Gravitational waves from first-order phase transitions: ultra-supercooled transitions and the fate of relativistic shocks

Ryusuke Jinno (DESY)

Based on 1905.00899

with Hyeonseok Seong (IBS & KAIST), Masahiro Takimoto (Weizmann), Choong Min Um (KAIST)

09.2019 @ Nordita workshop

TALK PLAN

1. Introduction

2. Ultra-supercooled transitions

3. Ultra-supercooled transitions:

Effective description of fluid propagation & Implications to GW production

4. Summary

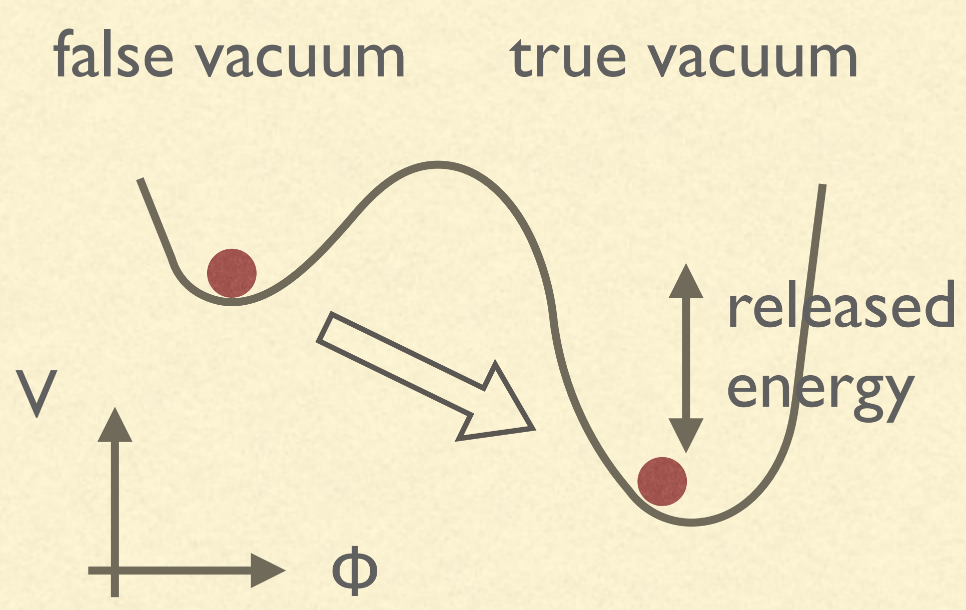
Introduction

FIRST-ORDER PHASE TRANSITION & GWS

- Rough sketch of 1st-order phase transition & GW production

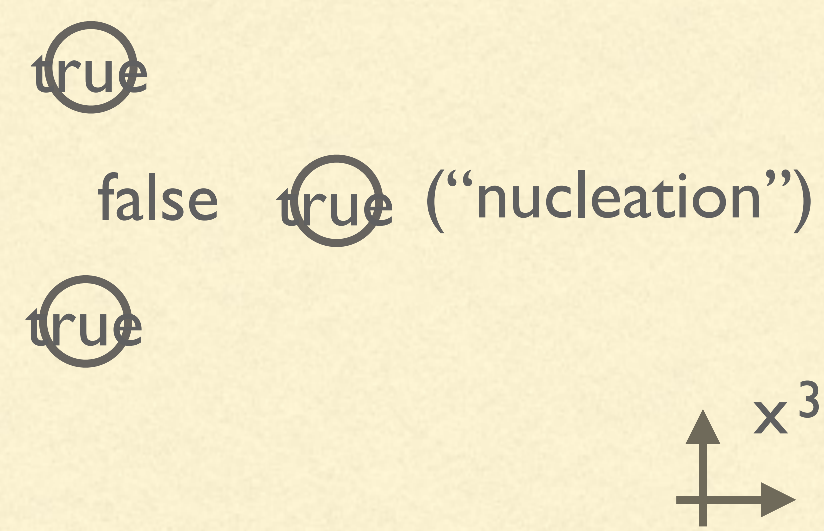
Bubbles nucleate, expand, collide and disappear, accompanying fluid dynamics

Field space



Quantum tunneling

Position space



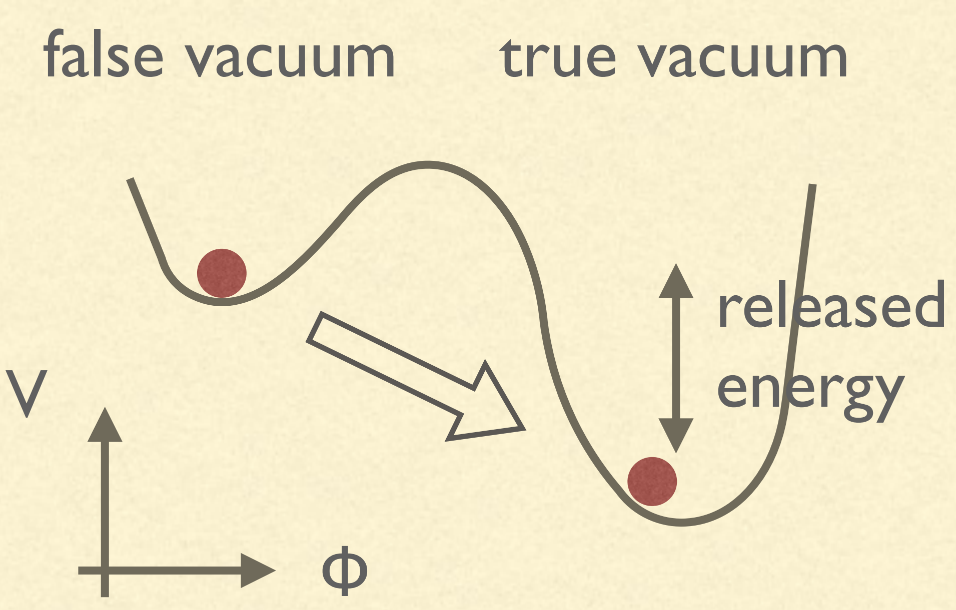
Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

- Rough sketch of 1st-order phase transition & GW production

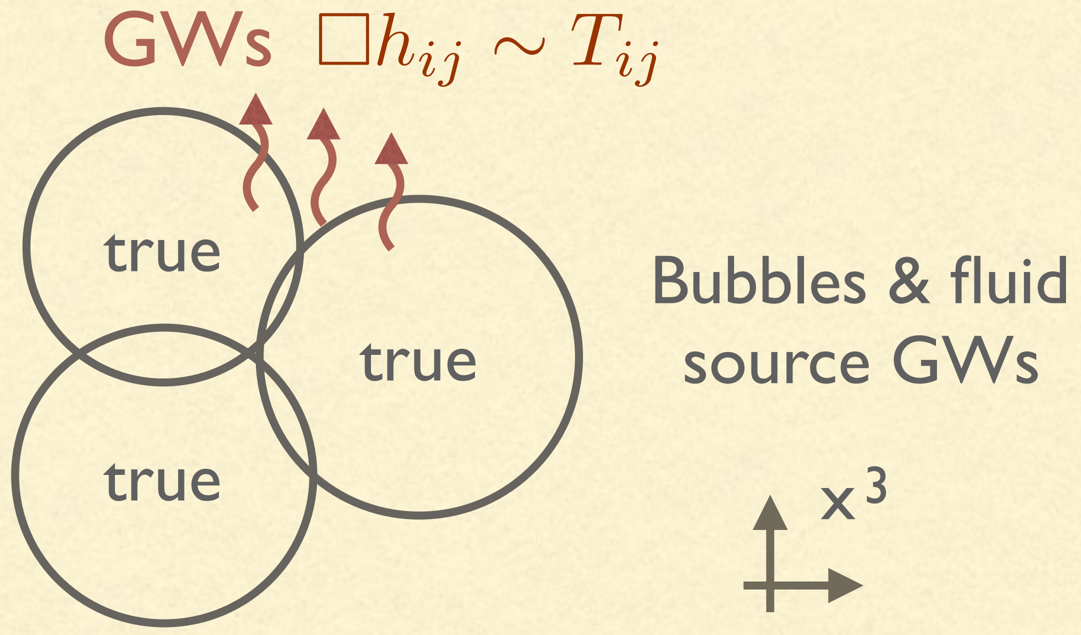
Bubbles nucleate, expand, collide and disappear, accompanying fluid dynamics

Field space



Quantum tunneling

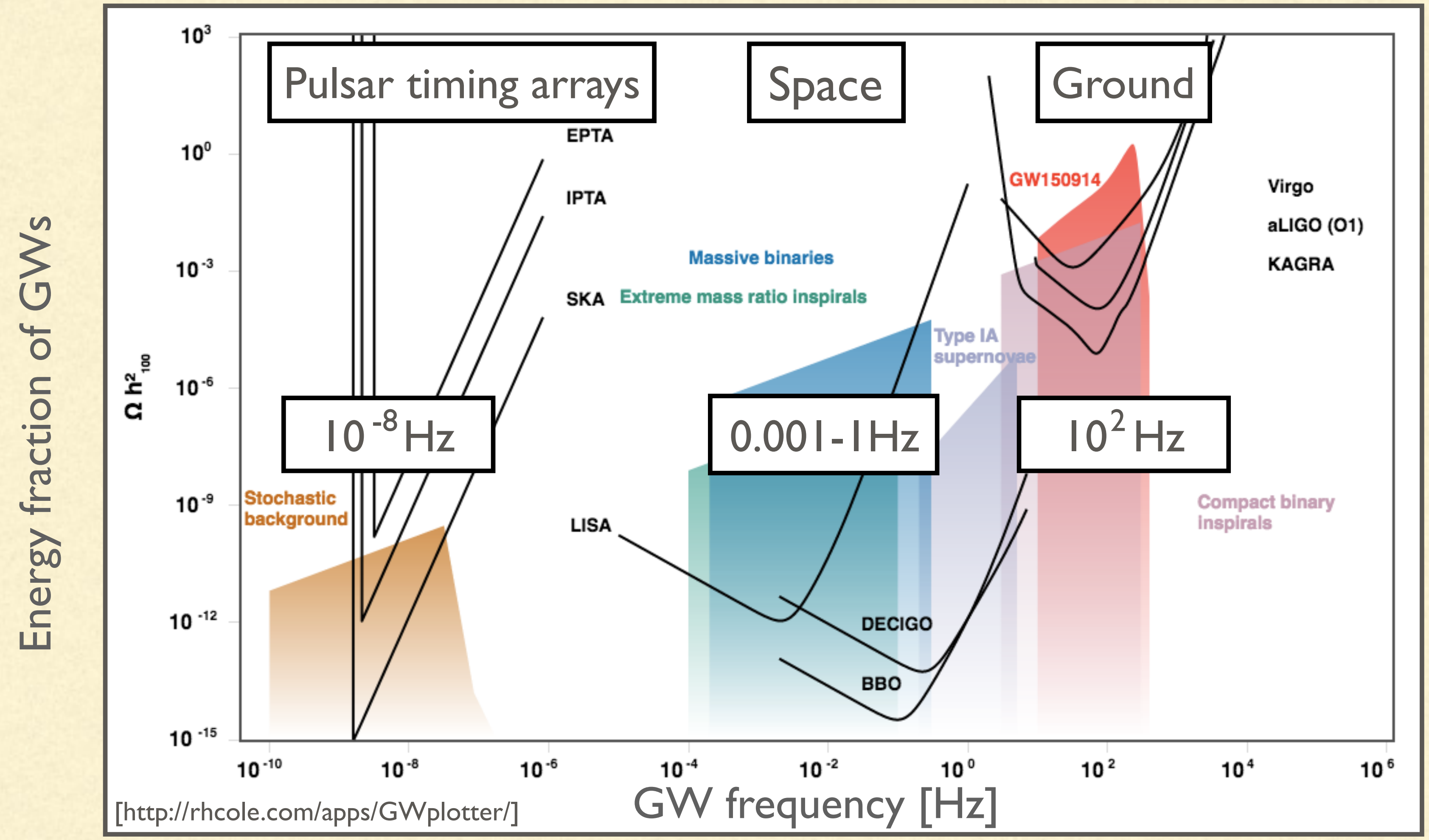
Position space



Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

- 10^{-3} ~ 1Hz GWs correpond to electroweak physics and beyond

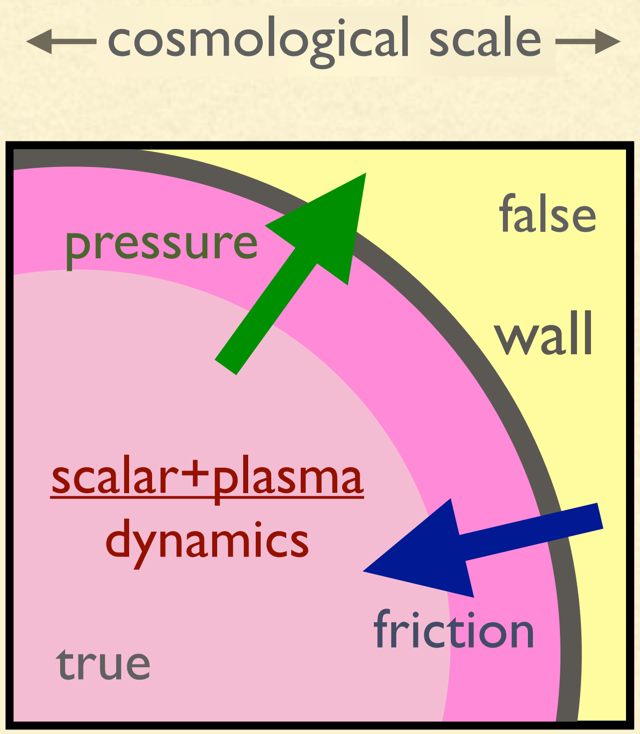


Temperature of the Universe ←

@ transition time MeV GeV TeV PeV

BUBBLE DYNAMICS BEFORE COLLISION

- "Pressure vs. friction" determines behavior of bubbles



- Two main players : **scalar field and plasma**

- Walls want to expand ("pressure")

  Parametrized by $\alpha \equiv \dfrac{\rho_{\rm vac}}{\rho_{\rm plasma}}$

- Walls are pushed back by plasma ("friction")

  Parametrized by coupling η btwn. scalar and plasma

- Let's see how bubbles behave for different α

  (with fixed coupling η)

