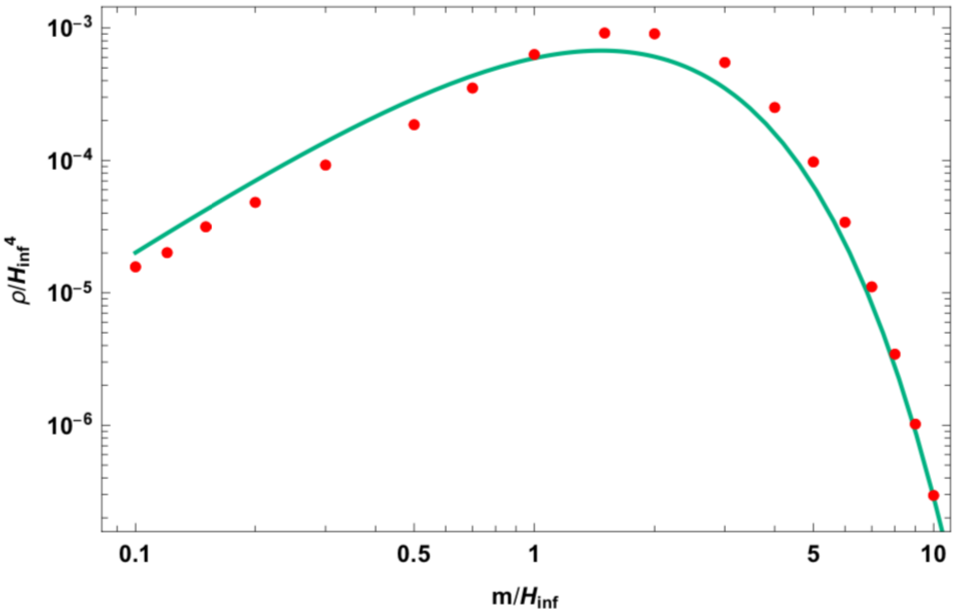
$$\rho \cong C' e^{-4m\Delta t} m^2 H_{\rm inf}^2 a^{-3}$$

m : Fermion mass

Δt : Transition time scale

$H_{\rm inf}$: Hubble parameter during inflation

$$C' \cong 2 \times 10^{-3} \approx 10 C_{\rm scalar}$$



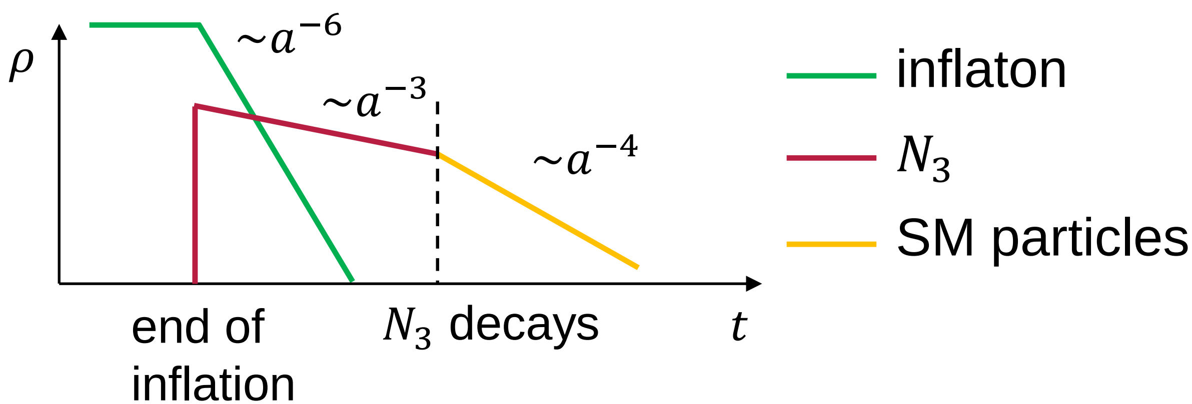
$\boldsymbol{N_3}$ for reheating

- Decay of N_3

  N_3 decays into SM particles with decay rate Γ_3

$$\Gamma_3 = \frac{1}{4\pi} \sum_{\alpha} \left| \tilde{h}_{i\alpha} \right|^2 M_3$$

  Since N_3 is much heavier than any SM particles, resultant SM particles are relativistic



- Reheating temperature

$T_{RH} \cong 6$

- Gravitons (quantum GWs) must be sufficiently diluted

$$N_{\rm eff,GW} = 2.36 \frac{r_{GW}}{r_{\rm rad}} = 2.36 \frac{r_{GW}}{r_A}\Bigg|_d < 0.72$$

$$\Rightarrow \left(\sum_\alpha \left|\tilde{h}_{3\alpha}\right|^2\right)^{-\frac{1}{3}} e^{-4M_3\Delta t}\left(\frac{M_3}{H_{\rm inf}}\right)^{\frac{5}{3}} > 1.0 \times 10^2$$

$$\Rightarrow \boldsymbol{\sum_\alpha \left|\tilde{h}_{3\alpha}\right|^2 < 8.5 \times 10^{-11}}$$

