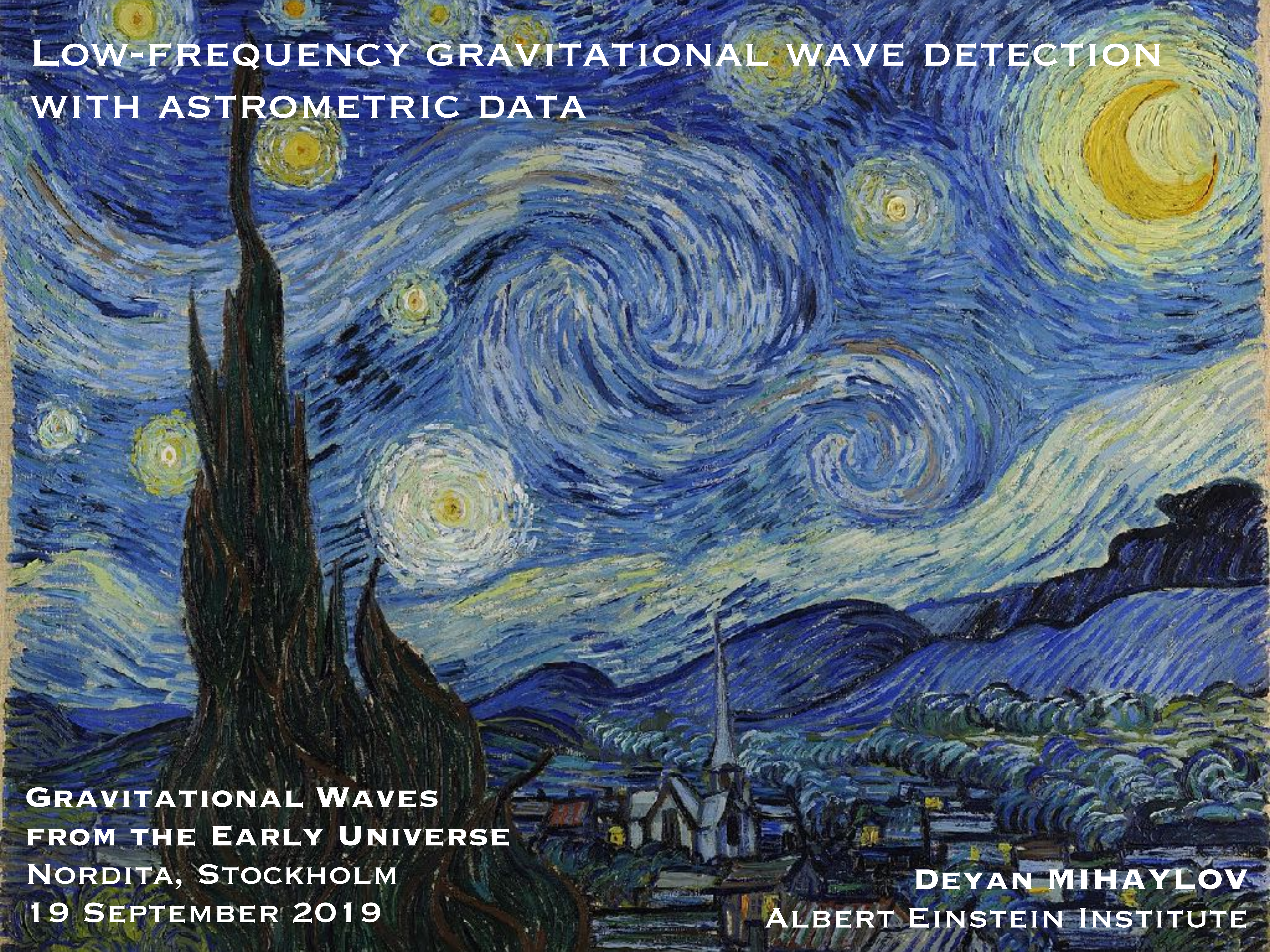
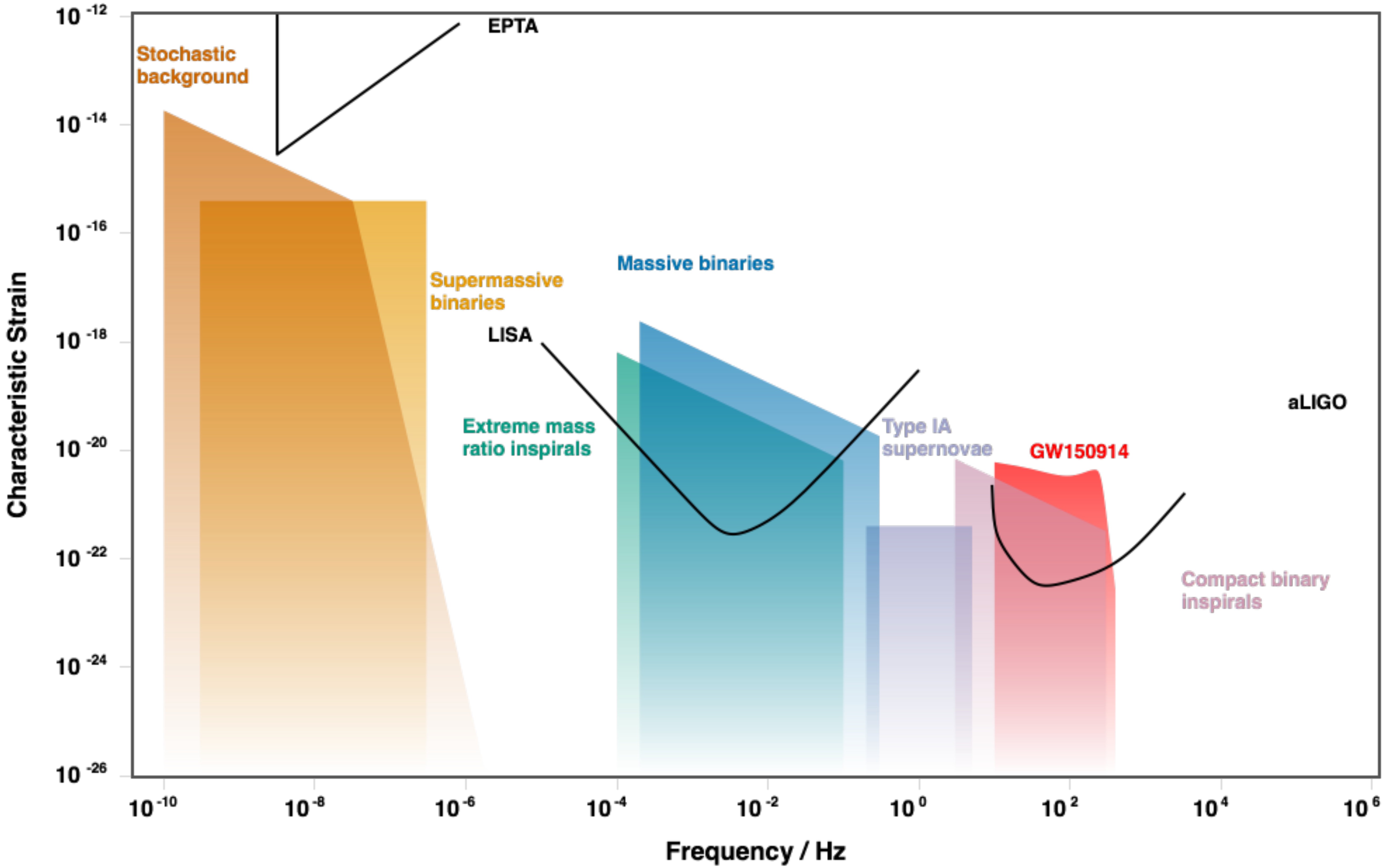
Low-frequency gravitational wave detection with astrometric data

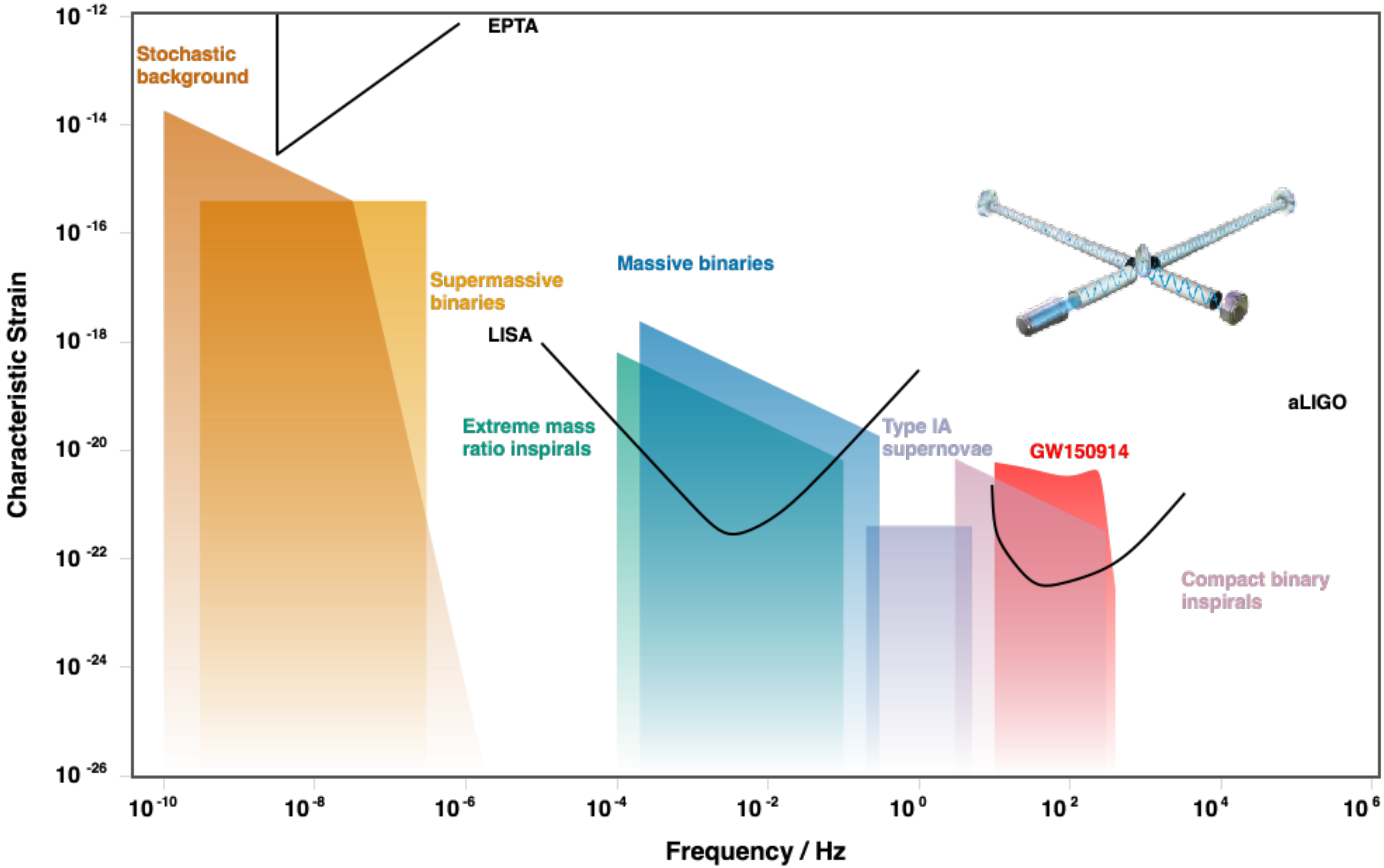


Gravitational Waves from the Early Universe
Nordita, Stockholm
19 September 2019

Deyan Mihaylov
Albert Einstein Institute



EPTA
Stochastic background
Supermassive binaries
Massive binaries
LISA
Extreme mass ratio inspirals
Type IA supernovae
GW150914
aLIGO
Compact binary inspirals
Characteristic Strain
Frequency / Hz
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}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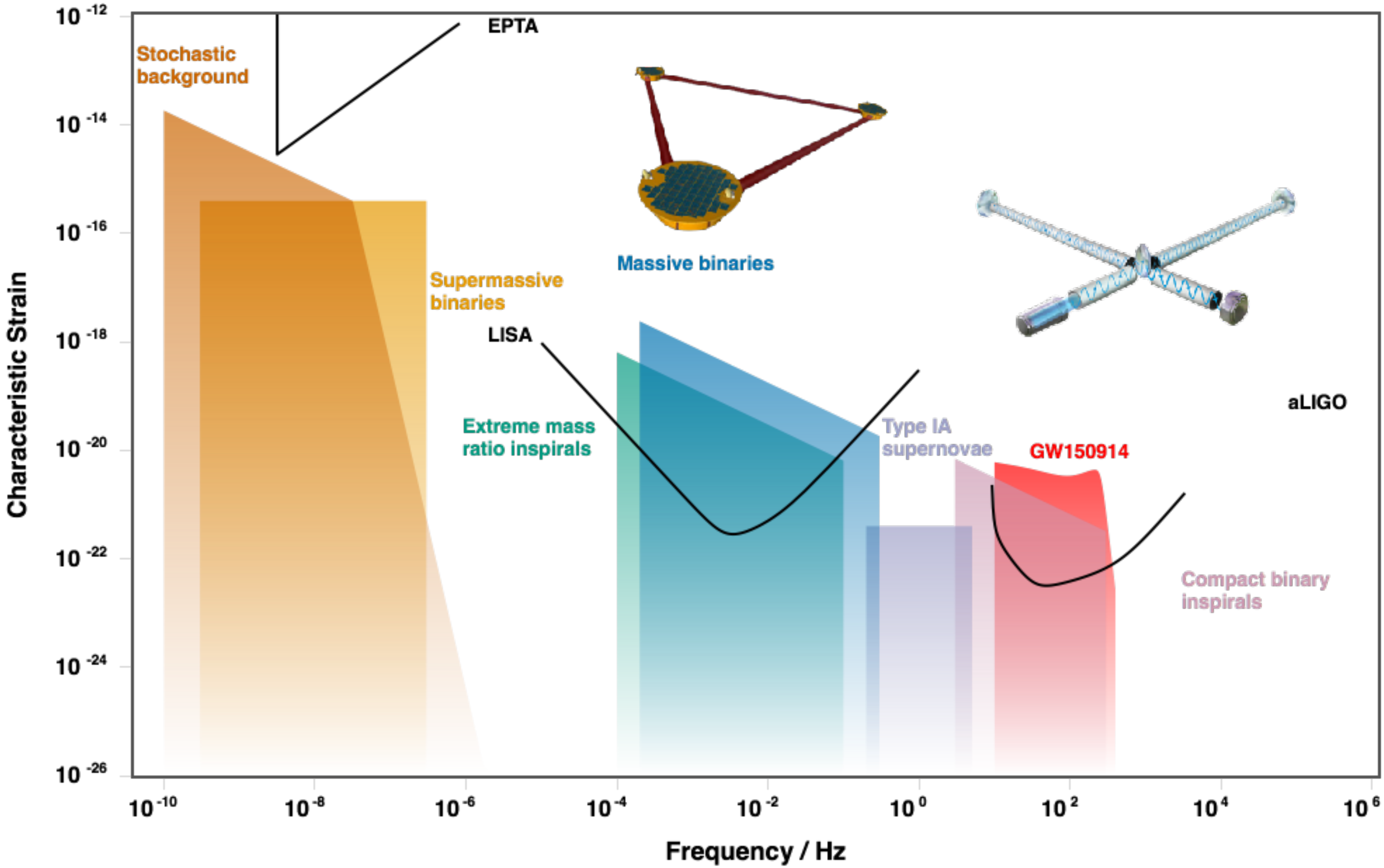
10^-8
10^-6
10^-4
10^-2
10^0
10^2
10^4
10^6
Frequency / Hz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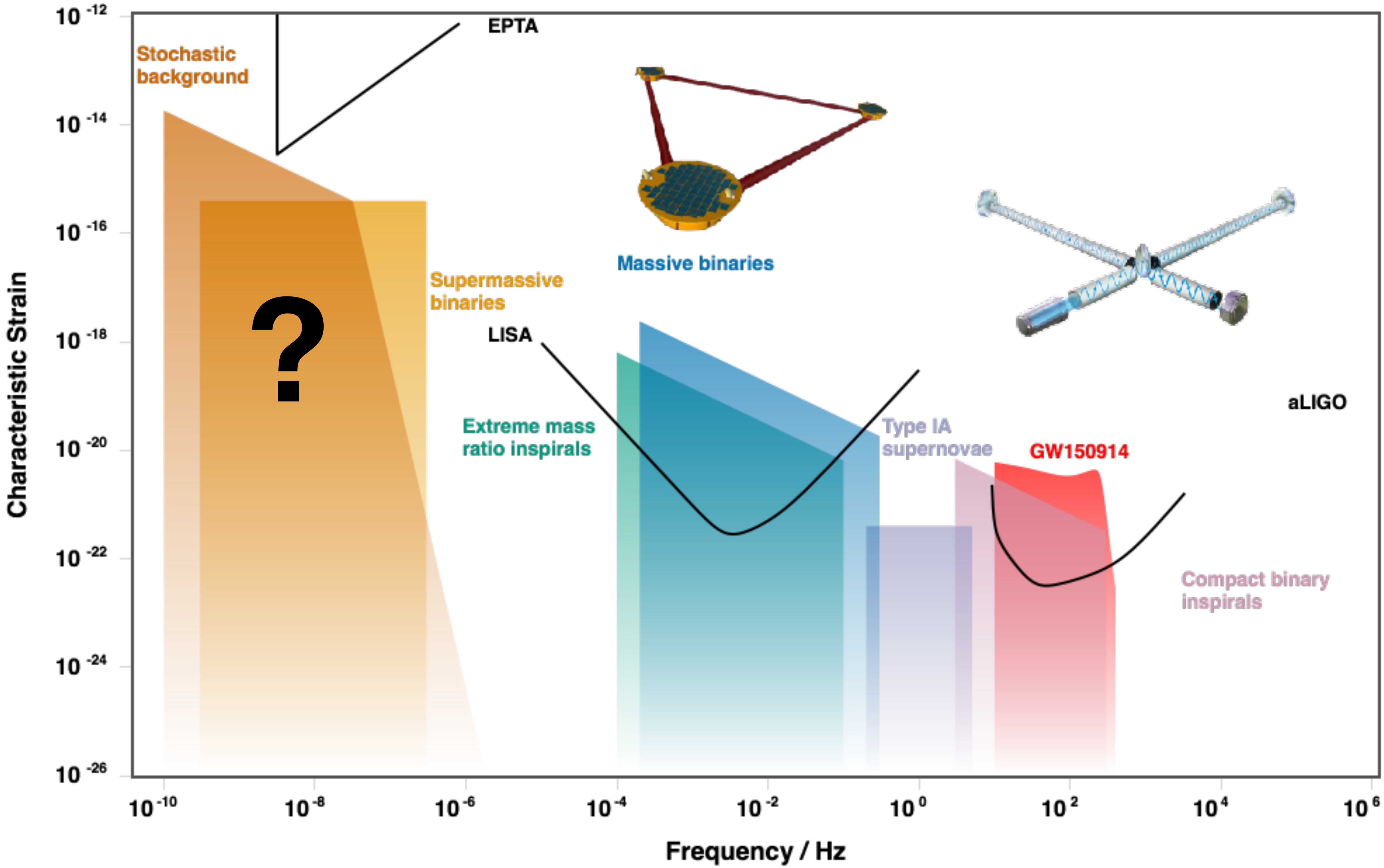

