Phase transitions and the lattice

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Phase transitions at Electroweak scale

Interesting physics:

- 1st order transition?
  - $\rightarrow$ Gravitational waves?
  - $\rightarrow$ Baryon number generation?
  - $\rightarrow$ Magnetic fields?
- Baryon number violation stops
- No phase transitions in the Standard Model (at $\mu = 0$)
  - QCD and EW "phase transitions" are cross-overs
- Many BSM models may have a first order EW phase transition $\mapsto$
  - MSSM
  - 2HDM
  - Composite Higgs, Technicolor ...

In this talk:

- Why non-perturbative?
- 3d effective theory
- Precision results of the Standard Model:
  - Equation of state - “softness”, width of the cross-over
  - Sphaleron rate
  - Why study SM? Background e.g. to leptogenesis, precision physics
- MSSM
- 2HDM

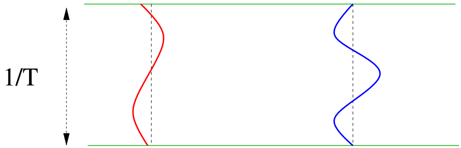
Loops at $T \neq 0$:

Finite T ensemble: euclidean with imaginary time extent $1/T$, with (anti)periodic b.c for bosons (fermions) $\Rightarrow$

$$\frac{1}{p^2} \longrightarrow \frac{1}{\bar{p}^2 + \omega_n^2}, \quad \omega_n = \begin{cases} 2n\pi T & n \in Z \quad \text{Bosons} \\ (2n+1)\pi T & n \in Z \quad \text{Fermions} \end{cases}$$

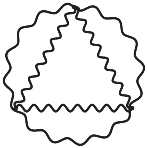
$$\int \frac{d^4 p}{(2\pi)^4} \longrightarrow T \sum_n \int \frac{d^3 \vec{p}}{(2\pi)^3}$$

Thus, all $n \neq 0$ Bosonic modes and all Fermionic modes acquire a "mass" $\sim \pi T$. Clearly, only bosonic $n = 0$ modes are infrared sensitive.



Momenta $k \sim g^2 T$ non-perturbative:

Let us consider vacuum diagram (pressure) at finite T, where we add a (fictitious) mass term m to keep track of the scale: [Linde 80]



$$N \text{ loops} \rightarrow \begin{cases} (N-1) & \text{4-vertices} \\ (2N-2) & \text{propagators} \end{cases}$$

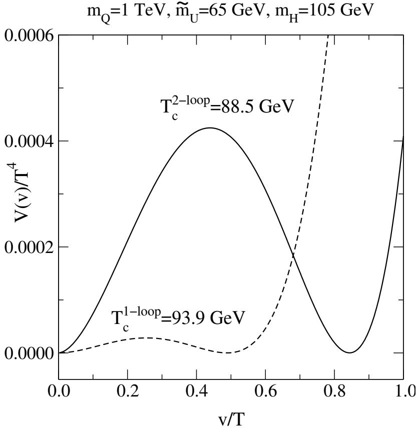
$$\left[T \int d^3p\right]^N (g^2)^{N-1} \left[\frac{1}{q^2+m^2}\right]^{2N-2} \propto g^6 T^4 \left[\frac{g^2 T}{m}\right]^{N-4}$$

If $m = g^2 T$ ("magnetic" scale), all loops contribute to pressure at g^6!
Perturbatively $m = 0$ for magnetic gauge modes $\Rightarrow$
Loop expansion fails when $\boxed{k \lesssim g^2 T}$ (3d confinement).