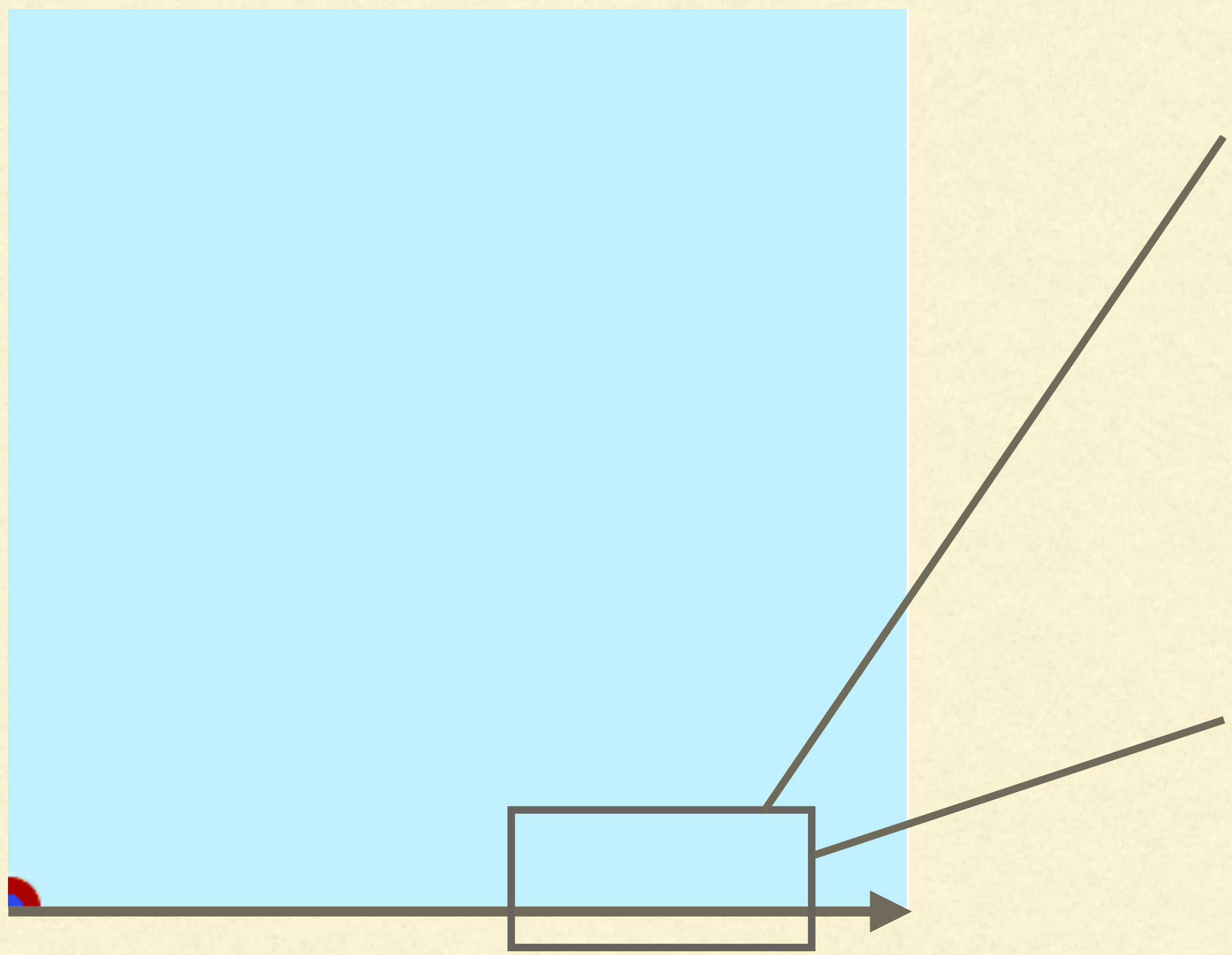
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

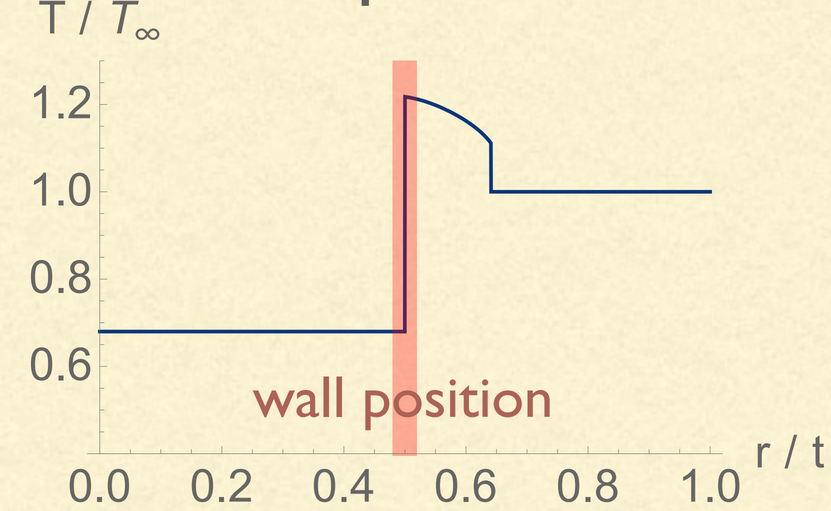
[Espinosa, Konstandin, No, Servant '10]

- Small α (say, $\alpha \lesssim \mathcal{O}(0.1)$)

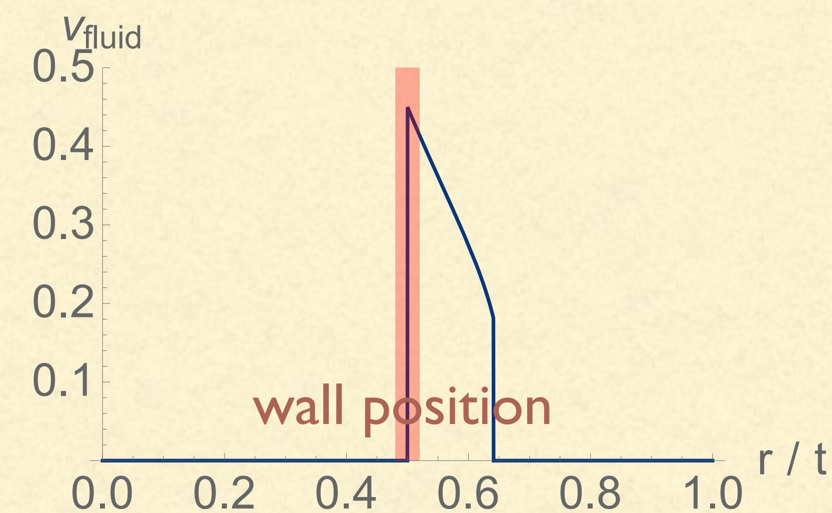
"deflagration"



Temperature



Fluid outward velocity

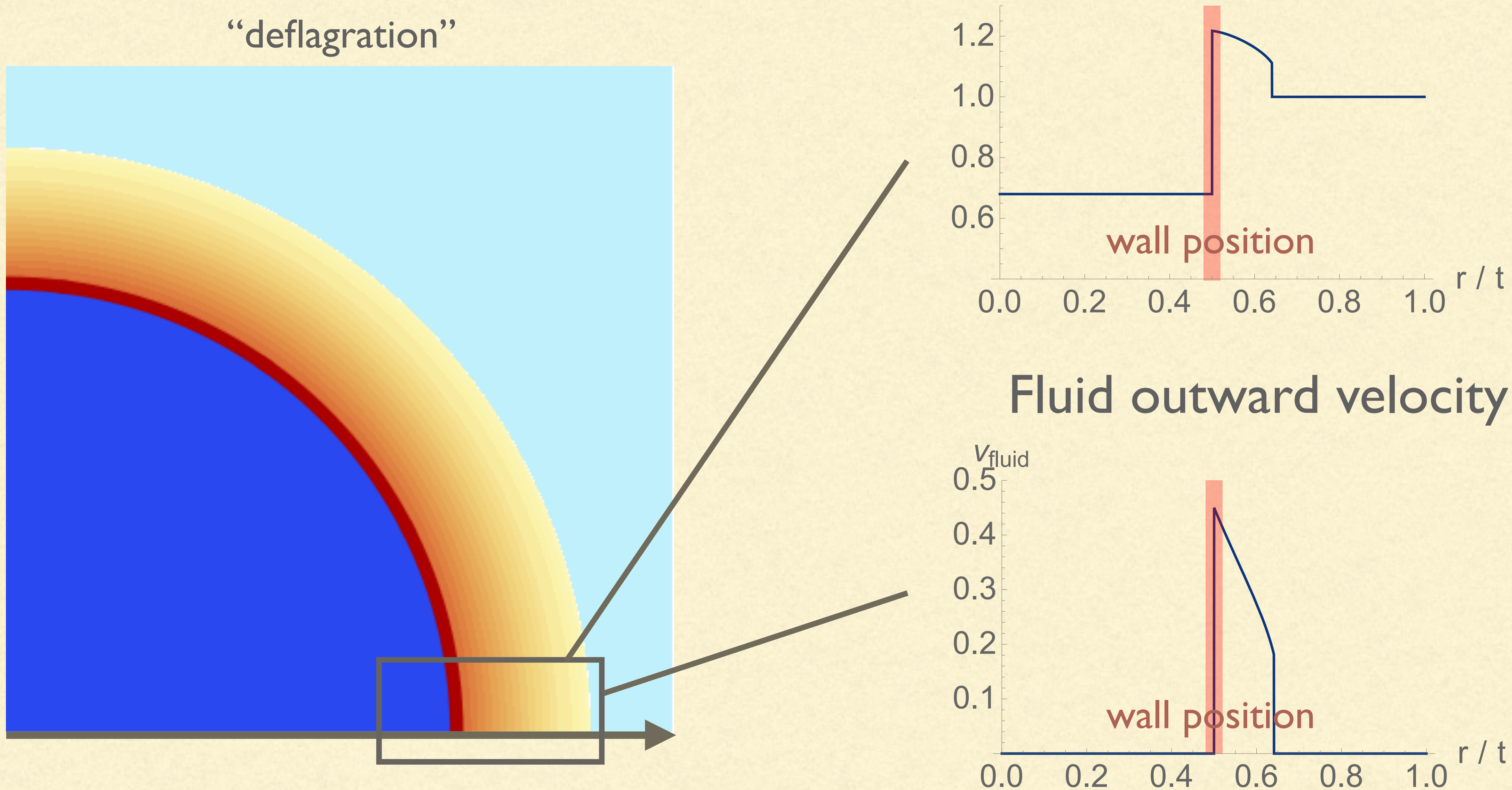


BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

[Espinosa, Konstandin, No, Servant '10]

- Small α (say, $\alpha \lesssim \mathcal{O}(0.1)$)



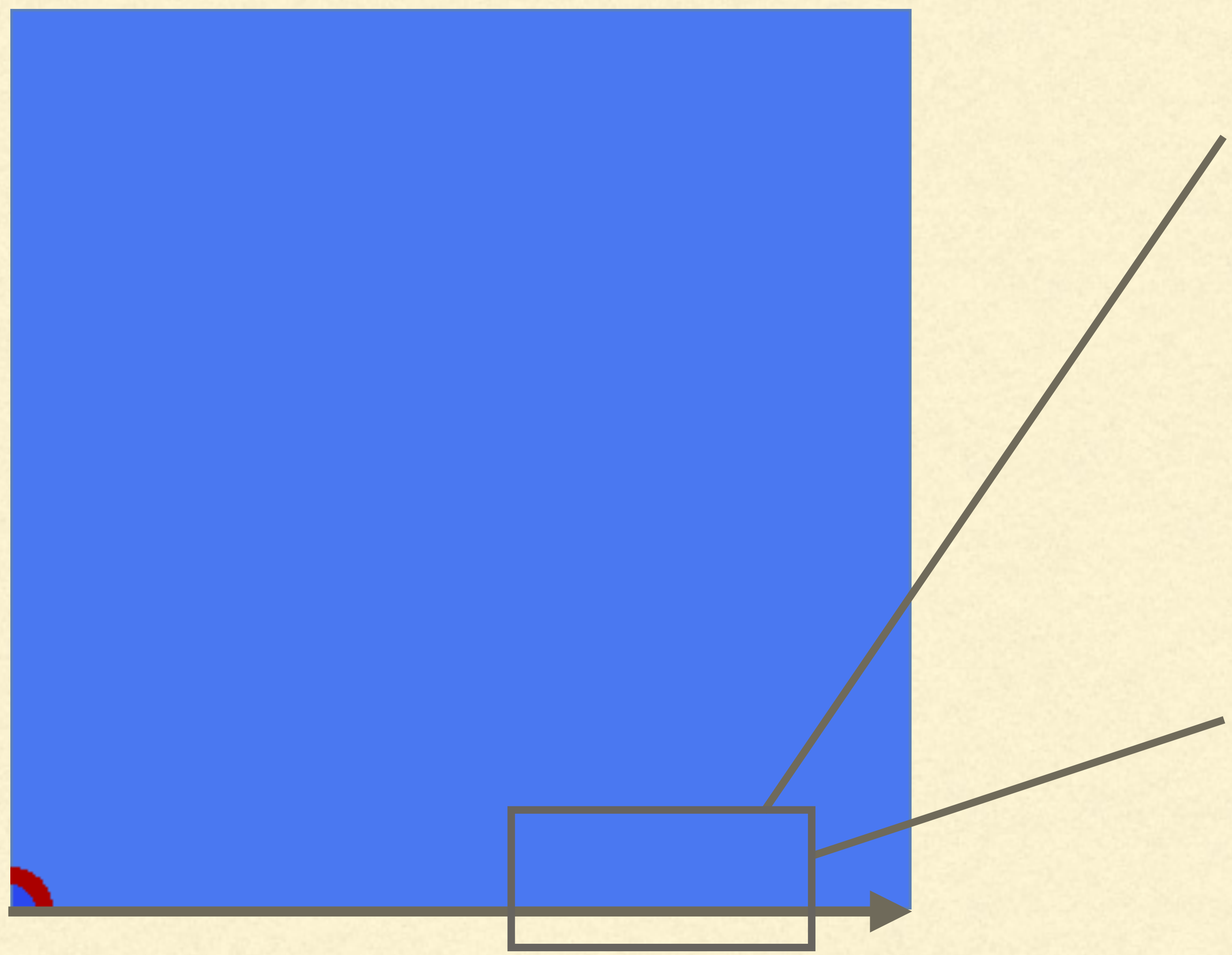
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\text{vac}}}{\rho_{\text{plasma}}}$$

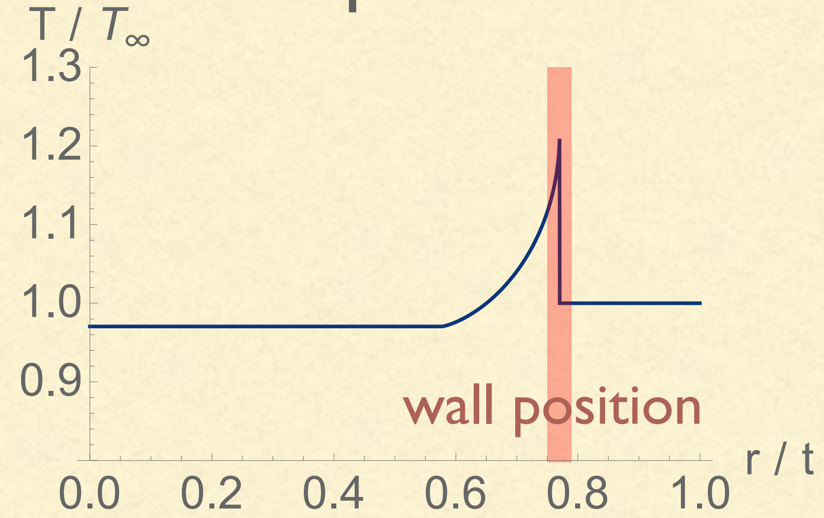
[Espinosa, Konstandin, No, Servant '10]

