Gravitational Wave Spectra from Primordial Magnetohydrodynamic Turbulence

Nordita program on "Gravitational Waves from the Early Universe"
Aug. 26 – Sep. 20 2019

Alberto Roper Pol (PhD candidate)
Research Advisor: Axel Brandenburg
Collaborators: Tina Kahniashvili, Arthur Kosowsky, Sayan Mandal

University of Colorado at Boulder

Laboratory for Atmospheric and Space Physics (LASP)

August 26, 2019

Overview

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during the electroweak phase transition (EWPT)
- Source of information from earlier times than recombination epoch (Cosmic Microwave Background)

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during the electroweak phase transition (EWPT)
- Source of information from earlier times than recombination epoch (Cosmic Microwave Background)
- MHD sources of GWs:
  - Hydrodynamic turbulence from phase transition bubbles nucleation
  - Magnetic turbulence from primordial magnetic fields

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during the electroweak phase transition (EWPT)
- Source of information from earlier times than recombination epoch (Cosmic Microwave Background)
- MHD sources of GWs:
  - Hydrodynamic turbulence from phase transition bubbles nucleation
  - Magnetic turbulence from primordial magnetic fields
- Possibility of GWs detection with:
  - Space mission LISA
  - Polarization of CMB
  - Pulsar Timing Arrays (PTA)

Introduction and Motivation

- Generation of cosmological gravitational waves (GWs) during the electroweak phase transition (EWPT)
- Source of information from earlier times than recombination epoch (Cosmic Microwave Background)
- MHD sources of GWs:
  - Hydrodynamic turbulence from phase transition bubbles nucleation
  - Magnetic turbulence from primordial magnetic fields
- Possibility of GWs detection with:
  - Space mission LISA
  - Polarization of CMB
  - Pulsar Timing Arrays (PTA)
- Numerical simulations using Pencil Code to solve:
  - Relativistic MHD equations
  - Gravitational waves equation

Gravitational waves equation

GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe
- Friedmann–Lemaître–Robertson–Walker (FLRW) metric η_{ij}
- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = \eta_{ij} + a^2 h_{ij}^{\mathrm{phys}}$$

- GWs equation is[1]

$$\left(\partial_t^2 - c^2\nabla^2\right) h_{ij} = \frac{16\pi G}{ac^2} T_{ij}^{\mathrm{TT}}$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\mathrm{phys}}$
  - Comoving spatial coordinates ∇ and conformal time t
  - Comoving stress-energy tensor components T_{ij}
  - Radiation-dominated epoch such that $a'' = 0$

[1] L. P. Grishchuk, *Sov. Phys. JETP*, 40, 409-415 (1974)

Gravitational waves equation

Properties

- Tensor-mode perturbations are gauge invariant
- h_{ij} has only two degrees of freedom: h^{+}, $h^{\times}$
- The metric tensor is traceless and transverse (TT gauge)

Contributions to the stress-energy tensor

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$
  Relativistic equation of state:
  $p = \rho c^2/3$
- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

Normalized GW equation[2]

$$\left(\partial_t^2 - \nabla^2\right) h_{ij} = 6\, T_{ij}^{\mathrm{TT}} / t$$

Properties

- All variables are normalized and non-dimensional
- Conformal time is normalized with t_*
- Comoving coordinates are normalized with c/H_*
- Stress-energy tensor is normalized with $\mathcal{E}_{\mathrm{rad}}^*$
- Scale factor is $a_* = 1$

[2] A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.*, DOI:10.1080/03091929.2019.1653460, arXiv:1807.05479 (2019)

Traceless and transverse projection

- For computational reasons, we solve GWs equation sourced by the stress-energy tensor[3]

$$\left(\partial_t^2 - c^2 \nabla^2\right) h_{ij} = \frac{16\pi G}{ac^2} T_{ij}$$

- Projection of h_{ij}^{TT} only at specific times, not at every time-step
- Projection requires Fourier transform $\tilde{h}_{ij}$ (computationally expensive)
- TT projection:

$$\tilde{h}_{ij}^{\mathrm{TT}} = \left(P_{il} P_{jm} - \frac{1}{2} P_{ij} P_{lm}\right) \tilde{h}_{lm}$$

where $P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$

[3] A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.*, DOI:10.1080/03091929.2019.1653460, arXiv:1807.05479 (2019)

