Constraints on the parameters of efficiently radiating relativistic jets

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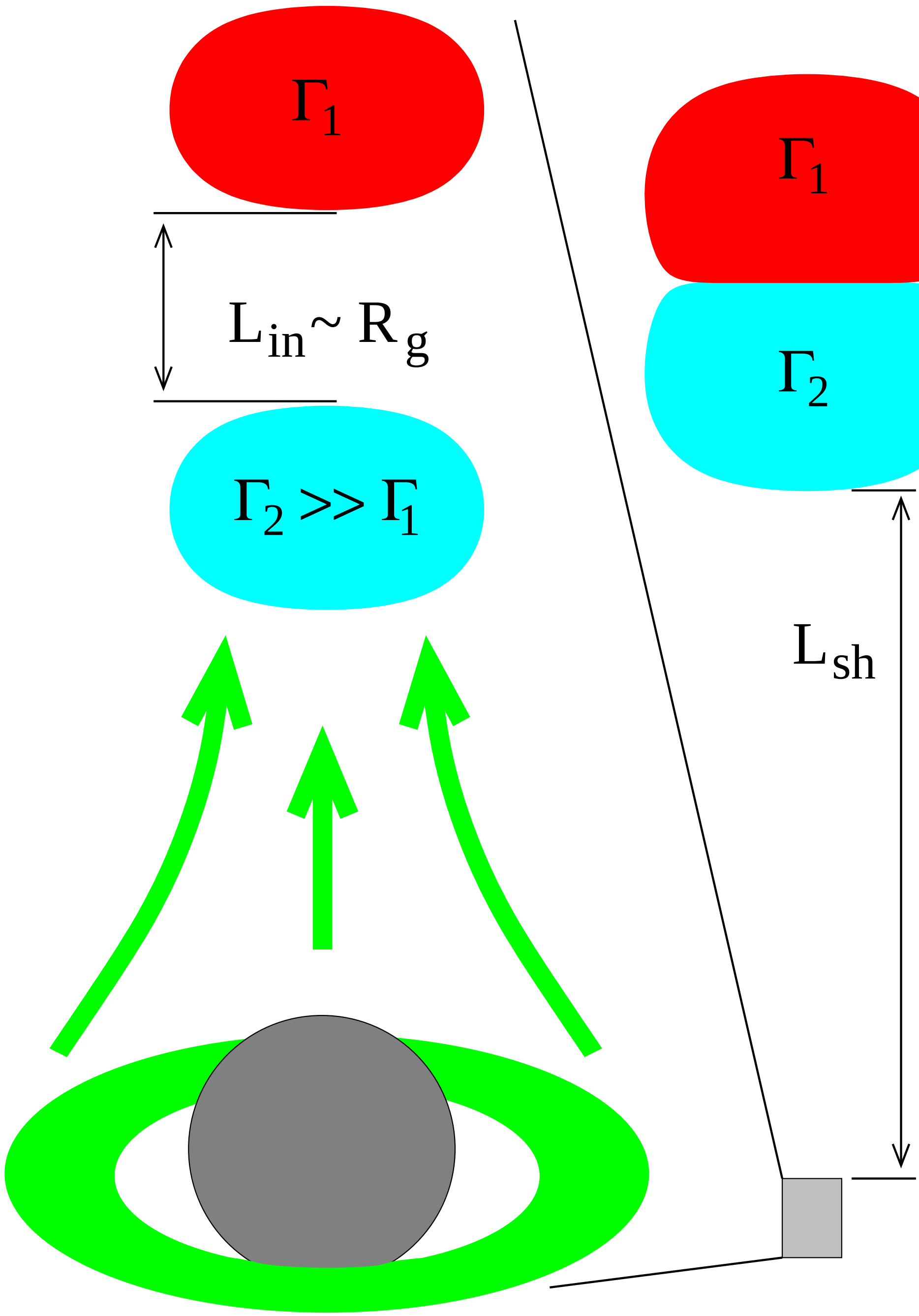
Physical limit to variability timescale



The central engine can change its state in a time $\sim R_g/c$

R_g – Schwarzschild radius

The faster blob catches up with the slower one at the distance

$$L_{\rm sh} = c\,\frac{L_{\rm in}}{v_2 - v_1} \simeq 2\,\Gamma_1^2\,L_{\rm in}$$

The minimum apparent duration of the light burst (at $\theta = 0$):

$$\tau \simeq \frac{L_{\rm sh}}{2\,\Gamma^2\,c} \sim \frac{R_g}{c}$$

An (arguable) assumption

The variability timescale measured in the comoving frame is approximately equal to the light-crossing time.