- Small but slightly increased α

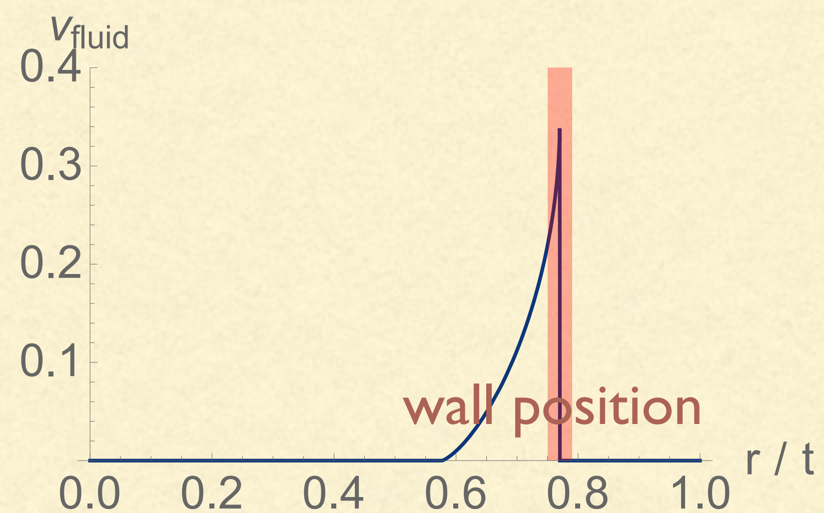
"detonation"



Temperature



Fluid outward velocity

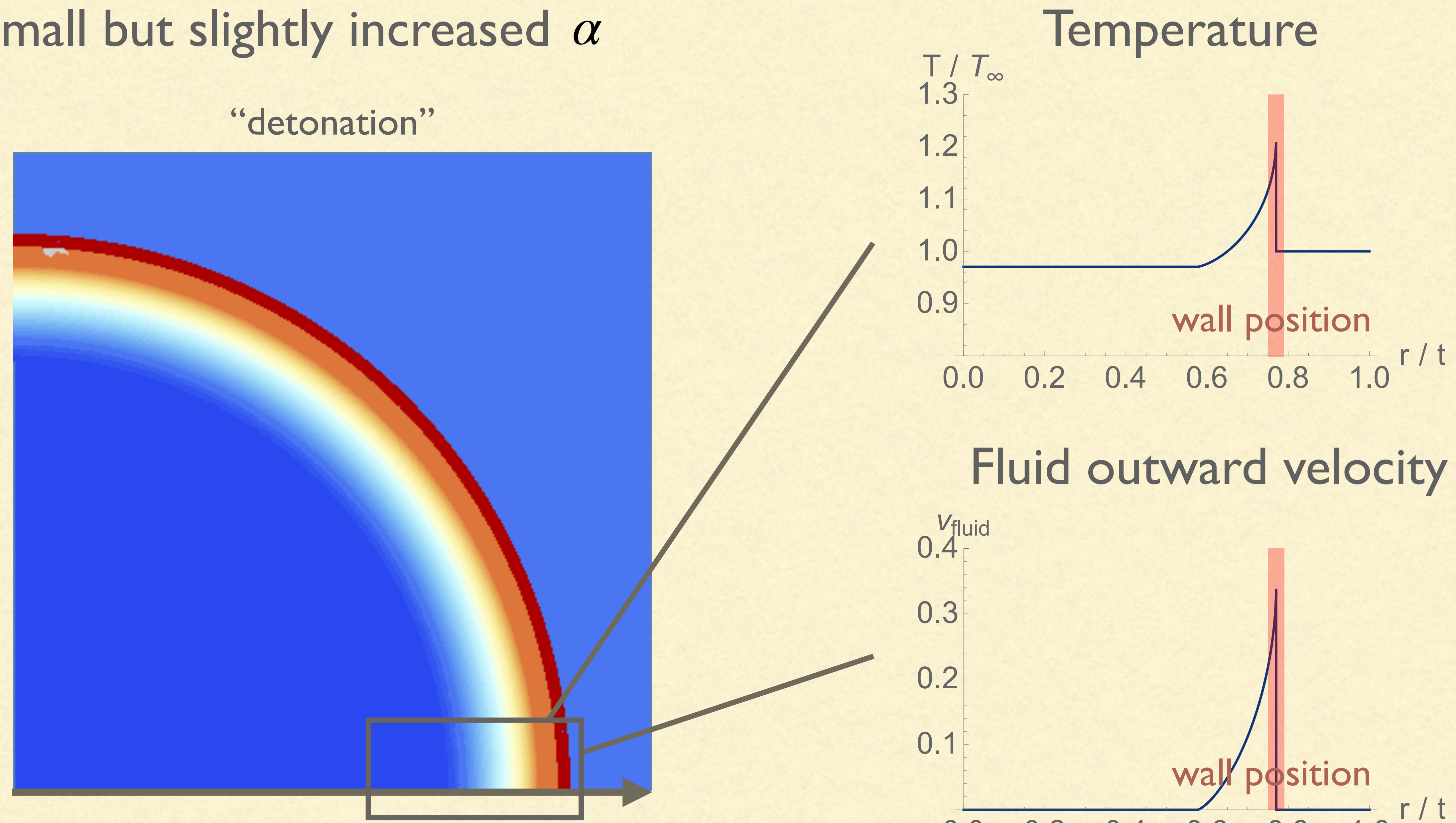


BUBBLE DYNAMICS BEFORE COLLISION

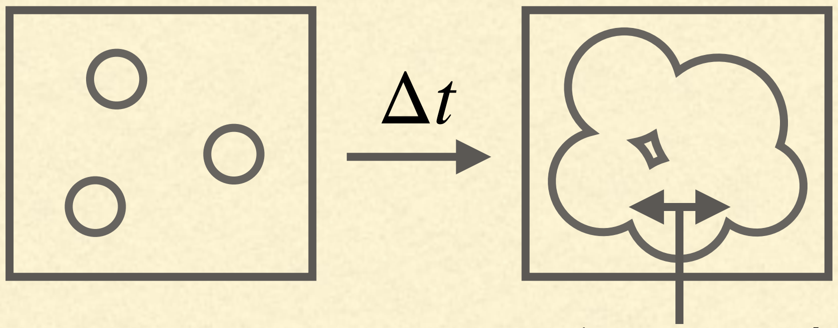
$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

[Espinosa, Konstandin, No, Servant '10]

- Small but slightly increased α

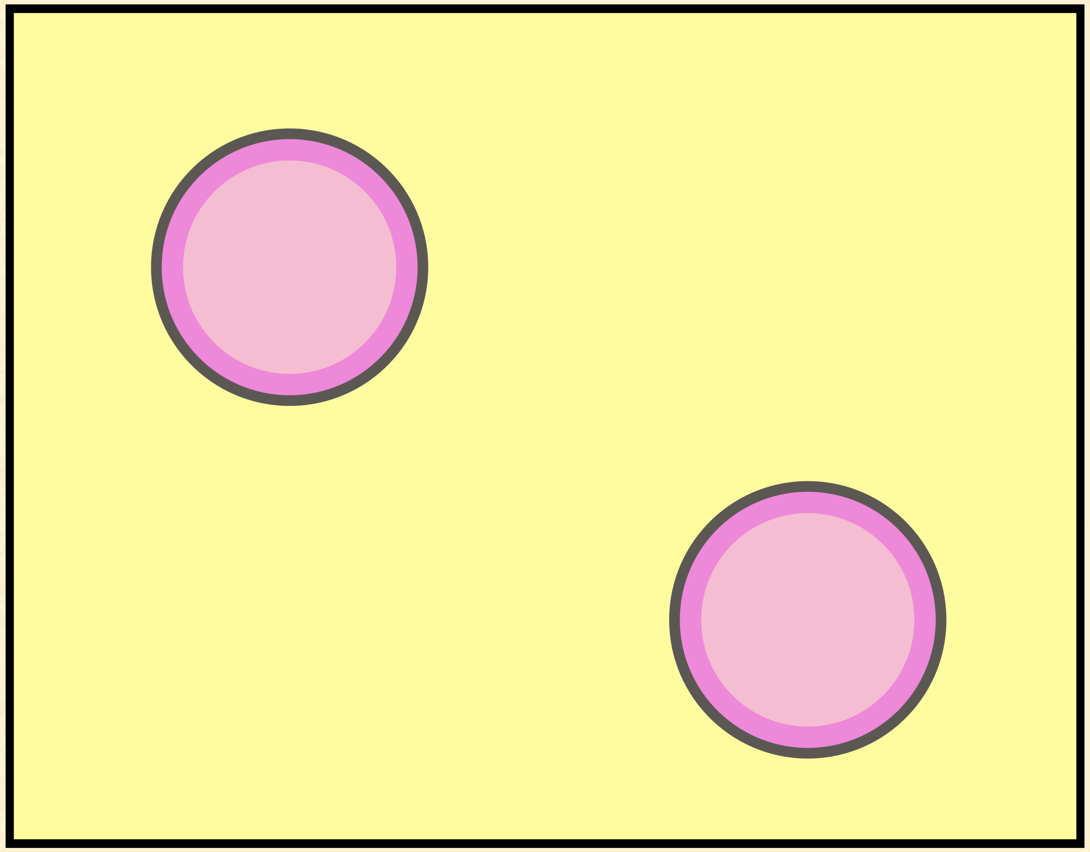


PARAMETERS CHARACTERIZING THE TRANSITION

	Definition	**Properties**
α	$\rho_{\rm vac}/\rho_{\rm plasma}$	Strength of the transition
β	Bubble nucleation rate Taylor-expanded around the transition time t_* $\Gamma(t) \propto e^{\beta(t-t_*)}$	Bubbles collide $\Delta t \sim 1/\beta$ after nucleation  Typical bubble size $\sim v_w \Delta t \sim v_w/\beta$
v_w	Wall velocity	Determined by the balance btwn. pressure & friction
T_*	Transition temperature	

DYNAMICS AFTER COLLISION

Bubbles nucleate & expand



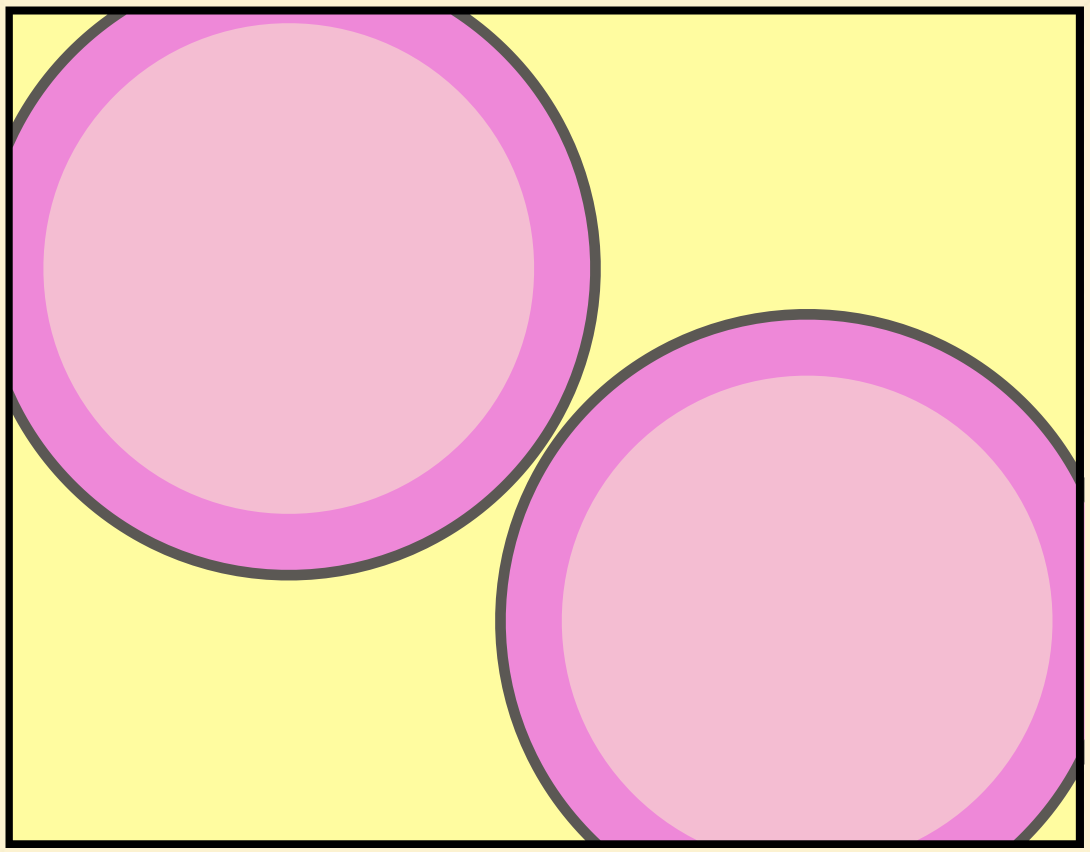
- Nucleation rate (per unit time & vol)

$$\Gamma(t) \propto e^{\beta(t-t_*)}$$

- Typically the released energy is carried by fluid motion [Bodeker & Moore '17]
- Collide $\Delta t \sim 1/\beta$ after nucleation

DYNAMICS AFTER COLLISION

Bubbles nucleate & expand



- Nucleation rate (per unit time & vol)

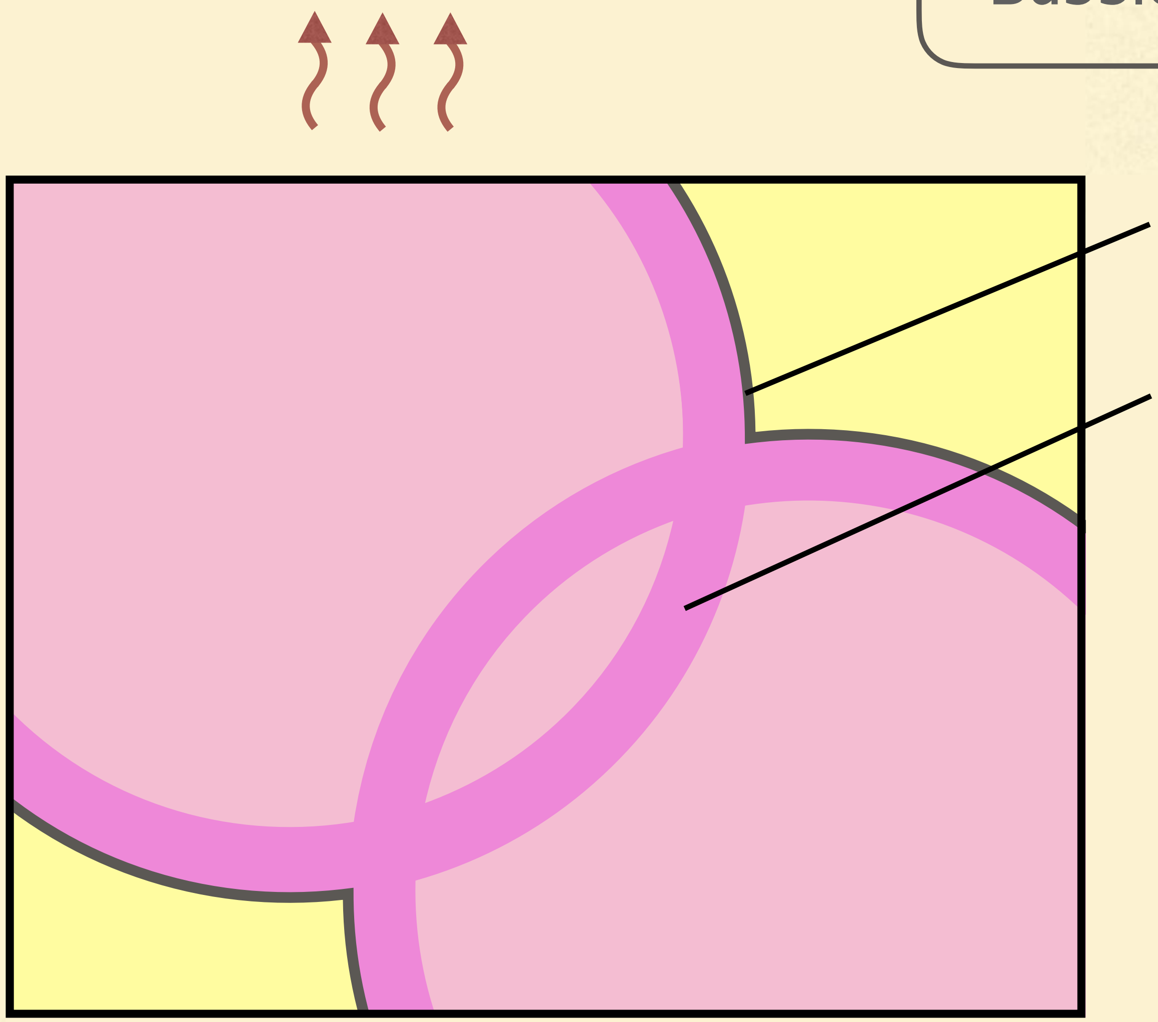
$$\Gamma(t) \propto e^{\beta(t-t_*)}$$

- Typically the released energy is carried by fluid motion [Bodeker & Moore '17]

