Confinement

and gravitational waves

Djuna Lize Croon (TRIUMF)
Nordita, September 2019
dcroon@triumf.ca | djunacroon.com

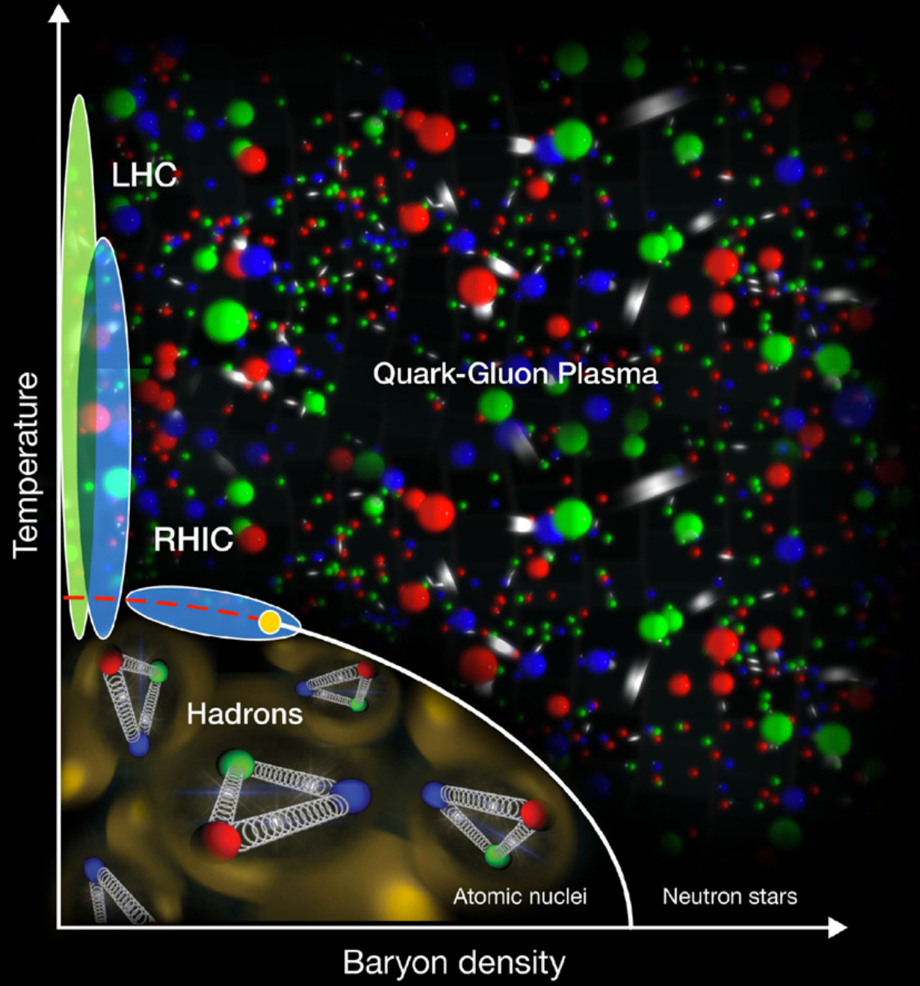
Based on
DC, Houtz, Sanz [JHEP, arXiv:1904.10967]
DC, Howard, Ipek, Tait, [arXiv:19XX.XXXX]
DC, Dror, Muruyama, White, [arXiv:19XX.XXXX]

Confinement in the Standard Model

- QCD confines when $\alpha_s > 4\pi$

- Confinement scale (MS-bar scheme): $\Lambda_{\mathrm{QCD}} \sim 300\ \mathrm{MeV}$
- At $300\ \mathrm{MeV}$, (2+1) dynamical flavors in the SM

- Transition is crossover → no GW (or other) signature



Motivations for QCD’ confinement

Either a *new strong sector*, or *modified QCD confinement*

- Strong CP problem (TeV axions)

e.g. S. Dimopoulos, A. Hook, J. Huang, G. Marques-Tavares, [JHEP, arXiv:1606.03097]
M. K. Gaillard, M. B. Gavela, R. Houtz, P. Quilez and R. Del Rey [EPJ, arXiv: 1805.06465]
P. Agrawal and K. Howe, [JHEP, arXiv:1712.05803]

- Baryogenesis

e.g. Ipek, Tait, PRL (2019)
Servant, PRL (2014)

- Axion relic abundance

e.g. Barr and Kyae, PRD (2005)

- PBH production

e.g. Jedamzik, PRD (1996)
Davoudiasl, PRL (2019)

- Dynamical generation of scales

e.g. Technicolor, Composite Higgs models
Many papers, typically ~TeV scale strong sector

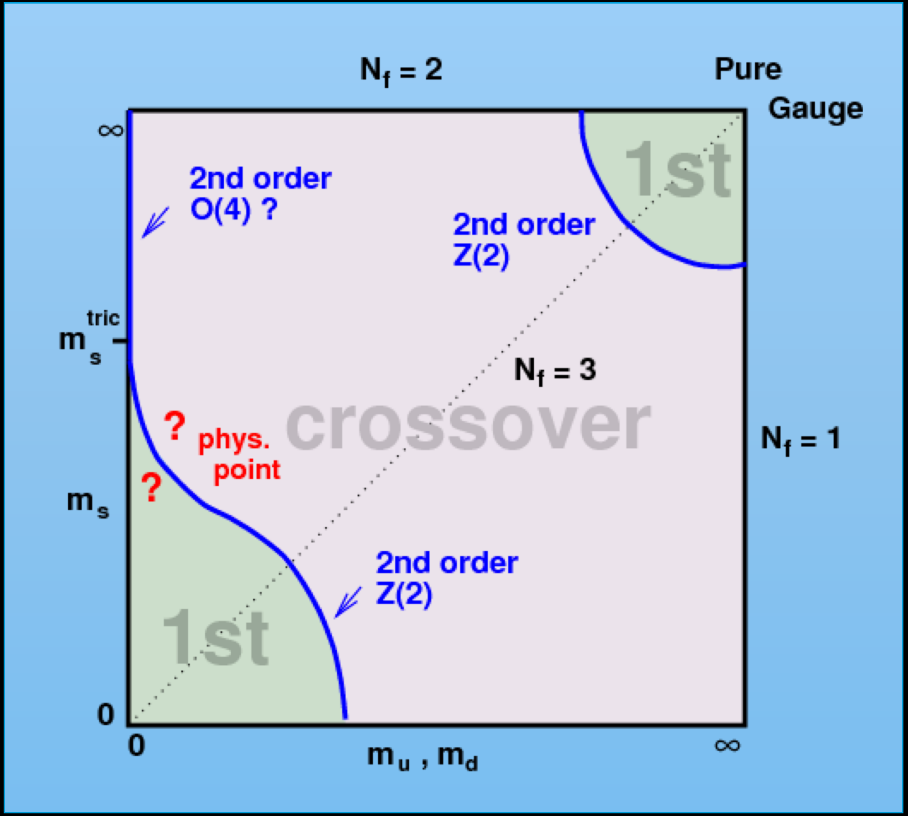
Chiral symmetry breaking (“the χPT –PT”)

