Entropy Rain and the Solar Convective Conundrum: Dilution and Compression of Individual Convective Downflows
Evan H. Anders, Daniel Lecoanet, Benjamin P. Brown
Heat Transfer of Stars

> 1.5 solar masses

0.5 - 1.5 solar masses

< 0.5 solar masses

Convection Zone
Radiation Zone
The classic view of envelope convection

Corona
Surface Convection Zone
Deep Convection Zone
Radiative Zone

Big swirls
Little swirls

[Image courtesy of Daniel Lecoanet]
Solar Convective Flows

Granules:
- \(~1 \text{ Mm}\)
- \(~5 \text{ min}\)

Supergranules:
- \(~30 \text{ Mm}\)
- \(~1 \text{ day}\)

Visible (Photosphere)  Ca II (Chromosphere)
Flows in Simple Stratified Convection

[Anders & Brown 2017; PRF 2]
The Solar Convective Conundrum: Horizontal Surface Flows

Hathaway et al. 2015
The Solar Convective Conundrum: Helioseismology

Expected from simulations

Near surface

Greer et al. 2015

Averaging kernel sensitivity (scaled)

Velocity (m/s)

Harmonic Degree

0.25 Mm
5 Mm
15 Mm
30 Mm
The Convective Conundrum cont’d

Red = surface observations
Blue = simulations
Grey = helioseismology

[Hanasoge et al 2015]
Extra motivation in the asteroseismic age

Many observations which rely on stellar structure models.

→ Stellar structure models rely on convection parameterizations like MLT.

→ We need to understand stellar envelope convection better.

[ Astro2020 white paper: Huber et al 2019 ]
Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
What’s going on?

Some options:
1) The observations are wrong
What’s going on?

Some options:
1) The observations are wrong
2) Some dynamical process masks giant cells
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3) Giant cells are not driven
What’s going on?

Some options:
1) The observations are wrong
2) Some dynamical process masks giant cells
3) Giant cells are not driven

Entropy Rain Hypothesis: Cold downflows from the solar surface carry the solar luminosity to the bottom of the CZ, not warm upflows which would manifest as giant cells.

[Spruit 1997; Brandenburg 2016; and results of Käpylä et al. 2017]
Thermals as a model for Entropy Rain

Starts as a blob of buoyant fluid

Evolves into a buoyant vortex ring

Entrains fluid from environment

[Suggested by Brandenburg 2016]
Thermals as a model for Entropy Rain
3D Visualization of an Evolved Thermal
Flows in Simple Stratified Convection

Anders & Brown 2017; PRF 2
Flows in Simple Stratified Convection

[Anders & Brown 2017; PRF 2]
Thermals as a model for Entropy Rain

Fundamental Questions:

1.) Can entropy rain survive throughout the depth of the CZ?

2.) Can entropy rain carry the stellar luminosity?
Thermal Entrainment in the Incompressible Limit

<table>
<thead>
<tr>
<th>Depth:</th>
</tr>
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<tbody>
<tr>
<td>$z \propto t^{1/2}$</td>
</tr>
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</table>

(implies deceleration)

growth of thermal through transport of environmental fluid into the thermal

[Lecoanet & Jeevanjee 2019]
Thermal Entrainment in the Incompressible Limit

Volume:
\[ V \propto r^3 \text{ (measured)} \]
\[ V \propto z^3 \text{ (spheroidal)} \]

[Lecoanet & Jeevanjee 2019]
Volume: $V \propto r^3 \propto z^3$, so
Radius: $z \propto r \propto t^{1/2}$

Even though thermals buoyantly accelerate, they entrain enough to stall themselves.
Fundamental problem of thermals as a model for entropy rain:

*If thermals entrain & slow down, how can they cross the solar CZ?*
Stratification breaks symmetry

**Cold thermals:**
- Boussinesq-like entrainment
- *stratification*-enduced *compression*

**Hot thermals:**
- Boussinesq-like entrainment
- *stratification*-enduced *expansion*
Stratification effects without buoyancy

Compression should go as:

- \( r \propto \rho^{-1/2} \) for *horizontal* compression
- \( r \propto \rho^{-1/3} \) for *spherical* compression

For \( \rho \), the atmospheric density, and \( r \), the radius:

[Brandenburg 2016]
Buoyantly neutral Hill vortices

Non-buoyant hill vortex simulations were between expected limits (slope of -0.8).