~ Yukawa coupling of electron

Baryogenesis through leptogenesis

(Fukugita & Yanagida 86)

$$\frac{n_B}{s} = \frac{28}{79}\frac{n_L}{s}$$

$$\approx 1 \times 10^{-3} \frac{\mathrm{Im}\left[\left\{\left(\tilde{h}\tilde{h}^{\dagger}\right)_{32}\right\}^2\right]}{\left(\tilde{h}\tilde{h}^{\dagger}\right)_{33}} \left(e^{-M_3 \Delta t} \ln \frac{M_3}{M_2}\right) \left(\sum_{\alpha} \left|\tilde{h}_{3\alpha}\right|^2\right)^{\frac{1}{4}} \frac{M_2}{M_3} \left(\frac{M_3}{H_{\mathrm{inf}}}\right)^{-\frac{1}{4}}$$

$$= 9 \times 10^{-11}$$

CP violation

Interference between N_3 & N_2

with δ the phase caus[...] violation.
We apply the dela[...]y mechanı[...] [8] tc[...]-
nerate the baryon asy[...]y ın the Universe. The
[...]t-of-equilıbrium co[...]ıs satısfıed, ıf the temp-
[...]ture T is smaller t[...]r[...]s M_1 so t[...] the in-

H H H N_3 N_3 N_2

$$M_2 \gtrsim 10^{11}\ \mathrm{GeV} \quad \text{and} \quad \tilde{h}_{22} \text{ or } \tilde{h}_{23} \gtrsim 10^{-3}\sqrt{M_3/M_2}$$

N_1 For Dark Matter in split seasaw scenario

(Kusenko, Takahashi & Yanagida 10)

$\sim 10\text{keV}$ sterile neutrino can account for whole dark matter!

$$\theta^2 \simeq \sum_\alpha \left|\tilde{h}_{1\alpha}\right|^2 \frac{v^2}{2M_1^2} \sim 10^{-11}$$

$$v = 246\text{GeV}$$

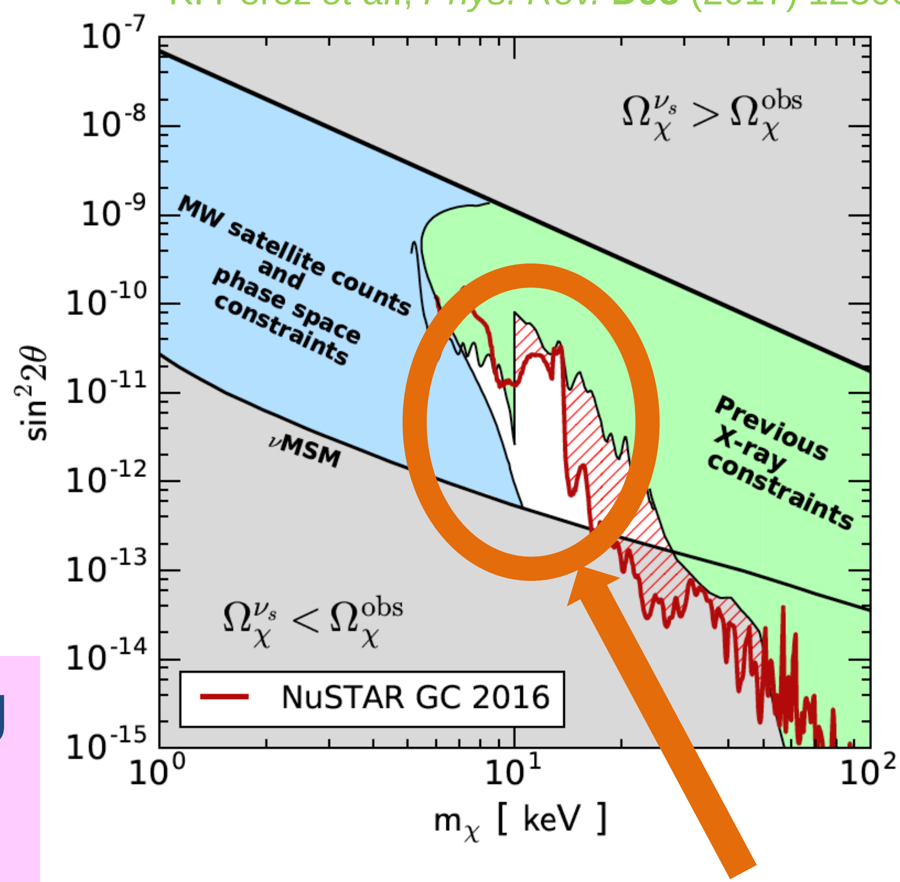
For $M_1 \sim 10$ keV,

$$\sum_\alpha \left|\tilde{\boldsymbol{h}}_{1\alpha}\right|^2 < 10^{-26}$$

This small Yukawa coupling can be naturally explained in Randall-Sundrum type Brance world scenario.

Allowed region for sterile neutrino dark matter

K. Perez *et al.*, *Phys. Rev.* **D95** (2017) 123002.



$M_{N_1} \gg 10\text{keV}$

Since the mass of N_1 is too light, its gravitational production is too inefficient to account for dark matter.

This problem can be solved by introducing a coupling to the scalar curvature as

$$\frac{R}{\mu}\bar{\psi}\psi \qquad \mu \text{ : constant with unit mass dimension}$$

Then the abundance is given by $n \cong 1.1 \times 10^{-1} H_{\rm inf}^5/\mu^2$ right after inflation with $\Delta t \approx H_{\rm inf}^{-1}$.

Taking $m \gg 10^{15}\,\mathrm{GeV}$, we can realize sufficient production for dark matter.

Conclusion

Cosmological gravitational wave is a useful probe of the early universe even if it is not detected yet.