I. Gravitational Wave Production from Inflaton Collapse
II. Stringent Limits on Cosmic Magnetic Fields from the CMBR

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I. Gravitational wave production from Inflaton Collapse

KJ, J. Martin, M. Lemoine, JCAP 1009 (2010) 034 and JCAP 1004 (2010) 021

Motivation

- After inflation the Universe is dominated by a scalar field
- Reheating occurs when this scalar field decays
- The reheating temperature may be low and/or the inflationary scale may be high

-> interested in this potentially long period between the end of inflation and reheating

Evolution of perturbations on sub-Hubble scales

much is known about the evolution of super-Hubble perturbations during this phase, but not much of sub-Hubble scales

longitudinal gauge

$$\mathrm{d}s^2 = a^2\big((1+2\Phi)\mathrm{d}\eta^2 - \big[(1+2\Phi)\delta_{ij}\big]\mathrm{d}x_i\mathrm{d}x_j\big)$$

$$\ddot{\Phi} + (H - 2\frac{\ddot{\varphi}}{\dot{\varphi}})\dot{\Phi} + (\frac{k^2}{a^2} + 2\dot{H} - 2H\frac{\ddot{\varphi}}{\dot{\varphi}})\Phi = 0$$

-> singularities better take Mukhanov variable $v = \delta\varphi + \frac{\dot{\varphi}}{H}\Phi$ (Finelli & Brandenberger 1999)

$$\ddot{\tilde{v}}_{\mathbf{k}} + \left[\frac{k^2}{a^2} + \frac{\mathrm{d}^2 V}{\mathrm{d}\varphi^2} + 3\kappa\dot{\varphi}^2 - \frac{\kappa^2}{2H^2}\dot{\varphi}^4 + \frac{3\kappa}{4}\left(\frac{\dot{\varphi}^2}{2} - V\right) + 2\kappa\frac{\dot{\varphi}}{H}\frac{\mathrm{d}V}{\mathrm{d}\varphi}\right]\tilde{v}_{\mathbf{k}} = 0\,.$$

with $\tilde{v}_{\mathbf{k}} = a^{1/2} v_{\mathbf{k}}$

Parametric Resonance for sub-Hubble scales

having

$$\varphi(t) \simeq \varphi_{\rm end} \left(\frac{a_{\rm end}}{a}\right)^{3/2} \sin\left(mt\right) \ ,$$

the evolution equation becomes

$$\frac{\mathrm{d}^2 \tilde{v}_{\mathbf{k}}}{\mathrm{d}z^2} + \left[1 + \frac{k^2}{m^2 a^2} - \sqrt{6\kappa}\varphi_{\rm end} \left(\frac{a_{\rm end}}{a}\right)^{3/2} \cos\left(2z\right)\right] \tilde{v}_{\mathbf{k}} = 0 \, ,$$

with $z \equiv mt + \pi/4$. This has the form of the Mathieu equation

$$\frac{\mathrm{d}^2 v_{\mathbf{k}}}{\mathrm{d}z^2} + \left[A_{\mathbf{k}} - 2q\cos(2z)\right] = 0$$

with

$$A_{\mathbf{k}} = 1 + \frac{k^2}{m^2 a^2} \qquad q = \frac{\sqrt{6\kappa}}{2}\varphi_{\rm end} \left(\frac{a_{\rm end}}{a}\right)^{3/2} \ .$$

Instabilities and the Mathieu equation

Mathieu equation has instability bands. i.e for

$$1 - q < A_{\mathbf{k}} < 1 + q$$

This implies instability for all modes $0 < \frac{k}{a} < \sqrt{3Hm}$.
These modes grow as

$$\tilde{v}_{\mathbf{k}} \propto \exp\left(\int \frac{q}{2} \mathrm{d}z\right) \propto a^{3/2} .$$

This implies that the curvature perturbations

$$\zeta_{\mathbf{k}} = \sqrt{\kappa/2} v_{\mathbf{k}} / (a \sqrt{-\frac{\dot{H}}{H^2}}) \tag{1}$$

for $\tilde{v} \propto a^{3/2}$ stay constant for superhorizon scales (known) and a large range of subhorizon scales (not known)

Growth of small-scale density perturbations

From

$$\delta_{\mathbf{k}} = -\frac{2}{5}\left(\frac{k^2}{a^2H^2} + 3\right)\zeta_{\mathbf{k}} \, , \tag{2}$$

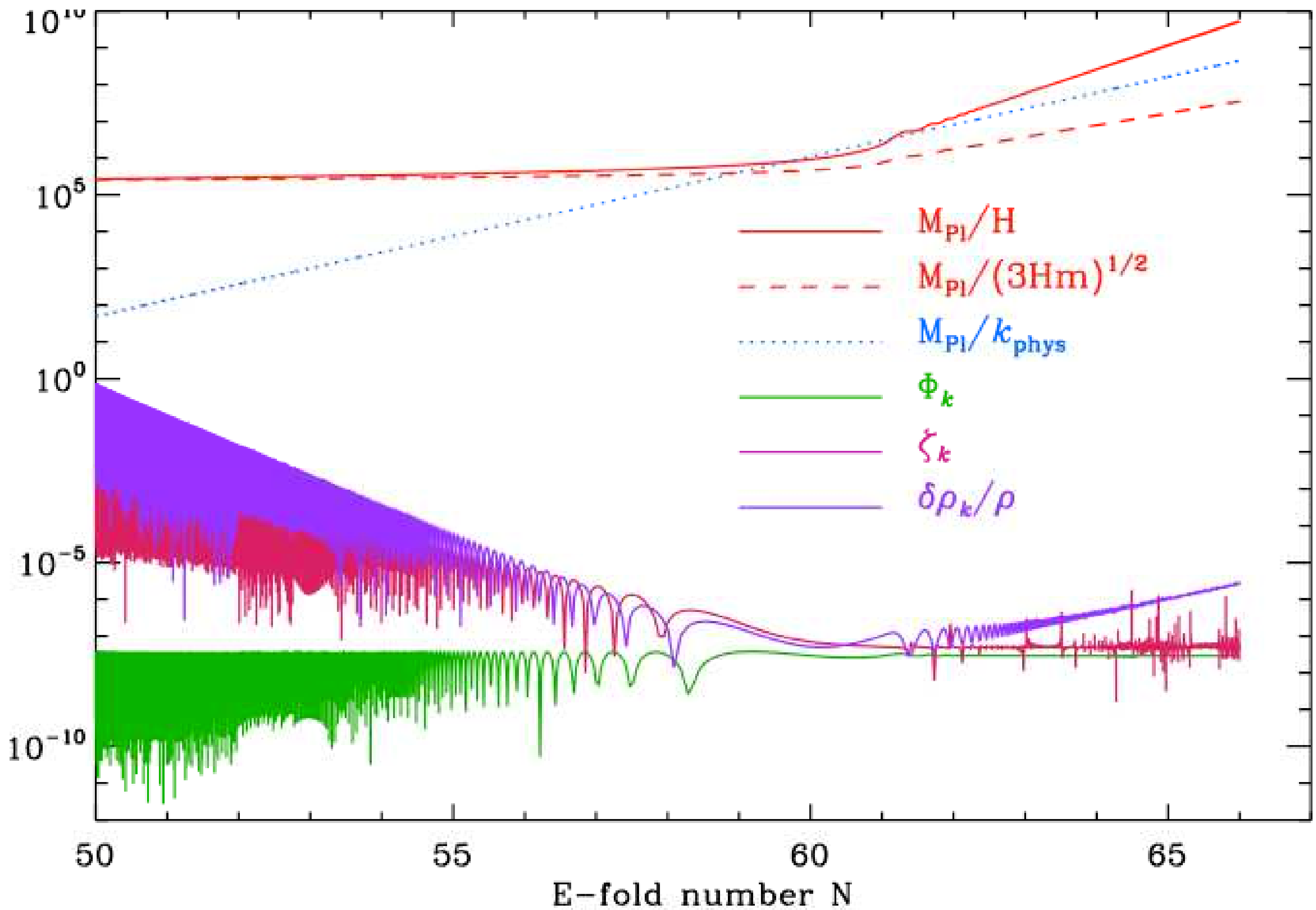
and $\zeta_{\mathbf{k}} \sim$ const one finds

$$\delta_{\mathbf{k}} \propto a$$

-> growing sub-Hubble density perturbations

not really a surprise, since "matter domination", i.e. $H \propto a^{-3/2}$ (Wands)

