Nonperturbative analysis of the electroweak phase transition and its gravitational wave signal

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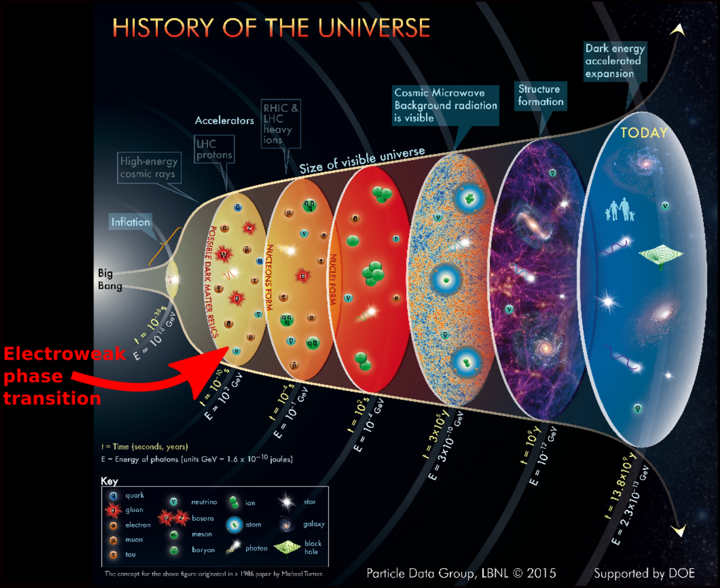
largely based on
arXiv:1903.11604
OG, Jonathan Kozaczuk, Lauri Niemi, Michael J. Ramsey-Musolf, Tuomas V. I. Tenkanen and David J. Weir

Outline

Electroweak phase transition



Electroweak crossover

Standard Model phase diagram

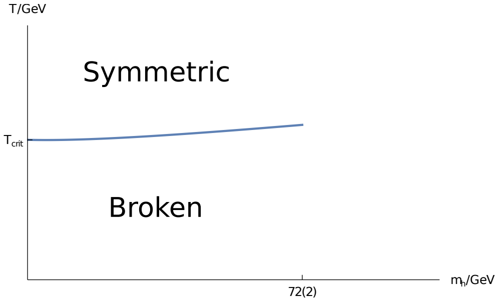


Figure: Phase diagram determined nonperturbatively.

Kajantie et al. '96, Gurtler et al. '97, Rummukainen et al. '98

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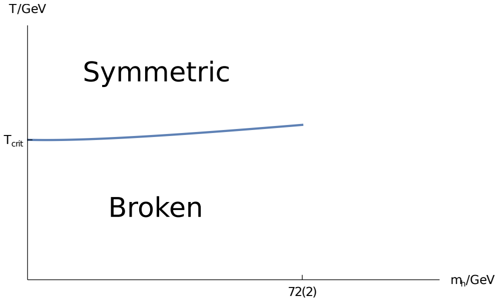

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Perturbation theory wrongly predicts a 1st order transition.

A first order transition?

First order transitions in many models

Need new dynamics coupled to the Higgs:

- Singlet or triplet scalar extensions (xSM and ΣSM)
- Two Higgs doublet
- Higgs portals
- Composite Higgs
- Scale invariant/dilaton-like scalar extensions
- SUSY models
- Low scale higher dimension operators

many refs

Such extensions are also motivated by electroweak baryogenesis (along with some new source of CP violation).