$\Rightarrow$ Any quantity is perturbatively computable only up to some fixed order!
Note: Loop expansion OK when
$k \sim \pi T$ (hard scales, $n \neq 0$)
$k \sim gT$ (Debye, electric scales)

Slow convergence

- Perturbative effective potential converges slowly: big differences between 1 and 2 loops: (MSSM)



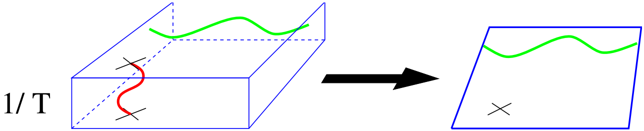
[Laine, K.R. 2002]

- Perturbatively weak 1st order transitions may vanish altogether (this happens in the Standard Model)

Method: 3d effective theory

- Tool for perturbative and lattice computations
- Modes $p > g^2 T$ are perturbative (at weak coupling): can be integrated out in stages:

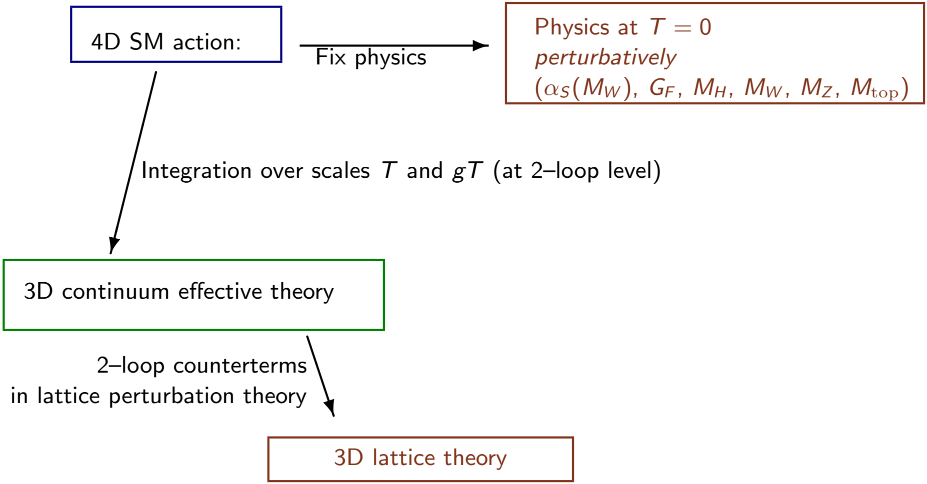
  1. $p \gtrsim T$: fermions, non-zero Matsubara frequencies
  $\rightarrow$ 3d theory (dimensional reduction)



  2. Electric modes $p \sim gT$

- Obtain a “magnetic theory” for modes $p \lesssim g^2 T$. Contains fully the non-perturbative thermal physics.

3d effective theory

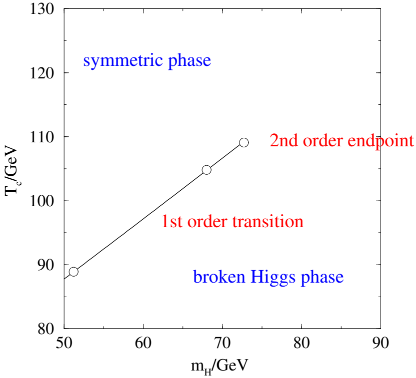


Standard Model

Phase diagram of the SM

- After lots of activity on and off the lattice:

→ No phase transition at all, smooth "cross-over" for $m_{\rm Higgs} \gtrsim 72$ GeV



[Kajantie,Laine,K.R.,Shaposhnikov,Tsypin 95–98]

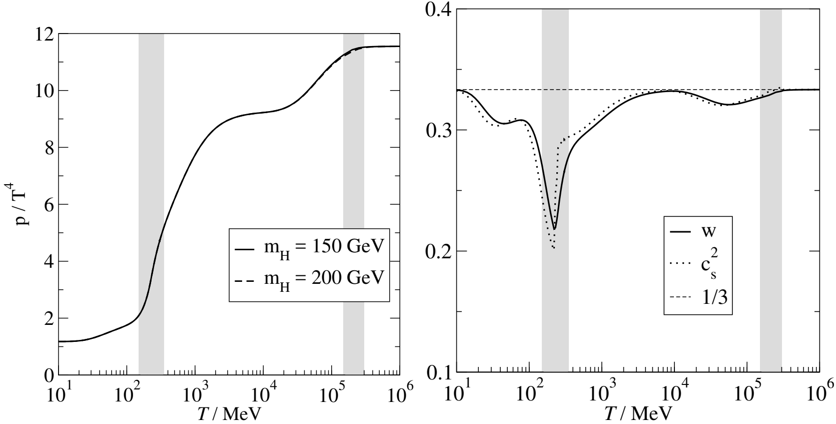
see also

[Csikor,Fodor, Heitger]