- Confinement implies chiral symmetry breaking (N_f dynamical fermions):

$$SU(N_f)\times SU(N_f)\rightarrow SU(N_f)$$

- Analytic argument (based on the linear Σ-model) suggests the chiral PT is first order for
  - $N_f \geq 3$
  - $N_f = 0$ (pure gauge)

Pisarski, Wilczek, PRD (1984)



Studying the chiral phase transition

- Very quickly run into general issues with the calculability of a strongly coupled theory

- Proposed methods:
    - Linear Σ-model
    - Nambu-Jona-Lasinio (NJL or PNJL) models
    - Lattice simulations
    - MIT bag model

… each with strengths and weaknesses

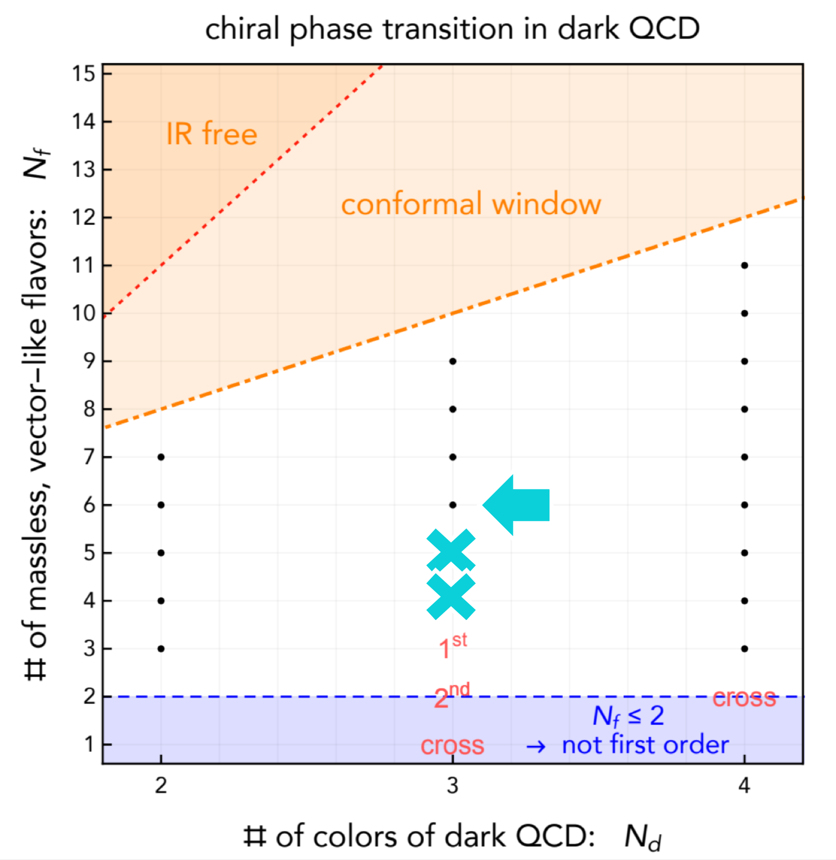


Image: Long, Bai, Lu, arXiv:1810.04360

DC, R. Houtz, and V. Sanz
[arXiv:1904.10967]

Chiral symmetry breaking: linear Σ-model

- Low energy effective theory $(\Sigma_{ij} \sim \langle \bar{\psi}_{Rj} \psi_{Li} \rangle)$

$$V(\Sigma) = -m_\Sigma^2 \;\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) - \left(\mu_\Sigma \;\det\Sigma + h.c.\right) + \frac{\lambda}{2}\left[\;\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\;\mathrm{Tr}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right)$$

- Note that if $\mu_\Sigma = 0$, there is an enhanced $\mathrm{SU(N_f)\times SU(N_f)\times U(1)_A}$ global flavor symmetry

- The μ_Σ terms are generated by instantons, which anomalously break the $\mathrm{U(1)_A}$ subgroup

't Hooft, PRD (1976)

DC, R. Houtz, and V. Sanz
[arXiv:1904.10967]

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- Decompose in terms of scalar mesons

$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}}\delta_{ij} + X^a T^a_{ij} + i\pi^a T^a_{ij}$$

Order parameter

Dynamical axion

η' is the pGB of $\mathrm{U}(1)_\mathrm{A}$
Anomalously coupled to $G\tilde{G}$
Gets a mass from the instantons of $\mathrm{SU}(\mathrm{N_C})$
('t Hooft 1976)

- One-loop (thermal) contributions from all mesons

DC, R. Houtz, and V. Sanz
[arXiv:1904.10967]
See also Long, Bai, Lu,
[arXiv:1810.04360]

The thermal linear sigma model

- One-loop thermal potential for the diagonal field φ calculated in the usual way
- m_i runs over meson masses; for example for $\mathrm{N_f} = 3$:

$$\begin{cases} V(\Sigma, T) = V(\Sigma) + V_\chi(\Sigma) + V_{T\neq 0}, \\ V_{T\neq 0} = \sum_i \frac{T^4}{2\pi^2} n_i J_B\left(\frac{m_i^2 + \Pi_i}{T^2}\right), \\ J_B(m^2) = \int_0^\infty dx\, x^2 \log\left(1 - e^{-\sqrt{x^2+m^2}}\right) \end{cases}$$

$$m_\varphi^2 + \Pi_\varphi = \frac{1}{6}\left(\varphi\left(3\kappa\varphi + 9\lambda\varphi - 2\sqrt{6}\mu_\Sigma\right) - 6m_\Sigma^2 + T^2(3\kappa + 5\lambda)\right)$$