Linear polarization modes h^+ and $h^\times$

Linear polarization basis

$$e_{ij}^+ = (\boldsymbol{e}_1 \times \boldsymbol{e}_1 - \boldsymbol{e}_2 \times \boldsymbol{e}_2)_{ij}$$

$$e_{ij}^\times = (\boldsymbol{e}_1 \times \boldsymbol{e}_2 + \boldsymbol{e}_2 \times \boldsymbol{e}_1)_{ij}$$

Orthogonality property

$e_{ij}^A e_{ij}^B = 2\delta_{AB}$, where $A, B = +, \times$

h^+ and $h^\times$ modes

$$\tilde{h}^+ = \frac{1}{2} e_{ij}^+ \tilde{h}_{ij}^{\mathrm{TT}}$$

$$\tilde{h}^\times = \frac{1}{2} e_{ij}^\times \tilde{h}_{ij}^{\mathrm{TT}}$$

GWs energy density:

$$\Omega_{\rm GW}(t) = \mathcal{E}_{\rm GW}/\mathcal{E}^*_{\rm rad}, \quad \mathcal{E}^*_{\rm rad} = \frac{3H_*^2 c^2}{8\pi G}$$

$$\Omega_{\rm GW}(t) = \int_{-\infty}^{\infty} \Omega_{\rm GW}(k,t)\,{\rm d}\ln k$$

$$\Omega_{\rm GW}(k,t) = \frac{k}{6H_*^2} \int_{4\pi} \left(\left| \dot{\tilde{h}}_+^{\rm phys} \right|^2 + \left| \dot{\tilde{h}}_\times^{\rm phys} \right|^2 \right) k^2 \,{\rm d}\Omega_k$$

Antisymmetric GWs energy density:

$$\Xi_{\rm GW}(t) = \int_{-\infty}^{\infty} \Xi_{\rm GW}(k,t)\,{\rm d}\ln k$$

$$\Xi_{\rm GW}(k,t) = \frac{k}{6H_*^2} \int_{4\pi} 2{\rm Im}\left(\dot{\tilde{h}}_+^{\rm phys} \dot{\tilde{h}}_\times^{\rm phys,*} \right) k^2 \,{\rm d}\Omega_k$$

$$H_* \approx 2.066 \cdot 10^{-11}\ {\rm s}^{-1} \left(\frac{T_*}{100\ {\rm GeV}} \right)^2 \left(\frac{g_*}{100} \right)^{1/2}$$

GWs amplitude:

$$h_{\mathrm{c}}(t) = \int_{-\infty}^{\infty} h_{\mathrm{c}}(k, t) \, \mathrm{d} \ln k$$

$$h_{\mathrm{c}}(k, t) = \frac{1}{2} \int_{4\pi} \left(\left| \tilde{h}_{+}^{\mathrm{phys}} \right|^2 + \left| \tilde{h}_{\times}^{\mathrm{phys}} \right|^2 \right) k^2 \, \mathrm{d}\Omega_k$$

Numerical results for decaying MHD turbulence[4]

Initial conditions

- Fully helical stochastic magnetic field
- Batchelor spectrum, i.e., $E_{\rm M} \propto k^4$ for small k
- Kolmogorov spectrum for inertial range, i.e., $E_{\rm M} \propto k^{-5/3}$
- Total energy density at t_* is $\sim 10\%$ to the radiation energy density[5]
- Spectral peak at $k_{\rm M} = 100 \cdot 2\pi$, normalized with $k_H = 1/(cH)$

Numerical parameters

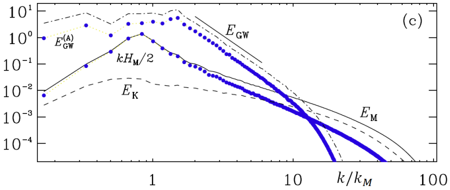
- 1152^3 mesh gridpoints
- 1152 processors
- Wall-clock time of runs is $\sim 1 - 5$ days

[4] A. Roper Pol, *et al.* arXiv:1903.08585

[5] limit from the Big Bang Nucleosynthesis bound

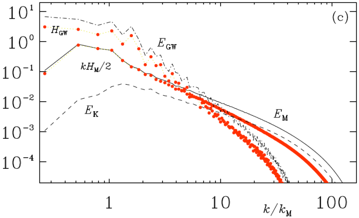
Numerical results for decaying MHD turbulence (old)