Numerical integration confirms

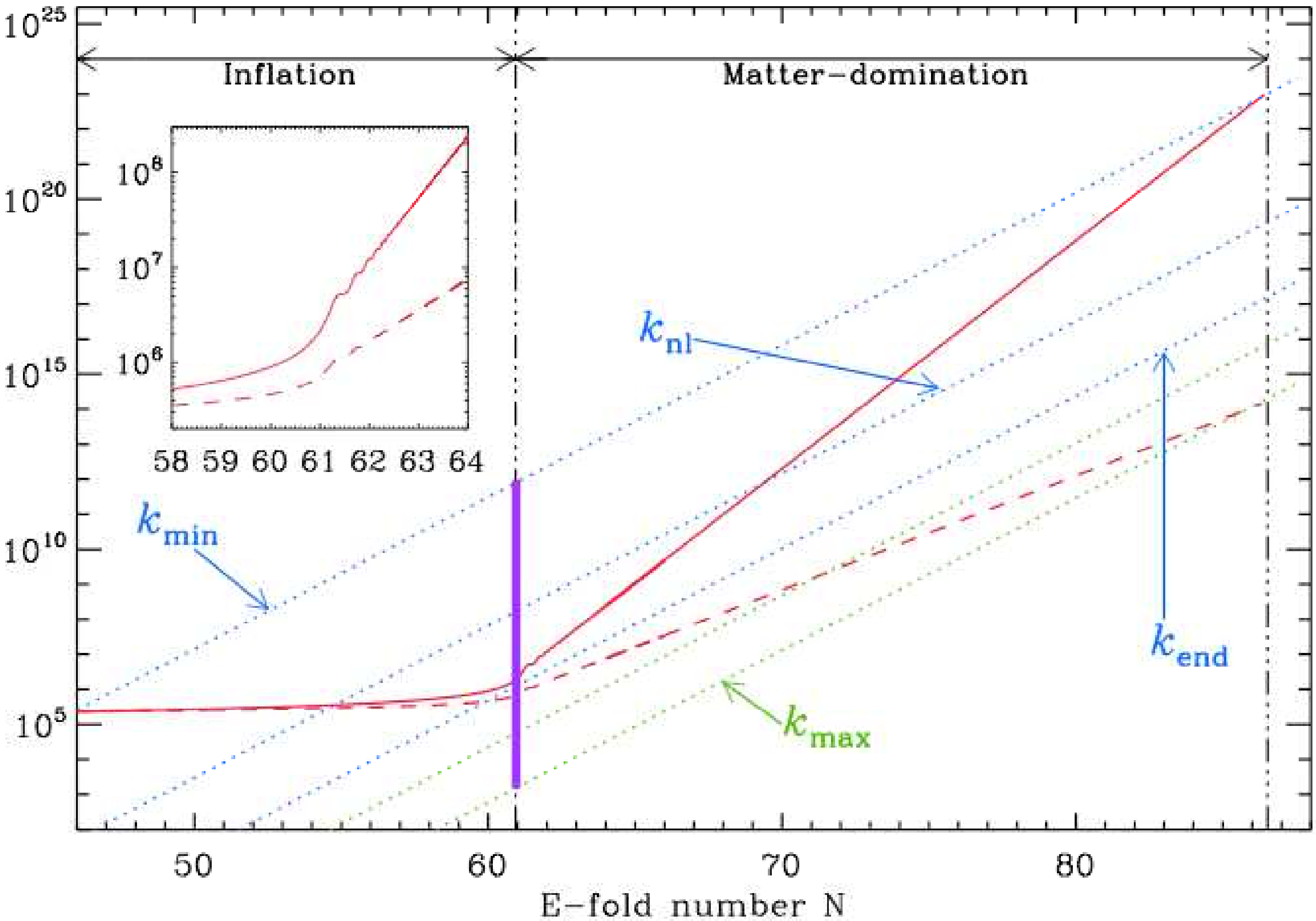


Structure formation before Reheating

-> small-scale perturbations grow and if reheating occurs late they become non-linear

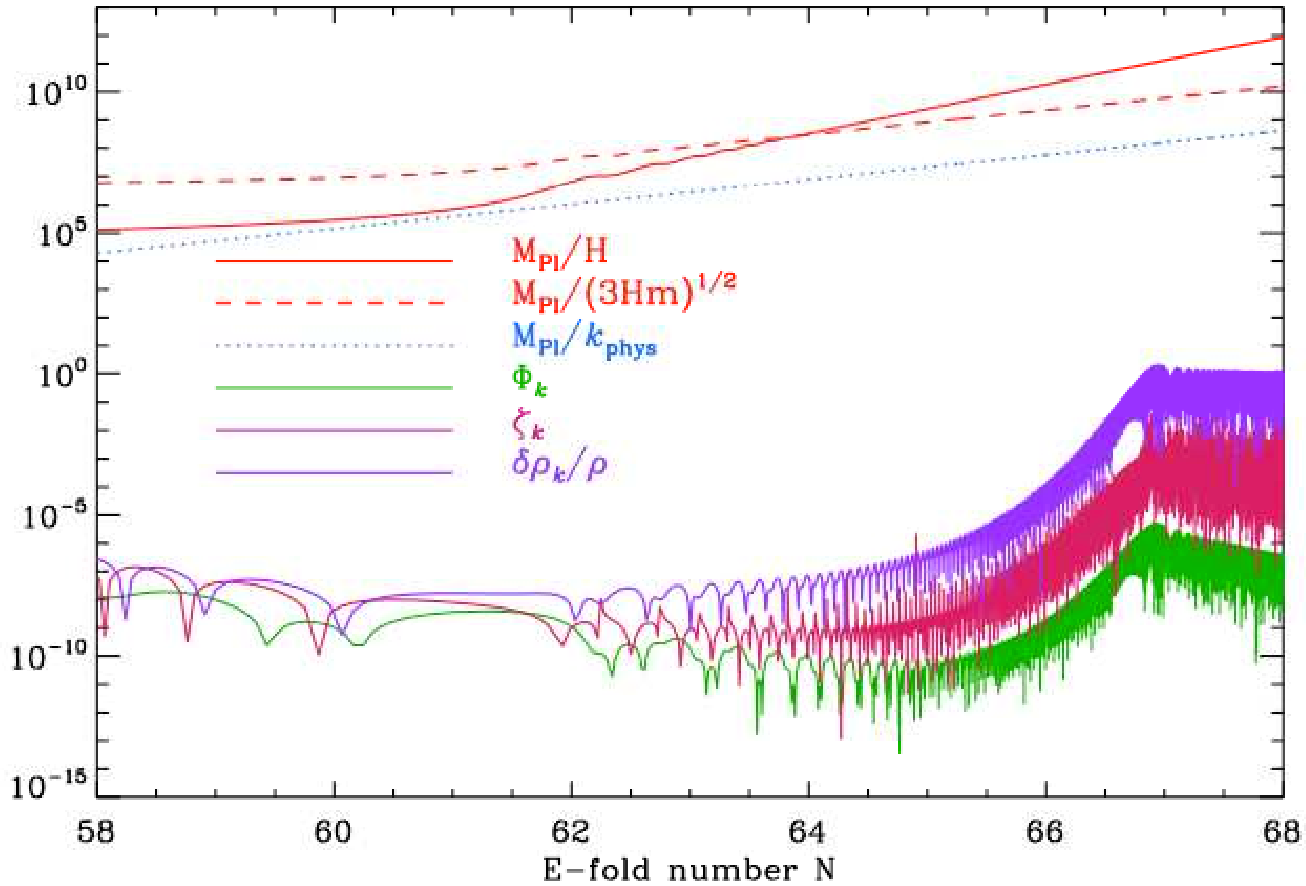
-> collapse and structure formation before reheating is possible

