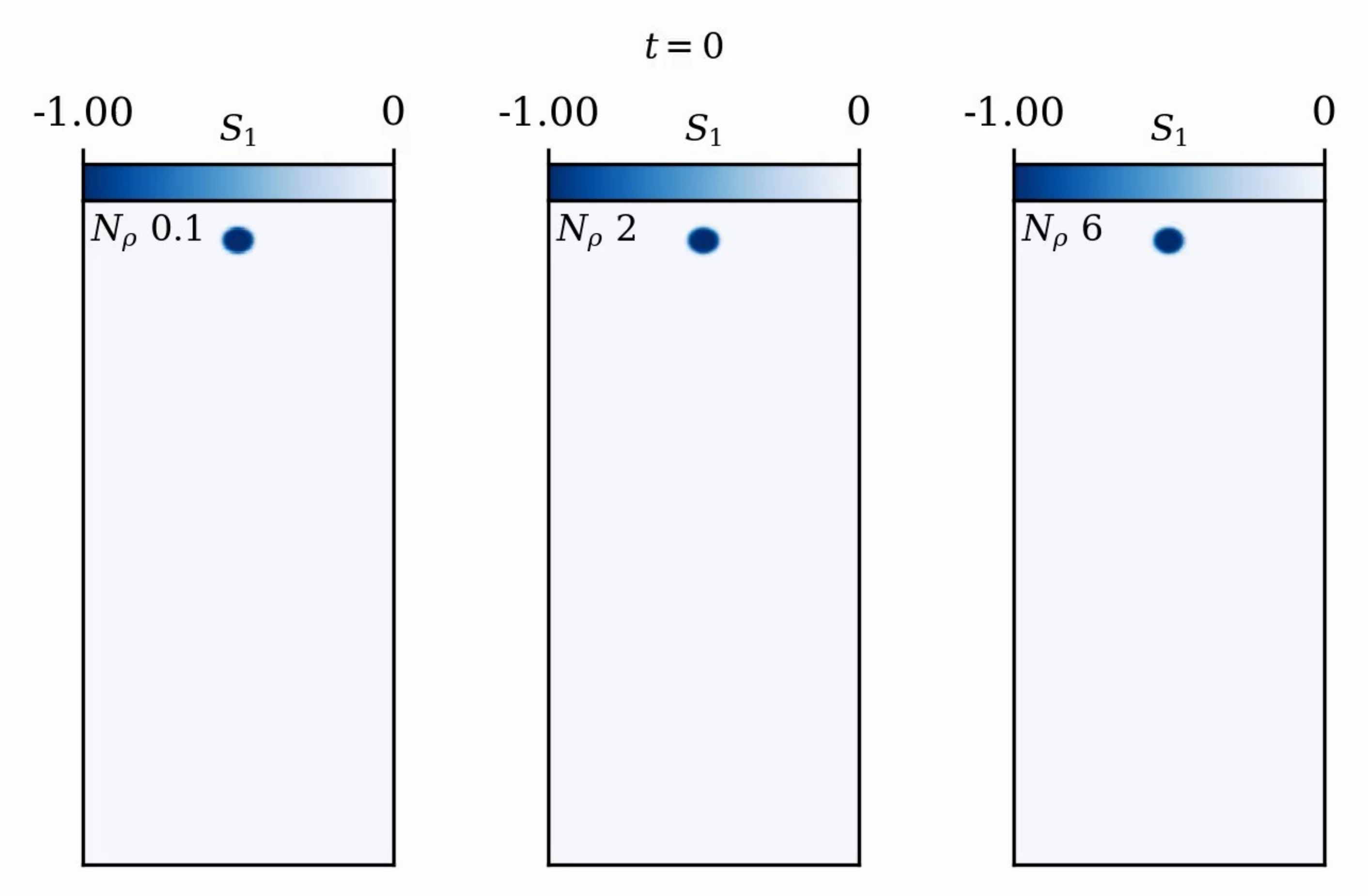
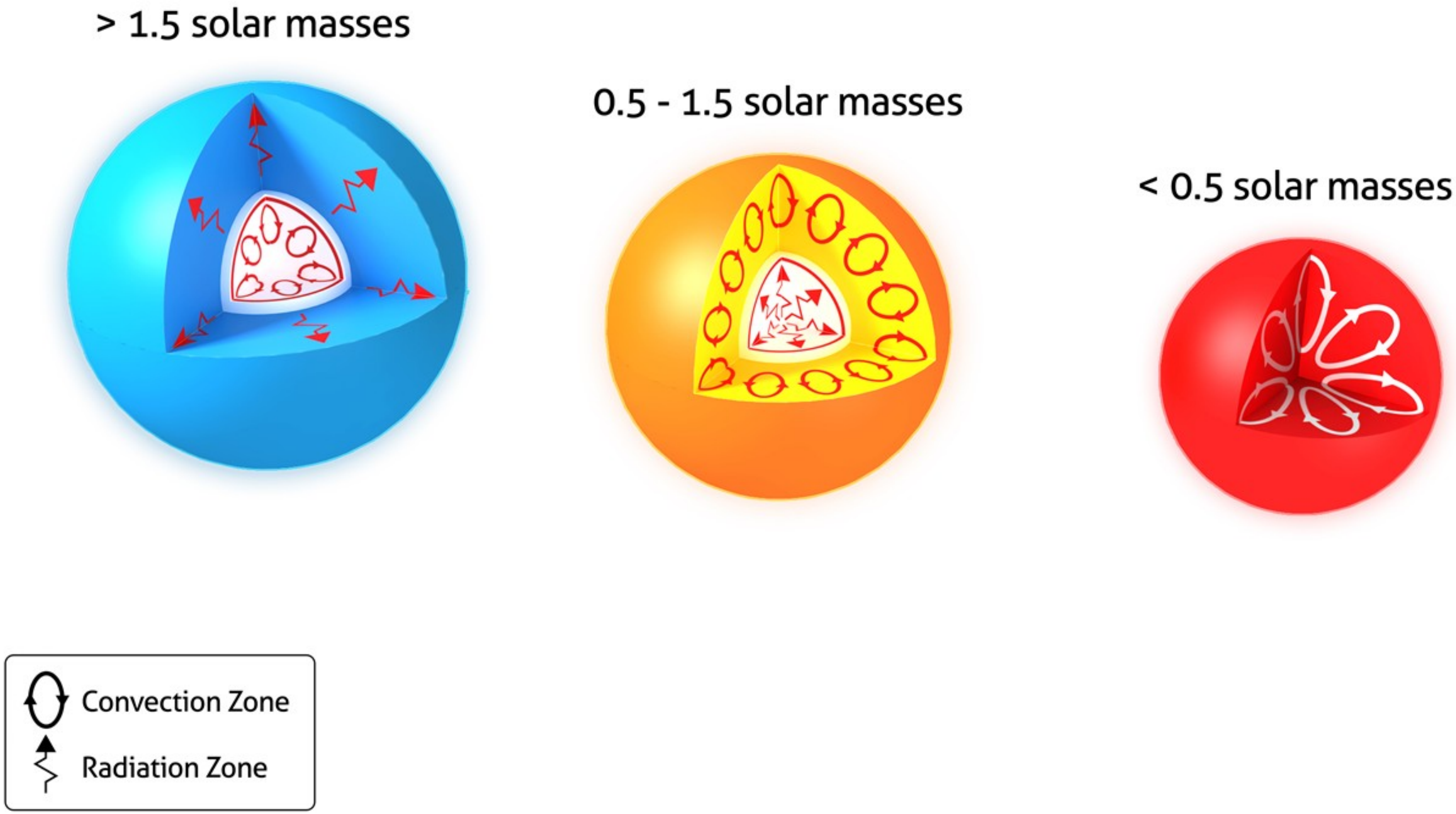
Entropy Rain and the Solar Convective Conundrum: Dilution and Compression of Individual Convective Downflows

Evan H. Anders, Daniel Lecoanet, Benjamin P. Brown

Laboratory for Atmospheric and Space Physics
University of Colorado Boulder

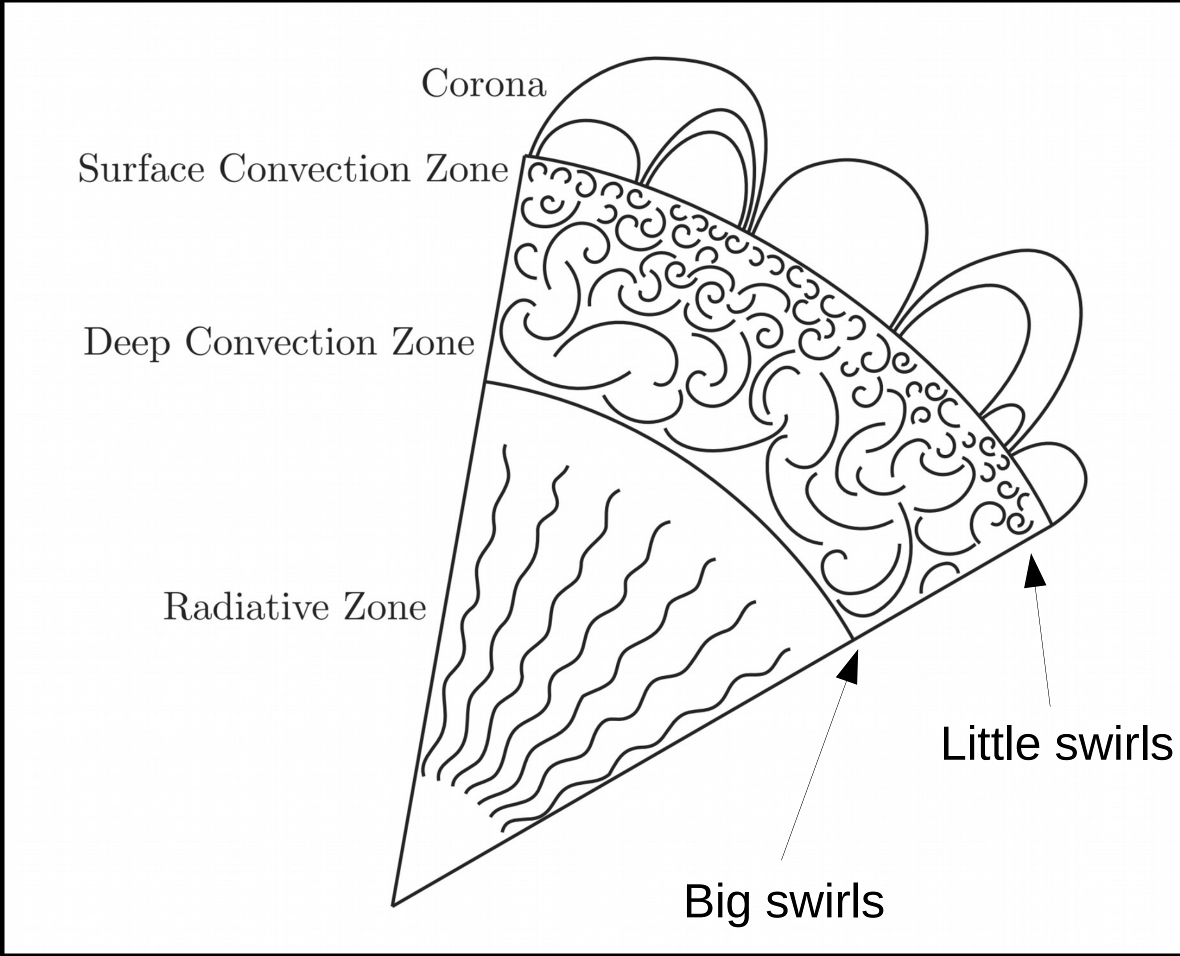


Heat Transfer of Stars



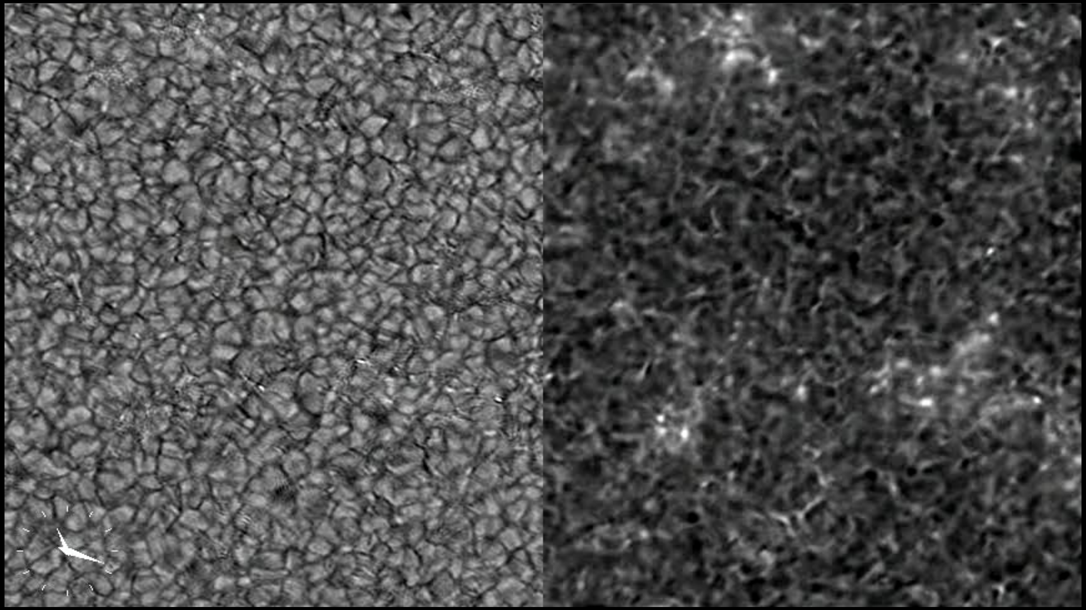
[Wikipedia]

