Re-evaluation of the γW-Box Correction to Neutron and Nuclear β-Decay

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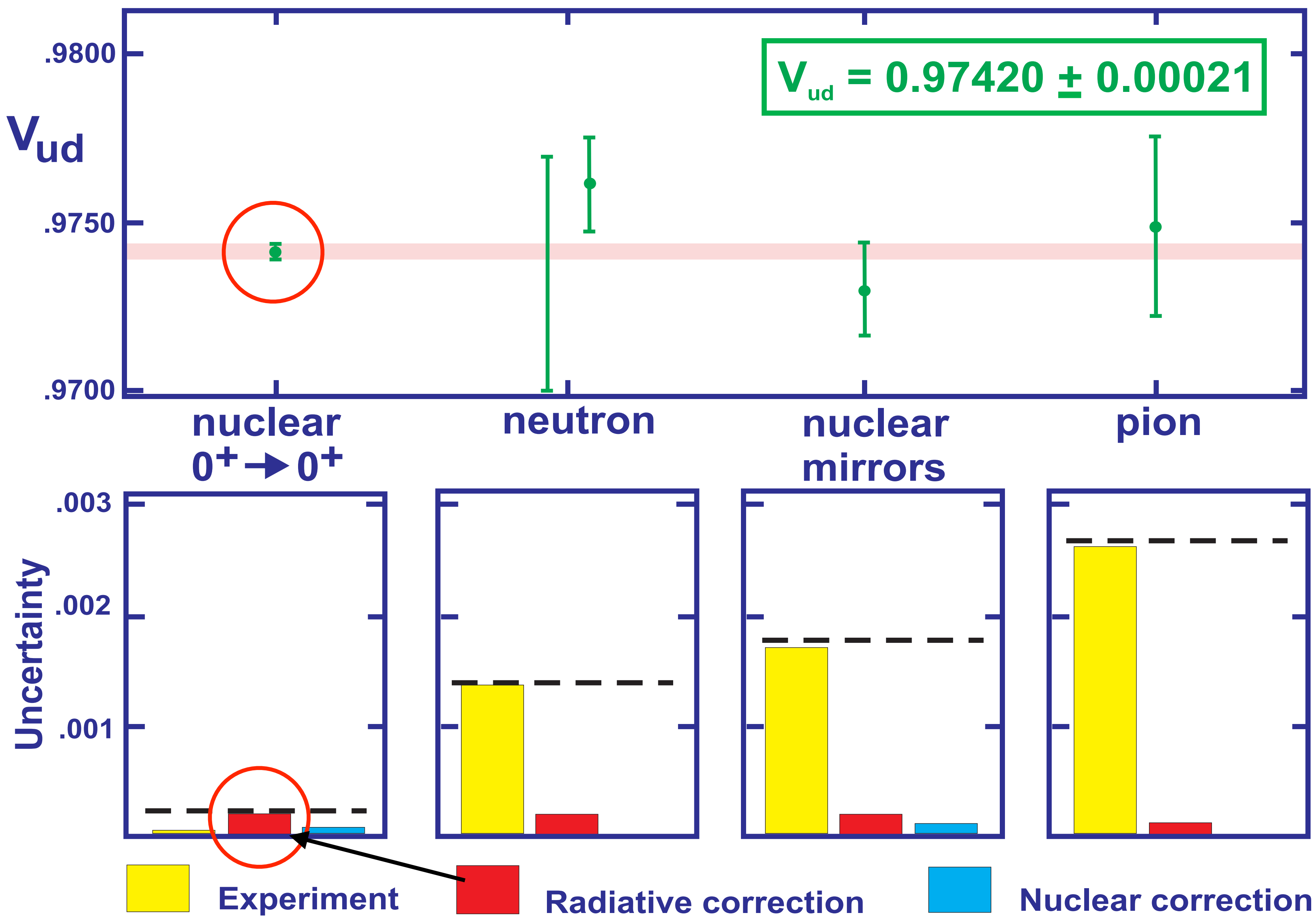
Based on 3 papers:

arXiv: 1807.10197
arXiv: 1812.03352
arXiv: 1812.04229

Particle Physics with Neutrons at the ESS — NORDITA, Stockholm, Sweden
December 10-14, 2018

Current status of V_{ud} and CKM unitarity



$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005$$

0^+-0^+ nuclear decays $|V_{ud}|^2 = 0.94906 \pm 0.00041$

K decays $|V_{us}|^2 = 0.05031 \pm 0.00022$

B decays $|V_{ub}|^2 = 0.00002$

CKM unitarity: V_{ud} the main contributor to the sum and to the uncertainty

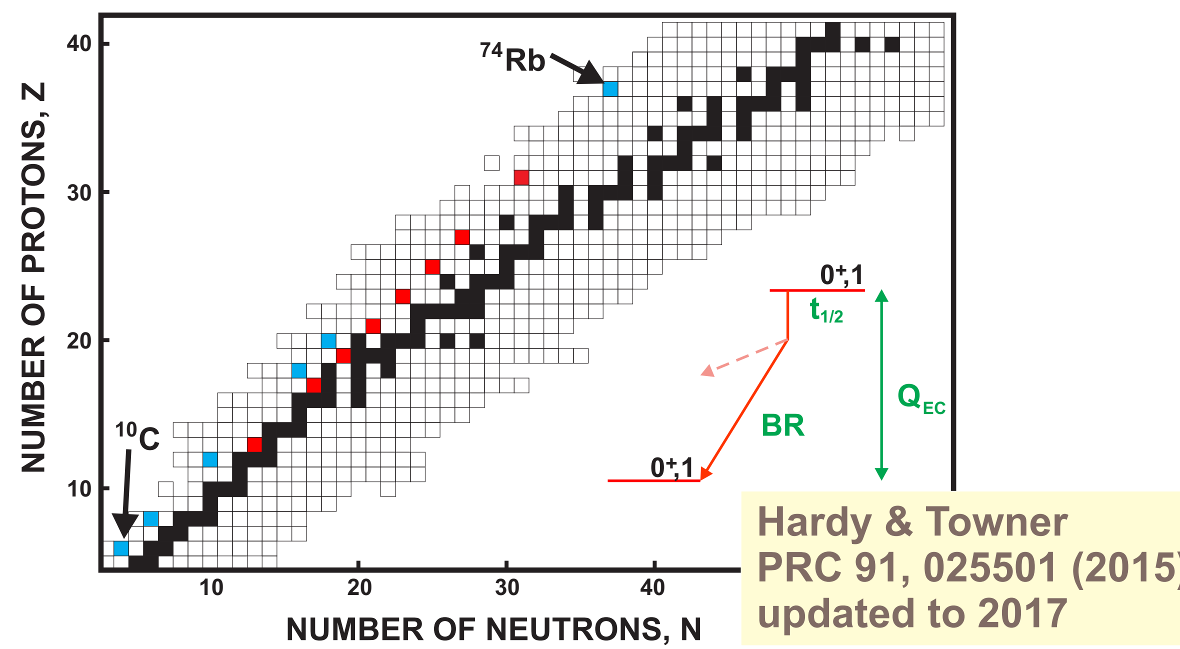
Why are superallowed decays special?

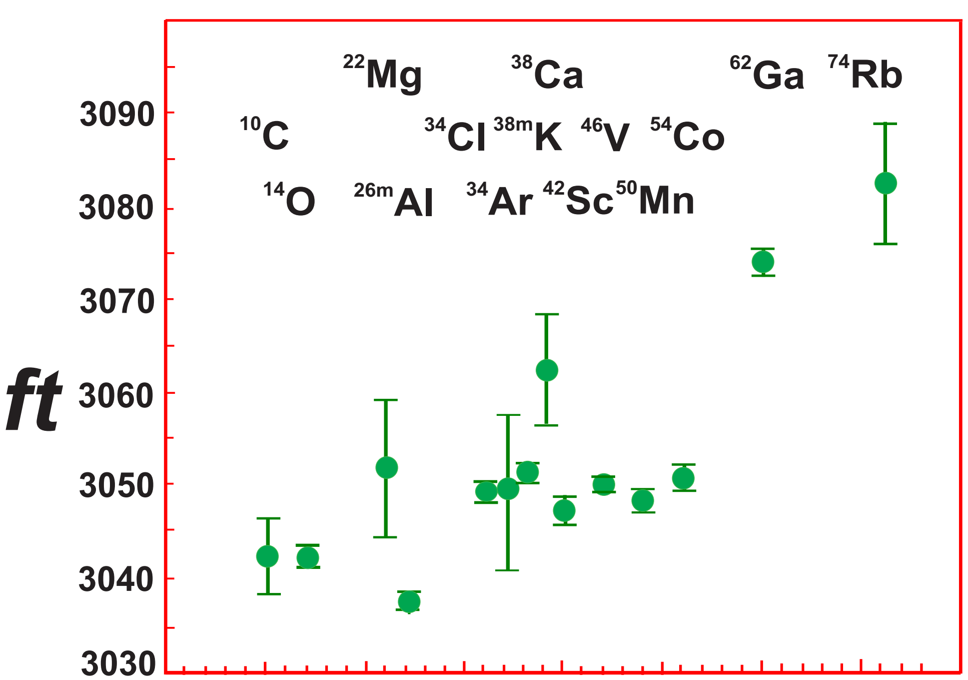
Superallowed 0^+-0^+ nuclear decays:

- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**





ft values: same within ~2% but not exactly!

