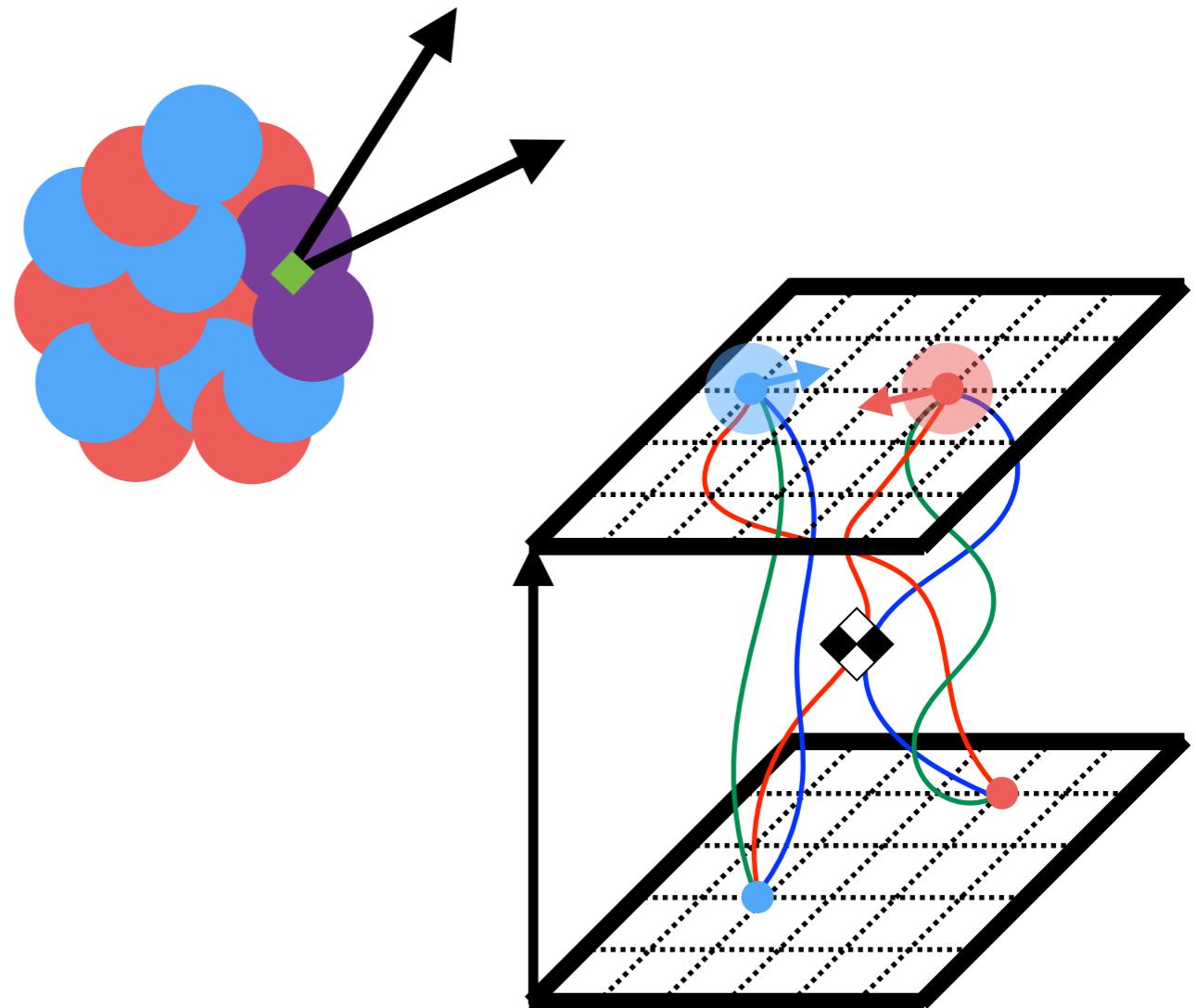


# Lattice Gauge Theory Calculations of NN Weak Amplitudes

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Evan Berkowitz

Institut für Kernphysik  
Institute for Advanced Simulation  
Forschungszentrum Jülich

Particle Physics with Neutrons at the ESS  
NORDITA  
11 December, 2018





Berkeley  
LBL



RBRC

David Brantley, Henry Monge Camacho, Chia  
Cheng (Jason) Chang, Ken McElvain, André  
Walker-Loud



JLab

Enrico Rinaldi



Liverpool  
Plymouth

EB



Office of  
Science

FZJ

Bálint Joó



LLNL

Pavlos Vranas

NERSC

Thorsten Kurth

UNC

Amy Nicholson



nVidia

Kate Clark



Glasgow

Chris Bouchard



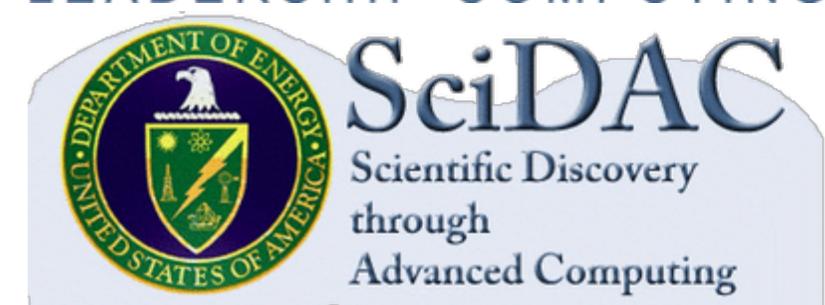
Rutgers

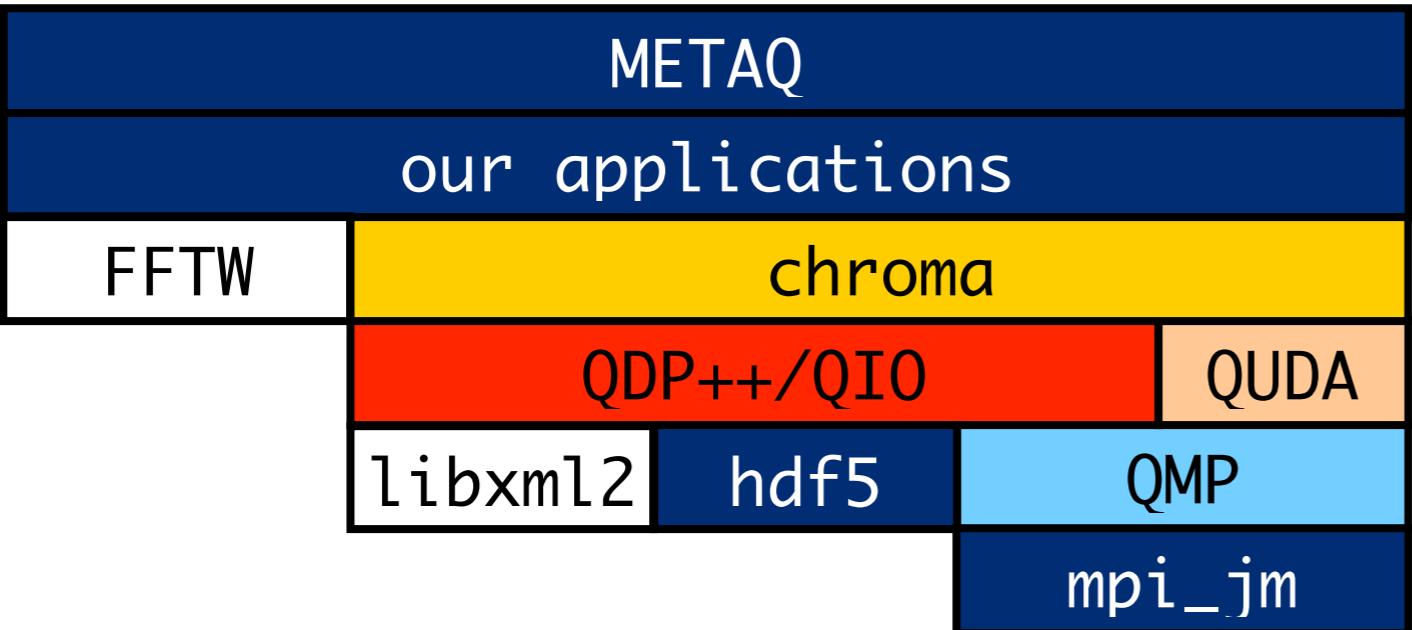
Chris Monahan



William &  
Mary

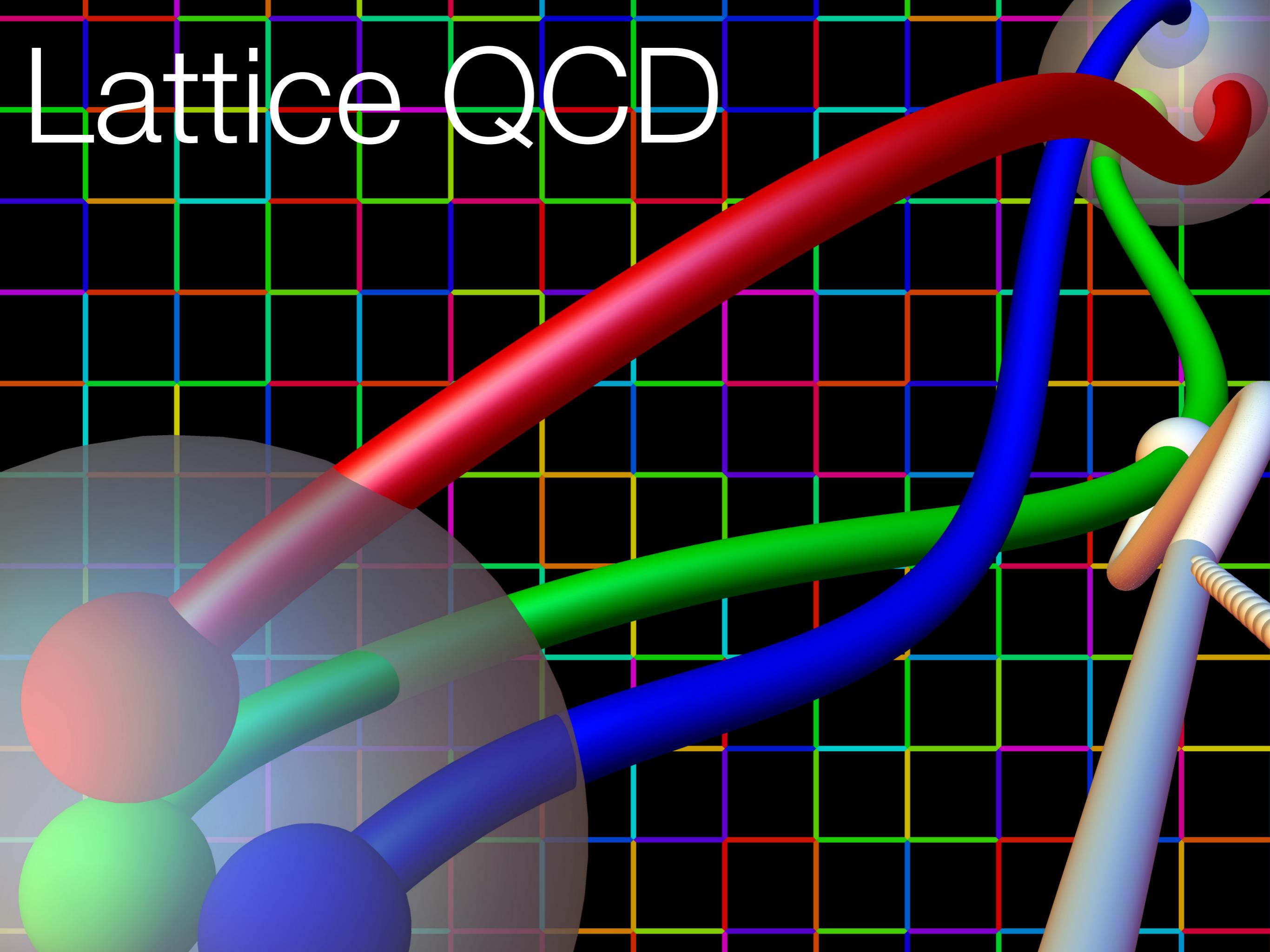
Kostas Orginos





Software	References
METAQ	Berkowitz arXiv:1702.06122 <a href="https://github.com/evanberkowitz/metaq">github.com/evanberkowitz/metaq</a> Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018)
chroma	Edwards and Joo (SciDAC, LHPC and UKQCD Collaborations) Nucl. Phys. Proc. Suppl 140, 832 (2005)
QDP++	Clark et al. Comput. Phys. Commun. 181 1517 (2010) Babich et al. Supercomputing 11, 70
hdf5 in QDP++	Kurth et al PoS LATTICE2014 045 (2015)
qmp	Chen, Edwards, and Watson et al. <a href="https://github.com/usqcd-software/qmp">https://github.com/usqcd-software/qmp</a>
mpi_jm	Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018) McElvain et al. <a href="https://github.com/kenmcelvain/mpi_jm/">https://github.com/kenmcelvain/mpi_jm/</a>

# Lattice QCD



# Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(iD + m)\psi$$

$$\begin{aligned} C(t) &= \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t)\mathcal{O}^\dagger(0)e^{-S[\bar{\psi},\psi,U]} \\ &= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(D + M) e^{-S[U]} \mathcal{O}(t)\mathcal{O}^\dagger(0) \end{aligned}$$

# Introduction to LQCD

---

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lattice  
finite volume

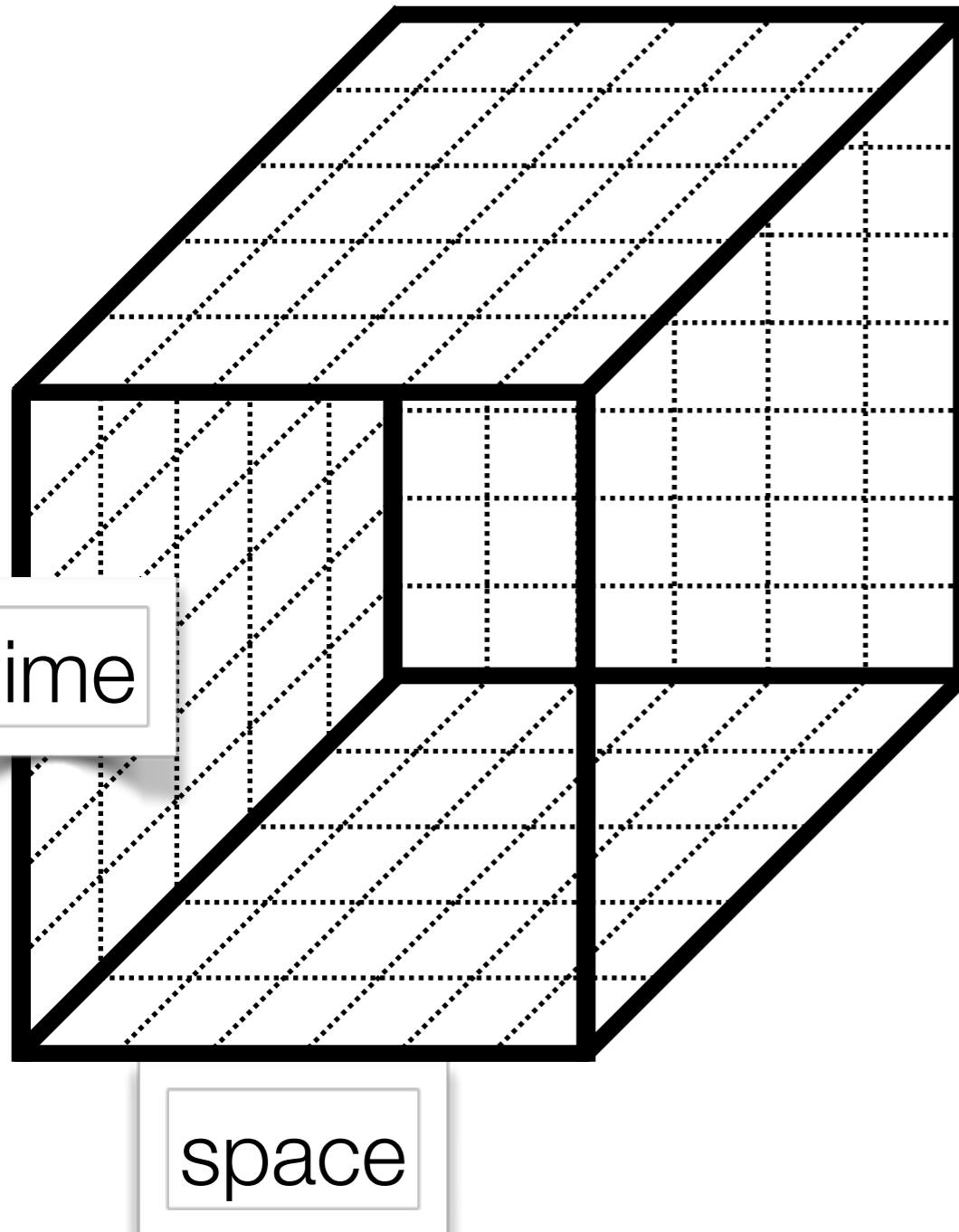
# Introduction to LQCD

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$$= \frac{1}{Z} \int \mathcal{D}U \det(D + M) e^{-S[U]} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

lattice  
finite volume



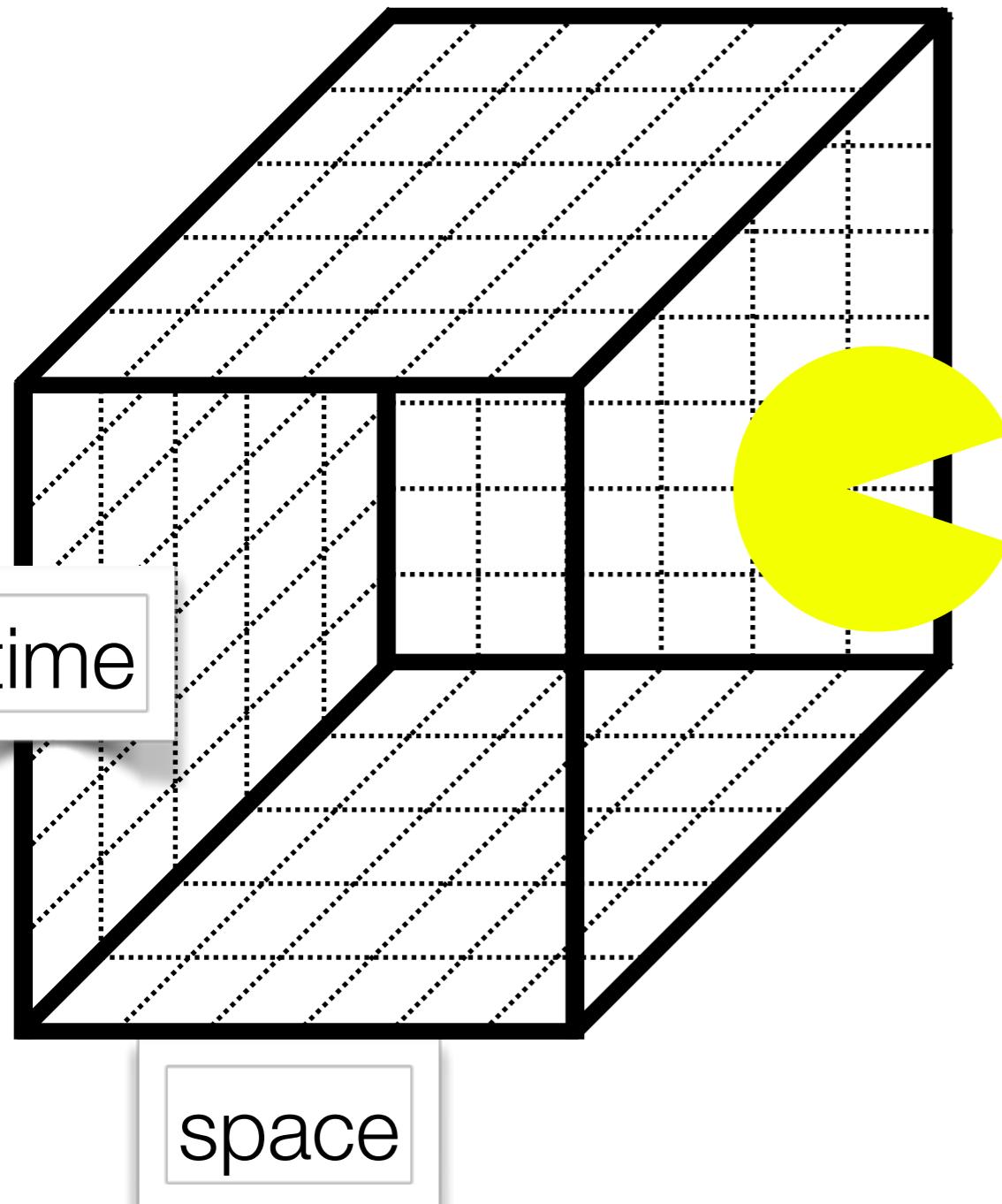
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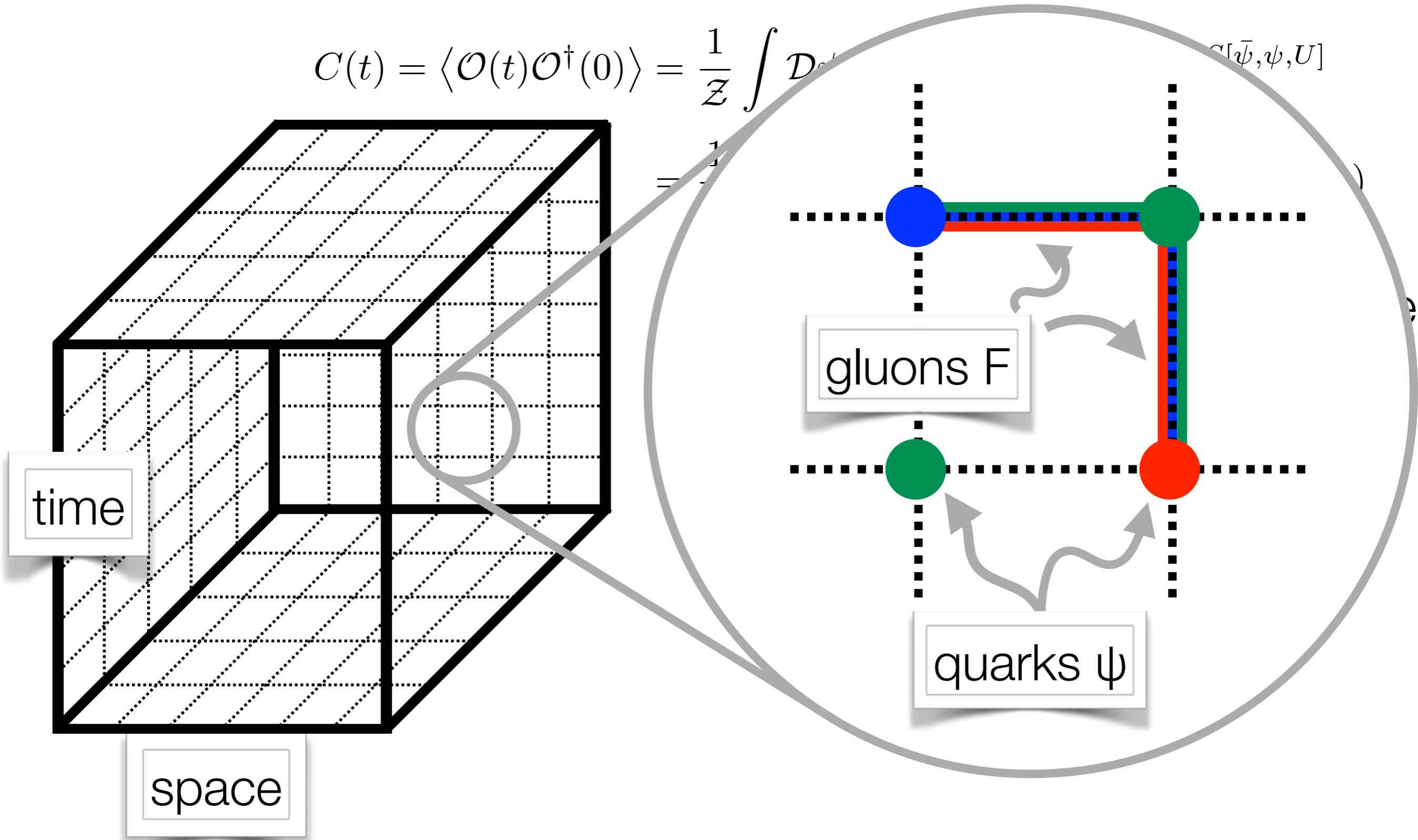
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lattice  
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# Introduction to LQCD