The classic view of envelope convection



[Image courtesy of Daniel Lecoanet]

Solar Convective Flows

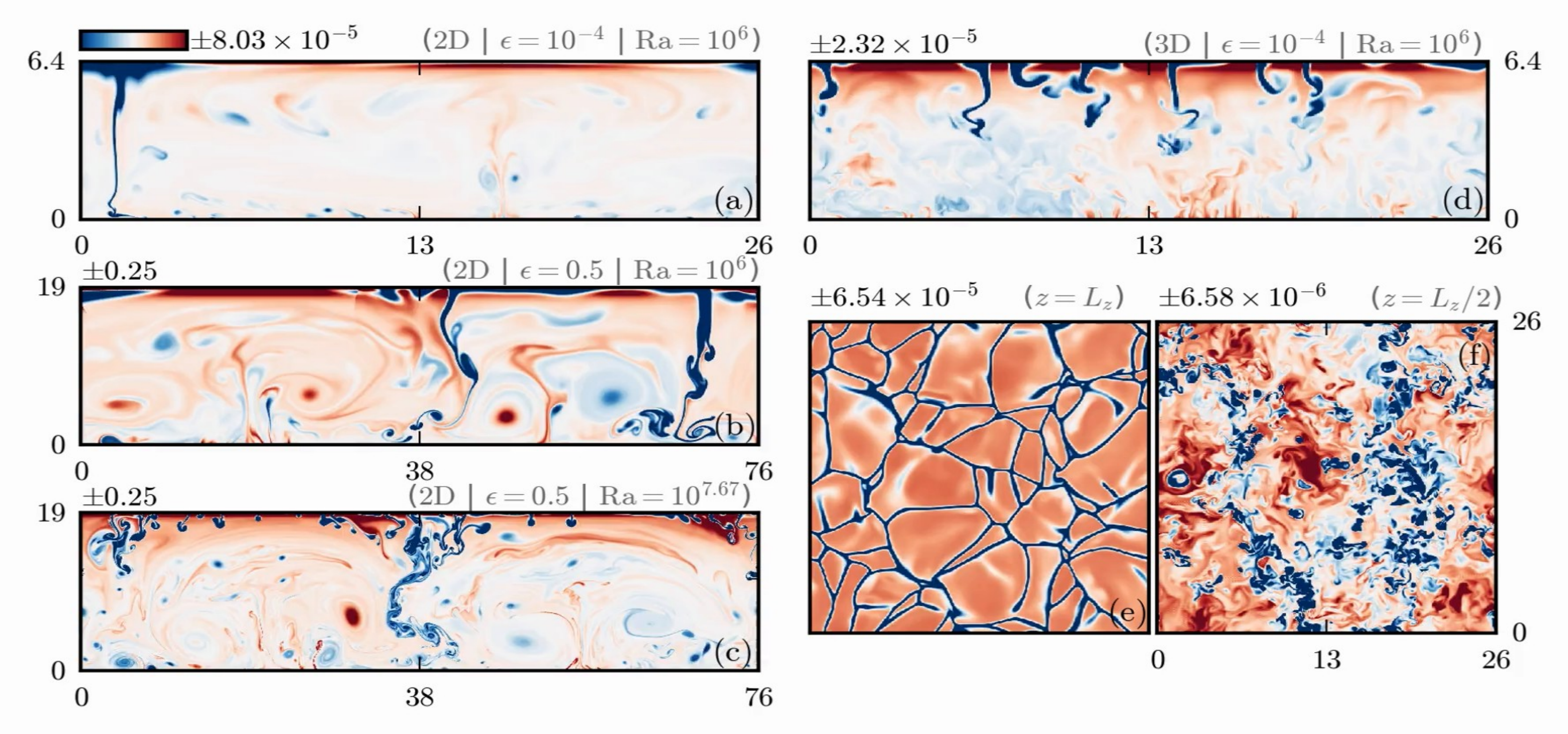


Visible (Photosphere) Ca II (Chromosphere)

Granules:
~1 Mm
~5 min

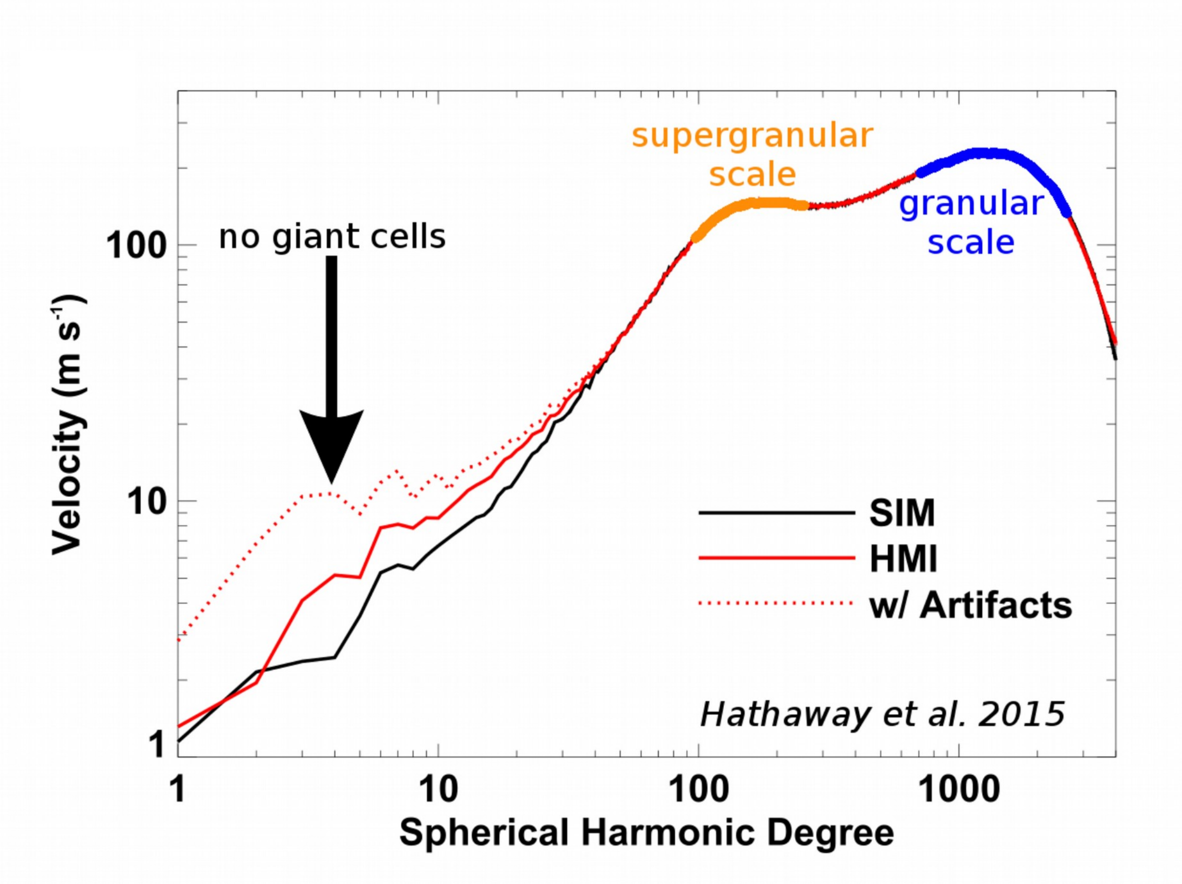
Supergranules:
~30 Mm
~1 day

Flows in Simple Stratified Convection

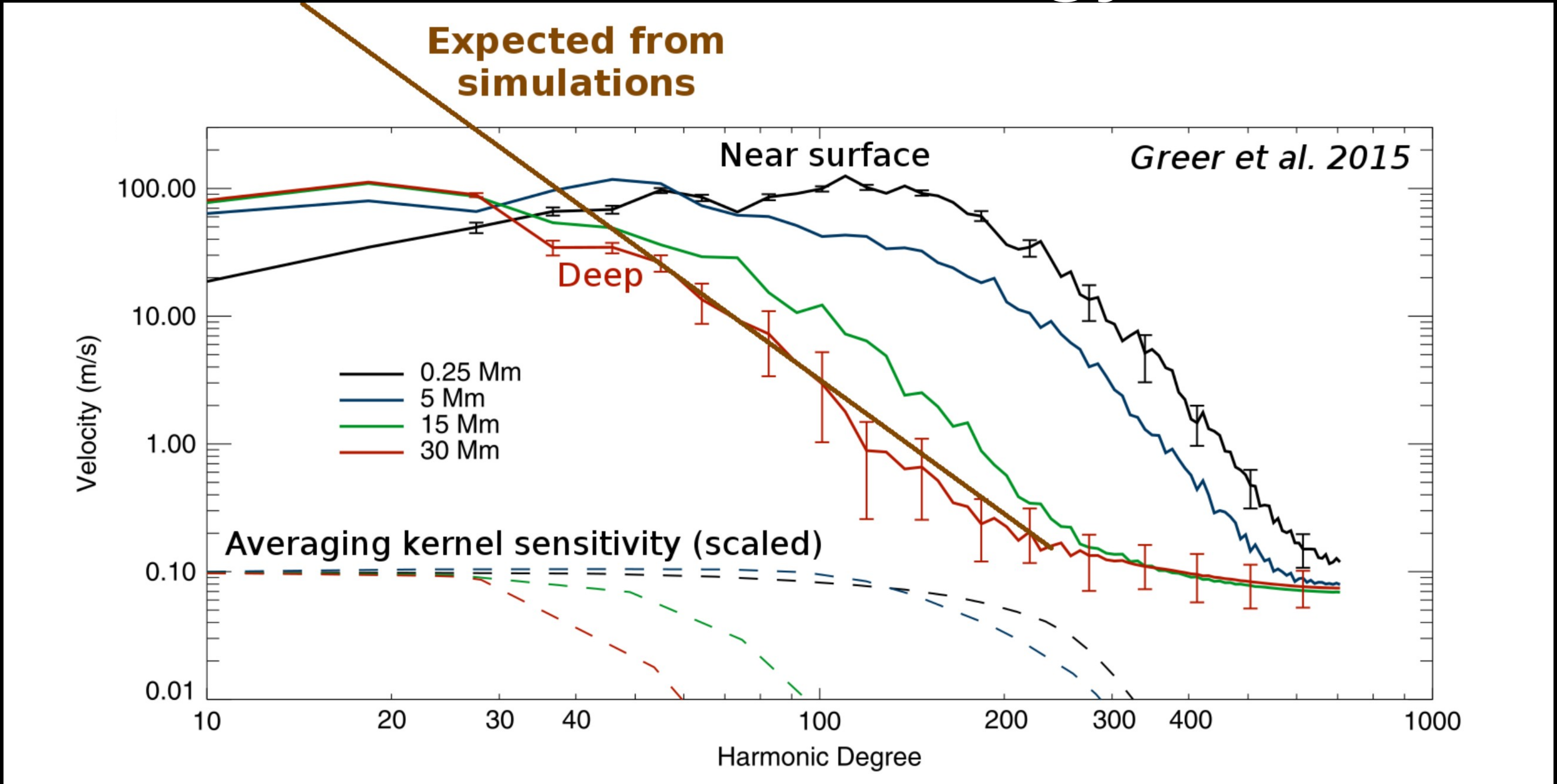


[Anders & Brown 2017; PRF 2]