Stochastic background
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Compact binary inspirals
Characteristic Strain
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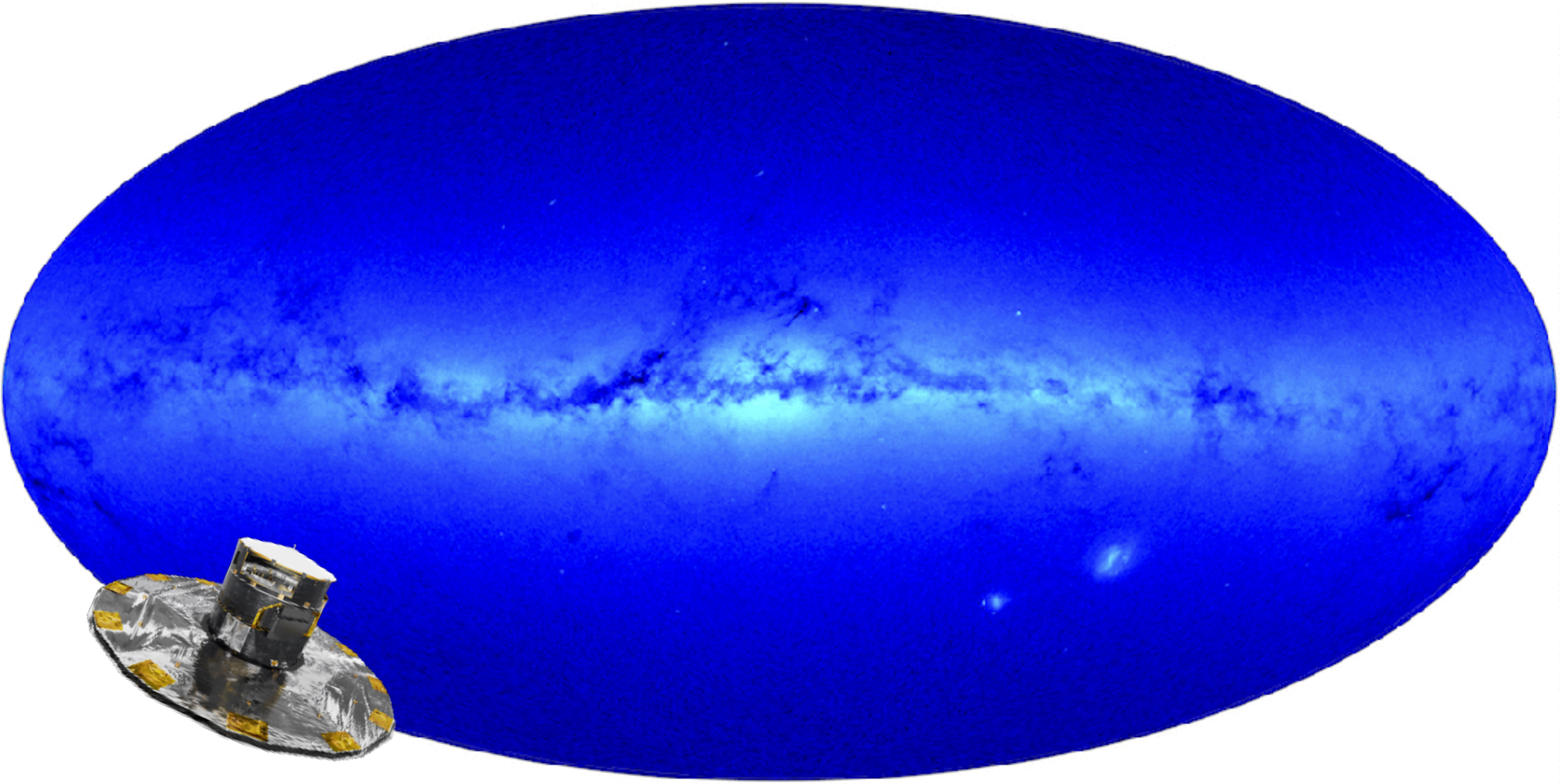
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Frequency / Hz



Gerry
Gilmore

Anthony
Lasenby

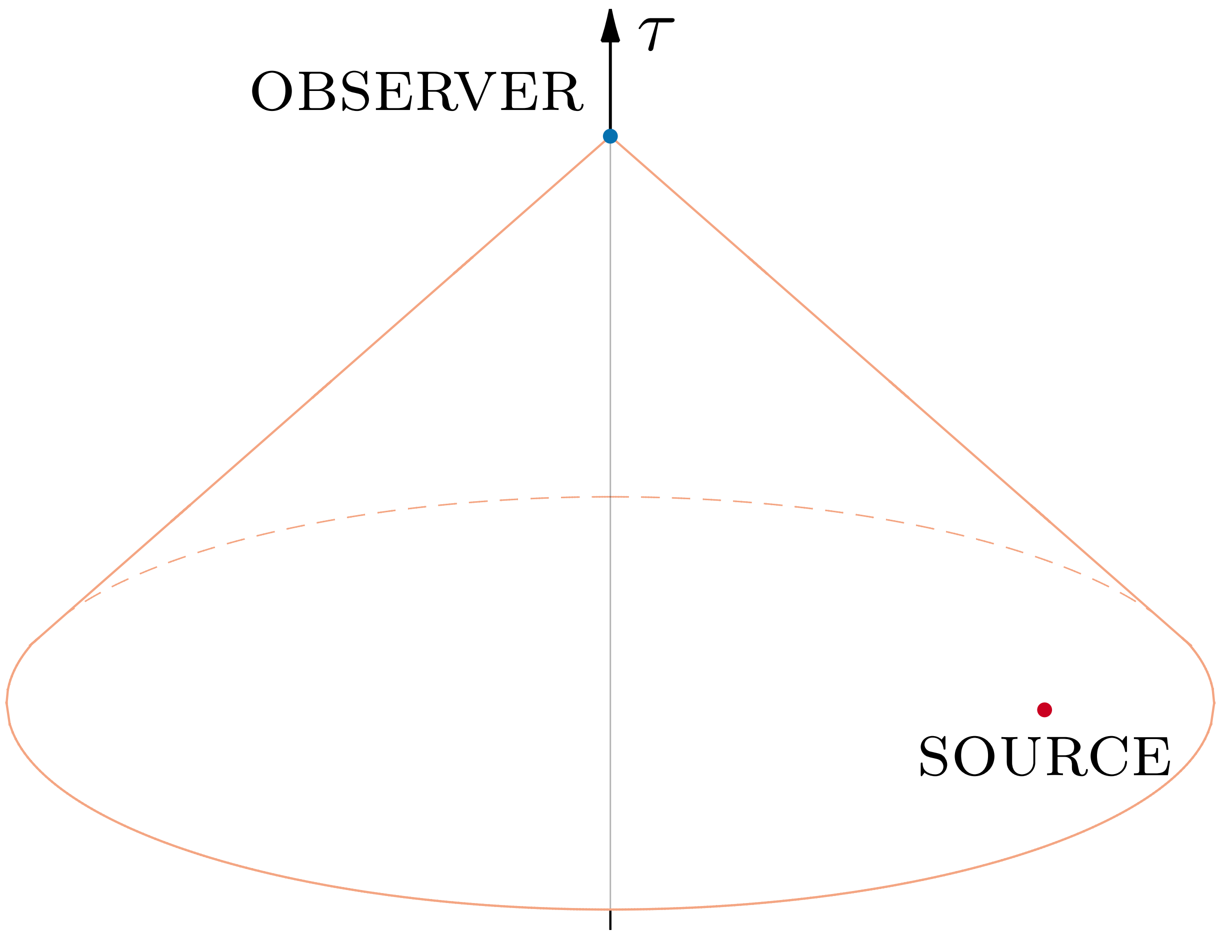
Jonathan
Gair

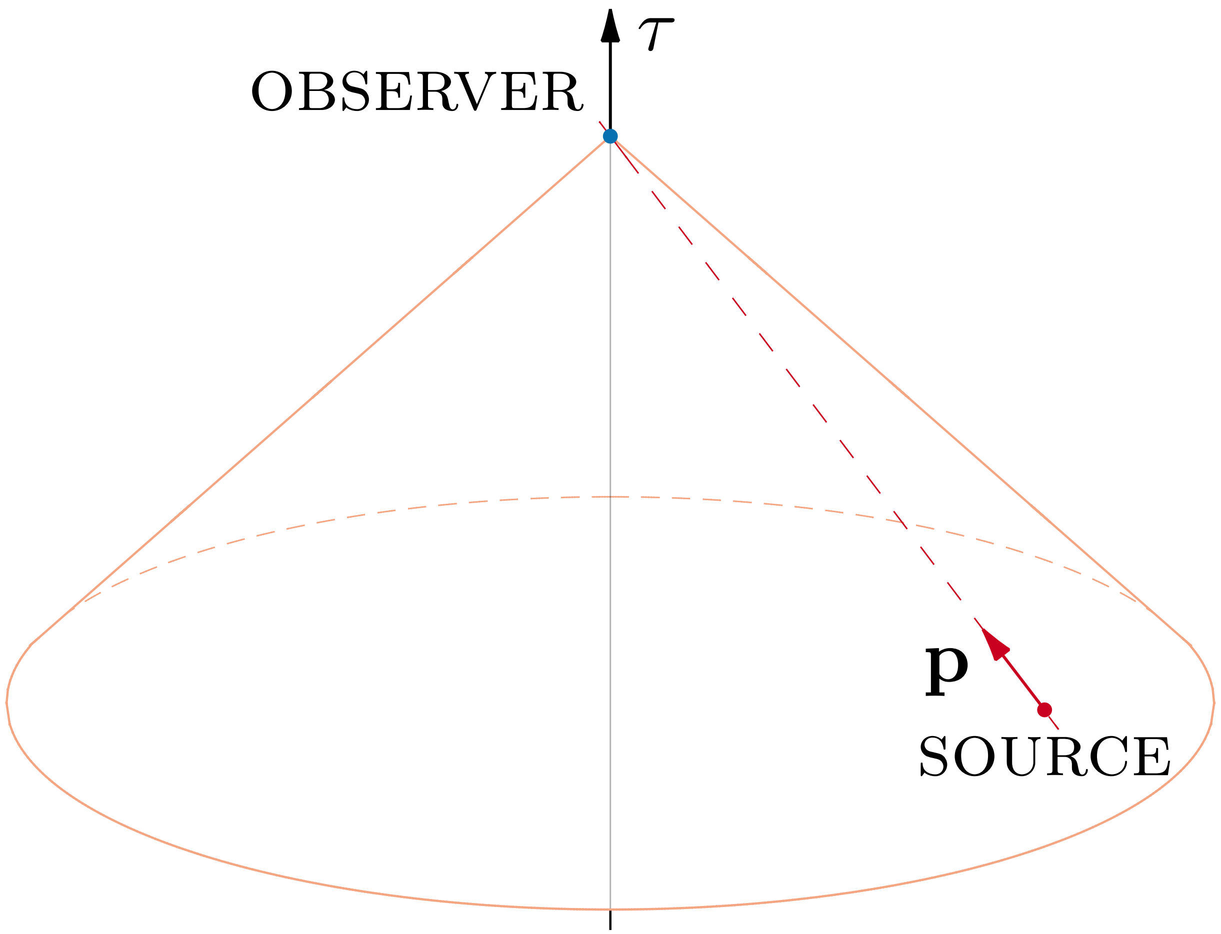
Christopher
Moore

Contents

Astrometric response of a Gravitational Wave

Observer (Earth) and photon source (star) are at rest in flat space

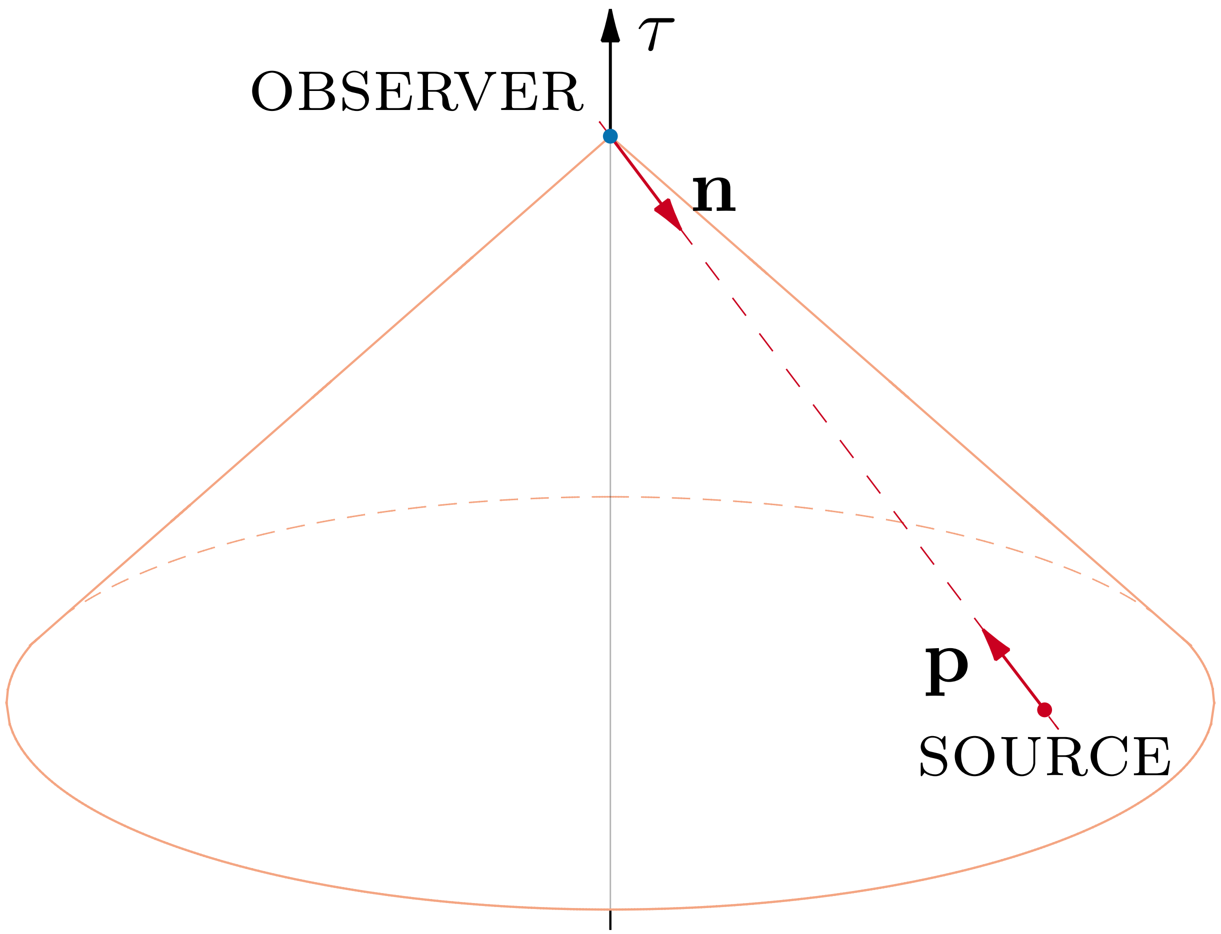




Observer (Earth) and photon source (star) are at rest in flat space

Joined by a null geodesic

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\,x^\mu(\lambda) = 0, \quad p^\mu = \frac{\mathrm{d}}{\mathrm{d}\lambda}\,x^\mu(\lambda) \equiv \text{const.}$$



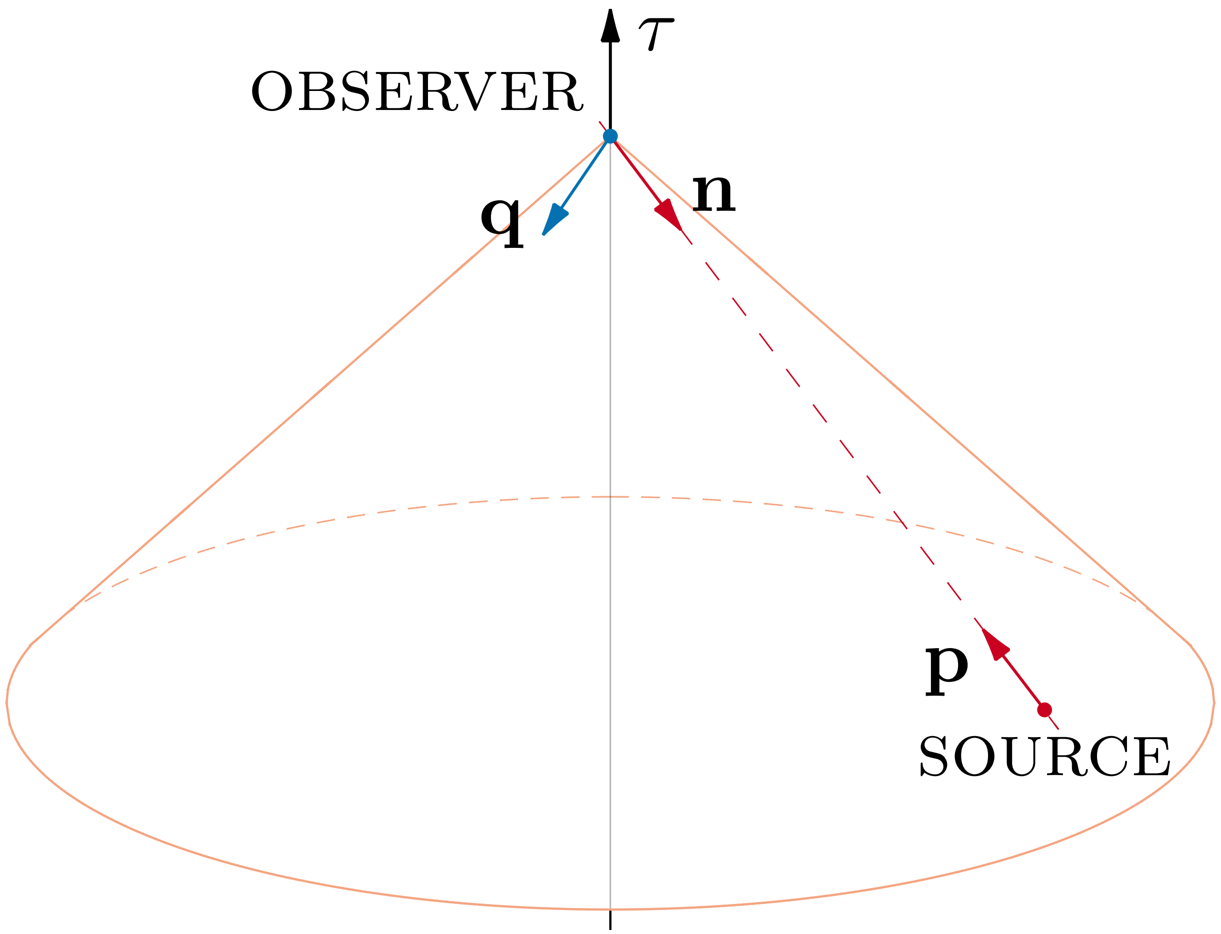
Observer (Earth) and photon source (star) are at rest in flat space