Gravitational wave production

Broken phase bubbles in symmetric phase

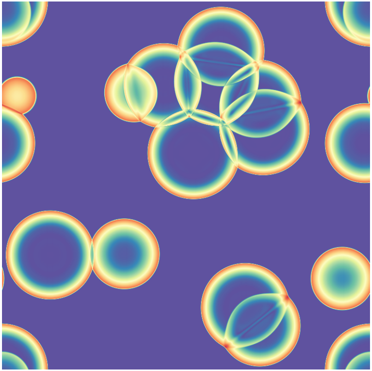
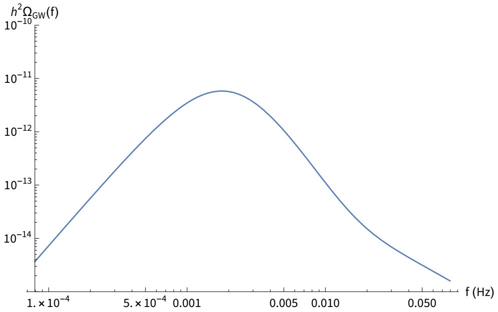


Figure: Magnitude of the fluid velocity during a strong first order phase transition.
Figure courtesy of Daniel Cutting and collaborators.

Gravitational wave spectra

Gravitational wave signals depend on BSM physics through:

- T_c, the critical temperature,
- L, the latent heat (or $\Delta\Theta$, the change in trace anomaly),
- v_w, the bubble wall speed,
- β/H_*, the (inverse) duration of the phase transition.



Gravitational wave detection

Laser Interferometer Space Antenna (LISA)

The space-based gravitational wave detector, LISA, due to be launched in 2034, will be sensitive to the right frequencies for observing such a signal.

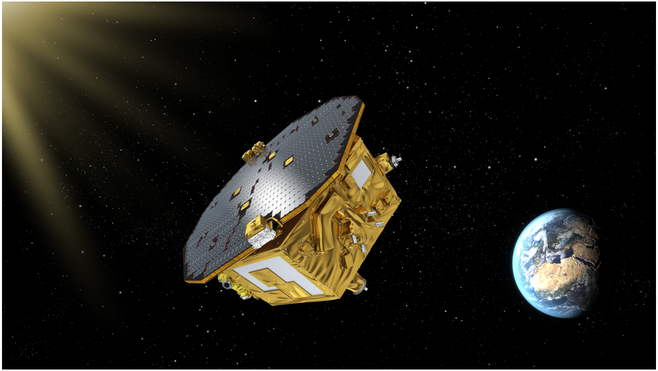


Figure: LISA pathfinder (2016) exceeded ESA's expectations.

Questions for this talk

- Perturbation theory does not reliably describe the phase transition. How can we overcome this?
- Gravitational wave signal depends on just a few parameters but BSM models may have many parameters. Is there a better way of organising things?

The Infrared Problem

Linde's infrared problem

At high temperatures, effective expansion parameter for light bosons is

$$\frac{g^2}{e^{E/T}-1} \approx \frac{g^2 T}{E} \geq \frac{g^2 T}{m}.$$

So for $m \lesssim g^2 T$, the perturbative expansion breaks down due to high occupancies of infrared bosonic modes. Linde '80

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Light bosons are nonperturbative at finite temperature

The problematic light bosons

In Matsubara formalism, bosons satisfy periodic boundary conditions in imaginary time, with period $1/T$,

$$\phi(\mathbf{k},\tau) = \sum_{n=0}^{\infty} \phi_n(\mathbf{k})\mathrm{e}^{-2\pi i T n \tau}.$$

The propagator for nth mode is

$$\frac{1}{\mathbf{p}^2 + (2\pi T n)^2 + m^2},$$

so only the $n = 0$ (static) modes are troublesome in infrared.

For fermions antiperiodic boundary conditions imply $n \to n + \frac{1}{2}$.

Perturbation theory

Infrared problem can be alleviated by resummation of ring diagrams, at least for strong transitions. Parwani '92, Arnold & Espinosa '93

- Two-loop accuracy difficult - sum-integrals, resummation, many fields.
- Strong dependence on RG scale and gauge fixing parameter.
- Even one-loop accuracy has not been acheived for bubble nucleation rate.