The Solar Convective Conundrum: Horizontal Surface Flows



The Solar Convective Conundrum: Helioseismology



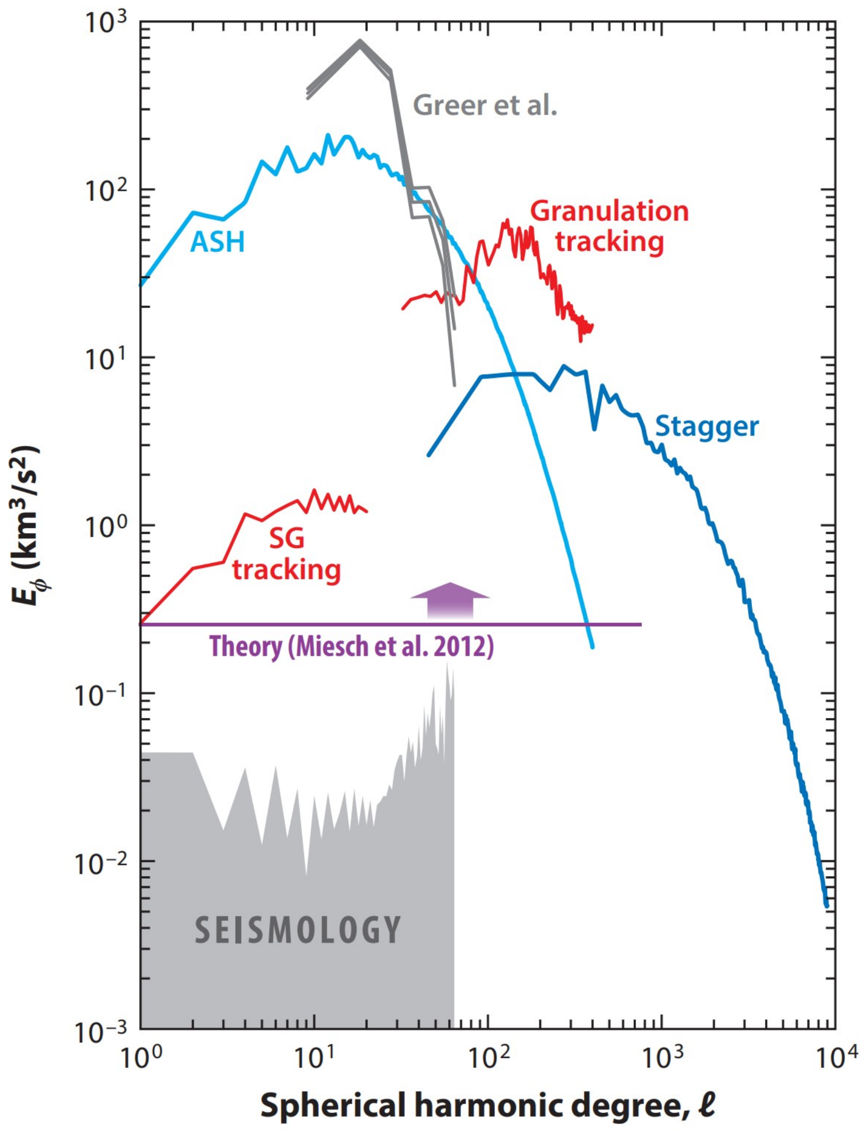
The Convective Conundrum cont'd

Red = surface observations

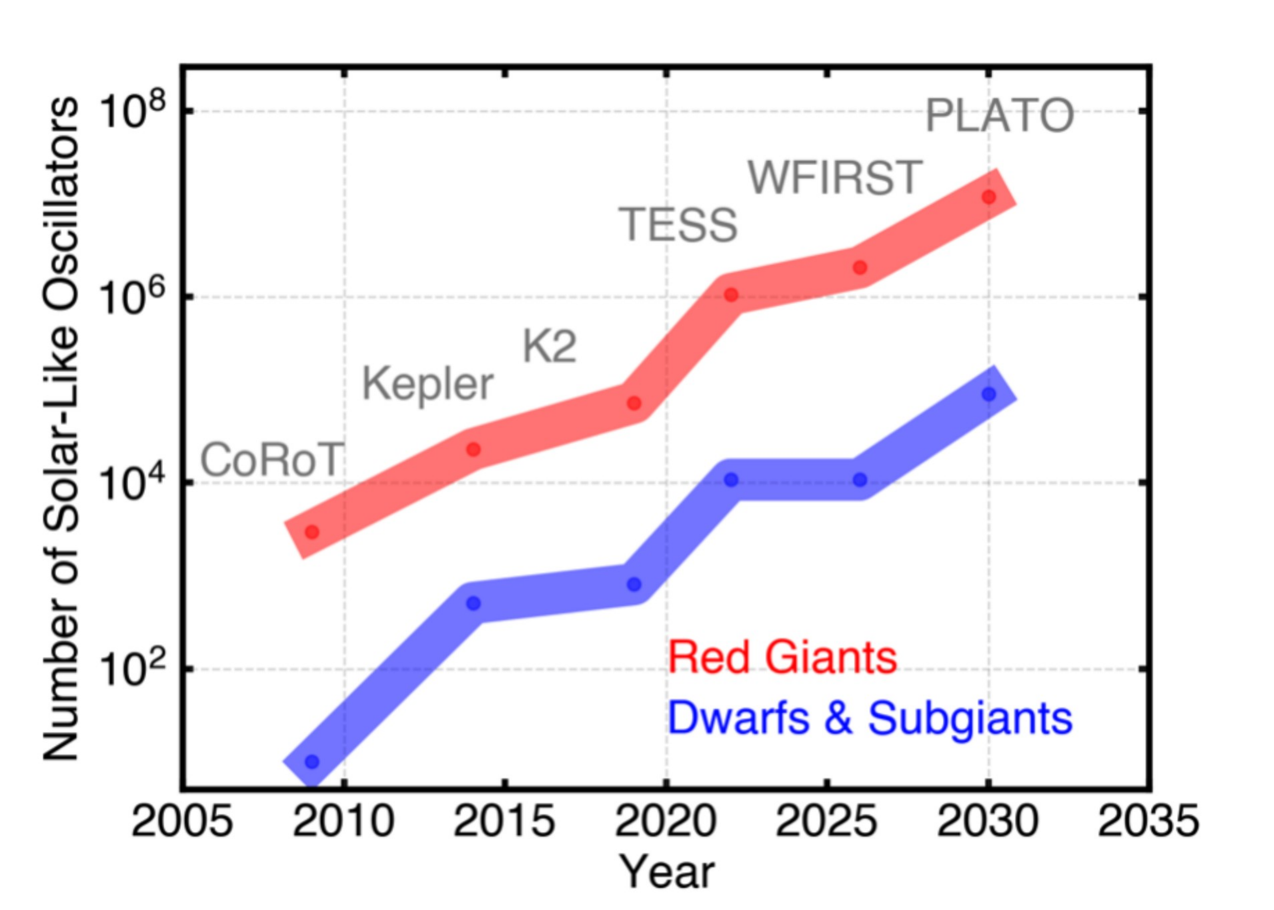
Blue = simulations

Grey = helioseismology

[Hanasoge et al 2015]



Extra motivation in the asteroseismic age



Many observations which rely on stellar structure models.

→Stellar structure models rely on convection parameterizations like MLT.

→We need to understand stellar envelope convection better.

[Astro2020 white paper: Huber et al 2019]

Takeaways

1) **There is an absence of large-scale convective power in observations of the Sun.**

What’s going on?

Some options:

1) The observations are wrong

What’s going on?

Some options:

1) ~~The observations are wrong~~
2) Some dynamical process masks giant cells

What’s going on?

Some options:

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3) Giant cells are not driven

What’s going on?

Some options:

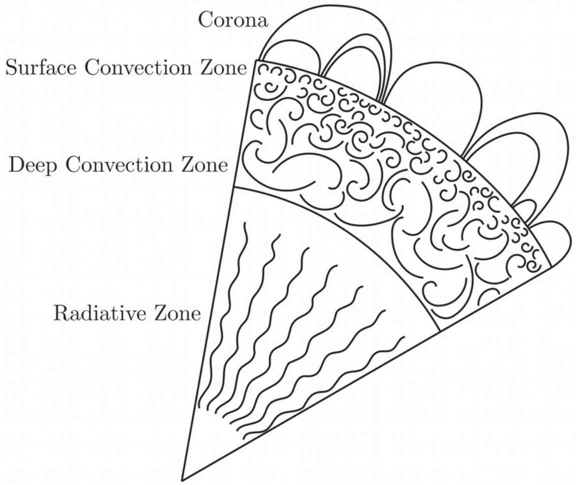
1) ~~The observations are wrong~~

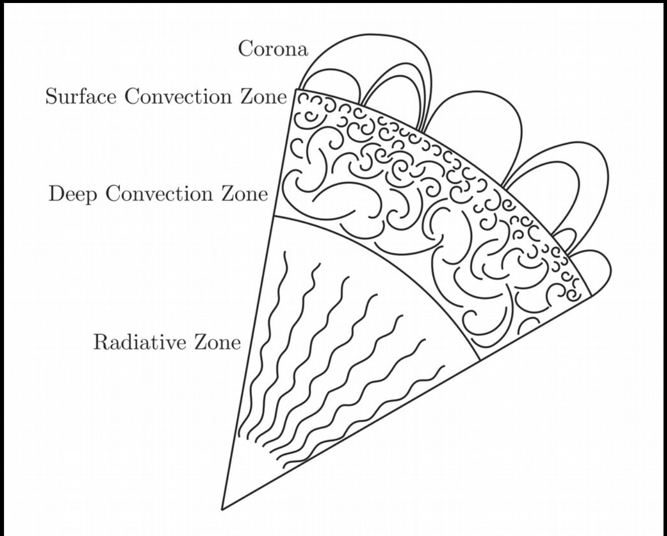
2) Some dynamical process masks giant cells

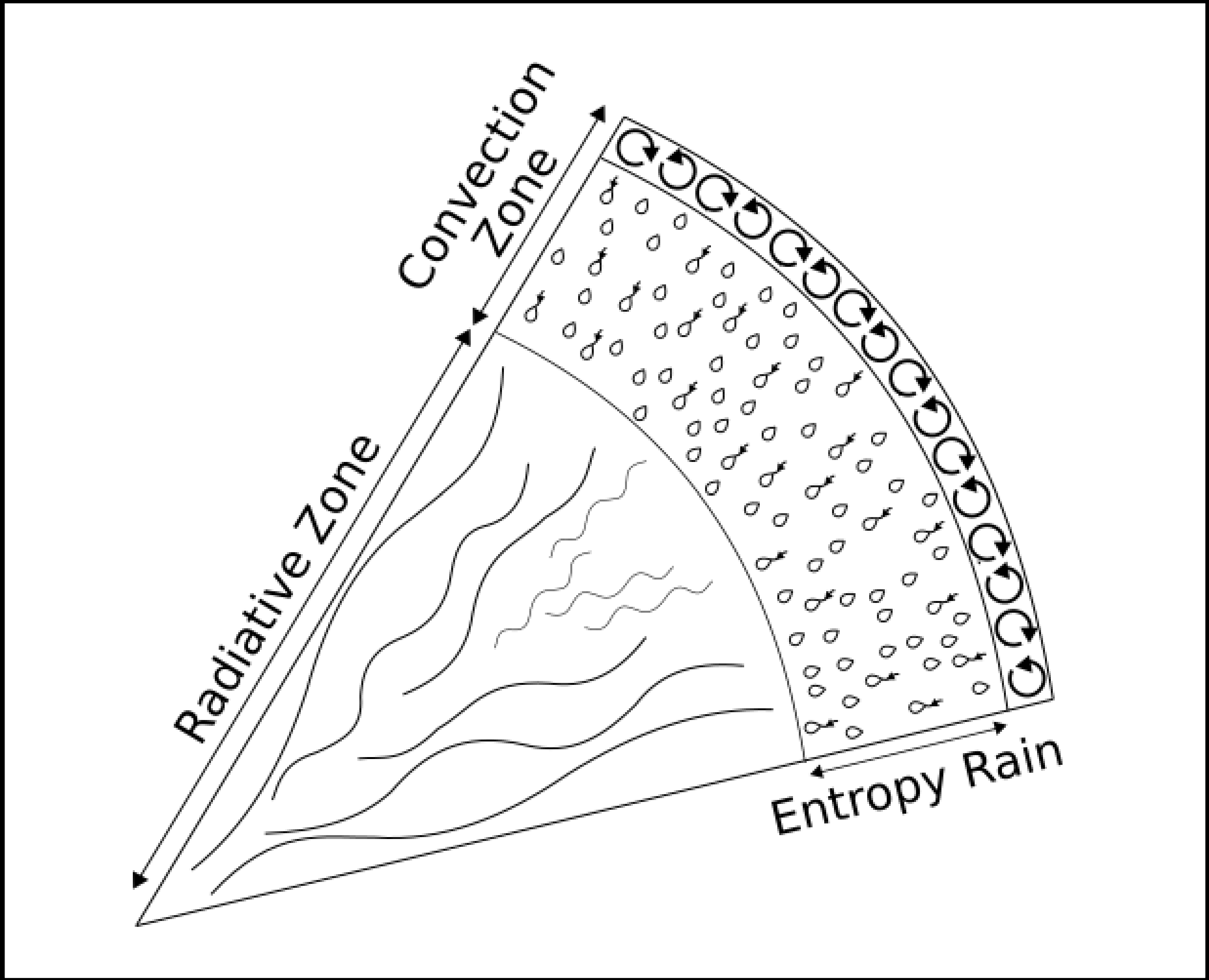
3) <u>Giant cells are not driven</u>

<u>Entropy Rain Hypothesis:</u> Cold downflows from the solar surface carry the solar luminosity to the bottom of the CZ, *not* warm upflows which would manifest as giant cells.