[Gürtler,Ilgenfritz,Schiller,Strecha]

Overall EOS

[Laine, Schröder 2006]



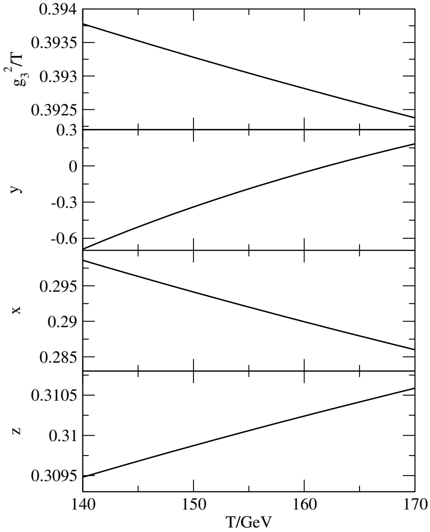
- Perturbation theory + Lattice QCD + Hadron RG
- Here EW transition "featureless" – can do better!

3d effective Lagrangian:

$$L = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} +$$
$$(D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

- SU(2) + U(1) gauge + Higgs
- Parameters:

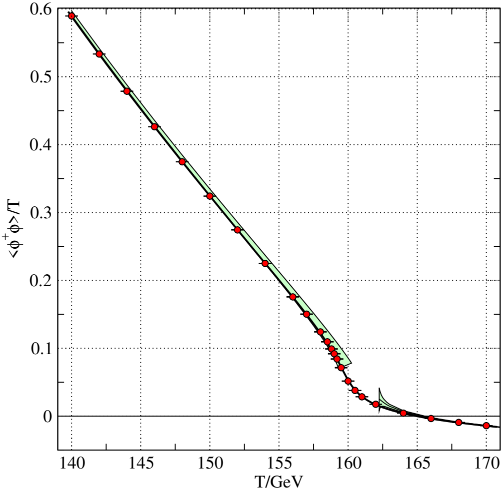
$$g_3^2 \sim g_W^2 T$$
$$x \equiv \lambda_3/g_3^2$$
$$y \equiv m_3^2/g_3^4$$
$$z \equiv g_3'^2/g_3^2$$



Precise mapping between T and 3d parameters

Higgs field expectation value

- Red dots: 3D lattice (continuum limit)
- Green bands:
  - Broken phase: 2-loop Coleman-Weinberg [Kajantie et al 95]
  - Symmetric phase: 3-loop [Laine and Meyer 2015]
- Agreement remarkably good away from the cross-over
- P.T. does not converge near the cross-over



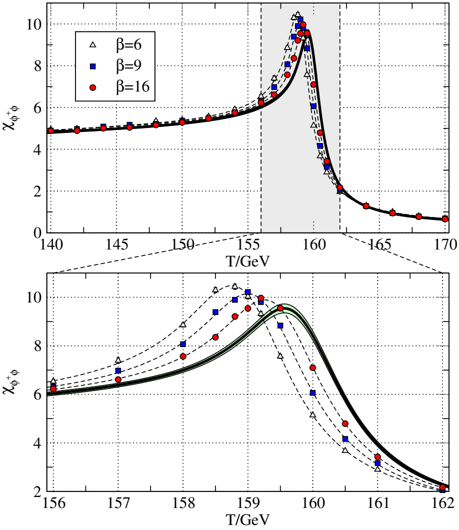
Higgs susceptibility & pseudocritical temperature

- Define pseudocritical T: maximum location of the $\langle\phi^\dagger\phi\rangle$ susceptibility $\chi_{\phi^\dagger\phi}$:

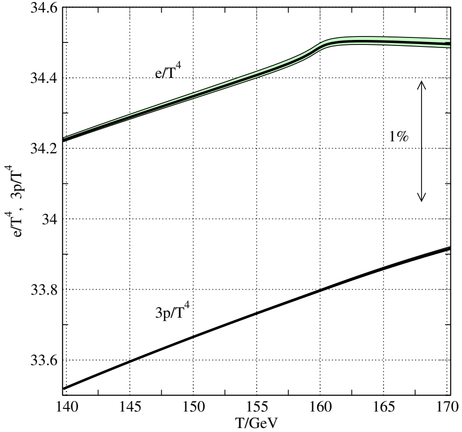
  $$T_c = 159.6 \pm 0.1 \pm 1.5 \text{ GeV}$$

- 1st error: lattice errors
- 2nd error: estimate of the systematic uncertainty in 3d action

Right: continuum extrapolation of χ



Pressure and energy density

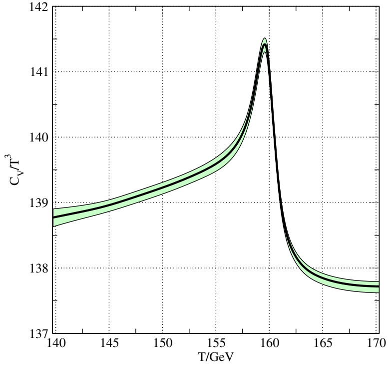


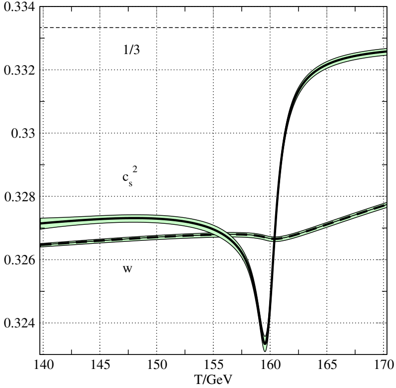
- Pressure:

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Delta(T')}{T'}$$

- Use reference temperature $T_0 = 140\,\mathrm{GeV}$,
- pert. pressure $p(T_0)/T_0^4 = 11.173$ [Laine, Schröder 2006]
- Energy density $e = \Delta + 3p$
- $p(T_0)$ has $\sim 1\%$ uncertainty $\rightarrow$ uncertainty for e and p

More thermodynamics



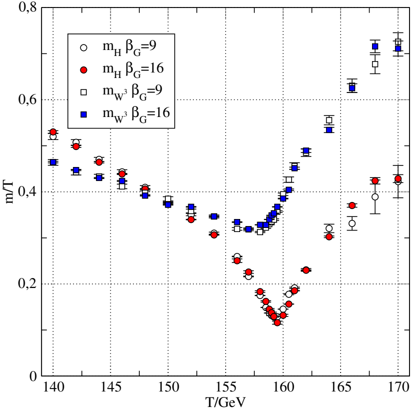


- Heat capacity $C_V = e'(T)$
- Speed of sound: $c_s^2 = p'/e'$
- EOS parameter $w = p/e$

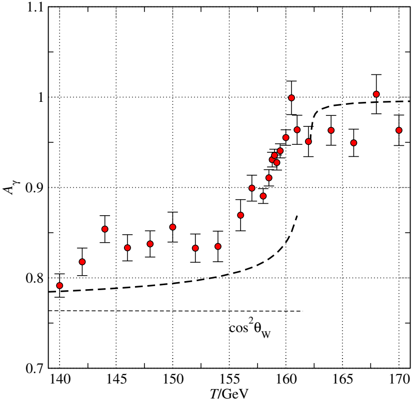
Cross-over well defined, but very soft!

Masses

Higgs and W^3 screening masses



Effective $\cos^2\theta_{\text{Weinberg}}$



Baryon number violation: sphaleron rate

- Anomaly: baryon number B and gauge topology are connected:

$$\Delta B = \Delta L = 3\Delta N_{\rm CS} = \frac{3}{32\pi^2}\int_0^t dt \int dV F^{\mu\nu}\tilde{F}_{\mu\nu}$$

- Baryogenesis
- Rate in thermal equilibrium:

$$\Gamma = \lim_{V,t\to\infty} \frac{\langle(\Delta N_{\rm CS}(t))^2\rangle}{Vt}$$

