Measurements of Neutron-Mirror Neutron Coupling Through Precession Experiments

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Outline

- Model for spin-independent neutron couplings to a mirror universe with mirror magnetic fields
- Estimate of the signal for a Ramsey Spin-Flip Experiment for neutrons when this coupling is present
- A notion of some limits from different experiments

Pre-preliminary Musings...

Discovered yesterday, idea is briefly covered in Berezhiani, “More about neutron-mirror neutron oscillations”, Eur. Phys. J. C **64**, 421-431 (2009), where detailed analysis of many different probes of mirror neutron couplings are explored...

Background

- Motivated by fact that spin-conserving mirror world interaction breaks into two, coupled two state problems when fields are along the quantization axis in respective universes
- Expect generically that eigenstates are effected quadratically by perturbations

Help checking algebra – (not done!) – E. D. Davis and R. Musedinovic

I’m effectively sharing my notes...

Spin-independent Couplings to Mirror Neutrons

Start with the model: Hamiltonian is presented in Berezhiani et al, Eur. Phys. J. C 72:1974 (2012) for a spin-independent n-n' coupling:

$$H = \begin{bmatrix} \mu B \sigma & \epsilon \\ \epsilon & \mu' B' \sigma \end{bmatrix} \tag{1}$$

which can be expanded to

$$H = \begin{bmatrix} \frac{\mu}{2}B & 0 & \epsilon & 0 \\ 0 & -\frac{\mu}{2}B & 0 & \epsilon \\ \epsilon & 0 & \frac{\mu'}{2}B' & 0 \\ 0 & \epsilon & 0 & -\frac{\mu'}{2}B' \end{bmatrix} \tag{2}$$

For fields along the z and z' axis

for the wavefunction

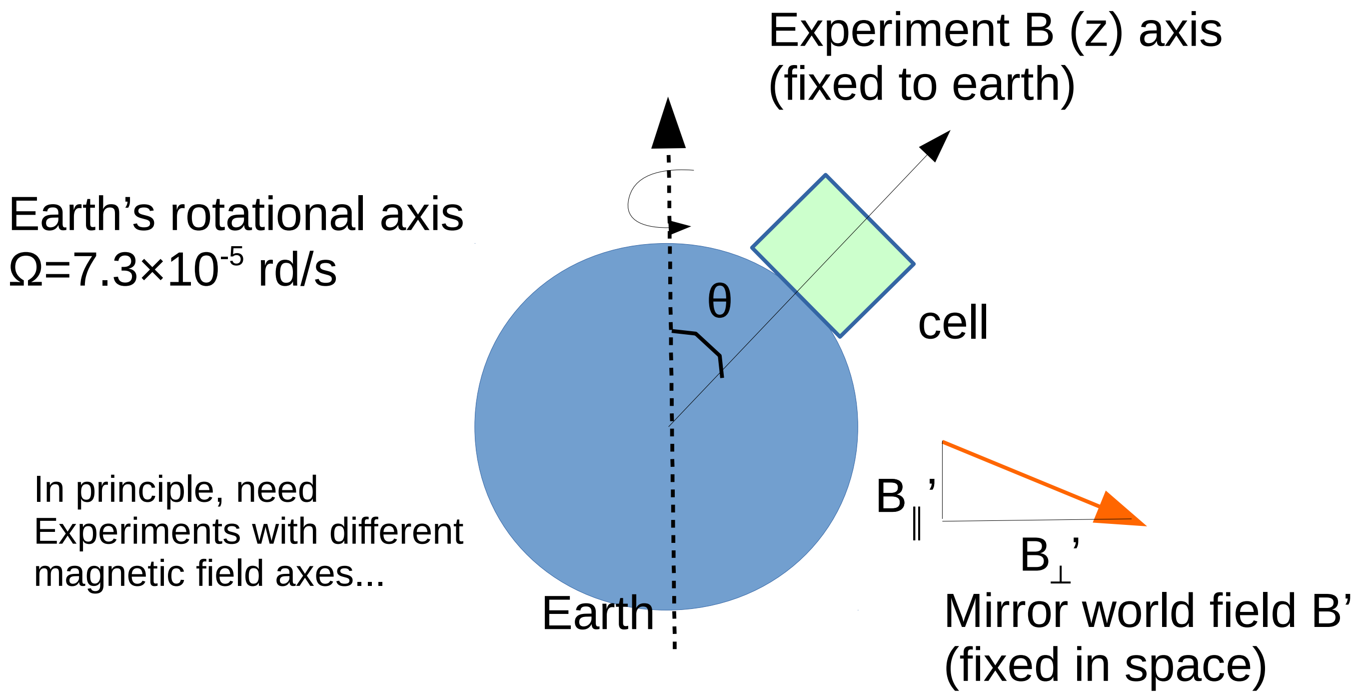
$$\Psi = \begin{bmatrix} \psi_{n+} \\ \psi_{n-} \\ \psi_{n'+} \\ \psi_{n'-} \end{bmatrix} \tag{3}$$

neutron: (spin up, spin down)