Joined by a null geodesic

$$\frac{d^2}{d\lambda^2} x^\mu(\lambda) = 0, \quad p^\mu = \frac{d}{d\lambda} x^\mu(\lambda) \equiv \text{const.}$$

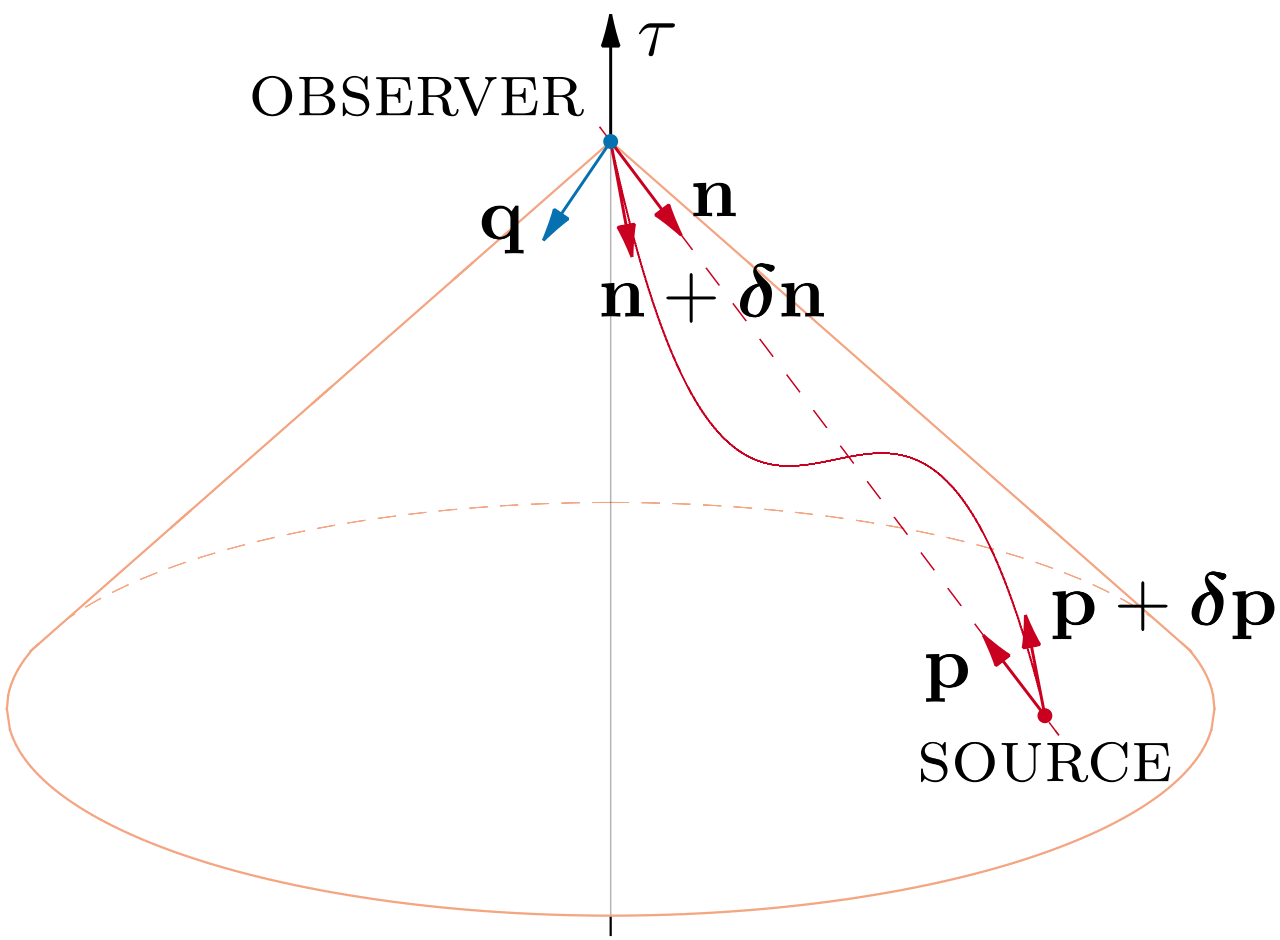
The Observer measures astrometric position and frequency

$n_{\hat{\imath}}, \quad \Omega$



Now consider perturbing the flat space-time with a GW

$$h_{\mu\nu}\left(t, x^i\right) = \Re\left\{ H_{\mu\nu} \exp\left(\mathrm{i} k_\rho x^\rho\right) \right\},$$
$$k^\rho = \omega\left(1, -q^i\right)$$



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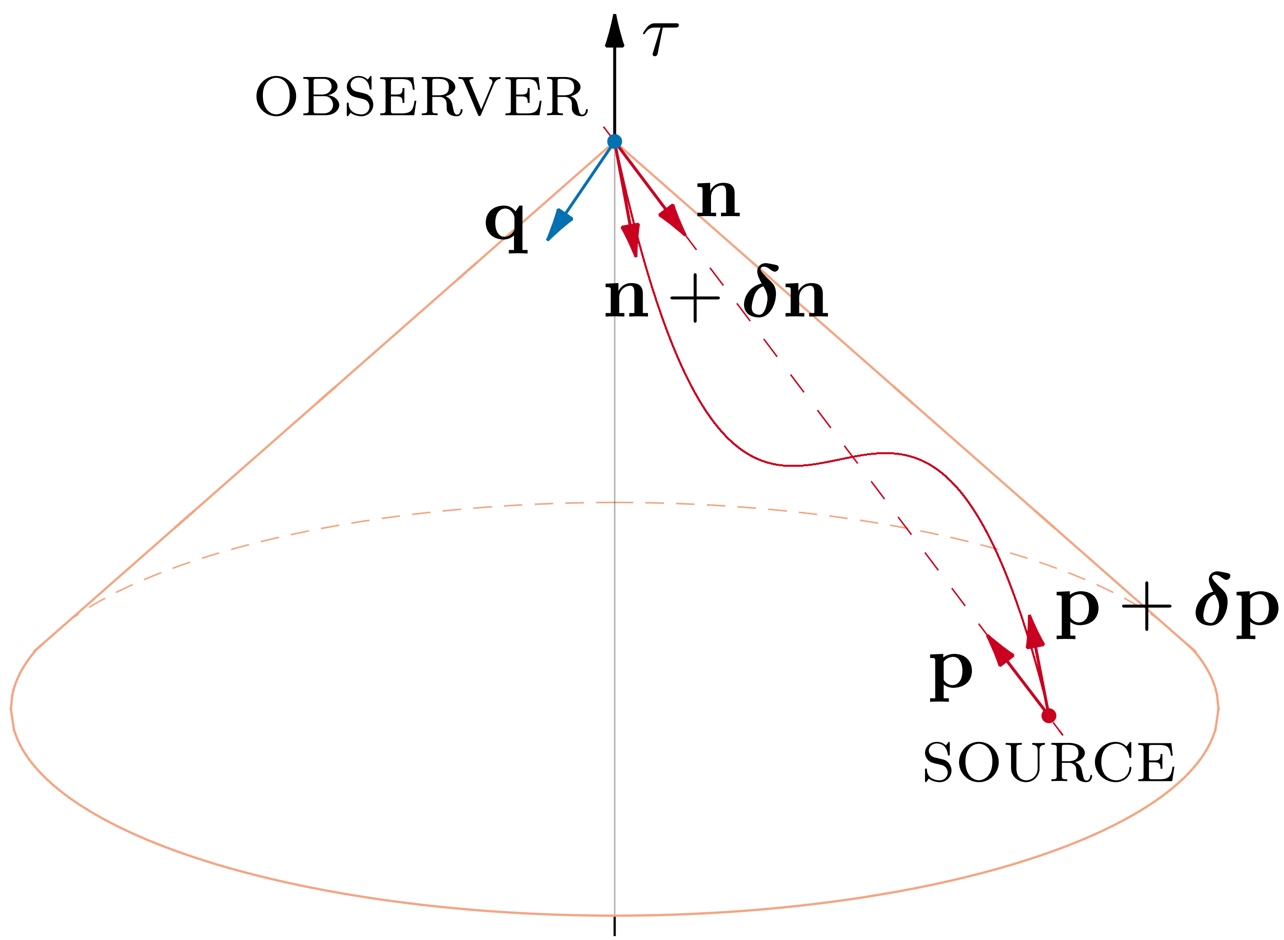
$$h_{\mu\nu}(t,x^i) = \Re\{H_{\mu\nu}\exp(\mathrm{i}k_\rho x^\rho)\},$$
$$k^\rho = \omega(1,-q^i)$$

Worldlines of **observer** and **source** are unaffected

Photon worldline is a geodesic in both metrics

$$x^\mu(\lambda) \mapsto x^\mu(\lambda) + \delta x^\mu(\lambda)$$



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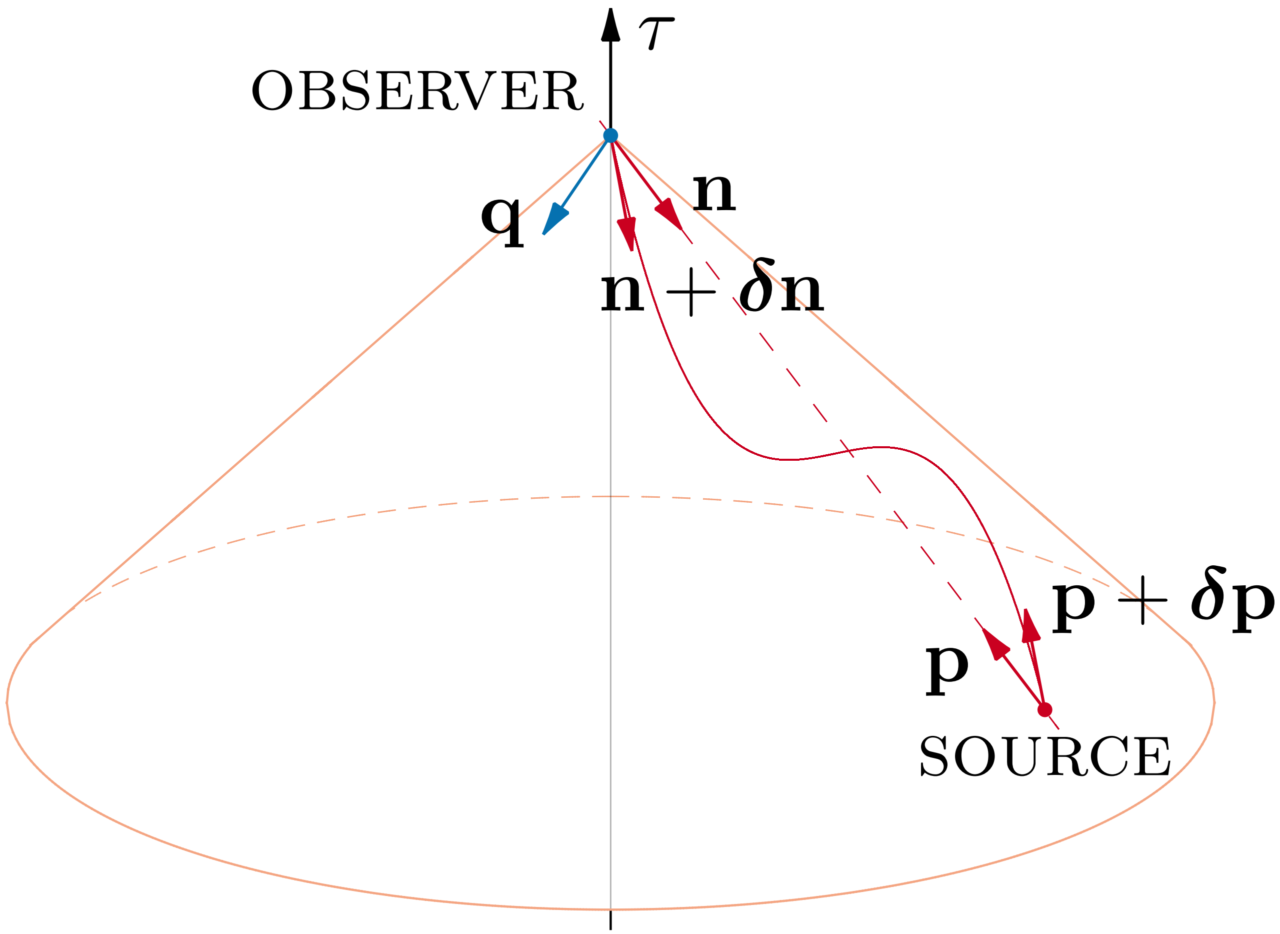
Worldlines of **observer** and **source** are unaffected

Photon worldline is a geodesic in both metrics

$$x^\mu(\lambda) \mapsto x^\mu(\lambda) + \delta x^\mu(\lambda)$$

Evolves according to the parallel transport equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2} \delta x^\mu_{t_0}(\lambda) = -\Gamma^\mu_{\nu\rho} p^\nu p^\rho$$

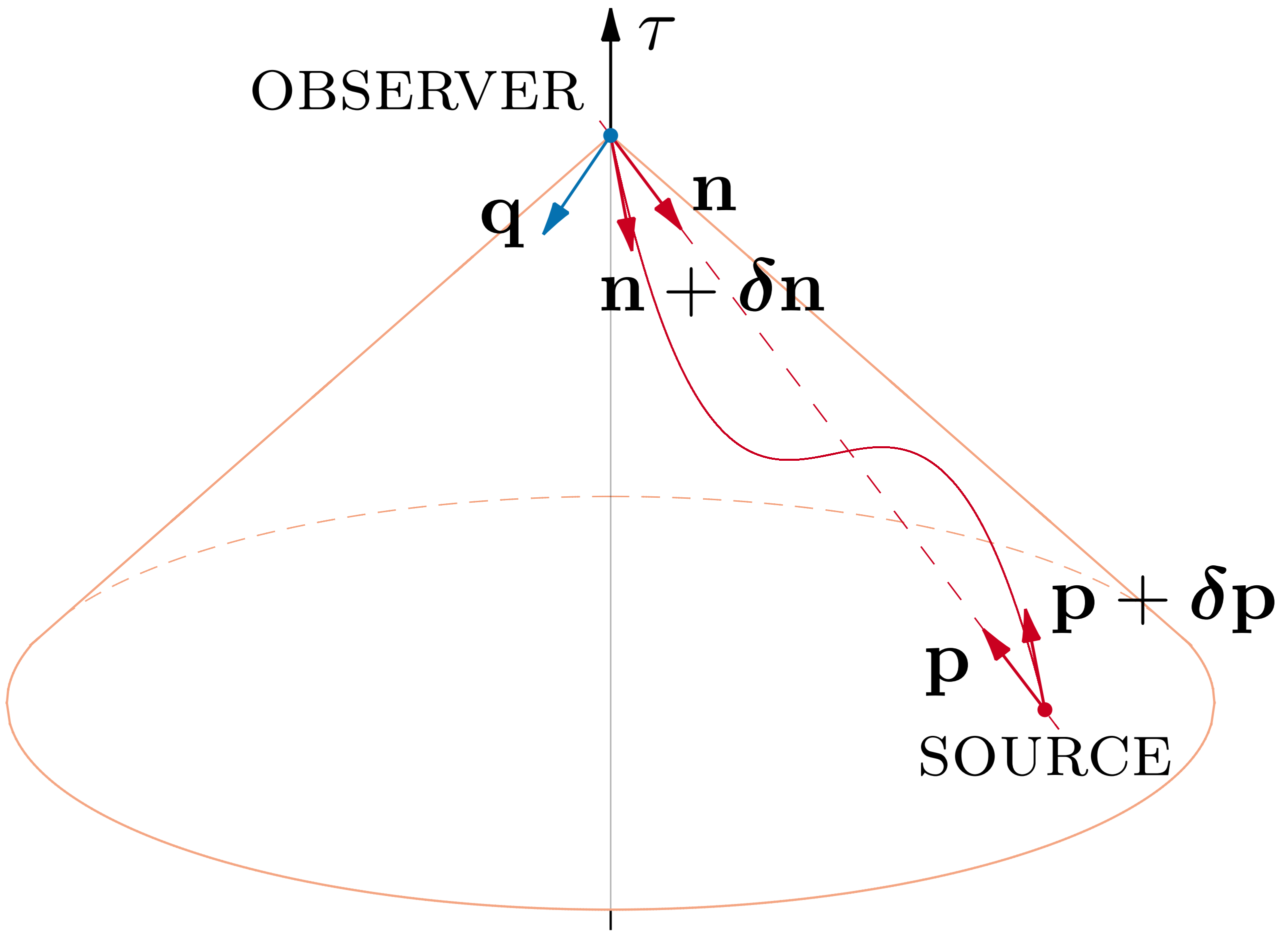


Integrate along the worldline of the photon

Boundary conditions:

A. Photon path is null

B. Photon path intersects source and observer worldlines



INTEGRATE ALONG THE WORLDLINE OF THE PHOTON

BOUNDARY CONDITIONS:

A. PHOTON PATH IS NULL

B. PHOTON PATH INTERSECTS SOURCE AND OBSERVER WORLDLINES

PERTURBED ASTROMETRIC POSITION AND FREQUENCY

$n_{\hat{\imath}} + \delta n_{\hat{\imath}}, \quad \Omega_{\text{obs}}$

Redshift

$$z = \frac{n^i n^j}{2\left(1 - n_k q^k\right)} \left[h_{ij}(\textsc{obs}) - h_{ij}(\textsc{source})\right]$$

Redshift

$$z = \frac{n^i n^j}{2\left(1 - n_k q^k\right)} \left[h_{ij}(\text{OBS}) - h_{ij}(\text{SOURCE})\right]$$

Astrometric shift

$$\delta n_{\hat{\imath}} = \Bigg[\Bigg(\Bigg\{ 1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right] \Bigg\} n_{\hat{\imath}} - \Bigg\{ 1 + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right] \Bigg\} q_{\hat{\imath}} \Bigg) \frac{H_{jk} n^j n^k}{2(1 - \vec{q}\cdot\vec{n})} - \Bigg\{ \frac{1}{2} + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right] \Bigg\} H_{\hat{\imath}j} n^j \Bigg] \exp(-\mathrm{i}\omega t_0)\,.$$

Astrometric shift

$$\delta n_{\hat{\imath}} = \Bigg[\Bigg(\Bigg(\Bigg\{1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} n_{\hat{\imath}}$$

$$- \Bigg\{1 + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} q_{\hat{\imath}}\Bigg) \frac{H_{jk}n^j n^k}{2(1 - \vec{q}\cdot\vec{n})}$$

$$- \Bigg\{\frac{1}{2} + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} H_{\hat{\imath}j}n^j\Bigg] \exp(-\mathrm{i}\omega t_0)\,.$$

$$= \delta n_{\hat{\imath}}\left(h(\textsc{obs}), h(\textsc{source})\right)$$

Astrometric shift

$$\delta n_{\hat{\imath}} = \Bigg[\Bigg(\Bigg\{1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\} n_{\hat{\imath}}$$