- In the symmetric phase, $\Gamma \propto \alpha_W^5 T^4$ [Arnold, Son, Yaffe 97] or rather $\Gamma \propto \alpha_W^5 \log(1/\alpha_W) T^4$ [Bödeker 98]
- In the broken phase the rate is exponentially suppressed
- Turning off of the rate is important for some baryogenesis scenarios

Calculation of the sphaleron rate

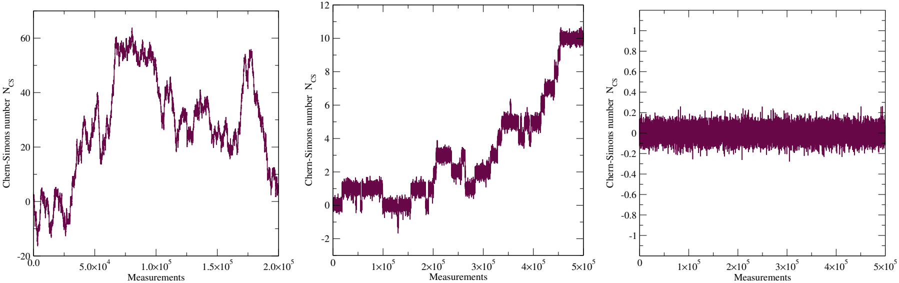
Non-perturbative $\Rightarrow$ real-time lattice simulations
Several methods:

- Classical EOM [Ambjorn, Krasnitz 95; + many]
  - UV modes (HTL) make result lattice spacing depedent [Arnold 97]
- Classical + HTL effective theories [Moore, Hu, Muller 97; Bodeker, Moore, K.R 99]
  - More control over UV modes, no continuum limit
  - Used also in studies of plasma instabilities in HIC
- Bödeker method: [Bödeker 98], heat bath version [Moore 98]
  - Fully dissipative evolution of soft (g^2T) modes
  - ✓ Exact to leading log order $\log(1/g^2)$, $\exists$ continuum limit, very simple to use
  - Same "action" and cont. limit as in 3D thermo simulation

$\exists$ lot of lattice results in pure gauge, few in broken EW phase.
Here: physical Higgs mass

Evolution of $N_{\rm CS}$

Symmetric $T = 152$GeV, broken $T = 145$GeV, deeply broken $T = 140$GeV (with $m_H = 113$GeV) [D'Onofrio et al 12]



Rate strongly suppressed in the broken phase. Use special "multicanonical" methods to overcome this.

Result: sphaleron rate

Symmetric phase:

$$\frac{\Gamma}{T^4} = (8.0 \pm 1.3) \times 10^{-7} \approx (18 \pm 3)\alpha_W^5$$

Broken phase: parametrized as

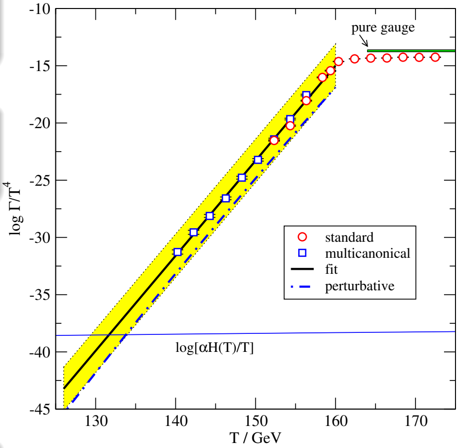
$$\log \frac{\Gamma}{T^4} = (0.83 \pm 0.01)\frac{T}{\text{GeV}} - (147.7 \pm 1.9)$$

Errors dominated by systematics

Cosmology: Hubble cooling $\dot{T} = -HT$
Freeze-out temperature T_* from
$\frac{\Gamma(T_*)}{T_*^3} = \alpha H(T_*)$ with $\alpha \approx 0.1015$ [Burnier et al 05]