Reason: SU(2) slightly broken

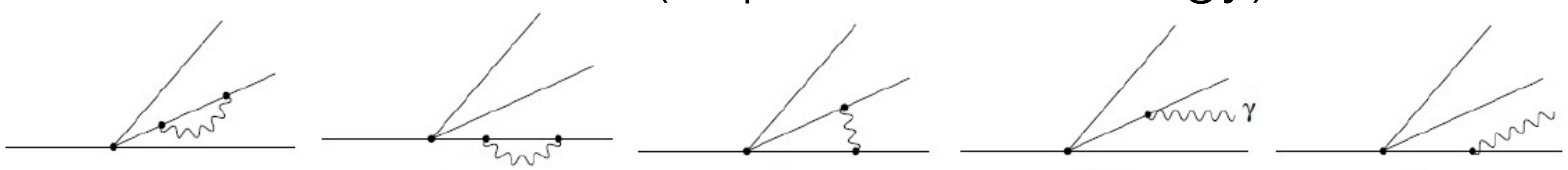
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
   (proton and neutron distribution not the same)

Why are superallowed decays special?

Modified ft-values to include these effects

$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C-\delta_{NS})]$$

δ'_R - “outer” correction (depends on e-energy) - QED



δ_C - SU(2) breaking in the nuclear matrix elements

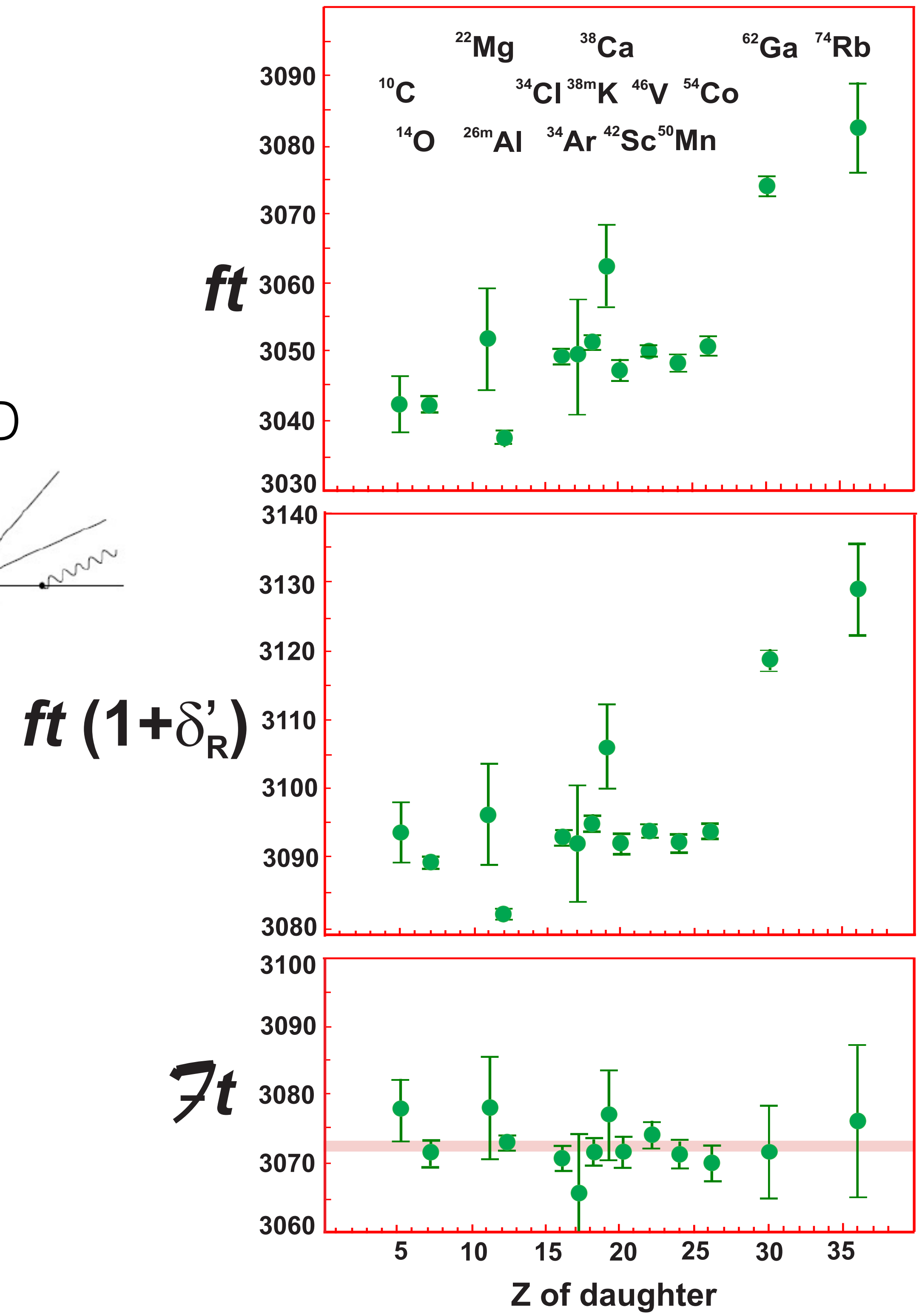
- mismatch of radial WF in parent-daughter
- mixing of different isospin states

δ_{NS} - RC depending on the nuclear structure

δ_C,δ_{NS} - energy independent

Average

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$



Hardy, Towner 1973 - 2018

Outline: RC to Beta Decay

$$|V_{ud}| = 0.97420(10)_{Ft}(18)_{\Delta_R^V}$$

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1+\Delta_R^V)}$$

$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C - \delta_{NS})]$$

Three caveats:

1. Calculation of the universal free-neutron RC Δ_R^V

2. Splitting the full nuclear RC into free-neutron Δ_R^V and nuclear modification δ_{NS}

3. Splitting the full RC into “outer” (energy-dependent but pure QED: no hadron structure) and “inner” (hadron&nuclear structure-dependent but energy-independent)
   - nucleon and nuclear case

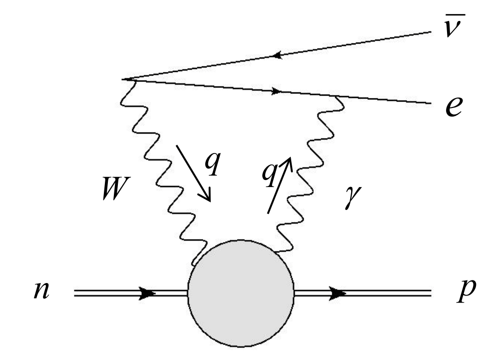
Will address each point

1. Check radiative corrections to the free neutron decay

C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1807.10197

γW-box

Box at zero energy and momentum transfer



$$T_{\gamma W} \;=\; \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e)\gamma^\nu (1-\gamma_5) v_\nu}{q^2[(k-q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T^{\gamma W}_{\mu\nu}$$

Hadronic tensor: two-current correlator

$$T^{\mu\nu}_{\gamma W} = \int dx e^{iqx} \langle p | T[J^\mu_{em}(x) J^\nu_W(0)] | n \rangle$$

General gauge-invariant decomposition (spin-independent)

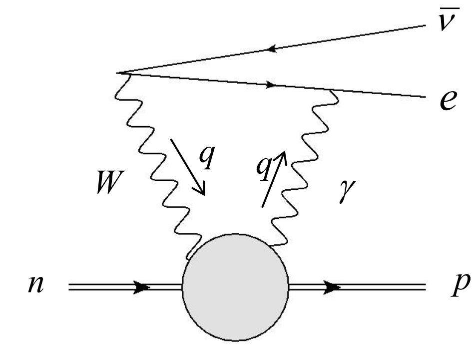
$$T^{\mu\nu}_{\gamma W} \;=\; \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{(p\cdot q)} \left(p - \frac{(p\cdot q)}{q^2} q\right)^\mu \left(p - \frac{(p\cdot q)}{q^2} q\right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3$$