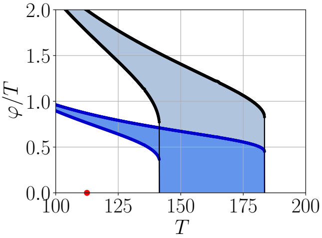

Image from Kainulainen, Keus, Niemi, Rummukainen, Tenkanen & Vaskonen '19

Dimensional reduction

Effective field theory to the rescue

Integrate out everything but the dangerous light bosons. The resulting effective field theory can then be studied on the lattice.

$\mathcal{L}\ (\phi, A_\mu, \psi, S, s)$

superheavy — πT — Integrate out $n > 0$ modes and $S_{n=0}$

$\mathcal{L}_3(\phi_3, A_i, A_0, s_3)$

heavy — gT — Integrate out A_0, s_3 fields

$\overline{\mathcal{L}}_3(\bar{\phi}_3, \bar{A}_i)$

light — $g^2 T$

Ginsparg '80,
Appelquist & Pisarski '81

Dimensional reduction is infrared safe.

Dimensional reduction by matching

4d theory

$$Z_{4\mathrm{d}} = \int \mathcal{D}A\mathcal{D}\phi\mathcal{D}S\mathcal{D}s\,\mathrm{e}^{-S_{4\mathrm{d}}[A,\phi,\psi,S,s]}$$

3d theory

$$Z_{3\mathrm{d}} = \int \mathcal{D}A_{3\mathrm{d}}\mathcal{D}\phi_{3\mathrm{d}}\,\mathrm{e}^{-S_{3\mathrm{d}}[A_{3\mathrm{d}},\phi_{3\mathrm{d}}]}$$

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Equate static correlation functions

Static correlation functions are matched for all light fields with momenta $p \lesssim g^2T$, e.g.

$$\langle \phi_0\phi_0 A_0^{a,i} A_0^{b,j}\rangle = \frac{1}{T^2}\langle \phi_{3\text{d}}\phi_{3\text{d}} A_{3\text{d}}^{a,i} A_{3\text{d}}^{b,j}\rangle + O(g^n),$$

Here the power g^n depends on the loop order at which the dimensional reduction is carried out.

Kajantie et al. '95

Electroweak effective field theory

The low energy theory

For the Standard Model, after integrating out all superheavy and heavy modes, the high temperature 3d EFT is,

$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4g_3^2}F^a_{ij}F^a_{ij} + \frac{1}{4g_3'^2}B_{ij}B_{ij} + (D\Phi)^\dagger D\Phi + m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 + \text{higher order operators}$$

N.B. fields and couplings have 3d mass dimensions,

$$[\Phi] = \mathrm{GeV}^{1/2},$$
$$[\lambda_3] = [m_3] = [g_3^2] = \mathrm{GeV}.$$

Electroweak effective field theory

Dimensions and scalings

The 3d theory only depends nontrivially on dimensionless ratios,

$$x \equiv \frac{\lambda_3}{g_3^2}, \qquad y \equiv \frac{m_3^2}{g_3^4}, \qquad z \equiv \frac{g_3'^2}{g_3^2}.$$

In which case we can write,

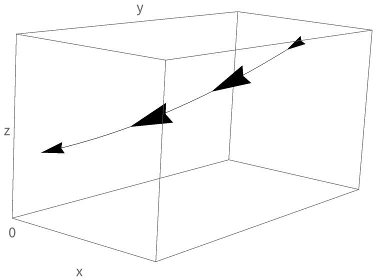
$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4}F_{ij}^a F_{ij}^a + \frac{1}{4z}B_{ij}B_{ij} + (D\Phi)^\dagger D\Phi + y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2$$

Map from 4d to 3d

Dimensional reduction (DR) gives a map from the 4d theory at a given temperature to the 3d effective theory,

$$\mathrm{DR} : (\{g_i\}, T) \to (x, y, z, g_3).$$

- x determined by Higgs interactions
- y varies strongly with temperature
- z fixed by $\tan\theta_w$



Nonperturbative results

Important quantities have already been determined nonperturbatively in the 3d effective theory:

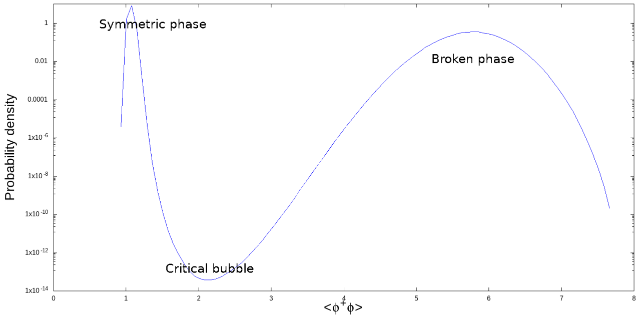
- Phase diagram — Kajantie et al. '96, Gurtler et al. '97, Rummukainen et al. '98
- Higgs vev, latent heat, surface tension — Kajantie et al. '95
- Nucleation rate — Moore & Rummukainen '00
- Sphaleron rate — Moore '98, D'Onofrio et al. '12

Lattice Monte-Carlo study of 3d EFT

- Histogram of measurements in Monte-Carlo Markov chain $\rightarrow$ probability distribution.
- Multicanonical methods necessary for strong transitions.

Berg & Neuhaus '91

- No fermions, only 3d $\rightarrow$ can go to large lattices.

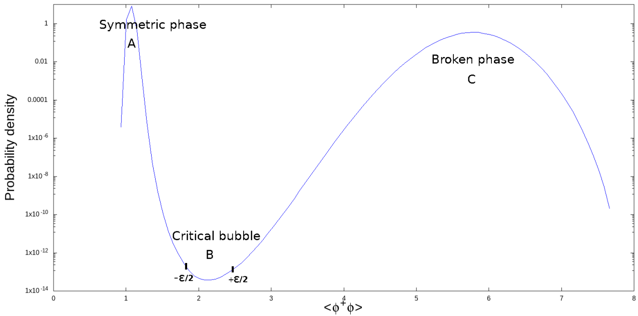


Bubble nucleation on the lattice

Rate of bubble nucleation,

$$\Gamma = \left\langle \frac{\#\ \mathsf{A} \to \mathsf{C}}{\#\ \mathsf{crossing\ B}} \right\rangle \left\langle \left| \frac{\Delta \phi^2_{\text{bub}}}{\Delta t} \right| \right\rangle \frac{P(\text{bub} \pm \frac{\epsilon}{2})}{\epsilon P(\text{sym})}$$

involves real-time evolution of critical bubble.

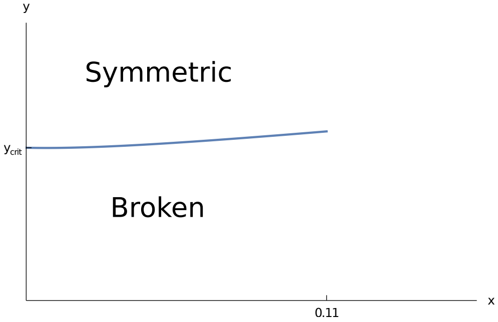


The phase diagram

There are first order phase transitions for

$$x \equiv \frac{\lambda_3}{g_3^2} \lesssim 0.11,$$

with transitions getting stronger as $x \to 0_+$.



How can BSM physics change this?

New electroweak physics

heavy/superheavy?
$(m \gtrsim gT)$

modified couplings
in 3d EFT
but no new fields

light?
$(m \lesssim g^2T)$

additional fields
in 3d EFT
(requires new
lattice simulations)

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xSM - real singlet extension

Consider an additional heavy/superheavy, real, gauge-singlet, σ, in the 4d theory,

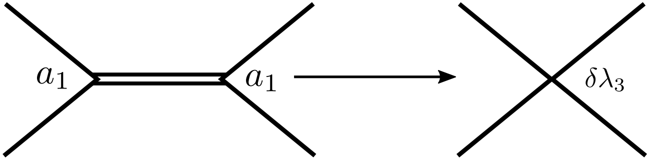
$$\delta\mathcal{L}_{\text{scalar}} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}\mu_\sigma^2\sigma^2 + b_1\sigma + \frac{1}{3}b_3\sigma^3 + \frac{1}{4}b_4\sigma^4 + \frac{1}{2}a_1\sigma\phi^\dagger\phi + \frac{1}{2}a_2\sigma^2\phi^\dagger\phi,$$

where ϕ is the 4d Higgs field.