Baryon number freeze-out

$T_* = 131.7 \pm 2.3$ GeV



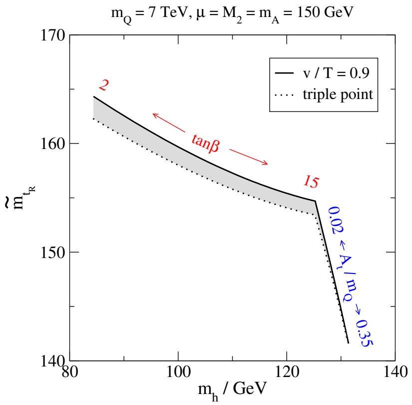
Overview of the Standard Model

- Standard Model equation of state solved to 1% level
- Pseudocritical temperature $T_c = 159.6 \pm 0.1 \pm 1.5$ GeV
- Cross-over weak but well defined
- Width of the cross-over region 2-3 GeV
- Baryon number freeze-out $T_* = 131.7 \pm 2.3$ GeV
- Symmetric phase sphaleron rate $\Gamma / T^4 = (18 \pm 3)\alpha_W^5$
- Broken phase rate can be parametrized as
  $\log \Gamma / T^4 = (0.83 \pm 0.01)\, T/\text{GeV} - (147.7 \pm 1.9)$
- . . . can be fed in to e.g. some leptogenesis scenarios

MSSM

[Laine, Nardini, K.R.]

Parameters:



Strong transition when right-handed stop U is light: $m_{\tilde{t}_R} < m_{\rm top}$
(U: Higgs for SU(3) color!)

Choose MSSM parameters
$\tilde{m}_U = 70.4$ GeV
$m_Q = 7$TeV
$\mu = M_2 = m_A = 150$GeV
$\tan\beta = 15$
$A_t/m_Q = 0.02$

These correspond to
Higgs mass $m_h = 126$ GeV and
the right-handed stop mass
$m_{\tilde{t}_R} \approx 155$ GeV

[Laine, Nardini, K.R.]

3d effective Lagrangian:

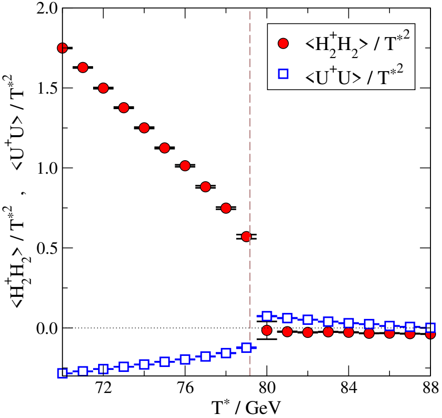
$$
\begin{aligned}
\mathcal{L}_{\rm 3d} \;=\;& \frac{1}{2}\,\mathrm{Tr}\, G_{ij}^2 + (D_i^s U)^\dagger (D_i^s U) + m_U^2(T) U^\dagger U + \lambda_U (U^\dagger U)^2 \\
+\;& \gamma_1 U^\dagger U H_1^\dagger H_1 + \gamma_2 U^\dagger U H_2^\dagger H_2 + \left[\gamma_{12} U^\dagger U H_1^\dagger H_2 + \mathrm{H.c.}\,\right] \\
+\;& \frac{1}{2}\,\mathrm{Tr}\, F_{ij}^2 + (D_i^w H_1)^\dagger (D_i^w H_1) + (D_i^w H_2)^\dagger (D_i^w H_2) \\
+\;& m_1^2(T) H_1^\dagger H_1 + m_2^2(T) H_2^\dagger H_2 + \left[m_{12}^2(T) H_1^\dagger H_2 + \mathrm{H.c.}\,\right] \\
+\;& \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 + \lambda_4 H_1^\dagger H_2 H_2^\dagger H_1 \\
+\;& \left[\lambda_5 (H_1^\dagger H_2)^2 + \lambda_6 H_1^\dagger H_1 H_1^\dagger H_2 + \lambda_7 H_2^\dagger H_2 H_1^\dagger H_2 + \mathrm{H.c.}\,\right].
\end{aligned}
$$

G_{ij}: SU(3) gauge
U: right-handed stop
F_{ij}: SU(2) gauge
H_1, H_2: two SU(2) scalars (Higgses)
Parameters $g_W^2; g_S^2; m_i^2; m_{12}^2; \lambda_j; \gamma_i; \gamma_{12}$ depend on the 4d physical parameters (incl. temperature T).