V-V correlator $T_{1,2}$: conserved vector-isovector current - model-independent
Sirlin 1967 - current algebra

Axial current not conserved -> A-V correlator T_3 - model-dependent

γW-box by Marciano & Sirlin

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2+Q^2} \frac{Q^2+\nu^2}{Q^4} \frac{T_3(\nu,Q^2)}{M\nu}$$



$Q^2 = -q^2$
$\nu = (pq)/M$

Marciano & Sirlin used loop techniques:

$$\Box^{VA}_{\gamma W} = \frac{\alpha}{8\pi} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2} F(Q^2)$$

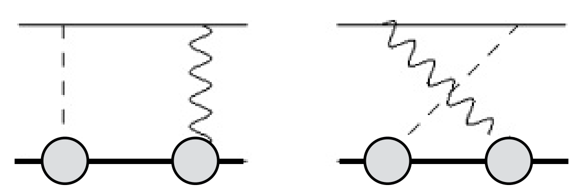
Short distance Q^2>> $F^{\mathrm{DIS}}(Q^2) = \frac{1}{Q^2}$

$$\Box^{\mathrm{DIS}}_{\gamma W} = \frac{\alpha}{8\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2} F^{\mathrm{DIS}}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda}$$

Finite Q^2 -
pQCD corrections:

$$F^{\mathrm{DIS}} = \frac{1}{Q^2} \to \frac{1}{Q^2}\left[1 - \frac{\alpha_s^{\overline{MS}}}{\pi}\right]$$

Long distance Q^2<< - elastic box



MS 1987: asymptotic + pQCD + Born

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\ln\frac{M_W}{\Lambda} + A_g + 2C_B\right]$$

~4.1 -0.24 1.85

Problem: connecting short and long distances

γW-box by Marciano & Sirlin

MS 2006 update

Short distance:
DIS to N³LO

$$F^{\mathrm{DIS}} = \frac{1}{Q^2} \to \frac{1}{Q^2}\left[1 - \frac{\alpha_s^{\overline{MS}}}{\pi} - C_2\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^2 - C_3\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^3\right]$$

GLS and Bjorken SR to N3LO
Larin, Vermaseren 1997

$Q^2 > Q_2^2$

Interpolate between them

Vector Dominance Model Ansatz

$$F^{\mathrm{INT}}(Q^2) = -\frac{1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2}$$

$Q^2 < Q_1^2$

Long distance: Born

$$F(Q^2) = F^B(Q^2)$$

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\underbrace{\ln\frac{M_W}{\Lambda}}_{\sim 3.86} + \underbrace{A_g}_{} + 2C_B\right]$$

~3.86 1.78

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\underbrace{\ln\frac{M_W}{\Lambda}}_{\sim 3.77} + A_g + C^{Int} + 2C_B\right]$$

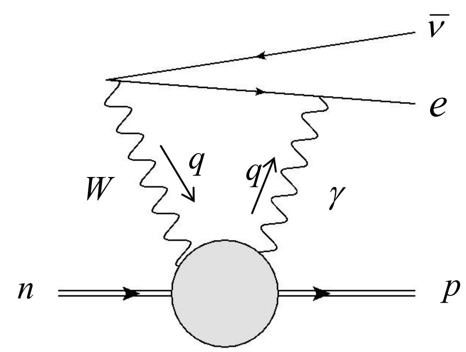
~ 3.77 0.14(14) 1.66

Uncertainty reduced by a factor ~2

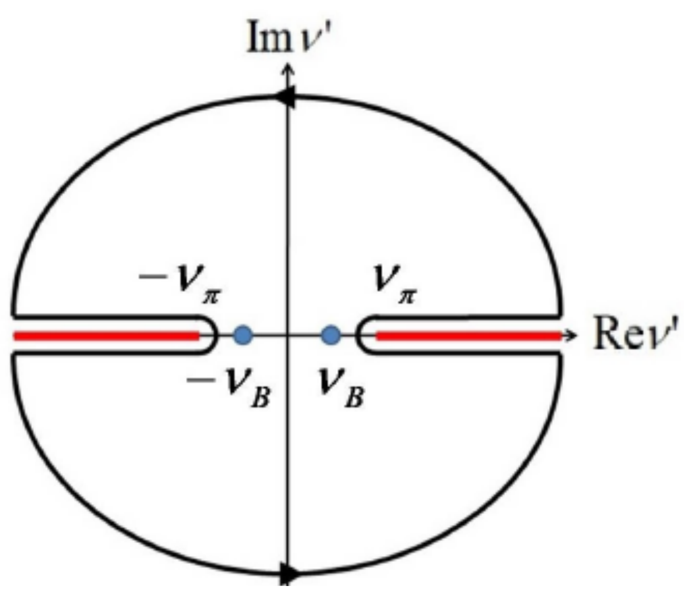
γW-box from Dispersion Relations

Check MS result + uncertainty independently

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2+Q^2} \frac{Q^2+\nu^2}{Q^4} \frac{T_3(\nu,Q^2)}{M\nu}$$



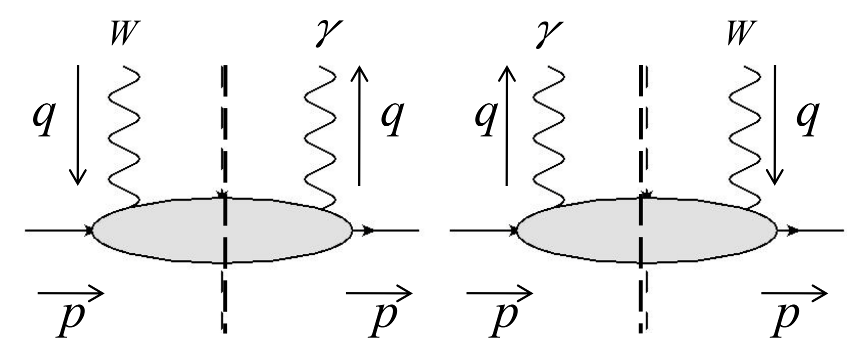
$Q^2 = -q^2$
$\nu = (pq)/M$



T_3 - analytic function inside the contour C in the complex ν-plane determined by its singularities on the real axis - poles + cuts

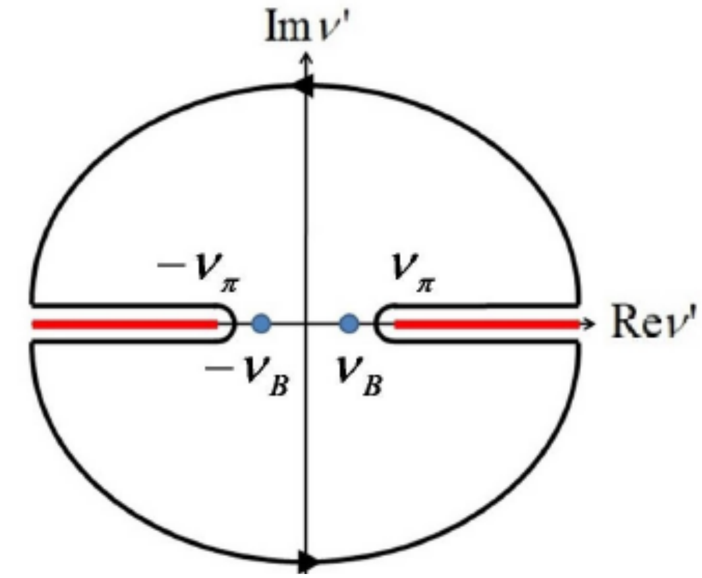
$$T_3(\nu,Q^2) = \frac{1}{2\pi i}\oint_C \frac{T_3(z,Q^2)dz}{z-\nu} \qquad \nu \in C$$

Forward amplitude T_3 - unknown;
Its absorptive part can be related to production of on-shell intermediate states a γW-analog of the SF F_3



$$\mathrm{Im}\, T_3^{\gamma W}(\nu,Q^2) = 2\pi F_3^{\gamma W}(\nu,Q^2)$$

γW-box from Dispersion Relations



Crossing behavior: photon is isoscalar or isovector

Different isospin channels behave differently

$$T_3^{\gamma W} = T_3^{(0)} + T_3^{(3)}$$

$$T_3^{(0)}(-\nu, Q^2) = -T_3^{(0)}(\nu, Q^2), \quad T_3^{(3)}(-\nu, Q^2) = +T_3^{(3)}(\nu, Q^2)$$