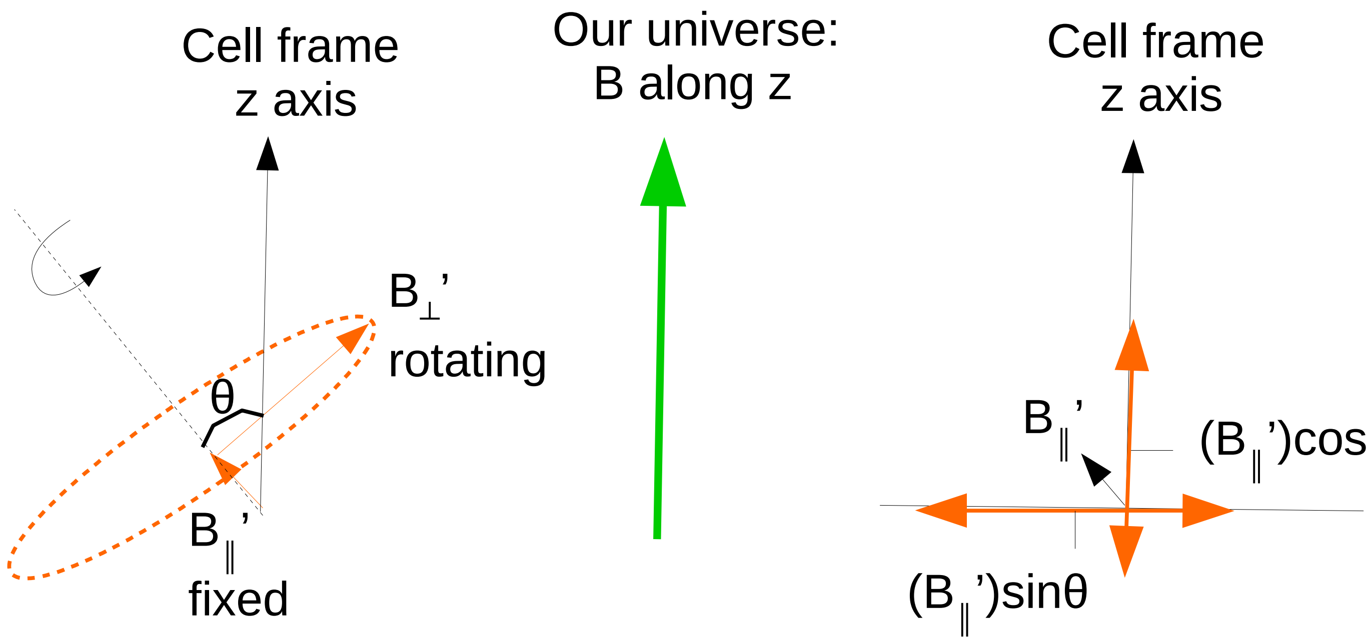
Mirror neutron: (spin up, spin down)

Assumptions:

- Fields in both neutron and mirror neutron frames have very small spatial variations (true for normal fields for precession experiments)
- Mirror neutrons are not confined by cell walls
- Magnetic field arranged along the z axis in our universe (the experiment)
- Each precession measurement takes ~100-300 s (T$_2$ or storage time limit)
- Negligible n' amplitude before first spin flip (flip in guide with very short collision time, short times between collisions compared to precession measurement



Cell Frame Fields



The cell sweeps out a trajectory, where is $B_{\parallel}'$ constant, and $B_{\perp}'$ rotates at freqency Ω

Average field: $(B_{\parallel}')\cos\theta$ along z
$(B_{\parallel}')\sin\theta$ along x

Fluctuating field: $(B_{\perp}')\cos\theta$ along z
$(B_{\perp}')\sin\theta$ along x-y

Assume variations in mirror fields small during a precession measurement

Spin-independent Couplings to the Mirror Neutrons

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which can be expanded to

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Although only z component in earth-frame, will also have fields in the x and y dirs in mirror frame!