Simulation volumes

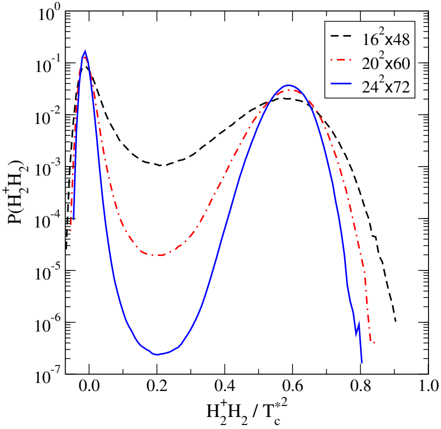
$\beta_w = 4/(g_W^2 Ta)$	volumes
8	12^3, 16^3
10	16^3
12	16^3, 20^3, 32^3, $12^2 \times 36$, $20^2 \times 40$
14	24^3, $14^2 \times 42$, $24^2 \times 48$
16	24^3, $16^2 \times 48$, $20^2 \times 60$, $24^2 \times 72$
20	32^3, $20^2 \times 60$, $26^2 \times 72$, $32^2 \times 64$
24	24^3, 32^3, 48^3, $24^2 \times 78$, $30^2 \times 72$
30	48^3

Results: Higgs field expectation value $\langle H_2^\dagger H_2 \rangle$

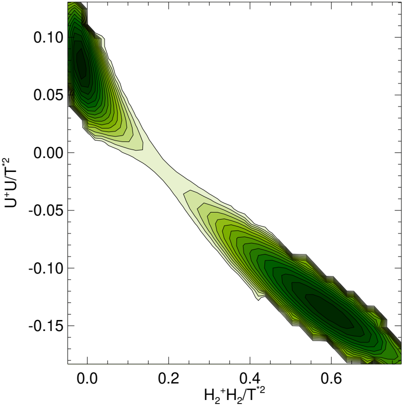


- Results extrapolated to continuum
- Strong transition
- Stop field U participates

Histograms

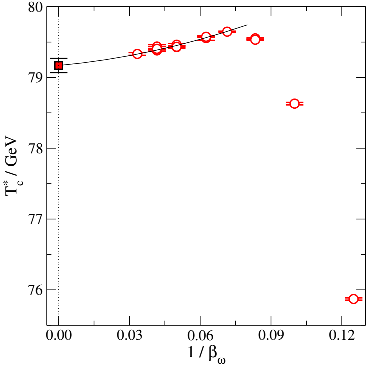


10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$P(H_2^+H_2)$
0.0
0.2
0.4
0.6
0.8
1.0
$H_2^+H_2 / T_c^{*2}$
16^2x48
20^2x60
24^2x72



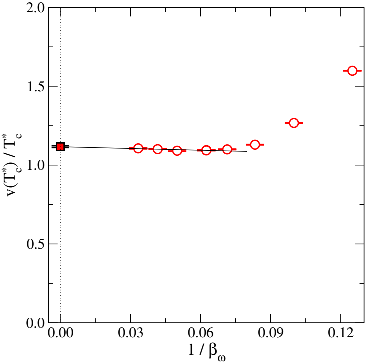
0.10
0.05
0.00
-0.05
-0.10
-0.15
U^+U/T^{*2}
0.0
0.2
0.4
0.6
$H_2^+H_2/T^{*2}$

Results: continuum limits



$T_c = 79.17 \pm 0.10 \quad \mathrm{GeV}$

2-loop pert. theory ~ 84.4 GeV



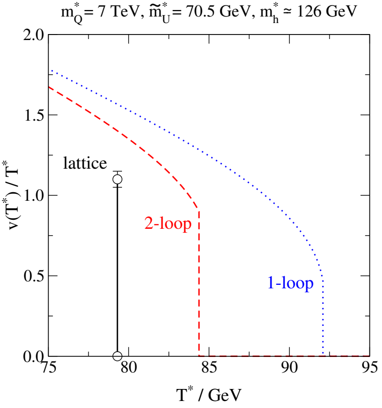
$v / T_c = 1.117 \pm 0.005$

pert. theory 0.9

Results compared with 2-loop perturbation theory

		lattice	perturbative (Landau gauge)
Transition temperature	T_c/GeV	79.17(10)	84.4
Higgs discontinuity	v/T_c	1.117(5)	0.9
Latent heat	L/T_c^4	0.443(4)	0.26
Surface tension	σ/T_c^3	0.035(5)	0.025

The transition is clearly stronger on the lattice.