[Spruit 1997; Brandenburg 2016; and results of Käpylä et al. 2017]


Corona
Surface Convection Zone
Deep Convection Zone
Radiative Zone


Corona
Surface Convection Zone
Deep Convection Zone
Radiative Zone

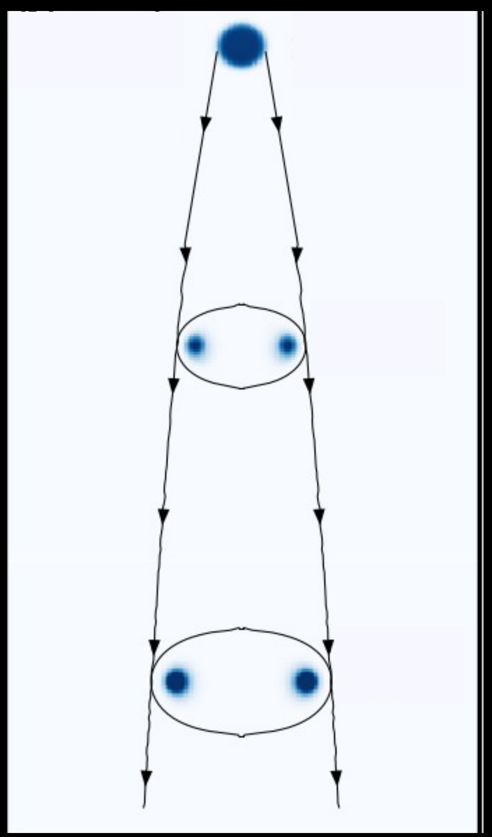

Convection Zone
Radiative Zone
Entropy Rain

Thermals as a model for Entropy Rain

Starts as a blob of buoyant fluid

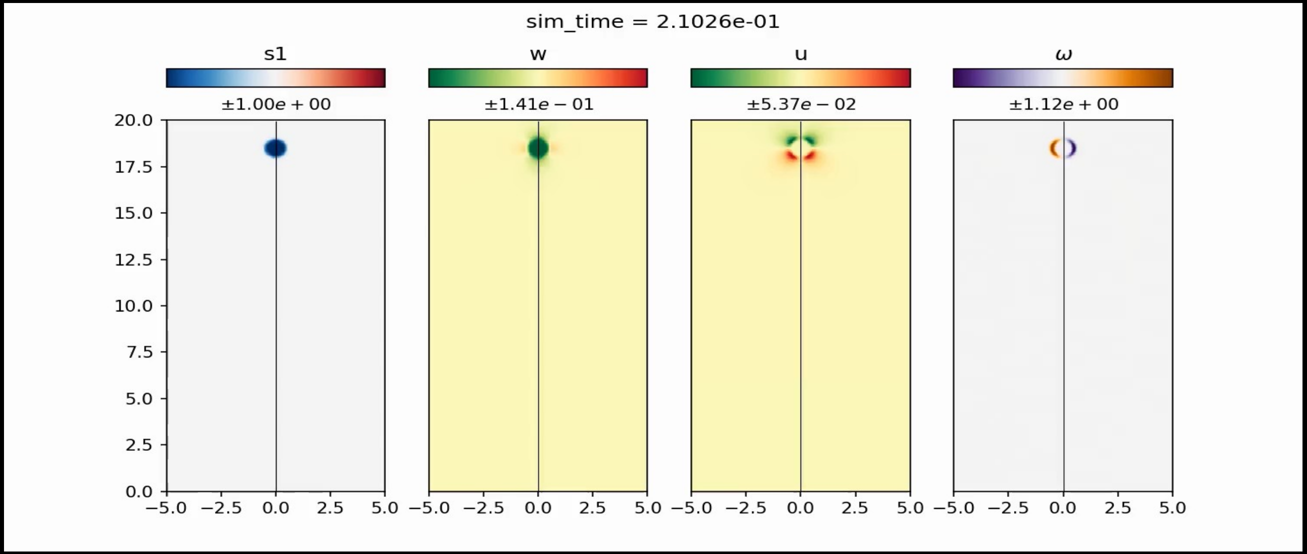
Evolves into a buoyant vortex ring

Entrains fluid from environment

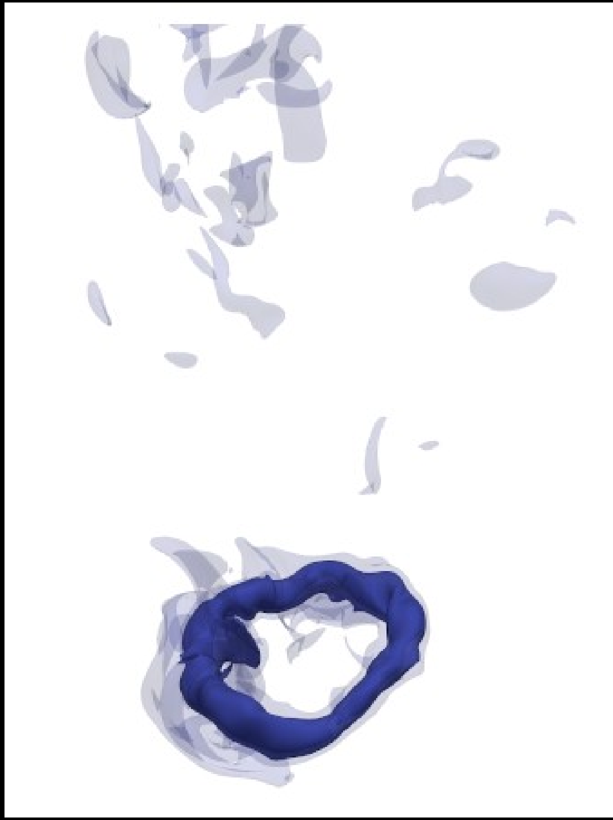


[Suggested by Brandenburg 2016]

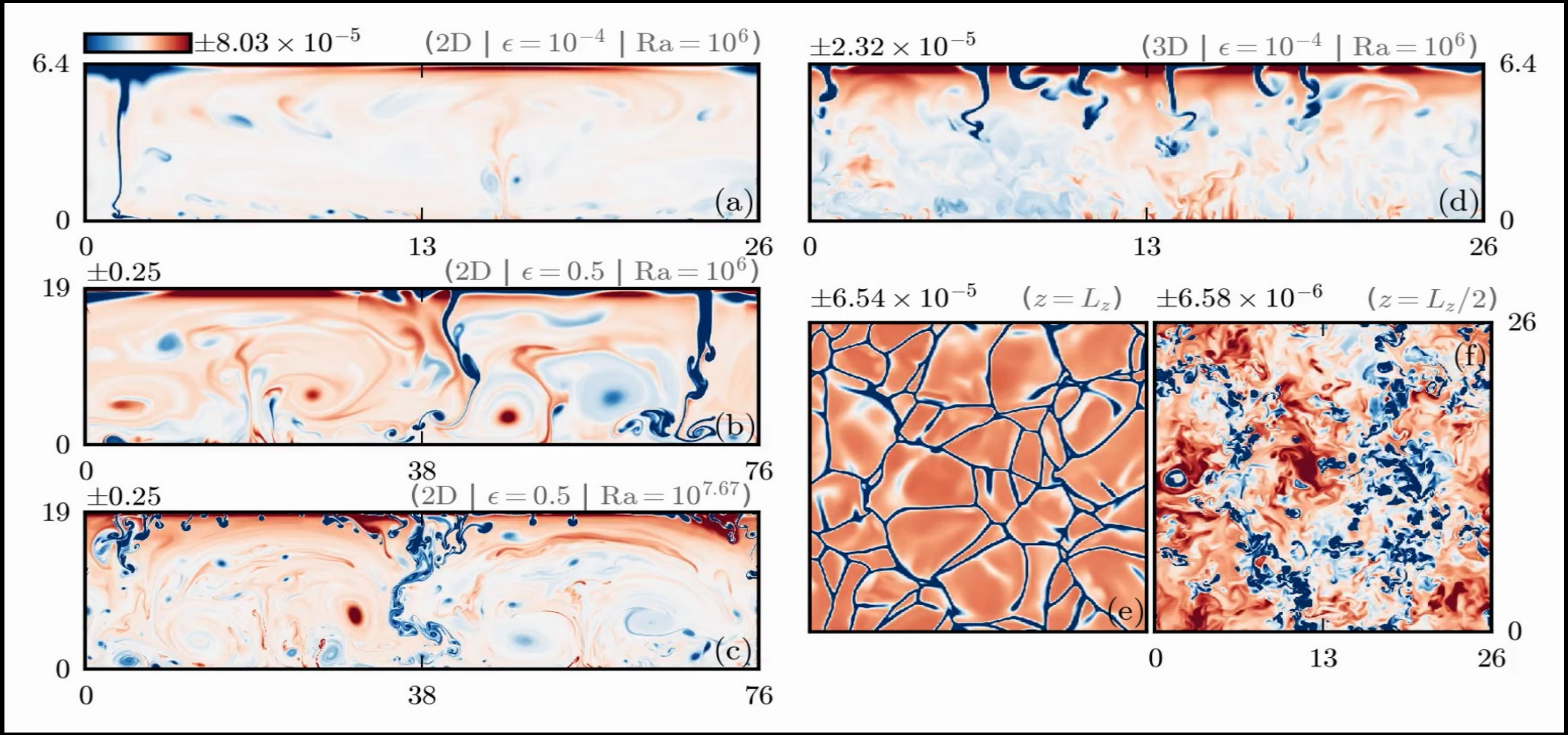
Thermals as a model for Entropy Rain



3D Visualization of an Evolved Thermal

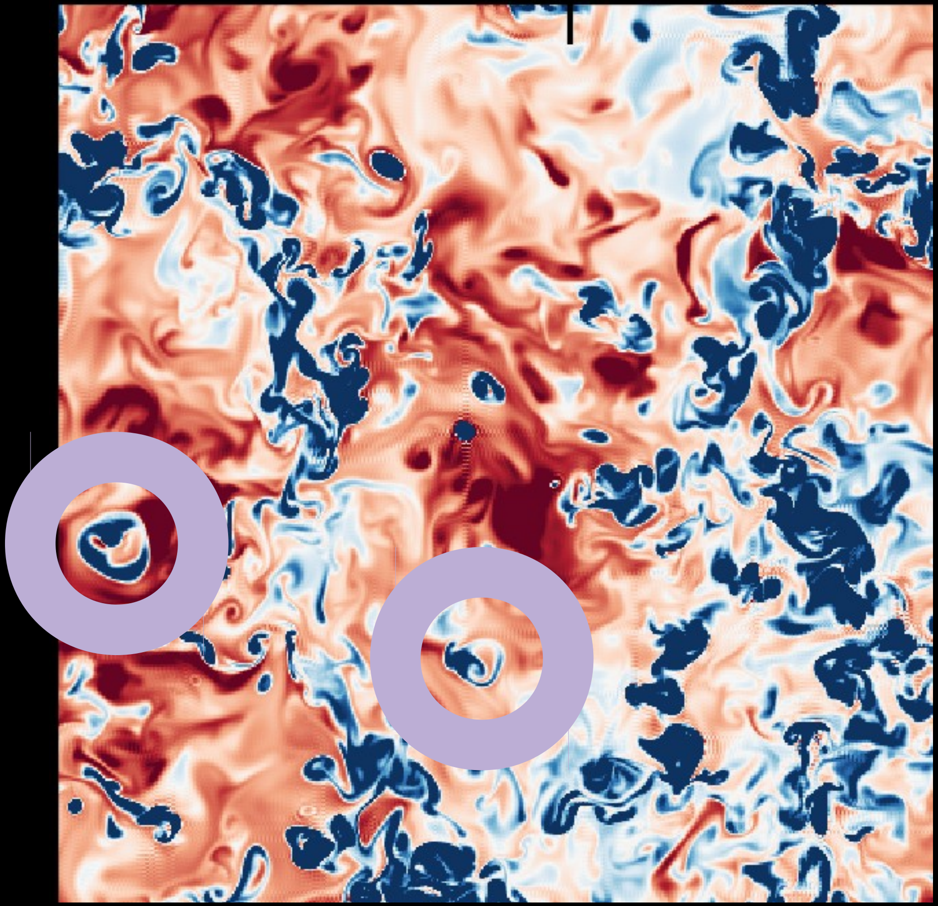


Flows in Simple Stratified Convection



[Anders & Brown 2017; PRF 2]