- Collide $\Delta t \sim 1/\beta$ after nucleation

DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$



Bubbles collide

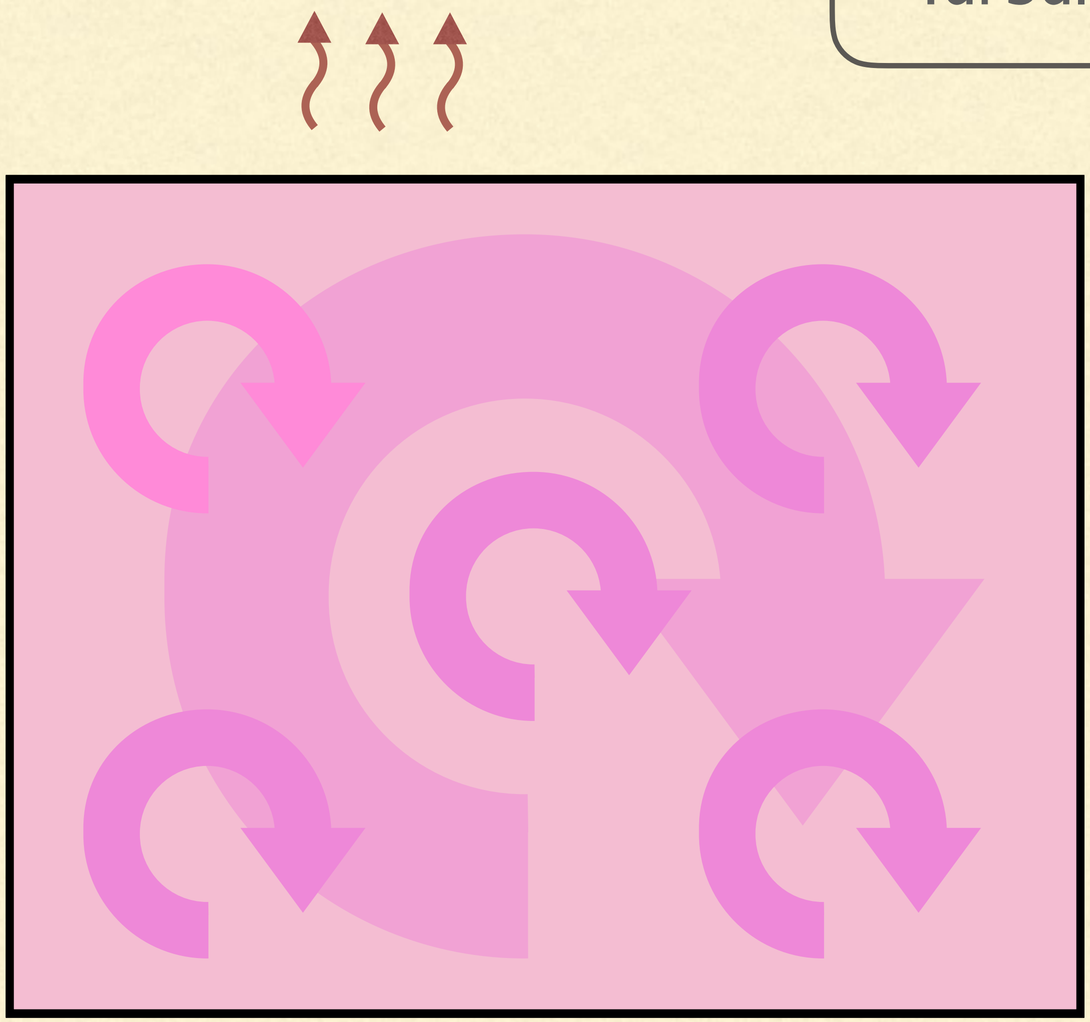
- Scalar field damps soon after collision
- For small α ($\lesssim \mathcal{O}(0.1)$), plasma motion is well described by linear approximation:

$$(\partial_t^2 - c_s^2 \nabla^2)\vec{v}_{\text{fluid}} \simeq 0 \quad \text{“sound waves”}$$

- In this case, fluid shell thickness is fixed at the time of collision

DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$



Turbulence develops

- Nonlinear effects becomes important at late times

"turbulence"

SOURCES OF GWS IN FIRST-ORDER PHASE TRANSITION

- Time evolution of the system

  Bubble nucleation & expansion → Collision → Sound waves → Turbulence

- Resulting GW spectrum is classified accordingly:

$$\Omega_{\rm GW} = \Omega_{\rm GW}^{\rm (coll)} + \Omega_{\rm GW}^{\rm (sw)} + \Omega_{\rm GW}^{\rm (turb)}$$

- Typically $\Omega_{\rm GW}^{\rm (sw)}$ is the largest because of different parameter dependence:

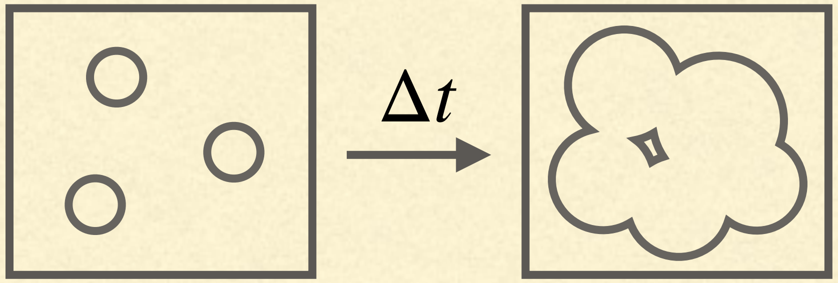
$\Omega_{\rm GW}^{\rm (coll)}$ (from scalar walls) $\propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-2}$

$\Omega_{\rm GW}^{\rm (sw)}$ (from fluid shells) $\propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-1}$

Note : $\frac{\beta}{H_*} \sim 10^{1-5} \gg 1$

GW ENHANCEMENT BY SOUND WAVES

- Reason for different dependence on β/H_*



Bubble collision

Bubbles collide and disappear within timescale $\Delta t \sim 1/\beta$

GWs are sourced during this preiod $h \propto \Delta t$

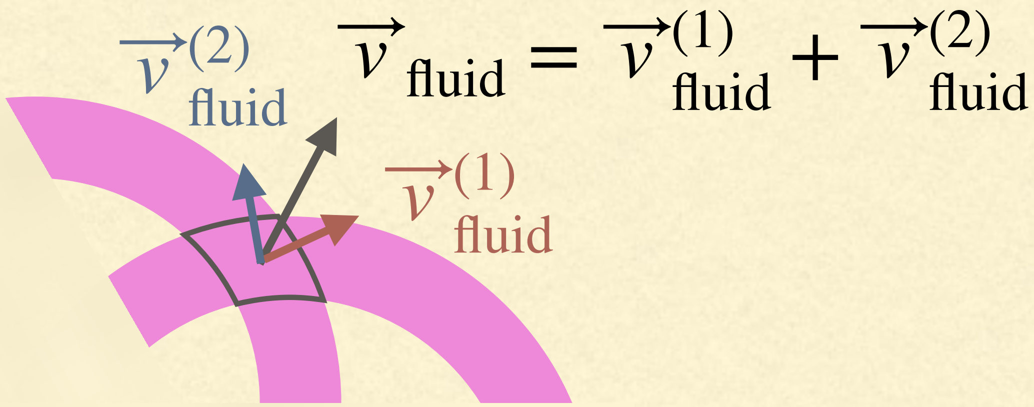
$\Omega_{\rm GW} \propto h^2 \propto \beta^{-2}$

Sound waves [See Mark's talk]

Shell overlap continuously creates

new velocity field during Hubble time

→ GW spectrum is enhanced by β/H_*



TALK PLAN

✔1. Introduction

2. Ultra-supercooled transitions

3. Ultra-supercooled transitions:

   Effective description of fluid propagation & Implications to GW production

4. Summary

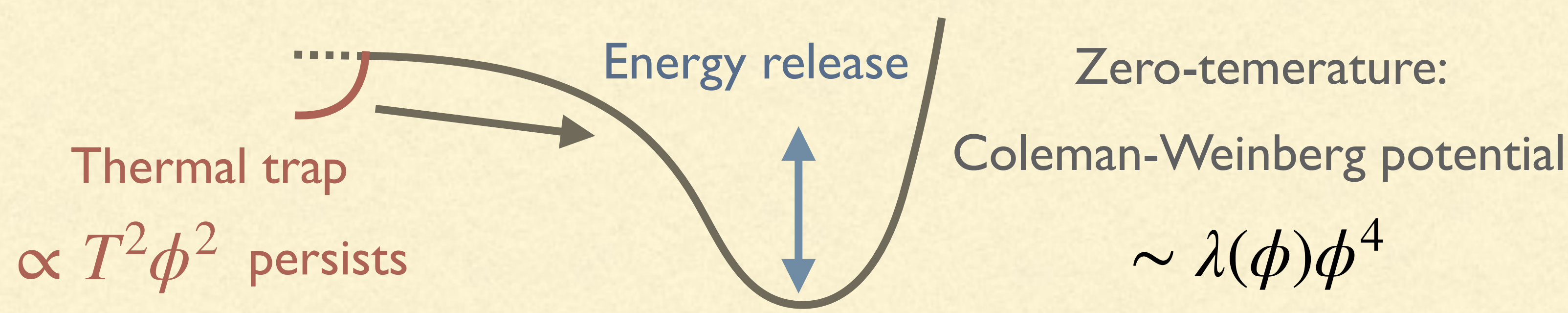
ULTRA-SUPERCOOLED TRANSITIONS

- $\alpha \gg 1$ occurs in a certain class of models [e.g. Randall & Servant '07, Konstandin & Servant '11]

  - Thermal trap persists even at low temperatures → $\alpha \gg 1$

  - These models also give small β / H_* (i.e. large bubbles)

one example:



- So, at least naively, large amplitude of GWs is expected

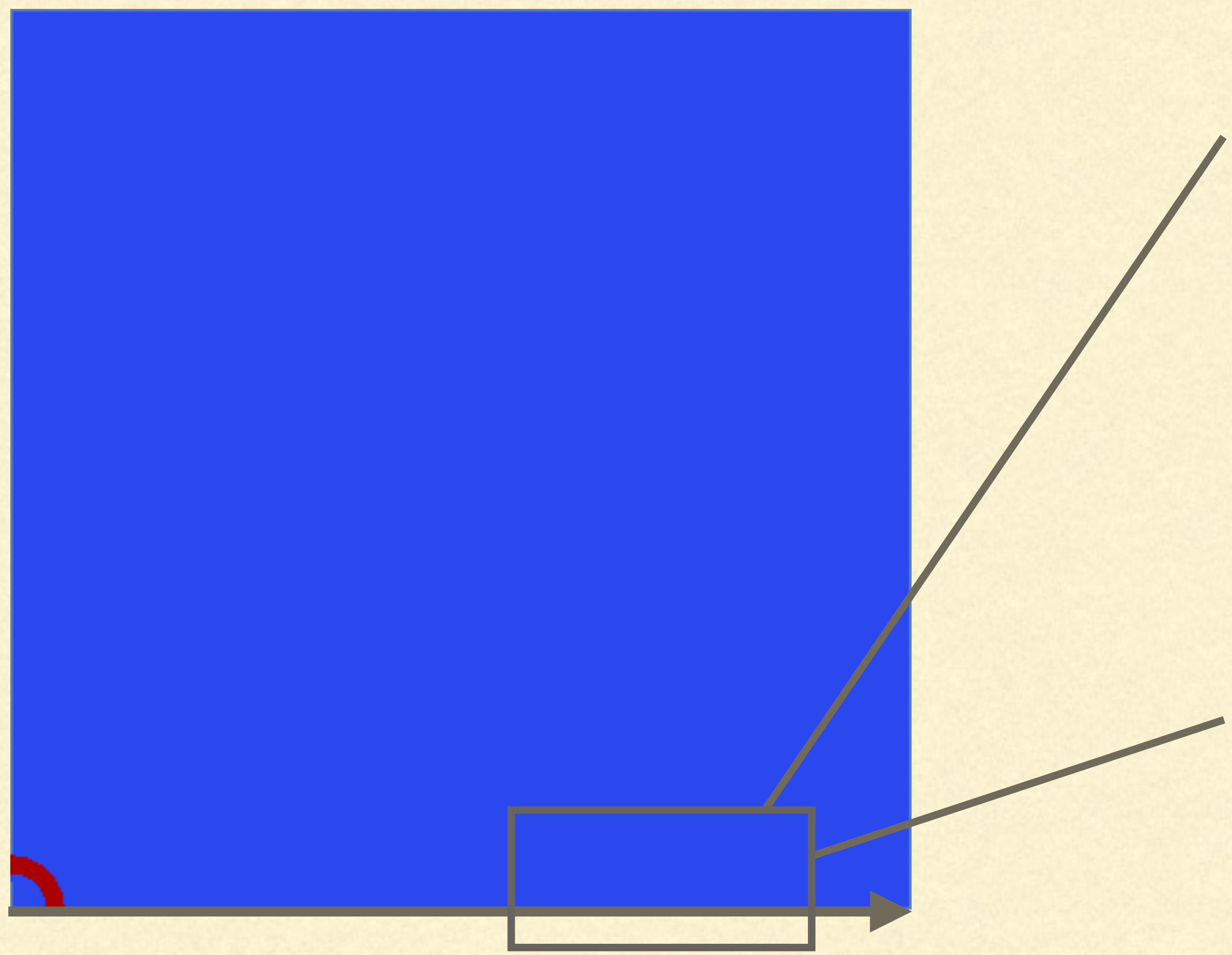
$$\Omega_{\rm GW}^{\rm (sw)} \propto \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{\beta}{H_*} \right)^{-1}$$

- However, the story is not so simple...