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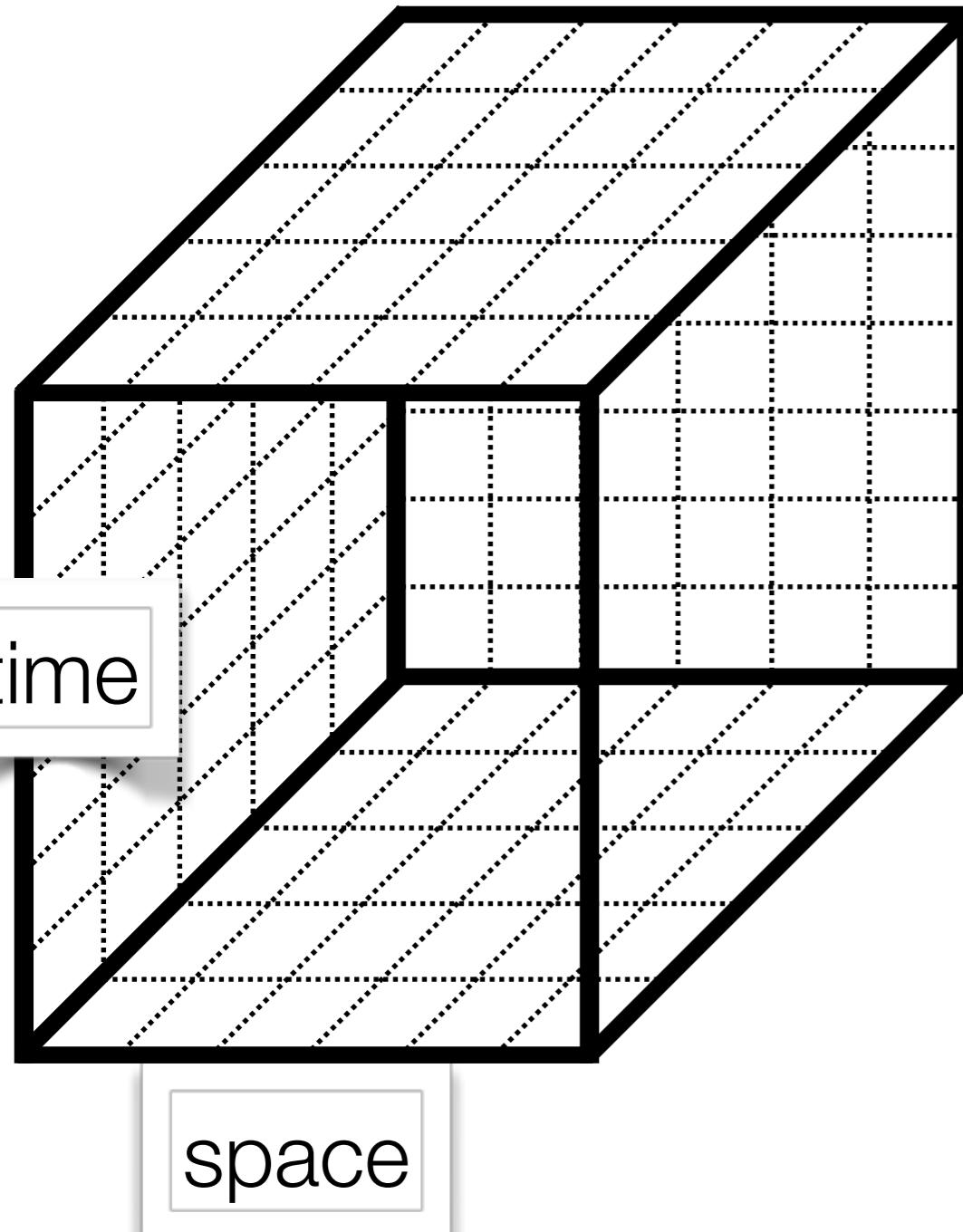
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lattice  
finite volume



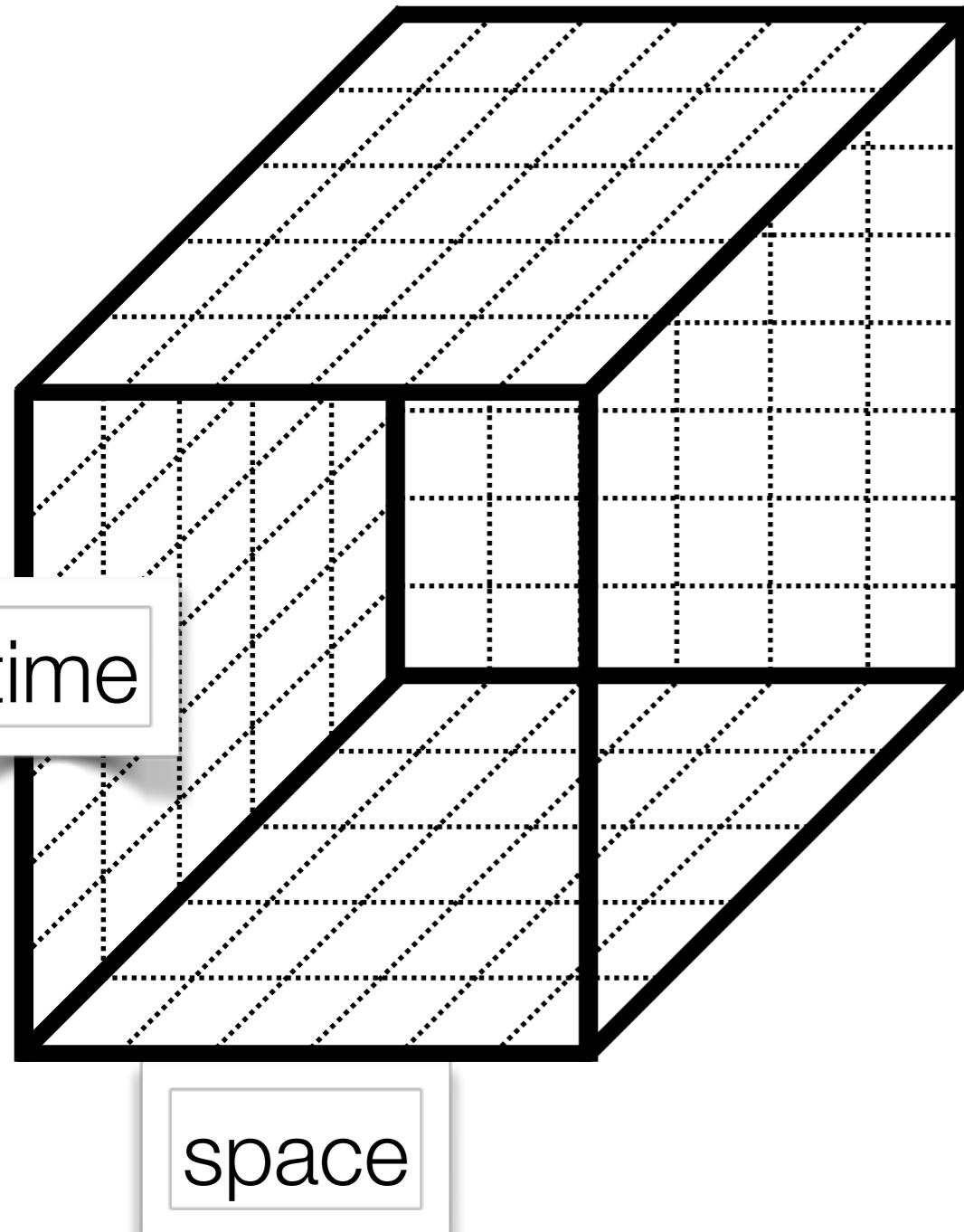
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Probability



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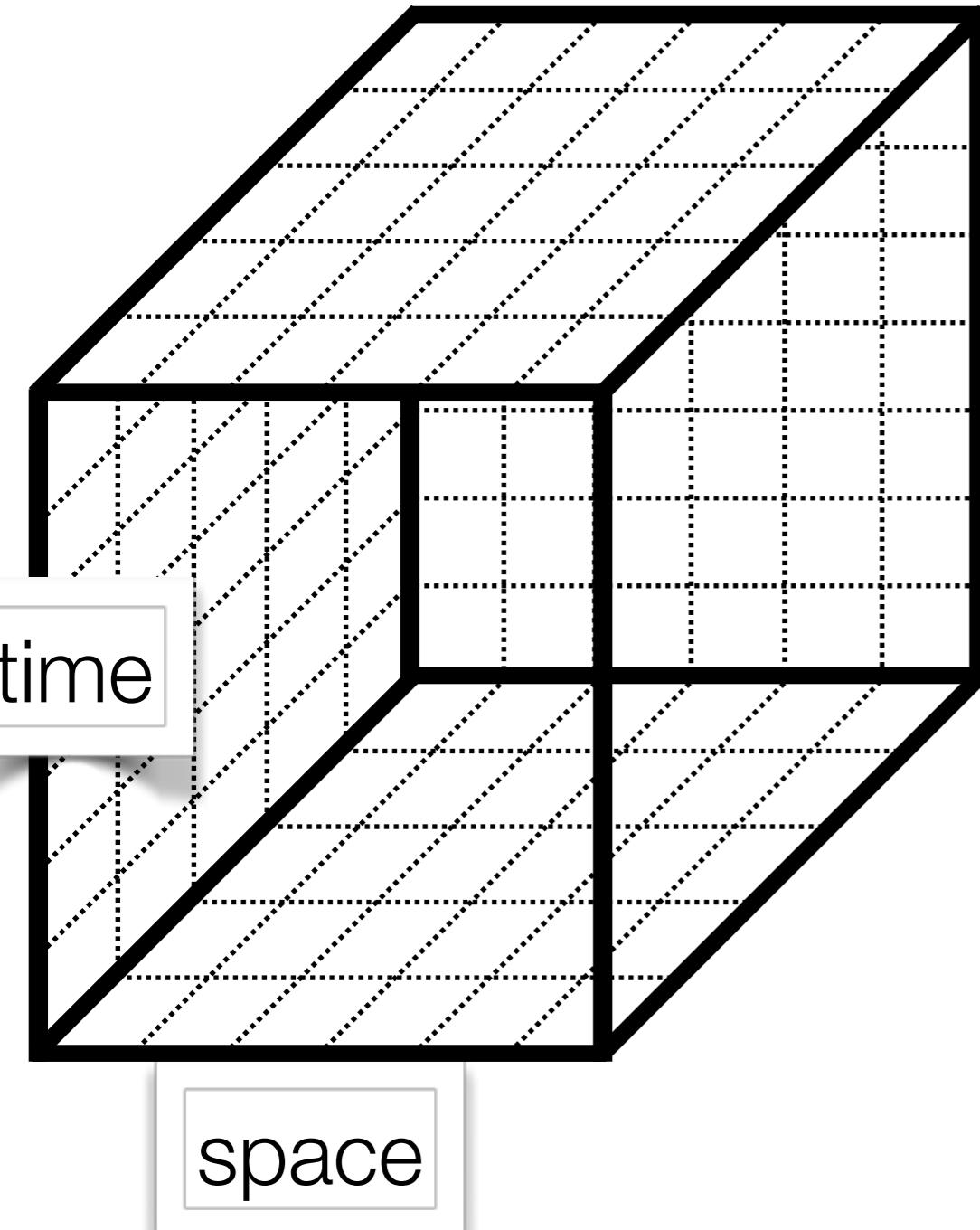
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo



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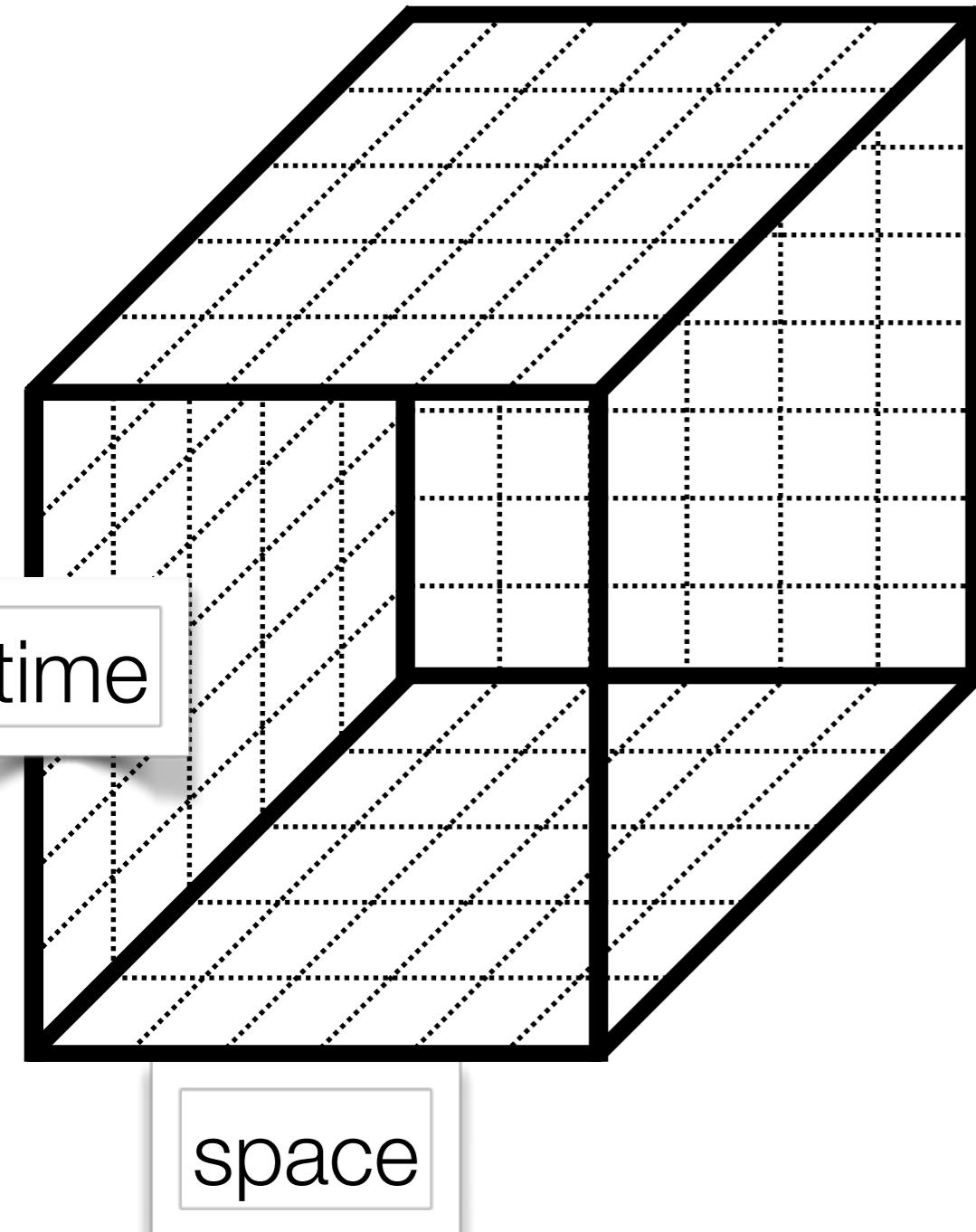
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i]$$



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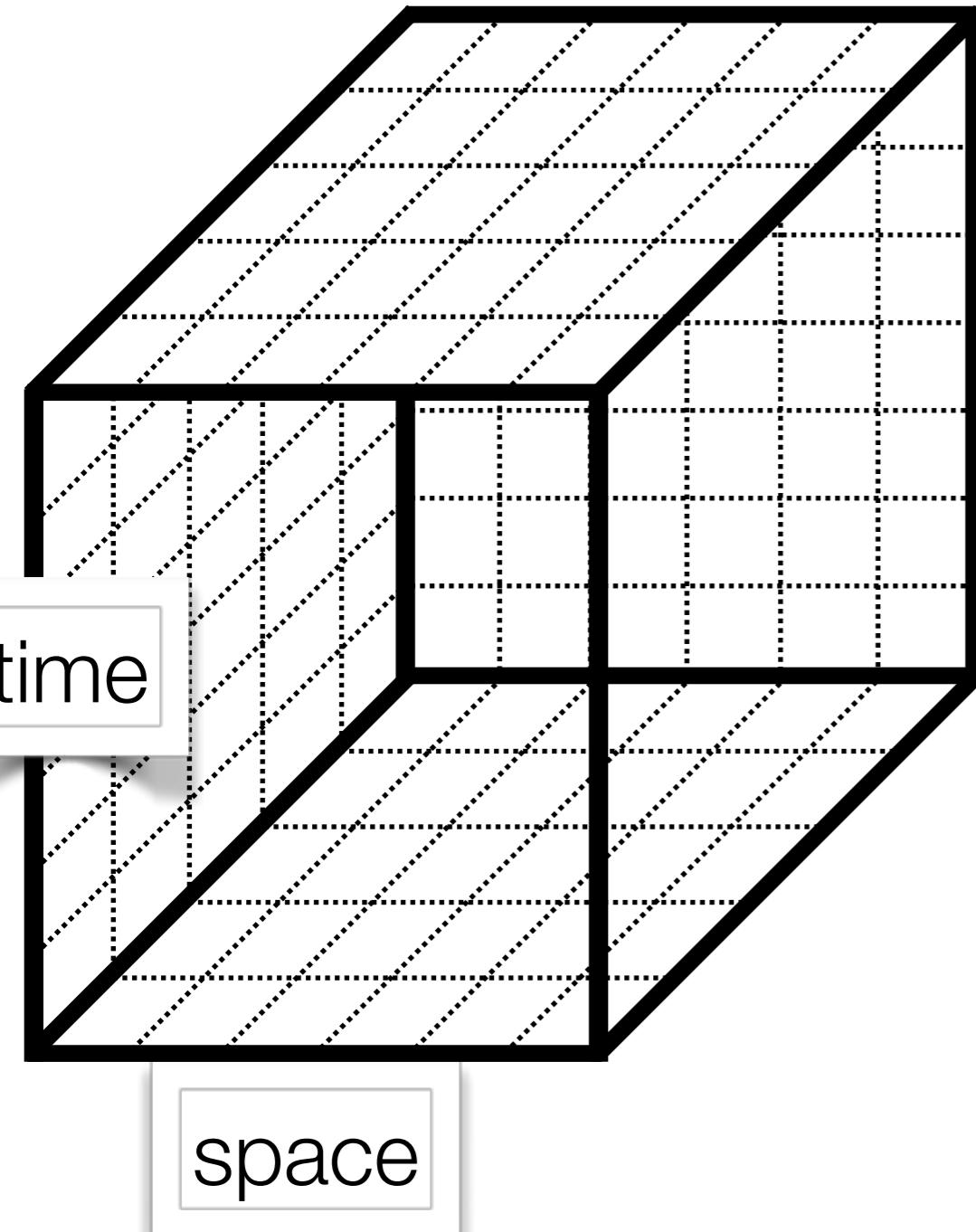
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

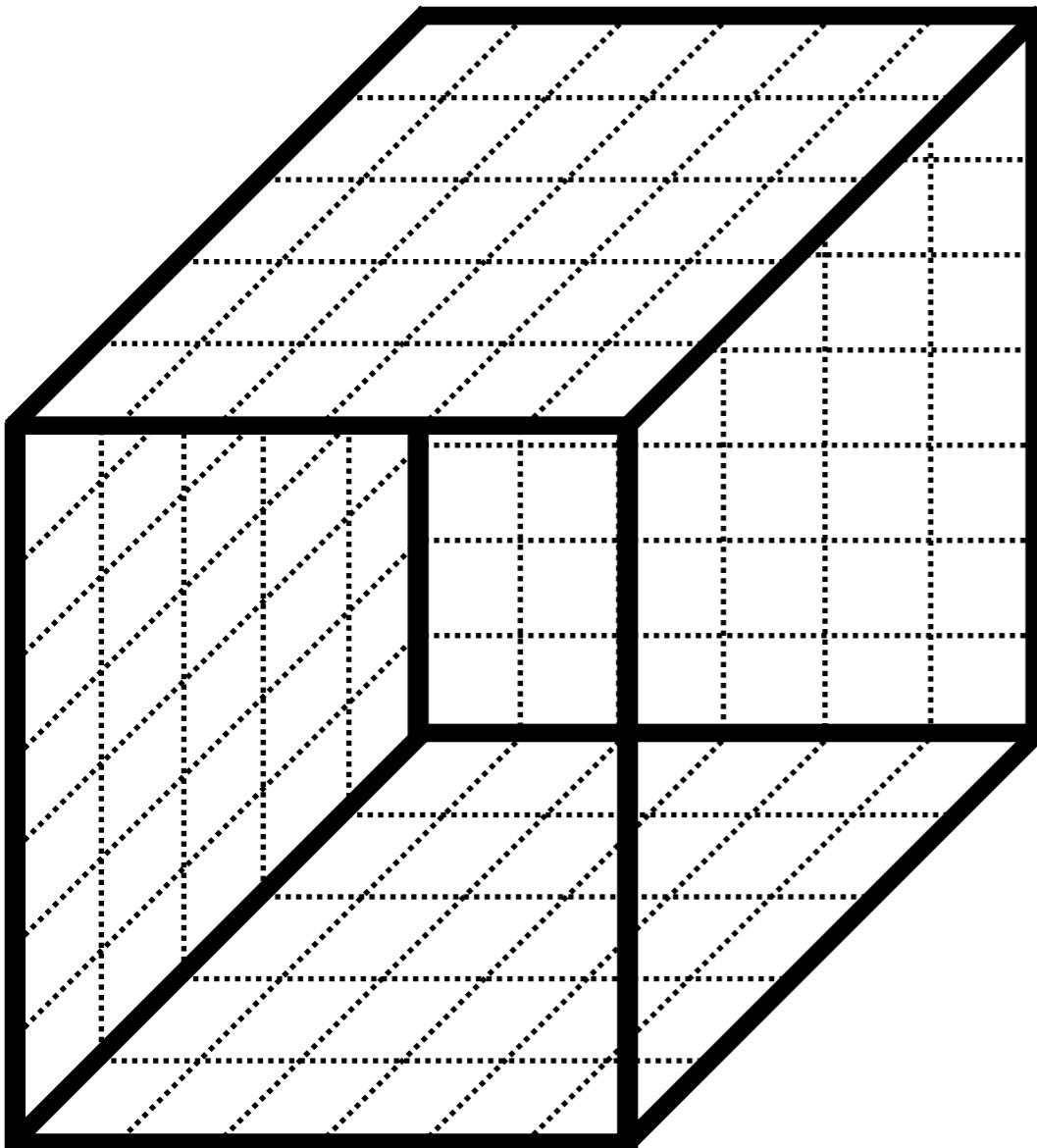
Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$



# Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(iD + m)\psi$$



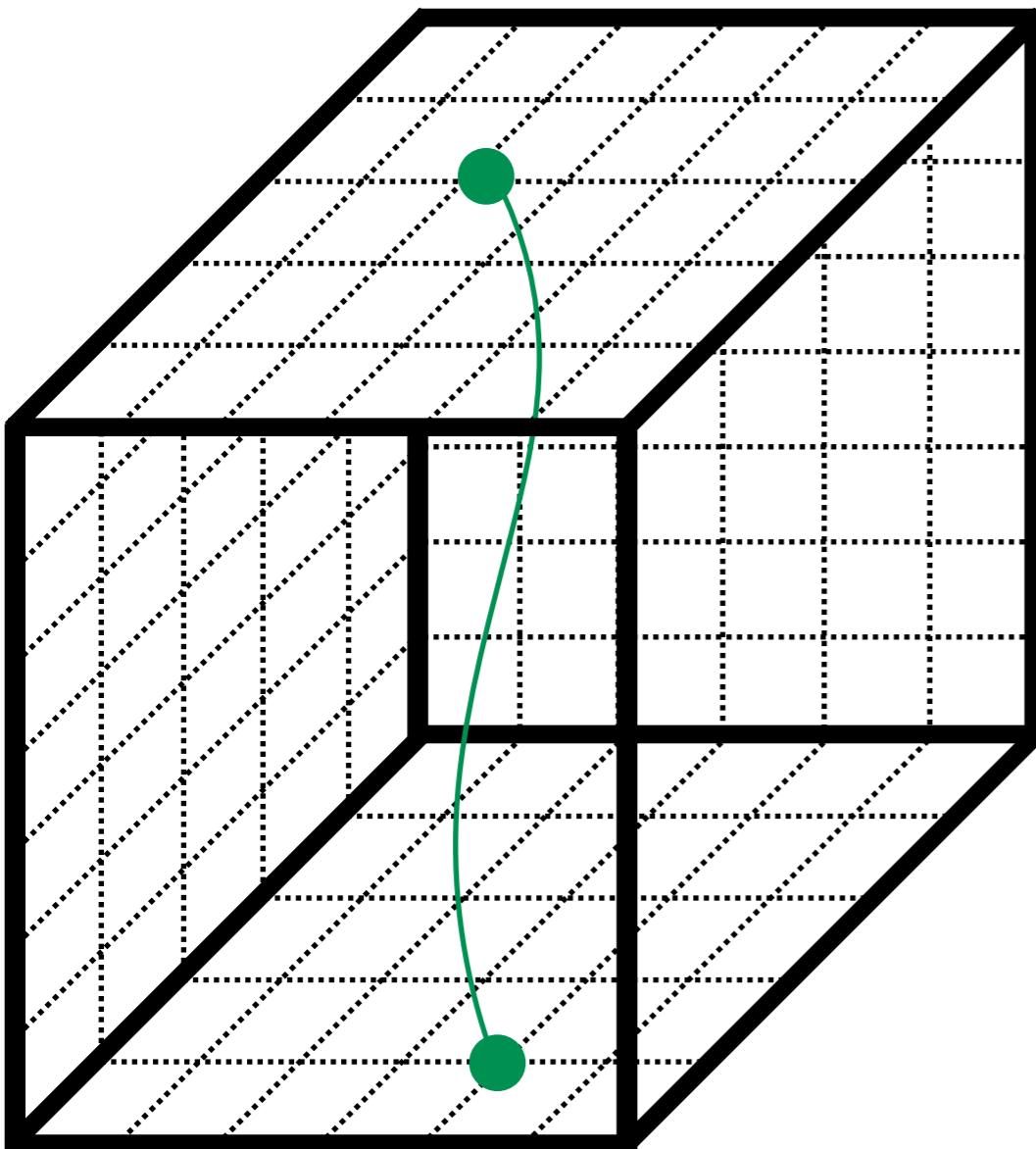
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D is a sparse matrix  
U-dependent  
( $N_x N_t N_c N_s$ ) on a side

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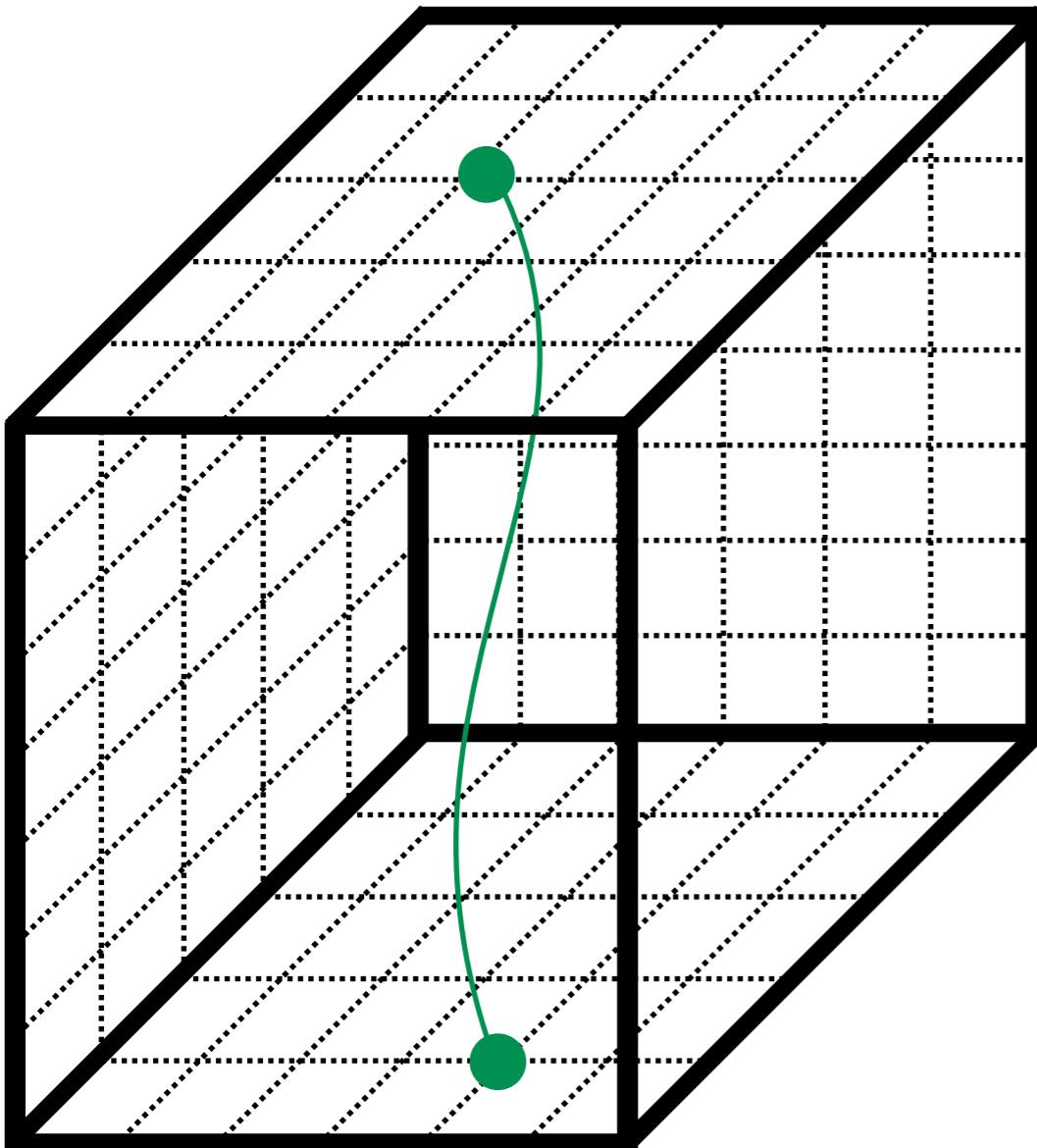


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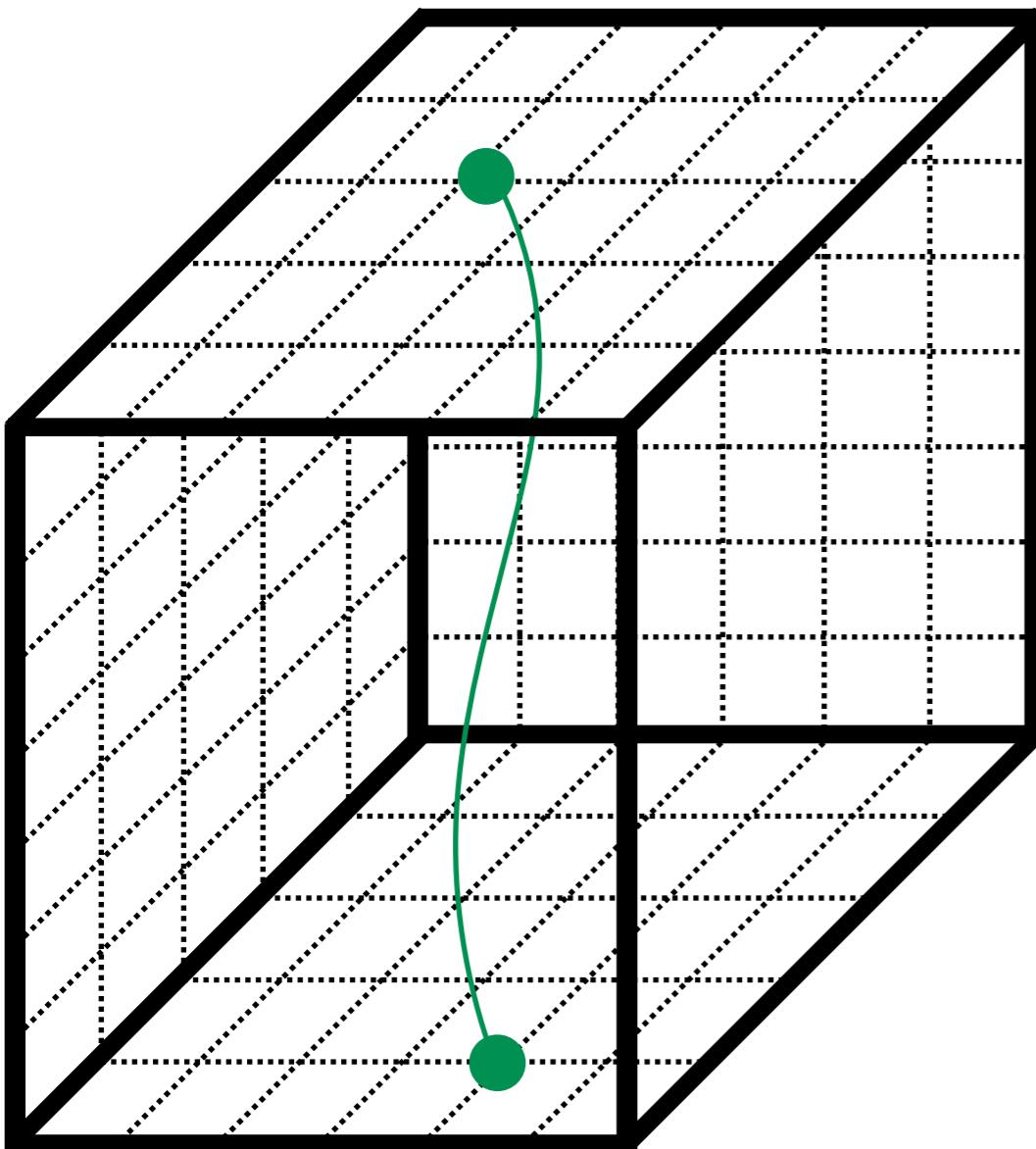


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$(48^3 \times 64 \times 3 \times 4) = 127\ 401\ 984$

# Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(iD + m)\psi$$



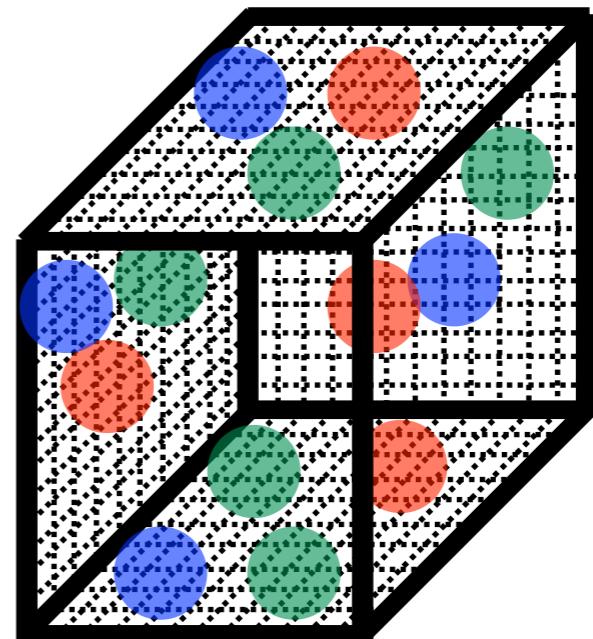
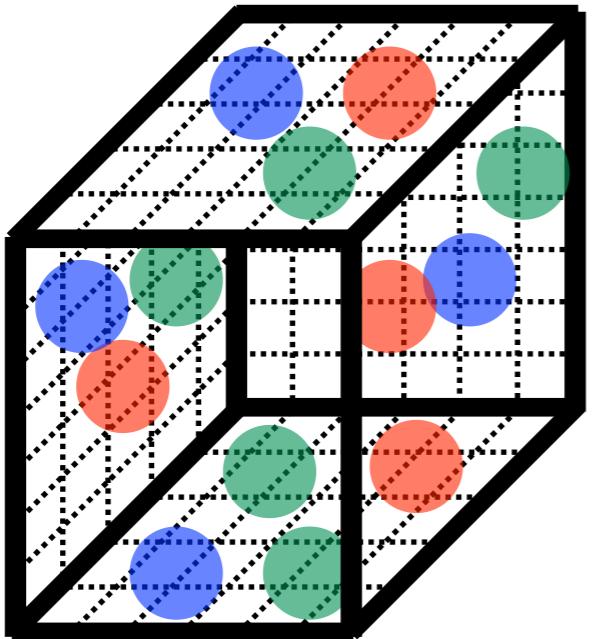
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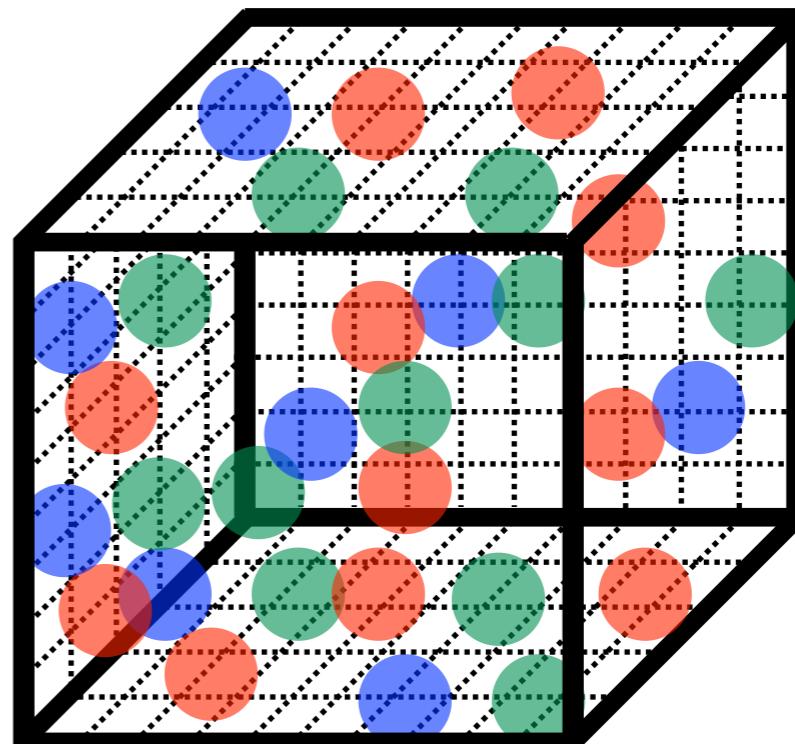
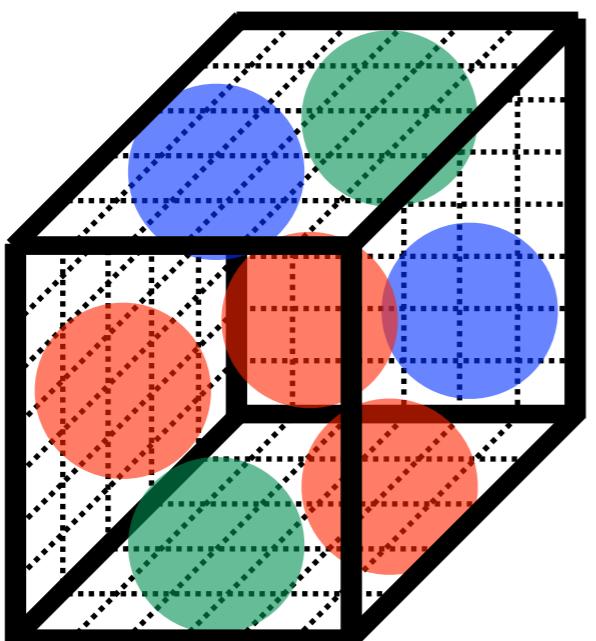
$$(48^3 \times 64 \times 3 \times 4) = 127\ 401\ 984$$

$$(D[U] + M)S_F(x \leftarrow y) = i\delta(x - y)$$

# LQCD Systematics

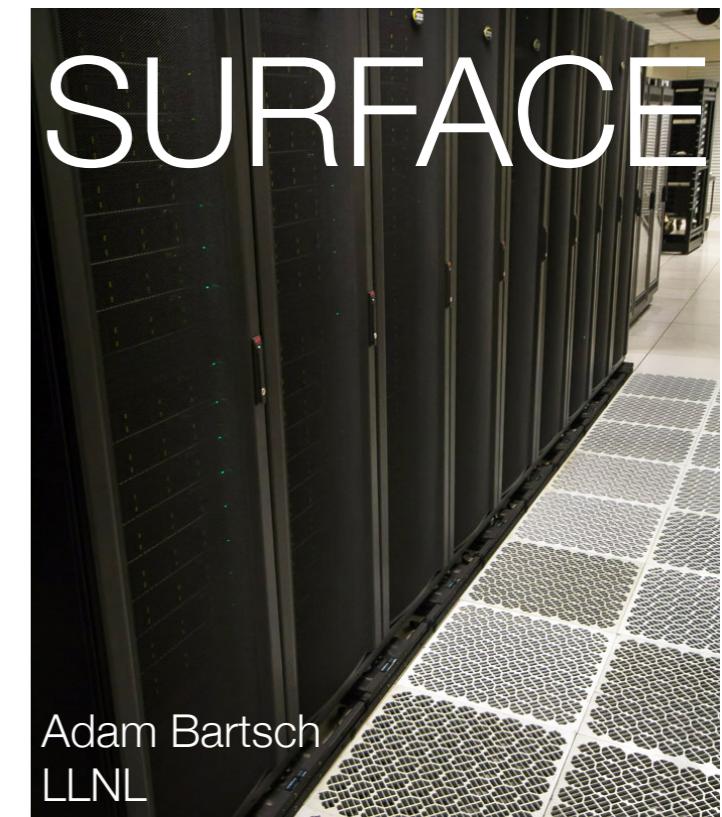
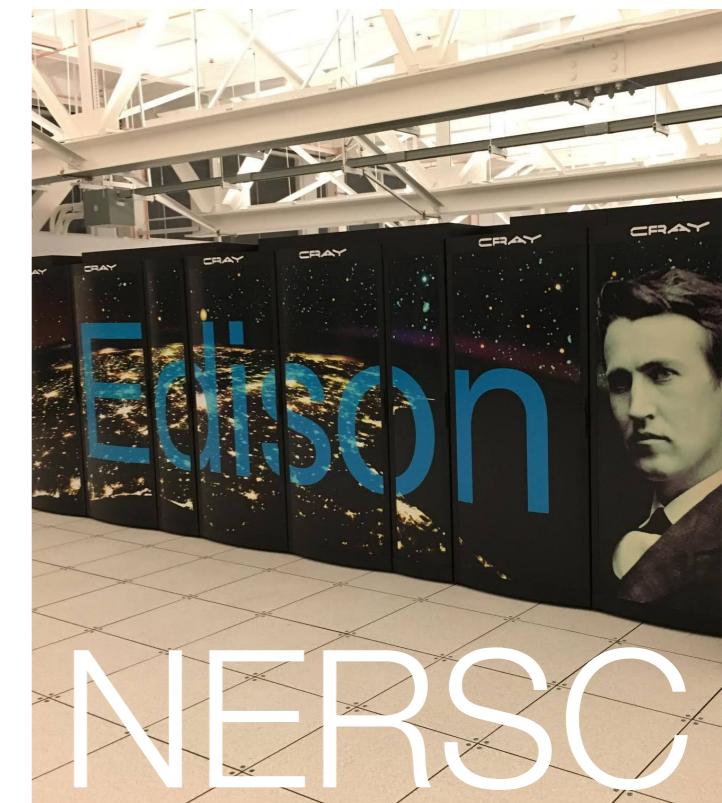