Increase Lorentz factor!

- **The emitting region moves further away**

$$L_{\mathrm{sh}} \propto \Gamma^2$$

- **Comoving photon density rapidly decreases**

$$w'_{\mathrm{ph}} \simeq \frac{1}{\Gamma^2}\,\frac{\mathcal{L}}{4\pi L_{\mathrm{sh}}^2 c} \simeq \frac{\mathcal{L}}{4\pi (c\tau)^2 \Gamma^6 c}$$

$\mathcal{L}$ – the isotropic luminosity

- **Two-photon absorption threshold increases**

An arbitrarily large Lorentz factor ...

(1) The synchrotron peak is at $\varepsilon = \Gamma \gamma'^2 \hbar \, \frac{eB}{m_e c}$

(2) Radiation flux is a fraction
of the magnetic-energy flux $w'_{\rm ph} = \eta_1 \frac{B^2}{8\pi}$

(3) Radiation efficiency is $\eta_2 \le \frac{4}{9} \frac{\Gamma \tau \, \gamma' \left(\frac{e^2}{mc^2}\right)^2 B^2}{m_e c}$

For a given isotropic luminosity $w'_{\rm ph} = \frac{\mathcal{L}\,\tau}{4\pi(\Gamma^2 c\tau)^3}$

τ – the dynamical timescale ($\approx$ the observed variability timescale)
ε – the observed photon energy
Γ – the jet's Lorentz factor
γ' – the electron's Lorentz factor in the jet comoving frame

... cannot be arbitrarily large.

Derishev, Kocharovsky, Kocharovsky, A&A **372**, 1071 (2001)
Begelman, Fabian, Rees, MNRAS Letters **384**, L19 (2008)

Substitute:

γ' from expression **(1)** into inequality **(3)**

B from expression **(2)** into inequality **(3)**

Obtain:

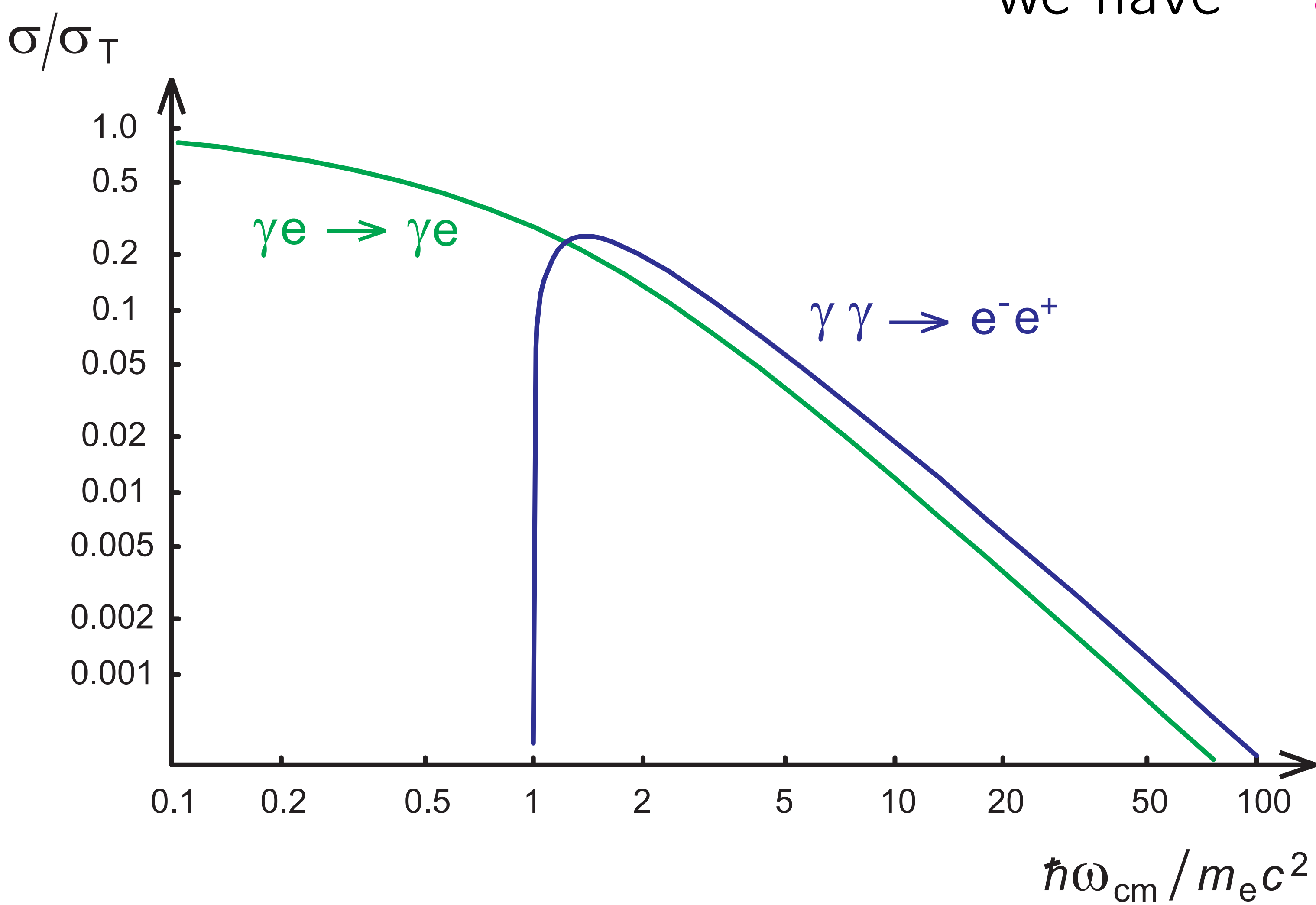
the upper limit to the Lorentz factor

$$\Gamma < \left(\frac{\left[2^{11}/3^{8}\right] \; e^{14} \; \varepsilon^{2} \; \mathcal{L}^{3}}{\eta_1^3 \eta_2^4 \, \hbar^2 \; (m_e c^2)^{10} \; c^7 \; \tau^2} \right)^{1/16} \simeq 55 \left(\frac{1}{\eta_1^3 \eta_2^4} \right)^{1/16} \frac{\varepsilon_6^{1/8} \, \mathcal{L}_{45}^{3/16}}{\tau_3^{1/8}}$$

Interlude: Two-photon absorption

In the limit $\varepsilon_\gamma \gg m_e c^2$

we have $\sigma_{\gamma\gamma} \approx 2\,\sigma_{e\gamma}$



Two-photon absorption

Optical depth for two-photon absorption

$$\tau_{\gamma\gamma}(\omega) \simeq \sigma_{\gamma\gamma}\, N_{ph}(\omega_*)\, R$$

Inverse Compton energy losses per particle

$$\dot{\varepsilon} \simeq \frac{1}{2}\varepsilon\, \sigma_{e\gamma}\, N_{ph}(\omega_*)\, c$$

Under assumption of high radiation efficiency ($\dot{\varepsilon} > \varepsilon/t$)

the optical depth of a source with size $R \simeq ct$ is

$$\tau_{\gamma\gamma} > 2\,\frac{\sigma_{\gamma\gamma}(\varepsilon/2)}{\sigma_{e\gamma}(\varepsilon)} \gg 1$$

$N_{ph}\,(\omega_*)$ – number density of photons with frequency $\sim \omega_*$

Self-Compton radiation

(1) The peak is at $\varepsilon = \Gamma \gamma'^{4} \hbar \frac{eB}{m_e c}$ (Thomson regime!)

(2) Radiation flux is a fraction of the magnetic-energy flux $w'_{\mathrm{ph}} = \eta_1 \frac{B^2}{8\pi}$

(3) Radiation efficiency is

$$\eta_2 \leq \frac{4}{9} \frac{\Gamma \tau \gamma' \left(\frac{e^2}{mc^2}\right)^2 B^2}{m_e c}$$

or

$$\eta_2 \leq \frac{4}{9} \eta_1^{1/2} \frac{\Gamma \tau \gamma' \left(\frac{e^2}{mc^2}\right)^2 B^2}{m_e c}$$

The limiting Lorentz factor (inverse Compton radiation)