Negative initial helicity



$k_M/k_H = 2$

Positive initial helicity

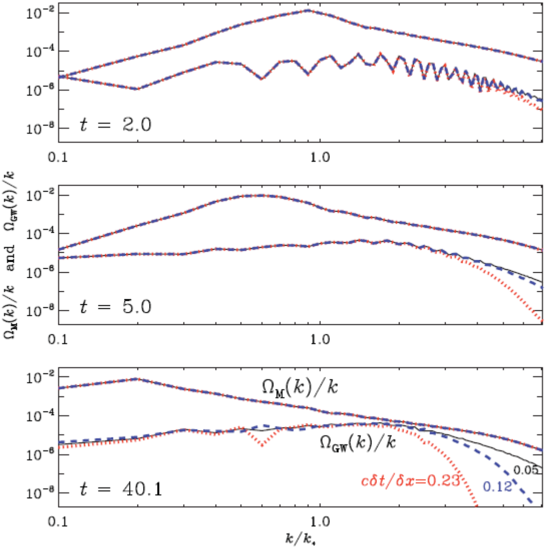


$k_M/k_H = 6$

Blue dots indicate negative helicity spectrum (magnetic fields) and negative antisymmetric spectrum (GW)

Red dots indicate positive helicity spectrum (magnetic field) and positive antisymmetric spectrum (GW)

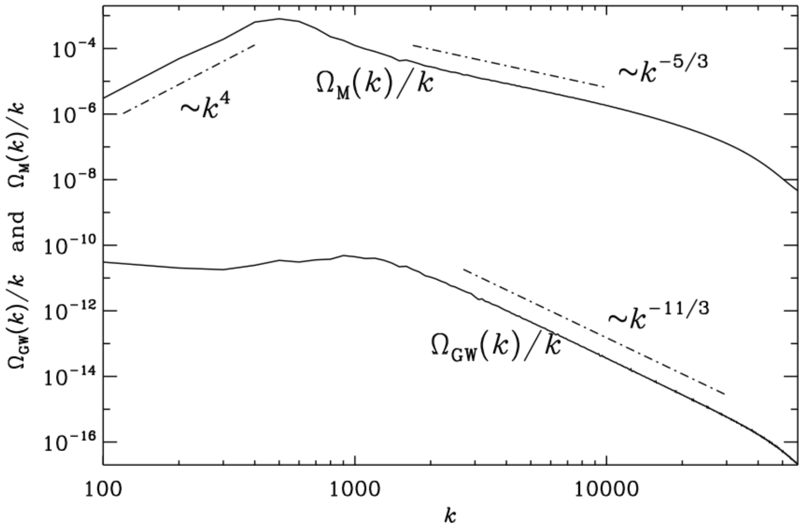
Numerical accuracy[4]



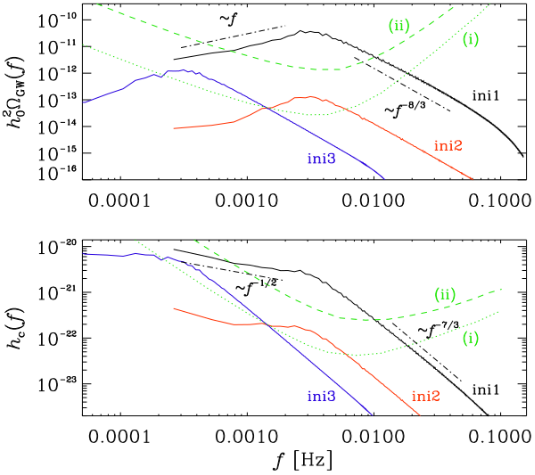
- CFL condition is not enough for GW solution to be numerically accurate
- $c\delta t/\delta x \sim 0.05 \ll 1$
- Higher resolution is required
- Hydromagnetic turbulence does not seemed to be affected

[4] A. Roper Pol *et al.*, *Geophys. Astrophys. Fluid Dyn.*, DOI:10.1080/03091929.2019.1653460, arXiv:1807.05479 (2019)