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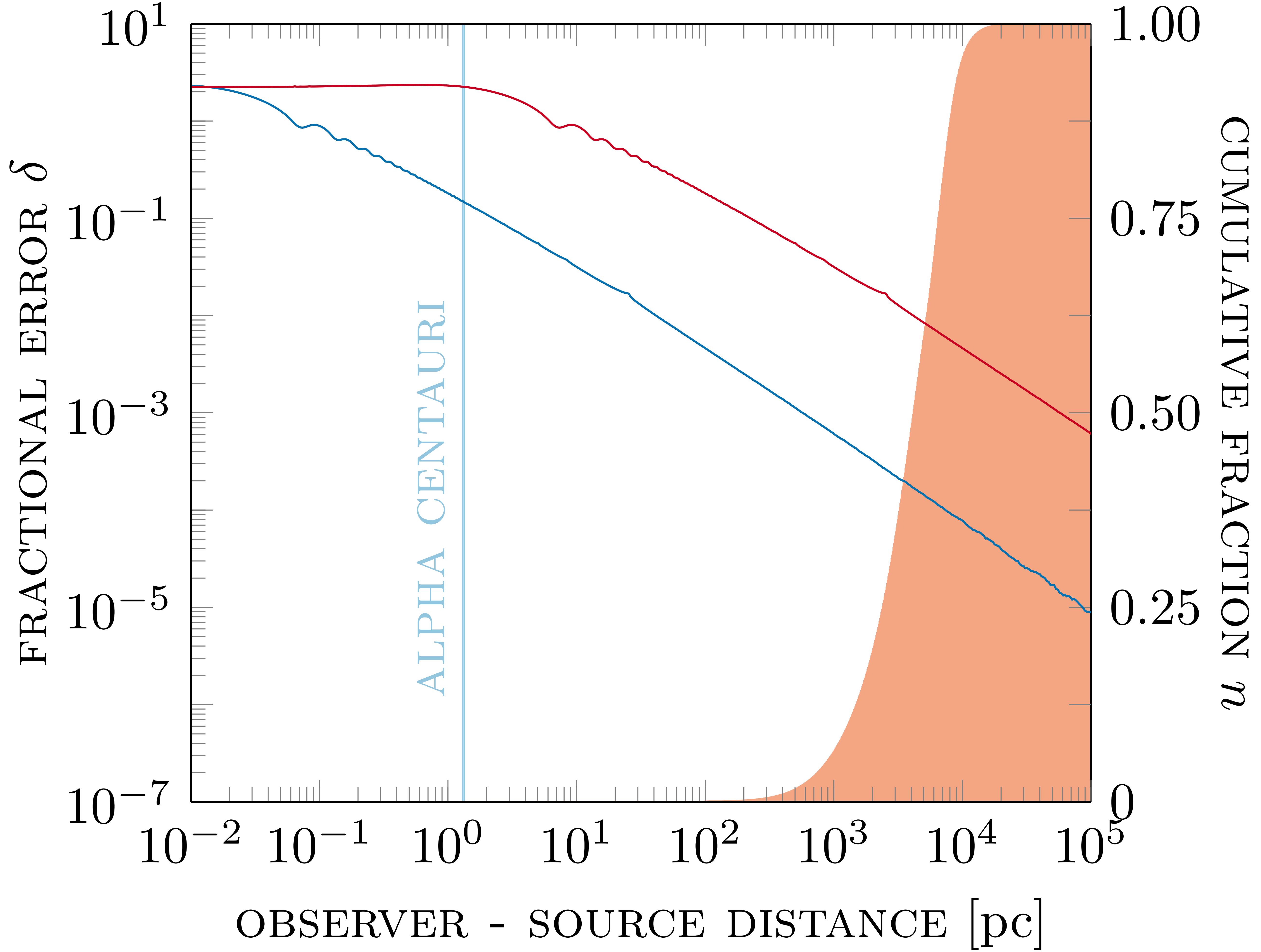
$$- \Bigg\{\frac{1}{2} + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\} H_{\hat{\imath}\jmath} n^{\jmath}\Bigg] \exp(-\mathrm{i}\omega t_0) \,.$$

$$= \delta n_{\hat{\imath}} \left(h(\textsc{obs}), h(\textsc{source})\right)$$

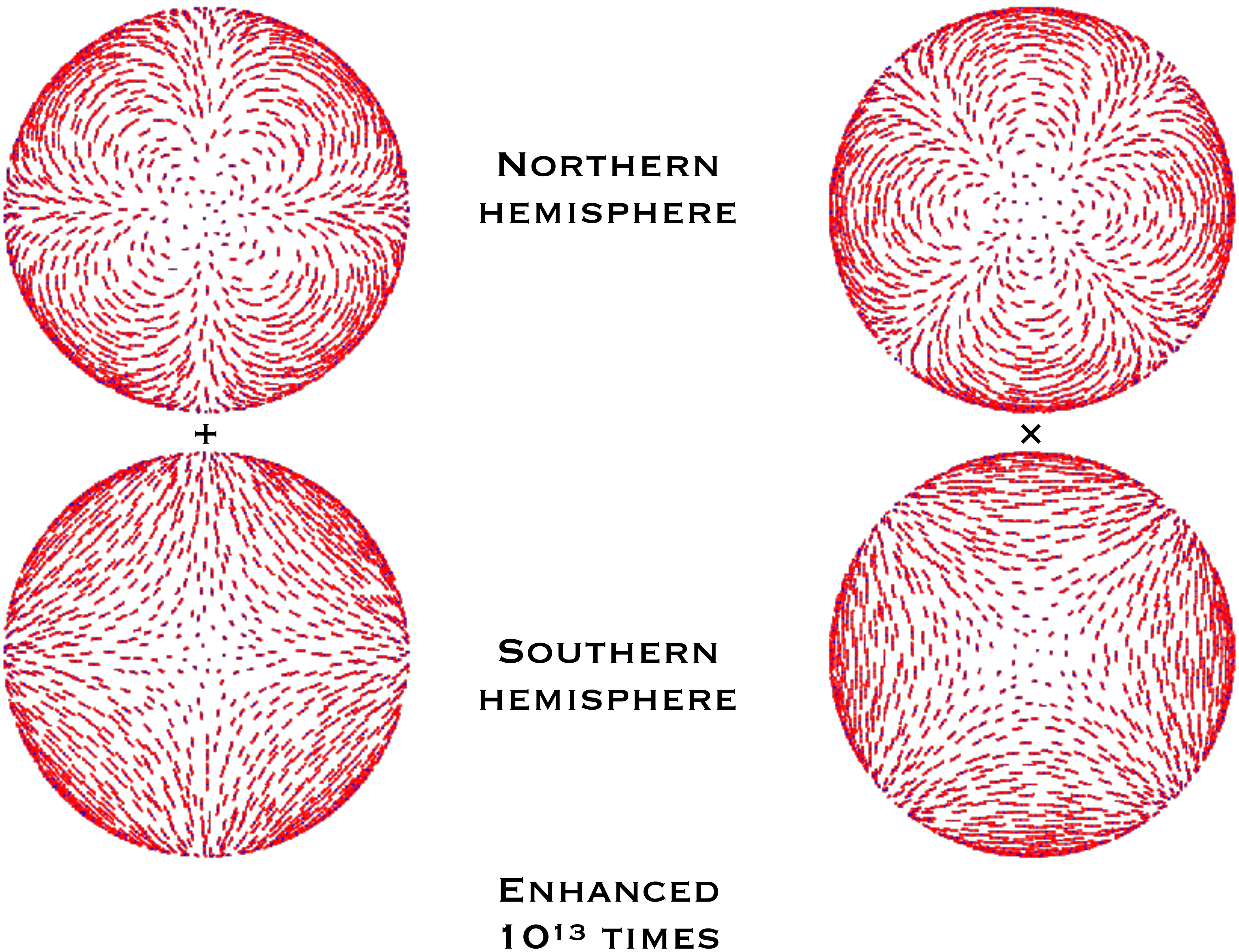
In the distant source limit

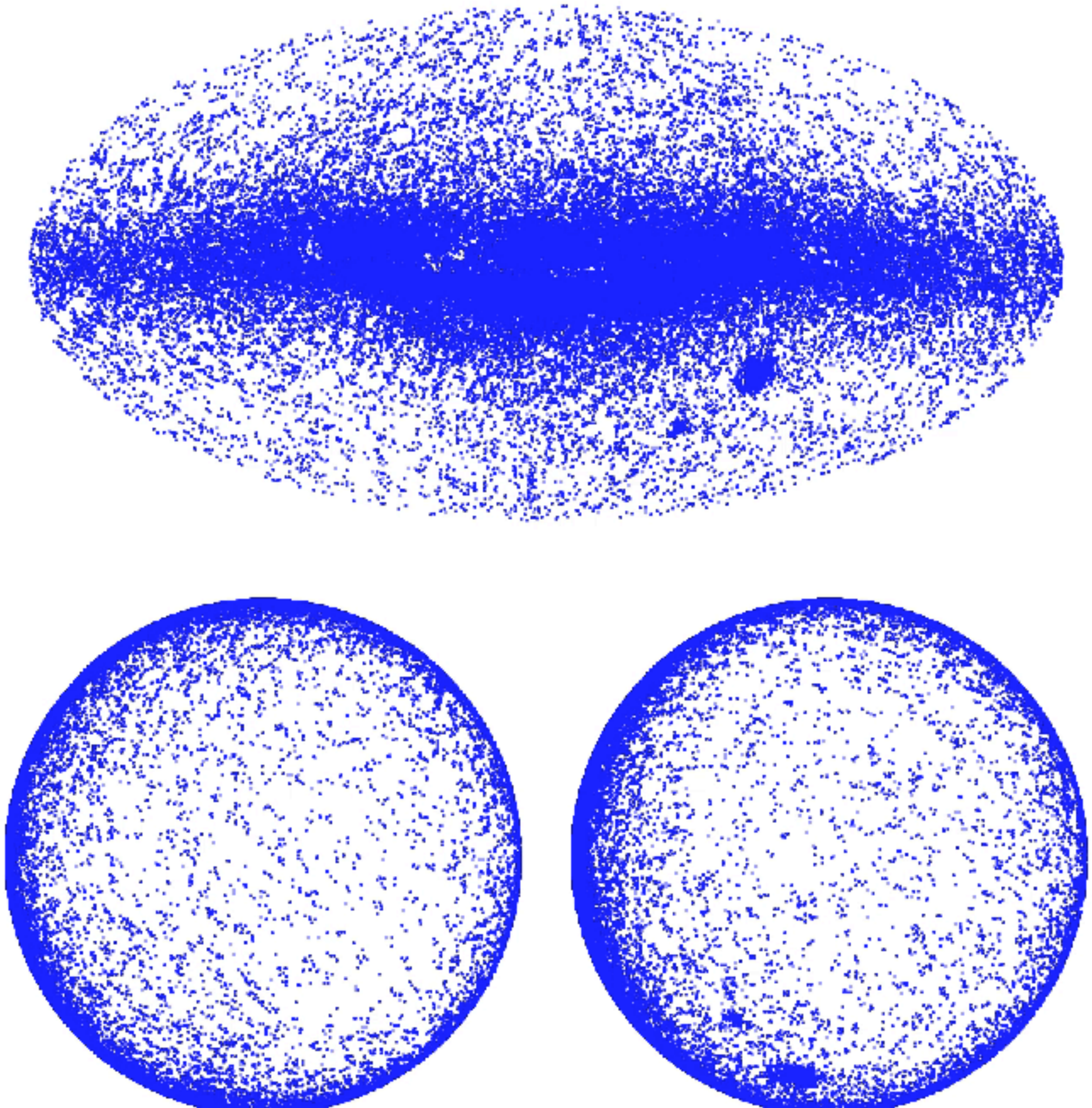
$$\delta n_{\hat{\imath}} = \frac{n_{\hat{\imath}} - q_{\hat{\imath}}}{2(1-\vec{q}\cdot\vec{n})} h_{\hat{\jmath}\hat{k}}(\textsc{obs}) n^{\hat{\jmath}} n^{\hat{k}} - \frac{1}{2} h_{\hat{\imath}\hat{\jmath}}(\textsc{obs}) n^{\hat{\jmath}} \,.$$



FRACTIONAL ERROR δ
CUMULATIVE FRACTION n
ALPHA CENTAURI
OBSERVER - SOURCE DISTANCE [pc]
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
1.00
0.75
0.50
0.25
0
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}

Effect on the sky


+
×
Northern hemisphere
Southern hemisphere
Enhanced 10^{13} times

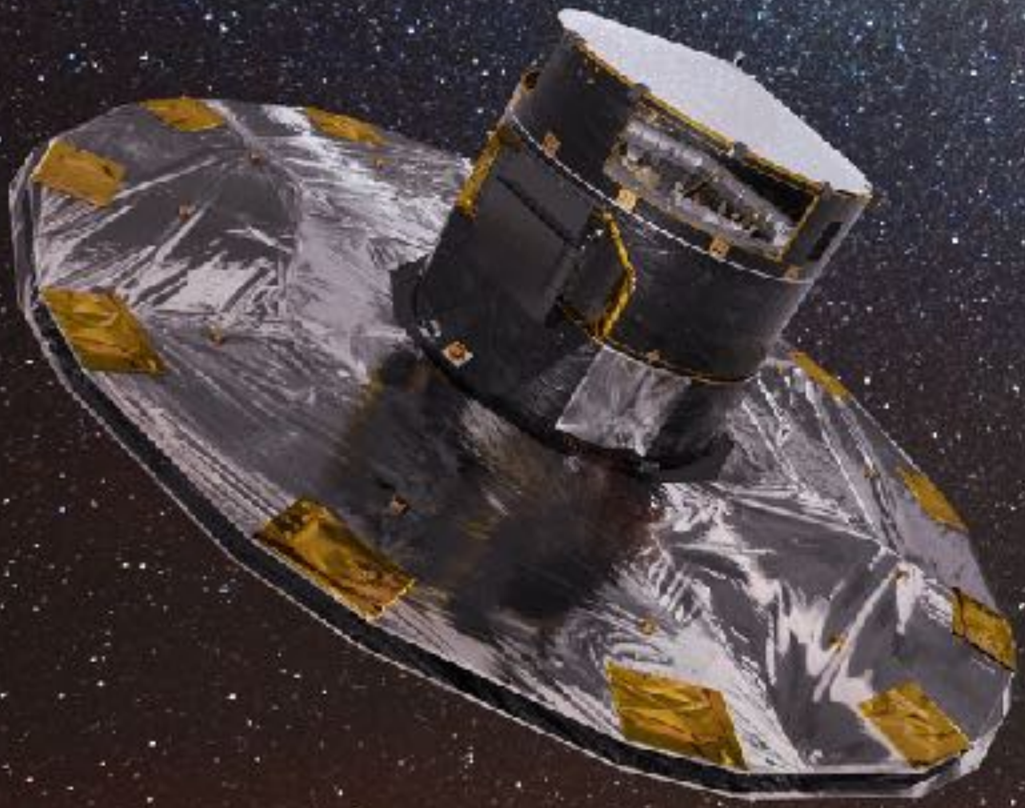


Enhanced
10^{13} times

Is this effect detectable?

Gaia

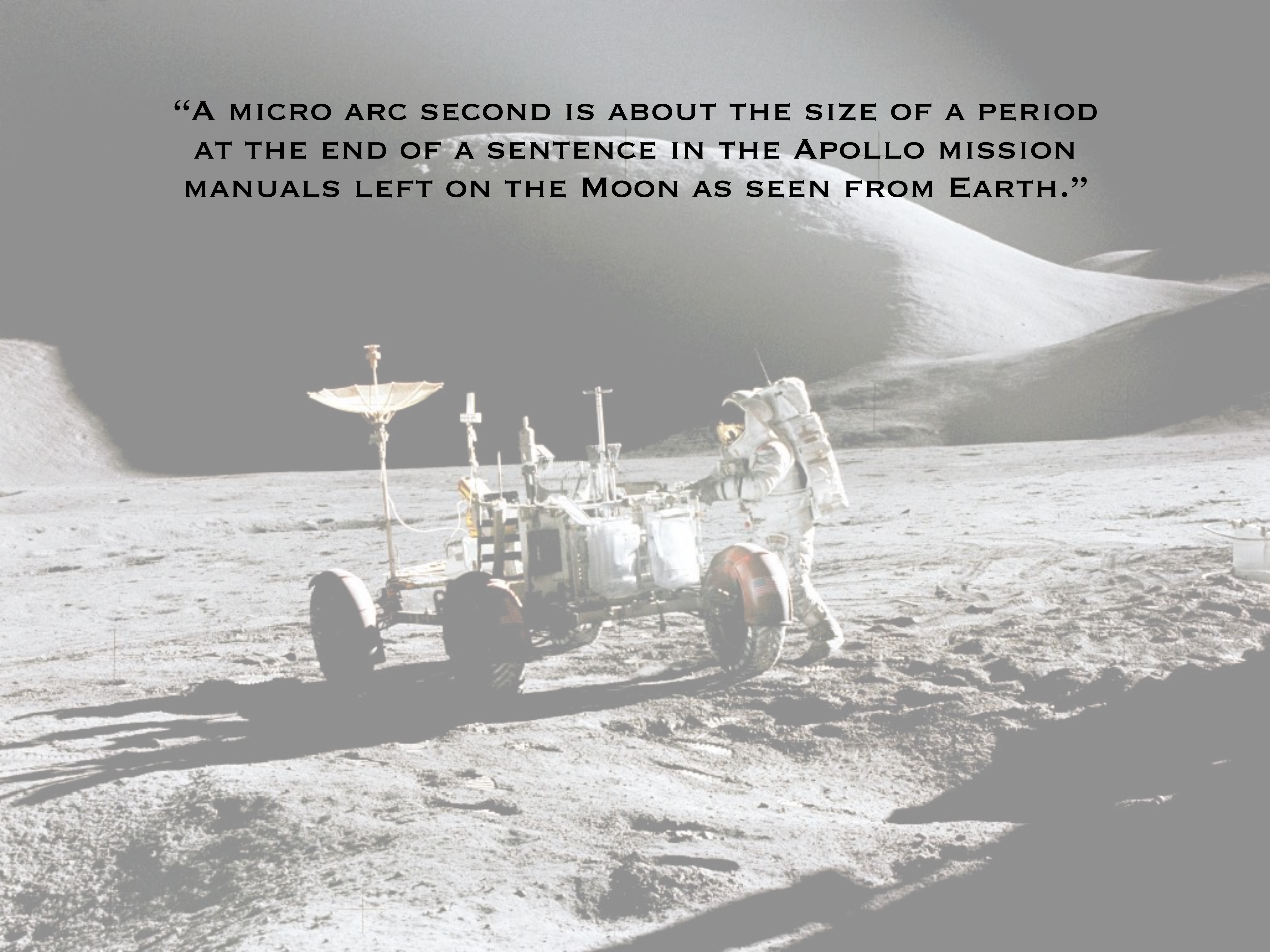
ESA mission for astrometry
in the Milky Way



Objectives:

Map ~10^9 objects ~70 times each

Astrometric accuracy
~10 micro arc seconds


"A micro arc second is about the size of a period at the end of a sentence in the Apollo mission manuals left on the Moon as seen from Earth."