[Brandenburg 2016]
Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary.
Experiment: Evolve thermals in atmospheres with different stratifications

Fundamental control parameter:
\[ S = \frac{\text{size of thermal}}{\text{atmospheric scale height}} \]

Limits:
\[ S \to 0 \text{ (Boussinesq)} \]
\[ S \to O(1) \text{ (stratification matters)} \]
To first order, we see what we expect: blended compression & entrainment.

Sims soon; First, some theory

Goal: Model vortex ring stage of thermal’s evolution

Important quantities:
- Buoyancy of the thermal, $B$
- Circulation of the vortex ring, $\Gamma$
- Volume of the thermal, $V$
- Radius of the thermal, $r$
- Impulse of the thermal, $I$
Theory of stratified vortex ring entrainment

Fundamental Quantity: \textbf{Impulse}

\[ I = \frac{1}{2} \int_{\mathcal{V}} x \times (\nabla \times (\rho \mathbf{u})) dV, \]
Theory of stratified vortex ring entrainment

...which is similar to the momentum

\[ I_z = (1 + k) \rho \mathcal{V} \mathcal{W}_{th}, \]
Theory of stratified vortex ring entrainment

...which is similar to the momentum

\[ I_z = (1 + k) \rho \mathcal{V}_\text{th}, \]

and which is approximately (thin core vortex ring)

\[ I_z \approx \pi \rho_0 r^2 \Gamma, \]
Theory of stratified vortex ring entrainment

Assume the volume-integrated buoyancy force of the thermal is $\sim$constant in time,

$$B \equiv \int_{V} \rho_0 S_1 \frac{g}{c_P} \, dV.$$
Theory of stratified vortex ring entrainment

Assume the volume-integrated buoyancy force of the thermal is \( \sim \) constant in time,

\[
B \equiv \int_V \rho_0 S_1 \frac{g}{c_p} \, dV.
\]

...and assume buoyancy is the only force acting on the thermal.
Theory of stratified vortex ring entrainment

Assume circulation is constant (no baroclinic torques)

\[
\frac{d\Gamma}{dt} = \oint_C g \frac{S_1}{c_P} \hat{z} \cdot d\mathbf{x},
\]
The theory of stratified vortex ring entrainment

We can solve impulse for radius vs time,

\[ I_z \approx \pi \rho_0 r^2 \Gamma, \]

\[ r = C \left( \sqrt{\frac{B_{th} t + I_0}{\rho_0}} \right) \]

→ \( r \propto \rho^{-1/2} \) for purely horizontal compression, and

→ \( r \propto t^{1/2} \) for Boussinesq regime.
Theory of stratified vortex ring entrainment

Assume spheroidal volume

\[ \mathcal{V} \approx m r^3 \]

Substitute in the definition of velocity,

\[ w_{th} \equiv d z_{th}/dt \]
The theory of stratified vortex ring entrainment

Step 6: Solve for depth vs time

\[ \frac{dz}{\rho(z)^{1/2}} = C \frac{dt}{(t + t_{\text{off}})^{1/2}} \]

(Simple ODE which can be solved for a prescribed density stratification)
Experiment: two types of simulations

1) 2D anelastic, cylindrically symmetric
   → Computationally cheap
   → Limit in which theory was derived

2) 3D fully compressible, cartesian boxes
   → The “true” solution
Simulations vs. Theory

Simulations vs. Theory

Boussinesq, $t^{1/2}$

Highly Stratified, $t^2$

Speed up

Slow down

Simulations vs. Theory

Simulations vs. Theory

Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary.
3) Buoyancy makes vortex rings expand & entrain, while stratification compresses them. Entropy rain feels both of these effects.
4) We develop a theory that describes the size & depth of entropy rain vs. time and verify that theory with simulations. There are two regimes: “stalling” and “falling”.
Extrapolation to solar parameters

<table>
<thead>
<tr>
<th>Simulation Output Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\rho$</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>2D Anelastic Simulations (Cylindrical)</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>3D Fully Compressible Simulations (Cartesian)</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

- **Roughly constant as stratification increases**
- **Changes with increasing stratification**

Stratification increases
Extrapolation to solar parameters

We calculated what $B$ would be for a solar downflow.