Will not analyze this case today...

for the wavefunction

$$\Psi = \begin{bmatrix} \psi_{n+} \\ \psi_{n-} \\ \psi_{n'+} \\ \psi_{n'-} \end{bmatrix} \qquad (3)$$

neutron: spin up, spin down

Mirror neutron: spin up, spin down

Time Independent Solutions

independent secular equation, $H\Psi = E\Psi$, can be written:

$$\begin{bmatrix} \frac{\mu}{2}B - E & 0 & \epsilon & 0 \\ 0 & -\frac{\mu}{2}B - E & 0 & \epsilon \\ \epsilon & 0 & \frac{\mu'}{2}B' - E & 0 \\ 0 & \epsilon & 0 & -\frac{\mu'}{2}B' - E \end{bmatrix} \begin{bmatrix} \psi_{n+} \\ \psi_{n-} \\ \psi_{n'+} \\ \psi_{n'-} \end{bmatrix} = 0 \quad (4)$$

The eigenvalue equation for this system is (with $f = \frac{\mu}{2}B$ and $d = -\frac{\mu'}{2}B'$:

$$(E^2 - f^2)(E^2 - d^2) - 2\epsilon^2 (E^2 + fd) + \epsilon^4 = 0 \quad (5)$$

With solutions (letting $\Gamma = \sqrt{(f-d)^2 + 4\epsilon^2}$):

$$E_{n\pm} = \pm\frac{1}{2}(\Gamma + (f+d))$$ Neutron in limit $\epsilon \to 0$

$$E_{n'\pm} = \mp\frac{1}{2}(\Gamma - (f+d))$$ Mirror neutron in limit $\epsilon \to 0$

(6)

$$v_{1(n+)} = \frac{1}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}}\{f-d+\Gamma, 0, 2\epsilon, 0\}$$

$$v_{2(n-)} = \frac{1}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}}\{0, f-d+\Gamma, 0, -2\epsilon\}$$

$$v_{3(n'+)} = \frac{1}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}}\{-2\epsilon, 0, f-d+\Gamma, 0\}$$

$$v_{4(n'-)} = \frac{1}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}}\{0, 2\epsilon, 0, f-d+\Gamma\}$$

Eigenfunctions

Where, for example, we can produce a pure neutron state with spin $= +1/2$ with the superposition:

$$\Psi_{n+} = \alpha v_1 + \beta v_3$$
$$\Psi_{n-} = \alpha v_2 + \beta v_4$$

Pure neutron state spin up (t=0)

Pure neutron spin down (t=0)

with

$$\alpha = \frac{f-d+\Gamma}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}},$$

$$\beta = \frac{-2\epsilon}{\left[(f-d+\Gamma)^2+4\epsilon^2\right]^{1/2}}.$$

Particle states now mixtures in our coupled basis, Evolving through eigen-energies E_1, E_2, E_3, E_4

Put in the time-dependent equations!

Time Dependent Amplitudes

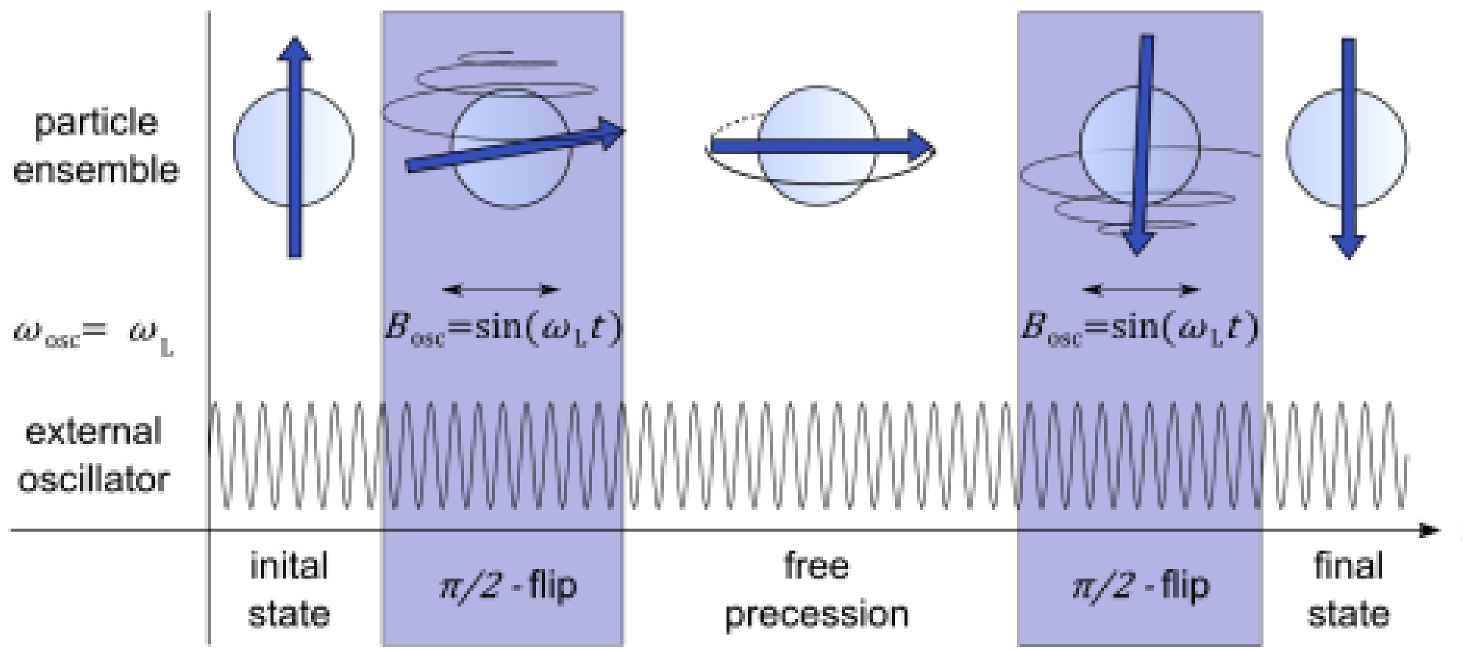
Neutron spin up state becomes: $\Psi_{n,+} = \alpha v_1 e^{iE_1 t} + \beta v_3 e^{iE_2 t}$