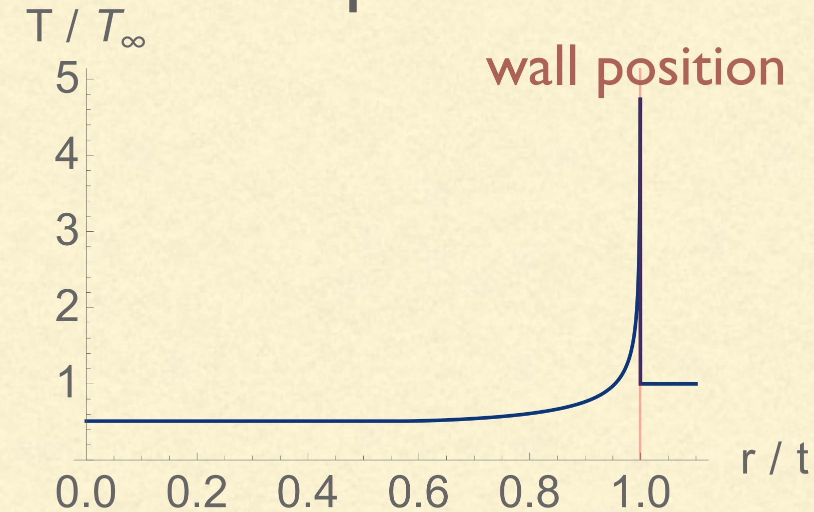
BUBBLE EXPANSION IN ULTRA-SUPERCOOLED TRANSITIONS

- Large α $(\gg 1)$

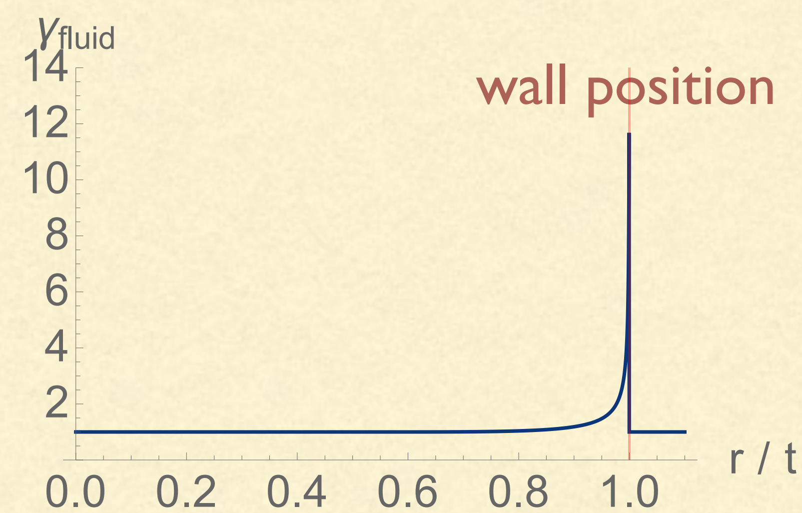
“strong detonation”



Temperature

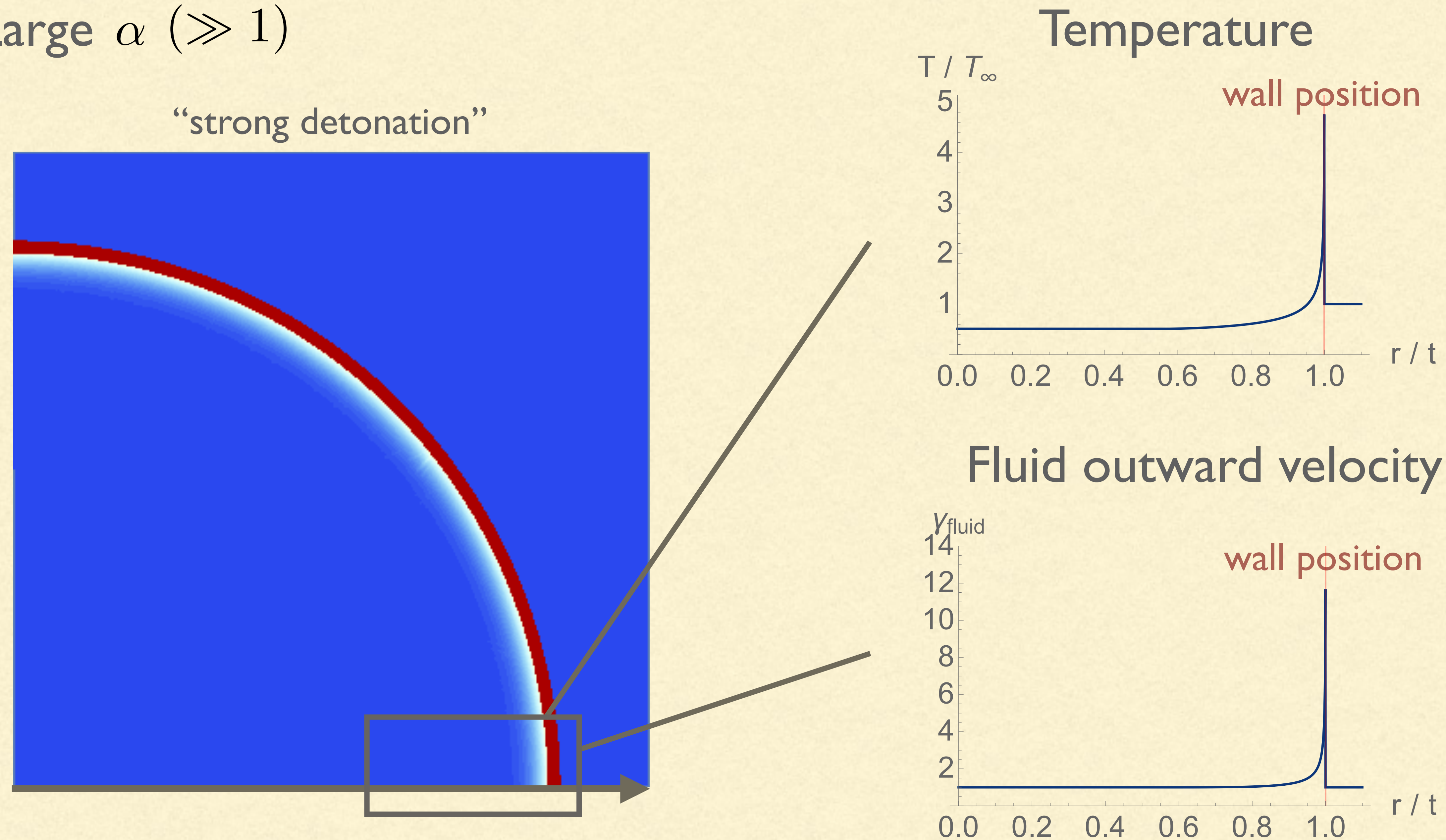


Fluid outward velocity



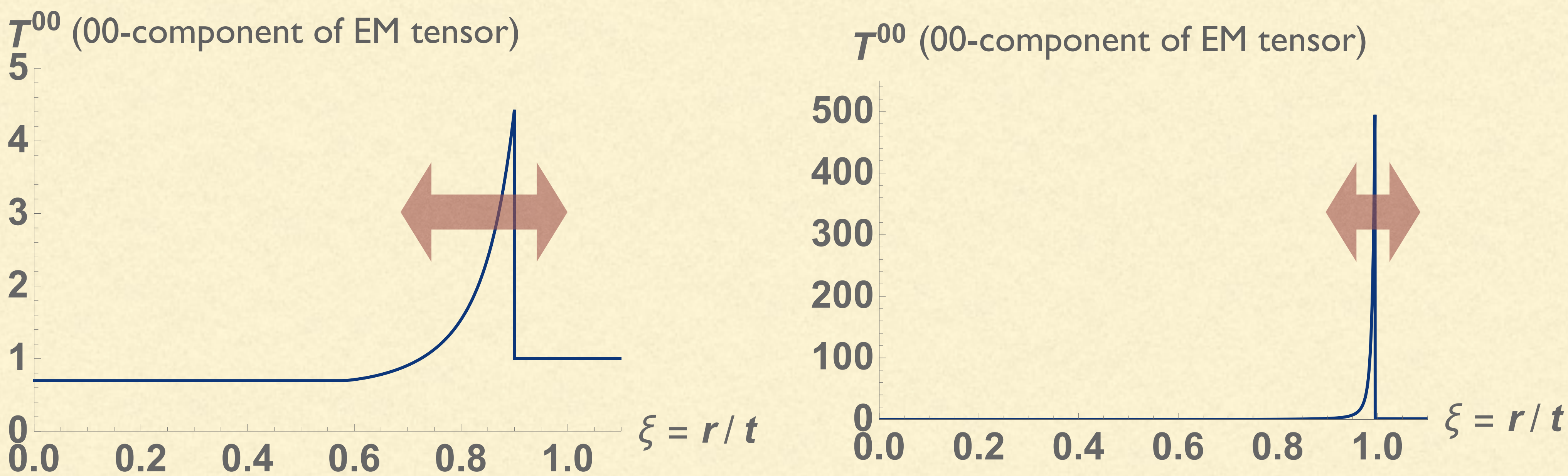
BUBBLE EXPANSION IN ULTRA-SUPERCOOLED TRANSITIONS

- Large α $(\gg 1)$



ENERGY LOCALIZATION IN ULTRA-SUPERCOOLED TRANSITIONS

- Fluid energy sharply localizes around bubble wall as α increases



$\alpha = 0.4, \quad v_w = 0.9$ $\qquad$ $\alpha = 10, \quad v_w = 0.995$

- In realistic ultra-supercooled transitions, α is much larger, e.g. $\alpha \sim 10^{10}$

- As a result, huge hierarchy appears between bubble size and energy localization

→ Hard to simulate fluid dynamics after bubble collisions numerically

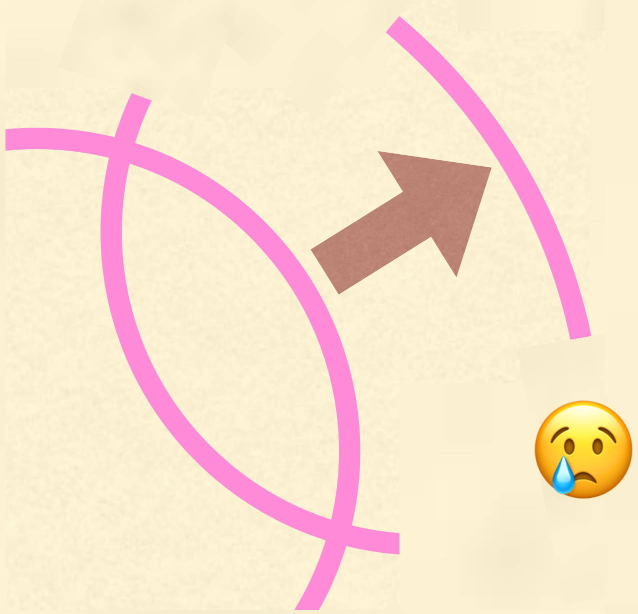
GW ENHANCEMENT CONDITION BY SOUND WAVES

- Necessary conditions to have GW enhancement by sound waves

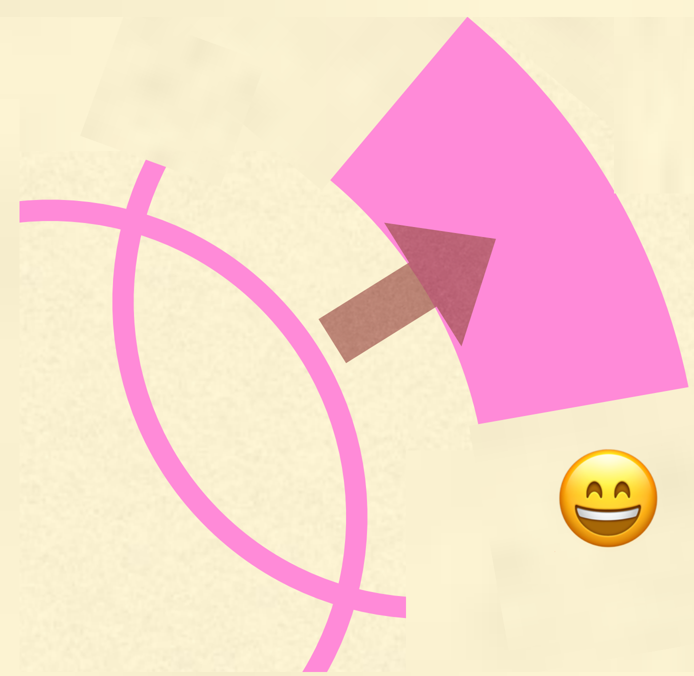
  - Delayed onset of turbulence

  - Sound shell overlap

- In order to have shell overlap, the energy localization has to break up:



or



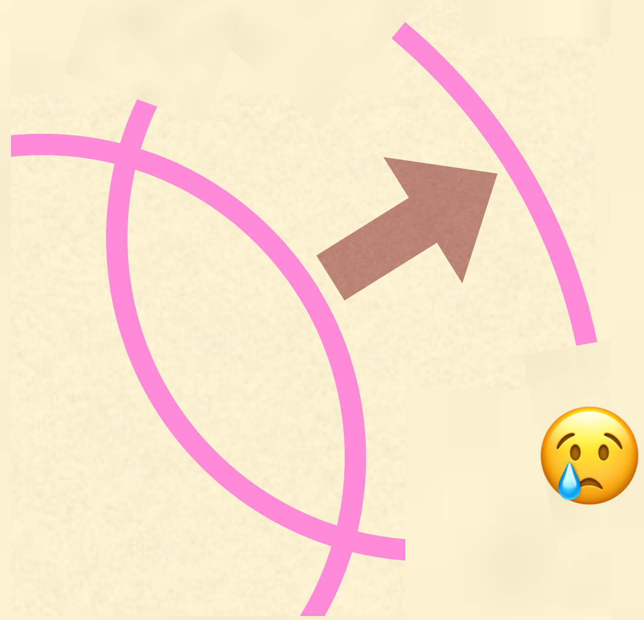
GW ENHANCEMENT CONDITION BY SOUND WAVES

- Necessary conditions to have GW enhancement by sound waves

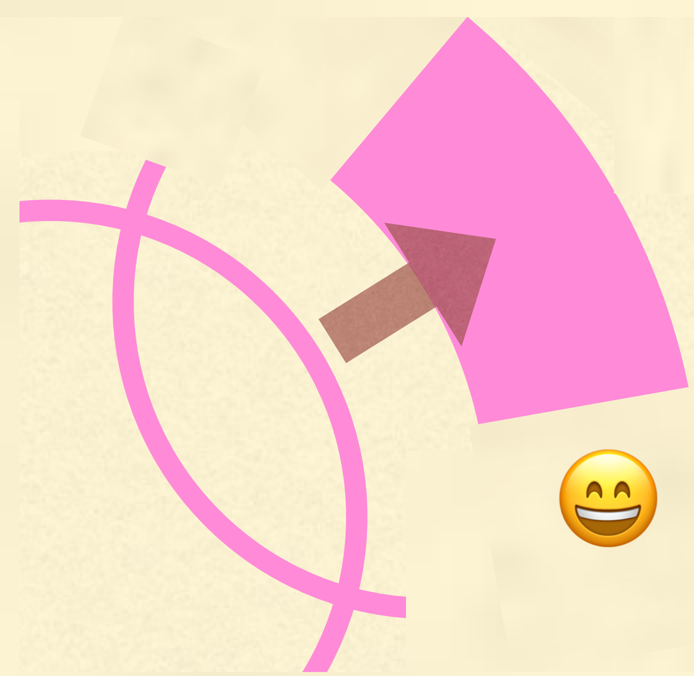
  - Delayed onset of turbulence

  - **Sound shell overlap**

- In order to have shell overlap, the energy localization has to break up:



or



SUMMARY OF MOTIVATION

- Ultra-supercooled transitions ($\alpha \gg 1$) occur in some class of models,

  and they are at least observationally interesting

- Does GW enhancement by sound waves occur in these transitions?