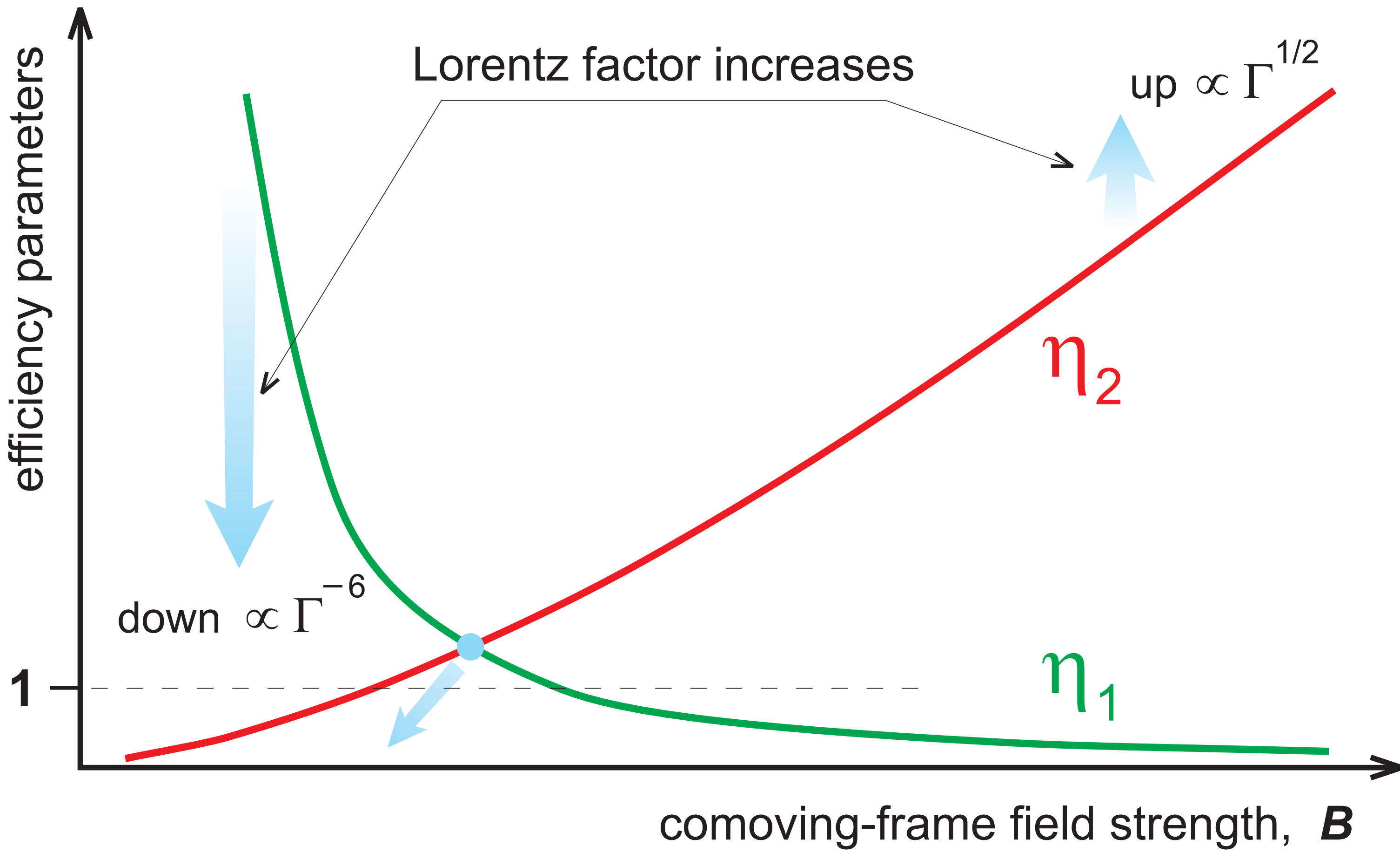
$$\Gamma < \left(\frac{\left[2^{23}/3^{16}\right] \ e^{30} \ \varepsilon^2 \ \mathcal{L}^7}{\eta_1^7 \eta_2^8 \, \hbar^2 \ (m_e c^2)^{22} \ c^{15} \ \tau^6} \right)^{\frac{1}{36}}$$

For active galactic nuclei

$$\Gamma \leq 21 \left(\frac{1}{\eta_1^7 \eta_2^8} \right)^{1/36} \frac{\varepsilon_{12}^{1/18} \ \mathcal{L}_{45}^{7/36}}{\tau_3^{1/6}}$$

For gamma-ray bursts

$$\Gamma \leq 1400 \left(\frac{1}{\eta_1^7 \eta_2^8} \right)^{1/36} \frac{\varepsilon_{6}^{1/18} \ \mathcal{L}_{51}^{7/36}}{\tau_{-3}^{1/6}}$$