Neutron spin down state becomes: $\Psi_{n,-} = \alpha v_2 e^{iE_3 t} + \beta v_4 e^{iE_4 t}$

A Precession Experiment with Polarized Neutrons

From B. Franke, “By-products of nEDM Searches”, Neutron Summer School 2018, Raleigh NC (2018):

TRIUMF "The neutron magnetometer"



Ingredients to extract f_n via the Ramsey method:

- 100 % polarized ensemble
- Magnetic field, ideally on single homogeneous component
- very precise external clock
- count neutrons depending on polarization state

Spin Flipping

$$a(t) = \frac{N\omega_1}{.\Omega} e^{-i\omega t/2} e^{i\phi_a} \left[-i \left(|b| e^{i\Delta\phi} + \frac{\Delta\omega}{\Omega} |a| \right) sin \frac{\Omega}{2} t + |a| cos \frac{\Omega}{2} t \right]$$

$$b(t) = -\frac{N\omega_1}{\Omega} e^{+\omega t/2} e^{i\phi_a} \left[\frac{\Omega}{\omega_1} |b| e^{i\Delta\phi} cos \frac{\Omega}{2} t + i \left(\frac{\Delta\omega}{\omega_1} |b| e^{i\Delta\phi} - |a| \right) sin \frac{\Omega}{2} t \right]$$

Use an RF field along the y axis – follow Gasciorowiscz (p. 245-249)
But included an arbitrary initial phase difference between a(0) and b(0), which is handy for the situation at the end of precession:

- Fields act on neutron component of state
- Neutron Spin up amplitude a(t), spin down amplitude b(t)
- Initial state $|a|e^{i\Phi a}$ and $|b|e^{i\Phi b}$, with normalization $|a|^2+|b|^2 = 1$ (no losses)
- Variables

ω = rf field circular frequency (rd/s)

$\omega_c = \mu B/h = 2f/h$

$\omega_1 = \mu B_1/h$ is the spin rotation rate around the rf field

$\Delta\omega = \omega - \omega_c$

$\Delta\Phi = \Phi_b - \Phi_a$

$\Delta\lambda = \Omega = (\omega - \omega_c)^2 + (\omega_1)^2$

Results of a Perfect π/2 Spin Flip (measurement)

$$|a(t_f)|^2 = \frac{N^2\omega_1^2}{\Delta\lambda^2}\left[\left(|b|^2 + 2|a||b|\frac{\Delta\omega}{\Omega}cos\Delta\phi + \frac{\Delta\omega^2}{\Omega^2}|a|^2\right)sin^2\frac{\Omega}{2}t_f - |a||b|sin\Delta\phi sin\Omega t_f + |a|^2cos^2\frac{\Omega}{2}t_f\right]$$

$$|b(t)|^2 = \frac{N^2\omega_1^2}{\Delta\lambda^2}\left[|b|^2\left(\frac{\Omega^2}{\omega_1^2}cos^2\frac{\Omega}{2}t_f + \frac{\Delta\omega^2}{\omega_1^2}sin^2\frac{\Omega}{2}t_f\right) + 2|a||b|\left(\frac{\Omega}{2\omega_1}sin\Delta\phi sin\Omega t_f - \frac{\Delta\omega}{\omega_1}cos\Delta\phi sin^2\frac{\Omega}{2}t_f\right) + |a|^2sin^2\frac{\Omega}{2}t_f\right] \quad (15)$$