continuum limit



physical quark masses

infinite volume limit

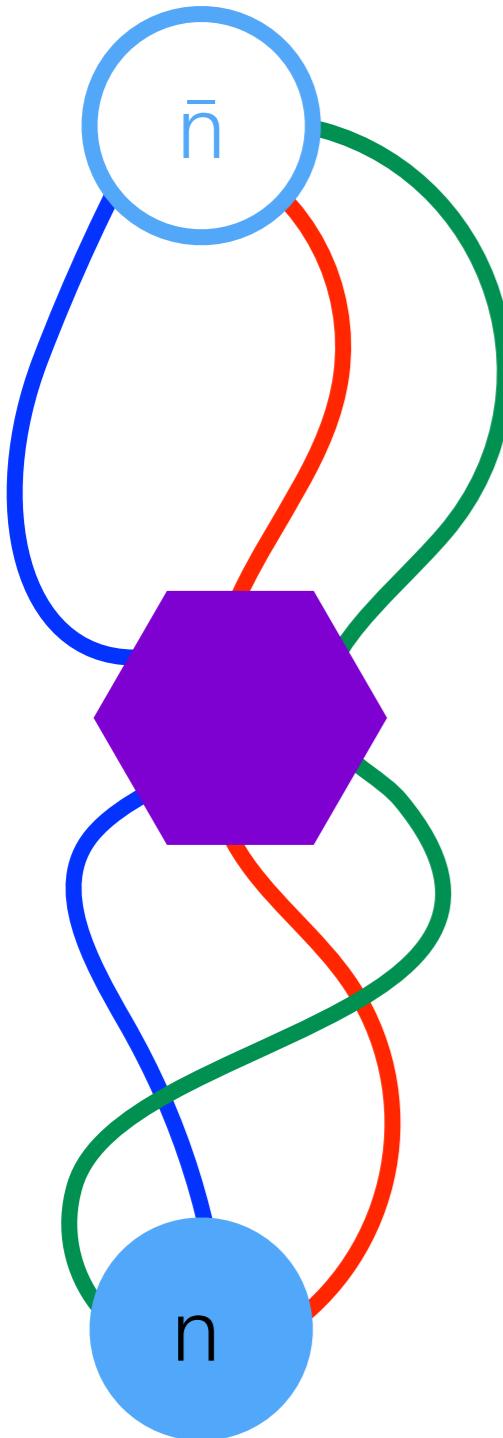


gA

Enrico's talk Thursday morning 12:10

## $\bar{n}n$ Oscillations

Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, and Wasem 1809.00246



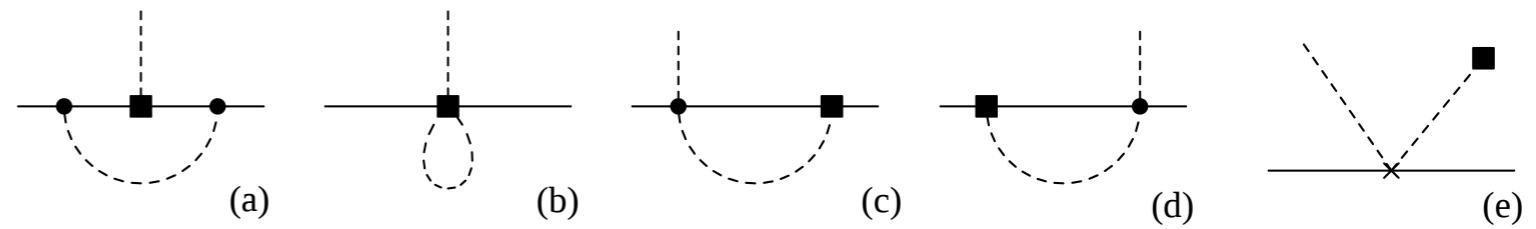
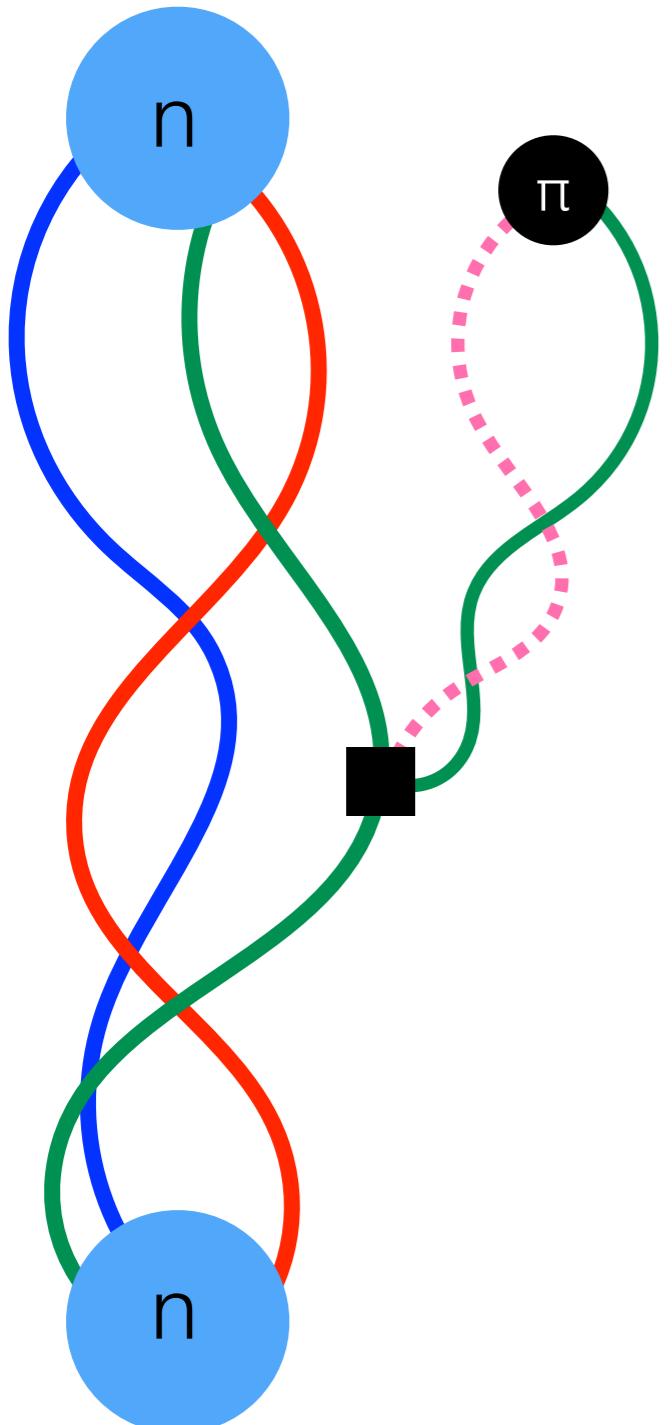
- ✓ Physical point
- ✗ continuum, infinite volume

Operator	$\overline{\text{MS}}(2 \text{ GeV}),$ $10^{-5} \text{ GeV}^6$	$\frac{\overline{\text{MS}}(2 \text{ GeV})}{\text{MIT bag B}}$	Bare, $10^{-5}$ l.u.	$\chi^2/\text{dof}$
$Q_1$	-44(19)	5.0	-3.7(1.6)	0.75
$Q_2$	140(40)	12.8	11.8(3.2)	0.69
$Q_3$	-79(23)	9.7	-6.6(1.9)	0.72
$Q_5$	-1.43(64)	2.1	-0.096(42)	0.73

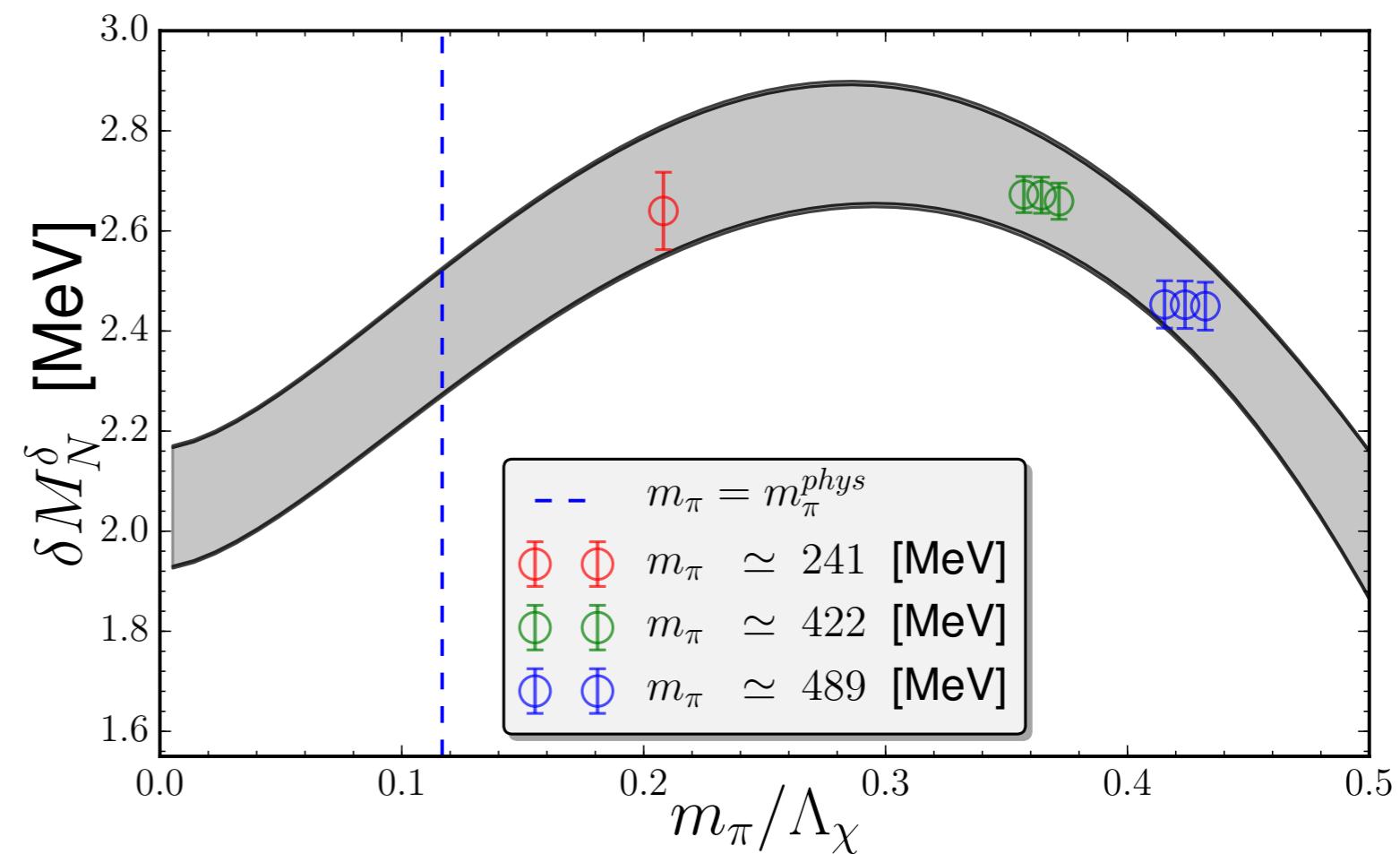
enhancement  
means experiments  
have greater reach

# CP Violating $\pi N$ Coupling

Brantley, Joo, Mastropas, Mereghetti, Monge-Camacho, Tiburzi, and Walker-Loud arXiv:1612.07733 [See also PLB 766 (2017) 254-262]



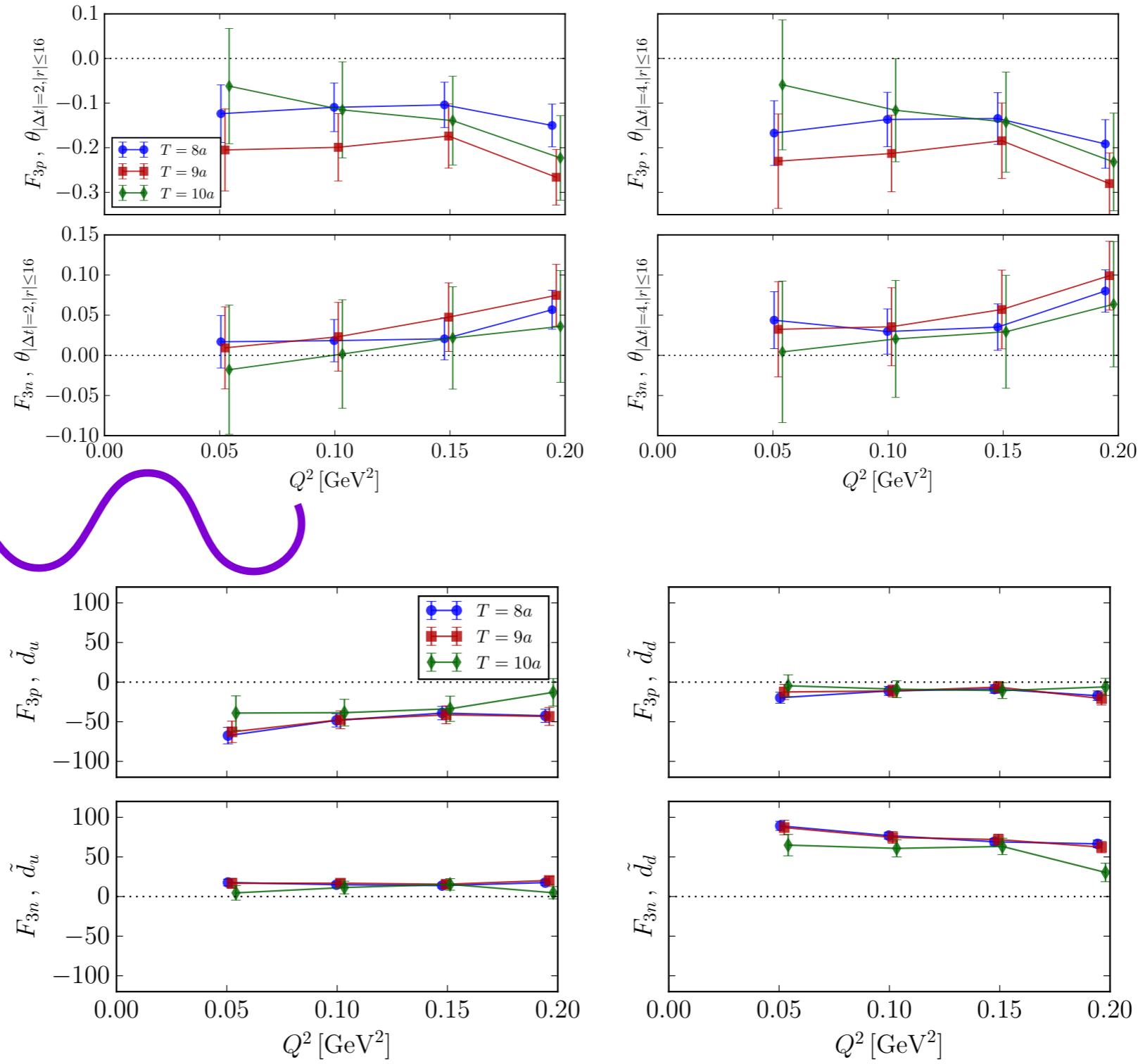
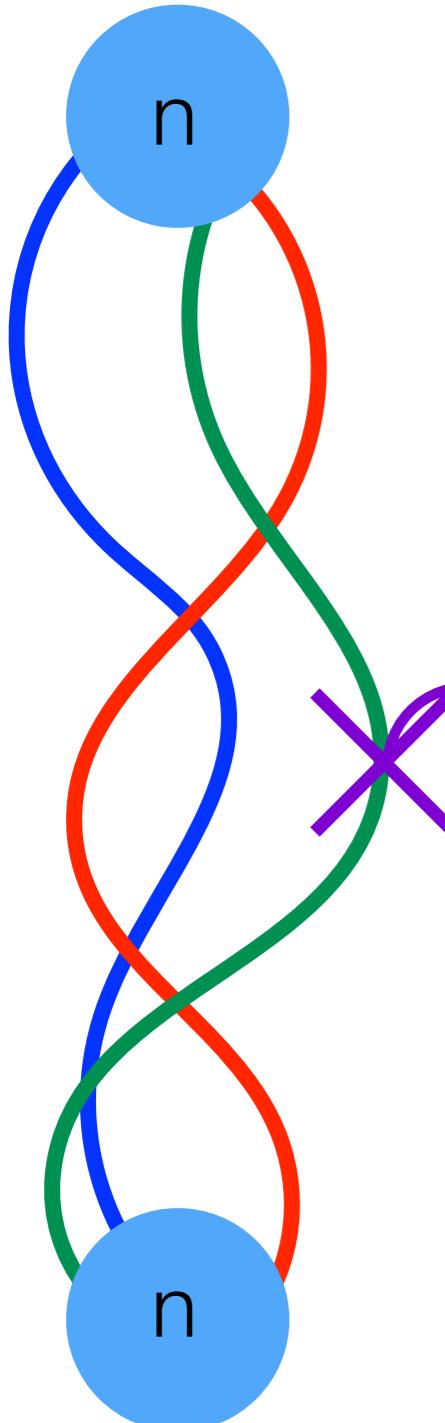
$$\bar{g}_0 = 14.7(1.8)^{\text{stat}}(1.4)^{\text{sys}} \cdot 10^{-3} \bar{\theta} f_\pi \sqrt{2}$$



# n EDM

Syritsyn, Ohki, Izubuchi CIPANP 2018 1810.03721

$V = 48^3 \times 96 (\times 24 \text{ DWF})$   
 $a = 0.114 \text{ fm}$   
 $m_\pi = 139.2 \text{ MeV}$

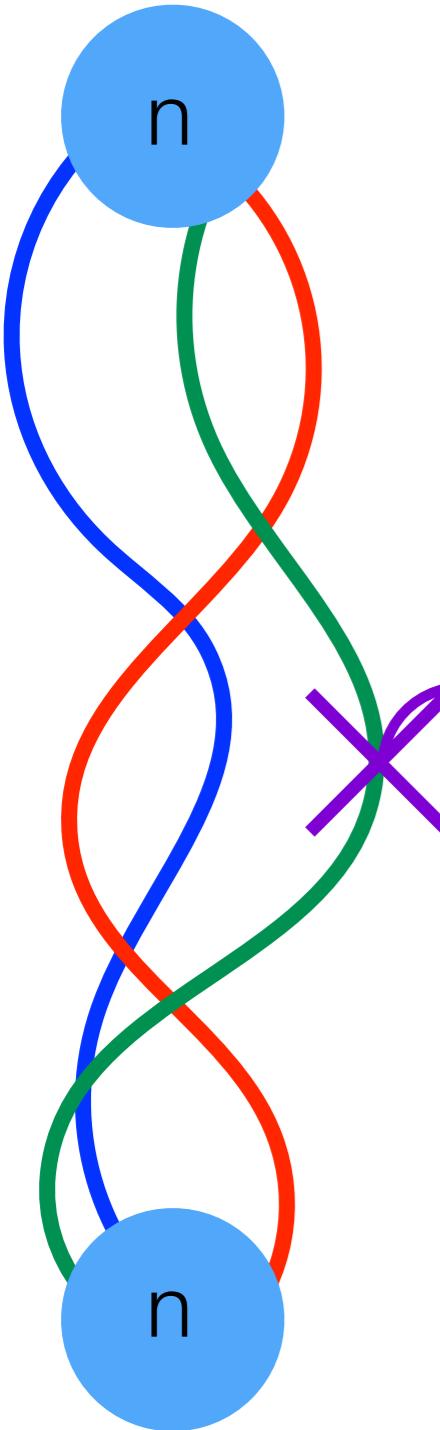


$\Theta_{\text{QCD}}$

cEDM

# n EDM

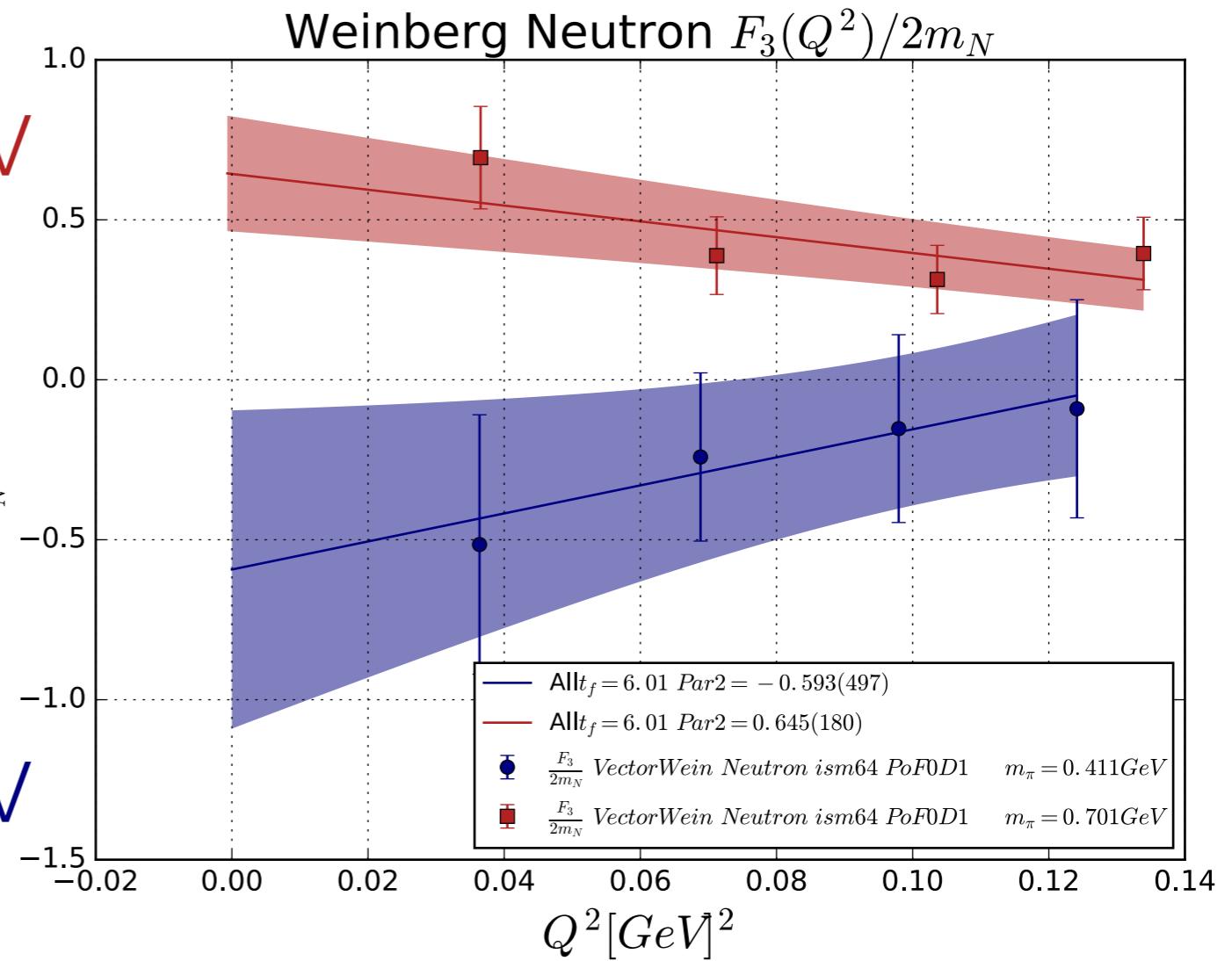
Dragos, Luu, Shindler, de Vries LATTICE 2017 EPJ Web Conf 175 (2018) 06018 arXiv:1711.04730