Evolution of length scales

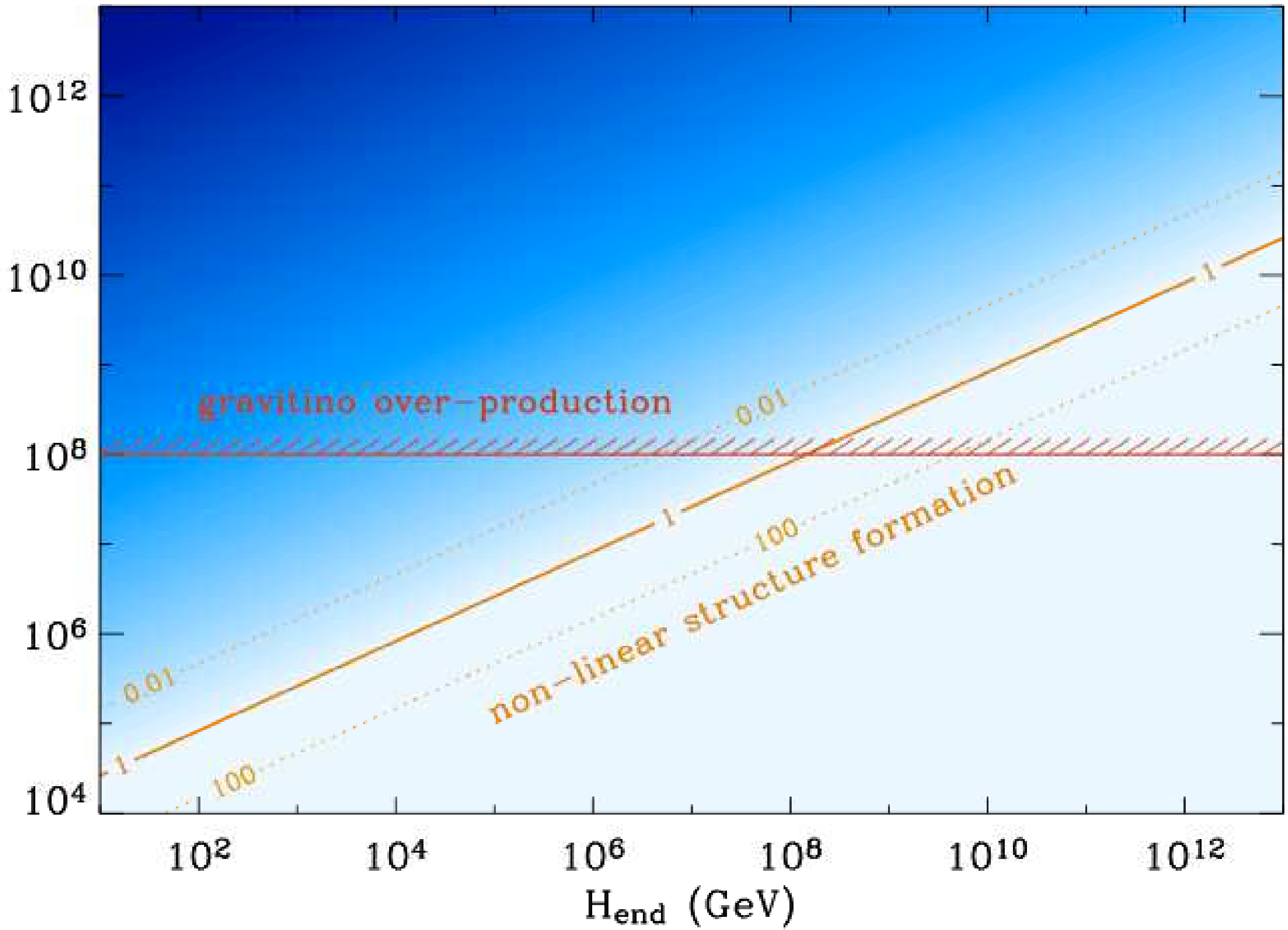


-> Result is generic for many inflationary models as the first non-vanishing term in the expansion of the inflaton field is $m^2\varphi^2$

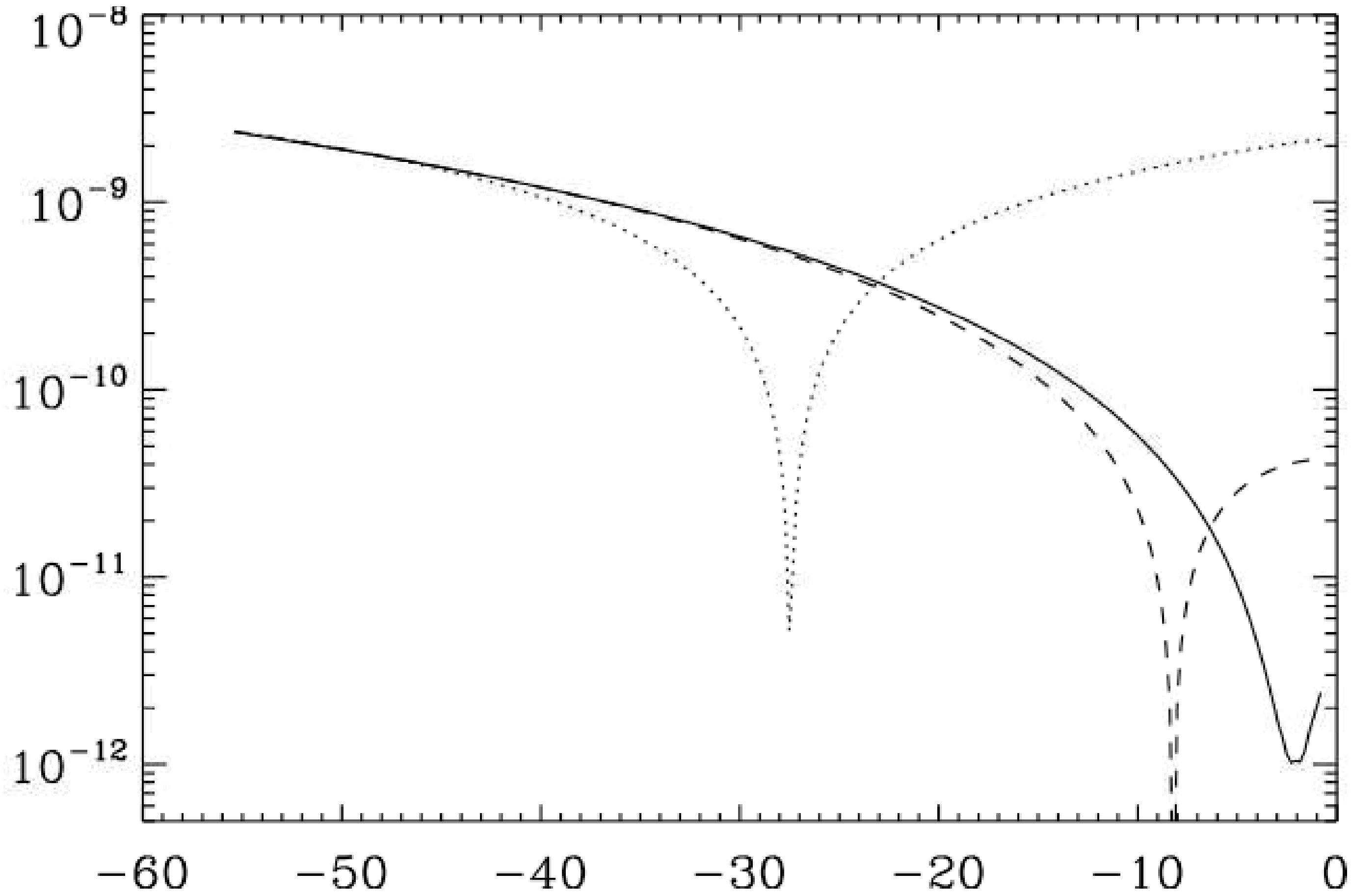
$\sim \lambda\varphi^4$ even more unstable



Non-linear structure formation before reheating



Curvature perturbations in chaotic inflation



Gravitational Wave Signal ?

Gravitational wave signal from very early structure formation ?