Note that, for a perfect spin flip on resonance, $\delta\omega = 0$, $\Omega = \omega_1$, so:

$$|a(t_f)|^2 = N^2\left[|b|^2sin^2\frac{\omega_1}{2}t_f - |a||b|sin\Delta\phi sin\omega_1 t_f + |a|^2cos^2\frac{\omega_1}{2}t_f\right]$$

$$|b(t_f)|^2 = N^2\left[|b|^2cos^2\frac{\omega_1}{2}t_f + |a||b|sin\Delta\phi sin\omega_1 t_f + |a|^2sin^2\frac{\omega_1}{2}t_f\right] \quad (16)$$

Note t_f is the time at the end of the spin-flip rf pulse

Free Precession of the Neutron State

- Typical cell dimensions taken to be about 50 cm. For an average velocity of 5 m/s, the time between wall collisions is around 0.1 s
- The spin precession measurement time is between about 100 and 300 s (so much longer)
- Important assumption: each wall collision “analyzes” the superposition state and eliminates mirror amplitudes (they pass through the wall) but preserves the relative phase of the spin amplitudes of the neutron (as we generally observe in well-designed precession experiments). At present, we have a few different pictures for this process...taking one...

Conjecture

I have not yet reconciled different methods of propagating neutron solutions through collisions. When I “reset” the amplitude, preserving the complex phase for the neutron part of the states, I get, after a collision:

The dominant amplitudes are **essentially unaffected** and continue to coherently precess

$$a_{n+}(t) = \alpha v_1(\cos^2\varphi e^{iE_1(t + t_p)} + \sin^2\varphi e^{i(E_1 t + E_3 t_p)})$$
$$+ \beta v_3(\sin^2\varphi e^{iE_3(t + t_p)} + \cos^2\varphi e^{i(E_3 t + E_1 t_p)})$$

$$a_{n-}(t) = \alpha v_2(\cos^2\varphi e^{iE_2(t + t_p)} + \sin^2\varphi e^{i(E_2 t + E_4 t_p)})$$
$$+ \beta v_4(\sin^2\varphi e^{iE_4(t + t_p)} + \cos^2\varphi e^{i(E_4 t + E_2 t_p)})$$

When $d \ll f$, $E_3 \ll E_1$, $\beta \sim 5\times10^{-4}$, $\sin^2\Phi \sim 2.5\times10^{-7}$

Assume small amplitudes create noise (after many collisions), but effectively random …w/ $N_c \sim 1000$

We will therefore assume that the neutron is spin-flipped from a perfectly polarized state with $|a|e^{i\phi_a} = 1$ and $|b|e^{i\phi_b} = 0$ with a $\pi/2$ flip ($\omega_1 t_f = \pi/2$) into a coherent super-position of spin states. with $a(t_f) = \frac{\sqrt{2}}{2}e^{-i\phi_o}$ and $b(t_f) = \frac{\sqrt{2}}{2}e^{\phi_o - \pi/2}$, with $\phi_o = \omega t_f = \frac{\pi\omega_o}{\omega_1}$. When couplings to the mirror neutrons are present, this physical state is therefore composed of:

$$\begin{aligned} a(t_f) &= \frac{\sqrt{2}}{2}e^{-i\phi_o}\left(\alpha v_1 e^{iE_1(t-t_f)/\hbar} + \beta v_3 e^{iE_3(t-t_f)/\hbar}\right) \\ b(t_f) &= \frac{\sqrt{2}}{2}e^{i(\phi_o-\pi/2)}\left(\alpha v_2 e^{iE_2(t-t_f)/\hbar} + \beta v_4 e^{iE_4(t-t_f)/\hbar}\right) \end{aligned} \tag{17}$$

and the full wavefunction is a coherent super-position of these two amplitudes:

$$\Psi(t_f) = \frac{\sqrt{2}}{2}e^{-i\phi_o}\left(\alpha v_1 e^{iE_1(t-t_f)/\hbar} + \beta v_3 e^{iE_3(t-t_f)/\hbar}\right) + \frac{\sqrt{2}}{2}e^{i(\phi_o-\pi/2)}\left(\alpha v_2 e^{iE_2(t-t_f)/\hbar} + \beta v_4 e^{iE_4(t-t_f)/\hbar}\right) \tag{18}$$

After precessing for time t_p, the wavefunction is:

$$\Psi(t_f) = \frac{\sqrt{2}}{2}e^{-i\phi_o}\left(\alpha v_1 e^{iE_1 t_p/\hbar} + \beta v_3 e^{iE_3 t_p/\hbar}\right) + \frac{\sqrt{2}}{2}e^{(\phi_o-\pi/2)}\left(\alpha v_2 e^{iE_2 t_p/\hbar} + \beta v_4 e^{iE_4 t_p/\hbar}\right). \tag{19}$$

$$E_{1,2} \approx \quad E_{n\pm} = \pm\frac{1}{2}\left(\Gamma + (f+d)\right)$$

Expect "dominant" amplitudes to coherently precess for entire measurement

Where now the energies encode the coupling to the mirror state, $\Gamma = \sqrt{4\varepsilon^2 - (f-d)^2}$