Dispersion representation of the γW-box correction at zero energy

$$\Box_{\gamma W}^{VA(0)} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty \frac{d\nu(\nu + 2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu, Q^2)$$

$$\Box_{\gamma W}^{VA(3)} = 0 \qquad q = \sqrt{\nu^2 + Q^2}$$

Connection to MS: first Nachtmann moment of F_3

$$M_3^{(0)}(1,Q^2) = \frac{4}{3}\int_0^1 dx \frac{1 + 2\sqrt{1 + 4M^2x^2/Q^2}}{(1 + \sqrt{1 + 4M^2x^2/Q^2})^2} F_3^{(0)}(x, Q^2)$$

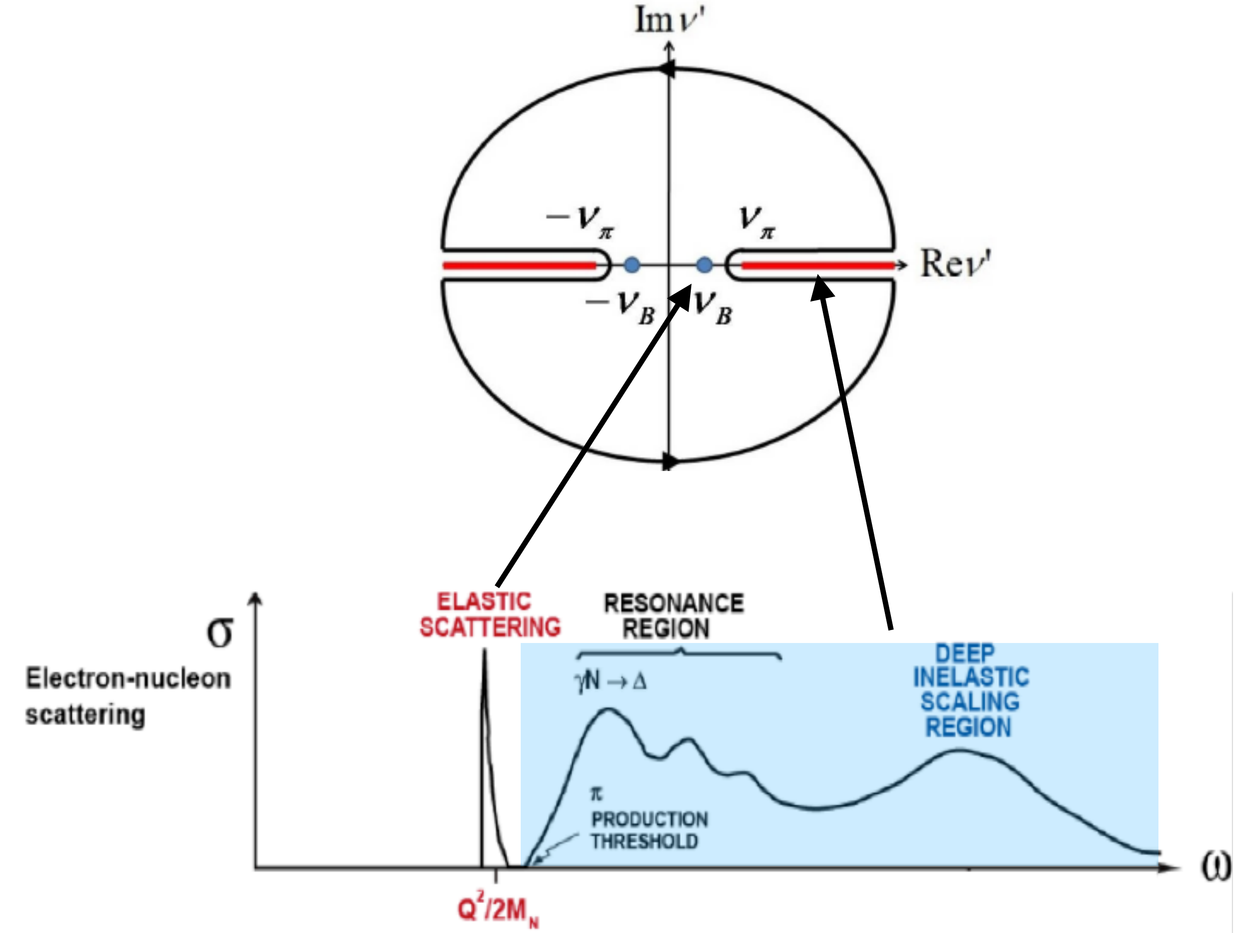
$$F_{MS}(Q^2) = \frac{12}{Q^2} M_3^{(0)}(1,Q^2)$$

$$\Box_{\gamma W}^{VA(0)} = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} M_3^{(0)}(1,Q^2)$$

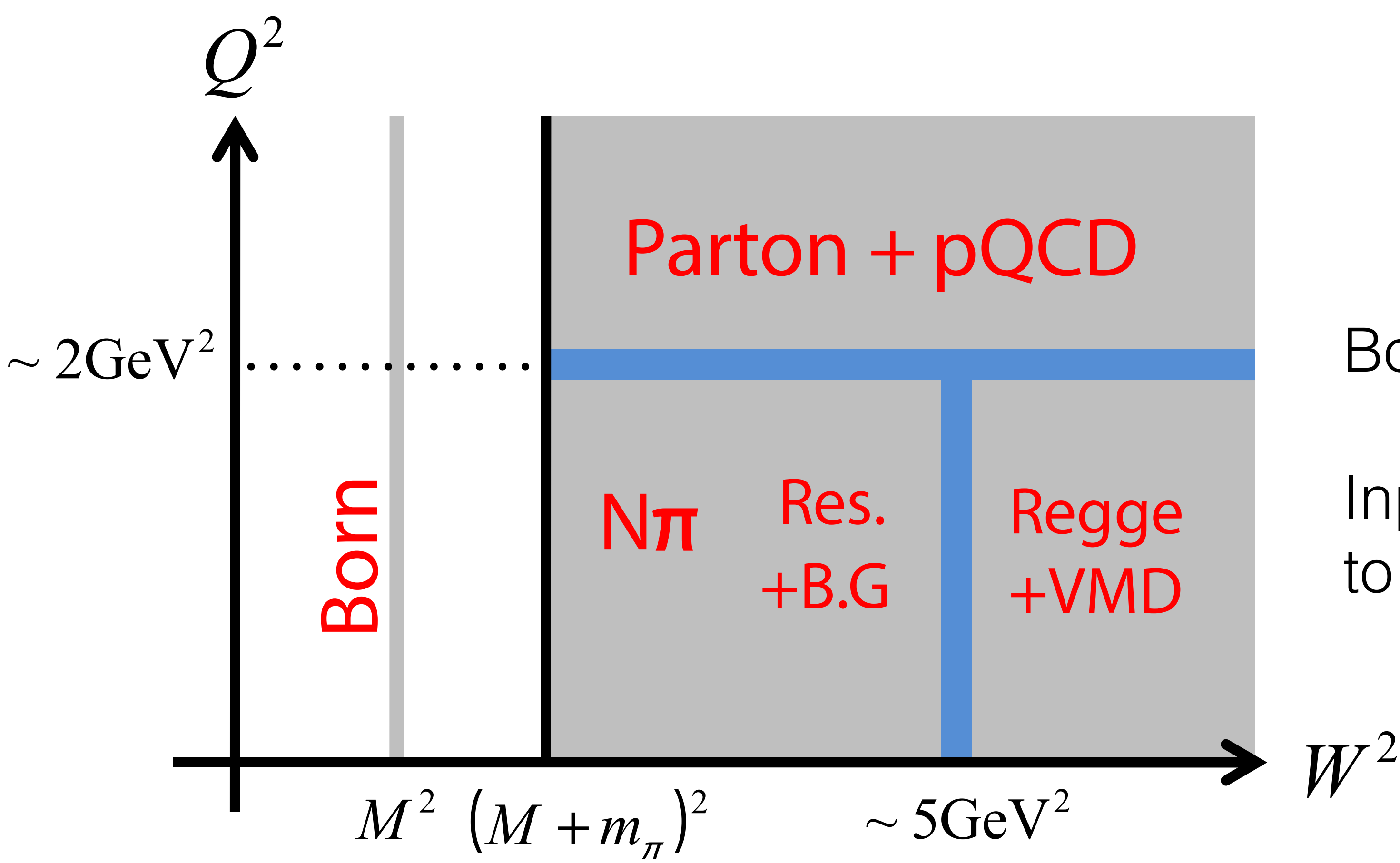
$$\Box_{\gamma W}^{VA(0)} = \frac{\alpha}{8\pi}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F_{MS}(Q^2)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q^2:
scanning dominant physics pictures