Transverse-traceless metric

$$\mathrm{d}s^2 = a^2\mathrm{d}\eta^2 - \left[\delta_{ij} + h_{ij}\right]\mathrm{d}x_i\mathrm{d}x_j\Big)$$

Evolution of metric coefficient

$$\bar{h}''_{ij} + \left(k^2 - \frac{a''}{a}\right)\bar{h}_{ij} = 2\kappa a T^{\mathrm{TT}}_{ij}(\eta, \mathbf{k})\,,$$

with $\bar{h} = ah$. Energy density in gravitational waves

$$\varrho_{\mathrm{gw}}(\eta) = \frac{1}{4\kappa a^4(\eta)}\sum_{ij}\left\langle \bar{h}'_{ij}(\eta,\mathbf{x})\bar{h}'_{ij}(\eta,\mathbf{x})\right\rangle\,.$$

(see also Assadullahi & Wands 09, Alabidi *et al.* 13, Kohri & Terada 18, Inomate *et al* 19)

Gravitational Waves from growing modes in linear regime

Present-day frequency, where $\hat{k} \sim 1$ for the most growing mode

$$f_0 \simeq 5 \times 10^5 \mathrm{Hz}\, \hat{k} \left(\frac{T_{\mathrm{rh}}}{10^9 \mathrm{GeV}} \right)^{1/3} (\text{chaotic inflation})$$

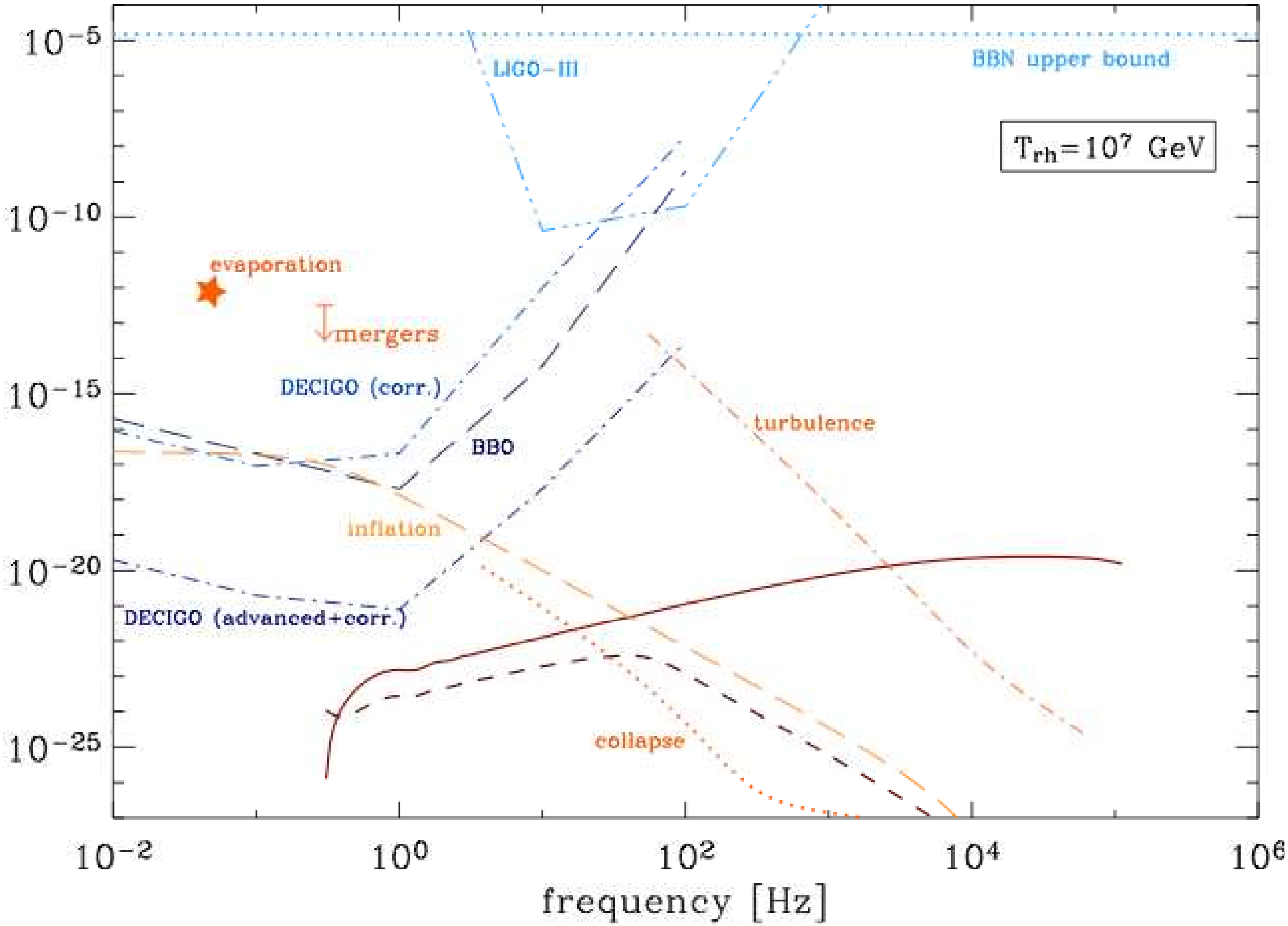
with energy density

$$\frac{\mathrm{d}\Omega_{\mathrm{gw}}}{\mathrm{d}\ln k} \simeq 2.8 \times 10^{-16} \Omega_{\gamma,0} \left(\frac{T_{rh}}{10^9 \mathrm{GeV}} \right)^{-4/3} \hat{k} \hat{\mathcal{J}}(\hat{k}) \hat{\mathcal{I}}_{11}(\hat{k})$$

where the time integral $\hat{\mathcal{J}}(\hat{k}) \to 1$ when T_{rh} small and where $\hat{\mathcal{I}}_{11}(\hat{k}) \equiv 10^{22} \hat{\mathcal{I}}(\hat{k})$ is an integral over the initial power spectrum

$$\hat{\mathcal{I}}(\hat{k}) \equiv \int_{\hat{q}_{\min}}^{\hat{q}_{\max}} \mathrm{d}\hat{q} \int_{-1}^{+1} \mathrm{d}\mu\, \mathcal{P}_\zeta(q) \mathcal{P}_\zeta \left(\sqrt{\hat{q}^2 + \hat{k}^2 - 2\hat{k}\hat{q}\mu} \right) \frac{\left(1-\mu^2\right)^2 \hat{q}^3}{\left(\hat{q}^2 + \hat{k}^2 - 2\hat{k}\hat{q}\mu\right)^{3/2}}$$

Present day gravtational wave energy density



Contributions from non-linear structure formation

Once non-linear structures exist signal may be enhanced

Needs really numerical simulations, however, back-of the envelope estimates via the quadrupole approximation

$$h \simeq \frac{G}{2}\left(\ddot{I}_{ij} - \frac{1}{3}\ddot{I}_{kk}\delta_{ij}\right)\frac{n_i n_j}{|\mathbf{x}|} ,$$

and

$$\ddot{I}_{ij} \simeq 2\int_h \varrho_{\rm h} v_i v_j \simeq M v^2 \simeq \frac{2GM^2}{R} ,$$

Contributions from non-linear structure formation

Collapse to halos when $\delta\varrho/\varrho \simeq 1$:

$$\frac{\mathrm{d}\Omega_{\rm gw}^{\rm coll}}{d\ln f} \sim 10^{-22}\hat{k}^{-2}\left(\frac{T_{\rm rh}}{10^9\,\mathrm{GeV}}\right)^{4/3}\left(\frac{\mathcal{P}_\zeta}{10^{-11}}\right)^{5/4}$$

Signal dominated by largest scales

$$\hat{k}_{\rm nl} \sim 0.1\left(\frac{T_{\rm rh}}{10^9\,\mathrm{GeV}}\right)^{2/3}\left(\frac{\mathcal{P}_\zeta}{10^{-11}}\right)^{-1/4} ,$$