The signal

After the second spin-flip we have

Spin up amplitude:

$$|a(t_f)|^2 = \left[|b|^2 sin^2\frac{\omega_1}{2}t_f - |a||b| sin\Delta\phi sin\omega_1 t_f + |a|^2 cos^2\frac{\omega_1}{2}t_f\right]$$

$$= \frac{\alpha^2 M^2}{2}\left[1 - sin\left(2\left(\phi_o + \gamma\right) + \left(E_2 - E_1\right) t_p/\hbar + \pi/2\right)\right]$$

Experiments were often set up to analyze the spin and just monitor spin-up (experiments have moved to analyzing both spin states in recent years)

The part of the phase that evolves with the total storage time is:

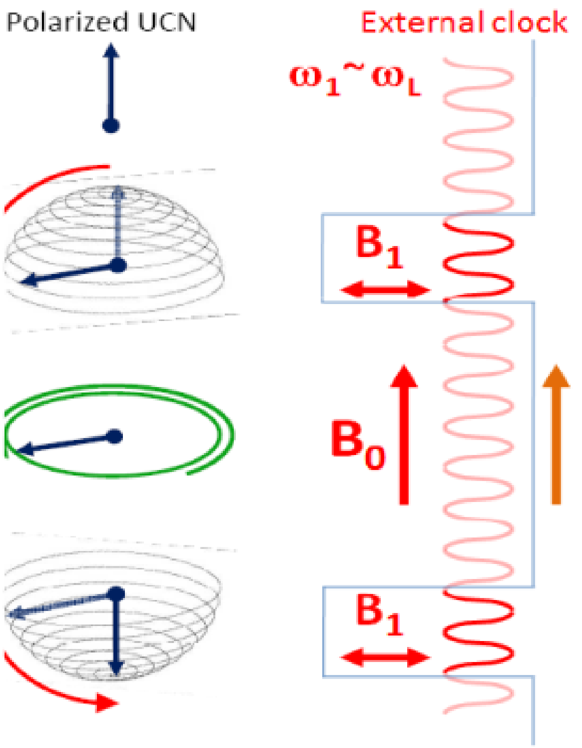
$$E_1 - E_2 = 2(f+d+\sqrt{4\varepsilon^2 + (f-d)^2})$$

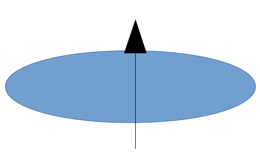
$$E_2 - E_1 \approx -2f\left(1 + \frac{2\epsilon^2}{(f-d)^2}\right),$$

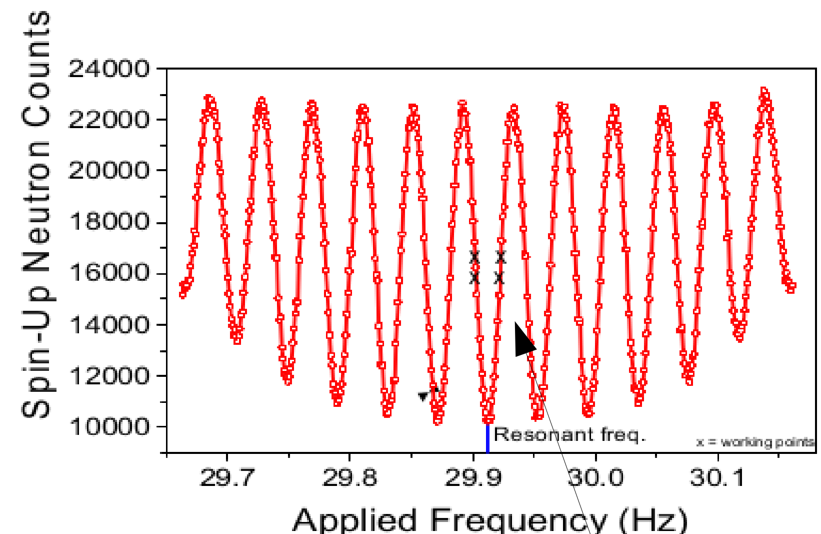
The effect of coupling to the mirror world is to change the effective size of the magnetic field! Effect is quadratic in couplings, like n-n' oscillation

Ramsey Fringes

The Ramsey's method of separated oscillating fields







$$\sigma(f_n) = \frac{\Delta\nu}{\alpha\sqrt{N}\pi}$$

Pick good points to
Measure slope is large...

Spin analyzer (depending on point in precession, spin somewhere between parallel and antiparallel spin analyzer axis)

Some Measurements One Can Perform*

*have already been performed...