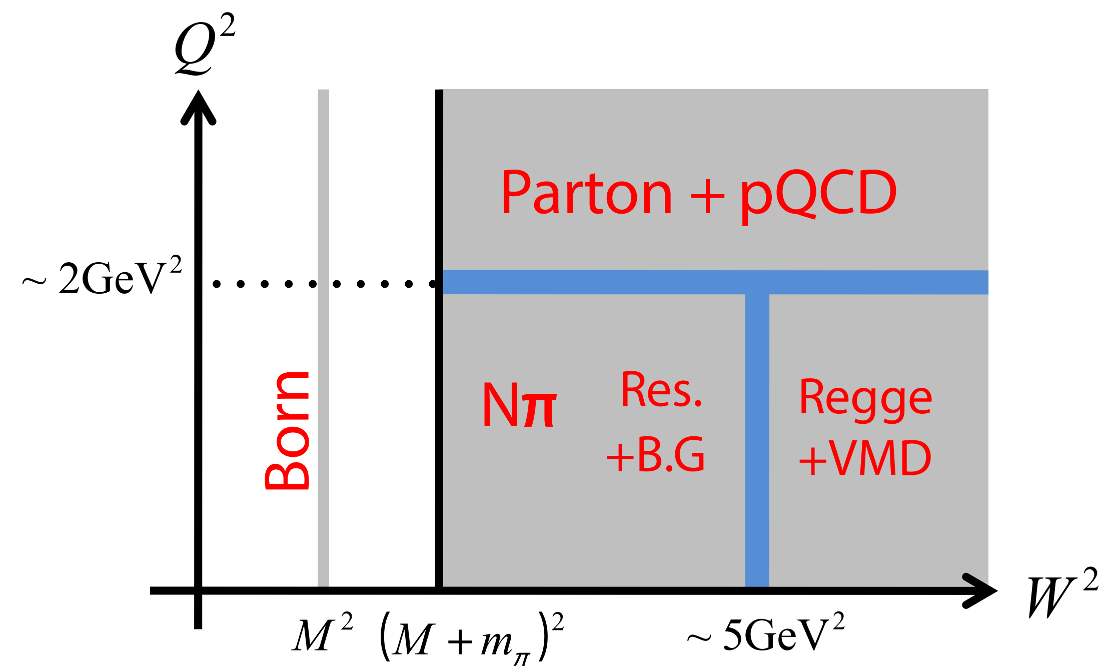
Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral

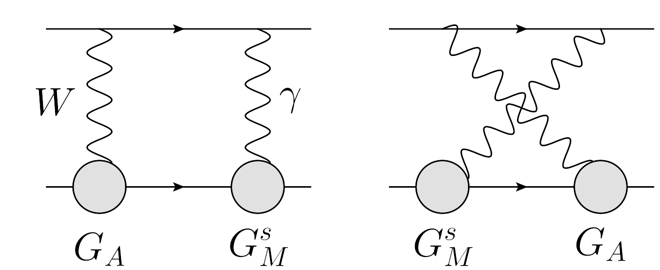
Our parametrization of the needed SF follows from this diagram

$$F_3^{(0)} = F_{\text{Born}} + \begin{cases} F_{\text{pQCD}}, & Q^2 \gtrsim 2 \text{ GeV}^2 \\ F_{\pi N} + F_{\text{res}} + F_{\mathbb{R}}, & Q^2 \lesssim 2 \text{ GeV}^2 \end{cases}$$



Born: elastic FF from e$^-$, ν scattering data

$$\square_{\gamma W}^{VA,\text{Born}} = -\frac{\alpha}{\pi}\int_0^\infty dQ \frac{2\sqrt{4M^2+Q^2}+Q}{\left(\sqrt{4M^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$



πN:

relativistic ChPT calculation plus nucleon FF

Resonances:

axial excitation from PCAC (Lalakulich et al 2006) - neutrino scattering

isoscalar photo-excitation from MAID and PDG - electron and γ inelastic scattering

Above resonance region:

multiparticle continuum economically described by Regge exchanges

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W\,(0)}$

Data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2F_1 + \left(1 - y - \frac{Mxy}{2E}\right)F_2 \pm x\left(y - \frac{y^2}{2}\right)F_3\right]$$

$$\sigma^{\nu p} - \sigma^{\bar{\nu}p} \sim F_3^{\nu p} + F_3^{\bar{\nu}p} = u_v^p(x) + d_v^p(x)$$

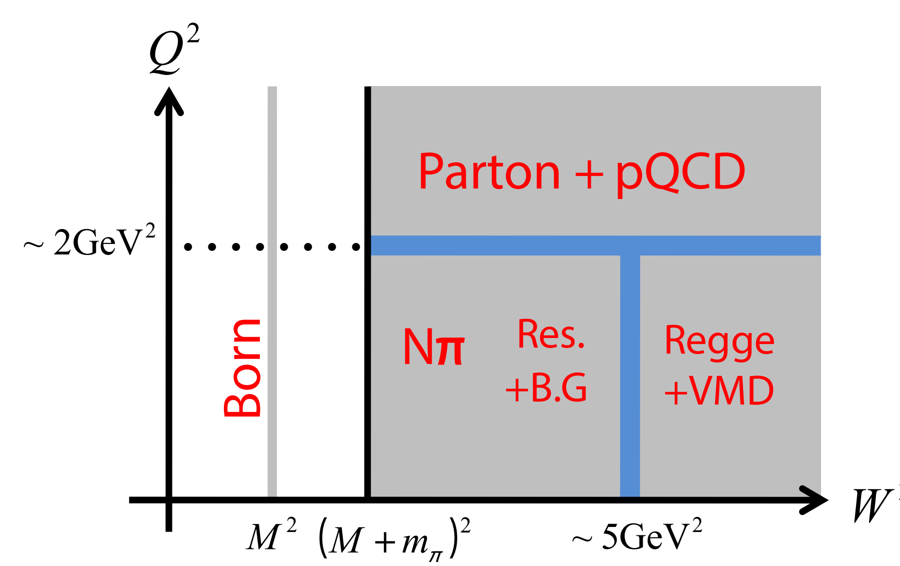
Gross-Llewellyn-Smith sum rule $\int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$

Validate the model for CC process; apply an isospin rotation to obtain γW

$$F_{3,\,\mathrm{low}-Q^2}^{\nu p+\bar{\nu}p} = F_{3,\,el.}^{\nu p+\bar{\nu}p} + F_{3,\,\pi N}^{\nu p+\bar{\nu}p} + F_{3,\,R}^{\nu p+\bar{\nu}p} + F_{3,\,\mathrm{Regge}}^{\nu p+\bar{\nu}p}$$

Low-W part of spectrum:
neutrino data from MiniBooNE, Minerva, …
- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^{\nu} \sim x^{-\alpha}$, $\alpha \sim$ 0.5-0.7



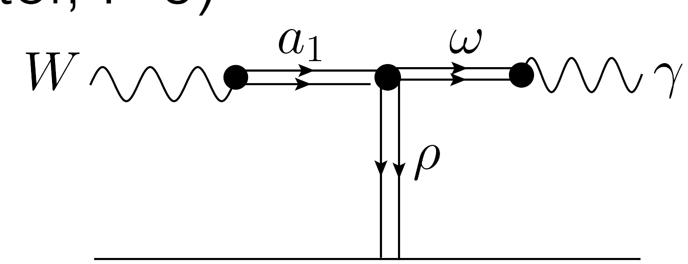
Inelastic states - low Q^2, high W

Scattering at high energy can be very effectively described by Regge exchanges

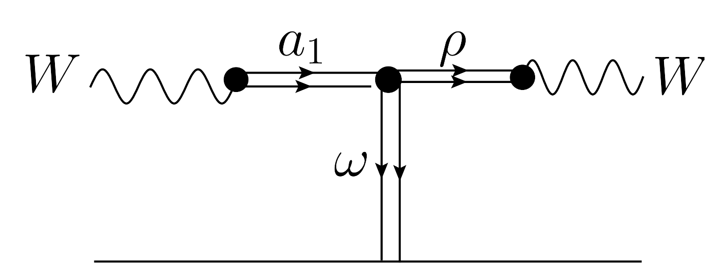
$$F_3^{(0),\mathrm{Regge}}(\nu, Q^2) = C_R(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$

Regge behavior in EW processes: hadron-like behavior of HE electroweak probes - Vector/Axial Vector Dominance is the proper language

γW-box: conversion of $W^\pm$ (charged, I=1, axial) to γ (neutral, vector, I=0)
requires charged vector exchange w. I=1 - $\rho^\pm$
effective a_1 - ρ - ω vertex



Inclusive ν scattering: conversion of $W^\pm$ (charged, I=1, axial) to $W^\pm$ (charged, I=1, axial)
requires neutral vector exchange w. I=0 - ω
effective a_1 - ω - ρ vertex



Minimal model for both reactions - check with data.