Merging and tidal interactions when structures exist:

$$\frac{\mathrm{d}\Omega_{\rm gw}}{\mathrm{d}\ln f} \sim 10^{-18}\epsilon\,(T_{\rm rh}/10^9\,\mathrm{GeV})^{-2}(\mathcal{P}_\zeta/10^{-11})^{5/2}$$

with typical frequency

$$f \sim 30\,\mathrm{Hz}\,(T_{\rm rh}/10^9\,\mathrm{GeV})$$

Contributions from evaporation of halos at T_{rh}

Evaporation of halos during reheating

$$\Omega_{\rm gw} \sim \epsilon\, \Omega_{\gamma,0}\, P_\zeta(k_{\rm nl})^{3/4}\,,$$

with

$$f \sim 30\,{\rm Hz}\,(T_{\rm rh}/10^9\,{\rm GeV})$$

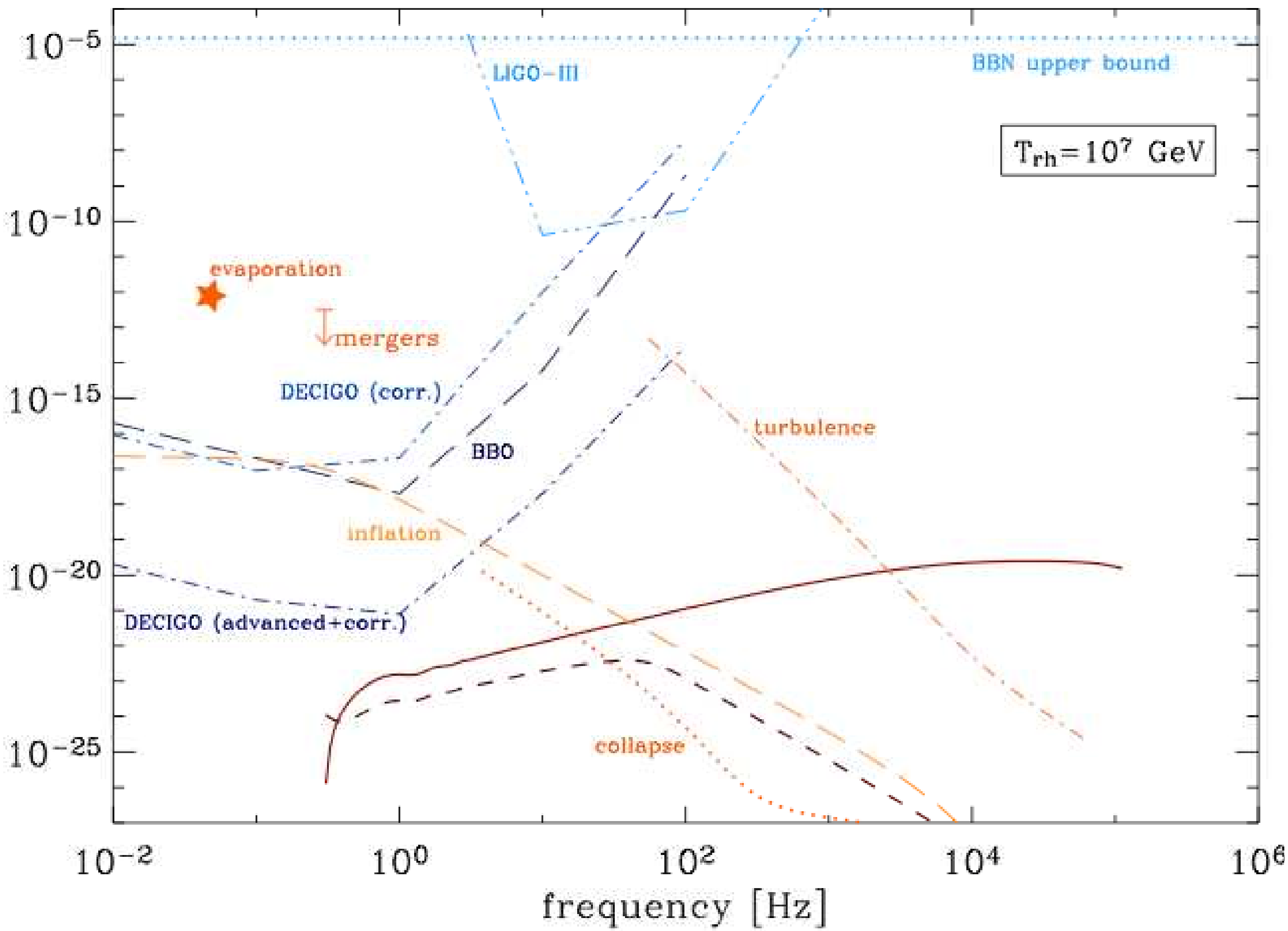
Ensuing radiation turbulence after evaporation of halos:

$$\frac{{\rm d}\Omega_{\rm gw}}{{\rm d}\ln f} \sim 2\times 10^{-14}\left(\frac{\mathcal{P}_\zeta}{10^{-11}}\right)^{3/4}\left(\frac{f}{f_{\rm s}}\right)^{-7/2}\,,$$

with frequency

$$f_{\rm s} \simeq 7\times 10^3\,{\rm Hz}\left(\frac{T_{\rm rh}}{10^9\,{\rm GeV}}\right)\left(\frac{\mathcal{P}_\zeta}{10^{-11}}\right)^{-1/6}\,.$$

Present day gravtational wave energy density



Conclusions

- Sub-Hubble scales after inflation are unstable to gravitational growth
- When the reheating temperature is low non-linear structure formation may occur before reheating
- These structures may lead to an interesting gravitational background signal from the early Universe

II. Stringent Limits on Cosmic Magnetic Fields from the CMBR

KJ and T. Abel JCAP 1310 (2013) 050 and KJ and A.Saveliev PRL 123 (2019), 021301

Motivation

- the early Universe may well have been magnetized
- observations of TeV blazars indicate the likely presence of an intergalactic magnetic field

$$B \gtrsim 10^{-15} \mathrm{Gauss}$$ Neronov & Vovk 10

- primordial magnetic fields of

$$B \sim 3 \times 10^{-12} \text{ Gauss}$$

would be sufficient to explain cluster magnetic fields

Known Sources of impact on the CMBR

- Faraday rotation
- dissipation of magnetic fields and μ and y blackbody spectral distortions
- direct generation of anisotropies below the Silk scale
- dissipation of magnetic fields shortly after recombination and changes in the Thomson optical depth
- non-Gaussian signatures (i.e bispectrum and trispectrum)
- reionization

Known CMB Limits on Scale-Ivariant primordial magnetic fields

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Principal Efffect	Upper Limit	References
spectral distortions	**30-40 nG**	Jedamzik *et al.* 2000 Kunze & Komatsu 2014
plasma heating	**0.63-3 nG**	Sethi & Subramanian 2004 Kunze & Komatsu 2014 Chluba *et al.* 2015 Planck collaboration 2015
direct TT anisotropies	**1.2 - 6.4 nG**	Subramanian *et al.* 1998, 2002, 2003 Yamazaki *et al.* 2010 Paoletti & Finelli 2010 Shaw & Lewis 2010 Caprini 2011 Paoletti & Finelli 2013 Planck collaboration 2015 Zucca *et al.* 2016 Sutton et al. 2017
non-Gaussianity bispectrum	**2-9 nG**	Brown & Crittenden 2005 Seshadri & Subramanian 2009 Caprini *et al.* 2009 Cai *et al.* 2010 Trivedi *et al.* 2010 Brown 2011 Shiraishi *et al.* 2011 Shiraishi & Sekiguchi 2014 Planck collaboration 2015
non-Gaussianity trispectrum	**0.7nG**	Trivedi *et al.* 2012
non-Gaussianity trispectrum with inflationary curvature mode	**0.05nG**	Trivedi *et al.* 2014
reionization	**0.36nG**	Sethi & Subramanian 2005 Schleicher *et al.* 2011 Vasiliev & Sethi 2014 Pandey *et al.* 2015 Bonvin *et al.* 2013