I have considered 3 kinds of experiments so far (with help from Beatrice Franke)

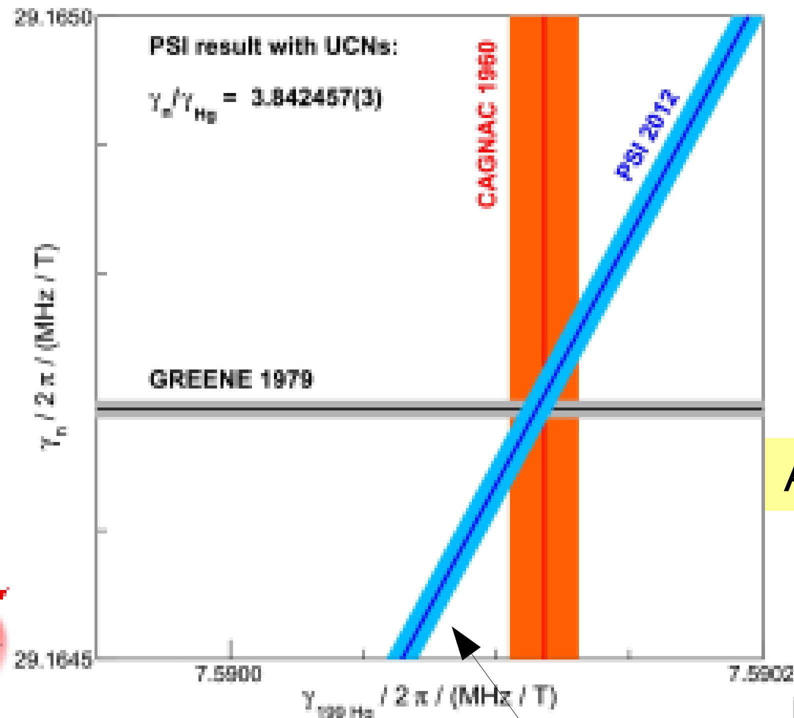
- Absolute average change in measured effective magnetic field due to “pseudo-magnetic field” from mirror couplings (expect deviation, especially if B’ on scales of few ~10^{-6} T where EDM experiments are performed with high precision)
- B-scaling measurements (look for non-linearity in B)
- Time varying fields

All three rely on the presence of **atomic co-magnetometer,** which can be used as a reference, and which is not affected by coupling to the mirror world (checked with Zurab, this appears to be reasonable), experiment measures R:

$$R = \frac{\gamma_n}{\gamma_M} \qquad \text{With } \omega = \gamma B$$

Pseudomagnetic field

TRIUMF Results from a clock comparison



[Afach et.al., PLB **739**, 128 (2015)]

Super precise measurement of magnetic moment of neutron relative to proton by Geoff Greene

Agreement of extracted field: 0.28 ± 0.53 pT !

Measurement of relative to mercury (correct for gravitational effect + gradients)

Limit on couplings

Our precession phase is parametrized in terms of energy differences for the energy states. When measuring precession in a magnetic field, the energy difference can be written

$$\Delta E = E_2 - E_1 = -\mu_n B_z - (\mu_n B_z) = 2|\mu_n|(B + b) = 2f(1 + \delta)$$

Spin down Spin up

Mirror field perturbation

$\delta = 2\varepsilon^2/(f - d)^2$

Given limits on a pseudomagnetic field of $b < 3\times10^{-20}$ eV with $B = 1\times10^{-6}$ T so $f = 6\times10^{-14}$ eV

$\varepsilon < 3\times10^{-17}$ eV

$\tau > 21$ s

For mirror fields significantly larger than 1 μT, these limits will be less stringent

B scaling

- Changing magnetic fields is difficult for nEDM experiments – they are carefully optimized for these fields (for PSI they are about 1 μT)
- If several measurements were made of precession at the current precision of nEDMs, then for mirror fields near 1 μT or lower, the limit will be at the uncertainty in determinations of R, equivalent to the relative uncertainty of the precession frequency (or the energy of the precessing state):

Convert EDM limit to effective magnetic splitting ($\Delta E = 2f + 2d_n E$):

$$\Delta E/2f = 2(\sim 2\times 10^{-26}\ \mathrm{ecm} * 10^{4}\ \mathrm{V/cm})/(2*6\times 10^{-14}\ \mathrm{eV}) = 3.3\times 10^{-9}$$