Flows in Simple Stratified Convection



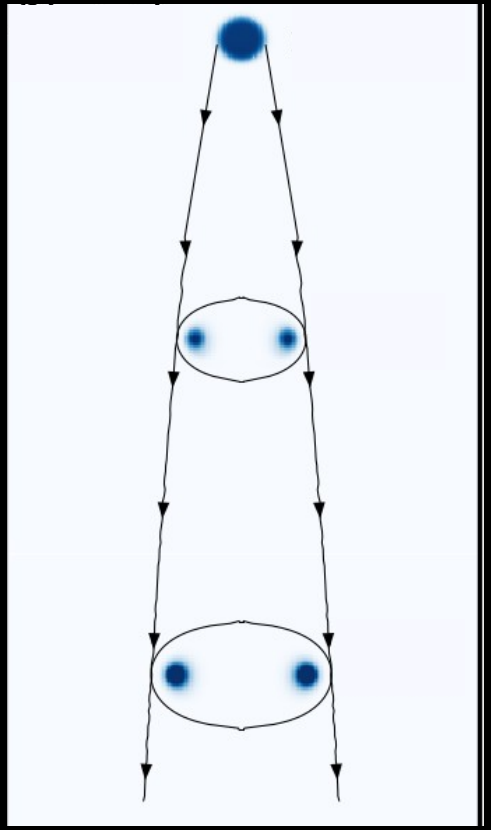
[Anders & Brown 2017; PRF 2]

Thermals as a model for Entropy Rain

Fundamental Questions:

1.) Can entropy rain survive throughout the depth of the CZ?

2.) Can entropy rain carry the stellar luminosity?



[Suggested by Brandenburg 2016]

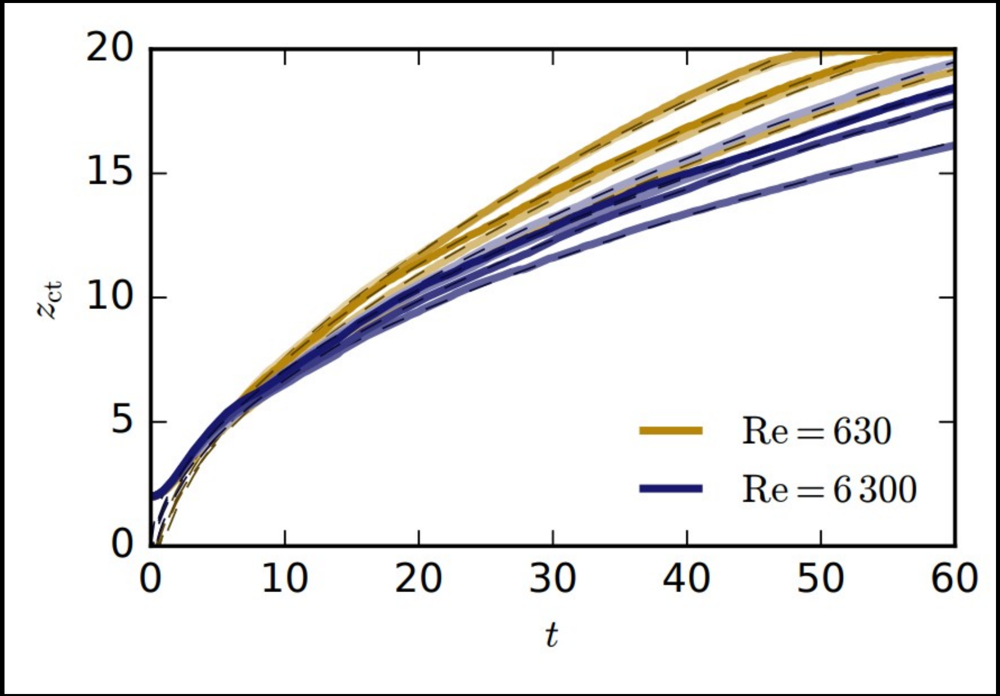
Thermal Entrainment in the Incompressible Limit

↳ growth of thermal through transport of environmental fluid into the thermal

Depth:

$$z \propto t^{1/2}$$

(implies deceleration)



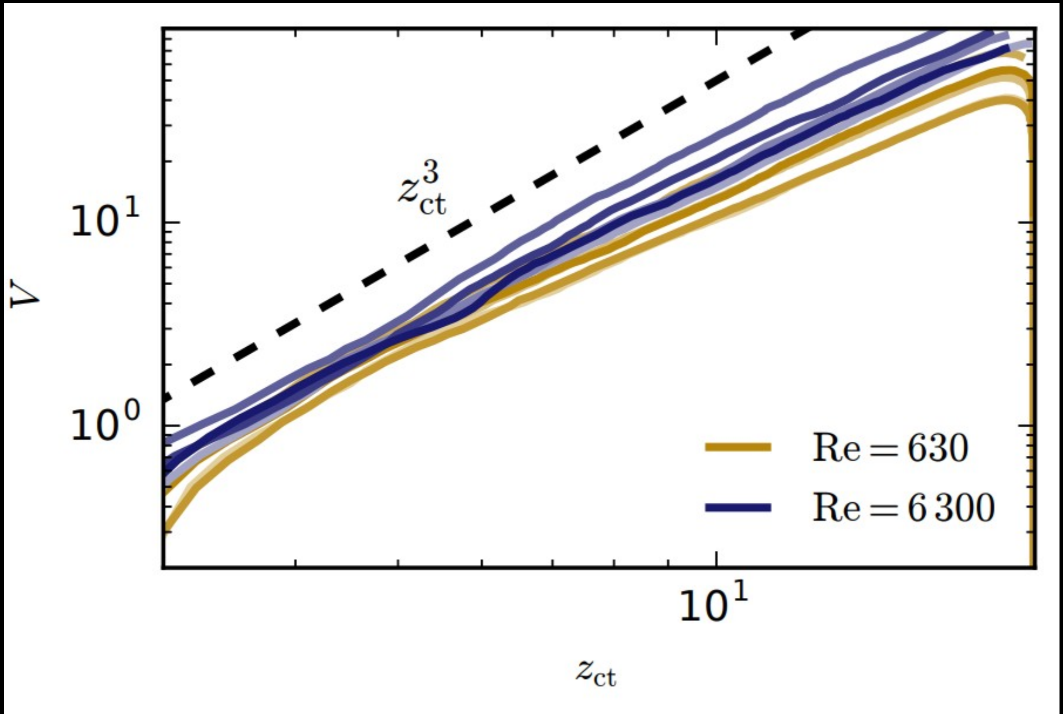
[Lecoanet & Jeevanjee 2019]

Thermal Entrainment in the Incompressible Limit

Volume:

V ∝ r³ (measured)

V ∝ z³ (spheroidal)

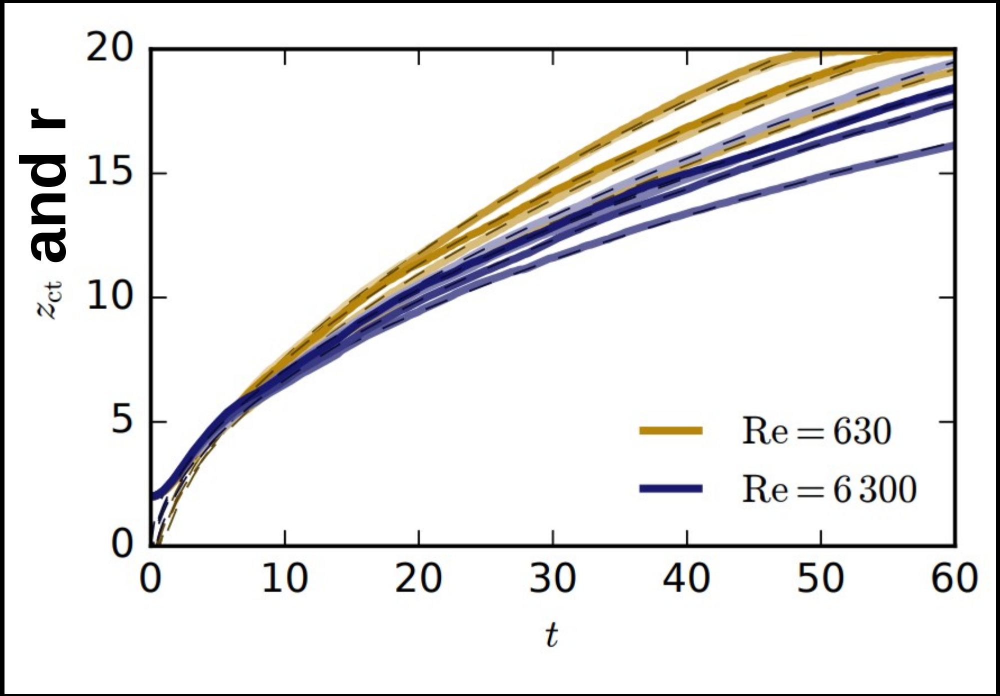


[Lecoanet & Jeevanjee 2019]

Thermal Entrainment in the Incompressible Limit

Volume: V ∝ r^3 ∝ z^3, so
Radius: z ∝ r ∝ $t^{1/2}$

Even though thermals buoyantly accelerate, they entrain enough to stall themselves.



[Lecoanet & Jeevanjee 2019]

Fundamental problem of thermals as a model for entropy rain:

If thermals entrain & slow down, how can they cross the solar CZ?

Stratification breaks symmetry

Cold thermals:

- Boussinesq-like entrainment
- ***stratification-enduced compression***

Hot thermals:

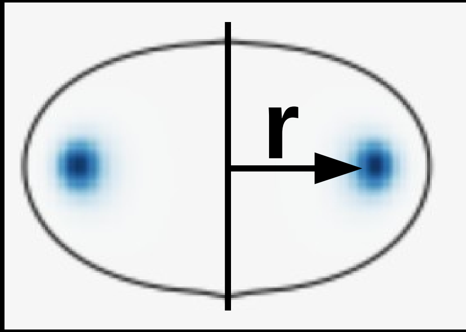
- Boussinesq-like entrainment
- ***stratification-enduced expansion***

Stratification effects without buoyancy

Compression should go as:

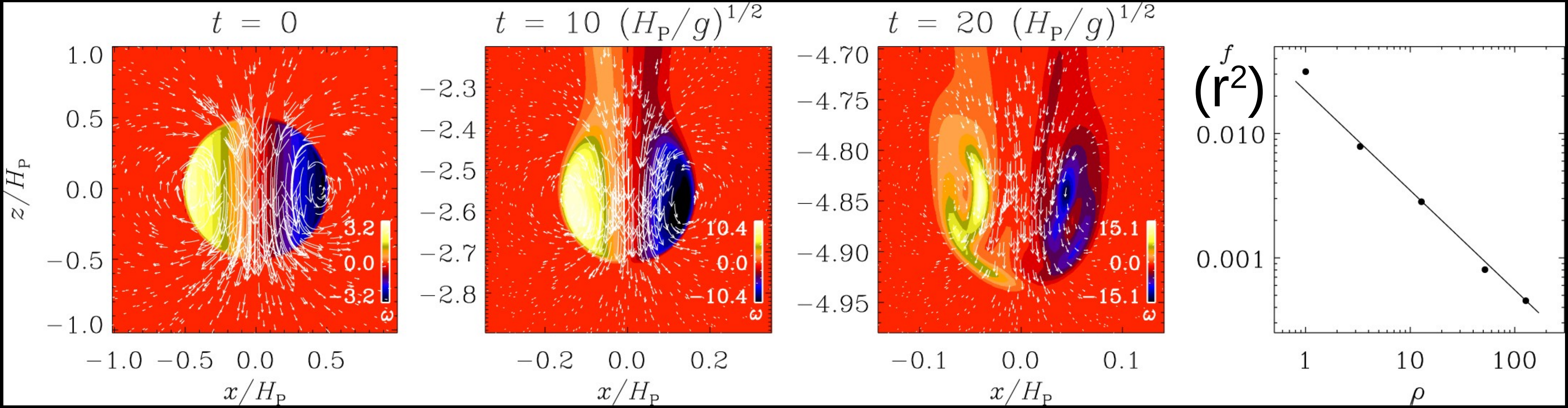
- $\mathbf{r} \propto \boldsymbol{\rho}^{-1/2}$ for ***horizontal*** compression
- $\mathbf{r} \propto \boldsymbol{\rho}^{-1/3}$ for ***spherical*** compression

For **ρ**, the atmospheric density,
and **r**, the radius:



[Brandenburg 2016]

Buoyantly neutral Hill vortices



Non-buoyant hill vortex simulations were between expected limits (slope of -0.8).