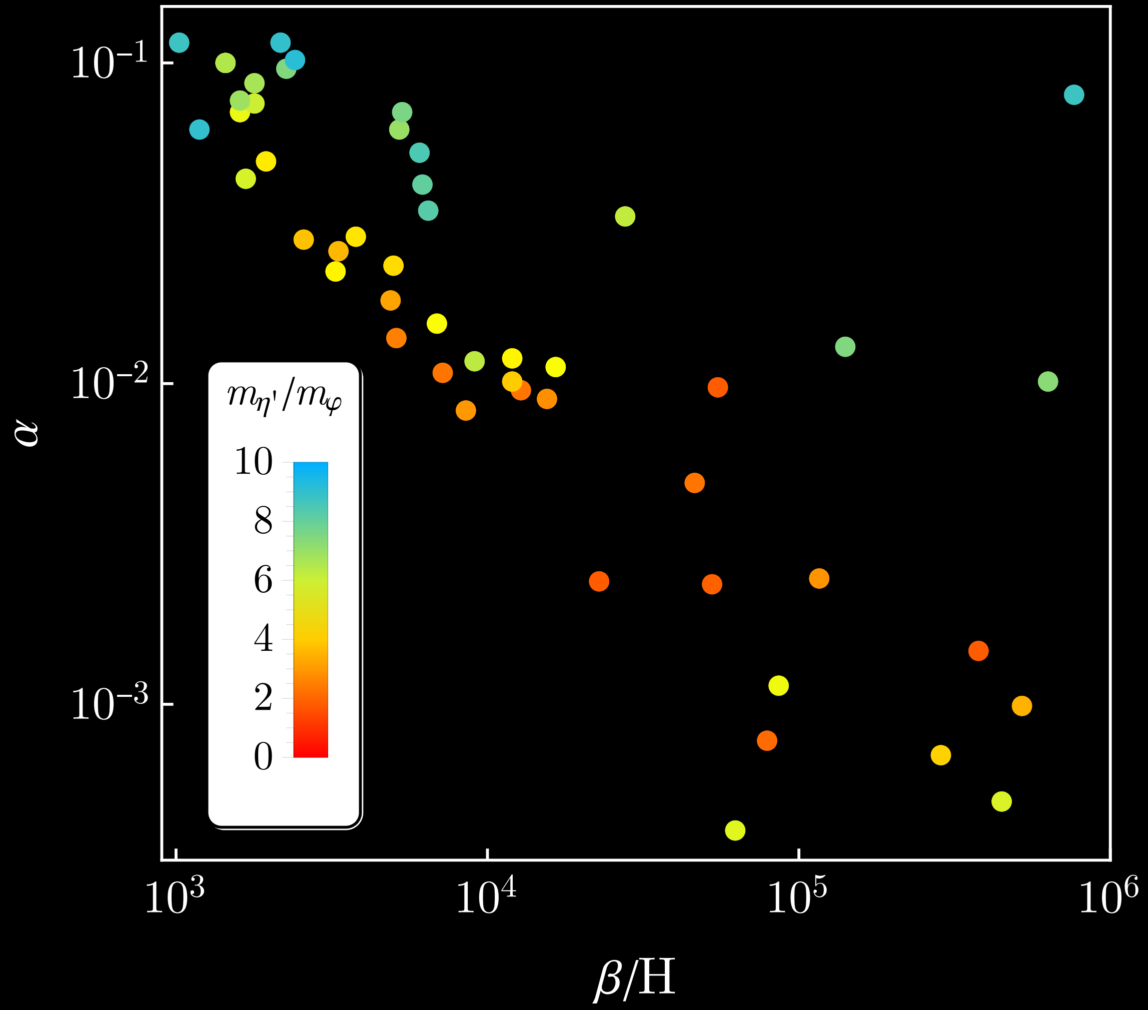
$$m_{\eta'}^2 + \Pi_{\eta'} = \frac{1}{6}\left(\varphi\left(\kappa\varphi + 3\lambda\varphi + 2\sqrt{6}\mu_\Sigma\right) - 6m_\Sigma^2 + T^2(3\kappa + 5\lambda)\right)$$

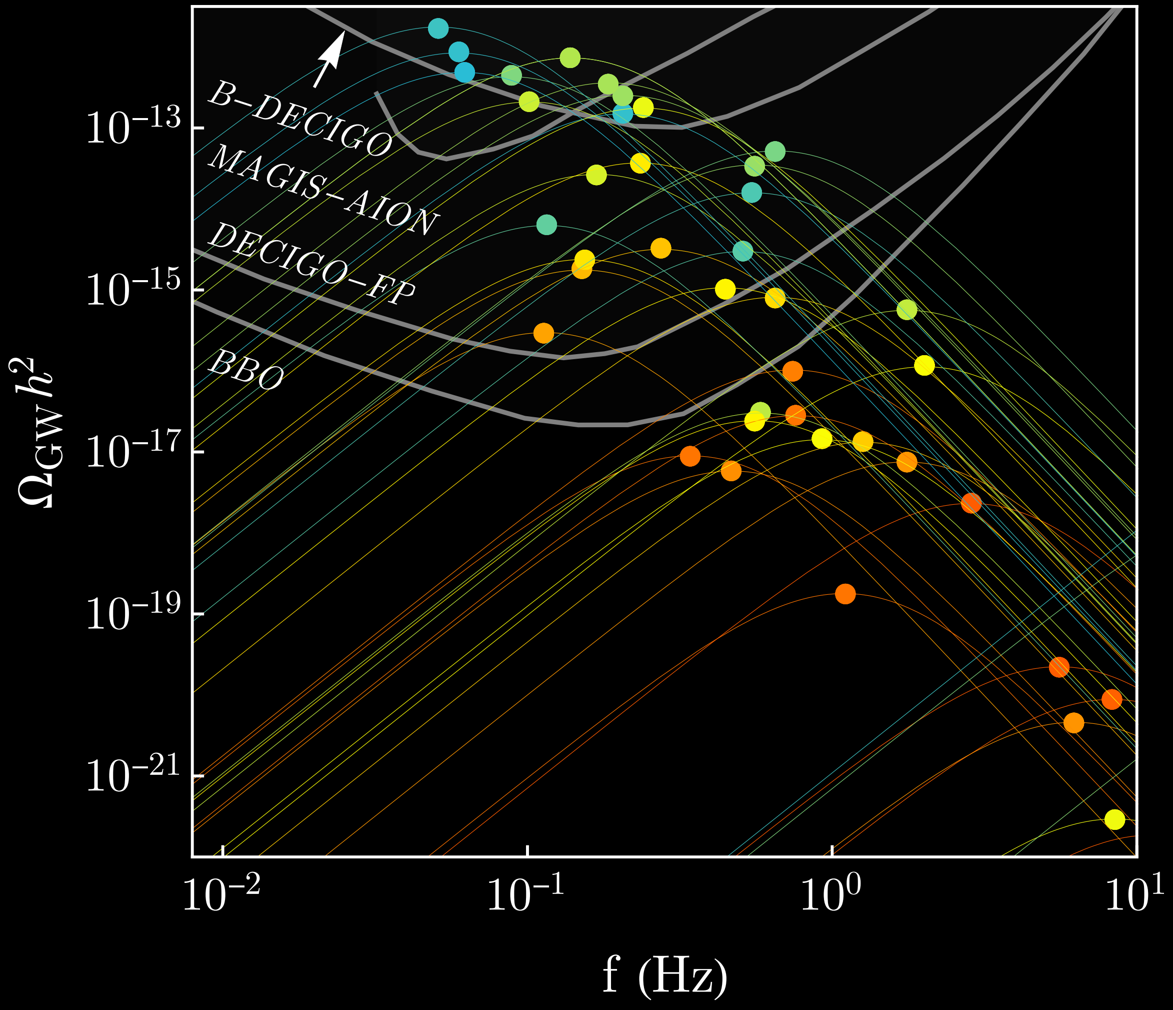
$$m_X^2 + \Pi_X = \frac{1}{6}\left(3\kappa\varphi^2 + 3\lambda\varphi^2 + \sqrt{6}\mu_\Sigma\varphi - 6m_\Sigma^2 - 18\xi + T^2(3\kappa + 5\lambda)\right)$$

$$m_\pi^2 + \Pi_\pi = \frac{1}{6}\left(\kappa\varphi^2 + 3\lambda\varphi^2 - \sqrt{6}\mu_\Sigma\varphi - 6m_\Sigma^2 - 18\xi + T^2(3\kappa + 5\lambda)\right)$$

DC, R. Houtz, and V. Sanz
[arXiv:1904.10967]

Dynamical axions and GW ($N_C = 3$, $N_f = 4$)





Takeaways and comments

- More explicit symmetry breaking ↔ larger η' axion mass leads to a greater GW amplitude
  - In some models, a large ratio $m_{\eta'}/m_{\varphi}$ is a natural prediction

e.g. Gavela, Ibe, Quilez, Yanagida [arXiv:1812.08174]

- New colored states would induce loop-level contributions to couplings of the φ and η' to gluons → dijet signatures @ LHC

- Predictions of the linear sigma model should be contrasted with other methods

Ipek, Tait, PRL (2019)