Numerical results for decaying MHD turbulence



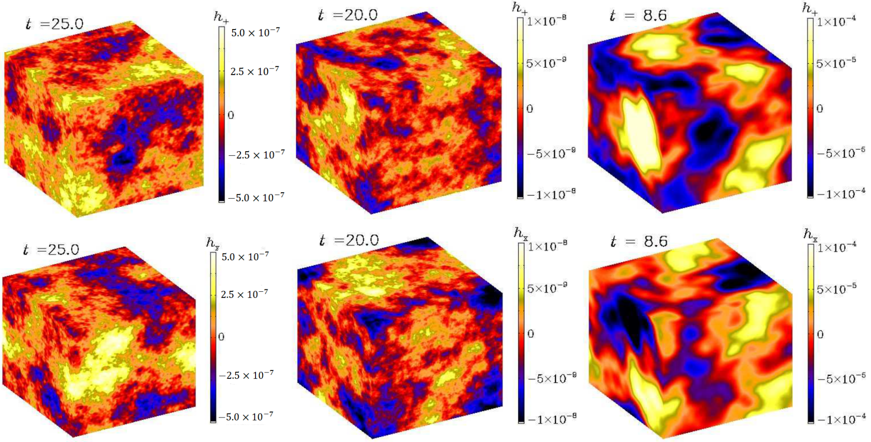
Numerical results for decaying MHD turbulence



- ini1: $k_{\rm M} = 100$, $\Omega_{\rm M} \approx 0.1$
- ini2: $k_{\rm M} = 100$, $\Omega_{\rm M} \approx 0.01$
- ini3: $k_{\rm M} = 10$, $\Omega_{\rm M} \approx 0.01$

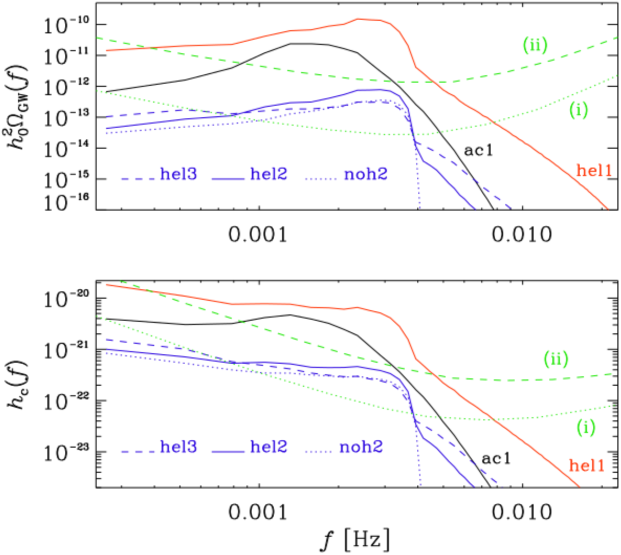
Numerical results for decaying MHD turbulence

Box results for positive initial helicity:

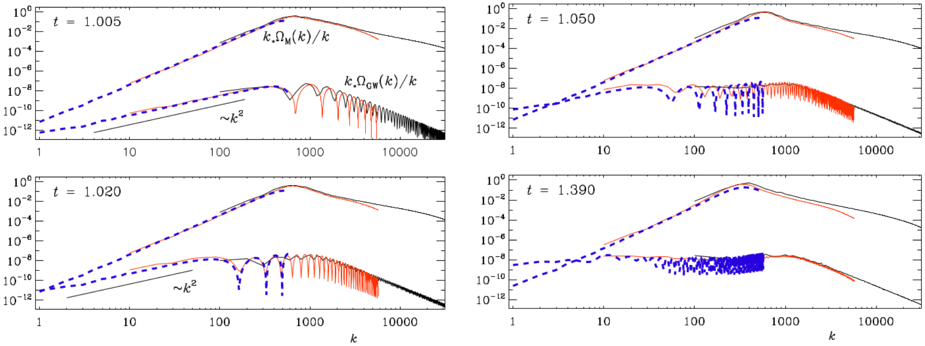


$k_M / k_H = 300$ $k_M / k_H = 60$ $k_M / k_H = 2$

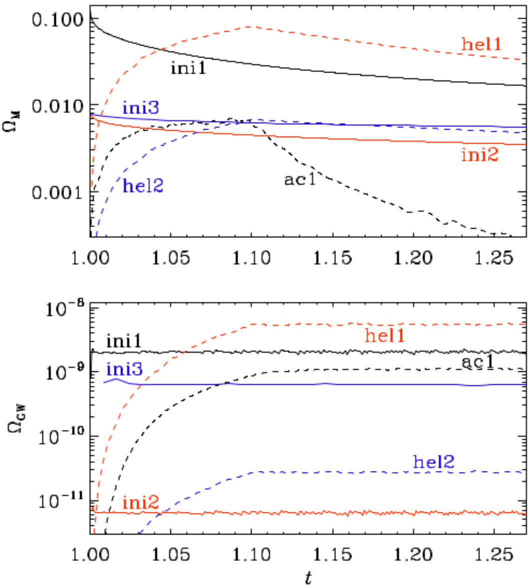
Forced turbulence (built-up primordial magnetic fields and hydrodynamic turbulence), low resolution