[Brandenburg 2016]

Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary.

Experiment: Evolve thermals in atmospheres with different stratifications

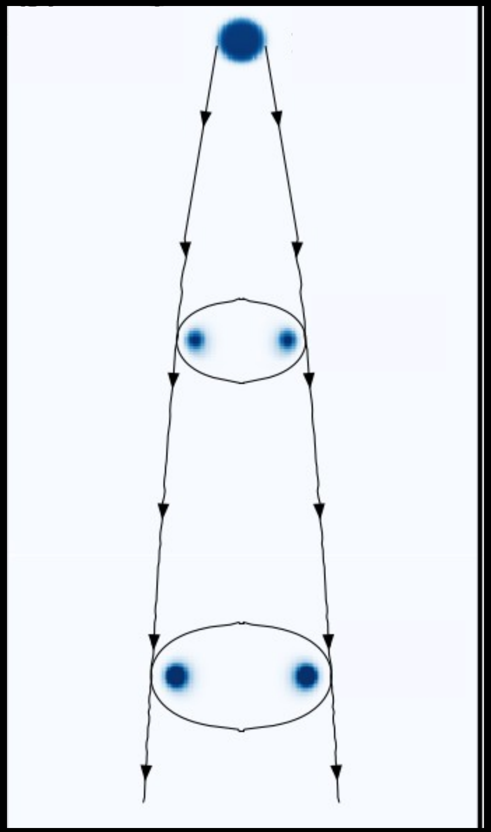
Fundamental control parameter:

S = (size of thermal) / (atmospheric scale height)

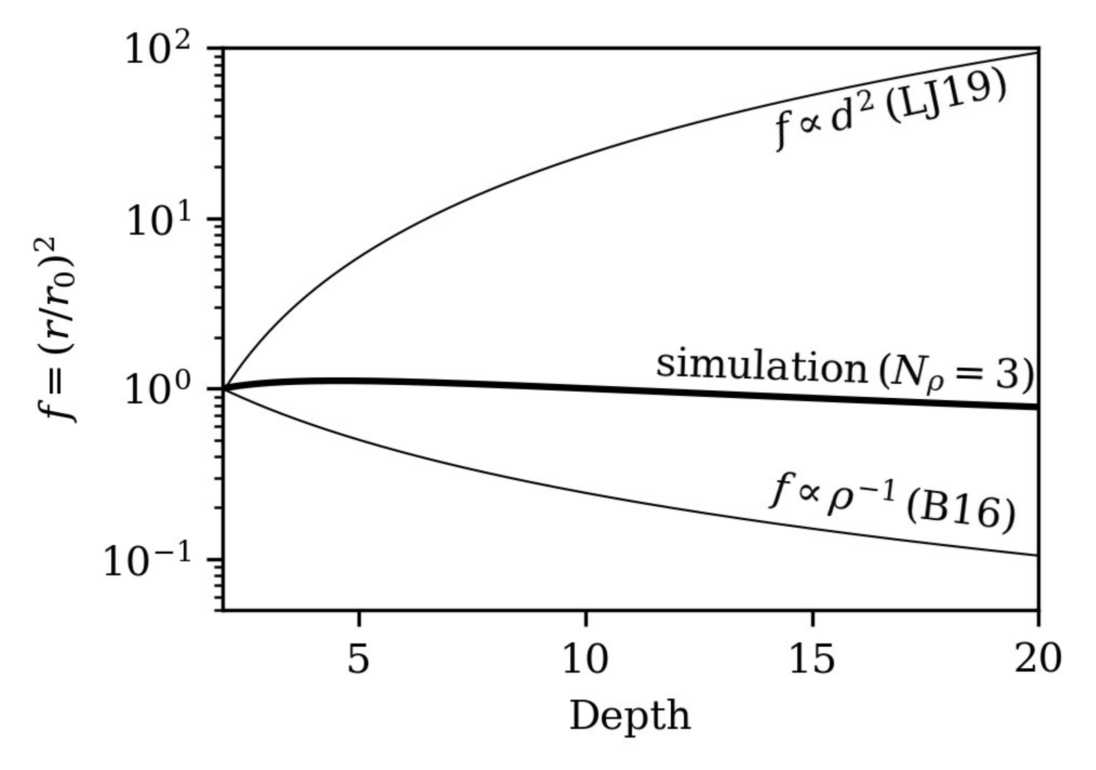
Limits:

S → 0 (Boussinesq)

S → O(1) (stratification matters)



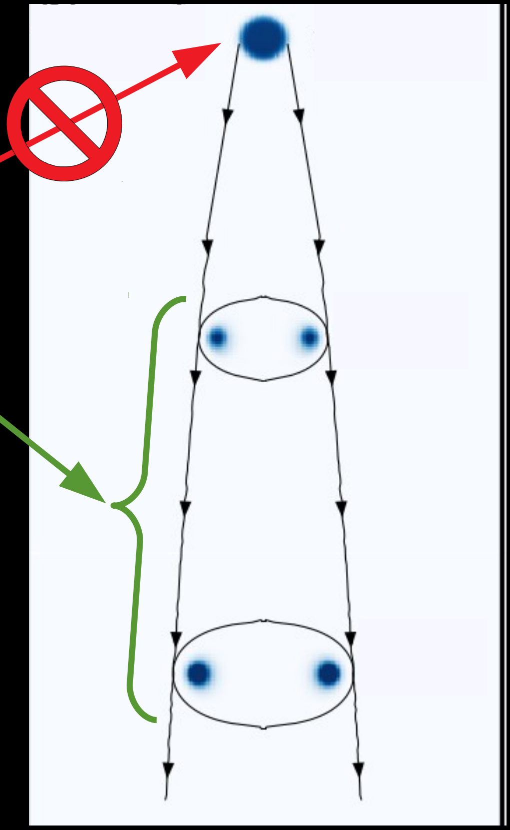
To first order, we see what we expect: blended compression & entrainment



[Anders, Lecoanet & Brown 2019; ApJ 884]

Sims soon; First, some theory

Goal: Model vortex ring stage of thermal's evolution



<u>Important quantities:</u>

- Buoyancy of the thermal, **B**
- Circulation of the vortex ring, **Γ**
- Volume of the thermal, **V**
- Radius of the thermal, **r**
- Impulse of the thermal, **I**

Theory of stratified vortex ring entrainment

Fundamental Quantity: **Impulse**

$$\boldsymbol{I} = \frac{1}{2}\int_{\mathcal{V}} \boldsymbol{x} \times (\nabla \times (\rho \boldsymbol{u}))\, dV,$$

Theory of stratified vortex ring entrainment

...which is similar to the **momentum**

$$I_z = (1 + k)\rho \mathcal{V} w_{\mathrm{th}},$$

Theory of stratified vortex ring entrainment

...which is similar to the **momentum**

$$I_z = (1 + k)\rho \mathcal{V} w_{\mathrm{th}},$$

and which is approximately (thin core vortex ring)

$$I_z \approx \pi \rho_0 r^2 \Gamma,$$

Theory of stratified vortex ring entrainment

Assume the volume-integrated buoyancy force of the thermal is ~constant in time,

$$B \equiv \int_{\mathcal{V}} \rho_0 \, S_1 \, \frac{g}{c_P} \, dV.$$

Theory of stratified vortex ring entrainment

Assume the volume-integrated buoyancy force of the thermal is ~constant in time,

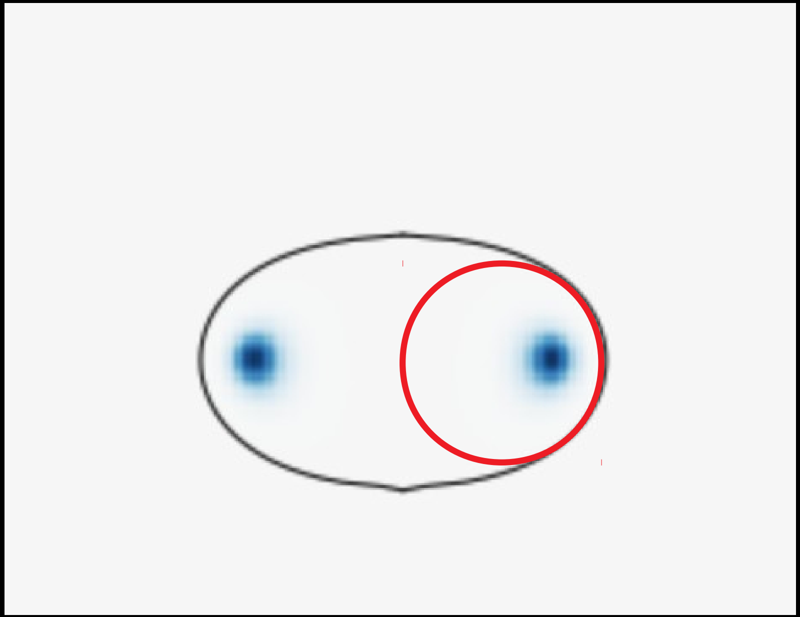
$$B \equiv \int_{\mathcal{V}} \rho_0 \, S_1 \, \frac{g}{c_P} \, dV.$$

...and assume buoyancy is the only force acting on the thermal.

Theory of stratified vortex ring entrainment

Assume circulation is constant (no baroclinic torques)

$$\frac{d\Gamma}{dt} = \oint_C g \frac{S_1}{c_P} \hat{z} \cdot d\boldsymbol{x},$$



Theory of stratified vortex ring entrainment

We can solve impulse for radius vs time,

$$I_z \approx \pi \rho_0 r^2 \Gamma, \longrightarrow r = C\sqrt{\frac{B_{\mathrm{th}} t + I_0}{\rho_0}}$$

→ **$r \propto \rho^{-1/2}$** for purely horizontal compression, and
→ **$r \propto t^{1/2}$** for boussinesq regime.