$m_\pi \sim 700 \text{ MeV}$

$m_\pi \sim 411 \text{ MeV}$

$$\frac{a^2 F_3(Q^2)}{2m_N} [efm]$$

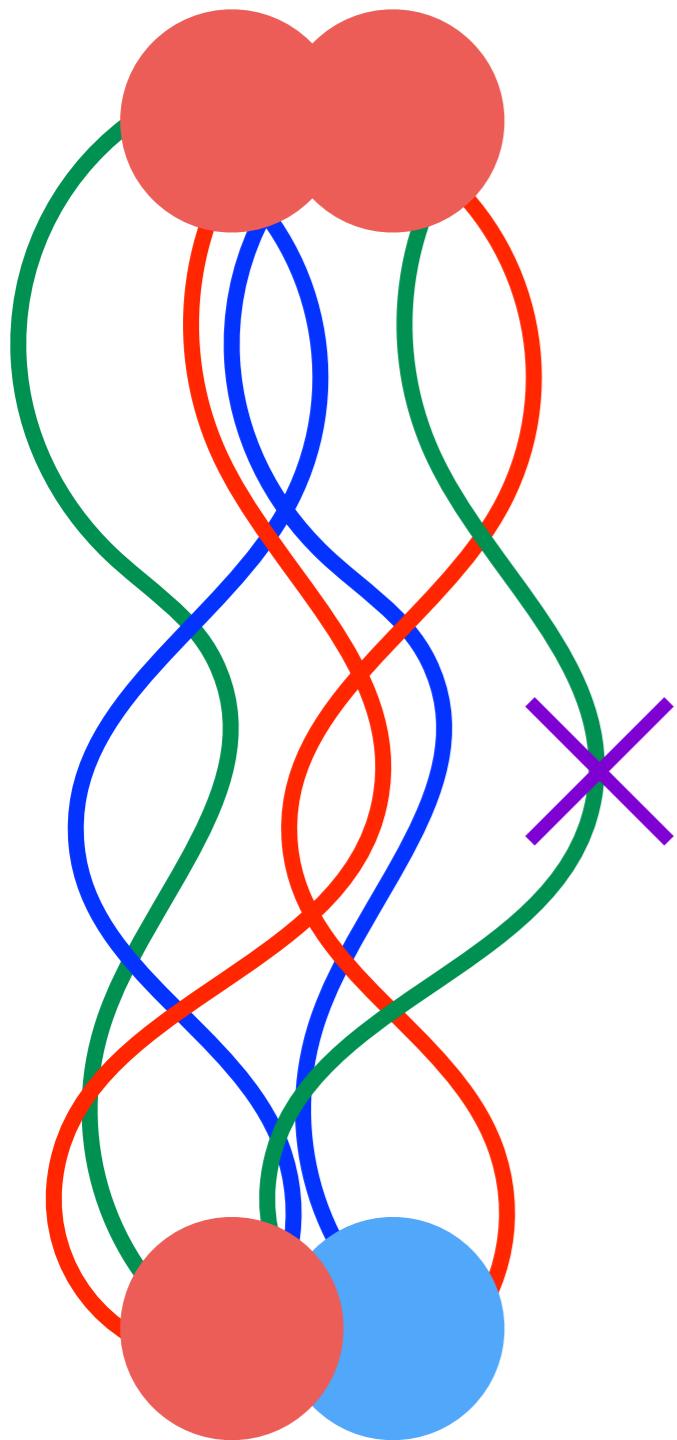


$$-i \frac{\alpha_{\tilde{G}}}{\Lambda^2} \frac{1}{3} f^{ABC} \tilde{G}_{\mu\nu}^A G_{\mu\rho}^B G_{\nu\rho}^C$$

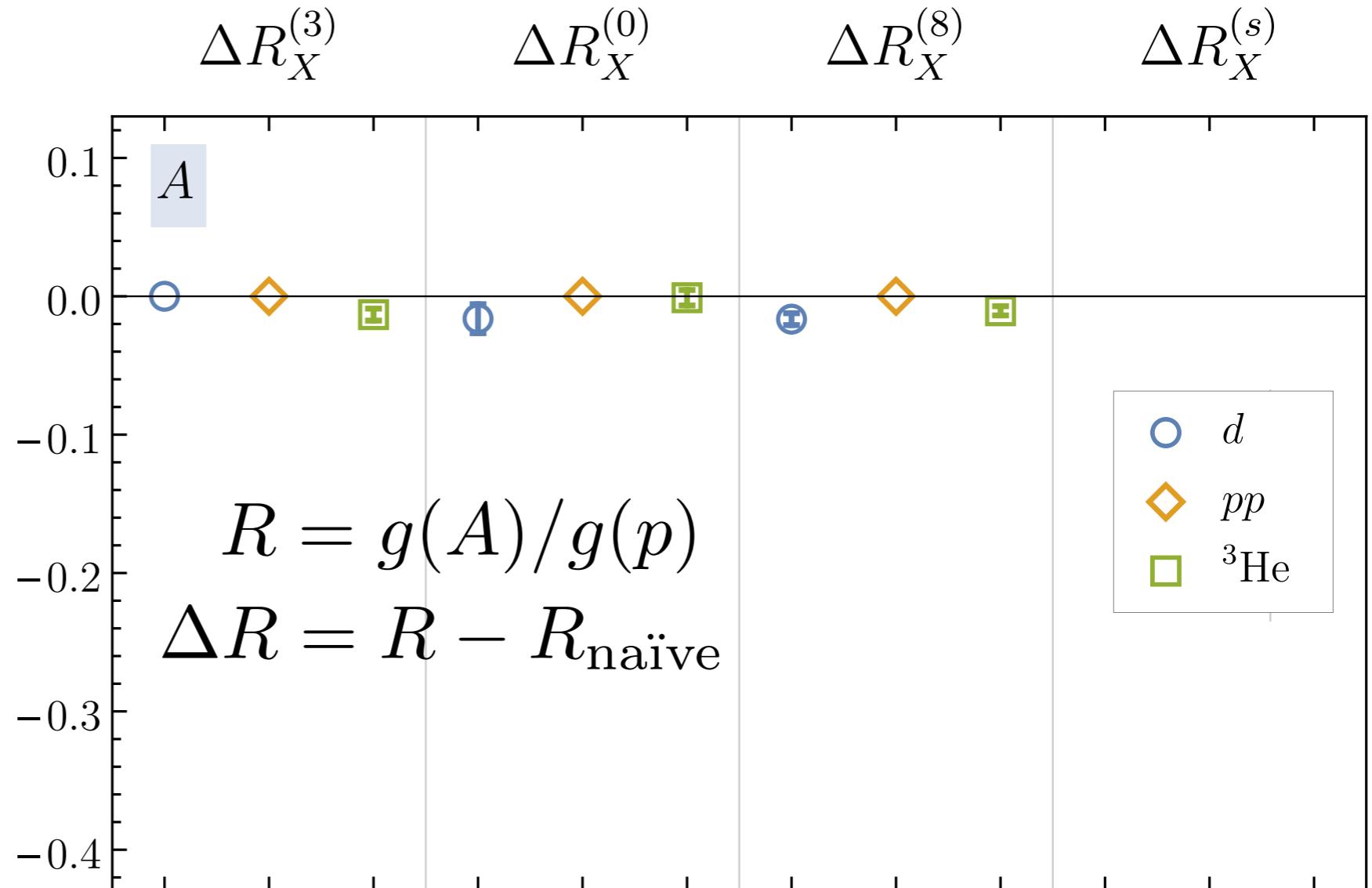
# $g_A$ Quenching

NPLQCD PRL 120 (2018) 15 152002 arXiv:1712.03221

$m_\pi \sim 800$  MeV;  $a \sim 0.145$  fm



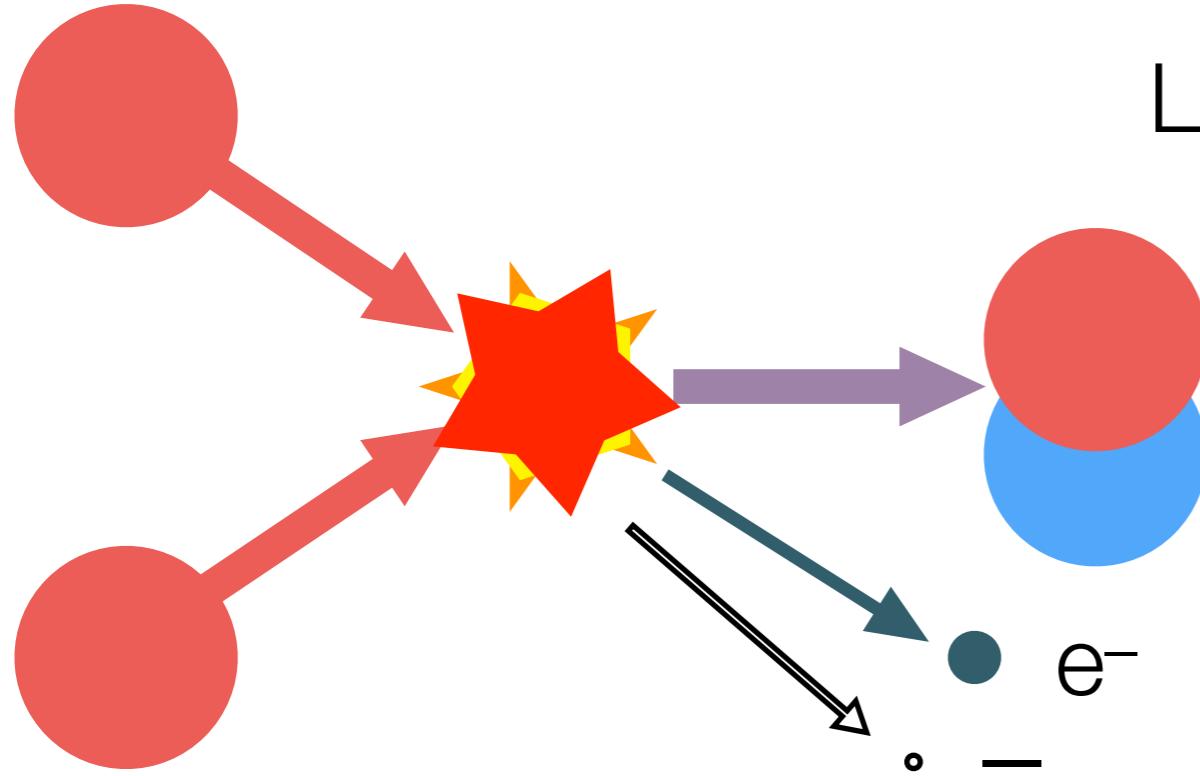
different flavor structures



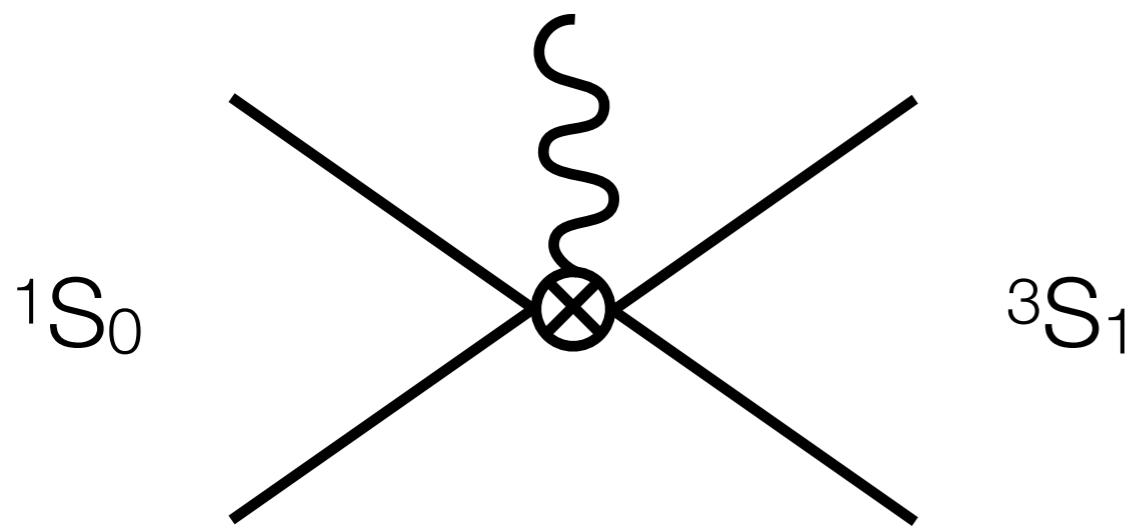
# pp Fusion

NPLQCD PRL 119 (2017) 06 062002 arXiv:1610.04545

$m_\pi \sim 800 \text{ MeV}; a \sim 0.145 \text{ fm}$



$$L_{1,A} = 3.9(0.2)^{\text{stat}}(1.0)^{\text{fit}}(0.4)^{\text{mass}}(0.9)^{\text{EFT}}$$

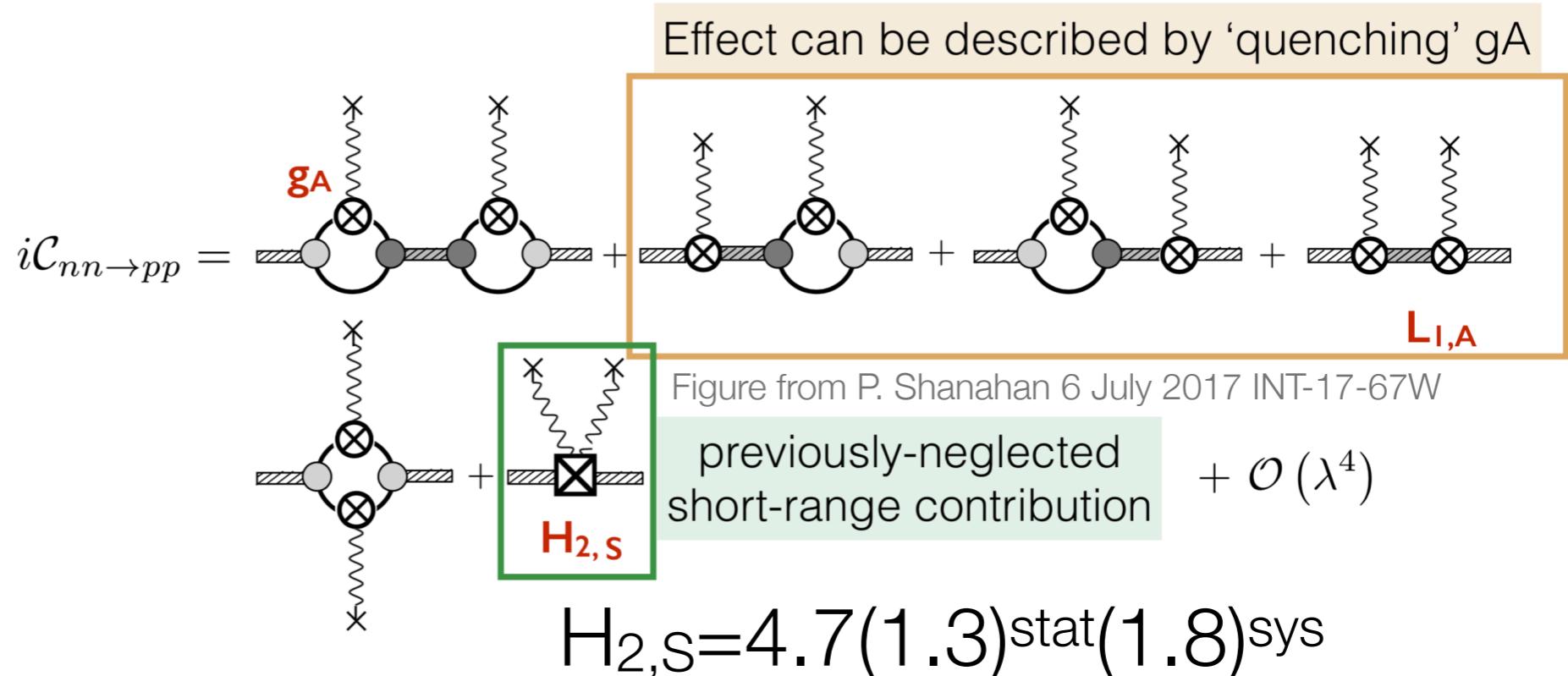
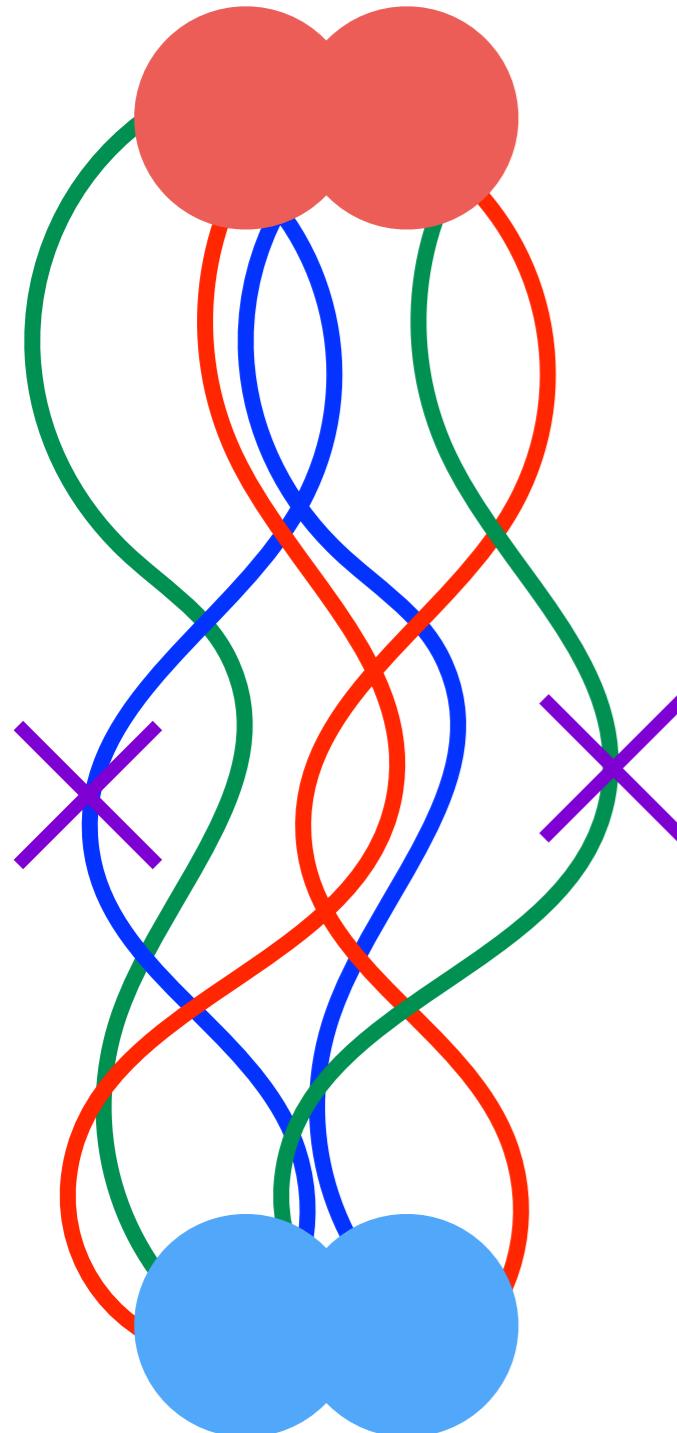


- Pionless EFT
- Dineutron bound at 800 MeV
- Just need binding energies, matrix element from the lattice

# Isotensor Polarizability ( $2\beta_{vv}$ )

NPLQCD PRL 119 (2017) 06 062003 arXiv:1701.03456

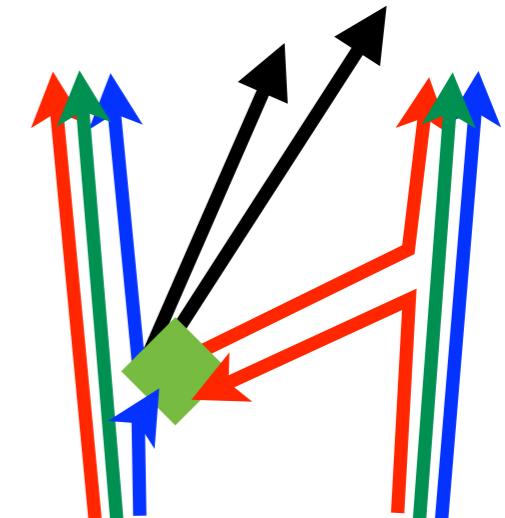
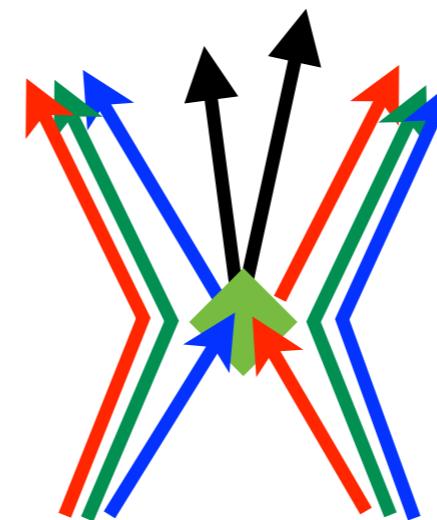
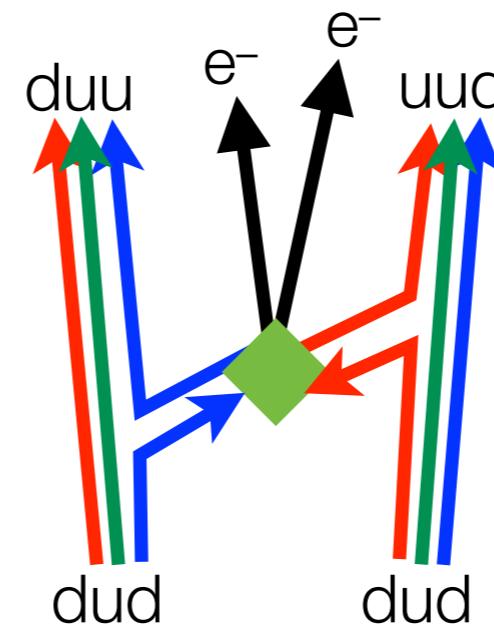
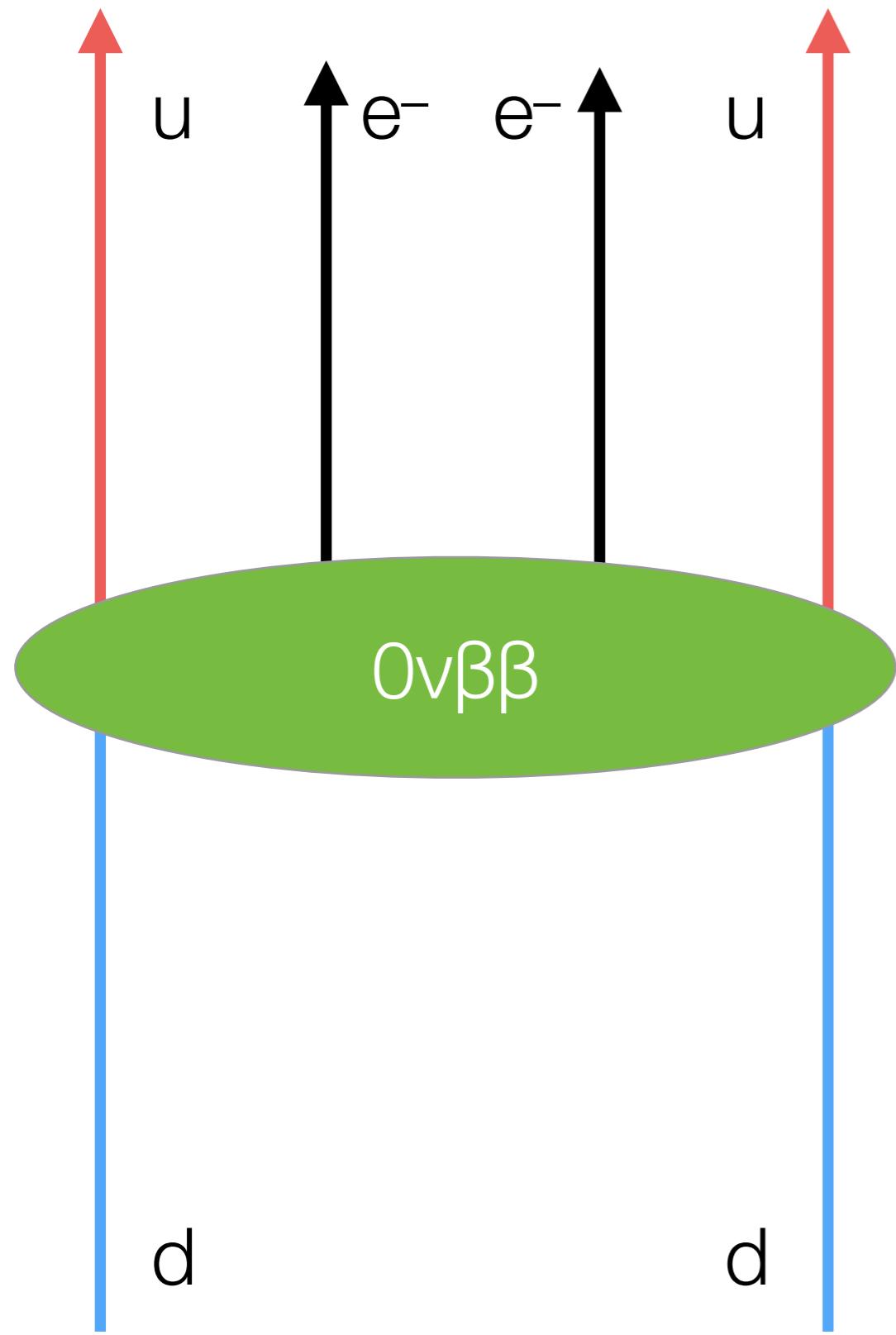
$m_\pi \sim 800 \text{ MeV}; a \sim 0.145 \text{ fm}$