Early QCD confinement

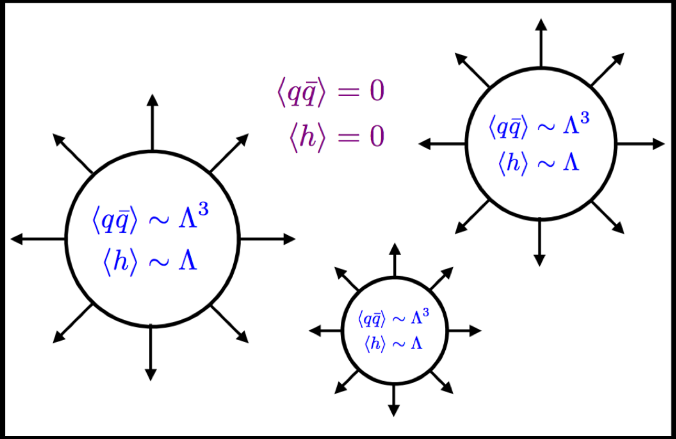
- Modified gluon kinetic term, $\boxed{-\frac{1}{4}\left(\frac{1}{g_{s0}^2}+\frac{S}{M}\right)G_{\mu\nu}G^{\mu\nu}}$

(1-loop MS-bar)

- When S evolves, QCD confines for $\Lambda_{\text{QCD}}(S)=\Lambda_0\,e^{\frac{24\pi^2}{2N_f-33}\frac{S}{M}}$

- Confinement triggers EWSB, because the meson condensate lead to a tadpole term for the Higgs:

$$V(h)\ni -y_t h\langle\bar{q}q\rangle\sim -y_t\frac{\Lambda^2}{4\pi}h\langle\Sigma\rangle$$



Ipek, Tait, PRL (2019), See also Kuzmin, Shaposhnikov, Tkatchev, PRD (1992), Servant, PRL (2014)

Early confinement and Baryogenesis

- Imagine the strong-CP problem is addressed by an axion
- Uncancelled strong CP-violating phase during the transition,

$$\left.\begin{array}{l} \dfrac{\alpha_s}{8\pi}\langle G\tilde{G}\rangle = m_a^2(T) f_a^2 \sin\bar{\theta}(T) \\ \mathcal{L}_{\text{eff}} \ni \dfrac{10}{f_\pi^2 m_{\eta'}^2}\dfrac{\alpha_s}{8\pi} G\tilde{G} \dfrac{\alpha_w}{8\pi} W\tilde{W} \end{array}\right\} \quad \mu_B = \frac{d}{dt}\left[\frac{10}{f_\pi^2 m_{\eta'}^2} m_a^2(T) f_a^2 \sin\bar{\theta}(T)\right]$$

- EW sphalerons produce baryon number, $n_B = \int_{t_i}^{t_f} dt\, \frac{\Gamma_{\text{sph}}(T)}{T}\mu_B$
  - Shut off for $v_h > T$ (inside the bubbles)

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]

Towards a realistic (minimal) model

- How does deconfinement (and re-confinement?) occur? Proposal:

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4}\left(\frac{1}{g_{s0}^2} + \frac{S}{M}\right) G_{\mu\nu}G^{\mu\nu} - V(S) - V(H) + \underbrace{b_1 S|H|^2 - b_2 S^2|H|^2}_{\text{New}}$$

- How can we study the physics in the confined phase?

- What are the observational signatures of early (de-)confinement?

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]

In the confined phase, quarks → mesons

- In terms of $U = e^{2i\,T^a\Pi^a/f_\pi}$ (T^a are the generators of $\mathrm{SU}(6)_\mathrm{V}$)

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U\right] + \alpha\,\mathrm{Tr}\left[UM\right] + \text{ H.c.}$$

- M includes the Yukawa couplings, approximately,

$$M = \mathrm{diag}\left(0,0,0,0,0,\frac{y_t h}{\sqrt{2}}\right)$$

→ This will give a tadpole term in the Higgs potential!

- $\mathrm{SU}(6)/\mathrm{SU}(5)$ gives 11 *top-flavored* pions ↔ 10 SU(6) generators have nonzero entries for T^{i6} or T^{6i}, 1 with T^{66}

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]

χPT in the confined phase

- Can calculate + relate the Higgs tadpole term to SM quantities,

$$\alpha \,\mathrm{Tr}\left[UM\right] + \text{ H.c.} = \frac{y_t}{y_u + y_d} \frac{m_0^2 f_0^2}{v_h} h \left(\frac{\Lambda}{\Lambda_{\mathrm{SM}}}\right)^3$$

- And the thermal potential, $V(h,T) \ni \sum_{i \in \text{mesons}} \frac{T^4}{2\pi^2} n_i J_B\left(\frac{m_i^2 + \Pi_i}{T^2}\right),$

Pion mass in SM QCD

$$m_{35}^2 = \frac{m_0^2}{1+5\sqrt{15}} \frac{y_t h}{(y_u+y_d)v_h}\left(\frac{\Lambda}{\Lambda_{\mathrm{SM}}}\right)$$

$$m_{25,\ldots,34}^2 = \frac{3m_0^2}{1+5\sqrt{15}} \frac{y_t h}{(y_u+y_d)v_h}\left(\frac{\Lambda}{\Lambda_{\mathrm{SM}}}\right)$$

Top-flavored pions

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]

Towards a minimal realistic model

- As announced, now introduce the following Higgs couplings,

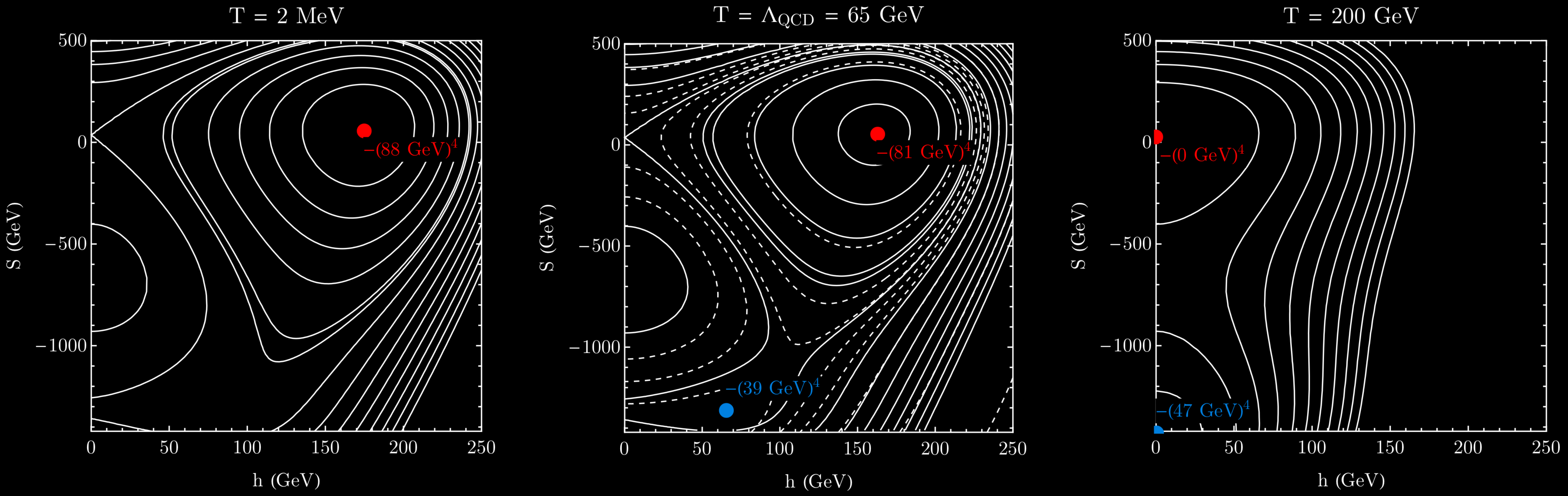
$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} - \frac{1}{4}\left(\frac{1}{g_{s0}^2} + \frac{S}{M}\right) G_{\mu\nu}G^{\mu\nu} - V(S) - V(H) + b_1 S|H|^2 + b_2 S^2|H|^2$$