Damping of pre-existing magnetic fields

- Pre-existing magnetic fields excite fluid motions
- These fluid motions are broken up into smaller and smaller eddies until they are damped by dissipation
- in this way the magnetic field energy is also drained
- magnetic diffusion is unimportant due to the high number of charged carriers
- due to the existence of radiation the speed of sound v_s is large and the evolution is incompressible

The fully turbulent regime

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v}\cdot\nabla)\cdot\mathbf{v} = \eta\nabla^2\mathbf{v} - \frac{1}{4\pi(\varrho+p)}\mathbf{B}\times(\nabla\times\mathbf{B})$$

$$\frac{d\mathbf{B}}{dt} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

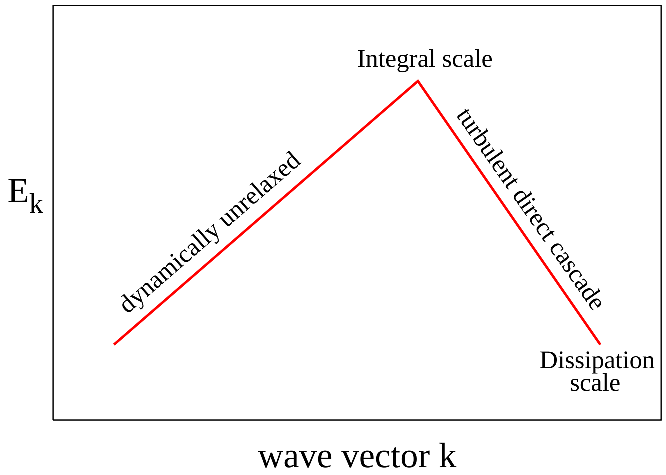
- The Alfven velocity: $v_A = \frac{B}{\sqrt{4\pi(\varrho+p}}$
- When fully turbulent, on larger scales, away from the dissipation scale, $v_A \approx v$
- The time needed to excite an eddy is given by $t_{\mathrm{eddy}} \approx L/v_A$

$\rightarrow$ Eddies on scale L are excited when $v_A/L \approx H$

The integral scale

The integral scale L is defined by the condition

$$v_A/L \approx H$$

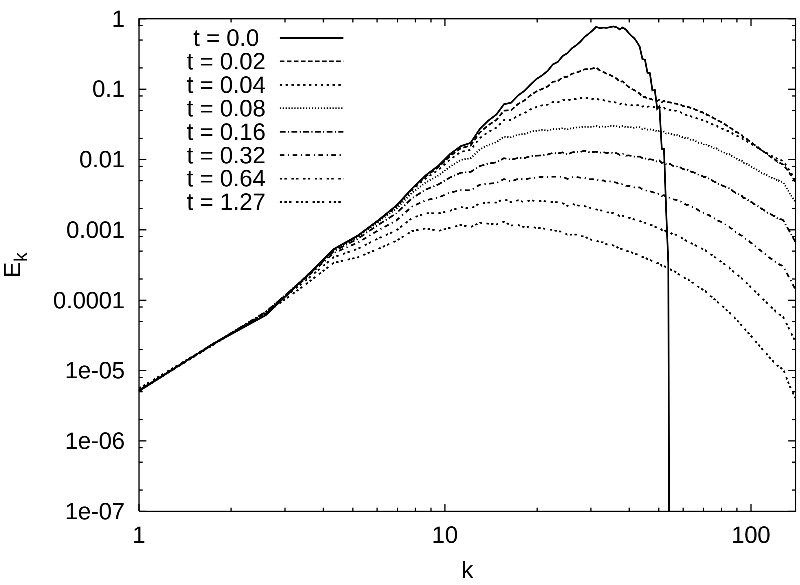


The growth of the integral scale

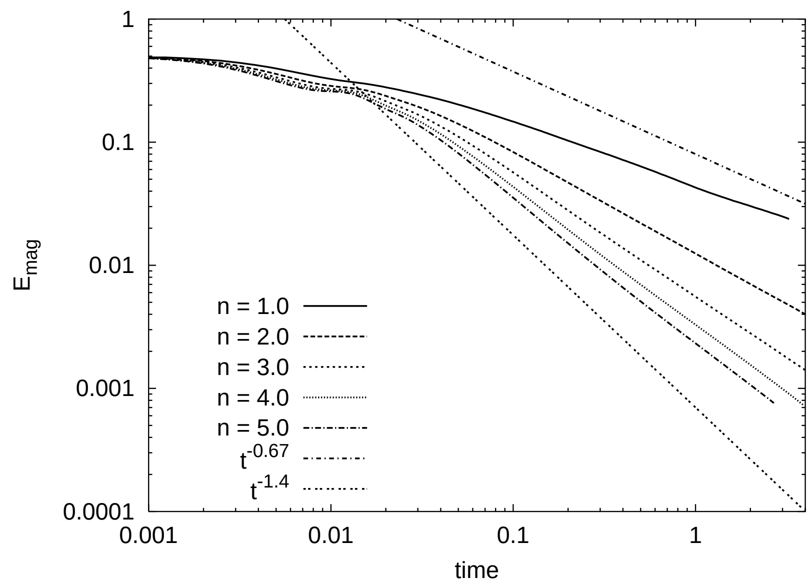
$$\frac{t_{\text{eddy}}}{t_{\text{Hubble}}} \approx \frac{L/v_{\text{A}}}{t_{\text{Hubble}}} \propto \frac{a}{a^2} \propto 1/a$$

Comparison to numerical simulations

Spectrum with time



Energy with time



Christensson, Hindmarsh, & Brandenburg 01, Banerjee & K.J. 04, Saveliev, K.J., & Sigl 13, Kahniashvili *et al* 13, Brandenburg, Kahniashvili, & Tevzadze 15

The growth of the integral scale after recombination

$$\frac{t_{\mathrm{eddy}}}{t_H} \approx \frac{L/v_{\mathrm{A}}}{t_H} \propto \frac{a/1/a^{1/2}}{a^{3/2}} \propto a^0$$

$\rightarrow$ after recombination essentially no more evolution

$\rightarrow$ what has not dissipated until recombination will remain to today

Predicted correlation length of primordial magnetic fields:

$$B_0 \lesssim 5 \times 10^{-12}\,\text{Gauss} \left(\frac{L_c}{\mathrm{kpc}}\right)$$

But wait ?

Magnetic fields on scales 1-10 kpc around recombination, that's below the photon mean free path ~ 2 Mpc at recombination

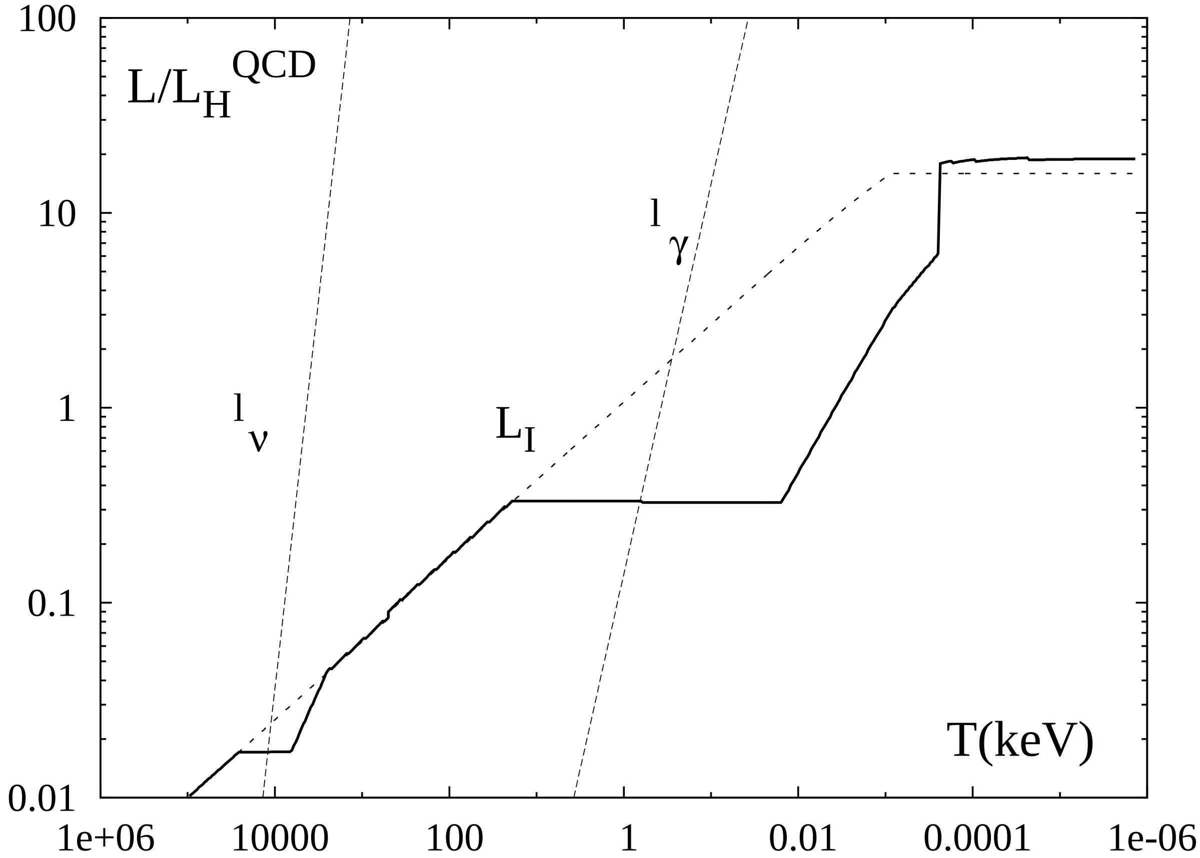
$\rightarrow$ Photons do not participate in the fluid motions