Parameters of the Regge model from neutrino scattering

Low $Q^2 < 0.1$ GeV^2: Born + $\Delta(1232)$ dominate
Not fitted: modern data more precise but cover only limited energy range
Fit driven by 4 data points between 0.2 and 2 GeV^2

Model & Uncertainty fully specified
- compare M&S vs This work

$M_3^{WW}(1,Q^2)$

Isospin symmetry

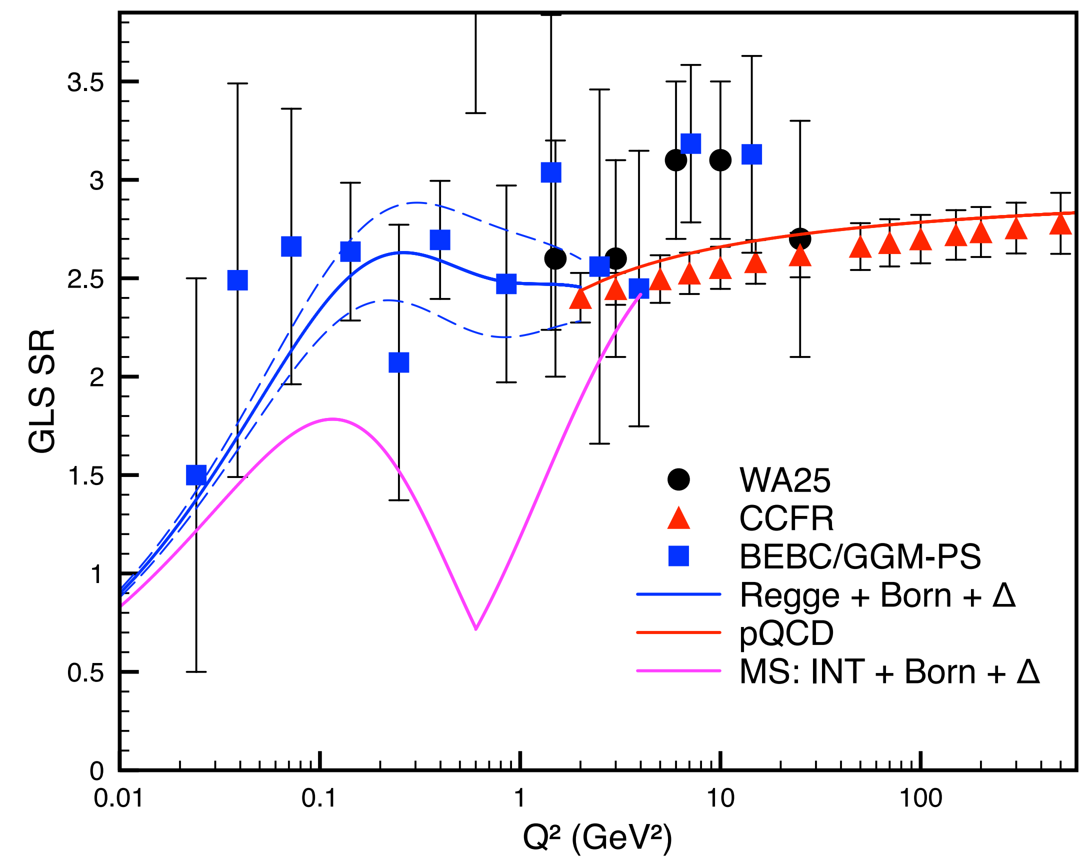
$M_3^{\gamma W}(1,Q^2)$

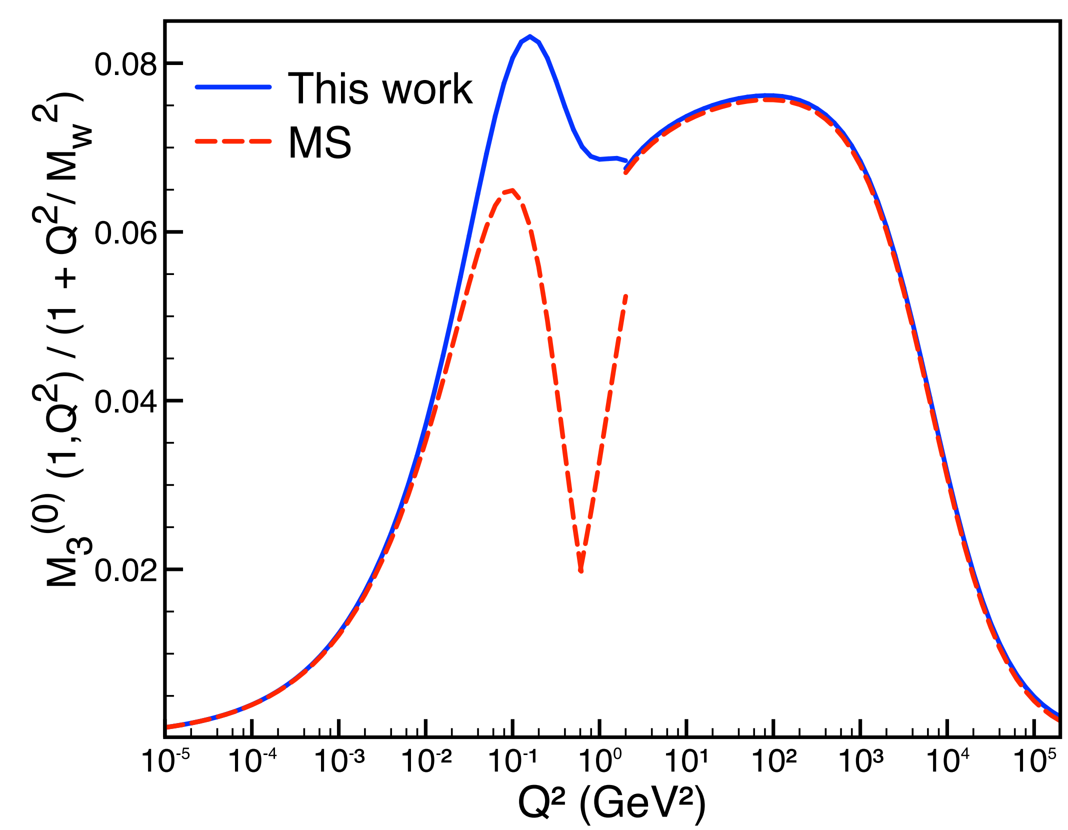
M&S: integrand discontinuous at Q^2 = 2.25 GeV^2

Log scale for x-axis: integral = surface under the curve

$$\text{MS Total}: \quad \Box_{\gamma W}^{(0)} = 0.00324 \pm 0.00018$$
$$\text{New Total}: \quad \Box_{\gamma W}^{(0)} = 0.00379 \pm 0.00010$$

Uncertainty reduced by almost factor 2;
~ 3-5 sigma shift from the old value





Universal γW-box

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Marciano & Sirlin 2006

$$\Delta_R^V = 0.02361(38)$$

$$|V_{ud}| = 0.97420(10)_{Ft}(18)_{\Delta_R^V}$$

Dispersion relations

$$\Delta_R^V = 0.02467(22)$$

$$|V_{ud}| = 0.97370(10)_{Ft}(10)_{\Delta_R^V}$$

DR allowed to reduce the uncertainty in Δ_R^V by almost factor of 2
due to the use of neutrino data

But the shift is more significant than anticipated from the uncertainty estimate by MS

Tension with CKM unitarity

Before

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005$$

After

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004$$

2.Radiative corrections to nuclear decays: Nuclear structure modification of the free-n RC

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

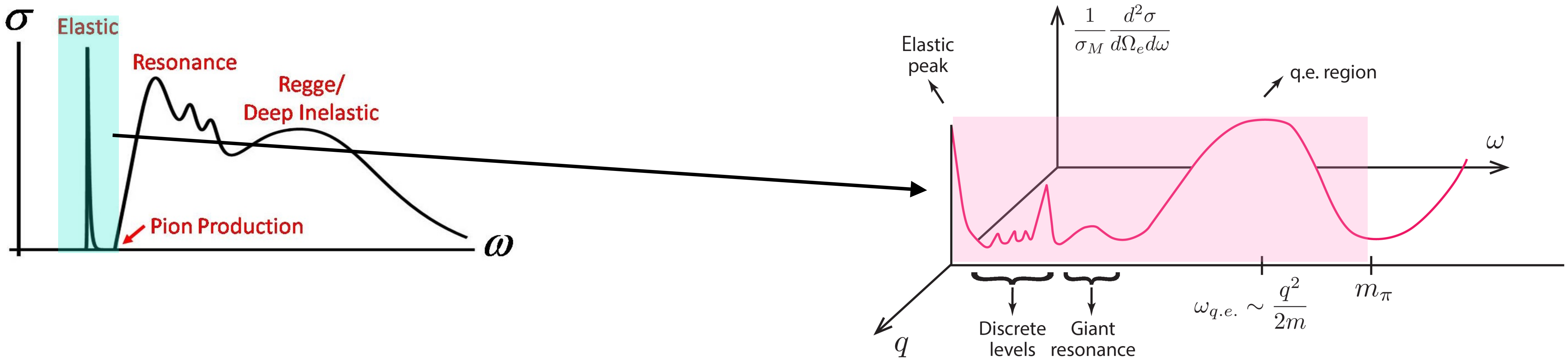
Splitting the γW-box into Universal and Nuclear Parts