Observed parameters:
$\mathcal{L}$, τ, $\varepsilon_{\rm sy}$

Assumption: $R \approx c\tau$

Free parameter: Γ

$$\eta_1 = \frac{\text{radiated energy}}{\text{magnetic field energy}} \propto \frac{\mathcal{L}\,\tau}{R^3 B^2} \propto \frac{\mathcal{L}}{\tau^2 B^2} \quad \approx \text{Compton } y \text{ parameter}$$

$$\eta_2 = \frac{\text{dynamical timescale}}{\text{particle cooling timescale}} \propto \gamma\, B^2 \tau \propto \varepsilon_{\rm sy}^{1/2} B^{3/2} \tau$$

2nd interlude: SSC vs ERC

- Efficient cooling means that $$w'_{\rm ph} > \frac{m_e c}{\frac{32\,\pi}{9}\,\gamma'\left(\frac{e^2}{m_e c^2}\right)^2 \Gamma\tau}$$
- Photons' occupation number $$K \simeq w'_{\rm ph}\frac{2\pi^2(\hbar c)^3}{\varepsilon_*^4}$$
- Comptonization in the Thomson regime, i.e.

$\varepsilon_* < m_e c^2/\gamma'$ and $\Gamma\gamma' > \varepsilon/m_e c^2$

Hence,

$$K > \frac{9\,\pi}{16}\left(\frac{\varepsilon}{m_e c^2}\right)^3 \frac{\lambda_c}{\alpha^2\Gamma^4\,c\tau} \qquad \text{(for SSC)}$$

$$K > \frac{9\,\pi}{16}\left(\frac{\varepsilon}{m_e c^2}\right)^3 \frac{\lambda_c}{\alpha^2\Gamma^2\,c\tau} \qquad \text{(for ERC)}$$

ε_* – the energy of comptonized photon in the comoving frame
α – the fine-structure constant
λ_c – the electron Compton wavelength

SSC vs ERC

Assume:

the photons' occupation number does not exceed its magnitude at the peak of black-body spectrum, i.e. $K < 0.02$

obtain:

independent *lower* limit to the Lorentz factor

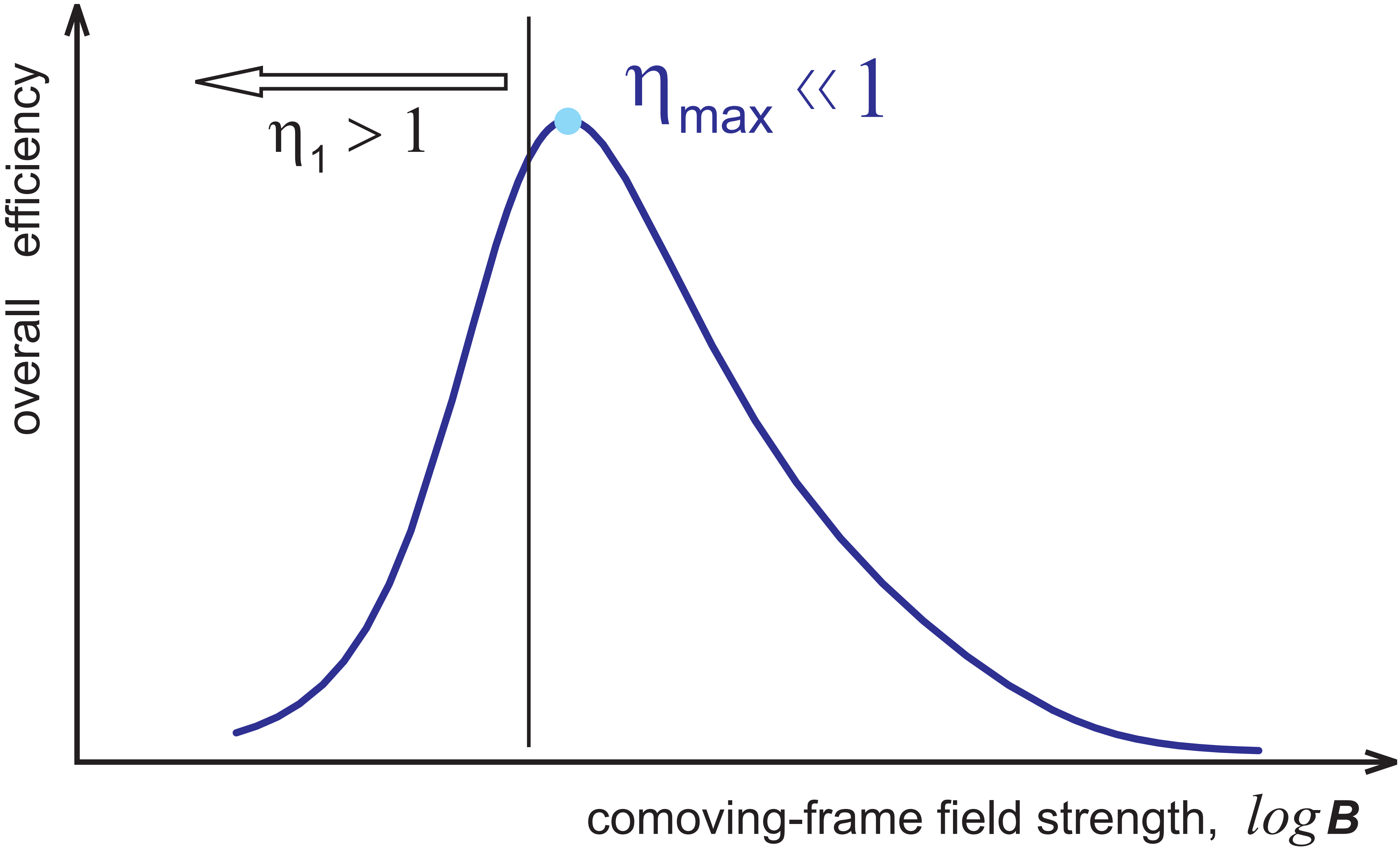
- For synchrotron self Compton

$$\Gamma > \frac{3}{\alpha^{1/2}} \left(\frac{\varepsilon}{m_e c^2} \right)^{3/4} \left(\frac{\lambda_c}{c\,\tau} \right)^{1/4} \simeq 2\, \frac{\varepsilon_{12}^{3/4}}{\tau_3^{1/4}}$$