  More precisely: When does the energy localization break up and shell overlap start?

- Numerically difficult to study because of hierarchy in scales

What can we do?

TALK PLAN

✔1. Introduction

✔2. Ultra-supercooled transitions

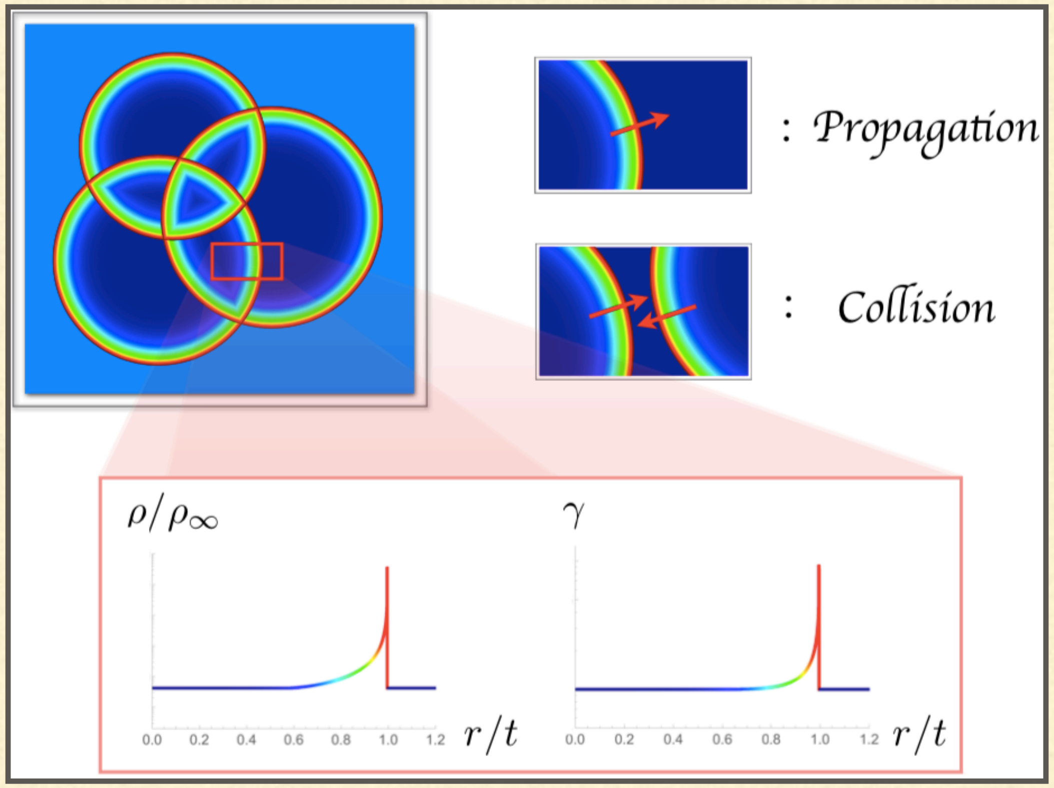
3. Ultra-supercooled transitions:

   Effective description of fluid propagation & Implications to GW production

4. Summary

REDUCING THE PROBLEM

- Let's devide the problem into small pieces:



(1) propagation of relativistic fluid

(2) collision of relativistic fluid

- Even propagation is nontrivial due to nonlinearity in fluid equation.

  We study propagation effects.

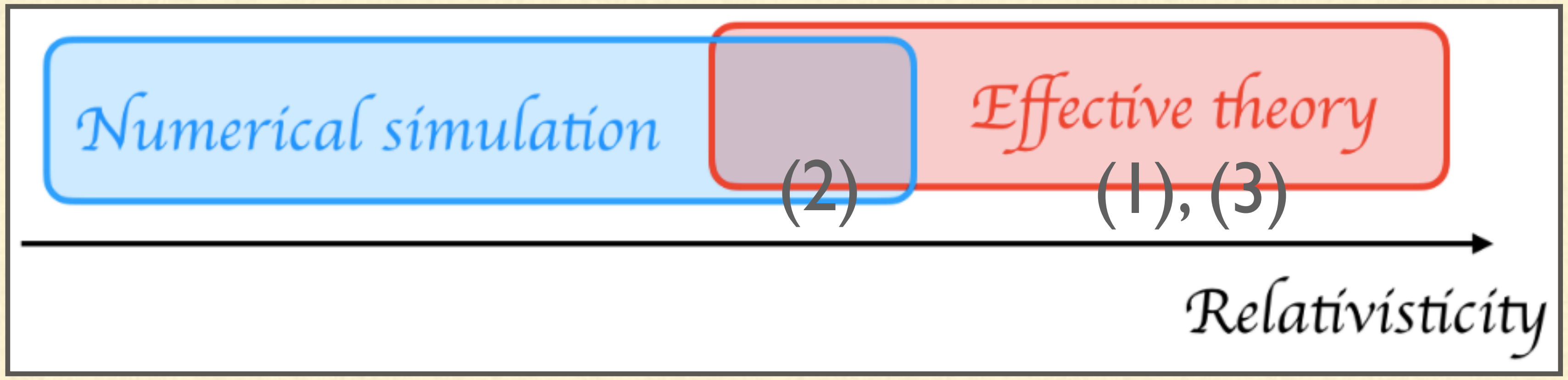
STRATEGY

- Our strategy:

(1) Develop an effective description of fluid propagation valid in highly relativistic regime

(2) Check the theory against simulation in mildly-relativistic regime

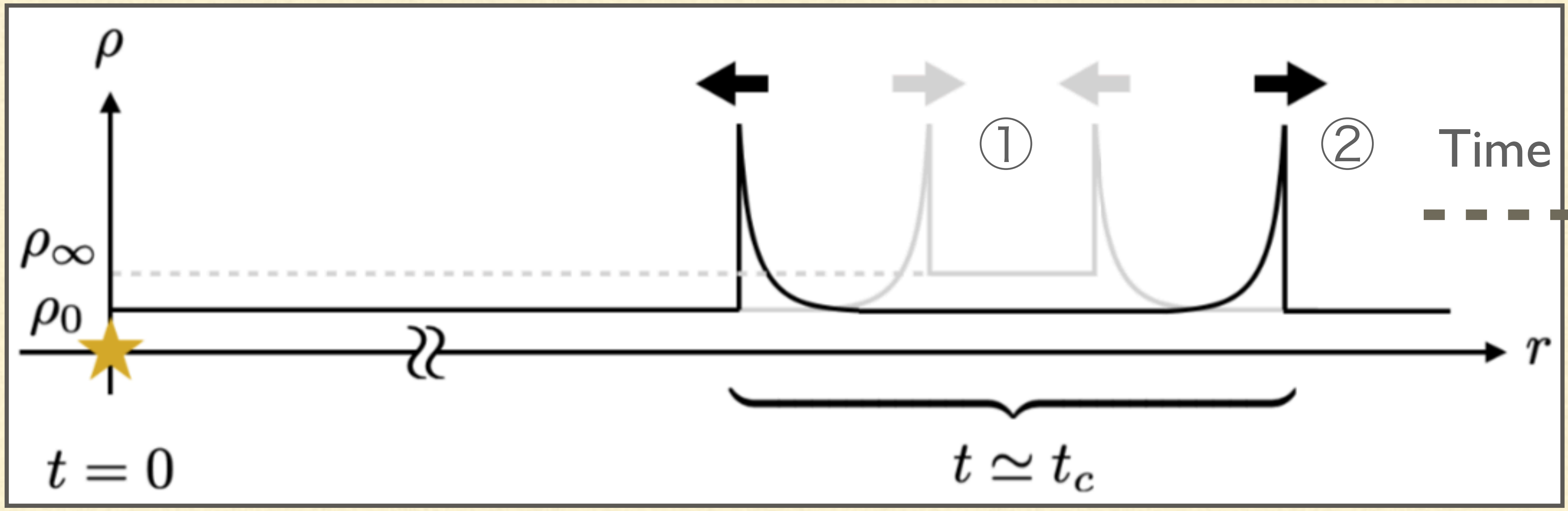
(3) Study implications of the effective description to GW production



(or simply the strength of transition α)

STRATEGY

- The setup we study



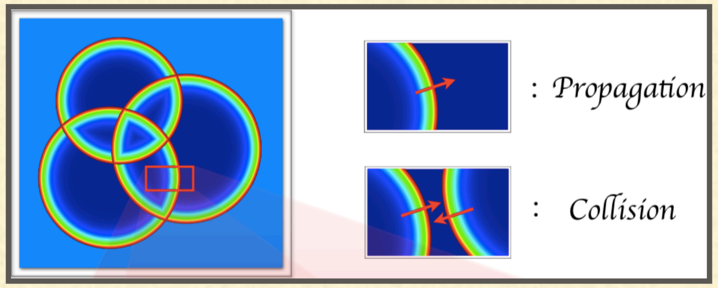
Time evolution

① Fluid profile just before collision: calculated from [Espinosa, Konstandin, No, Servant '10]

↓ Assumption: the first fluid collision does not change the profile significantly

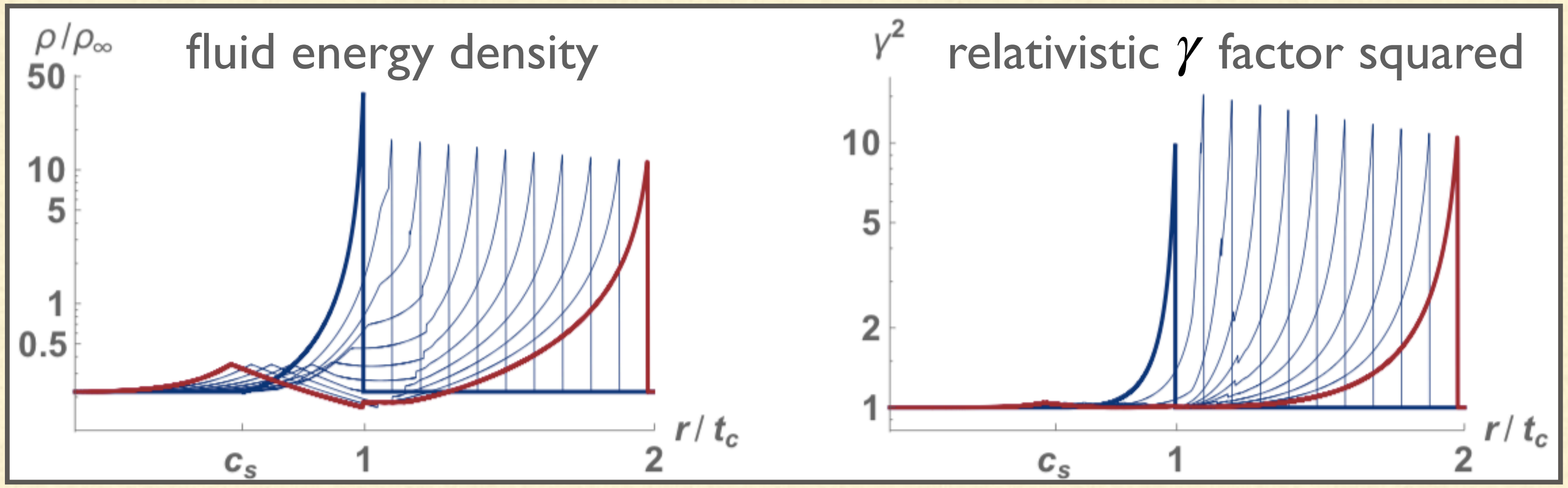
② Fluid profile just after collision: our interest is in the time evolution from here

EFFECTIVE THEORY OF FLUID PROPAGATION

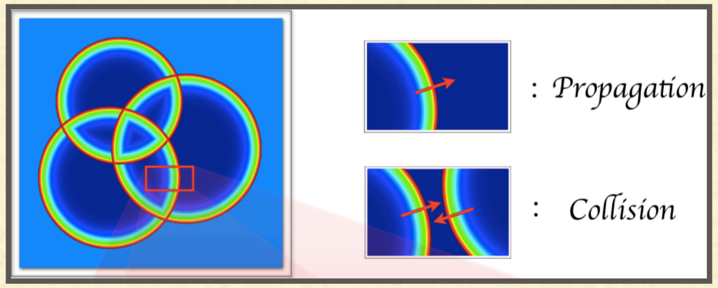


- Before constructing a theory, let's see the result of numerical simulation

(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)

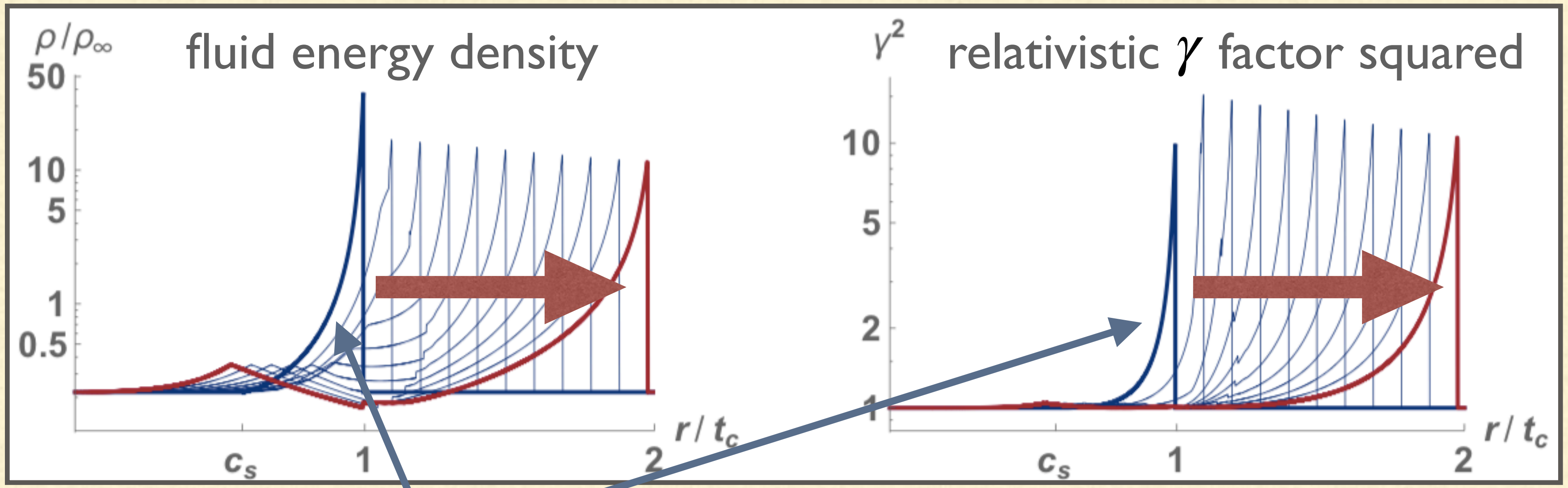


EFFECTIVE THEORY OF FLUID PROPAGATION



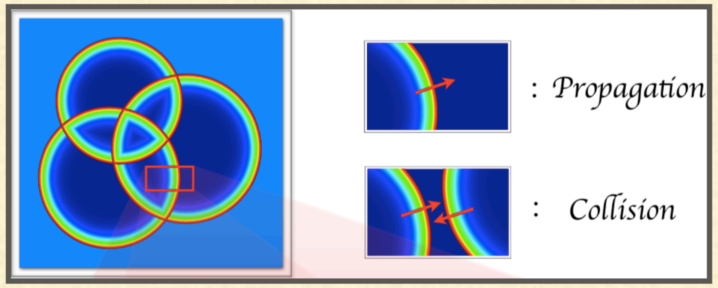
- Before constructing a theory, let's see the result of numerical simulation