General structure of RC for nuclear decay

$$ft(1+RC) = Ft(1+\delta_R')(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus

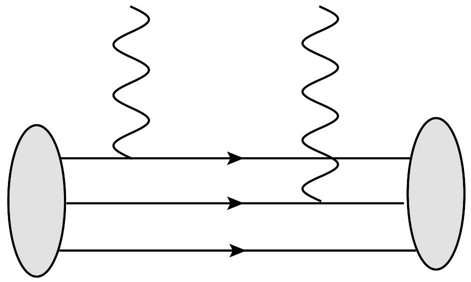


$$\Box_{\gamma W}^{\mathrm{VA,\ Nucl.}} = \Box_{\gamma W}^{\mathrm{VA,\ free\ n}} + \left[\Box_{\gamma W}^{\mathrm{VA,\ Nucl.}} - \Box_{\gamma W}^{\mathrm{VA,\ free\ n}}\right]$$

Universal vs. Nuclear Corrections

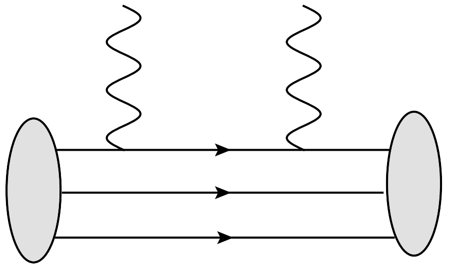
$$\Box_{\gamma W}^{VA,\,Nucl.} = \frac{\alpha}{N\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}\; F_{3,\,\gamma W}^{(0),\,\mathrm{Nucl.}}(\nu,Q^2)$$

Coupling to two different nucleons within a nucleus:
not present in the free nucleon case - new.
Shell model calculations: $\delta_{NS} \sim -0.3\% - 0$



Hardy, Towner '15, '18

Coupling to the same nucleon:
Low energy - quasielastic vs. free nucleon Born



$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\ln\frac{M_W}{\Lambda} + A_g + C^{Int} + 2C_B\right]$$

Modification - identically

universal

$$C_B^{\mathrm{Nucl.}} = C_B^{\mathrm{free\,n}} + [C_B^{\mathrm{Nucl.}} - C_B^{\mathrm{free\,n}}]$$

nuclear δ_{NS}

Universal vs. Nuclear Corrections

universal

Towner 1994 and ever since: $$C_B^{\rm free\ n} \to C_B^{\rm Nucl.} = C_B^{\rm free\ n} + [q_S^{(0)} q_A - 1] C_B^{\rm free\ n}$$

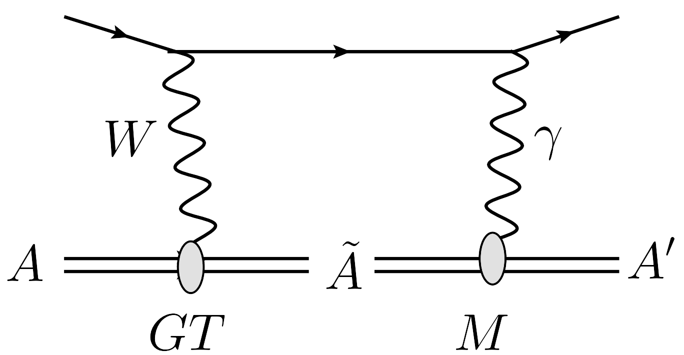
nuclear δ_{NS}

Idea: calculate Gamow-Teller and magnetic nuclear transitions in the shell model;
Insert the single nucleon spin operators —> predict the strength of nuclear transitions
"Quenching of spin operators in nuclei": shell model overestimates those strengths!

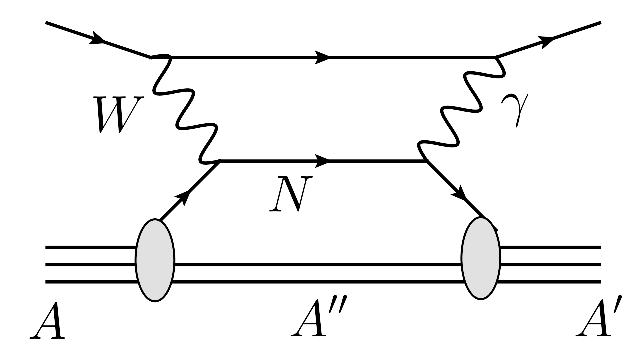
Each vertex is suppressed by 10-15%
Hardy, Towner: just rescale the Born contribution to the γW-box by that quenching,
assume the integral to be the same (nucleon form factors)

Numerically: on average $[q_S^{(0)} q_A - 1] C_B = -0.25$

$$\delta_{NS}^{quenched\,Born} = \frac{\alpha}{\pi}[q_S^{(0)} q_A - 1] C_B \approx -0.055(5)\,\%$$ used since 1998



But from dispersion relation perspective it corresponds
to a contribution of an excited nuclear state,
not to the modified box on a free nucleon!
The correct estimate should base on quasielastic knockout
with an on-shell N + spectator in the intermediate state



Modification of C_B in a nucleus - QE

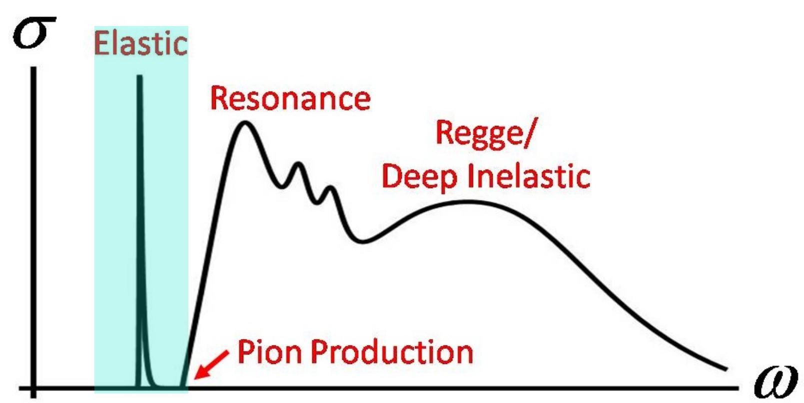
Integral is peaked at low ν, Q^2

$$\Box_{\gamma W}^{VA,\,Nucl.} = \frac{\alpha}{N\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}\; F_{3,\,\gamma W}^{(0),\,\mathrm{Nucl.}}(\nu,Q^2)$$

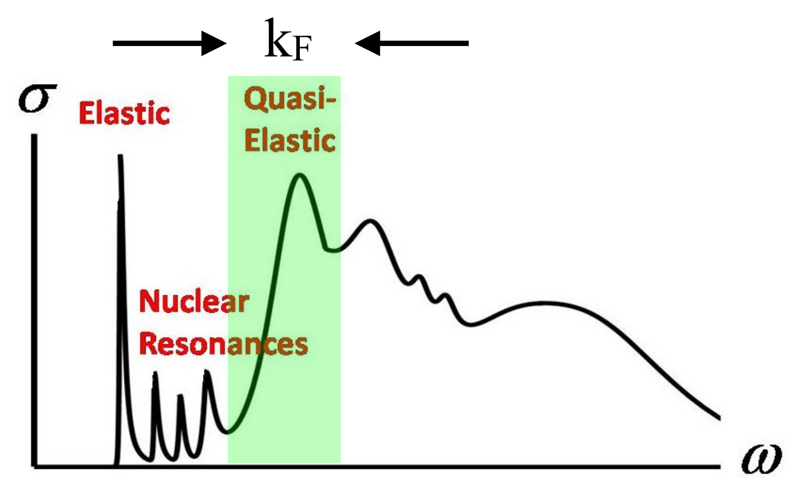
Born on free n:

$$F_3^{(0),\,B} = -\frac{Q^2}{4}G_A G_M^S \delta(2M\nu - Q^2)$$

Reduction for QE:
finite threshold ϵ (binding energy) + Fermi momentum k_F



$\nu = Q^2/2M$



$\nu \geq Q^2/2M + \epsilon$

QE calculation in free Fermi gas model with Pauli blocking
(nucleon momenta distributed uniformly within a sphere of radius k_F)