- As before, $\Lambda_{\mathrm{QCD}}(S) = \Lambda_0\, e^{\frac{24\pi^2}{2N_f - 33}\frac{S}{M}}$

- Note that we are free to define Λ_0, as long as $\Lambda_{\mathrm{QCD}}(\upsilon_{\mathrm{S,T=0}}) = \Lambda_{\mathrm{SM}}$

We look for the couplings that realize the following,

DC, Howard, Ipek, Tait, [arXiv:19XX.XXXX]



$T = 2$ MeV

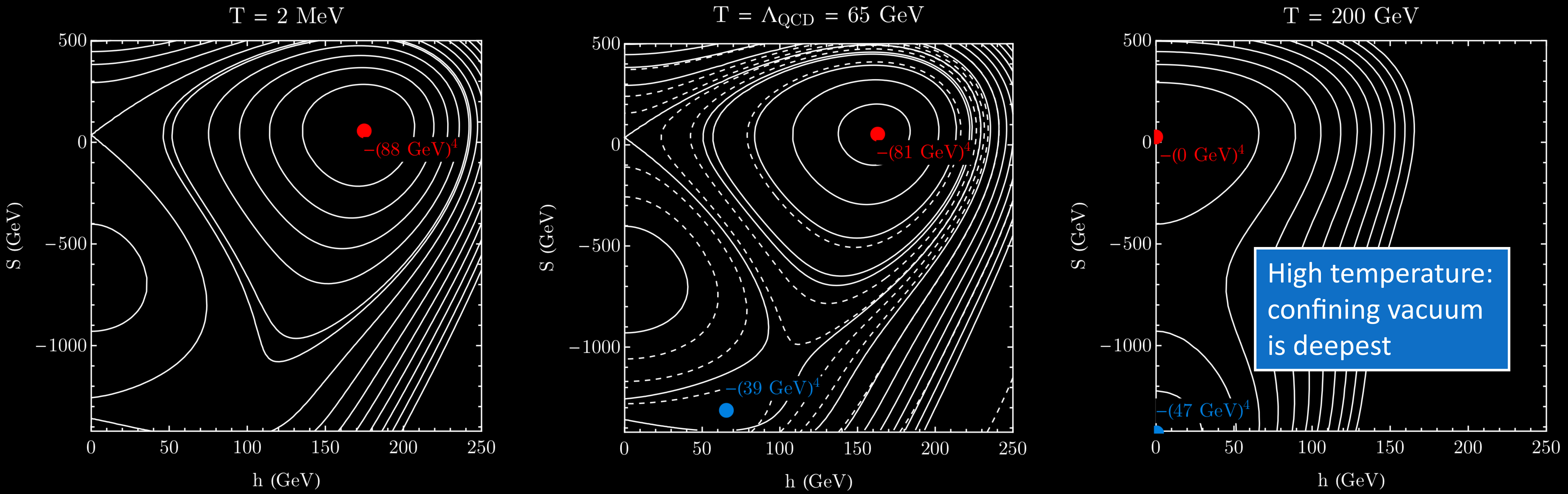
$T = \Lambda_{QCD} = 65$ GeV

$T = 200$ GeV

Dashed (solid) lines: potential in the unconfined (confined) phase

We look for the couplings that realize the following,

DC, Howard, Ipek, Tait, [arXiv:19XX.XXXX]



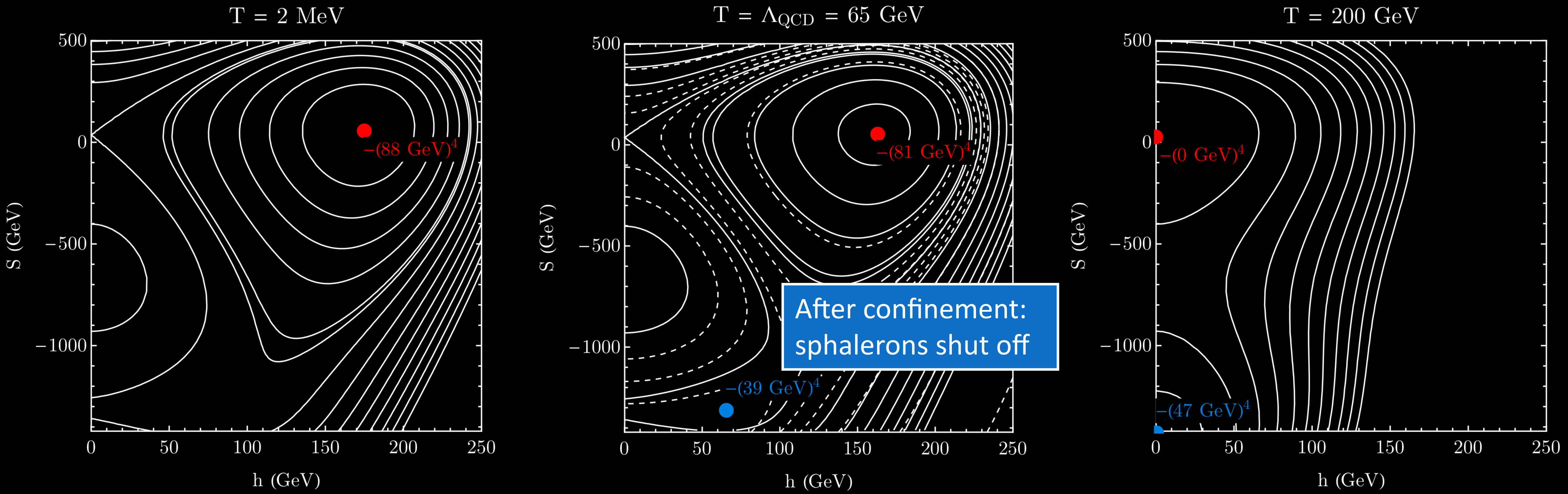
High temperature: confining vacuum is deepest

Dashed (solid) lines: potential in the unconfined (confined) phase

We look for the couplings that realize the following,

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]