<table>
<thead>
<tr>
<th>$N_{\rho}$</th>
<th>$B_{th}$</th>
<th>$\Gamma_{th}$</th>
<th>$m$</th>
<th>$\eta$</th>
<th>$k$</th>
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<tbody>
<tr>
<td></td>
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<td>(2D Anelastic Simulations (Cylindrical))</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>0.1</td>
<td>-0.548</td>
<td>-2.17</td>
<td>8.05</td>
<td>1.04</td>
<td>0.732</td>
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<tr>
<td>0.5</td>
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<td>8.65</td>
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<tr>
<td>2</td>
<td>-0.713</td>
<td>-1.89</td>
<td>9.23</td>
<td>0.842</td>
<td>0.682</td>
</tr>
<tr>
<td>3</td>
<td>-0.947</td>
<td>-1.73</td>
<td>9.81</td>
<td>0.807</td>
<td>0.654</td>
</tr>
<tr>
<td>4</td>
<td>-1.47</td>
<td>-1.59</td>
<td>10.2</td>
<td>0.794</td>
<td>0.642</td>
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<tr>
<td>5</td>
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<td>10.7</td>
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<td>8.98</td>
<td>1.06</td>
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<tr>
<td>0.5</td>
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<td>-2.12</td>
<td>8.79</td>
<td>0.978</td>
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<tr>
<td>1</td>
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<td>8.87</td>
<td>0.907</td>
<td>0.689</td>
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<tr>
<td>2</td>
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<td>-1.89</td>
<td>9.31</td>
<td>0.828</td>
<td>0.680</td>
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<td>3</td>
<td>-0.949</td>
<td>-1.75</td>
<td>9.99</td>
<td>0.815</td>
<td>0.648</td>
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Stratification increases

Roughly constant as stratification increases

Changes with increasing stratification
Extrapolation to solar parameters

We estimate that solar downflows are in this range, which is in the “falling” regime.

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<td>-1.73</td>
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</tr>
<tr>
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<td>6</td>
<td>-5.730</td>
<td>-3.40</td>
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Extrapolation to solar parameters

Thermals travel this way (from surface to RZ)

Behavior of range of estimated thermals

$r_{th}$ (Mm)

$10^{-5}$ $10^{-3}$ $10^{-1}$

thermals

$0.7$ $0.8$ $0.9$

radius ($R_\odot$)

$r \propto \rho^{-1/2}$
Extrapolation to solar parameters

\begin{align*}
\text{temperature} & \sim \rho^{-1/2} \\
\text{sound speed} & \sim \rho^{-1/2} \\
\text{velocity} & \sim \rho^{-1/2}
\end{align*}
Extrapolation to solar parameters

A single thermal carries 0.01% of the solar luminosity at base of CZ.
Extrapolation to solar parameters

Only a small fraction of the area of the CZ is required to carry the solar luminosity using thermals.
Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.

2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary.

3) Buoyancy makes vortex rings expand & entrain, while stratification compresses them. Entropy rain feels both of these effects.

4) We develop a theory that describes the size & depth of entropy rain vs. time and verify that theory with simulations. There are two regimes: “stalling” and “falling.”

5) The enthalpy fluxes carried by solar downflows is sufficient to carry the solar luminosity in the deep CZ

For more specifics, see the paper:
Preliminary Turbulent Results

- Entrainment behavior ~the same
- Acceleration & velocity behavior is quite different than the laminar cases
- Hunch: baroclinic torques

\[
\frac{d \Gamma}{dt} = \int_C g \frac{S_1}{c_P} \hat{z} \cdot \, d \mathbf{x},
\]

+ viscous heating

\[
\rho \frac{\partial S_1}{\partial t} \sim \rho \frac{\nu}{T} \nabla^2 |u|^2; \mu = \rho \nu
\]