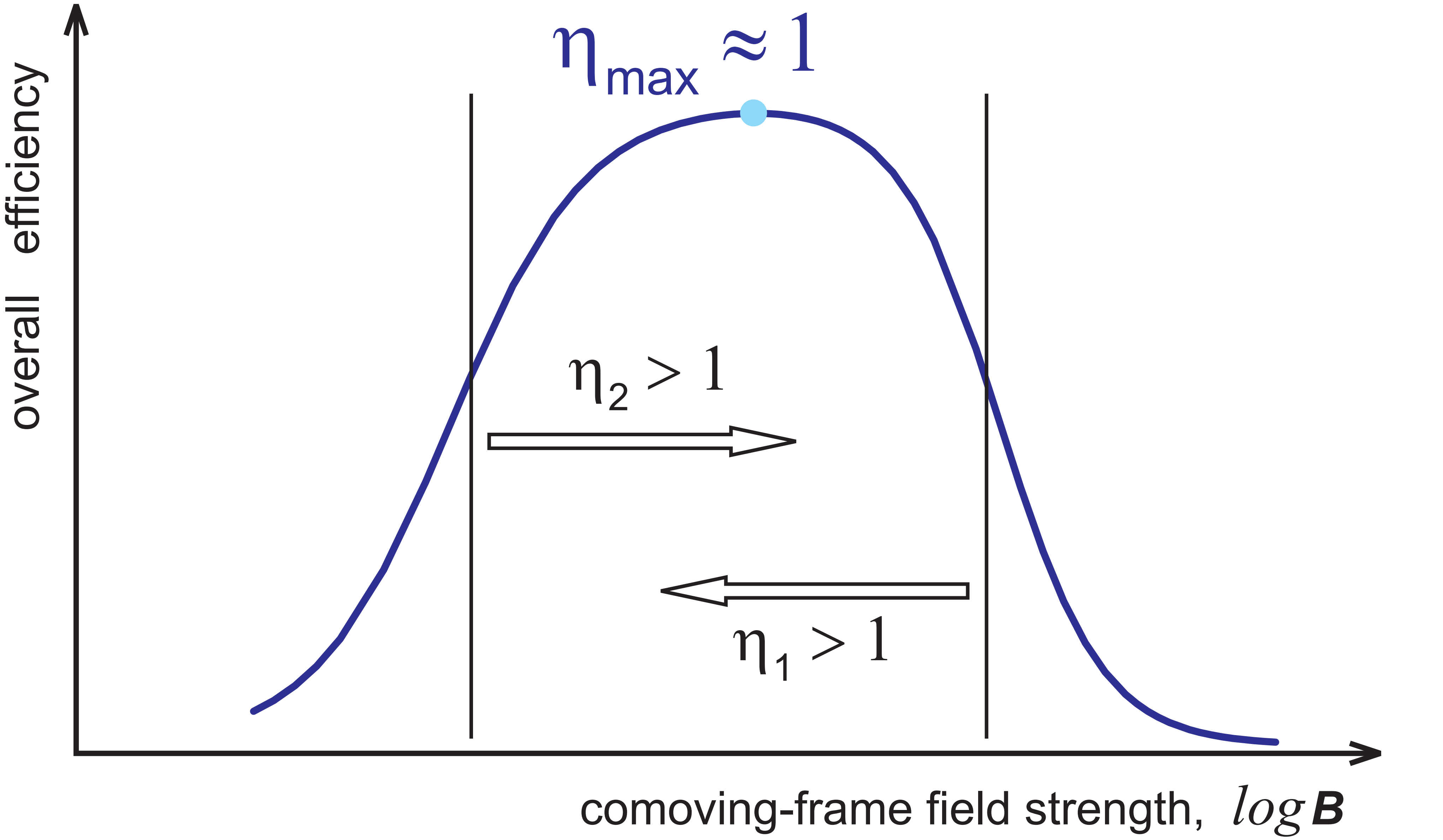
- For external radiation Compton

$$\Gamma > \frac{9}{\alpha} \left(\frac{\varepsilon}{m_e c^2} \right)^{3/2} \left(\frac{\lambda_c}{c\,\tau} \right)^{1/2} \simeq 4\, \frac{\varepsilon_{12}^{3/2}}{\tau_3^{1/2}}$$