SM-like vacuum is deeper, but tunneling is suppressed

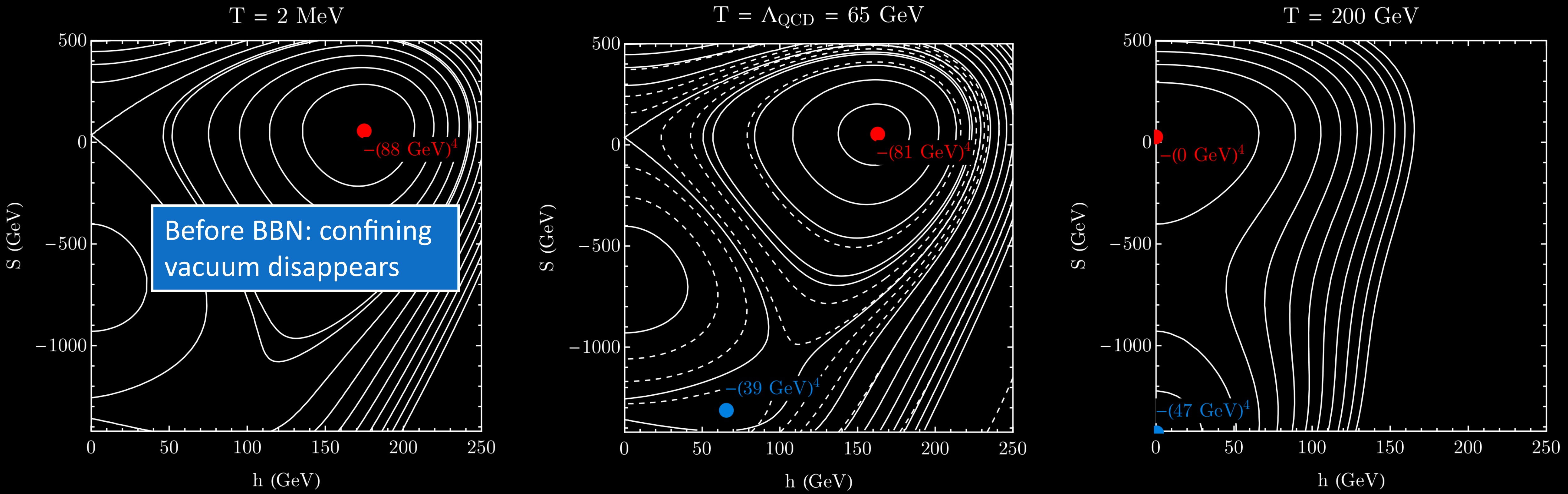


After confinement: sphalerons shut off

Dashed (solid) lines: potential in the unconfined (confined) phase

We look for the couplings that realize the following,

DC, Howard, Ipek, Tait, [arXiv:19XX.XXXX]

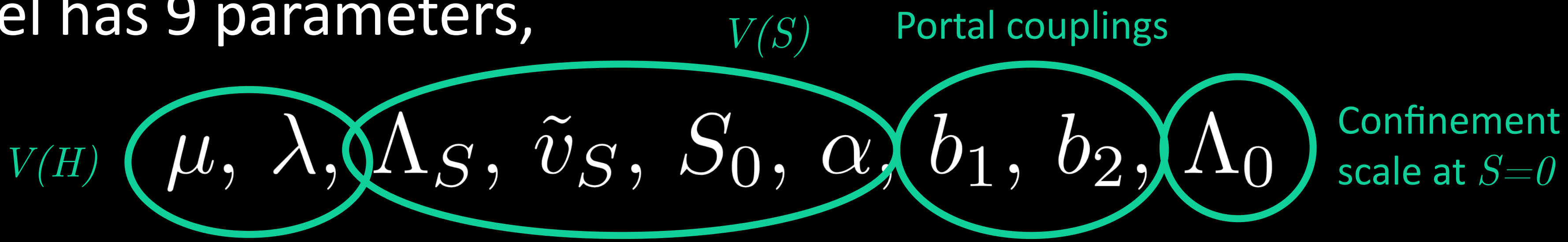


Before BBN: confining vacuum disappears

Dashed (solid) lines: potential in the unconfined (confined) phase

Towards a minimal realistic model

- The model has 9 parameters,



V(S) Portal couplings

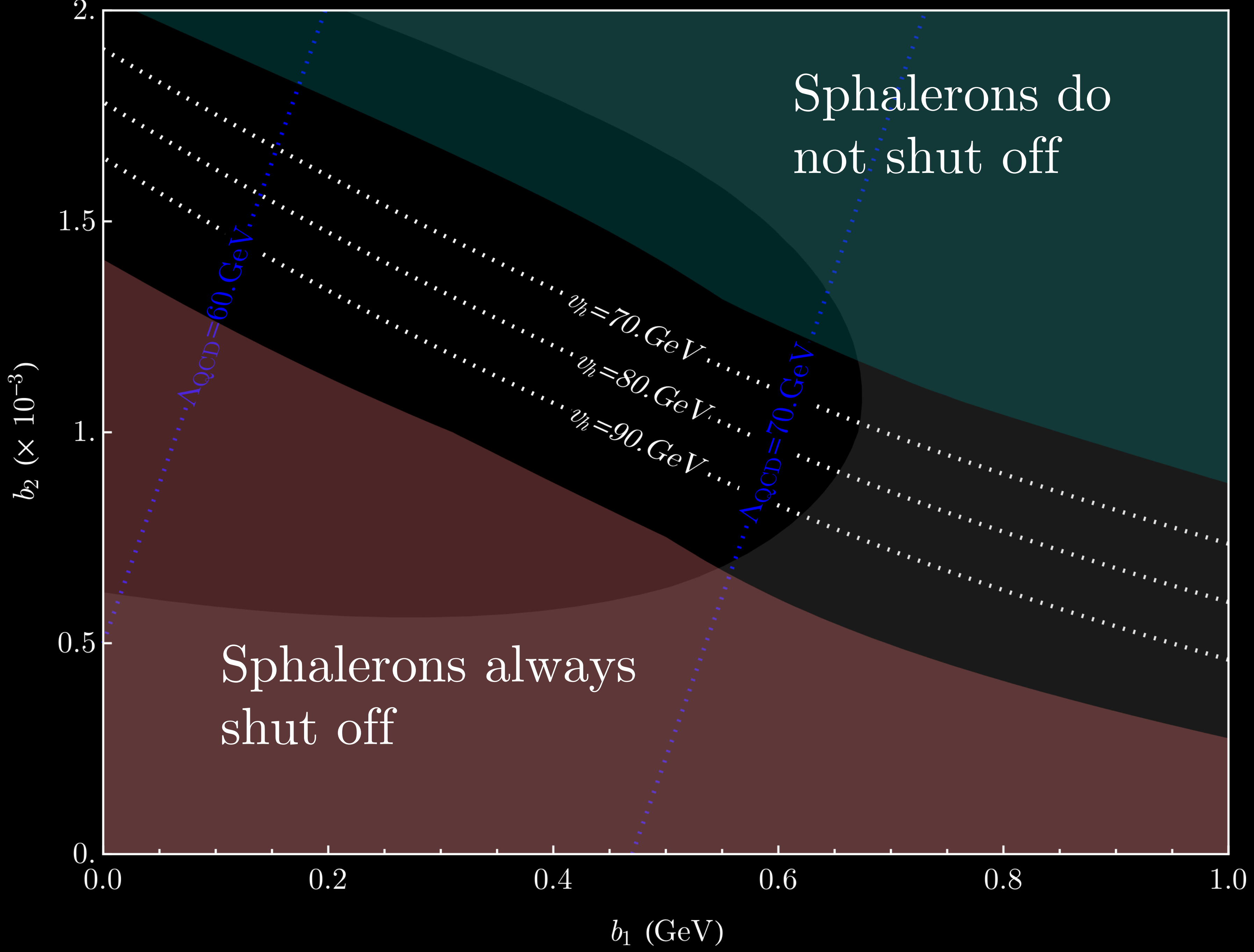
V(H) $\mu,\ \lambda,\ \Lambda_S,\ \tilde{v}_S,\ S_0,\ \alpha,\ b_1,\ b_2,\ \Lambda_0$