(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)



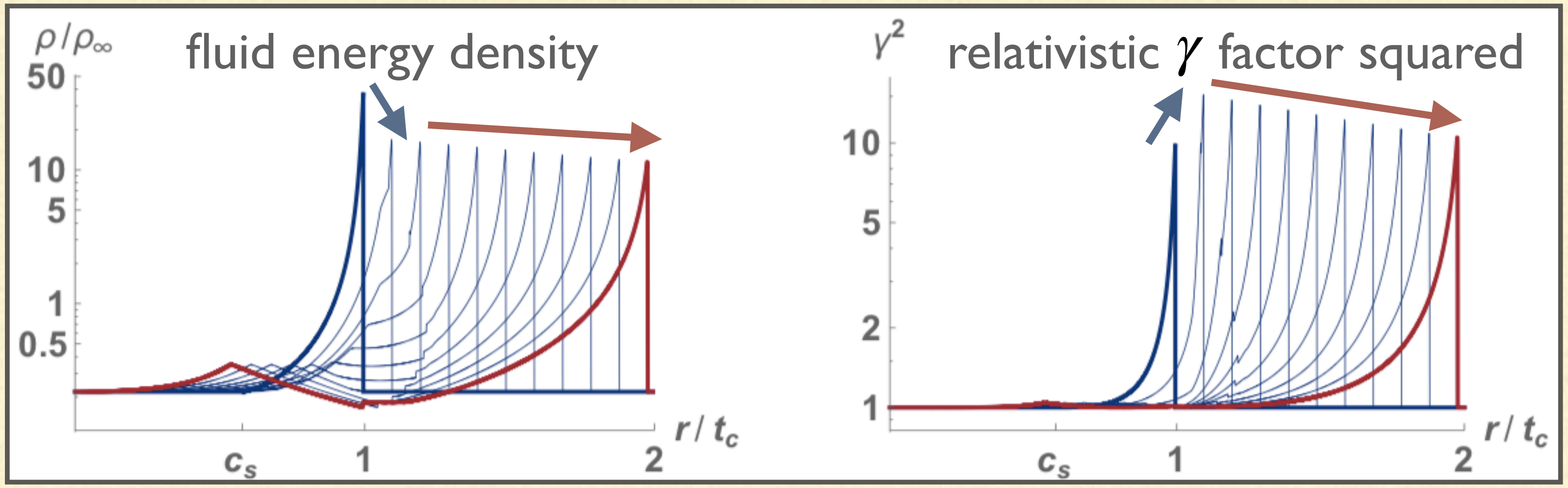
- Initial fluid profile (blue) propagates inside the other bubble (red)

EFFECTIVE THEORY OF FLUID PROPAGATION



- Before constructing a theory, let's see the result of numerical simulation

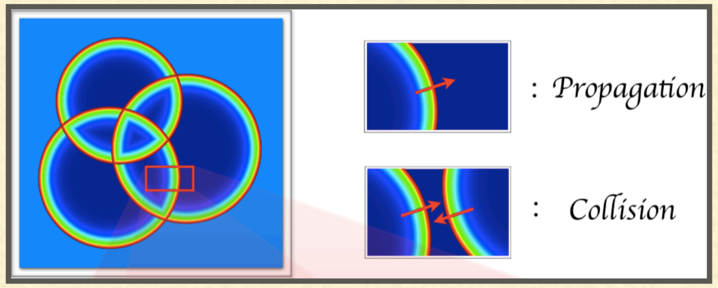
(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)



- Initial fluid profile (blue) propagates inside the other bubble (red)

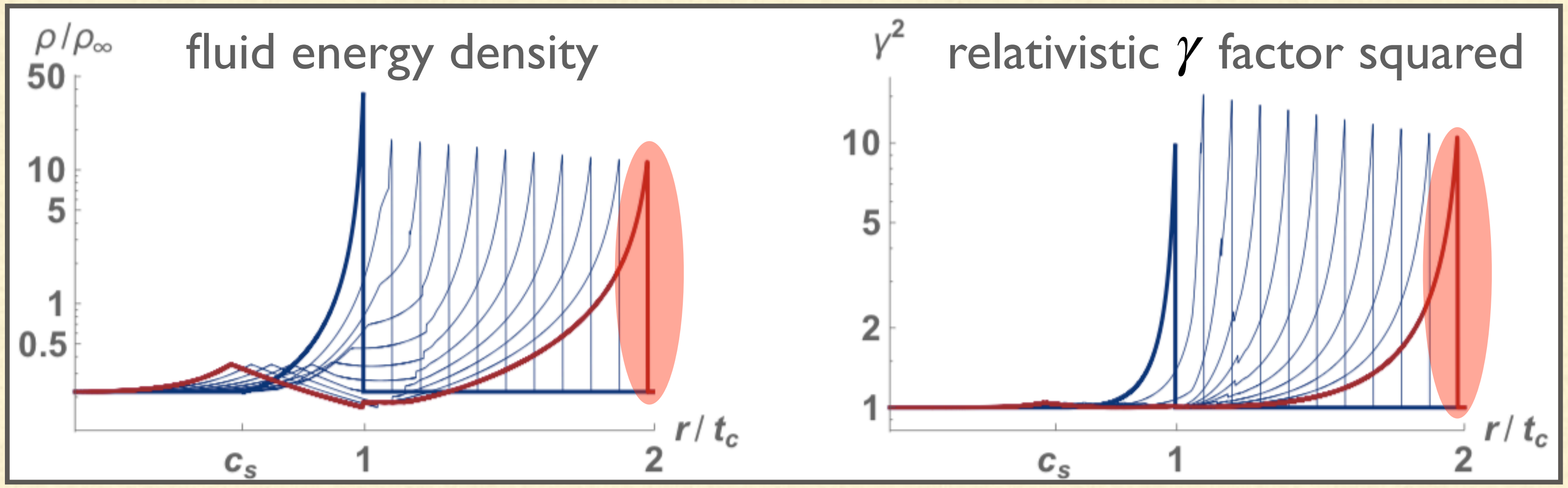
- Peaks rearrange to new initial values, and gradually become less energetic

EFFECTIVE THEORY OF FLUID PROPAGATION



- Before constructing a theory, let's see the result of numerical simulation

(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)

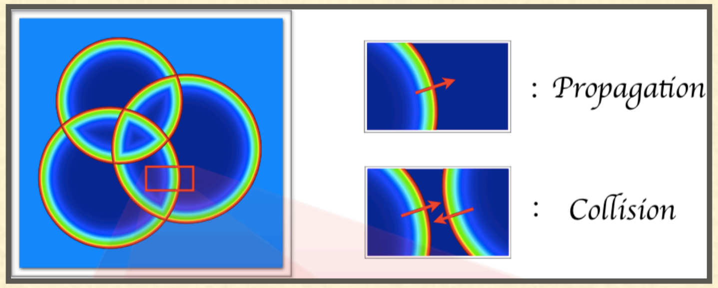


- Initial fluid profile (blue) propagates inside the other bubble (red)

- Peaks rearrange to new initial values, and gradually become less energetic

- Strong shocks (i.e. discontinuities) persist during propagation

EFFECTIVE THEORY OF FLUID PROPAGATION



- Can we construct an effective description?

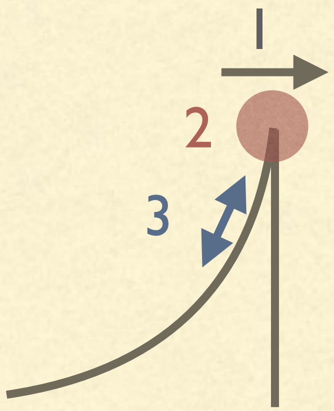
  - From the viewpoint of GW production, we are interested only in PEAKS, not TAILS

  - Can we describe the time evolution of peak-related quantities?

    1) Shock velocity: v_s

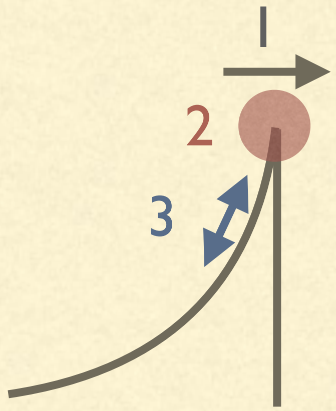
    2) Peak values: $\rho_{\rm peak}, v_{\rm peak}$ (equivalently $\rho_{\rm peak}, \gamma^2_{\rm peak}$)

    3) Derivatives at the peak: $\dfrac{d\rho_{\rm peak}}{dr}, \dfrac{dv_{\rm peak}}{dr}$ @ peak



  - We would like to construct a closed system for these quantities

HOW TO CONSTRUCT A CLOSED SYSTEM



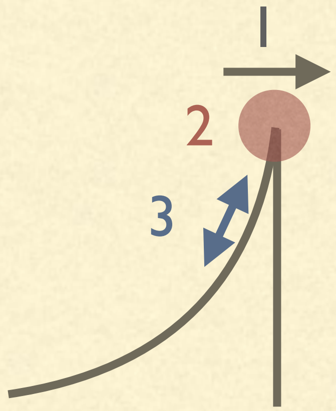
- Closed system for 5 quantities $\gamma_s^2,\ \rho_{\rm peak},\ \gamma_{\rm peak}^2,\ \dfrac{d\rho_{\rm peak}}{dr},\ \dfrac{d\gamma_{\rm peak}^2}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

$$p_{\rm peak} = \frac{p_0 + \rho_0 v_{\rm peak} v_s}{1 - v_{\rm peak} v_s}, \qquad v_s = \frac{(p_{\rm peak} + \rho_{\rm peak}) v_{\rm peak}}{p_{\rm peak} v_{\rm peak}^2 + \rho_{\rm peak} - \rho_0 (1 - v_{\rm peak}^2)}$$

HOW TO CONSTRUCT A CLOSED SYSTEM



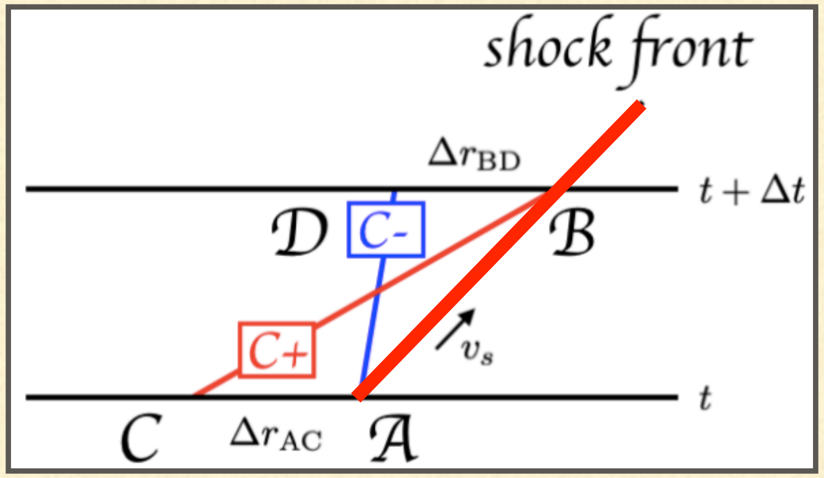
- Closed system for 5 quantities $\gamma_s^2, \rho_{\text{peak}}, \gamma_{\text{peak}}^2, \dfrac{d\rho_{\text{peak}}}{dr}, \dfrac{d\gamma_{\text{peak}}^2}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

  - Time evolution equations : 2 evolution equations

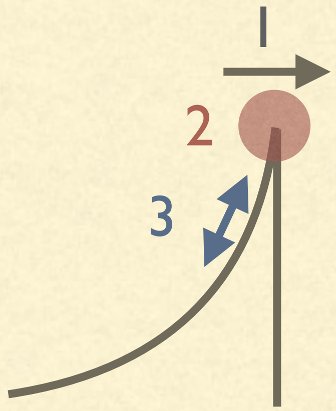
    (corresponding to temporal & spacial part of $\partial_\mu T^{\mu\nu}_{\text{fluid}} = 0$ behind the shock)



Advanced note

Easily derived from the conservation of Riemann invariants along C_+ & C_-

HOW TO CONSTRUCT A CLOSED SYSTEM



- Closed system for 5 quantities $\gamma_s^2,\ \rho_{\rm peak},\ \gamma_{\rm peak}^2,\ \dfrac{d\rho_{\rm peak}}{dr},\ \dfrac{d\gamma_{\rm peak}^2}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

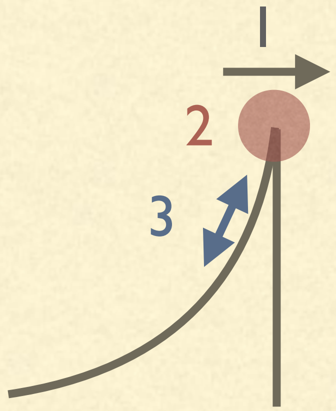
  - Time evolution equations : 2 evolution equations

    (corresponding to temporal & spacial part of $\partial_\mu T^{\mu\nu}_{\rm fluid} = 0$ behind the shock)

$$\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma_{\rm peak}^2 = -\frac{2\sqrt{3}-3}{4}\frac{1}{\gamma_{\rm peak}^2}\left[\frac{\sqrt{3}}{2}\ln\rho' + \ln\gamma^{2\prime}\right] - \frac{(\sqrt{3}-1)(d-1)}{t}$$

$$-\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma_{\rm peak}^2 = \frac{2\sqrt{3}+3}{4}\frac{1}{\gamma_{\rm peak}^2}\left[-\frac{\sqrt{3}}{2}\ln\rho' + \ln\gamma^{2\prime}\right] + \frac{(\sqrt{3}+1)(d-1)}{t}$$

HOW TO CONSTRUCT A CLOSED SYSTEM



- The last equation?