Sensitivity to individual events

~100 measurements of each object

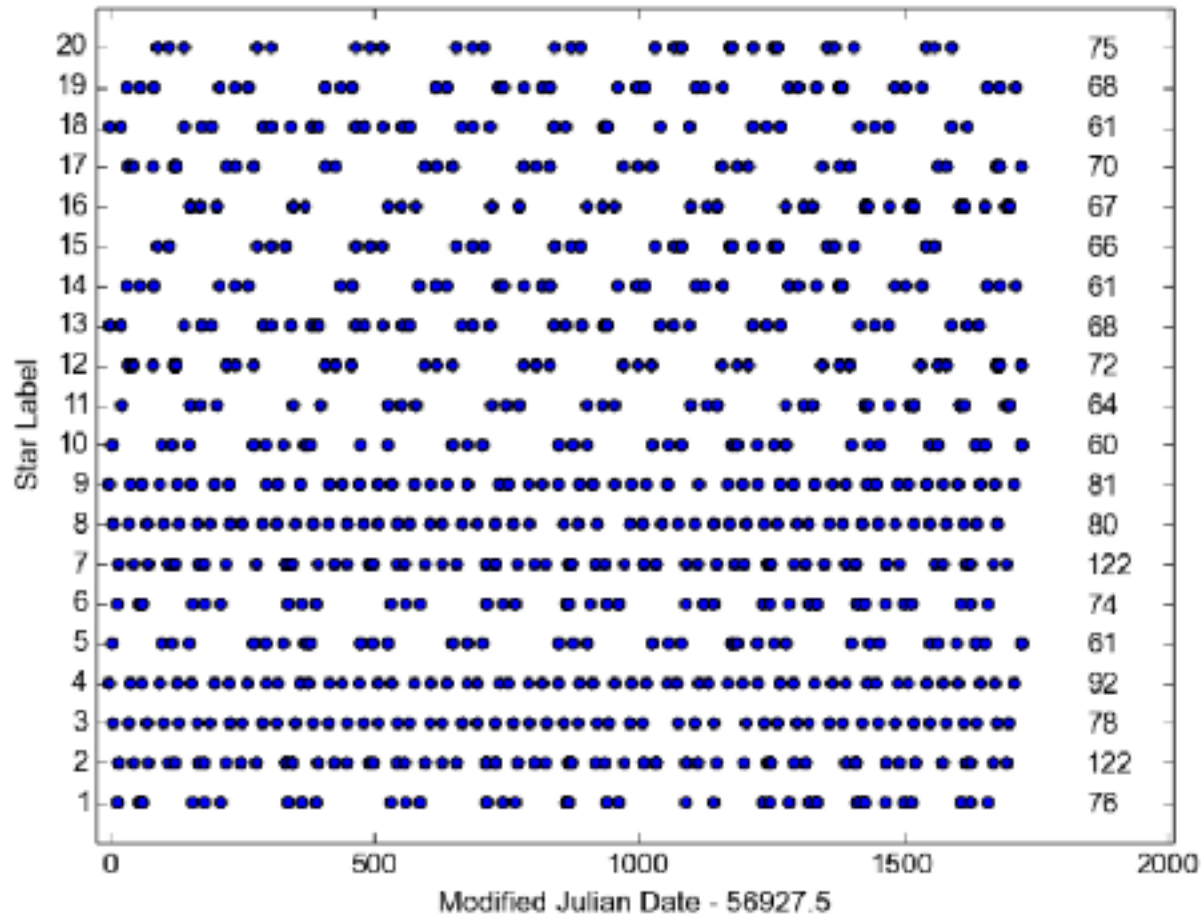
Mission duration 5-10 years

Sensitive in the range 10^{-8} - 3×10^{-7} Hz

Black hole binaries in the early PN inspiral, 10^{8} - 10^{10} solar masses

Caveats

Astrometric measurements are spread over 5 years



Idealised assumptions about the detector noise (Gaussian)

Gaia has a deadline of 5 years, PTA surveys continually improve

Detecting GWs with Gaia

It is only being seriously considered now, in the Gaia era

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Biggest challenge is the size of the data set

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Biggest challenge is the size of the data set

Data Releases 1 & 2 do not feature individual astrometric measurements, working with simulated data

Computational pipeline

Simulated data

Computational pipeline

Simulated data

Proper motion + Noise + GW signal

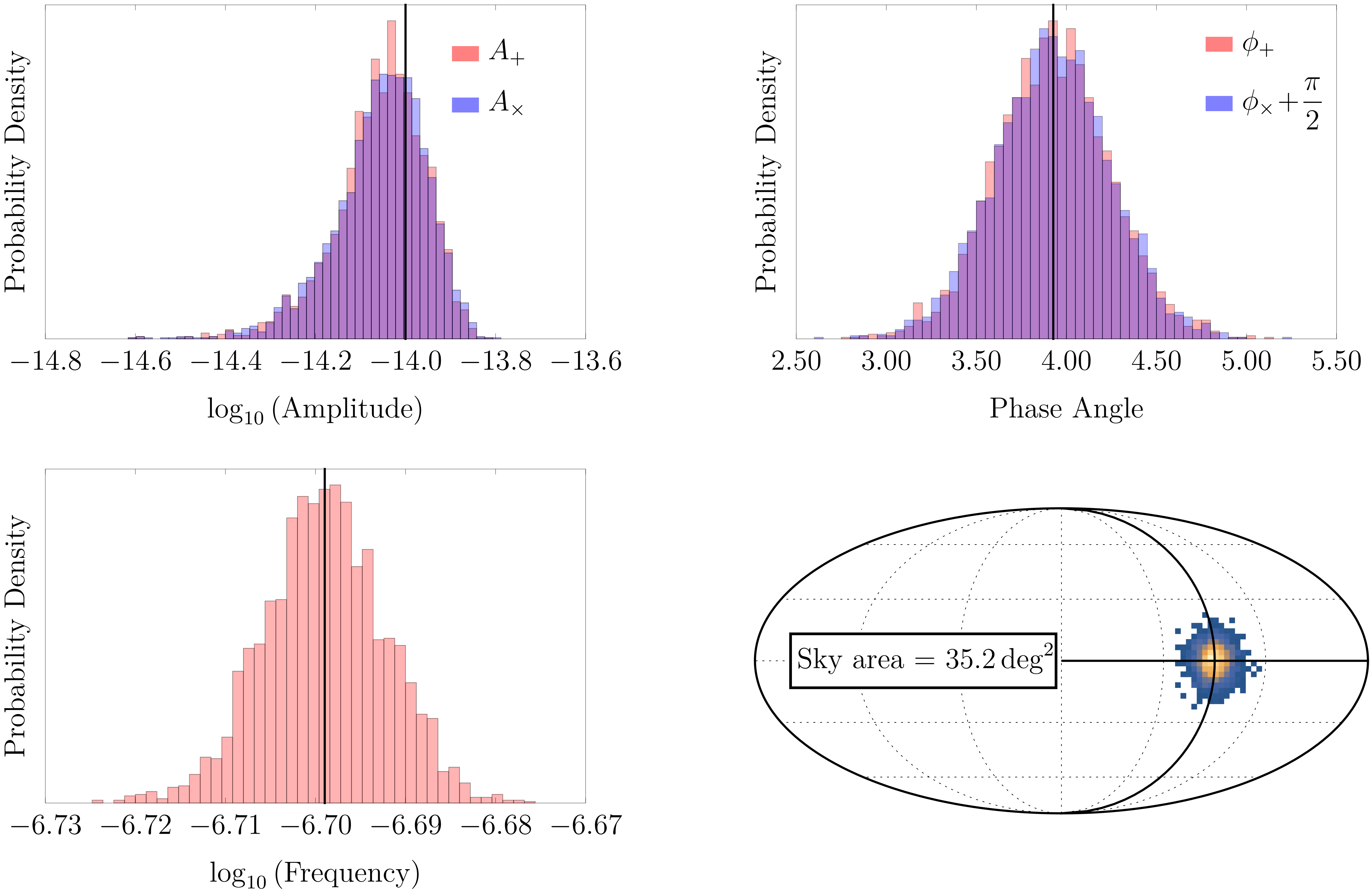
Computational pipeline

Simulated data

Noise + GW signal

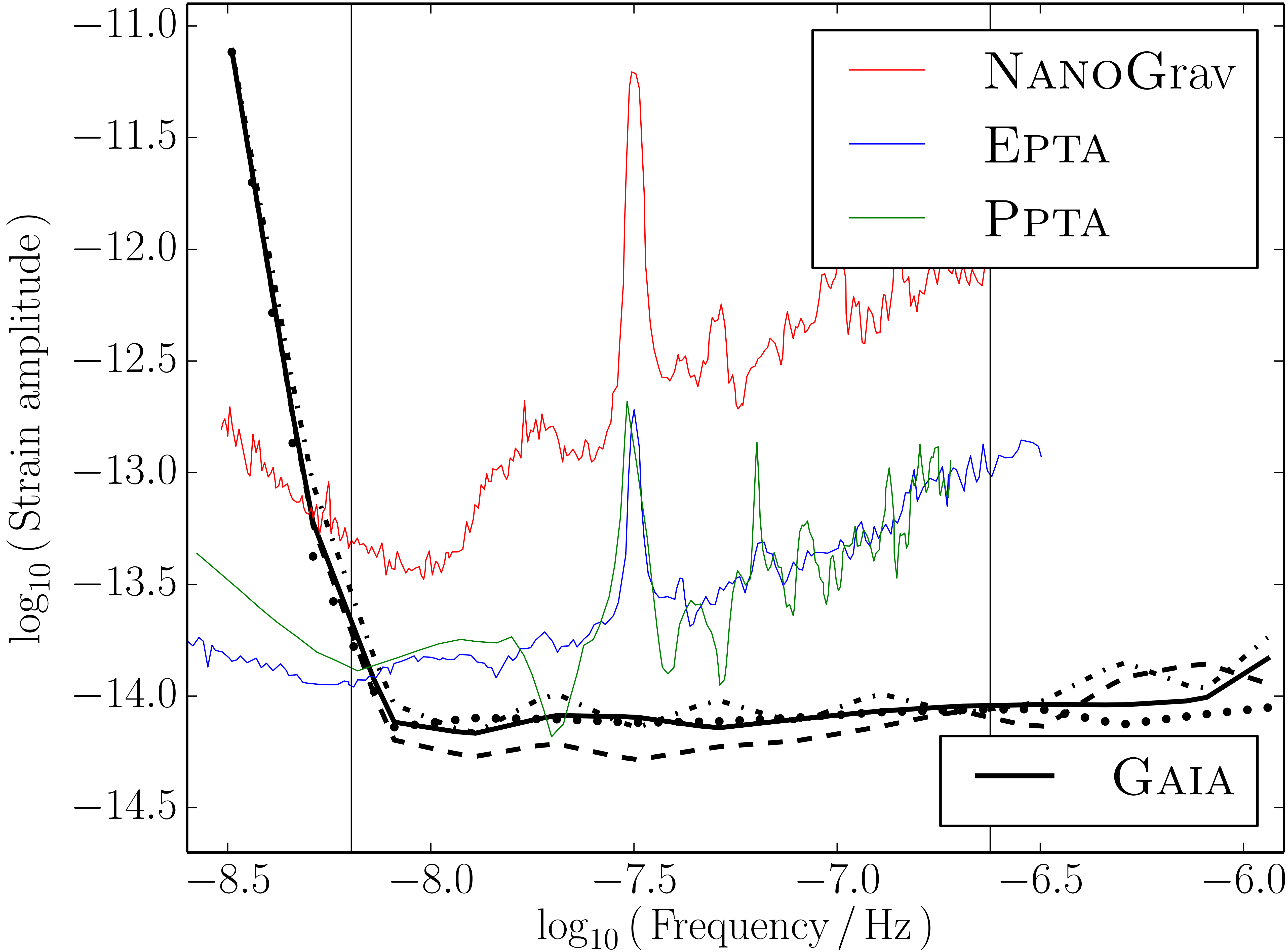
Pipeline for individual detections:

Bayesian inference on the parameter grid

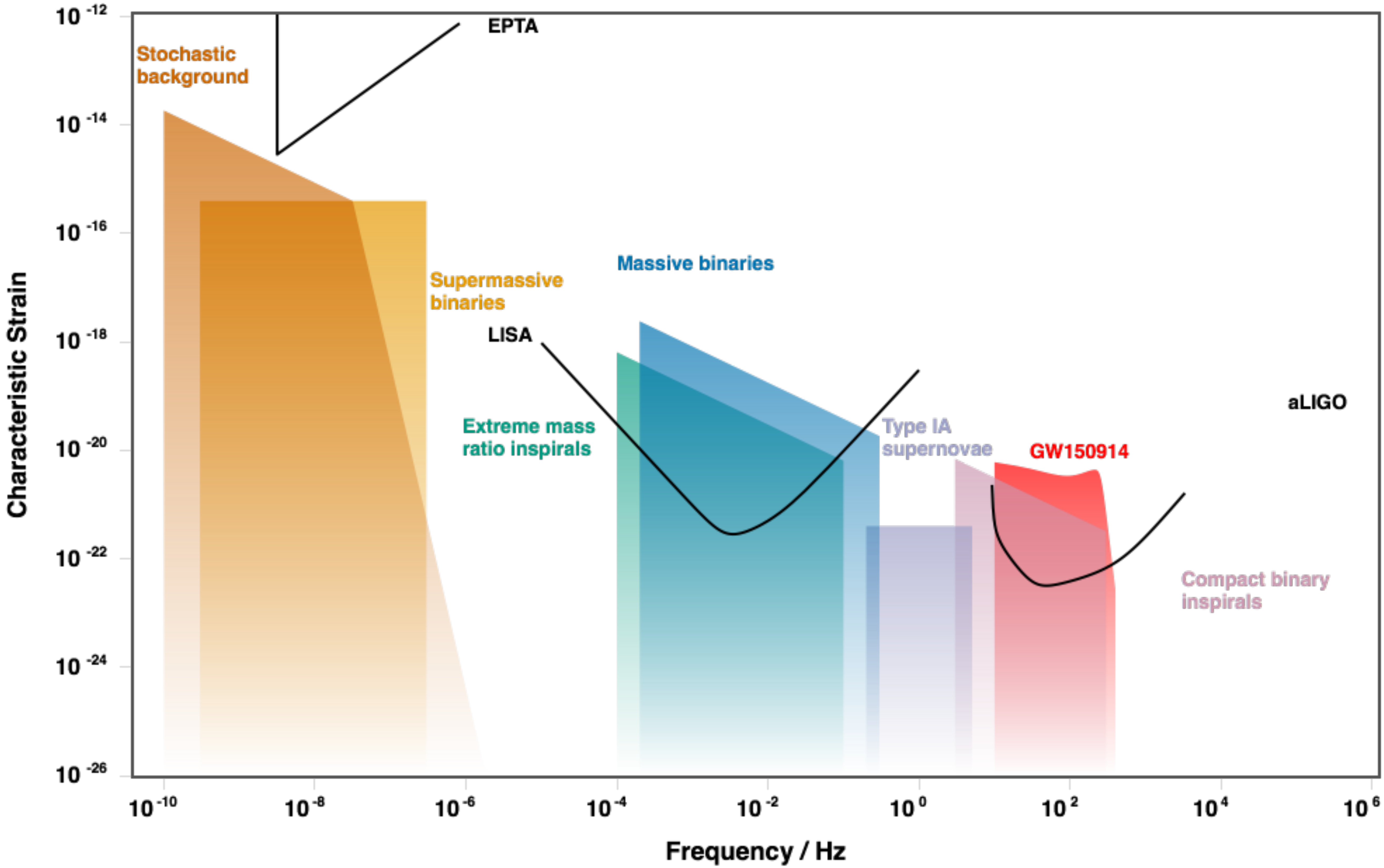


10^{10} SOLAR MASS BINARY 20 MPC AWAY

Frequency Sensitivity of Gaia



NANOGrav
EPTA
PPTA
GAIA
−11.0
−11.5
−12.0
−12.5
−13.0
−13.5
−14.0
−14.5
−8.5
−8.0
−7.5
−7.0
−6.5
−6.0
log10 (Strain amplitude)
log10 (Frequency / Hz)



Characteristic Strain
Frequency / Hz
10^{-12}
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10^{-24}
10^{-26}
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10^{-4}
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10^{4}
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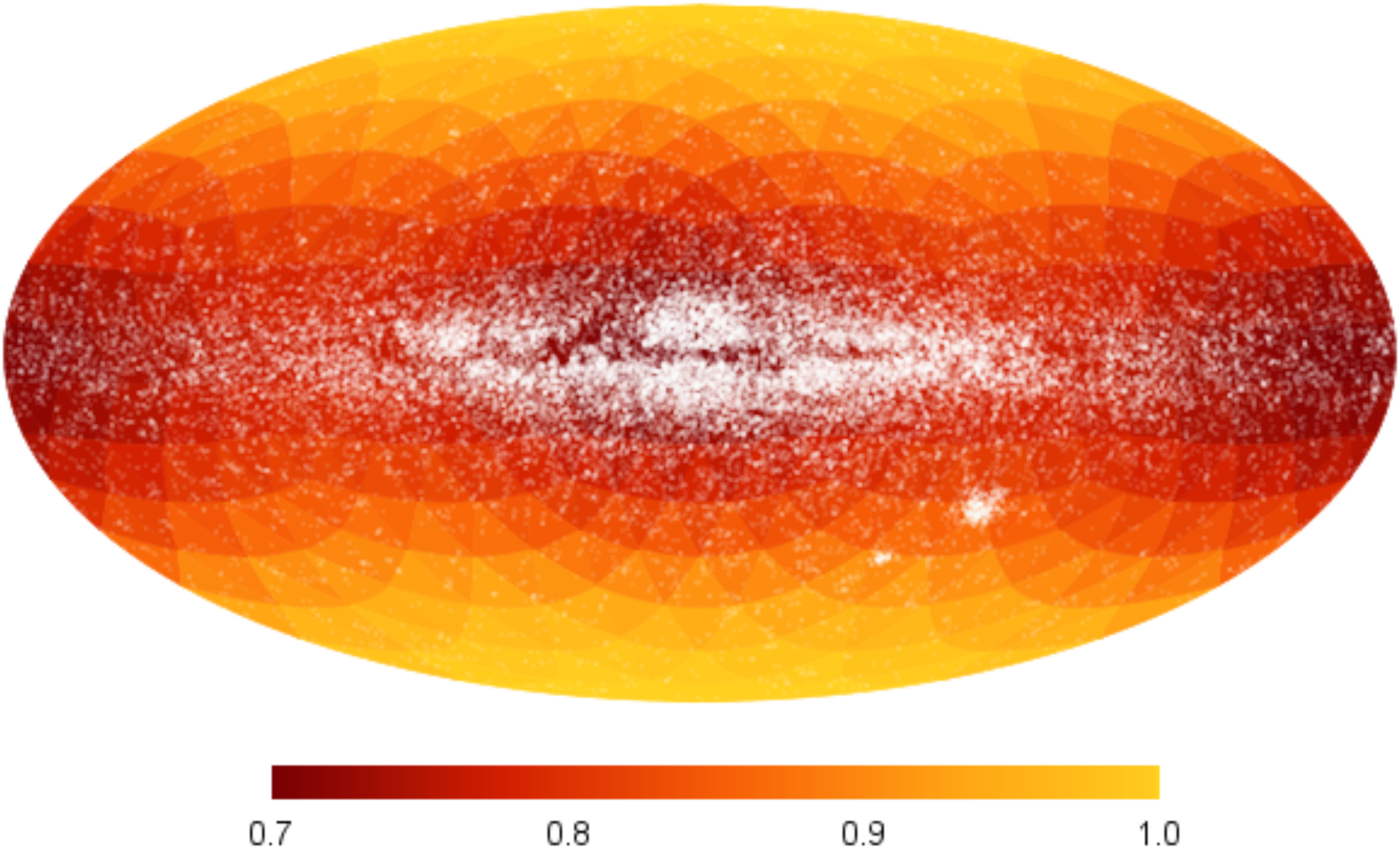
Directional sensitivity of Gaia