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

$C_{QE} - C_B = -0.45 \pm 0.04$ compare to the "quenched" estimate $[q_S^{(0)} q_A - 1]C_B = -0.25$

New δ^{QE}_{NS} ~ - 0.10(1)% instead of the previous estimate δ^{q}_{NS} ~-0.055(5)%

QE calculation - effect on Ft values and V_{ud}

Adopting a new estimate of the in-nucleus modification of the free-nucleon Born

Shifts the Ft value according to $\overline{\mathcal{F}t} \to \overline{\mathcal{F}t}(1 + \delta_{NS}^{new} - \delta_{NS}^{old})$

Numerically: $\mathcal{F}t = 3072.07(63)s \to [\mathcal{F}t]^{\mathrm{new}} = 3070.65(63)(28)s$

Will affect the extracted V_{ud} $|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$

Compensates for a part of the shift due to a new evaluation of Δ^V_R

$$V_{ud}^{\mathrm{old}} = 0.97420(21) \ \to \ V_{ud}^{\mathrm{new}} = 0.97370(14) \to V_{ud}^{\mathrm{new,\ QE}} = 0.97392(14)(04)$$

Brings the first row a little closer to the unitarity (4σ ➞ 3σ)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004 \quad \to |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9988 \pm 0.0004$$

Important message:
dispersion relations as a unified tool for treating hadronic and nuclear parts of RC

3.Splitting of the RC into inner and outer

MG, arXiv: 1812.04229

Splitting the RC into “inner” and “outer”

Radiative corrections ~ $\alpha/2\pi$ ~ 10^{-3} Precision goal: ~ 10^{-4}

When does energy dependence matter?
Correction ~ E_e/Λ, with Λ ~ relevant mass (m_e; M_p; M_A)
Maximal E_e ranges from 1 MeV to 10.5 MeV

Electron mass regularizes the IR divergent parts - (E_e/m_e important) - “outer” correction

If Λ of hadronic origin (at least m_π) —> E_e/Λ small, correction ~ 10^{-5} —> negligible
- certainly true for the neutron decay
- hadronic contributions do not distort the spectrum, may only shift it as a whole

However, in nuclei binding energies ~ few MeV — similar to Q-values

A scenario is possible when RC ~ $(\alpha/2\pi) \times (E_e/\Lambda^{Nucl})$ ~ 10^{-3}

Nuclear structure may distort the electron spectrum

With dispersion relations can be checked straightforwardly!

Nuclear structure distorts the β-spectrum!

With DR: can include energy dependence explicitly
Even and odd powers of energy - leading terms

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi N}\int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty d\nu \frac{F_3^{(0)}}{M\nu}\frac{\nu+2q}{(\nu+q)^2} + O(E^2)$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) = \frac{8\alpha E}{3\pi N M}\int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty \frac{d\nu}{(\nu+q)^3}\left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu+q)}{2Q^2}+1\right)\frac{M}{\nu}F_2^{(0)} + \frac{\nu+3q}{4\nu}F_3^{(-)}\right] + O(E^3)$$

E-dependent correction to the diff. decay rate in Fermi gas model:

$$\Delta_R(E) = 2\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) \ \sim (8\pm 8)\times 10^{-4}\left(\frac{E}{5\,\mathrm{MeV}}\right)$$

Correction to Ft values: integrate over spectrum (only total rate measured)

$$\Delta_E^{NS} = \frac{\int_{m_e}^{E_m} dE E p (Q-E)^2 \Delta_R(E)}{\int_{m_e}^{E_m} dE E p (Q-E)^2} \longrightarrow \tilde{\mathcal{F}}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS}+\Delta_E^{NS})$$

Nuclear structure distorts the β-spectrum!

$$\tilde{\mathcal{F}}t = ft(1+\delta'_R)(1-\delta_C+\delta_{NS}+\Delta_E^{NS})$$

Absolute shift in Ft values $\delta\mathcal{F}t = \mathcal{F}t \times \Delta_E^{NS}$

Decay	Q (MeV)	$\Delta_E^{NS}(10^{-4})$	$\delta\mathcal{F}t(s)$	$\mathcal{F}t(s)$ [3]
^{10}C	1.91	1.5	0.5	3078.0(4.5)
^{14}O	2.83	2.3	0.7	3071.4(3.2)
^{22}Mg	4.12	3.3	1.0	3077.9(7.3)
^{34}Ar	6.06	4.8	1.5	3065.6(8.4)
^{38}Ca	6.61	5.3	1.6	3076.4(7.2)
^{26m}Al	4.23	3.4	1.0	3072.9(1.0)
^{34}Cl	5.49	4.4	1.4	$3070.7^{+1.7}_{-1.8}$
^{38m}K	6.04	4.8	1.5	3071.6(2.0)
^{42}Sc	6.43	5.1	1.6	3072.4(2.3)
^{46}V	7.05	5.6	1.7	3074.1(2.0)
^{50}Mn	7.63	6.1	1.9	3071.2(2.1)
^{54}Co	8.24	6.6	2.0	$3069.8^{+2.4}_{-2.6}$
^{62}Ga	9.18	7.3	2.2	3071.5(6.7)
^{74}Rb	10.42	8.3	2.6	3076(11)

Shift due to Δ_E^{NS}: comparable to precision of 7 best-known decays

$$\overline{\mathcal{F}t} = 3072.07(63)\text{s} \rightarrow \overline{\mathcal{F}t} = 3073.6(0.6)(1.5)\text{s}$$

Decay electron polarizes the daughter nucleus

As a result the spectrum is slightly distorted towards the upper end

Positive-definite correction to Ft ~ 0.05%

Previously found: E-independent piece lowers the Ft value by about the same amount

$$\mathcal{F}t = 3072.07(63)s \rightarrow [\mathcal{F}t]^{\text{new}} = 3070.65(63)(28)s$$

Nuclear structure uncertainties might be underestimated

CKM first-row unitarity at a historic low.
Solutions: SM or beyond?

Discrepancy - BSM?

BSM explanation: non-standard CC interactions —> new V,A,S(PS),T(PT) terms

$$H_{S+V} = (\overline{\psi}_p \psi_n)(C_S \overline{\phi}_e \phi_{\overline{\nu}_e} + C'_S \overline{\phi}_e \gamma_5 \phi_{\overline{\nu}_e}) + (\overline{\psi}_p \gamma_\mu \psi_n) \left[C_V \overline{\phi}_e \gamma_\mu (1+\gamma_5) \phi_{\overline{\nu}_e}\right]$$

Scalar and Tensor interactions: distort the beta decay spectra

Complementarity to LHC searches (Martin's talk)

Exp. high precision measurement of ^{6}He spectrum (O. Naviliat-Cuncic, A. Garcia, …)

$$N(E)dE = p_e E (E_m - E)^2 \left[1 + C_1 E + b\frac{m_e}{E}\right]$$

C_1 = 0.00650(**7**) MeV^{-1} - effect of weak magnetism - positive slope
b ~ +- 0.001 - negative slope

Energy-dep. polarizability correction —> C'_1 ~ 0.00020(**20**) MeV^{-1} — at the level 3σ of C_1

Conclusions & Outlook

- The γW-box was evaluated in a new dispersion relation framework
- Confirmed dominant features of previous calculations but corrected subdominant ones
- Related the model-dependent contribution to neutrino data - systematically improvable!
- Hadronic and nuclear corrections in a unified framework
- Nuclear structure leaks in the outer correction, distorts the beta decay spectrum
- Nuclear uncertainties shift the emphasis on free neutron decay
- Tensions with CKM unitarity: $\Sigma_{i=d,s,b} |V_{ui}|^2 - 1 = -0.0016(4\text{-}6)$

Hadronic correction Δ_R^V
Neutrino data at low Q^2 used in this analysis are not precise
DUNE@Fermilab will provide better data for F_3 - direct check
Moments $M_3^{(0)}(N,Q^2)$ at 1 GeV^2 CAN and MUST be computed on the lattice

Nuclear correction δ_{NS}
DR allow to address hadronic and nuclear parts of the calculation on the same footing
Better calculations than free Fermi gas are needed
The full nuclear correction should be calculated (not just QE)

Decay spectra and nuclear polarizabilities
This novel effect needs a confirmation in more sophisticated models
Can contaminate the extraction of Fierz interference from precise spectra!