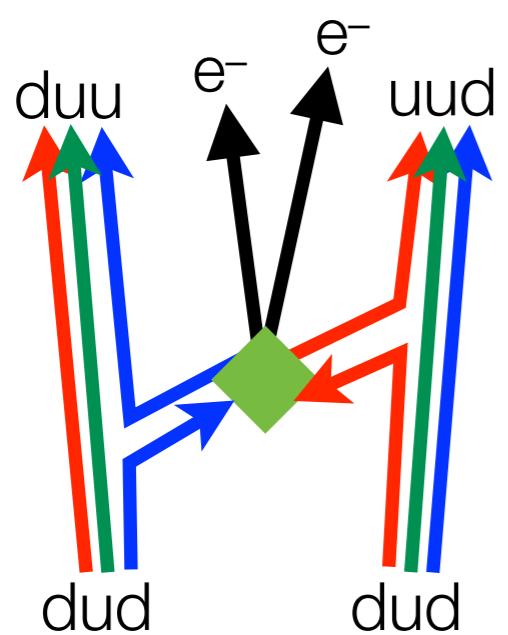
- Pionless EFT
- Dineutron bound at 800 MeV
- Binding energies, matrix element from the lattice

# Short Range $0\nu\beta\beta$

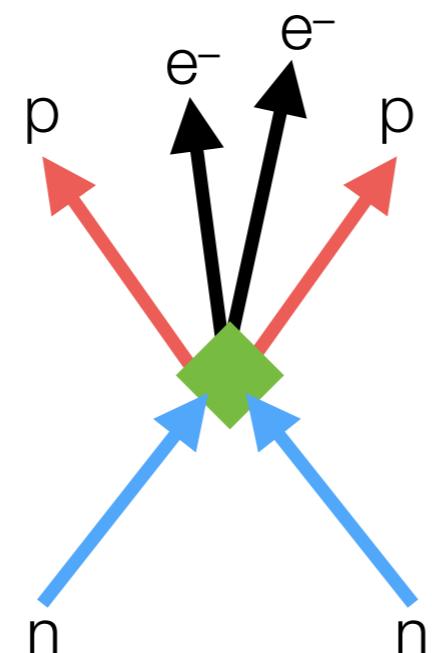
CalLat PRL 121 (2018) 17 172501 arXiv:1805.02634



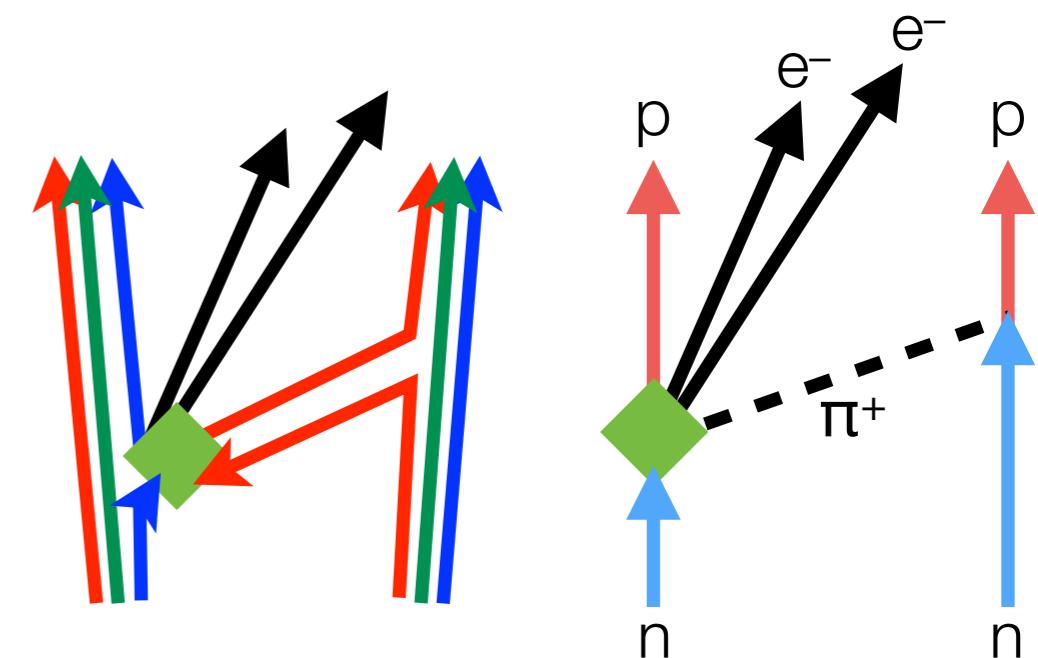
# In xPT



$\mathcal{O}(p^{-1})$  new  $\pi N$  vertex

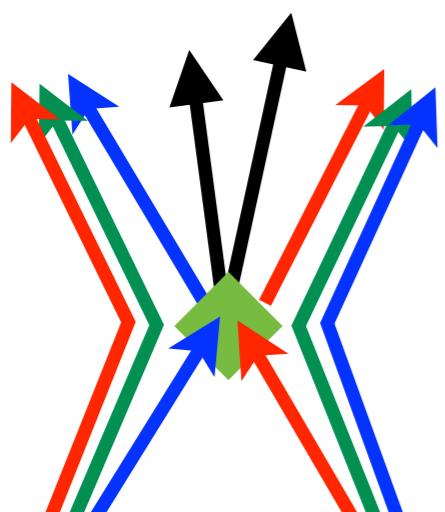


$\mathcal{O}(p^{-2})$  long-range  $\pi$  exchange



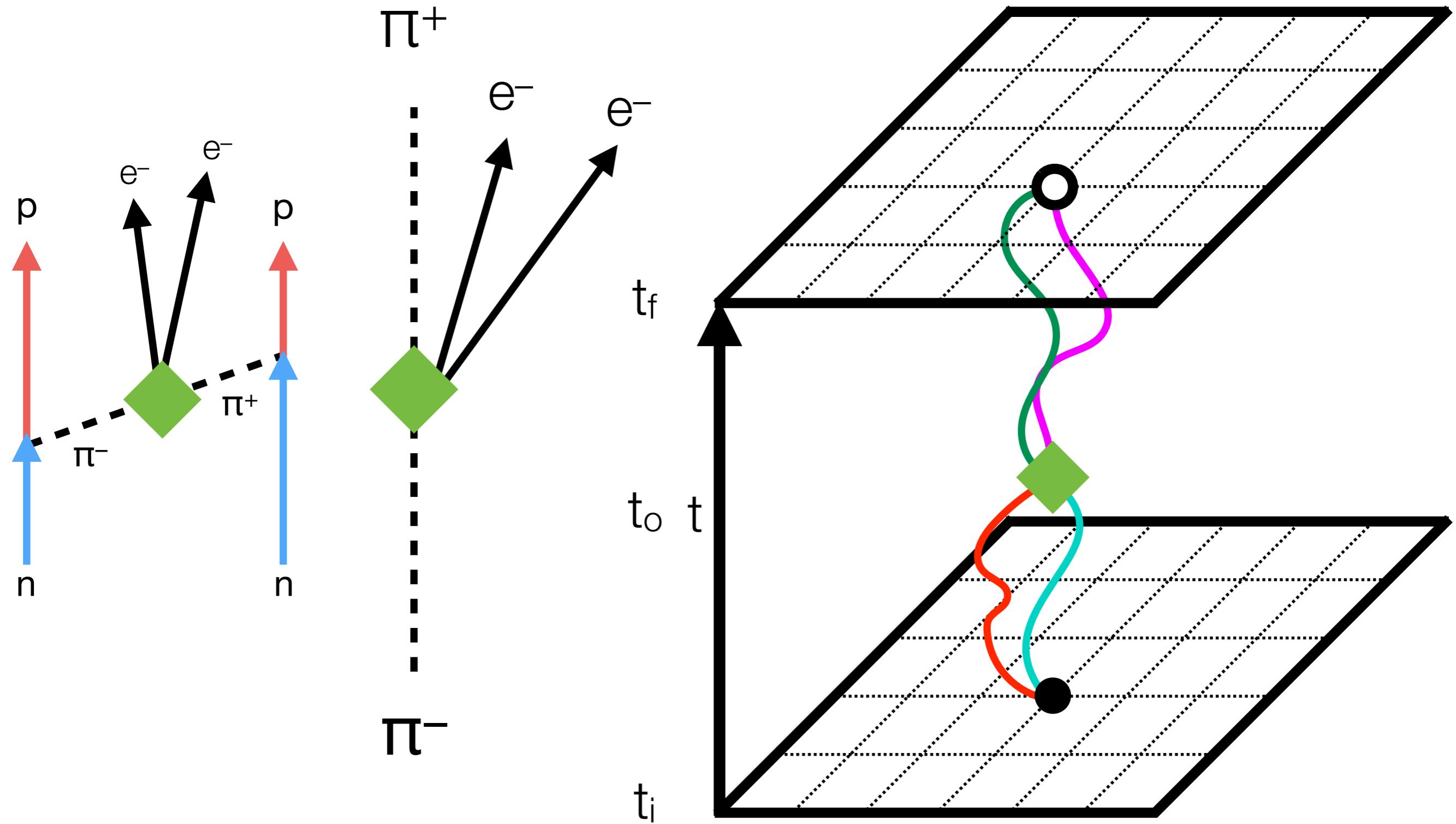
$\mathcal{O}(p^0)$  NN contact operator

Can be promoted, as in Cirigliano, Dekens, Mereghetti and Walker-Loud PRC 97 (2018) 06 065501 arXiv:1710.01729



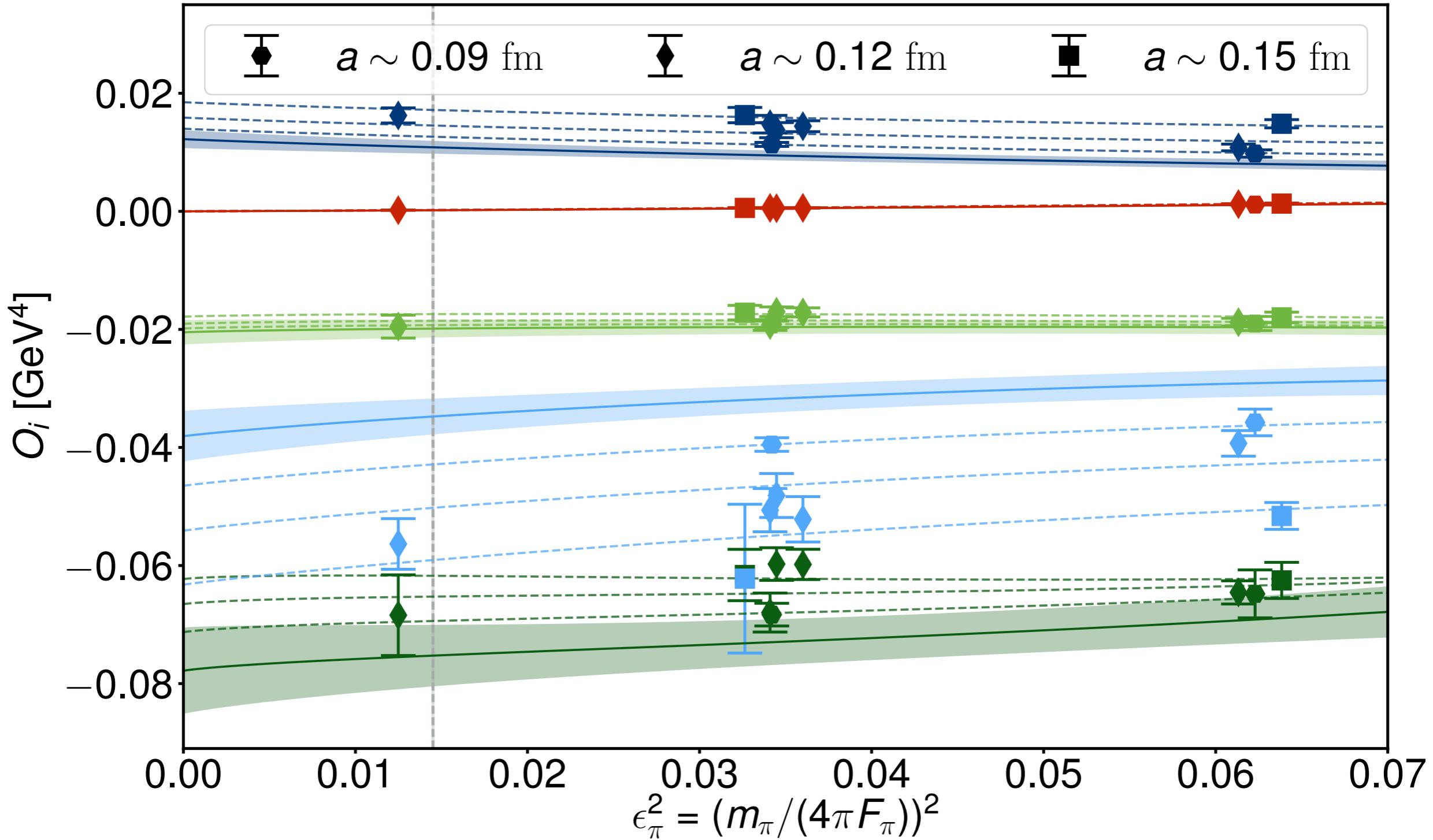
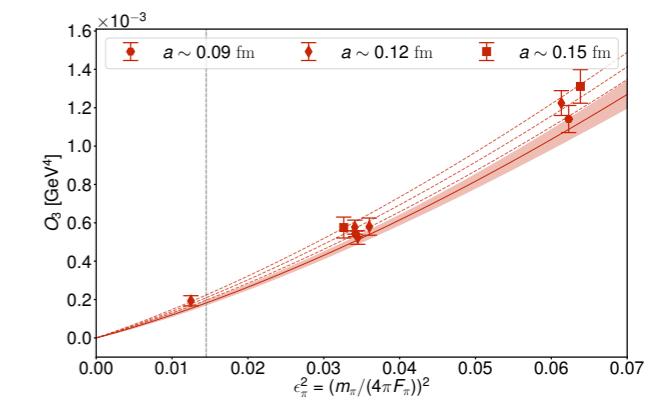
# Short Distance $0\nu\beta\beta$ $\pi^+ \pi^-$ Transition

CalLat PRL 121 (2018) 17 172501 arXiv:1805.02634



# Short-distance $0\nu\beta\beta$

CalLat PRL 121 (2018) 17 172501 arXiv:1805.02634



Data + jupyter notebook available on GitHub [https://github.com/callat-qcd/project\\_0vbb](https://github.com/callat-qcd/project_0vbb)

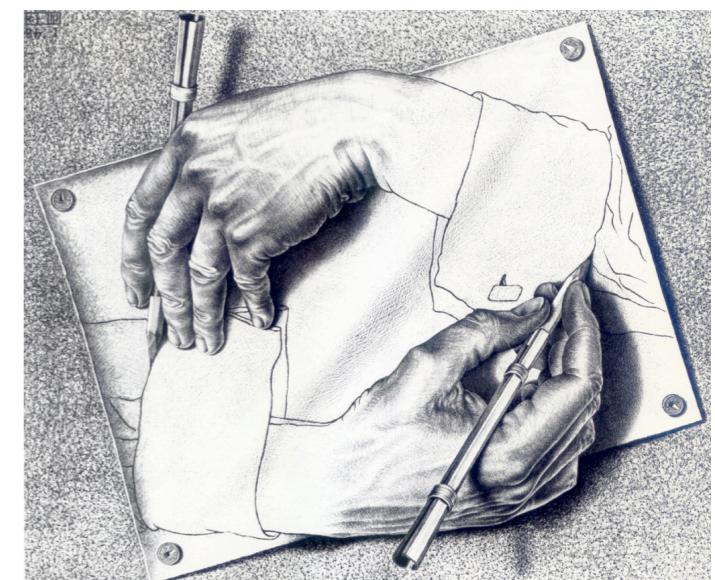
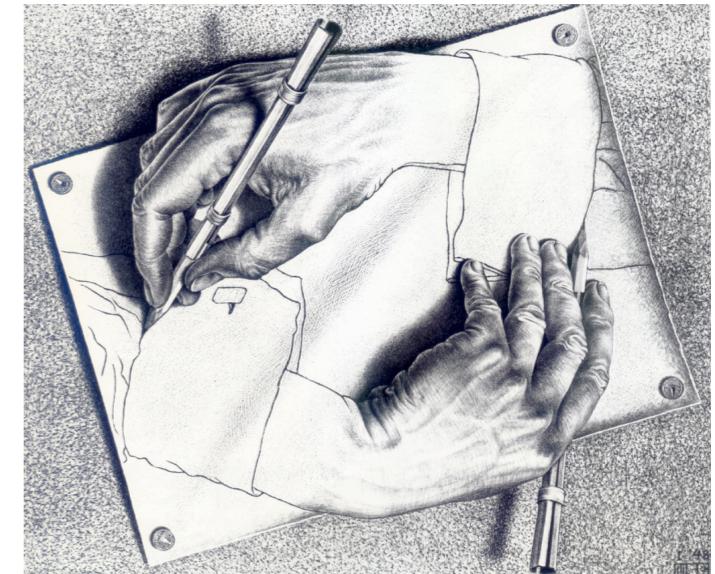
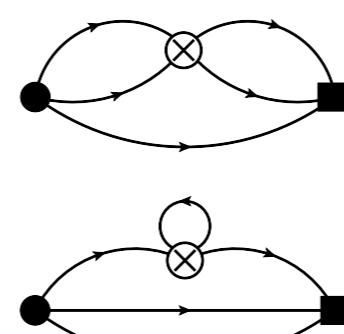
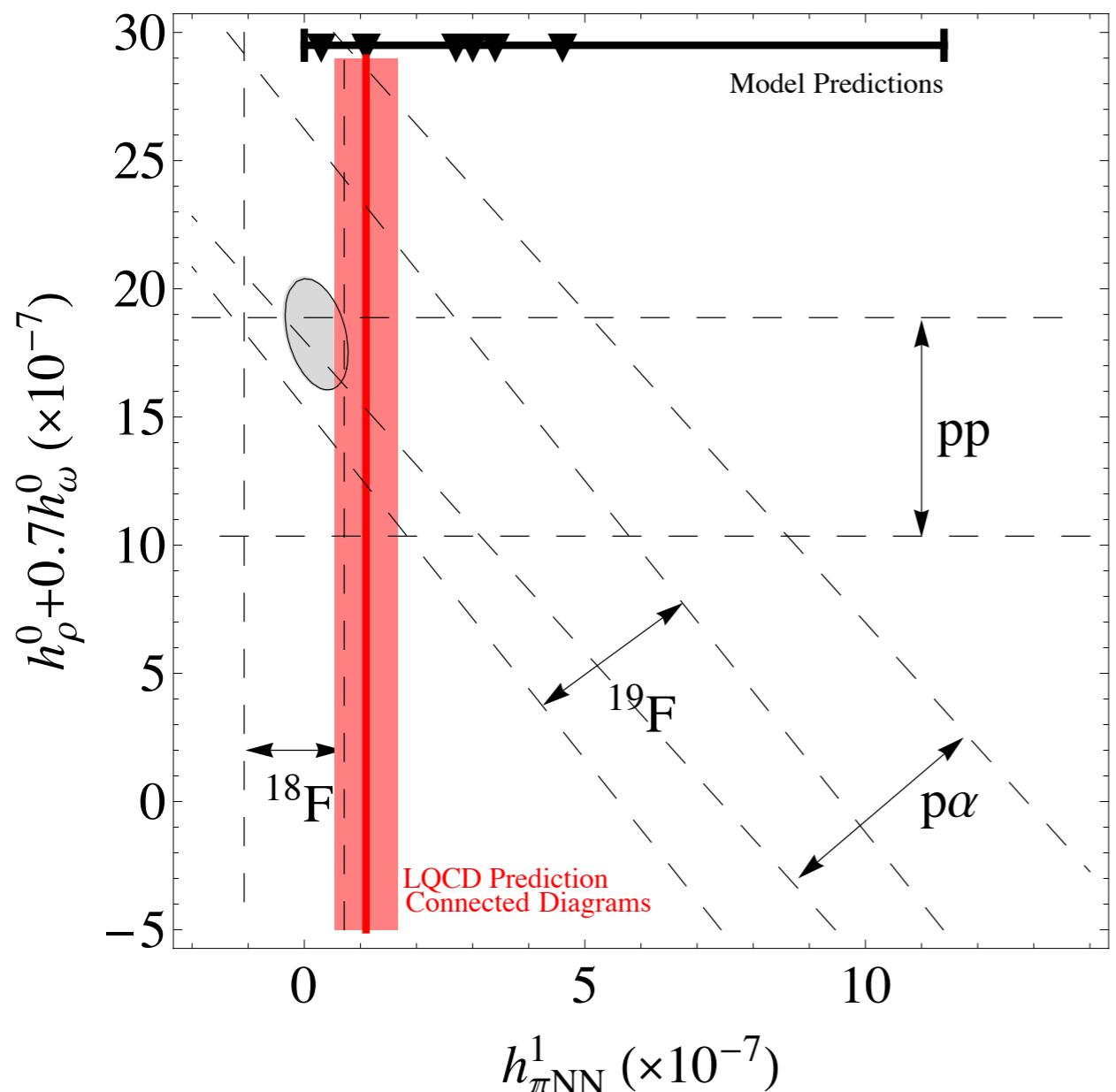
# Hadronic Parity Violation