Confinement scale at $S=0$

- But they are not all free,
  - Fix μ and λ using the Higgs (mass eigenstate and VEV) in the SM-like vacuum
  - Fix S_0 as a function of other parameters setting the SM-like QCD scale
  - Benchmark point:

Λ_0	Λ_S	$\tilde{v}_S$	α
$500\,\mathrm{MeV}$	$150\,\mathrm{GeV}$	$1.5\,\mathrm{TeV}$	0.51

DC, Howard, Ipek, Tait,
[arXiv:19XX.XXXX]



Takeaways and comments

- Early QCD confinement may form part of an EW Baryogenesis scenario
- In a minimal model, de-confinement (or relaxation to SM-like confinement) may be realized by portal couplings with the SM Higgs
- Both transitions are likely to be first order, and would fall within the LISA frequency window

DC, Dror, Muruyama, White, arXiv:19XX.XXXX

Froggatt-Nielsen confinement

- In FN models, quark masses arise from couplings of the form

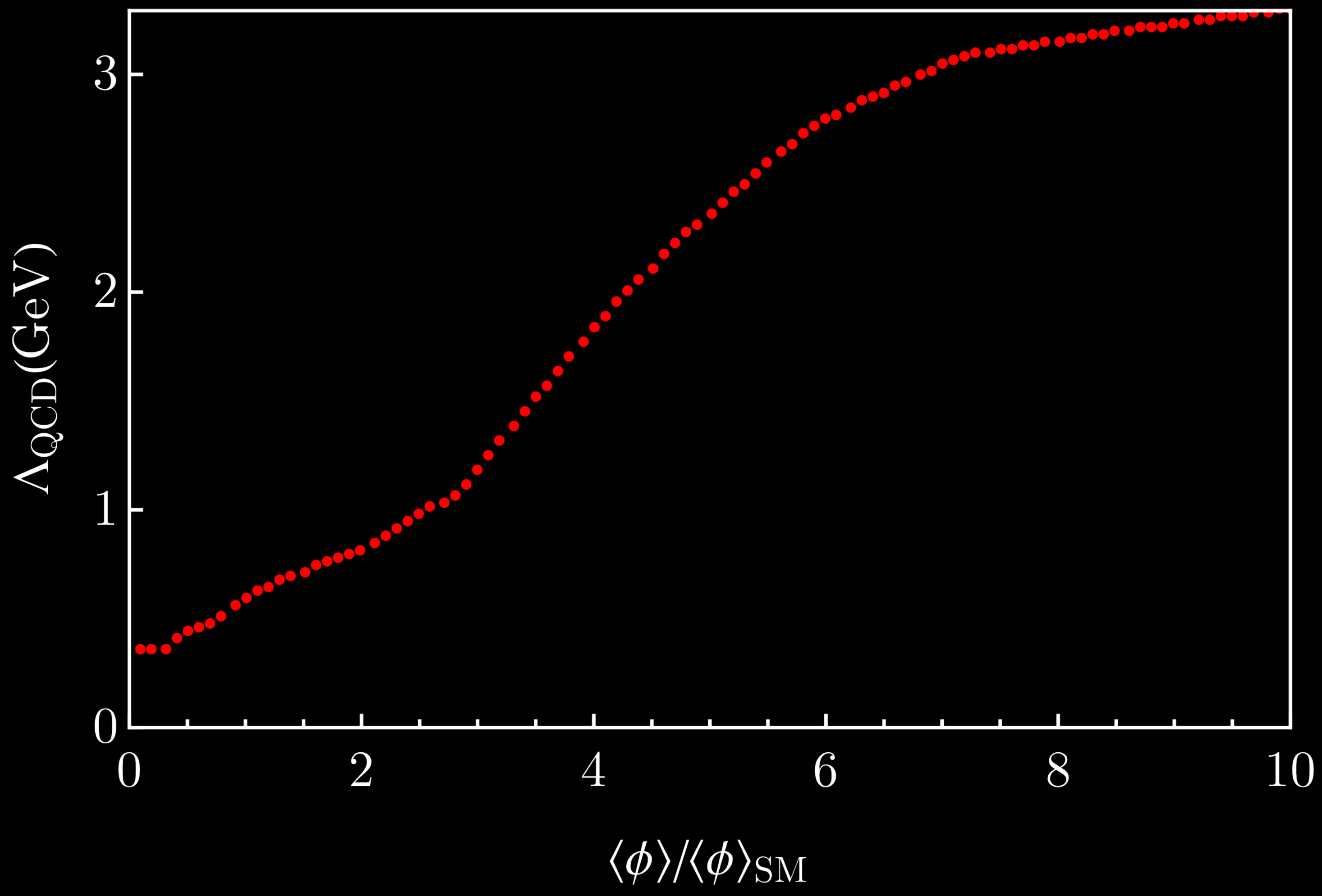
$$\mathcal{L} \supset y_{ij} v_h \left(\frac{\phi}{M}\right)^{n_{ij}} \bar{q}_i q_j$$

Quark masses arise when ϕ gets a VEV $v_\phi < M$, thus explaining flavor hierarchies by a horizontal flavor symmetry

- In such models, quark masses may evolve to their SM values only after QCD confines…
  - Confinement itself breaks the horizontal symmetry (via quark condensates)
  - The flavon field ϕ gets stuck by Hubble friction
  - ϕ starts evolving when its thermal potential becomes subdominant

Λ_{QCD} as a function of ϕ

Calculated at 3-loop MS-bar scheme



DC, Dror, Muruyama, White, arXiv:19XX.XXXX

Late origin of fermion masses- constraints

- FCNCs

$$\mathcal{L} \ni \frac{\phi}{M}\bar{q}_a U_{ai}\left[\frac{m_{ij}}{\langle\phi\rangle/M} n_{ij}\right] V^{\dagger}_{jb} q_b \longrightarrow \langle\phi\rangle \gtrsim 1.6 \times 10^7\ \mathrm{TeV}$$