MHD with strong photon drag

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v}\cdot\nabla)\cdot\mathbf{v} \;=\; -\alpha\mathbf{v} - \frac{1}{4\pi\varrho}\mathbf{B}\times(\nabla\times\mathbf{B})$$

- with strong photon drag only the RHS is important
- velocities $v \approx \frac{v_A^2}{L\alpha}$ are excited
- the magnetic energy dissipation $\dot{E} \sim -\alpha v^2 \sim \frac{v_A^4}{L^2\alpha}$ counterintuitively is smaller for larger drag α

The Evolution of the Magnetic Coherence Length



But wait again ?

Since photons do not participate anymore in the fluid motions the speed of sound is the much smaller baryonic one

$v_A \simeq v_s$ when $B \simeq 5 \times 10^{-11}$ Gauss

$\rightarrow$ Very weak magnetic fields on $\sim$kpc scales can excite density fluctuations

Production of density fluctuations before recombination

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v}\cdot\nabla)\cdot\mathbf{v} + v_s^2\frac{\nabla\varrho}{\varrho} = -\alpha\mathbf{v} - \frac{1}{4\pi\varrho}\mathbf{B}\times(\nabla\times\mathbf{B})$$

$$\frac{d\varrho}{dt} + \nabla(\varrho\mathbf{v}) = 0$$

- Very quickly small velocities $v \approx \frac{v_A^2}{L\alpha}$ are produced
- from the continuity equation one finds $\frac{\delta\varrho}{\varrho} \simeq \frac{Vt}{L} \simeq \frac{v_A^2 t}{L^2\alpha}$
- when the pressure term becomes important, i.e. $V_s^2(\delta\varrho/\varrho)/L \simeq \alpha v$
- $\rightarrow \delta\varrho/\varrho \simeq \frac{L\alpha v}{v_s^2} \simeq \frac{v_A^2}{v_s^2}$, but never much larger than 1 K.J. & Abel 2013

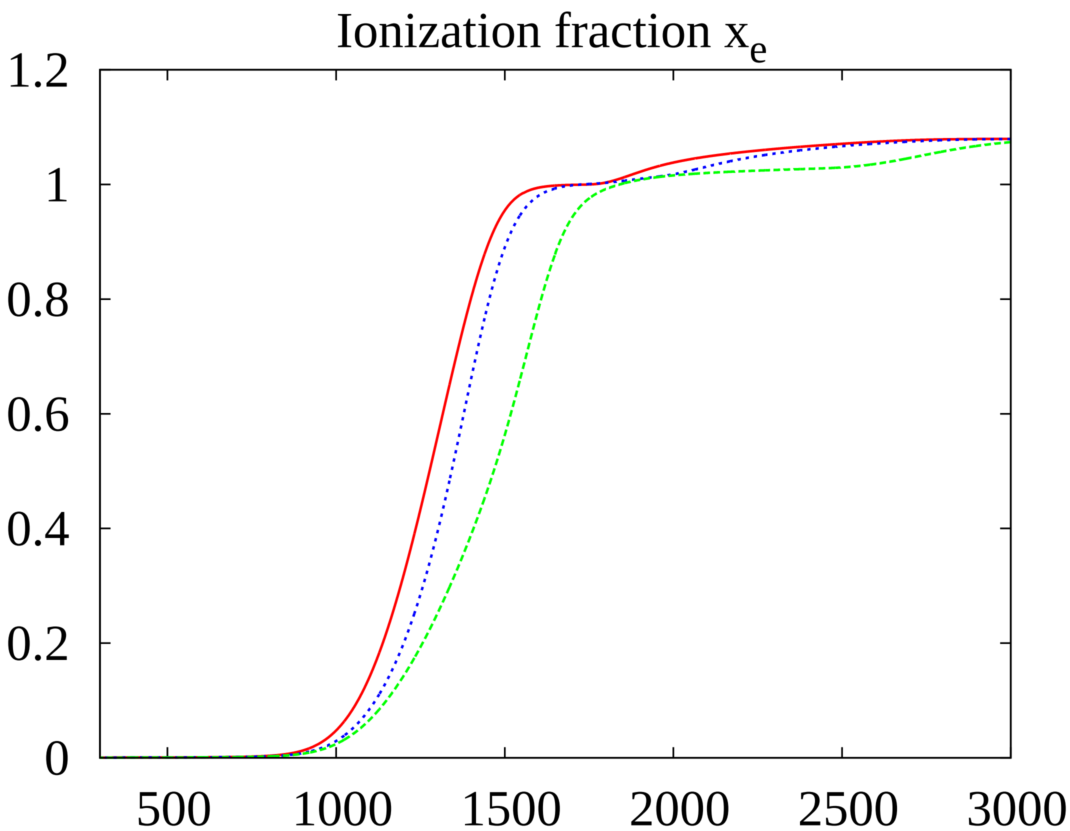
A "new" effect: Inhomogeneous recombination

recombination in inhomogeneous environment

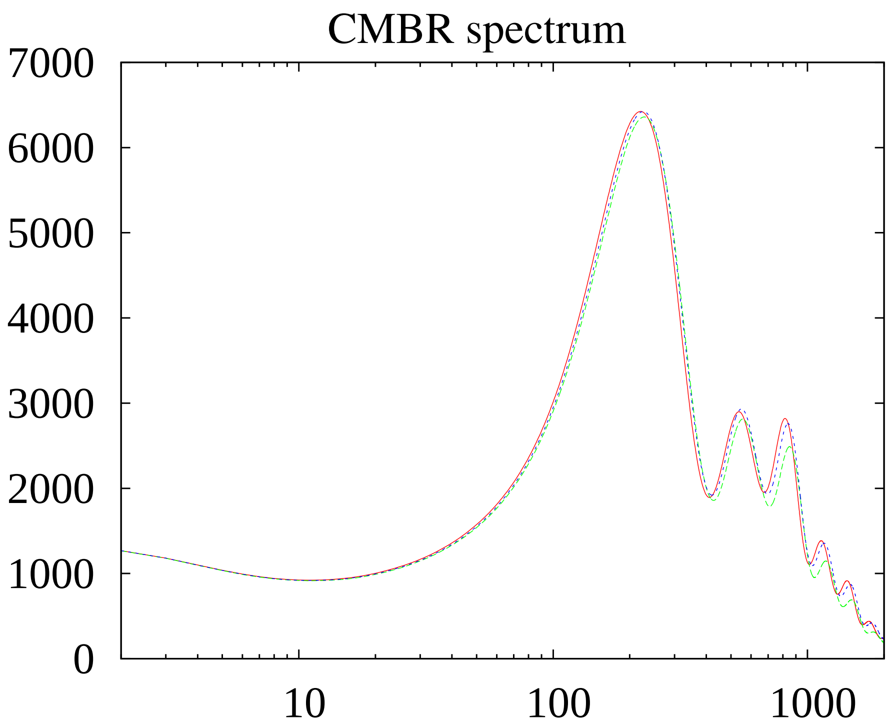
$$\frac{d\langle n_{H^0}\rangle}{dt}\Big|_{inhomo} =$$
$$\alpha_e\langle n_e n_p\rangle - \beta_e\langle n_{1s}\rangle \exp\left(\frac{-E_{\nu\alpha}}{kT}\right) \neq \frac{d\langle n_{H^0}\rangle}{dt}\Big|_{homo}$$ since
$$\langle n_e n_p\rangle \neq \langle n_e\rangle\langle n_p\rangle$$