Theory of stratified vortex ring entrainment

Assume spheroidal volume

$$\mathcal{V} \approx mr^3$$

Substitute in the definition of velocity,

$$w_{\mathrm{th}} \equiv dz_{\mathrm{th}}/dt$$

Theory of stratified vortex ring entrainment

Step 6: Solve for depth vs time

$$\frac{dz}{\rho(z)^{1/2}} = C\frac{dt}{(t + t_{\mathrm{off}})^{1/2}}$$

(Simple ODE which can be solved for a prescribed density stratification)

Experiment: two types of simulations

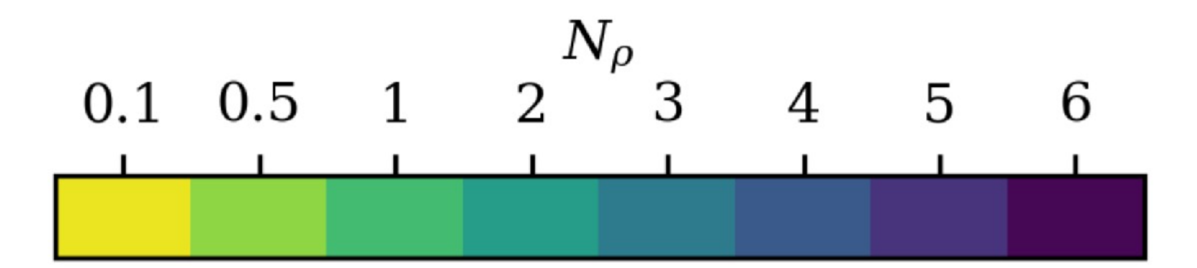
1) 2D anelastic, cylindrically symmetric

→ Computationally cheap

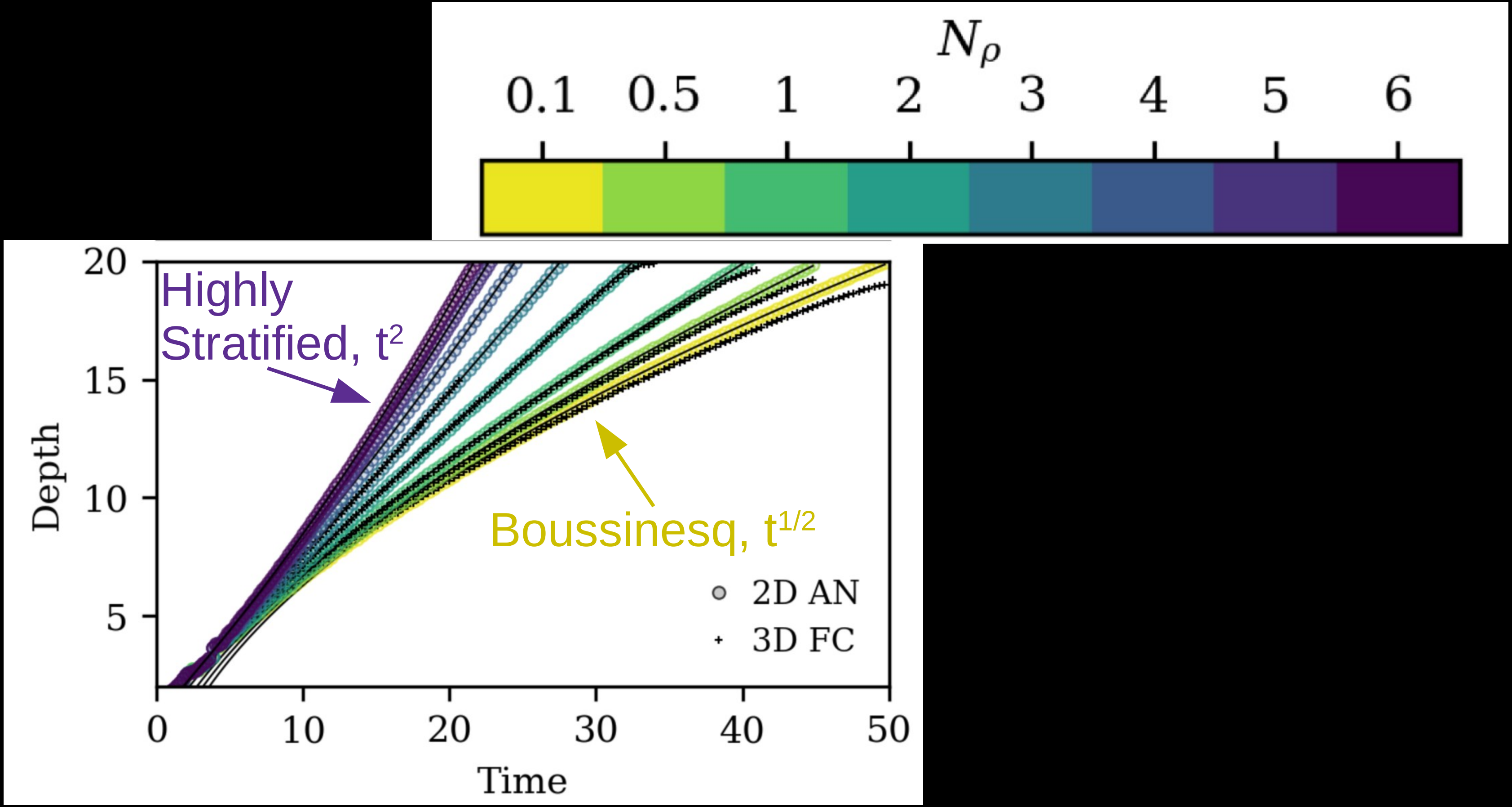
→ Limit in which theory was derived

2) 3D fully compressible, cartesian boxes

→ The “true” solution

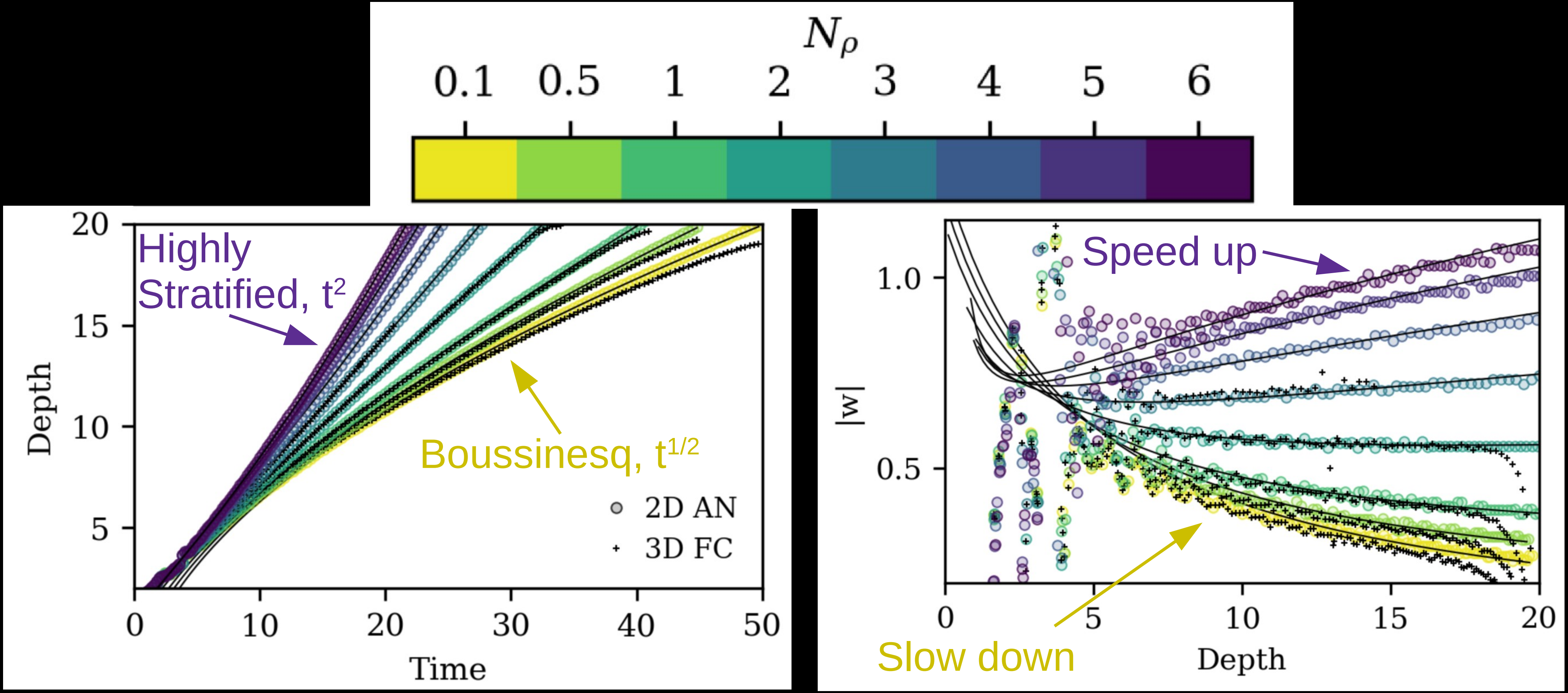


Simulations vs. Theory



[Anders, Lecoanet, & Brown 2019, ApJ 884]

Simulations vs. Theory



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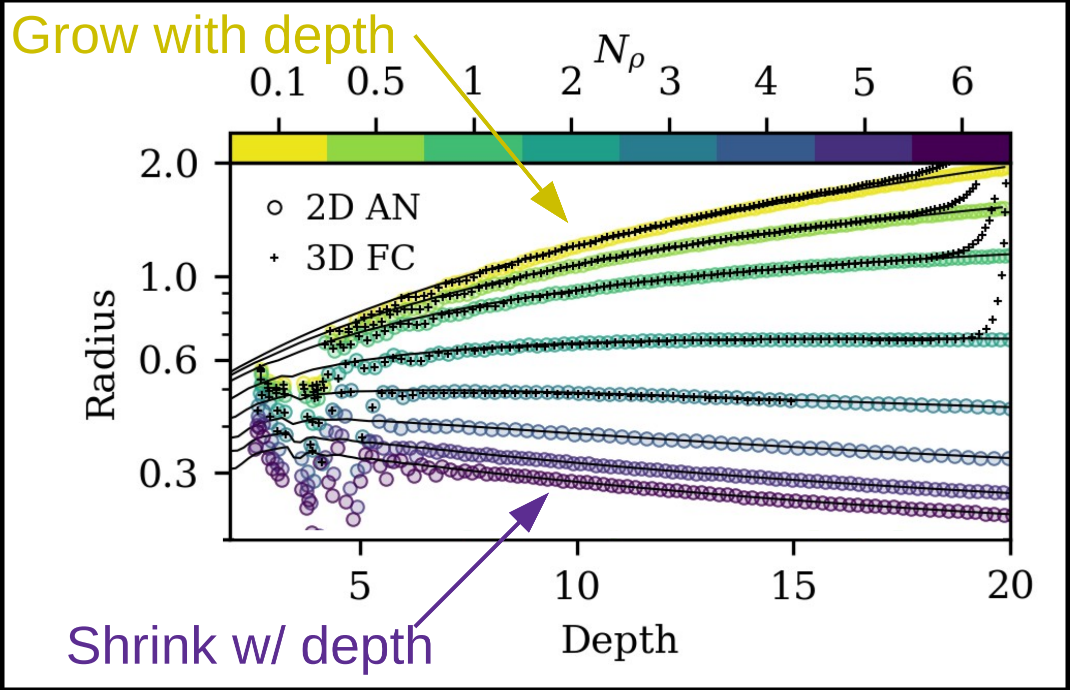