- 5th force constraints

Light scalar coupling to u and d quarks

- Relic abundance constraints

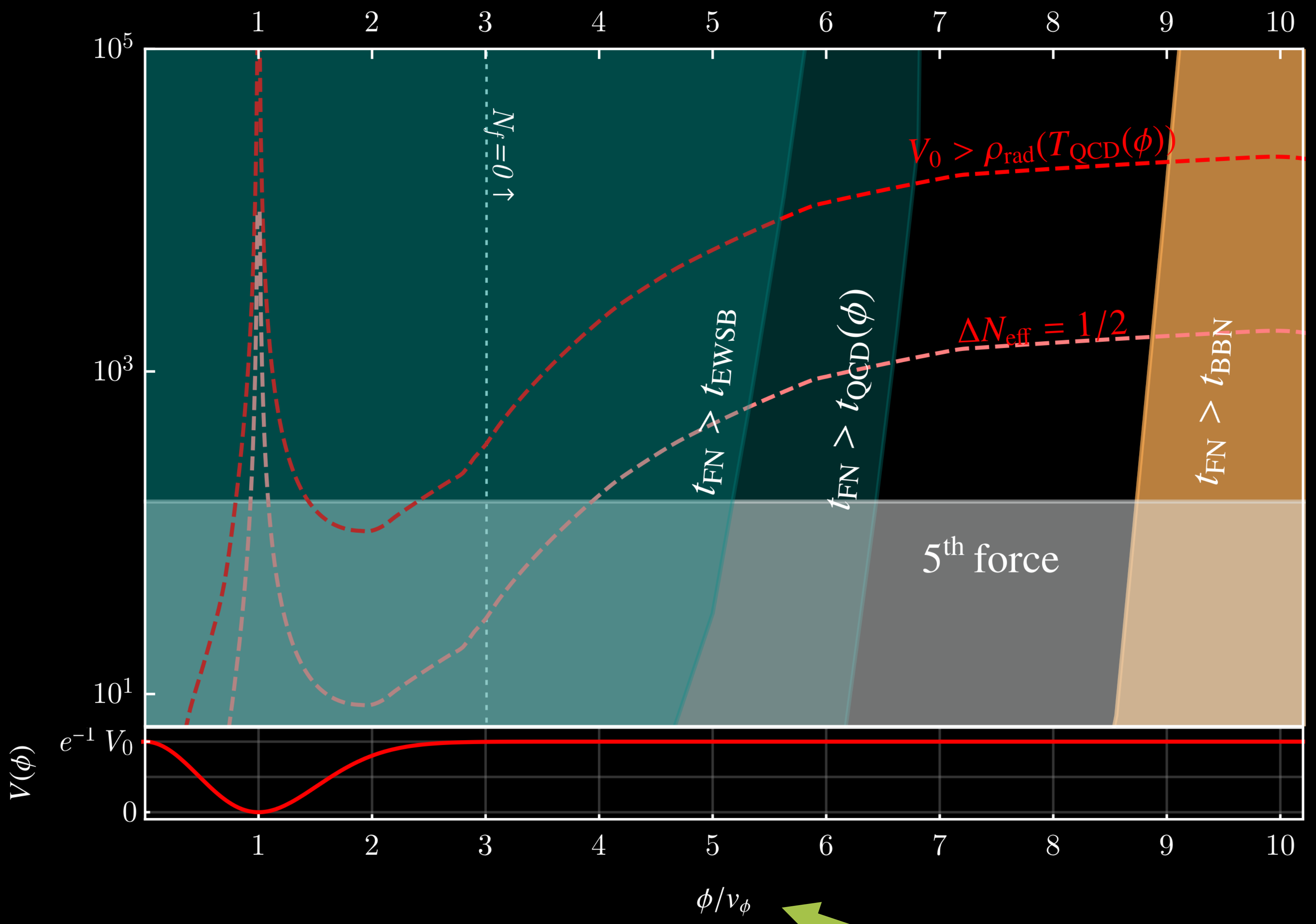
$$\left\{\begin{array}{l} \Omega_\phi \leq \Omega_{DM} \\ \text{No new radiation at BBN, } \Delta N_{eff} \end{array}\right.$$

- Late inflation constraints

$$V(\phi) < \rho_{\mathrm{rad}}$$

- Gravitational wave predictions

Derived from lattice results for $N_f = 0$



$\langle \phi \rangle \gtrsim 1.6 \times 10^7$ TeV (FCNCs)

$N_f = 0$ confinement and Gravitational Waves

- Thermal parameters from lattice fits

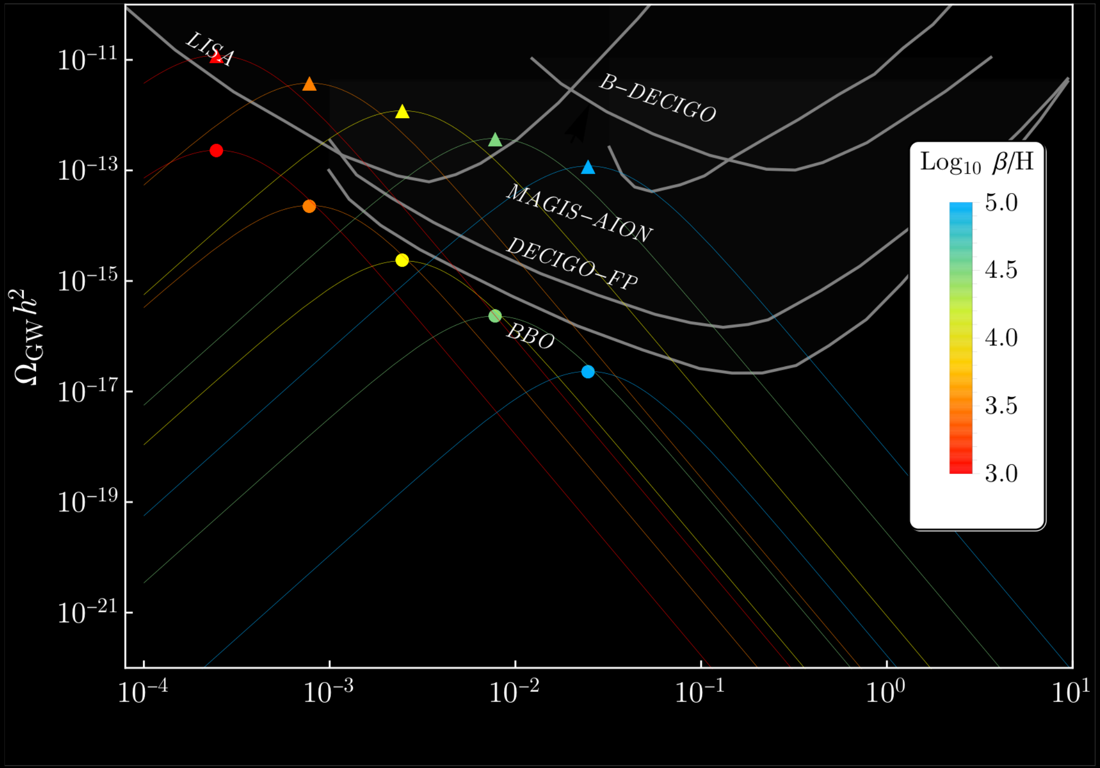
e.g. Lucini, Teper, Wenger, JHEP (2005)
Lucini, Rago, and Rinaldi, PLB (2012)

$$\alpha = \frac{(N_C^2 - 1) \times 30}{(g_* \times \pi^2)}(0.388 - 1.61/N_C^2)$$

$$T_C = (0.5949 + 0.458/N_C^2) \times 2.5 \times \Lambda_{\mathrm{QCD}}^{\bar{\mathrm{MS}}}$$

- Dynamical parameter β/H can be estimated from χPT

$$\beta/H \sim \mathcal{O}(10^4 - 10^5)$$



Final takeaways

- New confining phase transitions may occur in QCD’ models
  - Solutions to the strong CP problem
  - Models with new strongly interacting sectors, Baryogenesis
- QCD confinement itself may be modified
  - Effective coupling strength changed by a scalar
  - Late origin of quark masses
- Studying the (GW) phenomenology of such models is an interesting (and difficult) challenge