2HDM

[Kainulainen, Keus, Niemi, K.R., Tenkanen, Vaskonen]

Benchmark points

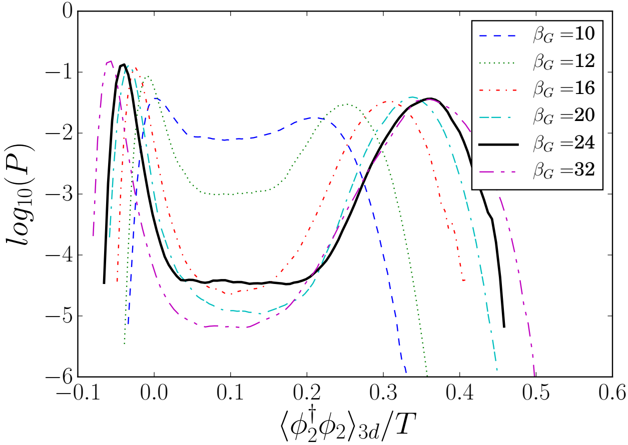
2HDM is not so strongly constrained than MSSM by collider phenomenology.
Strong phase transition $\rightarrow$ large scalar couplings λ_i

- $\rightarrow$ problems in perturbation theory; accuracy of eff. 3d description?
- $\rightarrow$ Landau pole is close

BM1	M_H	M_A	$M_{H\pm}$	μ	$\lambda_{345}/2$	λ_1	Λ_0
	66 GeV	300 GeV	300 GeV	0 GeV	1.07×10^{-2}	0.01	91 GeV
BM2	M_H	M_A	$M_{H\pm}$	μ	$\cos(\beta-\alpha)$	$\tan\beta$	Λ_0
	150 GeV	350 GeV	350 GeV	80 GeV	-0.02	2.75	265.018 GeV

BM1: "Inert doublet model", studied perturbatively by [Laine, Meyer, Nardini 2017]
BM2: Approaches [Dorsch et al.] point but more restricted λ_i

Strong transition



Heavy modes at $T_c \Rightarrow$ need to go to very small lattice spacing (large β_G)

Results

	Method	T_c/GeV	L/T_c^4	ϕ_c/T_c	L/GeV^4
BM1	1-loop Parwani resum.	134.0 ± 8.75	0.396 ± 0.002	1.01 ± 0.06	1.27×10^8
	1-loop A-E resum.	142.4 ± 6.88	0.33 ± 0.02	1.00 ± 0.07	1.37×10^8
	2-loop V_{eff} in 3d	111.6 ± 2.30	0.57 ± 0.10	0.98 ± 0.09	0.89×10^8
	3d lattice	116.40 ± 0.005	0.60 ± 0.02	1.08 ± 0.02	1.11×10^8
BM2	1-loop Parwani resum.	142.6 ± 18.0	0.29 ± 0.04	0.91 ± 0.06	1.19×10^8
	1-loop A-E resum.	162.5 ± 21.0	0.20 ± 0.03	0.88 ± 0.05	1.36×10^8
	2-loop V_{eff} in 3d	104.9 ± 2.30	0.61 ± 0.10	0.97 ± 0.06	0.74×10^8
	3d lattice	112.5 ± 0.01	0.81 ± 0.05	1.09 ± 0.03	1.29×10^8

Largish variation in results

Note: V_{eff} in 3d relies on the same 3d effective theory than 3d lattice

Transition stronger on the lattice

Conclusions:

- the Standard Model cross-over is resolved
- If couplings are small, 3d effective theory simulations give very accurate results
- Strong transitions seen in MSSM, 2HDM
- Effective theory method applicable to many "Higgs-like" models
- Bubble nucleation rate can be measured