30% VARIATION ACROSS THE SKY

Correlations of a Stochastic Background

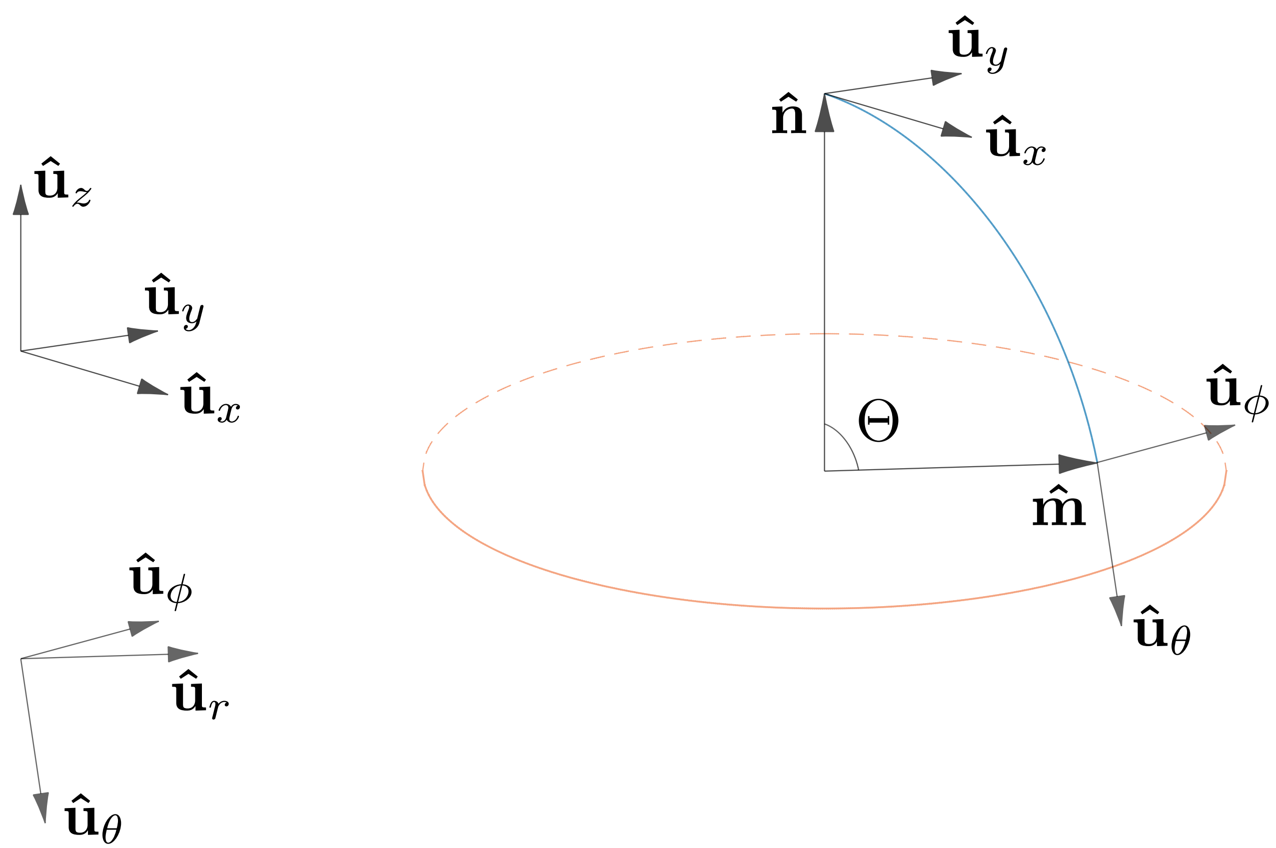
Stochastic GW background

$$\delta n_{\hat{\imath}} = \frac{n_{\hat{\imath}} - q_{\hat{\imath}}}{2(1 - \vec{q} \cdot \vec{n})} h_{\hat{\jmath}\hat{k}}(\mathrm{OBS}) n^{\hat{\jmath}} n^{\hat{k}} - \frac{1}{2} h_{\hat{\imath}\hat{\jmath}}(\mathrm{OBS}) n^{\hat{\jmath}} \,.$$

$$h_{ij}(t) = \Re \left\{ \sum_{P} \int_0^{\infty} \mathrm{d}f \int_{S^2} \mathrm{d}\Omega_{\mathbf{q}} \, A_P(\mathbf{q}, f) \, e^{-2\pi \mathrm{i} f t} \epsilon_{ij}^{P}(\mathbf{q}) \right\},$$

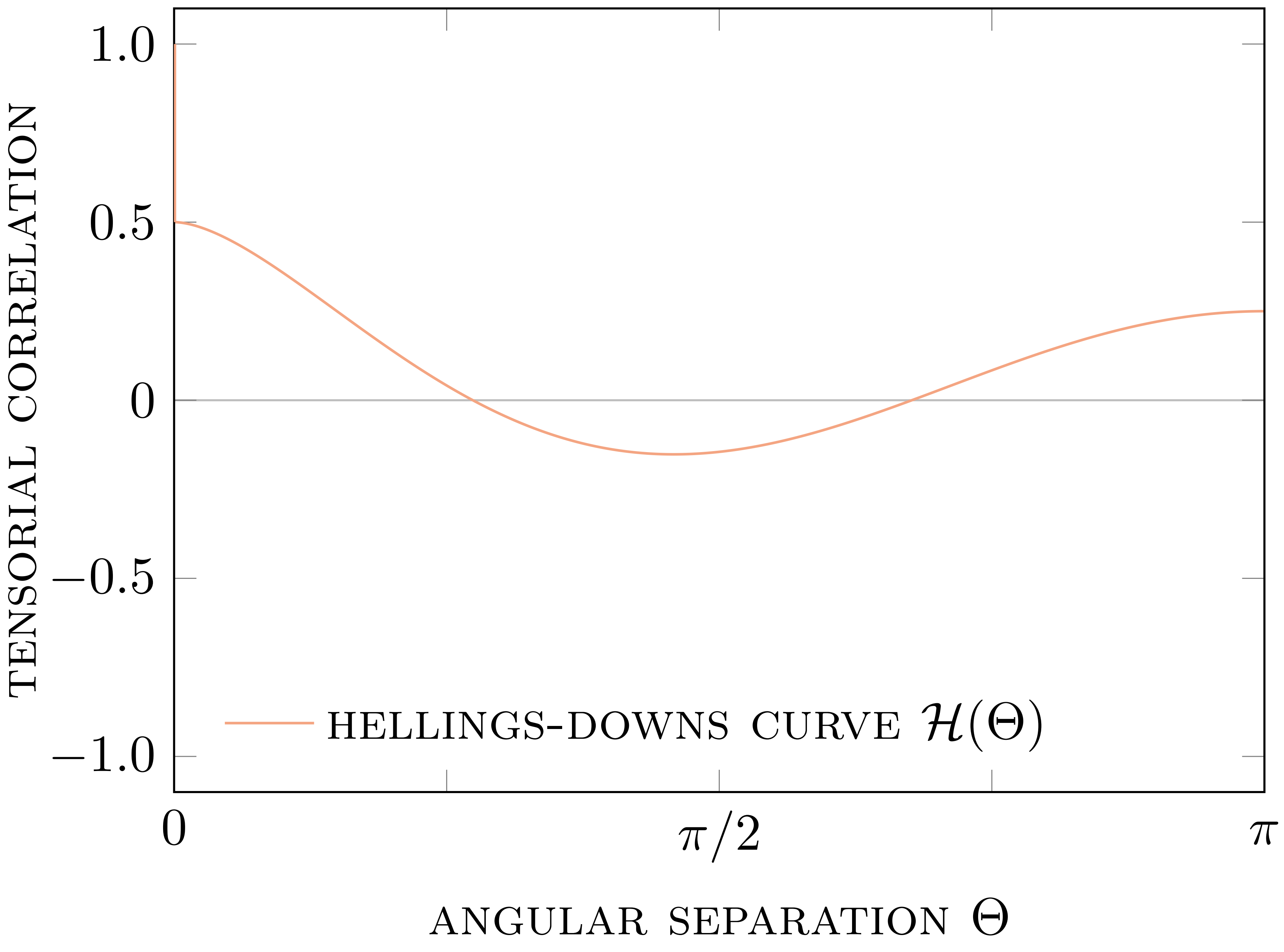
Investigate correlations of stars on the sky

$$\Gamma_{ij}^{P}(\Theta) \propto \int_{S^2} \mathrm{d}\Omega_{\mathbf{q}}\, \delta n_i(n_k, t)\, \delta m_j(m_\ell, t),$$



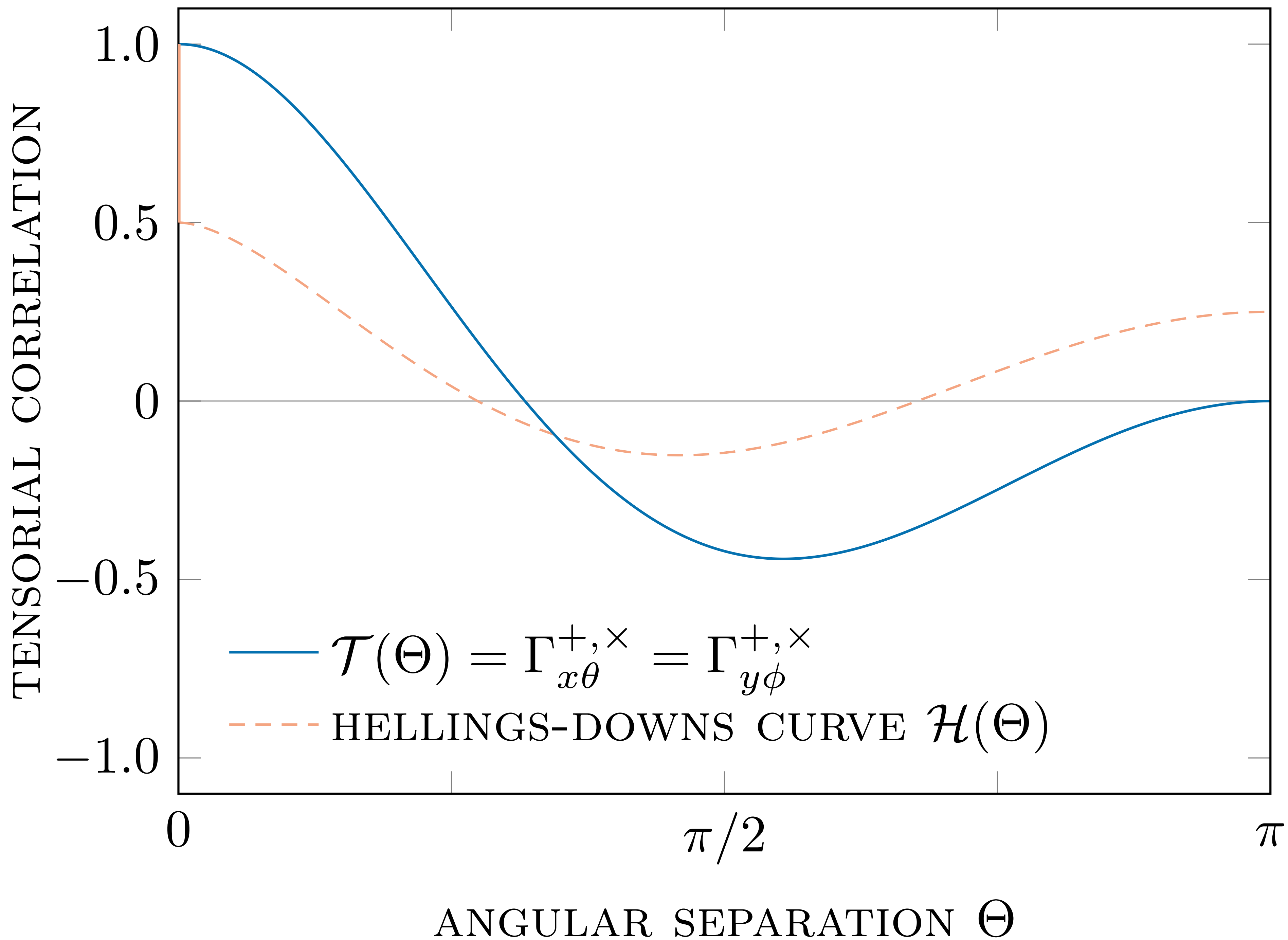
$$C = \left(\begin{array}{c|c} \Gamma_{x\theta} & \Gamma_{x\phi} \\ \hline \Gamma_{y\theta} & \Gamma_{y\phi} \end{array}\right)$$