Wasem PRC85 (2012) 022501 arXiv:1108.1151

$m_\pi \sim 389 \text{ MeV}; a \sim 0.123 \text{ fm}$

$$\mathcal{L}_{PV}^{\pi NN} = h_{\pi NN}^1 (\bar{p}\pi^+ n - \bar{n}\pi^- p)$$

$$h_{\pi NN}^{1,\text{con}} = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7}$$



$PV \neq VP$

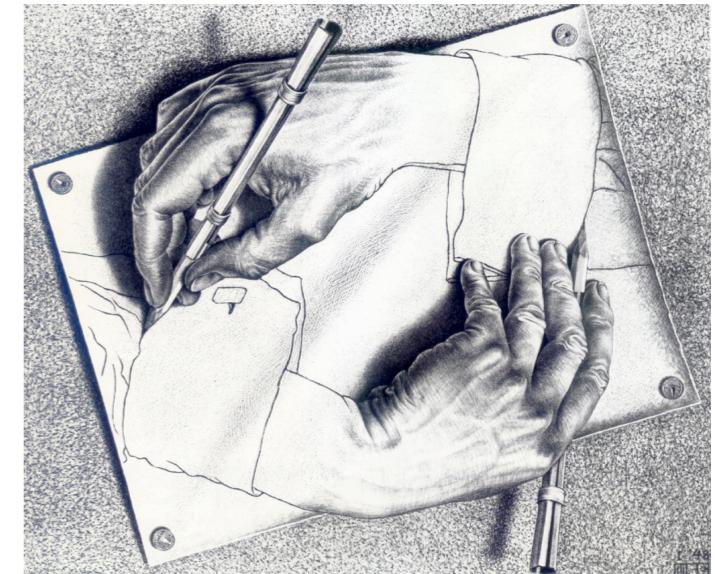
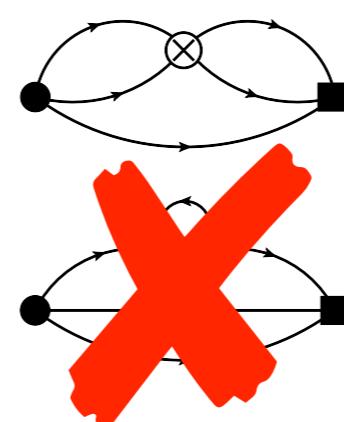
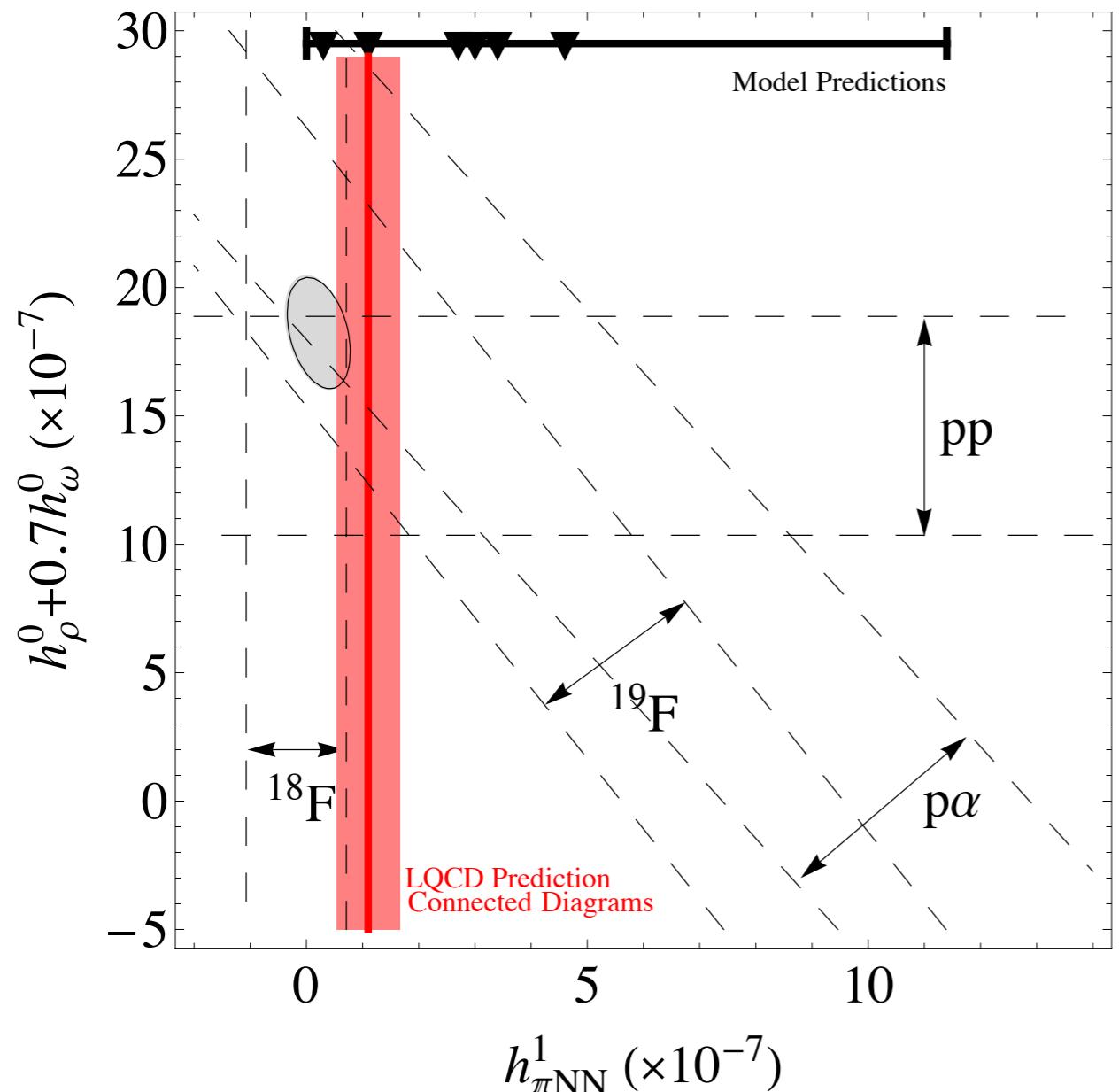
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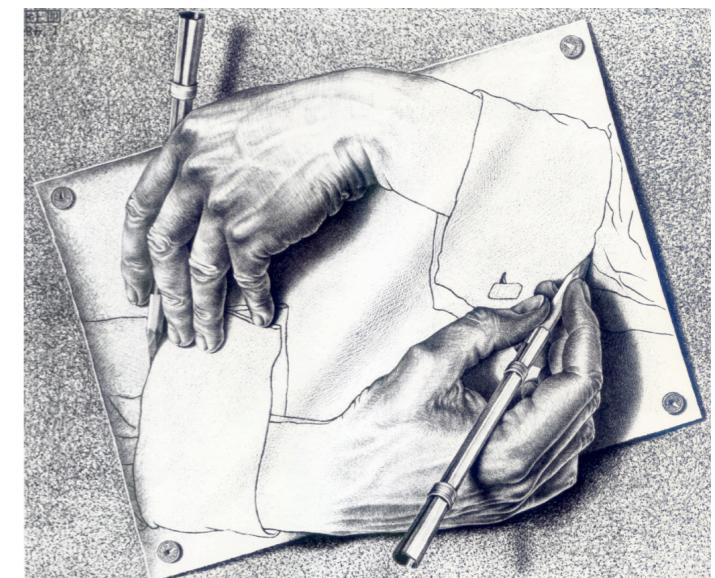
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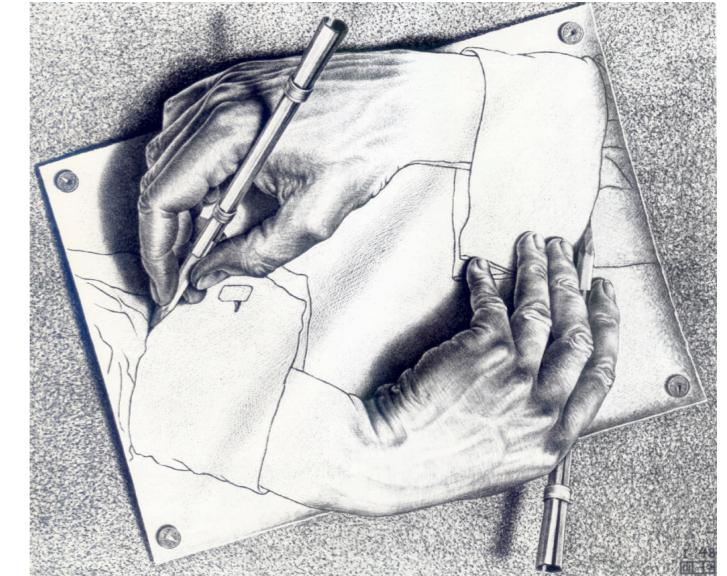
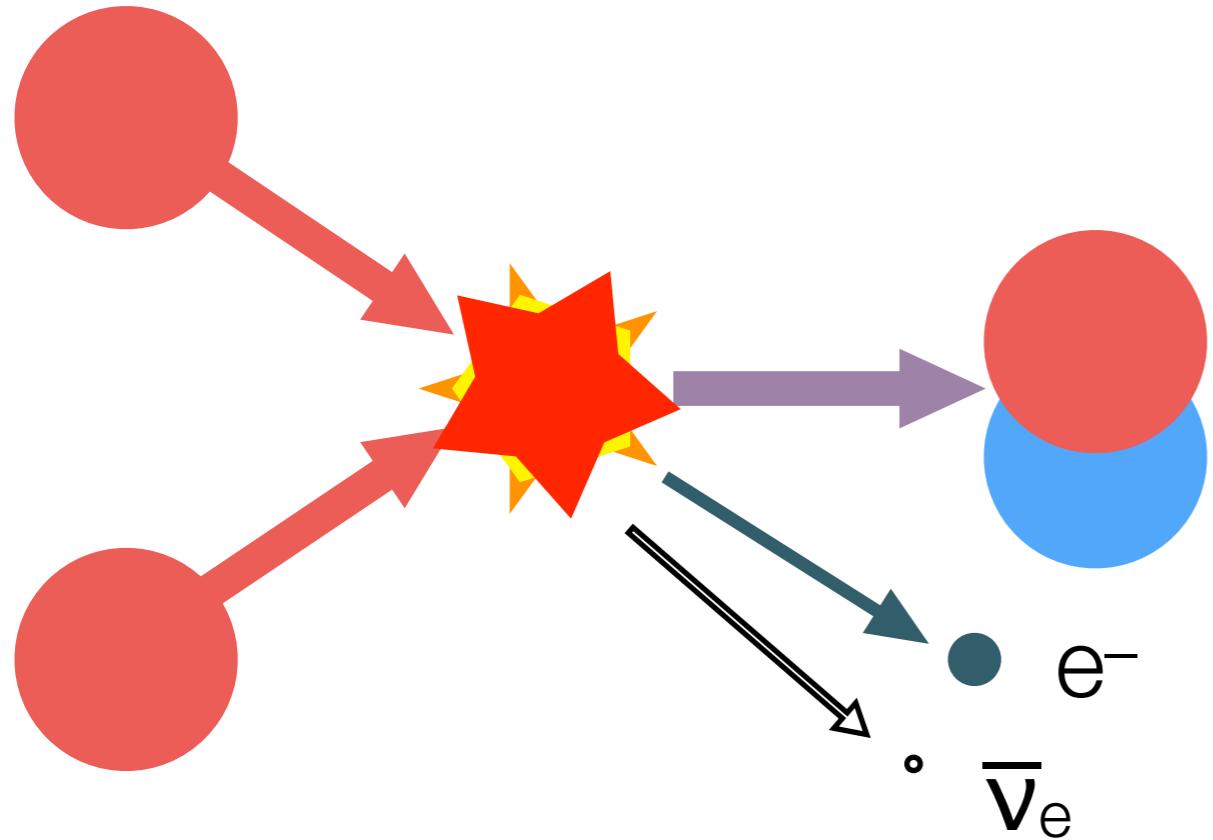
PV  $\neq$  VP



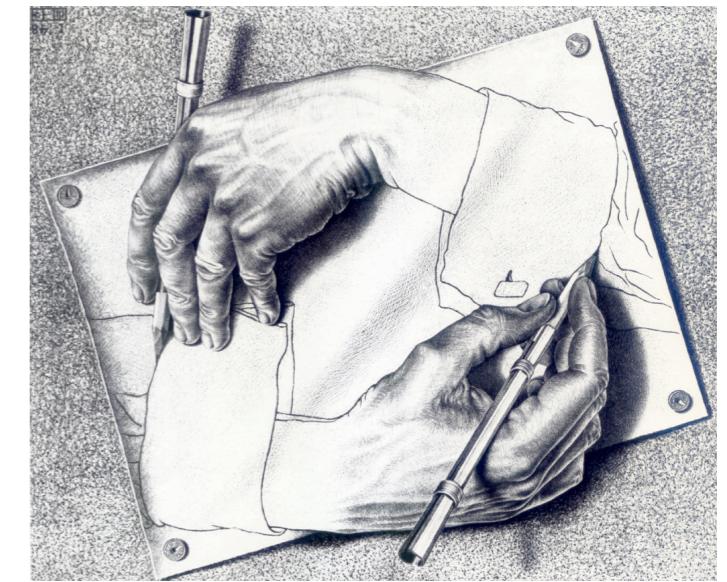
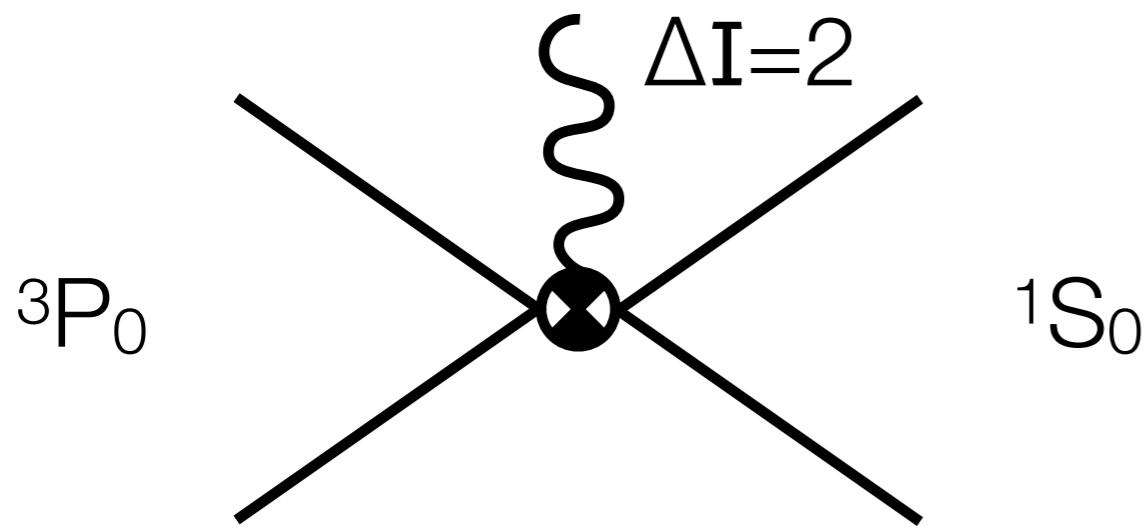
# Hadronic Parity Violation