Early time evolution of GW energy density spectral slope



Time evolution of GW energy density



Detectability with LISA

LISA

- Laser Interferometer Space Antenna (LISA) is a space based GWs detector
- LISA is planned for 2034
- LISA was approved in 2017 as one of the main research missions of ESA
- LISA is composed by three spacecrafts in a distance of $\sim$2M km

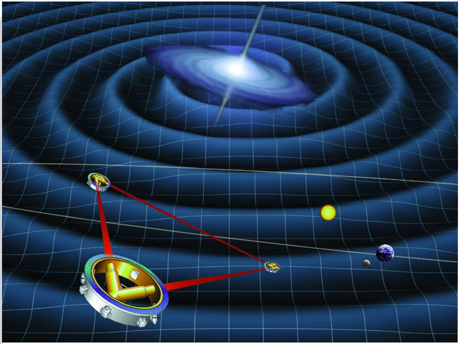


Figure: Artist's impression of LISA from Wikipedia

Detectability with LISA

- LISA sensitivity is usually expressed as $h_0^2 \Omega_{\rm GW}$
- $\Omega_{\rm GW}$ is the ratio of GWs energy density to critical energy density
- Critical energy density is
$$\mathcal{E}_{\rm crit} = \frac{3 H_0^2 c^2}{8\pi G}$$
- Current Hubble parameter is usually expressed as
$$H_0 = 100\, h_0 \ {\rm km\, s^{-1} Mpc^{-1}}$$
where h_0 represents the uncertainties in the actual value of H_0
- We consider two different LISA configurations [5]
  - 4-link configuration with 2×10^9 m arm length after 5 years of duration
  - 6-link configuration with 6×10^9 m arm length after 5 years of duration

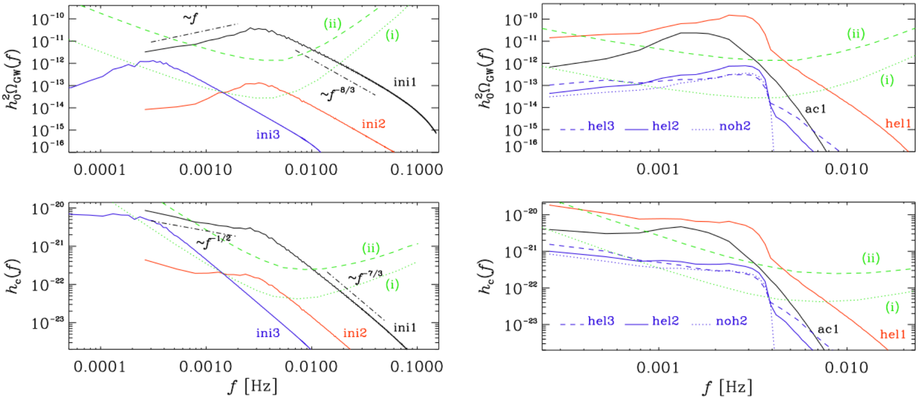
[5] C. Caprini et al., JCAP, 2016(04): 001001 (2016)

Detectability with LISA

GW energy density and characteristic amplitude

- Shifting due to the expansion of the universe:
  - $\Omega^0_{\rm GW}(k) = a_0^{-4}(H_*/H_0)^2 \Omega_{\rm GW}(k, t_{\rm end})$
  - $h^0_{\rm c}(k) = a_0^{-1} h_{\rm c}(k, t_{\rm end})$
  - $f^0 = a_0^{-1} f$

Detectability with LISA



Conclusions

- We have implemented a module within the Pencil Code that allows to obtain background stochastic GW spectra from primordial magnetic fields and hydrodynamic turbulence
- For some of our simulations we obtain a detectable signal by future mission LISA
- GW equation is normalized such that it can be easily scaled for different moments within the radiation-dominated epoch
- Bubble nucleation and magnetogenesis physics can be coupled to our equations for more realistic production analysis

The End
Thank You!