$\varepsilon < 2.4\times 10^{-18}$ eV

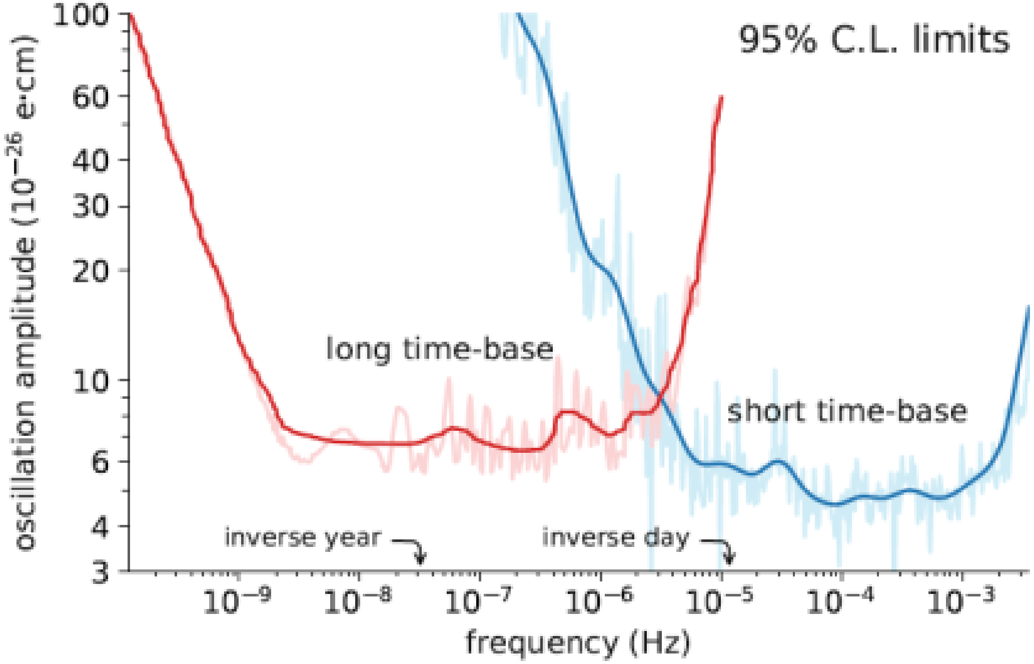
$\tau > 260$ s

Have measurements of this kind already been done?

They may be available, but I’m not aware of it! Different EDM experiments have run at different fields, which might provide a cross-reference of ratios of neutron to co-magnetometer to constrain in influence of mirror fields. Doing the scaling in an already optimized EDM experiment would have to be a set of dedicated runs (seems unlikely)...

Siderial Variation

Abel et al., Phys. Rev. X 7, 041034 (2017)



Analysis of precession data (but not organized into EDM sets, just daily variations) could make interesting limits!

$\varepsilon < \sim 4.2\times 10^{-18}$ eV

$\tau > \sim 149$ s

Summary

- We want to add the transverse fields in the mirror dimension. This has already been done by Berezhiani, but in a basis that confused me – I think that this will result in small changes, including producing a new source of T_1 and T_2 losses (needs to be checked – in some scenarios may produce observable signature for mirror fields)
- We should add the small losses due to oscillation to the mirror universe (and mirror neutrons oscillating in), but perhaps not critical
- Precession measurements may add a new tool (if this is right!) to probe interactions with a mirror universe. The sensitivitity seems comparable to beam and UCN disappearance measurements.
- Some data already taken can probably be cast into interesting limits (based on rough estimates)!
- These measurements may provide us with multiple tools to constrain models at the ESS on HIBEAM and ANNI

Free Precession of the Neutron State

- Typical cell dimensions taken to be about 50 cm. For an average velocity of 5 m/s, the time between wall collisions is around 0.1 s
- The spin precession measurement time is between about 100 and 300 s
- We assume each wall collision "analyzes" the superposition state and eliminates mirror amplitudes (they pass through the wall) but preserves the relative phase of the spin amplitudes of the neutron (as we generally observe in well-designed precession experiments). The result of a storage experiment for total time t is a neutron state with (noting short collision times mean small losses to mirror world) amplitudes:

$$a(t_f) = \frac{\sqrt{2}}{2} e^{-i\phi_o} \left(\alpha v_1 e^{iE_1(t-t_f)/\hbar} + \beta v_3 e^{iE_3(t-t_f)/\hbar} \right)$$

$$b(t_f) = \frac{\sqrt{2}}{2} e^{i(\phi_o - \pi/2)} \left(\alpha v_2 e^{iE_2(t-t_f)/\hbar} - \beta v_4 e^{iE_4(t-t_f)/\hbar} \right)$$

Neutron spin up and spin down states

$$\alpha = \frac{f - d + \Gamma}{\left[(f - d + \Gamma)^2 + 4\epsilon^2\right]^{1/2}},$$

$$\beta = \frac{-2\epsilon}{\left[(f - d + \Gamma)^2 + 4\epsilon^2\right]^{1/2}}.$$

Contributions from Ψ_3 and Ψ_4 suppressed