CalLat PoS(LATTICE 2015)329 arXiv:1511.02260

$m_\pi \sim 800 \text{ MeV}; a \sim 0.145 \text{ fm}$



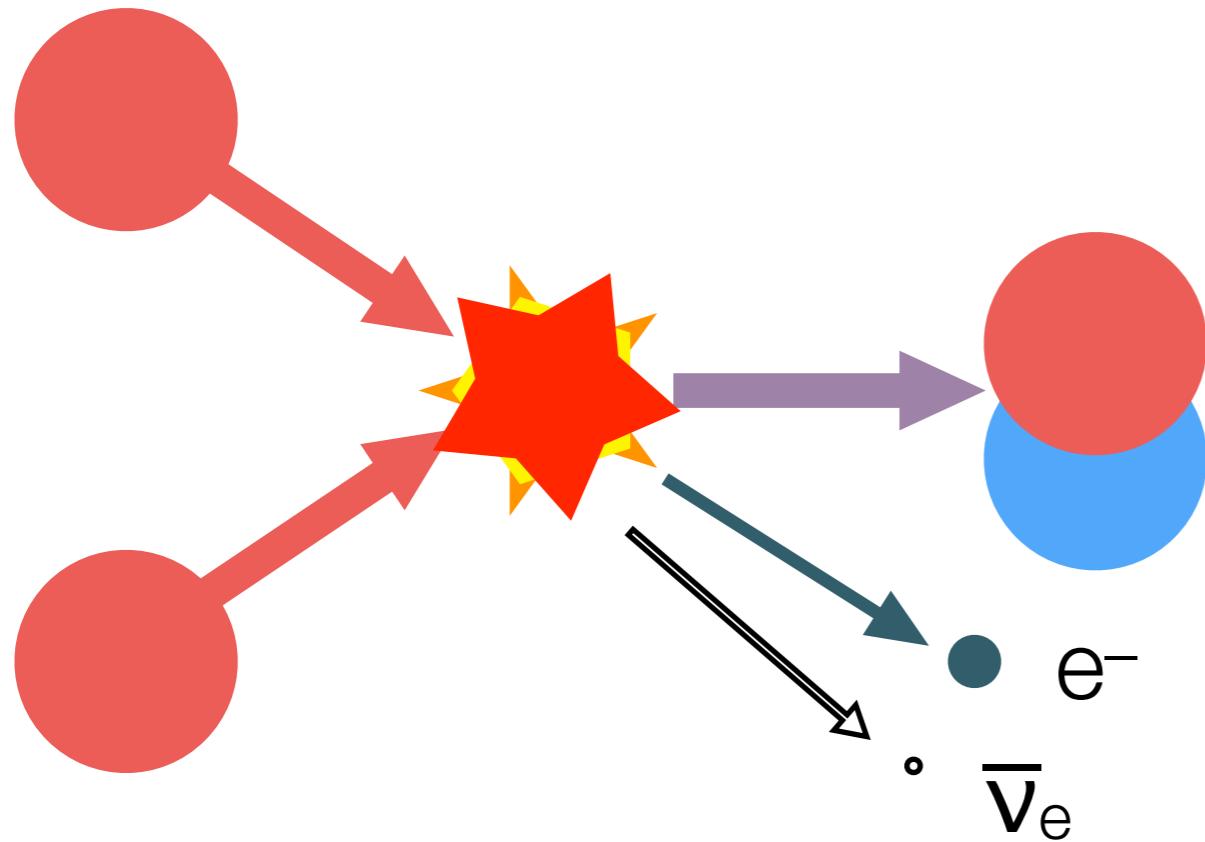
$$PV \neq VP$$



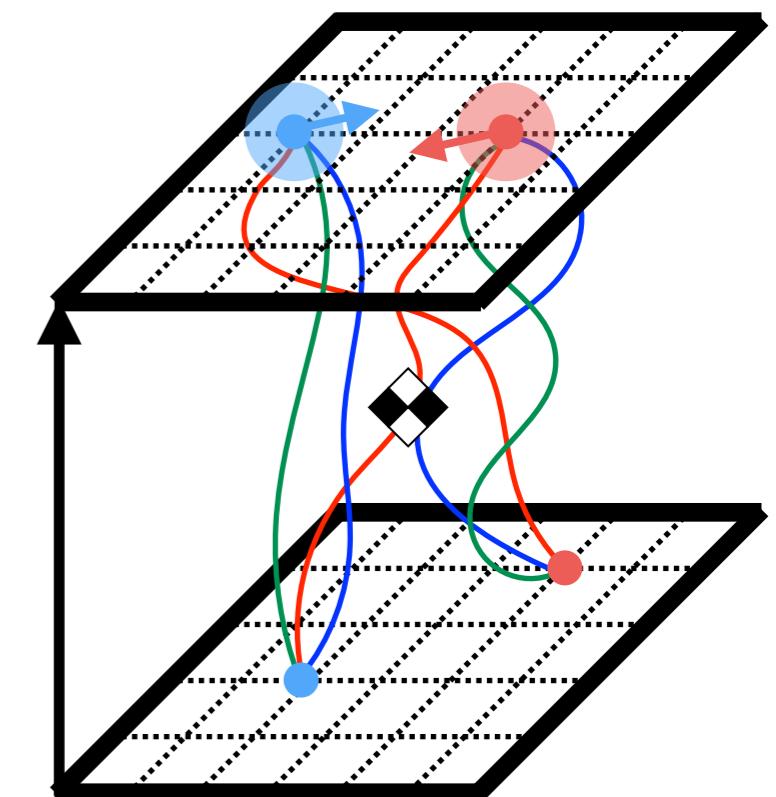
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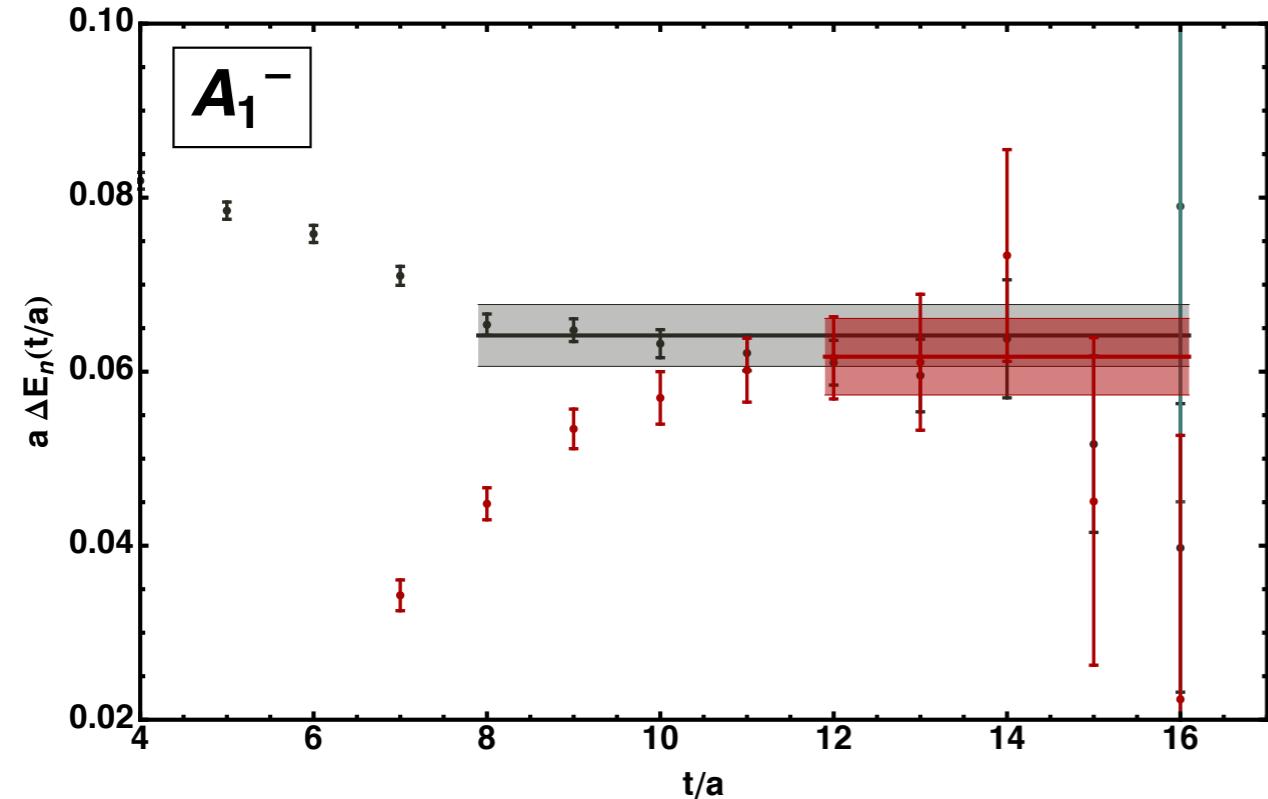
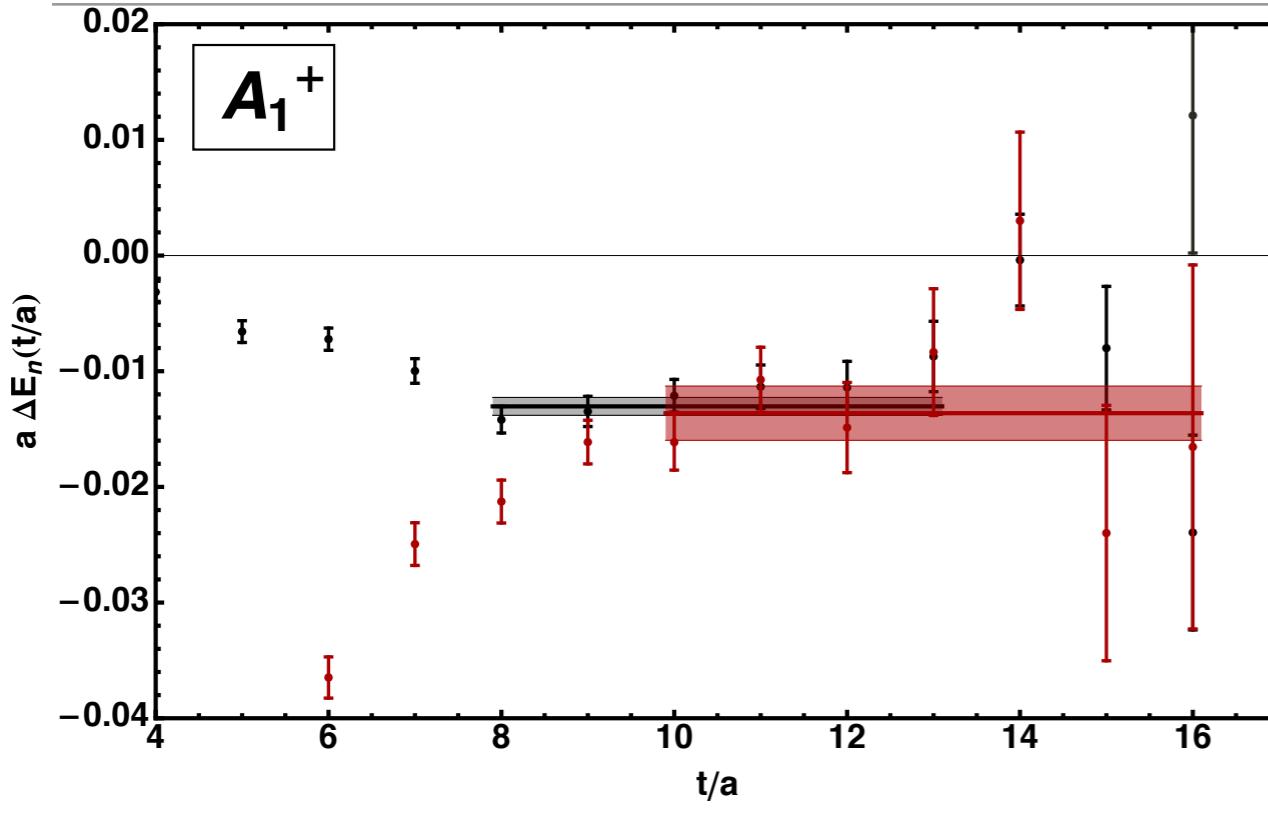


- $^3P_0$  is not bound
- Need  $^1S_0$  binding energy,  $^3P_0$  phase shift + derivative, and matrix element from the lattice

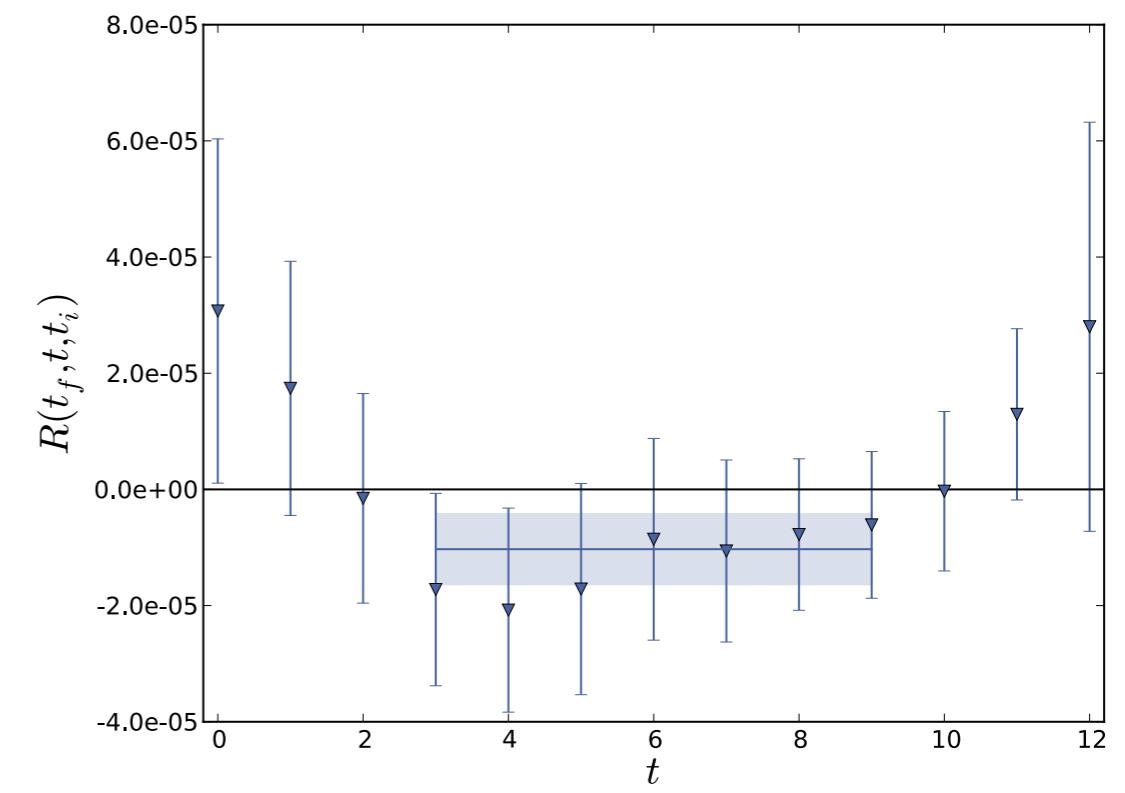
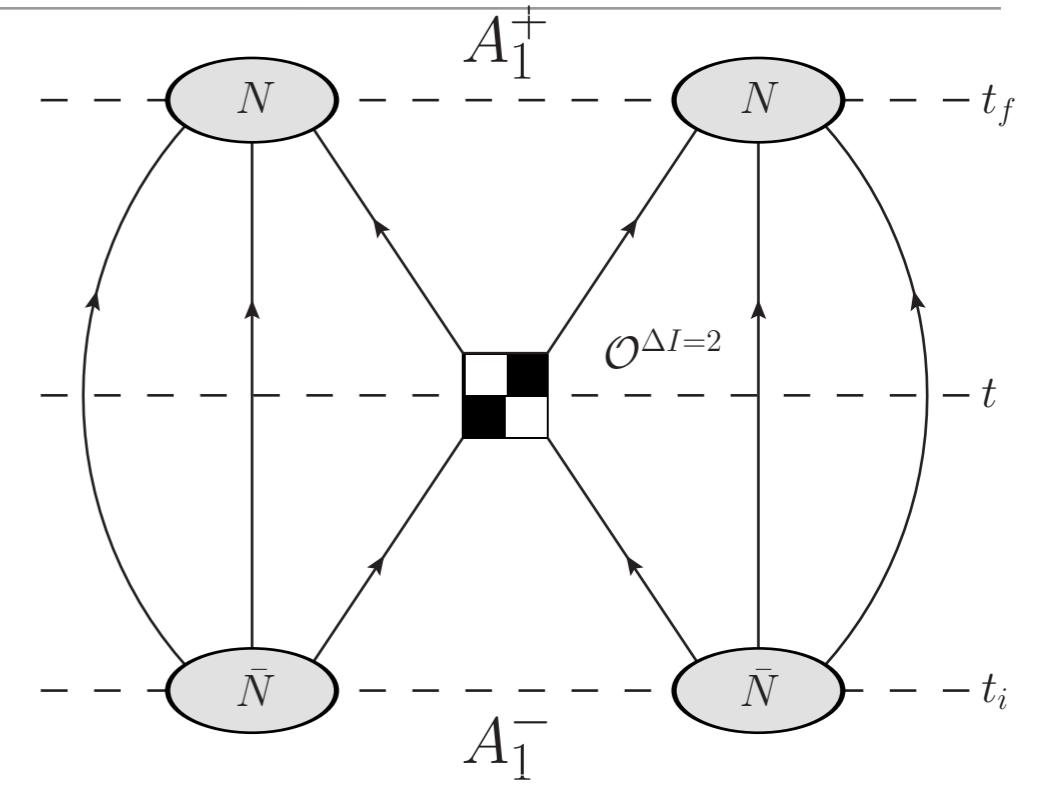


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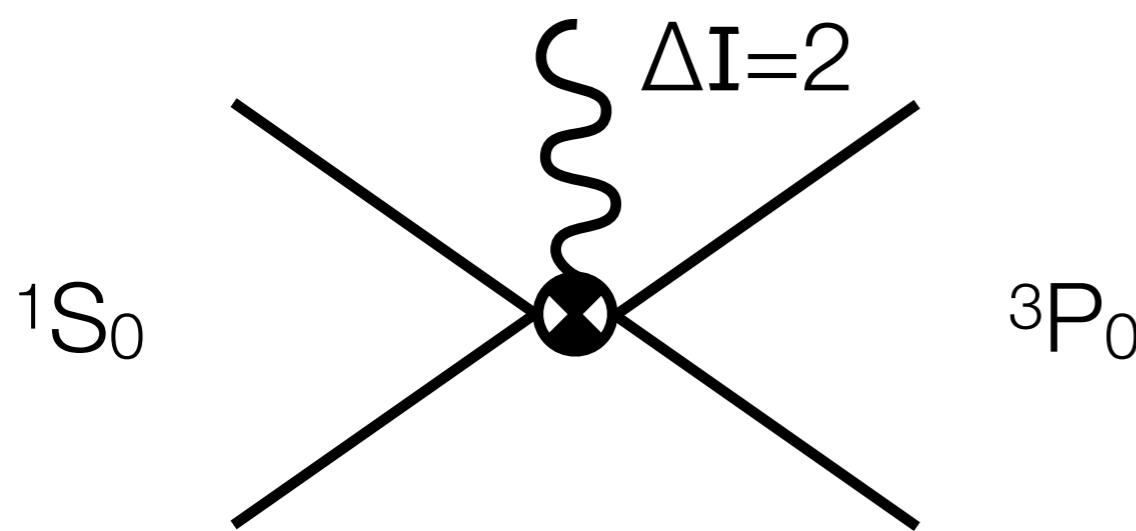
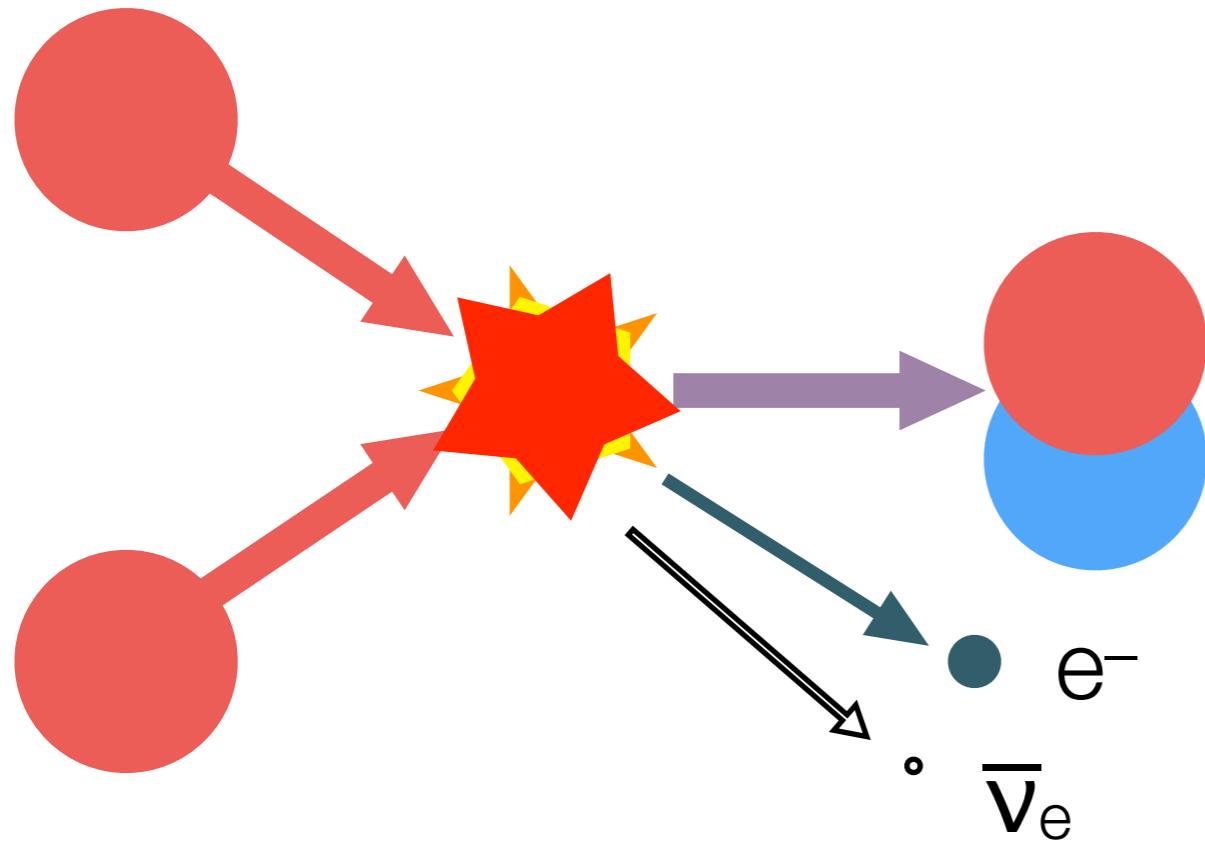
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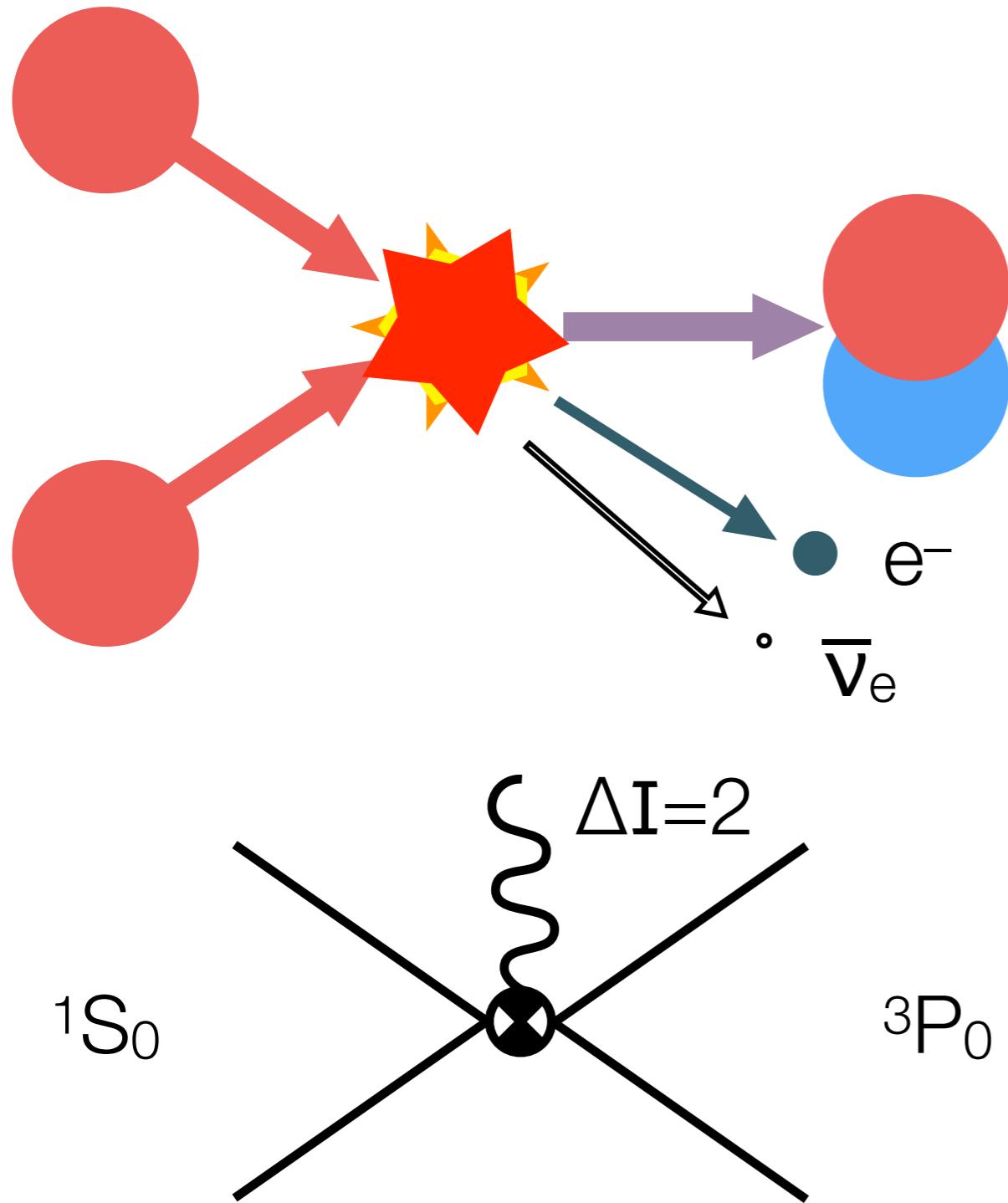


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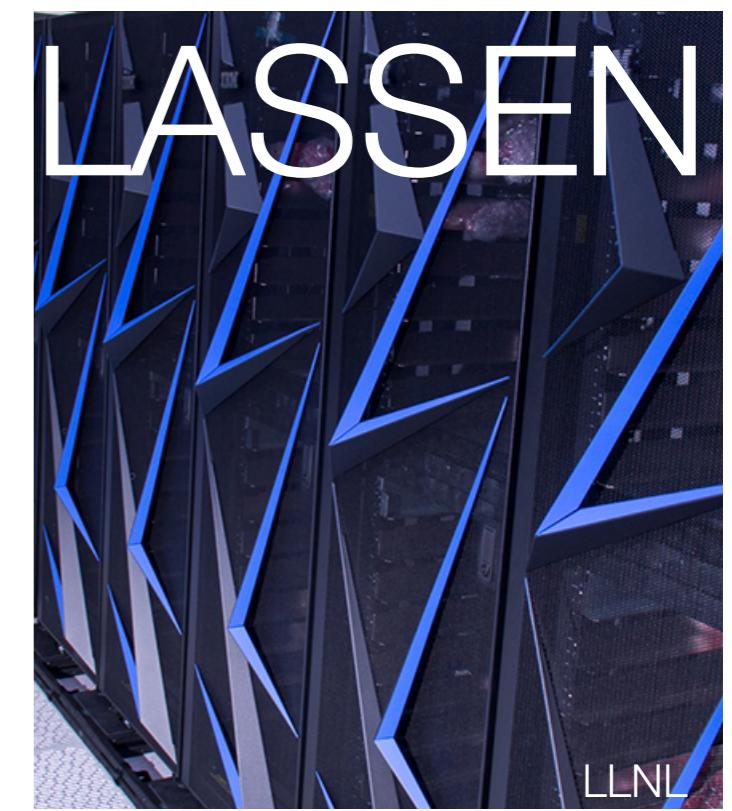
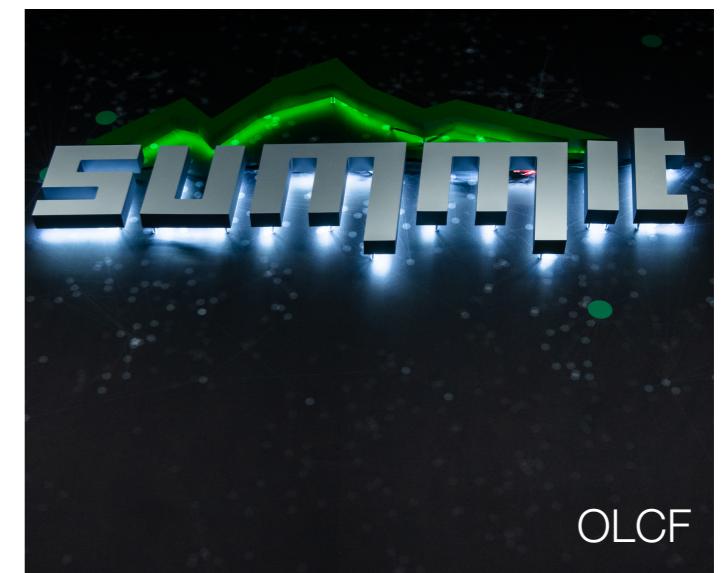
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- ${}^3P_0$  is not bound
  - Need  ${}^1S_0$  binding energy,  
 ${}^3P_0$  phase shift +  
derivative, and matrix  
element from the lattice
- new methods



# Summary

- $g_A$ ,  $n\bar{n}$ : Looking good, but you'll hear more Thursday
- $\theta$ EDM + cEDM: progress, but indistinguishable from zero; Weinberg operator: not as much progress
- CP violating  $\pi N$  coupling
- NN:  $g_A$  quenching, pp fusion, isotensor polarizability ( $2\nu\beta\beta$ )
- $\pi^+\pi^-$  transition ( $0\nu\beta\beta$ )
- $\Delta I=2$  Hadronic PV

# Outlook

- Single-nucleon quark-bilinear matrix elements: form factors, radii,  $\sigma$  term, polarizabilities
- $\pi$ EFT can help enormously
- four-quark operators require EFT understanding / new methods (meson transitions are much simpler, for example)
- New computers + methods will yield a dramatic improvement.