Effects of the heavy singlet

Away from the Z_2 limit, the effect of the heavy singlet is dominated by the following effects which strengthen the transition,

$$\frac{\lambda_3}{T} = \lambda - \frac{a_1^2}{8\mu_\sigma^2} - \frac{a_1^2 b_3 b_1}{4\mu_\sigma^6} + \frac{a_1 a_2 b_1}{2\mu_\sigma^4} + \text{loop effects.}$$



This effect can decrease λ_3 and hence x, moving the electroweak sector towards a first-order phase transition.

The phase diagram

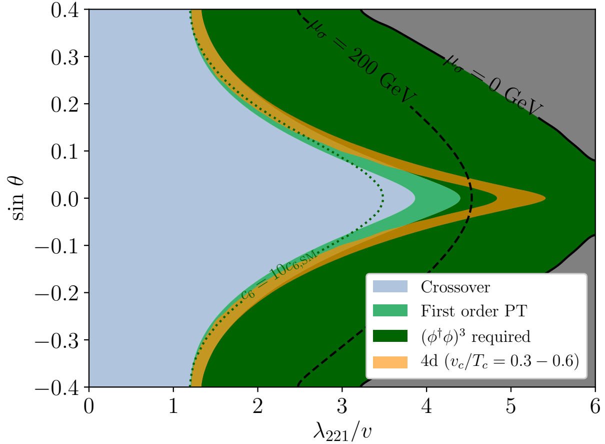


Figure: The phase diagram of xSM in parameter slice with $m_2 = 400$ GeV, $b_4 = 0.25$, $b_3 = 0$.

Dimensional reduction for the phase transition

The relation between the latent heat in 4d and 3d is

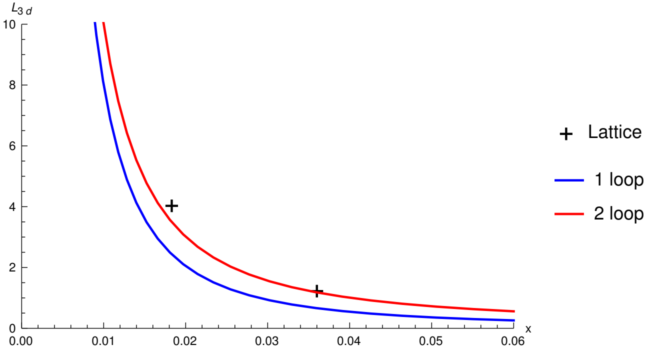
$$\frac{L_{4\mathrm{d}}(\{g_i\}, T_c)}{T_c^4} = (\text{DR dependent factor}) L_{3\mathrm{d}}(x, y, z),$$

and similarly for other quantities. Here the DR dependent factor is,

$$\left(\frac{g_3^6}{T_c^3} \frac{dy}{d \log T} \right) = O(g^4),$$

the order of magnitude being independent of the specific 4d theory.

Strong transitions



Very strong transitions require very small x. N.B. perturbation theory is reasonably good for strong transitions.