A "new" effect: Inhomogeneous recombination

$$\left(\frac{\delta\varrho}{\varrho}\right)_{rms} = 0,\ \sqrt{3},\ 10$$

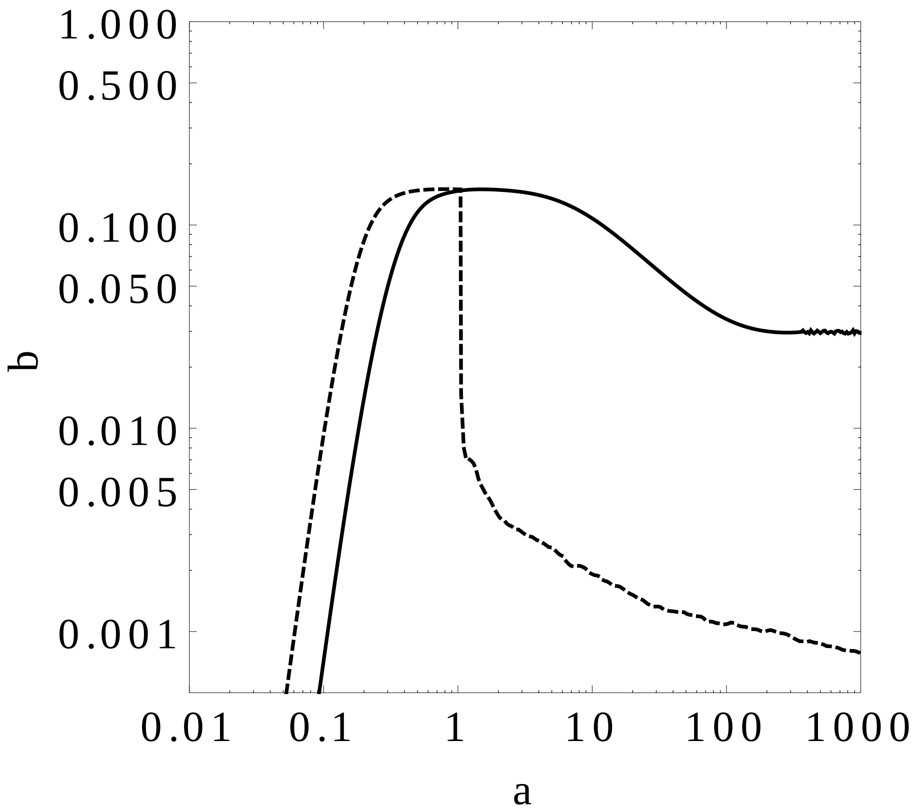


A "new" effect: Inhomogeneous recombination



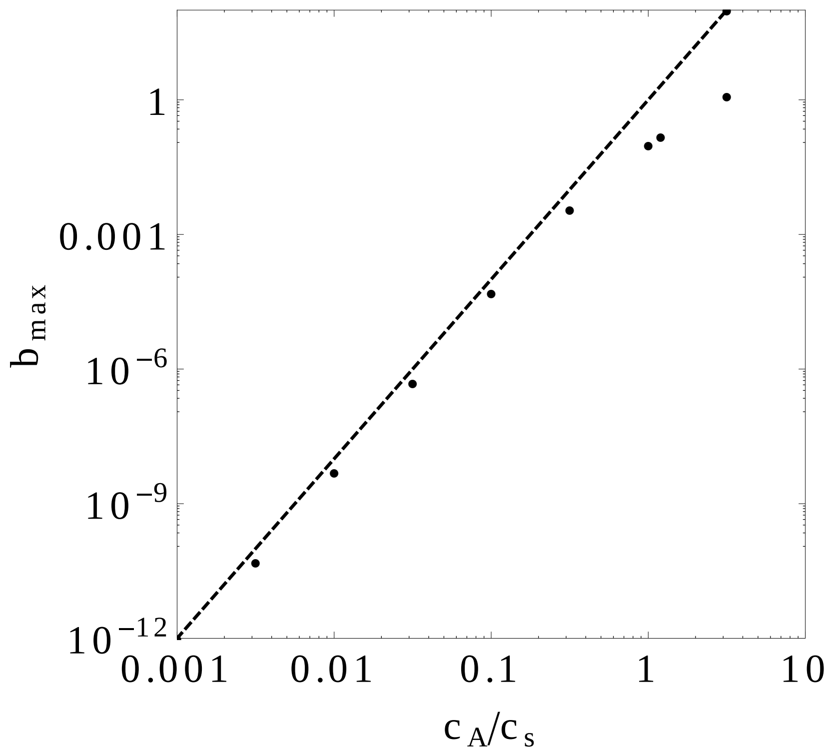
- earlier recombination, peaks shift to higher l
- enhanced Silk damping

Numerical simulations of compressible MHD before recombination

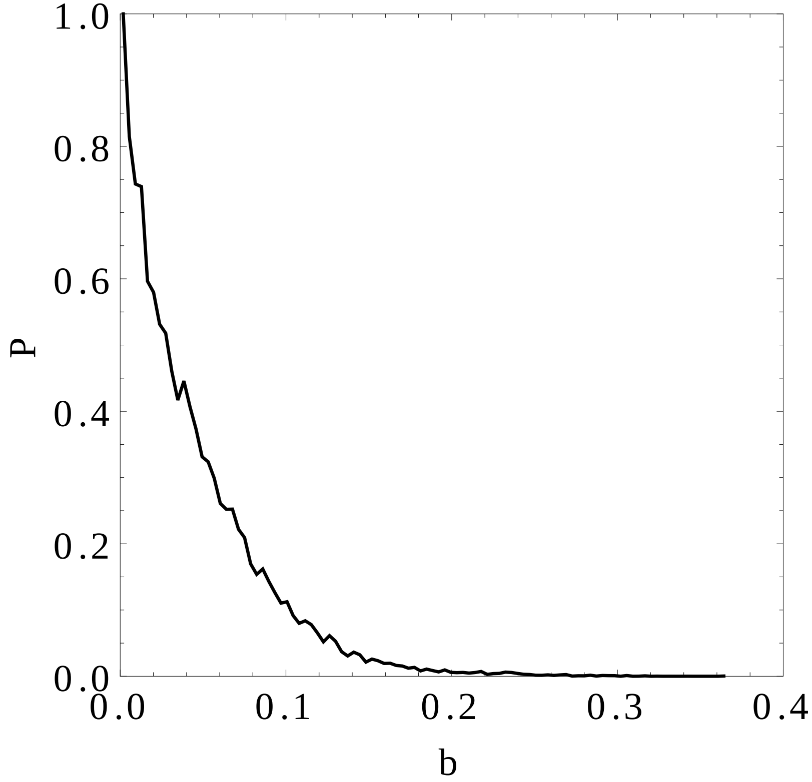


clumping factor $b = \frac{(\langle \varrho - \langle \varrho \rangle \rangle)^2}{\langle \varrho \rangle^2}$

Density fluctuations as a function of v_A



The marginalized probability for clumping from Planck data



Resulting limits on primordial magnetic fields

stringent new limits on primordial magnetic fields
from inhomogeneous recombination

$B \lesssim 0.0089$ nG (total field) at 95% confidence
for causual spectra

$B \lesssim 0.047$ nG
for scale-invariant spectra