Inefficient sources



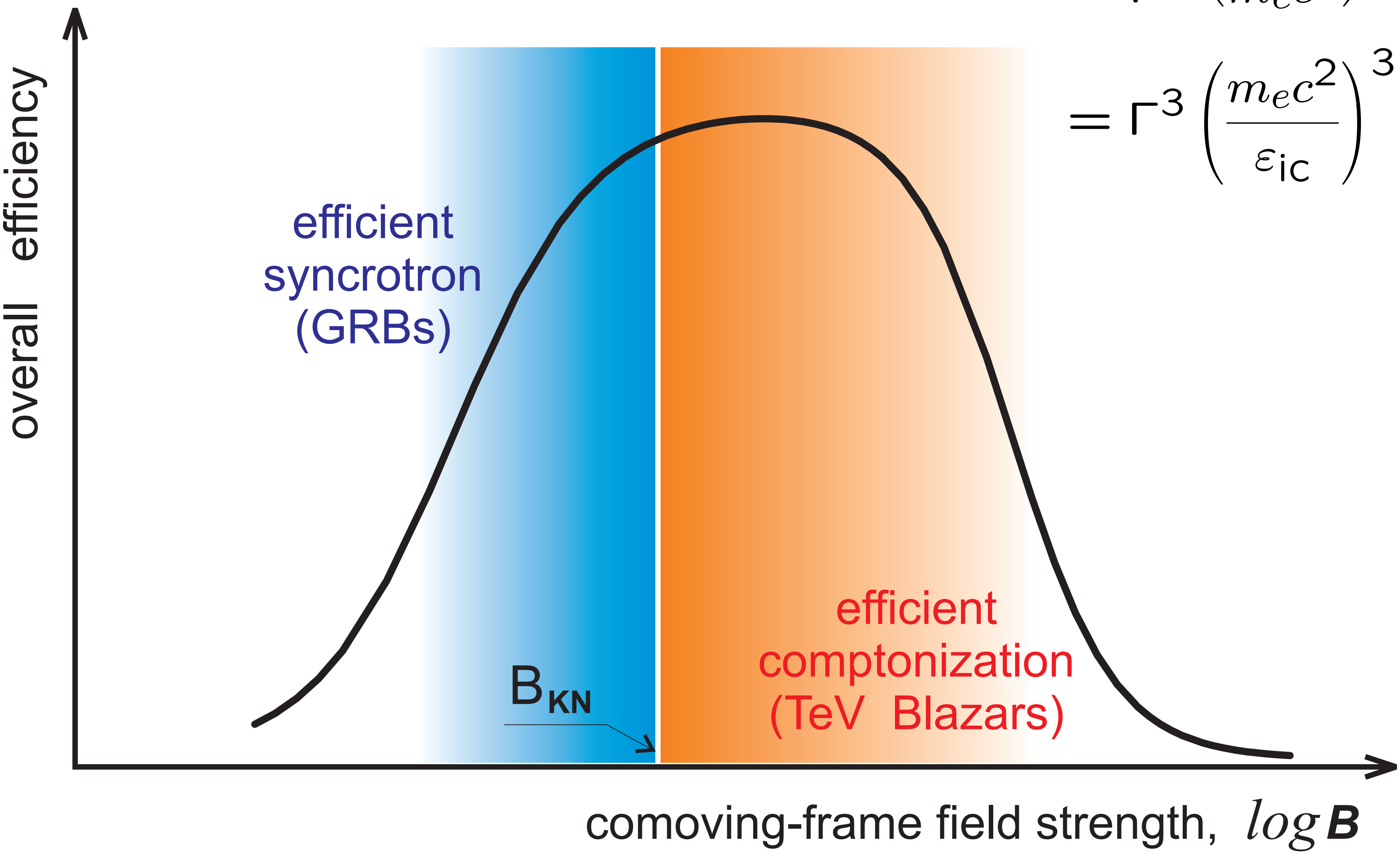
Efficient sources



Living rooms for GRBs and TeV Blazars

$$B_{\mathrm{KN}} = \frac{1}{\Gamma^3}\left(\frac{\varepsilon_{\mathrm{sy}}}{m_e c^2}\right)^3 \frac{m_e^2 c^3}{\hbar e}$$

$$= \Gamma^3 \left(\frac{m_e c^2}{\varepsilon_{\mathrm{ic}}}\right)^3 \frac{m_e^2 c^3}{\hbar e}$$



Gamma-ray Bursts

There is a window in the parameter space, where the sources are

- radiatively efficient
- opaque for the inverse Compton radiation

at the same time.

This leads to the condition $B < B_{\mathsf{KN}}$, **equivalent to**

$$\tau > \frac{1}{\alpha}\frac{r_e}{c}\left(\frac{2\,\mathcal{L}\,r_e/c}{m_e c^2}\right)^{1/2}\left(\frac{m_e c^2}{\varepsilon}\right)^{3} \simeq 3\times 10^{-5}\,\mathrm{s}\ \ \mathcal{L}_{51}^{1/2}\varepsilon_6^{-3}$$

α – the fine-structure constant
r_e – the classical electron radius

A byproduct limit for GRB sources

If a Gamma-Ray Burst is powered by a black hole,

then its mass must be

$$M > 3 \ \mathcal{L}_{51}^{1/2} \varepsilon_6^{-3} \quad M_\odot$$

TeV Blazars

A good TeV Blazar must be

- radiatively efficient
- *transparent* for the inverse Compton radiation

The window opens if $\eta_1(B_{\mathrm{KN}}) > 1$, **which may be treated in two ways**

(1) $$\tau < \frac{1}{\Gamma^6}\,\frac{1}{\alpha}\,\frac{r_e}{c}\left(\frac{2\,\mathcal{L}\,r_e/c}{m_e c^2}\right)^{\frac{1}{2}}\left(\frac{\varepsilon}{m_e c^2}\right)^3 \simeq 1.5\times 10^{12}\,\mathrm{s}\;\;\frac{\mathcal{L}_{45}^{1/2}\,\varepsilon_{12}^3}{\Gamma^6}$$

(2) $$\Gamma < \left(\frac{2\,\mathcal{L}\,r_e/c}{\alpha^2\, m_e c^2}\right)^{\frac{1}{12}}\left(\frac{r_e}{c\tau}\right)^{\frac{1}{6}}\left(\frac{\varepsilon}{m_e c^2}\right)^{\frac{1}{2}} \simeq 30\;\;\frac{\mathcal{L}_{45}^{1/12}\;\;\varepsilon_{12}^{1/2}}{\tau_3^{1/6}}$$

α — the fine-structure constant
r_e — the classical electron radius

A byproduct limit for blazars

If a TeV blazar is powered by a black hole,

then its mass must be

$$M < 1.5 \times 10^{17} \frac{\mathcal{L}_{45}^{1/2} \varepsilon_{12}^{3}}{\Gamma^{6}} \quad M_{\odot}$$