  - So far, less equations (4 eqs.) than the number of quantities (5 quantities)

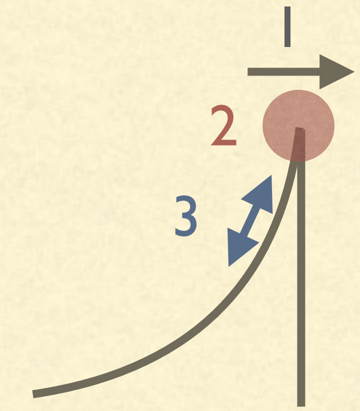
  - This is natural:

    the original system has infinite # of dof (i.e. # of spacial grids),

    so the system cannot be described strictly by finite number of dof

  - So, the last equality to close the system should be APPROXIMATE at best

HOW TO CONSTRUCT A CLOSED SYSTEM



- The last equation: energy domination by the peak

  - Any relation like "(peak T^{00}) × (thickness of the peak) = const." will work

  - In our parametrization, it will be like $\rho_{\rm peak}\gamma_{\rm peak}^2 \times \dfrac{1}{d\rho_{\rm peak}/dr \;\; \text{or} \;\; d\gamma_{\rm peak}^2/dr} = \text{const}\,.$

  - As an example, approximating $\rho_{\rm peak}$ and $\gamma_{\rm peak}$ to be exponential in r, we have

$$\sigma \simeq \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \int dr \, \frac{4}{3}\rho\gamma^2 = \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \frac{4}{3} \frac{\rho_{\rm peak}\gamma_{\rm peak}^2}{\ln\rho' + \ln\gamma^{2\prime}} \quad \text{for} \quad \begin{Bmatrix} d=1 \\ d=2 \\ d=3 \end{Bmatrix}$$

<u>Note</u> $d = 1,2,3$ corresponds to planar, cylindical, spherical

THEORY PREDICTION

- The resulting system can be solved analytically ($\delta = 10/13$)

1) Shock velocity:

$$\frac{1}{\gamma_s^2(t)} = \frac{8}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_s^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

2) Peak values:

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

$$\frac{1}{\gamma_{\rm peak}^2(t)} = \frac{16}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_{\rm peak}^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

3) Derivatives at the peak:

$$\ln \rho'(t) = \frac{448}{117}\left(\frac{\rho_0}{\sigma}\right) t^2 \gamma_{\rm peak}^4(t) + \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

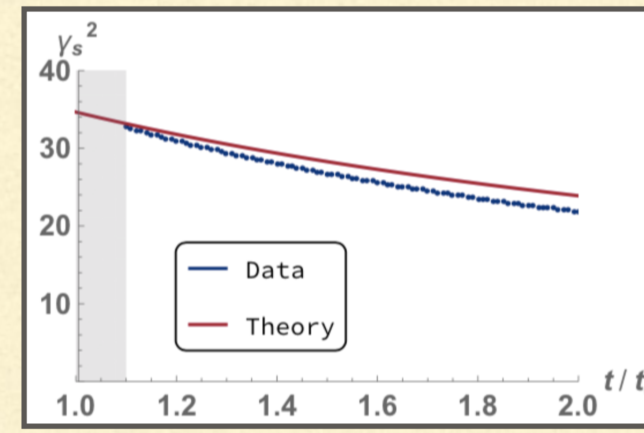
$$\ln \gamma^{2\prime}(t) = \frac{128}{39}\left(\frac{\rho_0}{\sigma}\right) t^2 \gamma_{\rm peak}^4(t) - \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

COMPARISON WITH NUMERICAL SIMULATION

- Analytic (red) vs. numerical (blue)

  with initial condition $\alpha = 10, \gamma_{\rm wall} = 10$

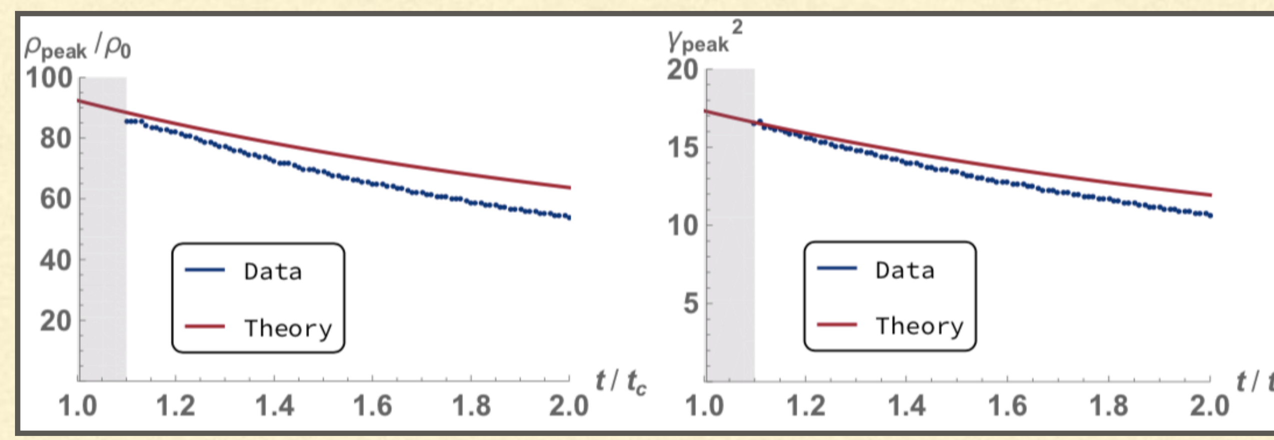
γ_s^2



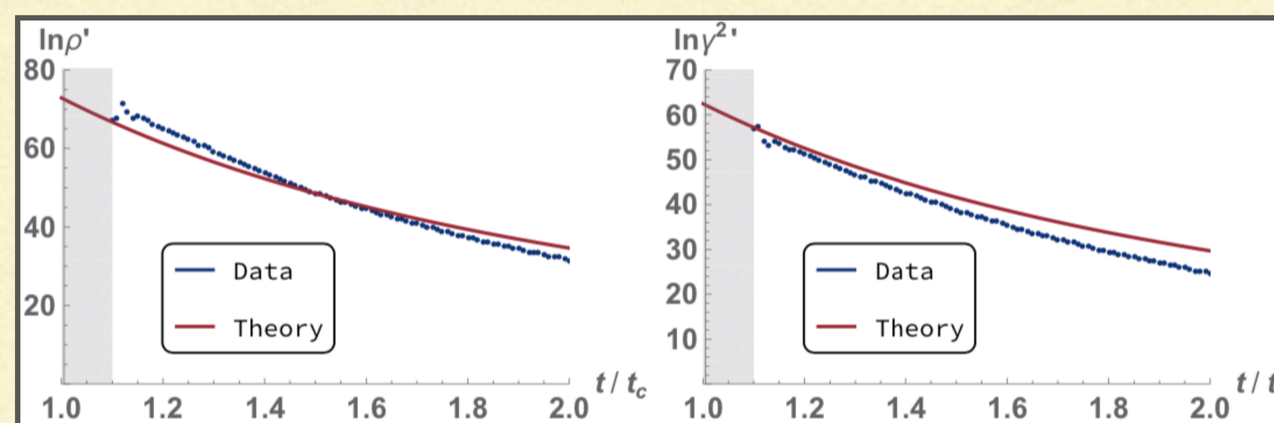
Qualitatively

OK!

$\rho_{\rm peak}, \gamma_{\rm peak}^2$



$$\frac{d\rho_{\rm peak}}{dr}, \frac{d\gamma_{\rm peak}^2}{dr}$$



IMPLICATIONS OF THE EFFECTIVE DESCRIPTION

- What can we learn?

  - All quantities have time dependence like

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^{\delta} t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^{\delta} \qquad \delta = 10/13$$

effect of increase in the surface area

effect of nonlinearity in fluid equation

  - Surface area effect wins (3 > 10/13).

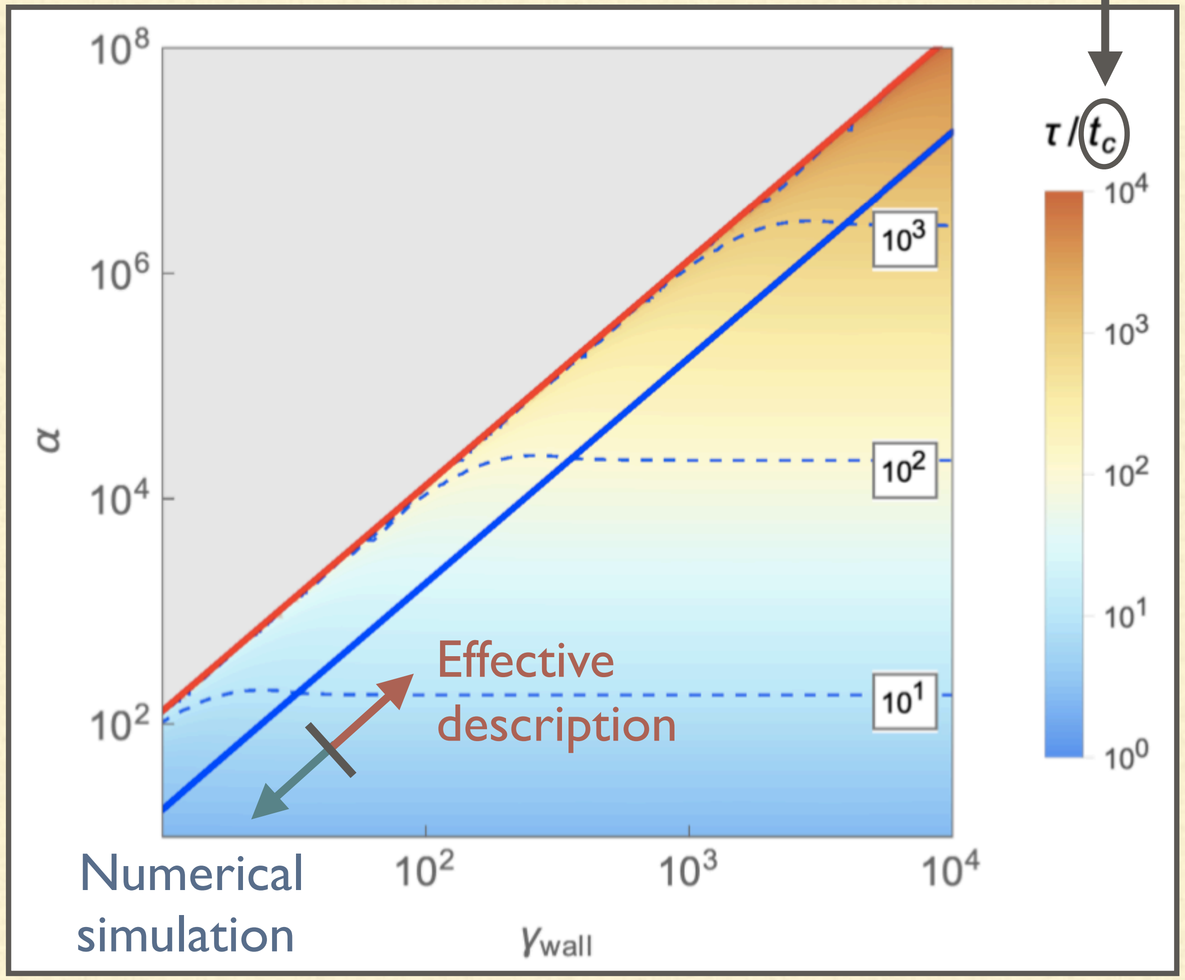
    In other words, nonlinearity is not effective in breaking up the energy localization.

  - Timescale for breaking up is controlled by $\tau \equiv \left(\sigma/\rho_0\right)^{1/3}$

IMPLICATIONS TO GW PRODUCTION

- Fluid profile remains to be relativistic and thin until late times, as long as we consider fluid propagation only
- In other words, we have to see fluid collisions in detail

Time from bubble nucleation to collision



TALK PLAN

✔ 1. Introduction

✔ 2. Ultra-supercooled transitions

✔ 3. Ultra-supercooled transitions:

Effective description of fluid propagation & Implications to GW production

4. Summary

SUMMARY

- GW production in ultra-supercooled transitions $\alpha \gg 1$ is interesting,

  but they are hard to simulate numerically

- We reduced the problem into (1) propagation and (2) collision, and tackled (1):

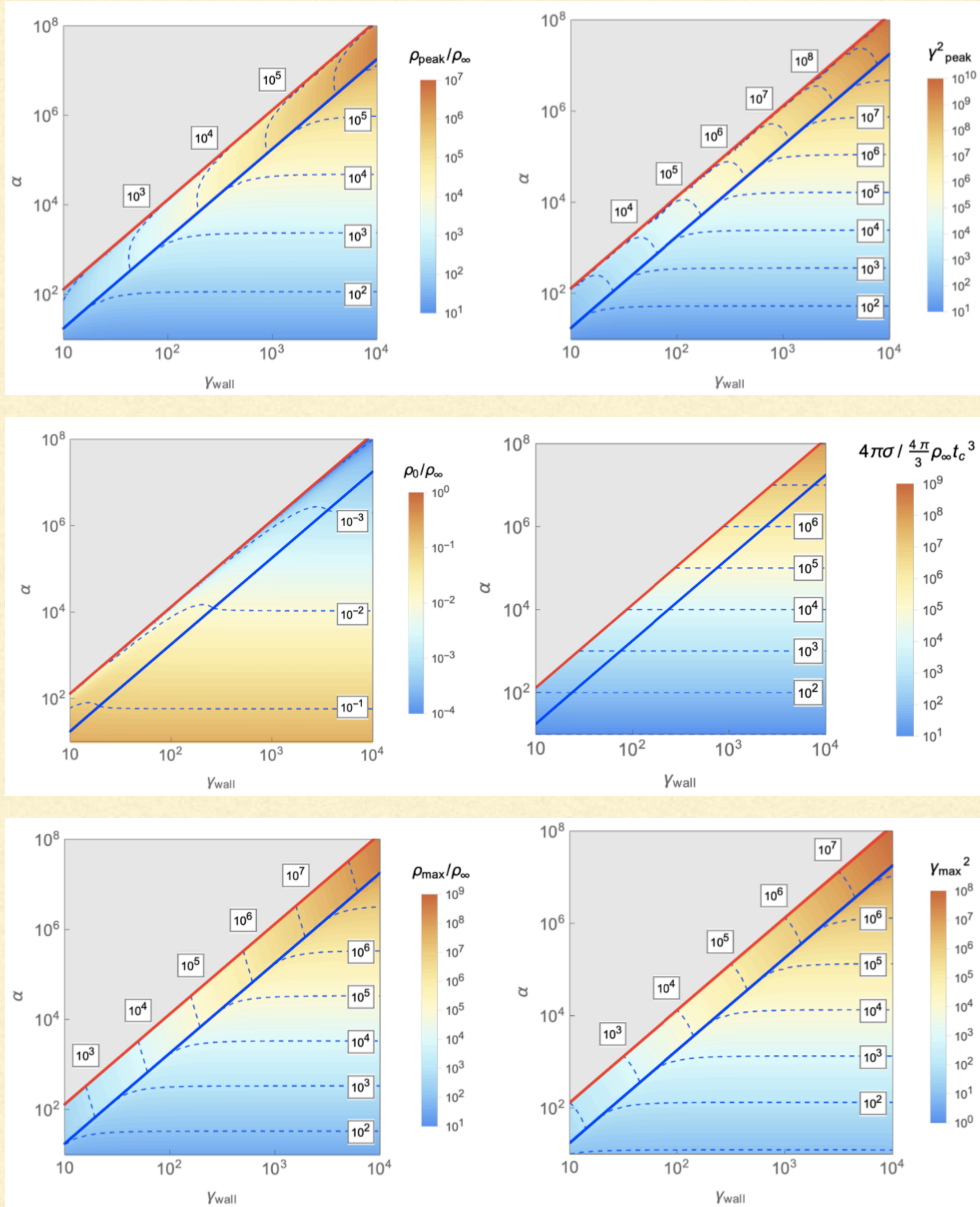
  - We constructed an effective description of relativistic fluid propagation,

    and cross-checked with numerical results in mildly-relativistic regime

  - We discussed implications to GW production, using the effective description

- Questions to be addressed: Effect of fluid collision / Effect of turbulence

Back up



ρ_{peak}/ρ_∞
$\gamma^2{}_{peak}$
ρ_0/ρ_∞
$4\pi\sigma / \frac{4\pi}{3}\rho_\infty t_c{}^3$
ρ_{max}/ρ_∞
$\gamma_{max}{}^2$
α
γ_{wall}