GR modes



Cf. Book and Flanagan, 2001

GR modes



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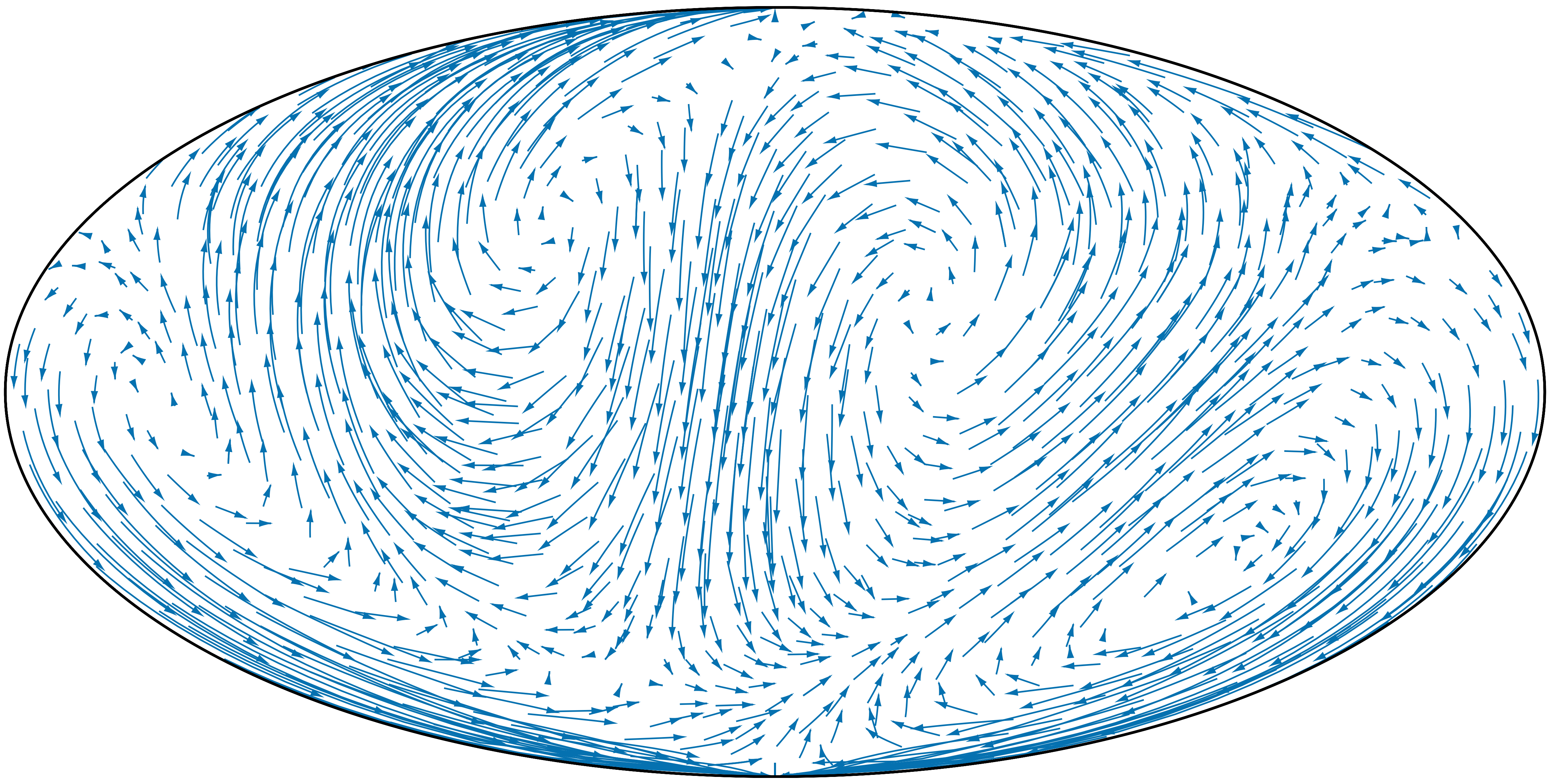
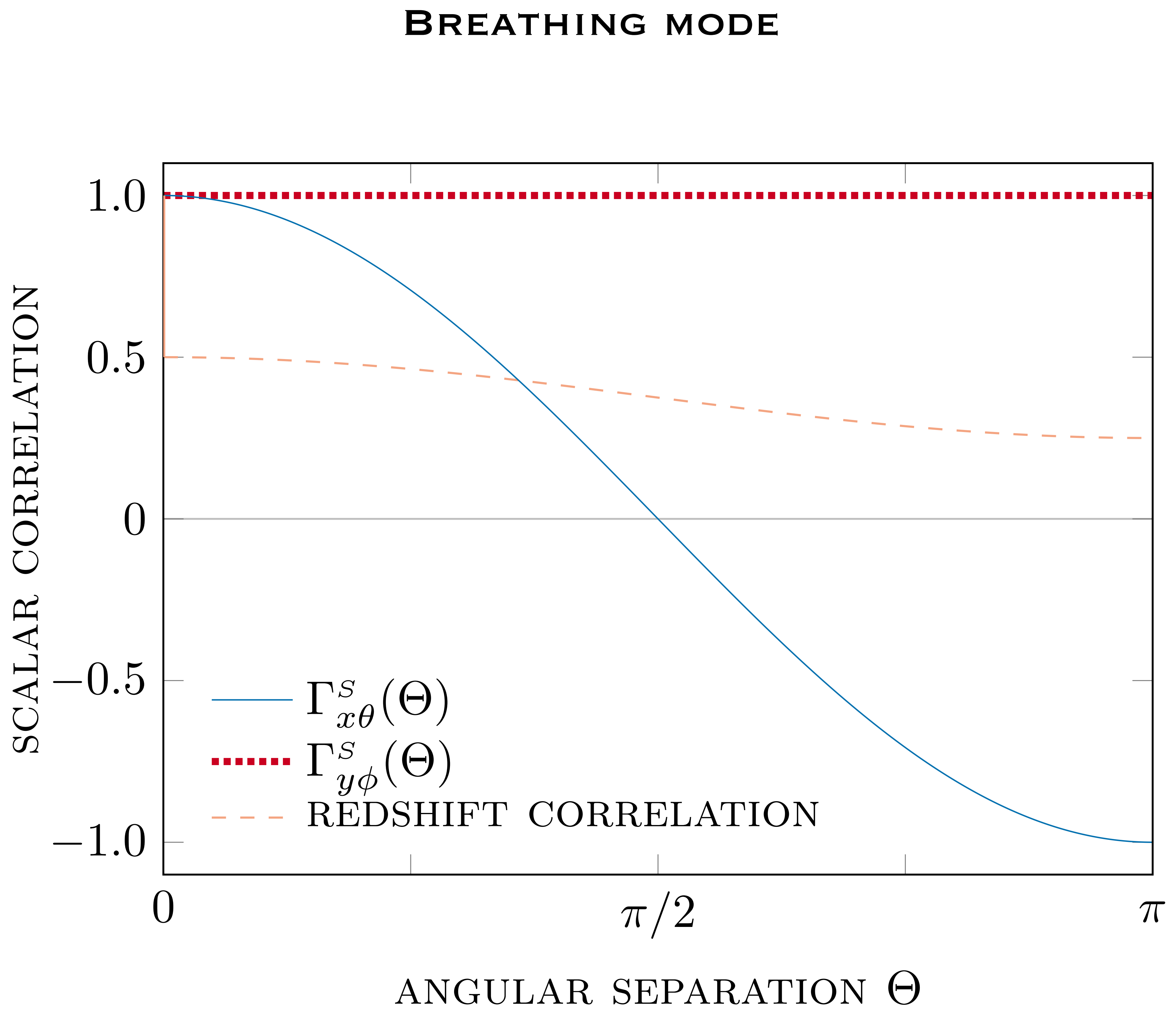
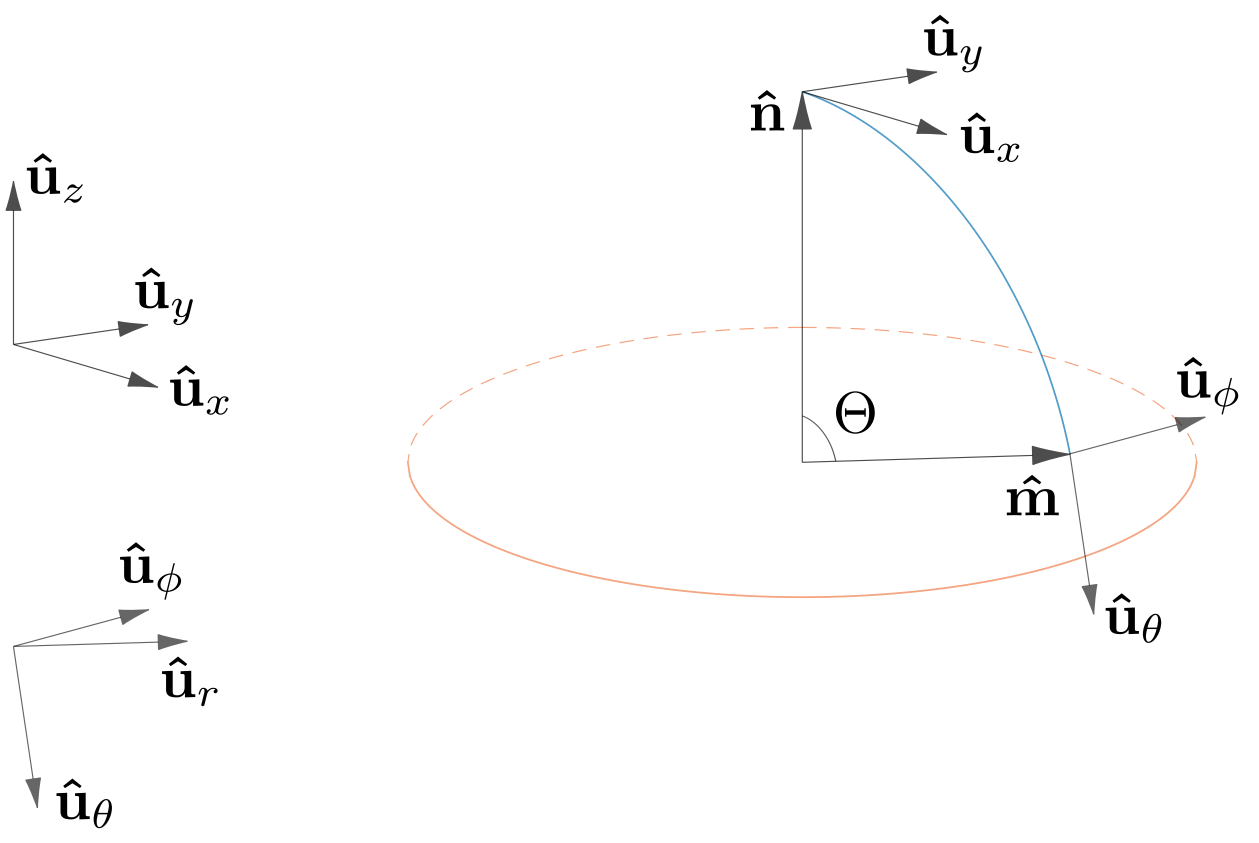
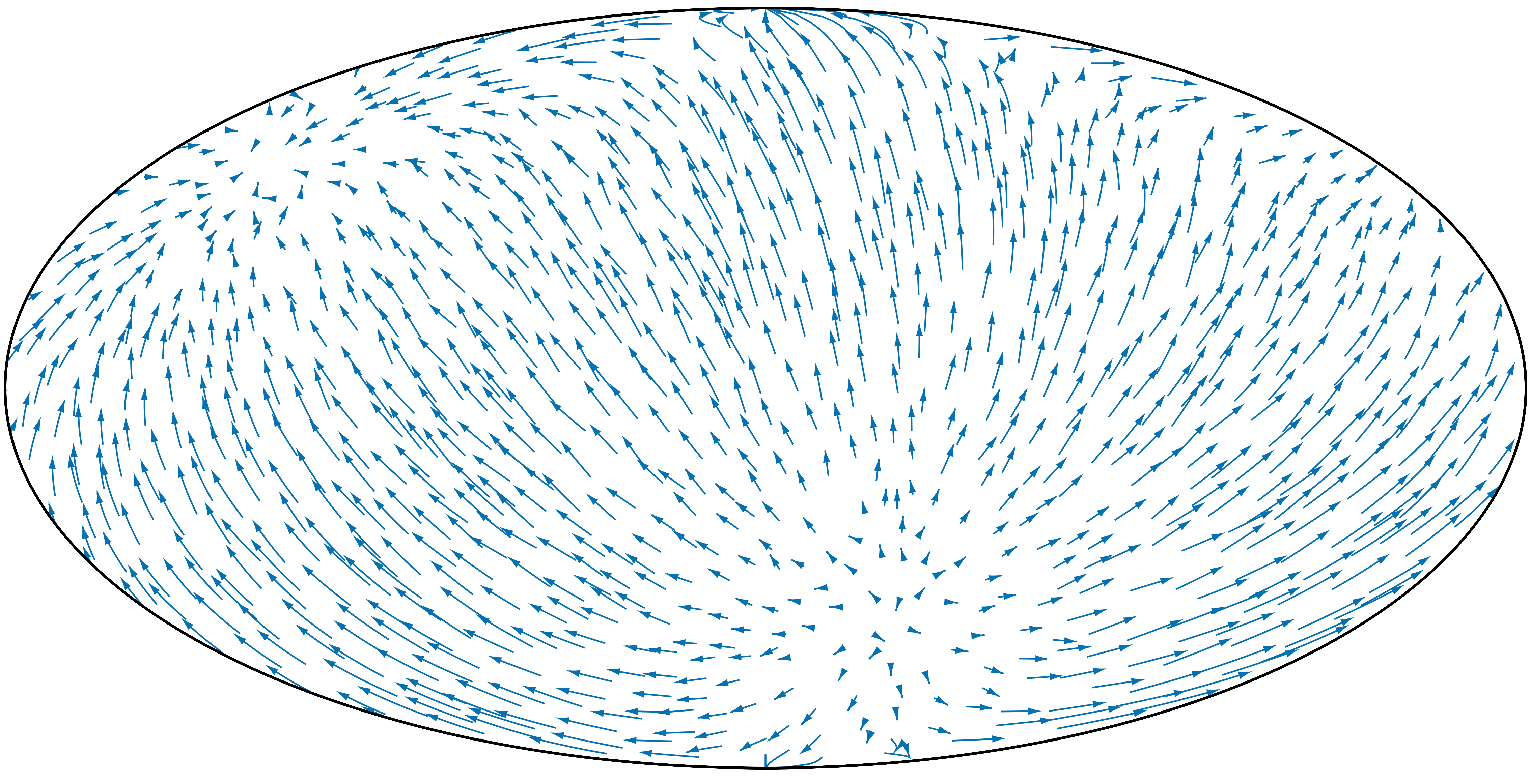


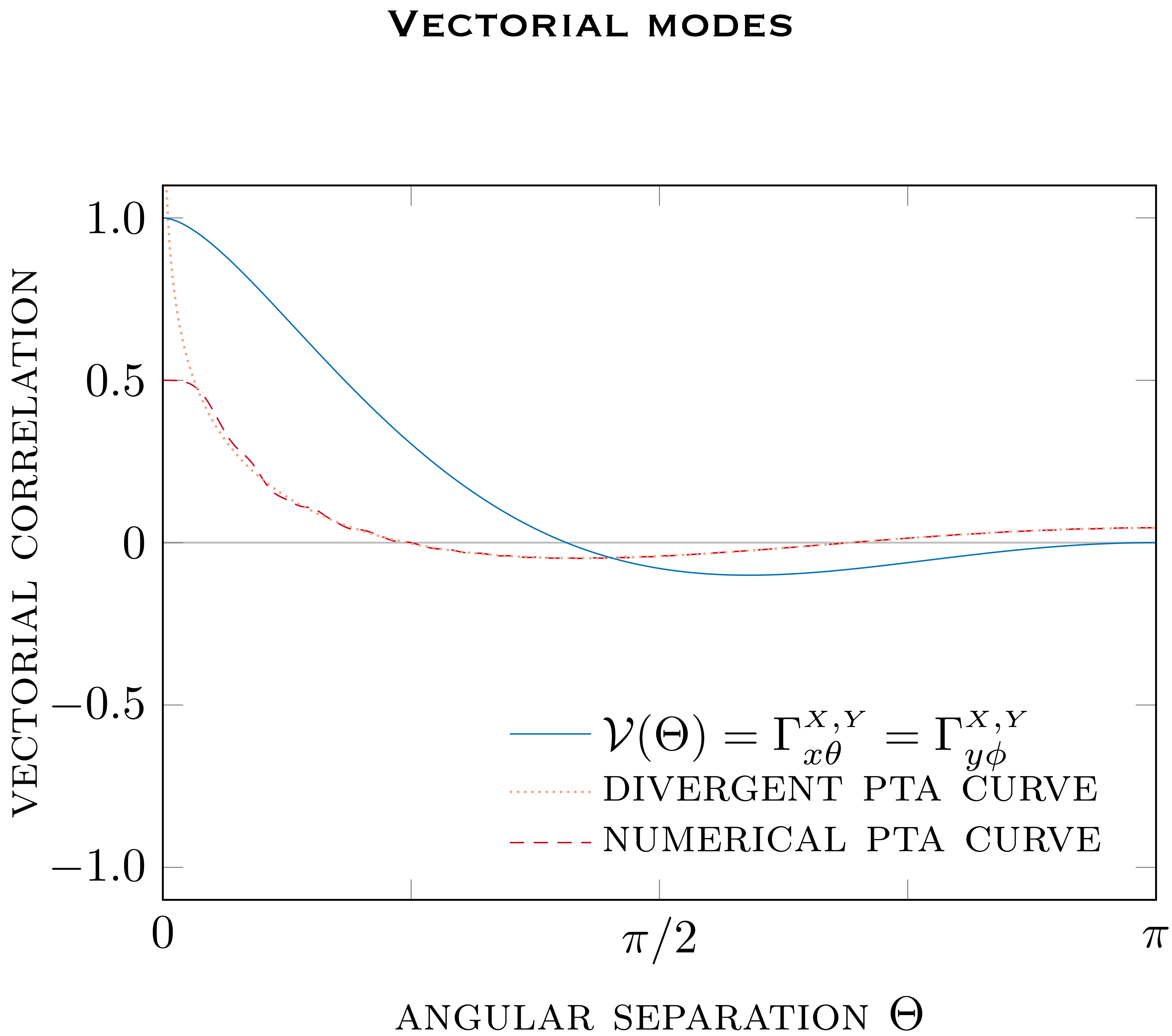

Breathing mode
1.0
0.5
0
−0.5
−1.0
0
π/2
π
scalar correlation
angular separation Θ
$\Gamma^S_{x\theta}(\Theta)$
$\Gamma^S_{y\phi}(\Theta)$
redshift correlation