Very strong transitions

But at very small x, higher order operators cannot be ignored,

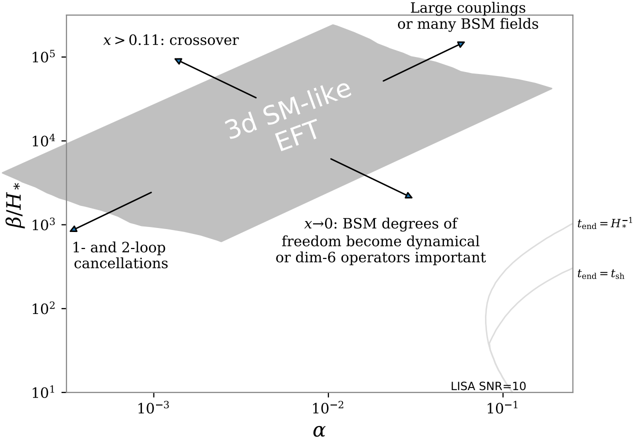
$$\mathcal{L}_{3\text{d}} = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4z}B_{ij}B_{ij} + (D\Phi)^\dagger D\Phi + y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 + c_6(\Phi^\dagger\Phi)^3 + \text{even higher order operators.}$$

The quartic term should be much larger than higher order terms,

$$x\langle(\Phi^\dagger\Phi)^2\rangle \gg c_6\langle(\Phi^\dagger\Phi)^3\rangle,$$

which for $c_6 = c_{6,\text{SM}}$ leads to $x \gg 0.01$. Thus the minimal 3d EFT can not give arbitrarily strong phase transitions.

The gravitational wave spectrum



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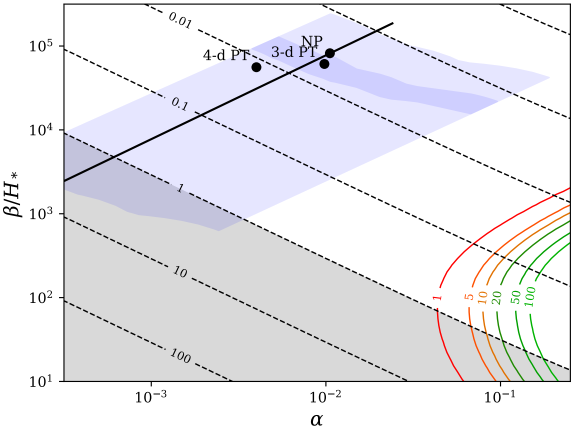


Including higher dimensional operators

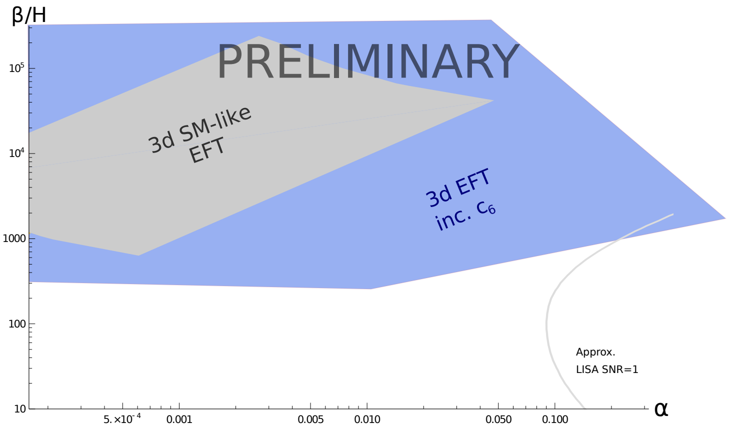
- Higher dimensional operators become important for very strong transitions in the minimal 3d SM-like EFT, where $x \lesssim 0.01$.
- The operator

$$c_6 \langle (\Phi^\dagger \Phi)^3 \rangle$$

  is the dominant higher dimensional operator for scalar extensions with large portal couplings, $c_6 = O(\lambda_{\text{portal}}^3)$.
- Including this, one gets strong transitions when $x < 0$, for,

$$\frac{c_6}{-x} \ll 1.$$

Including $c_6 \langle (\Phi^\dagger \Phi)^3 \rangle$



Djuna Croon, OG, Tuomas Tenkanen, Graham White, forthcoming

Conclusions

- The use of high temperature 3d effective theories helps to:
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  - new light ($m \lesssim g^2T$) bosonic fields coupled to the Higgs,
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Thank you for listening!