Breathing mode

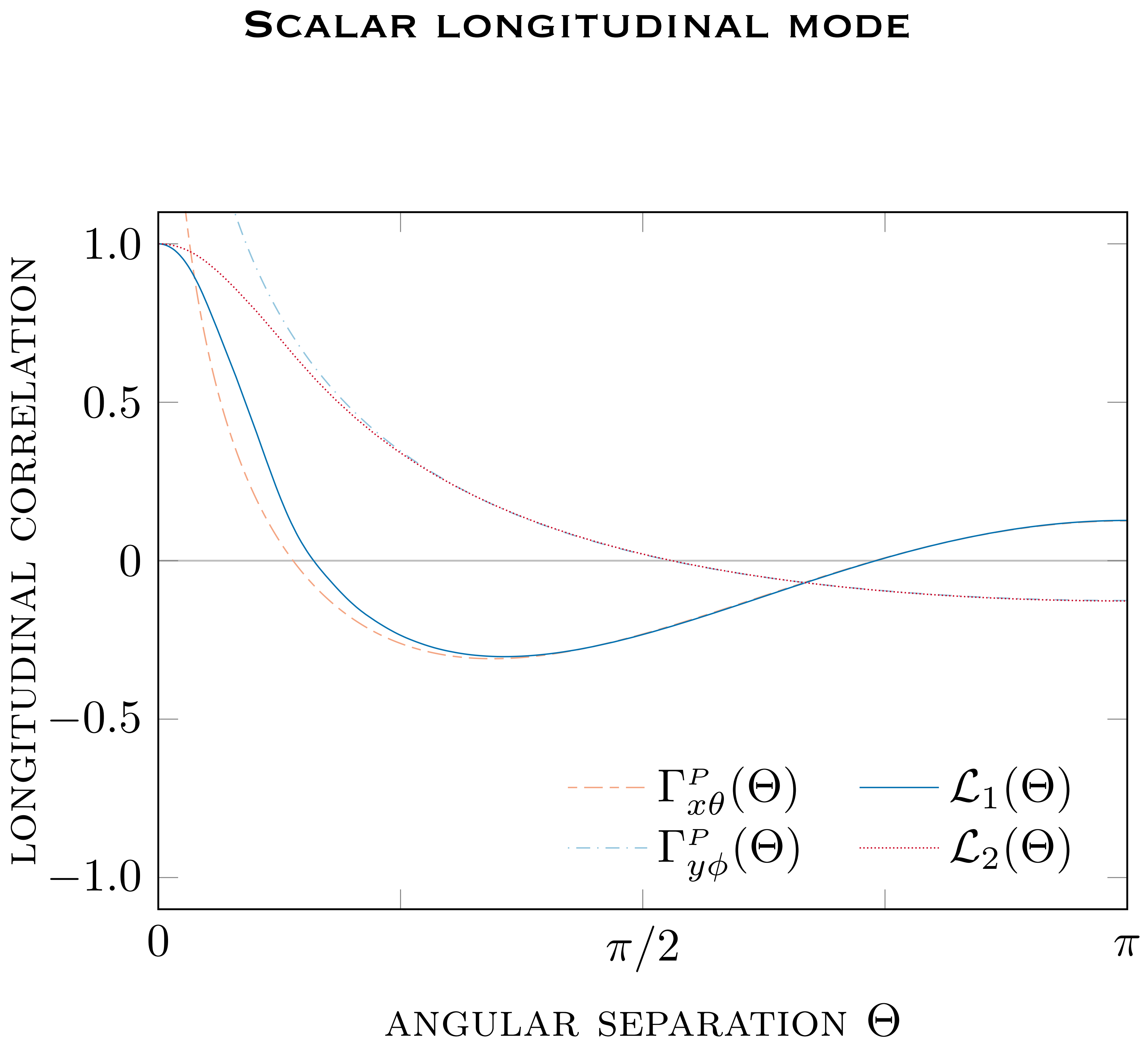


Breathing mode



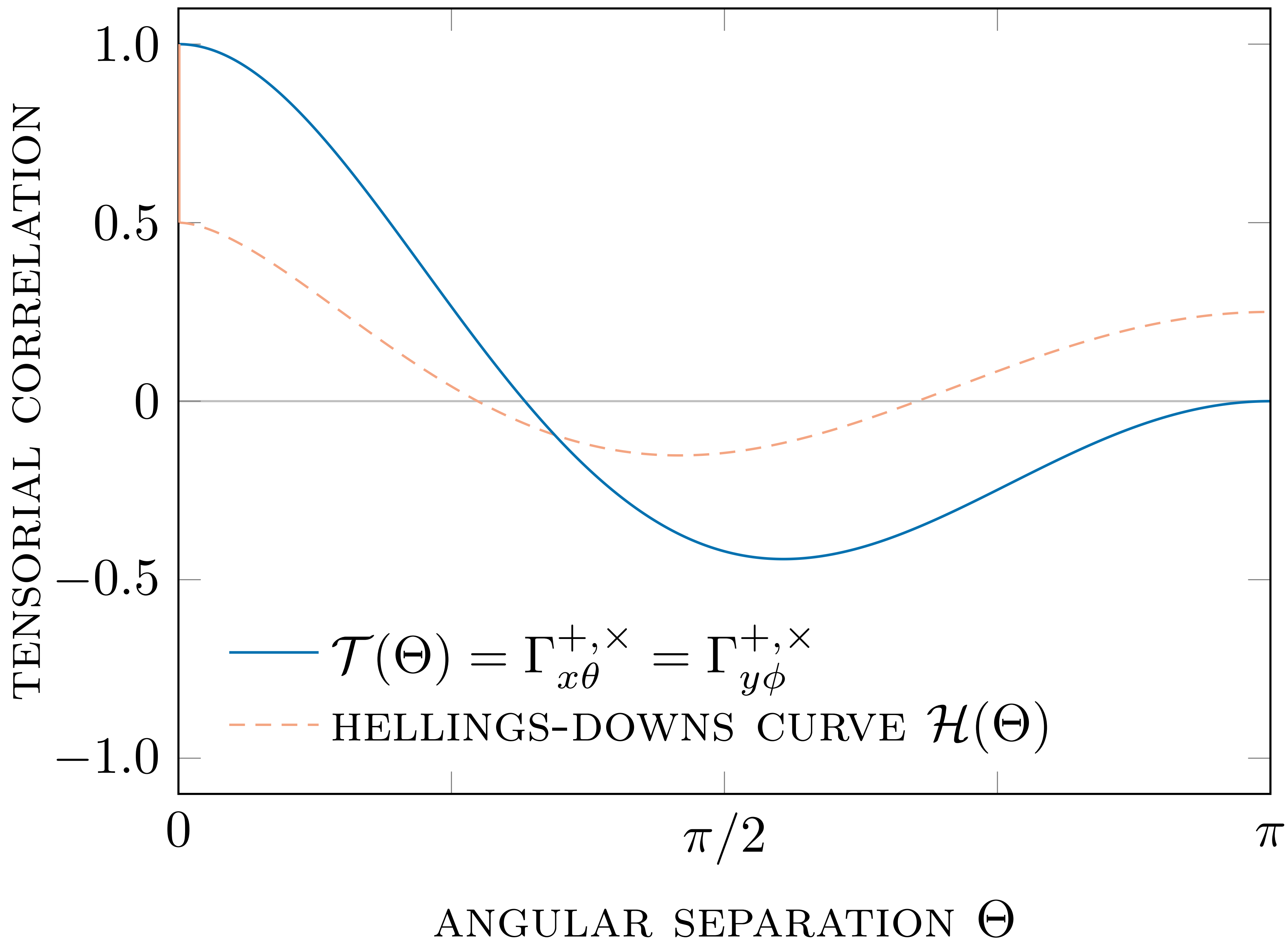


Vectorial modes
vectorial correlation
1.0
0.5
0
−0.5
−1.0
0
π/2
π
angular separation Θ
$\mathcal{V}(\Theta) = \Gamma^{X,Y}_{x\theta} = \Gamma^{X,Y}_{y\phi}$
divergent PTA curve
numerical PTA curve


Scalar longitudinal mode
longitudinal correlation
angular separation Θ
1.0
0.5
0
−0.5
−1.0
0
π/2
π
$\Gamma^P_{x\theta}(\Theta)$
$\Gamma^P_{y\phi}(\Theta)$
$\mathcal{L}_1(\Theta)$
$\mathcal{L}_2(\Theta)$

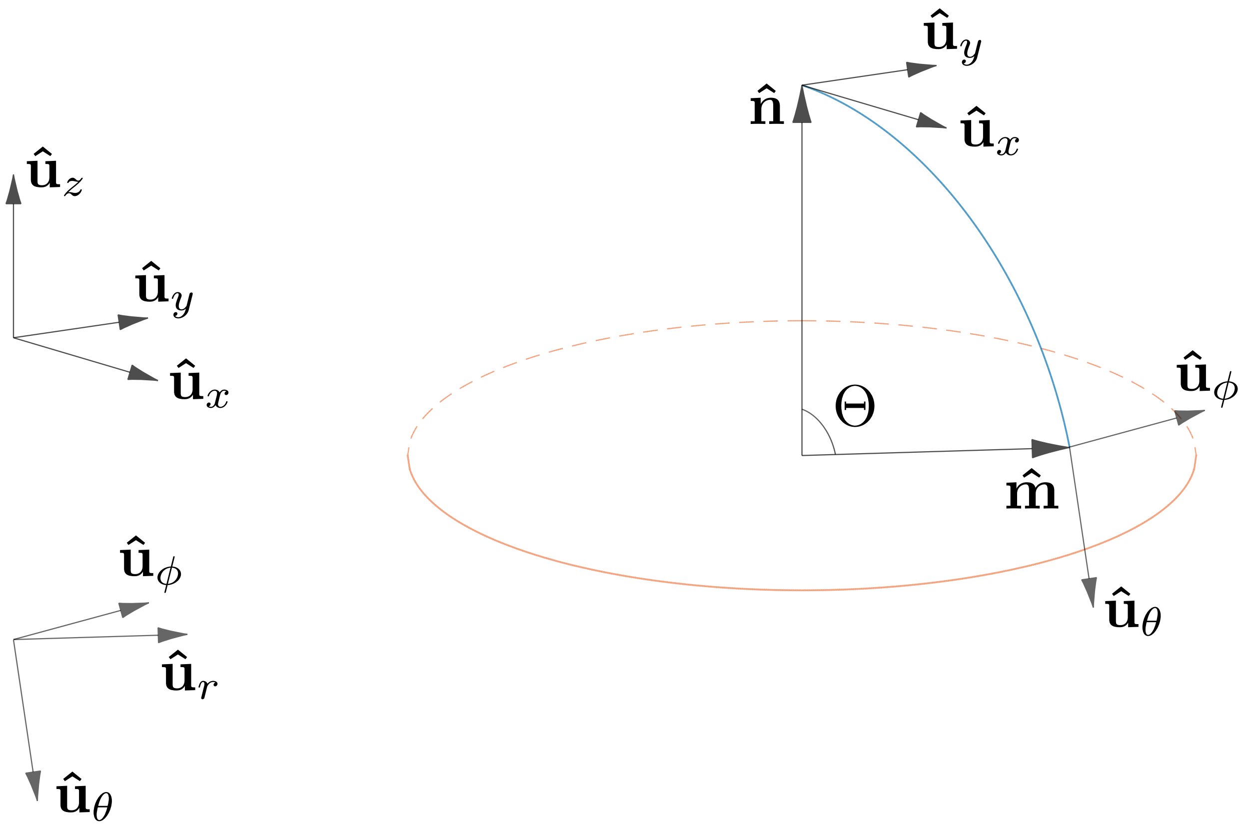
Redshift-astrometry Correction

GR modes

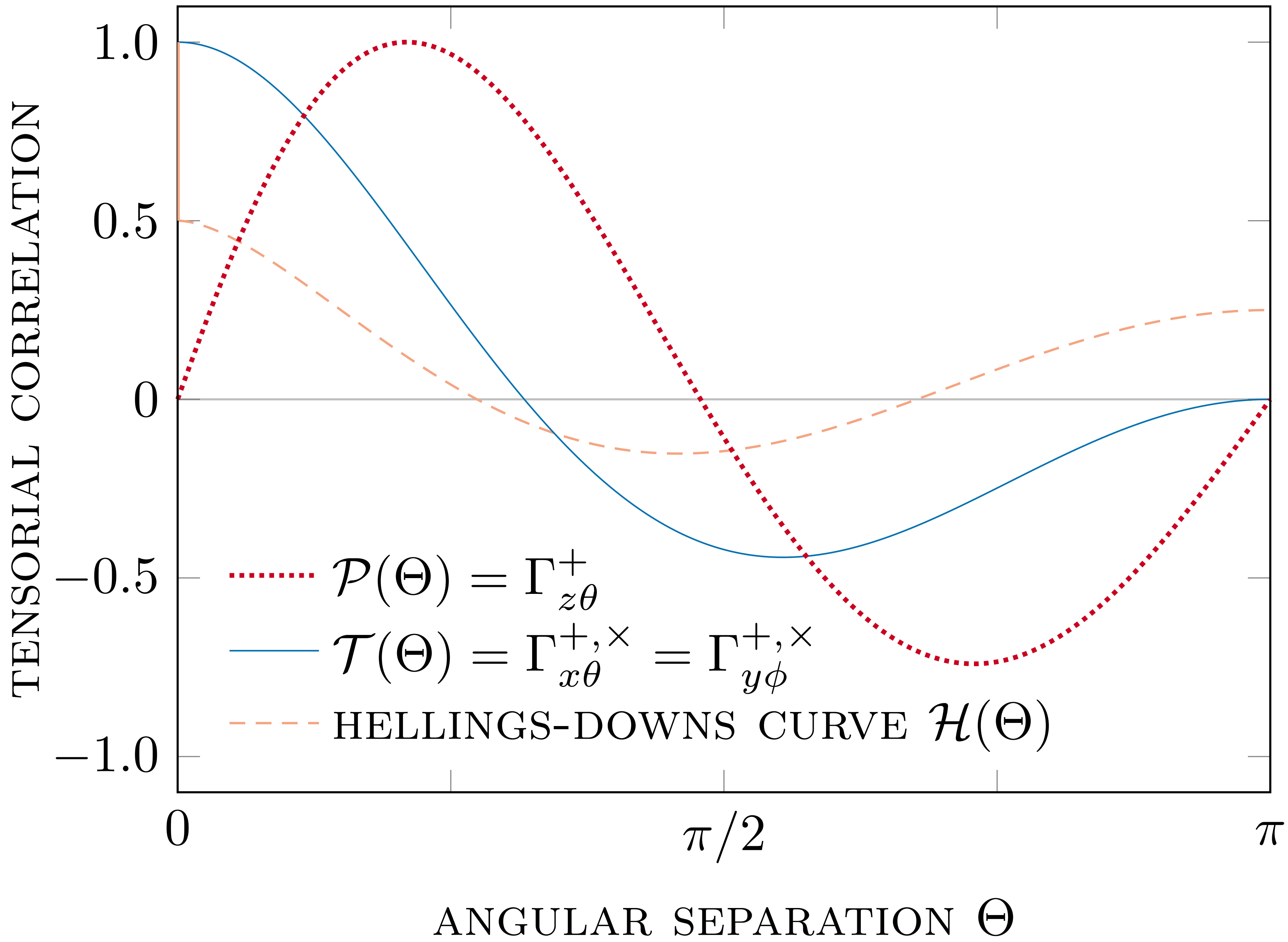


Cf. Book and Flanagan, 2001

Breathing mode

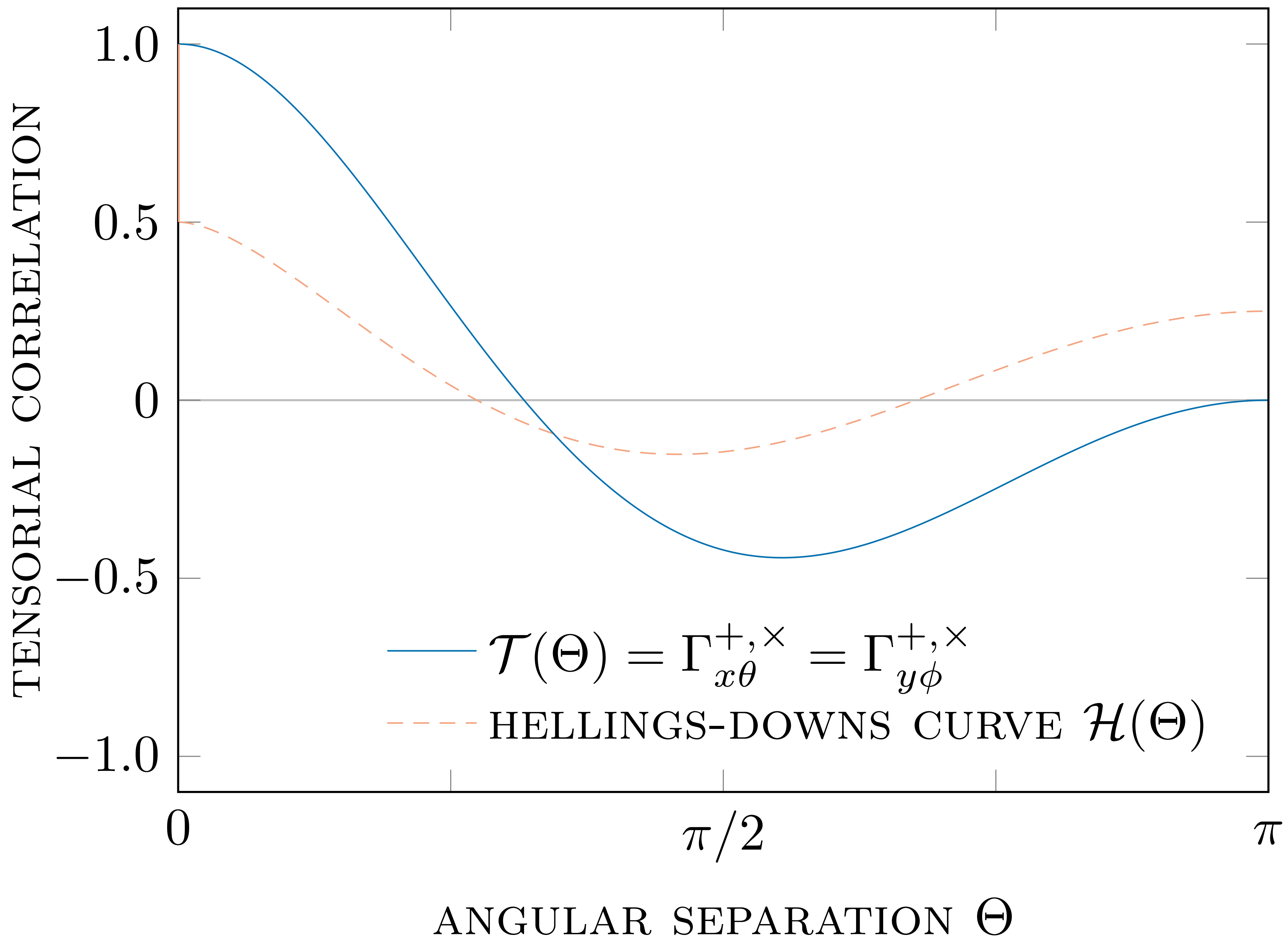


GR modes


1.0
0.5
0
−0.5
−1.0
tensorial correlation
0
π/2
π
angular separation Θ
$\mathcal{P}(\Theta) = \Gamma^{+}_{z\theta}$
$\mathcal{T}(\Theta) = \Gamma^{+,\times}_{x\theta} = \Gamma^{+,\times}_{y\phi}$
hellings-downs curve $\mathcal{H}(\Theta)$

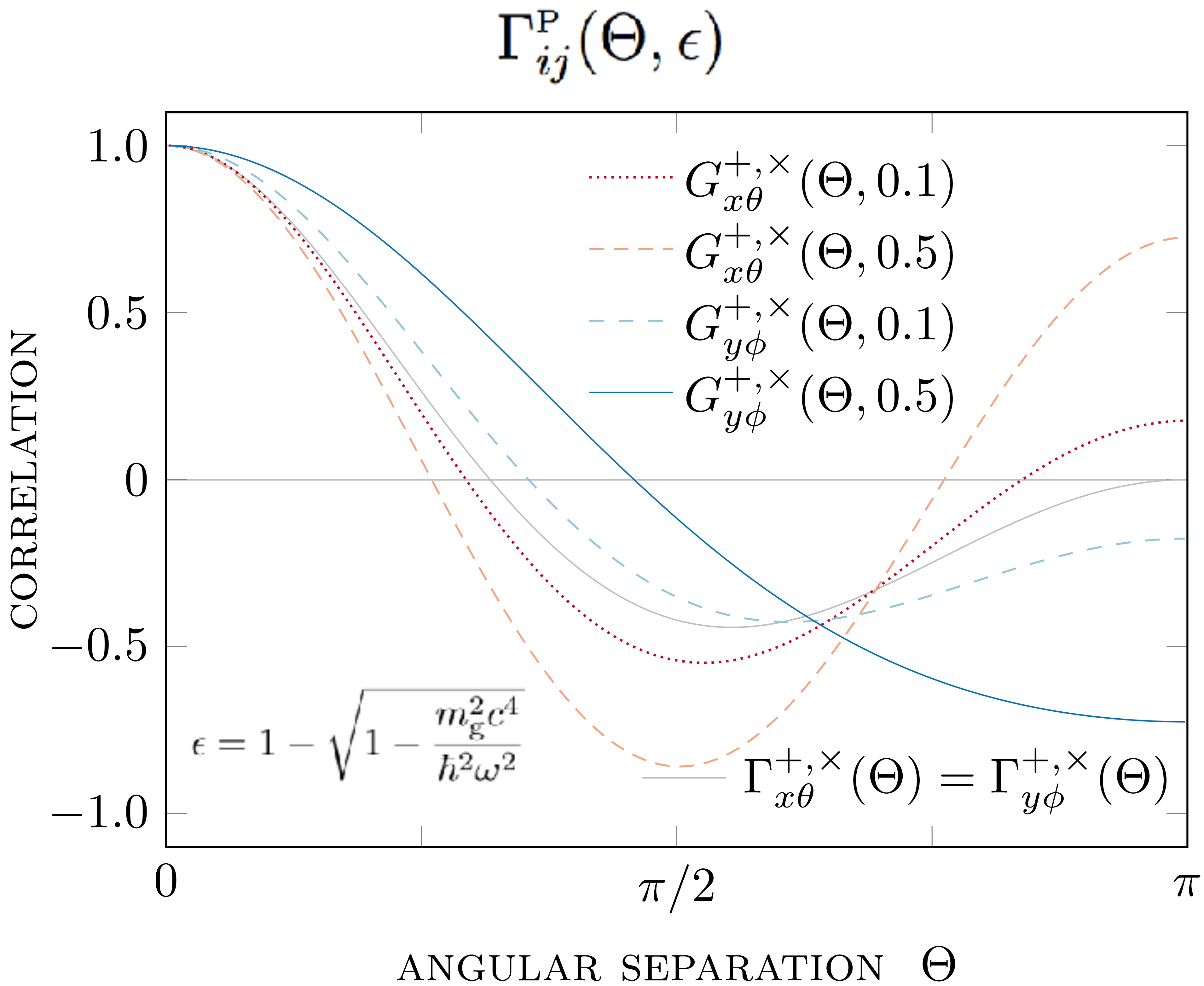
Massive Graviton Correction

GR modes



Cf. Book and Flanagan, 2001

GR modes with massive graviton corrections



Further work

Test incoming Gaia datasets - DR4 will be testable

Develop a numerical analysis for background correlations

Investigate anisotropic backgrounds — theory needed

Investigate current datasets for traces of GWs

Future astrometric mission requirements

Conclusions

1. GWs induce periodic perturbations in the astrometric measurements of stars

2. Gaia is the ideal tool to study this effect

3. We have developed a data analysis pipeline

5. Data can be compresses with little loss

6. Further data releases will allow GW searches to be performed.

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