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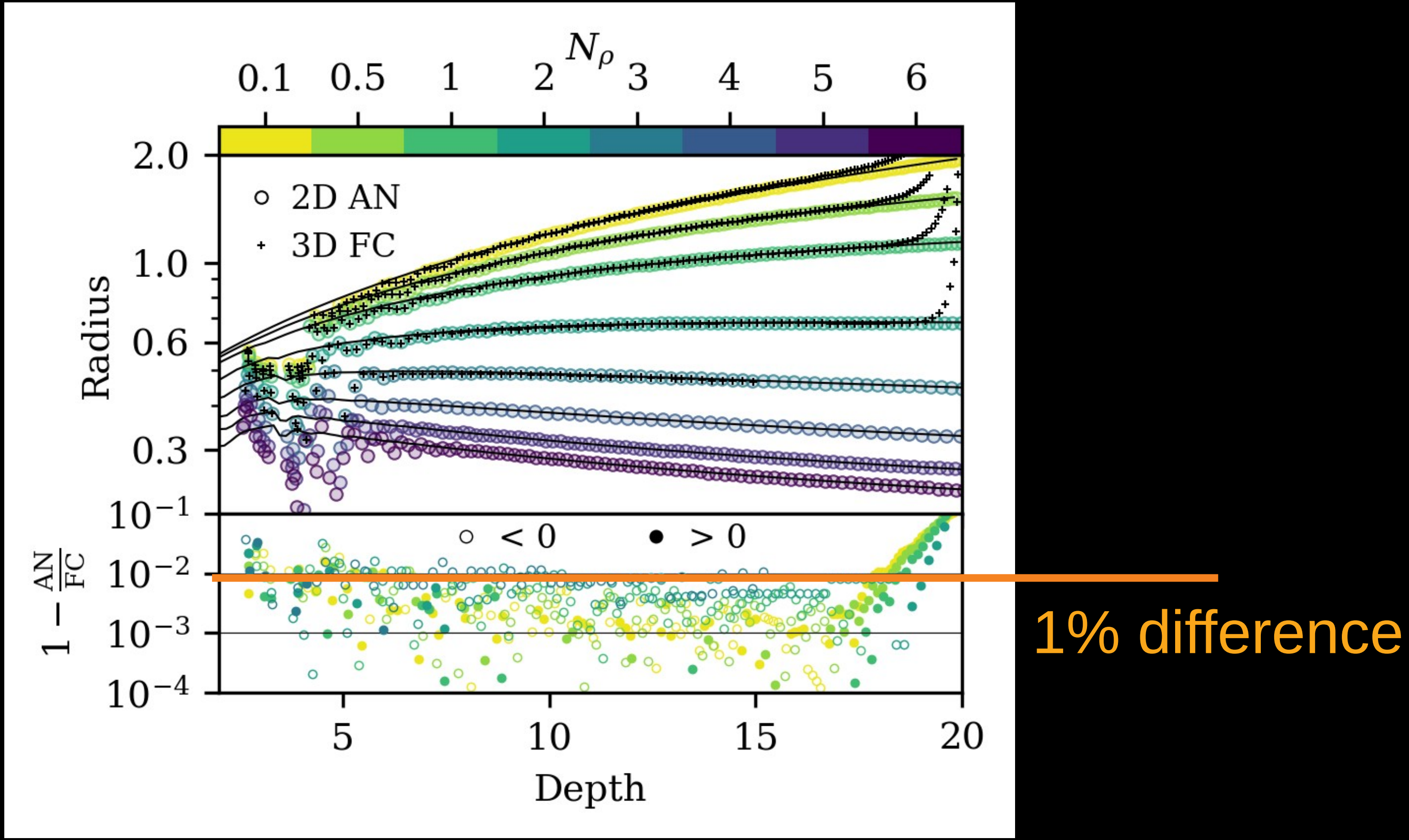
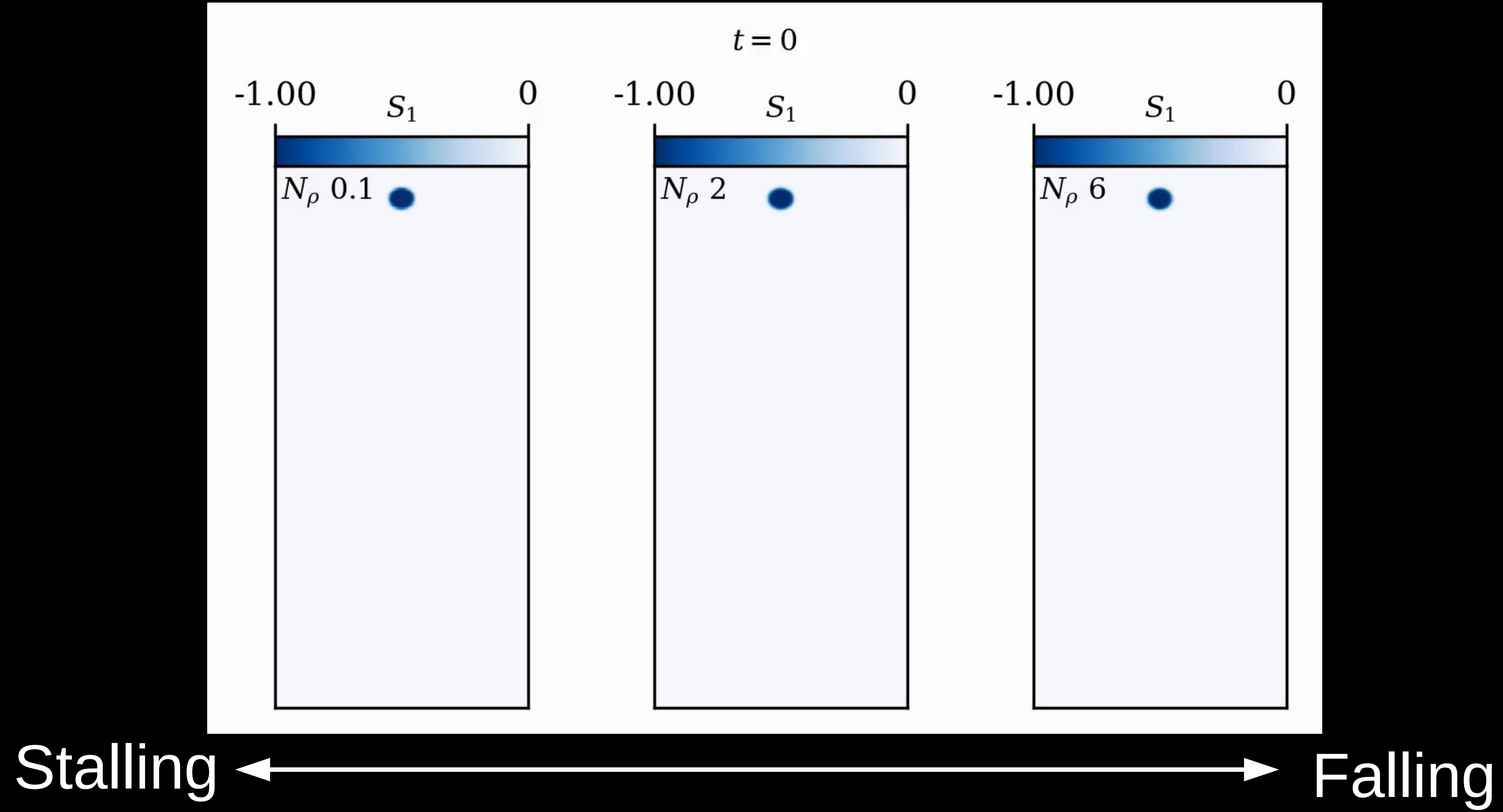
Simulations vs. Theory



[Anders, Lecoanet, & Brown 2019, ApJ 884]


$t = 0$
-1.00
S_1
0
-1.00
S_1
0
-1.00
S_1
0
N_ρ 0.1
N_ρ 2
N_ρ 6
Stalling
Falling

Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary
3) Buoyancy makes vortex rings expand & entrain, while stratification compresses them. Entropy rain feels both of these effects.
4) **We develop a theory that describes the size & depth of entropy rain vs. time and verify that theory with simulations. There are two regimes: “stalling” and “falling”.**

Extrapolation to solar parameters

Stratification increases

Simulation Output Parameterization

N_ρ	B_{th}	Γ_{th}	m	η	k
2D Anelastic Simulations (Cylindrical)					
0.1	−0.548	−2.17	8.05	1.04	0.732
0.5	−0.569	−2.12	8.34	0.977	0.715
1	−0.602	−2.05	8.65	0.915	0.703
2	−0.713	−1.89	9.23	0.842	0.682
3	−0.947	−1.73	9.81	0.807	0.654
4	−1.47	−1.59	10.2	0.794	0.642
5	−2.70	−1.49	10.7	0.781	0.609
6	−5.73	−1.43	10.8	0.787	0.616
3D Fully Compressible Simulations (Cartesian)					
0.1	−0.547	−2.17	8.98	1.06	0.636
0.5	−0.568	−2.12	8.79	0.978	0.678
1	−0.601	−2.05	8.87	0.907	0.689
2	−0.711	−1.89	9.31	0.828	0.680
3	−0.949	−1.75	9.99	0.815	0.648

Changes with increasing stratification

Changes with increasing stratification

Roughly constant as stratification increases

Extrapolation to solar parameters

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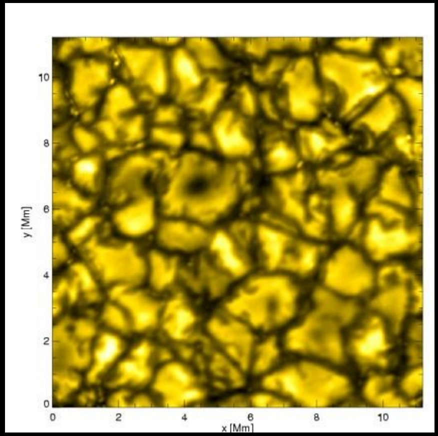
Roughly constant as stratification increases



↳ We calculated what **B** would be for a solar downflow….

Extrapolation to solar parameters

Simulation Output Par

N_ρ	$B_{\rm th}$	I			
2D Anelastic Simulations (Cylindrical)					
0.1	−0.548	[illegible]	[illegible]	[illegible]	[illegible]
0.5	−0.569	[illegible]	[illegible]	[illegible]	[illegible]
1	−0.602	[illegible]	[illegible]	[illegible]	[illegible]
2	−0.713	[illegible]	[illegible]	[illegible]	[illegible]
3	−0.947	−1.73	9.81	0.807	0.654
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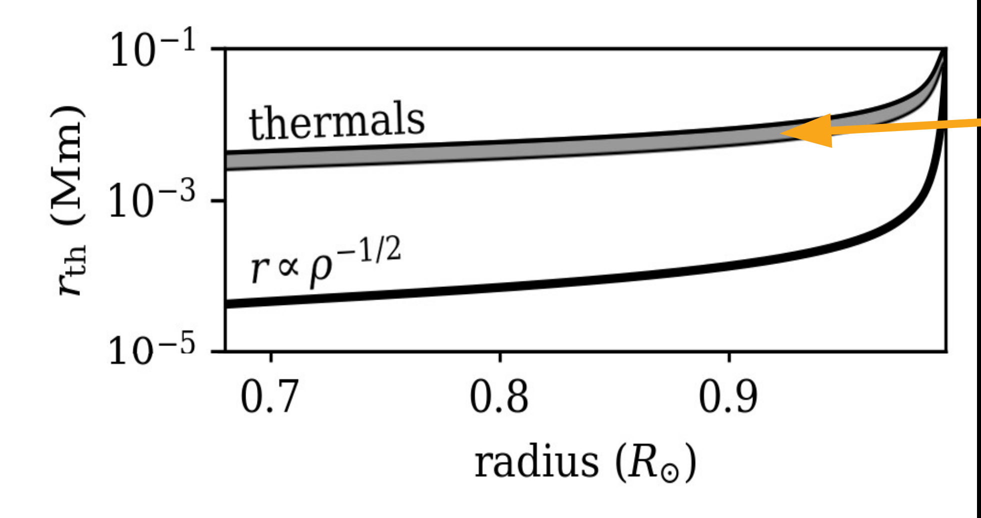
We estimate that solar downflows are in this range, which is in the "falling" regime.

[Anders, Lecoanet, & Brown 2019, ApJ 884]

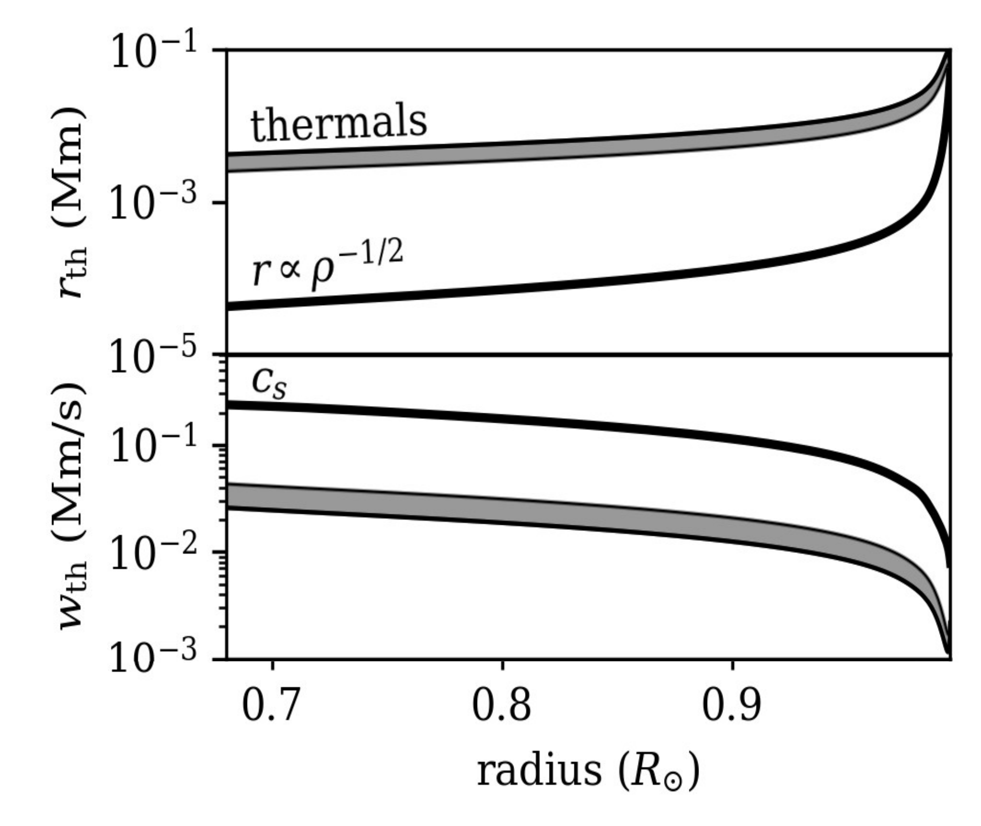
Extrapolation to solar parameters



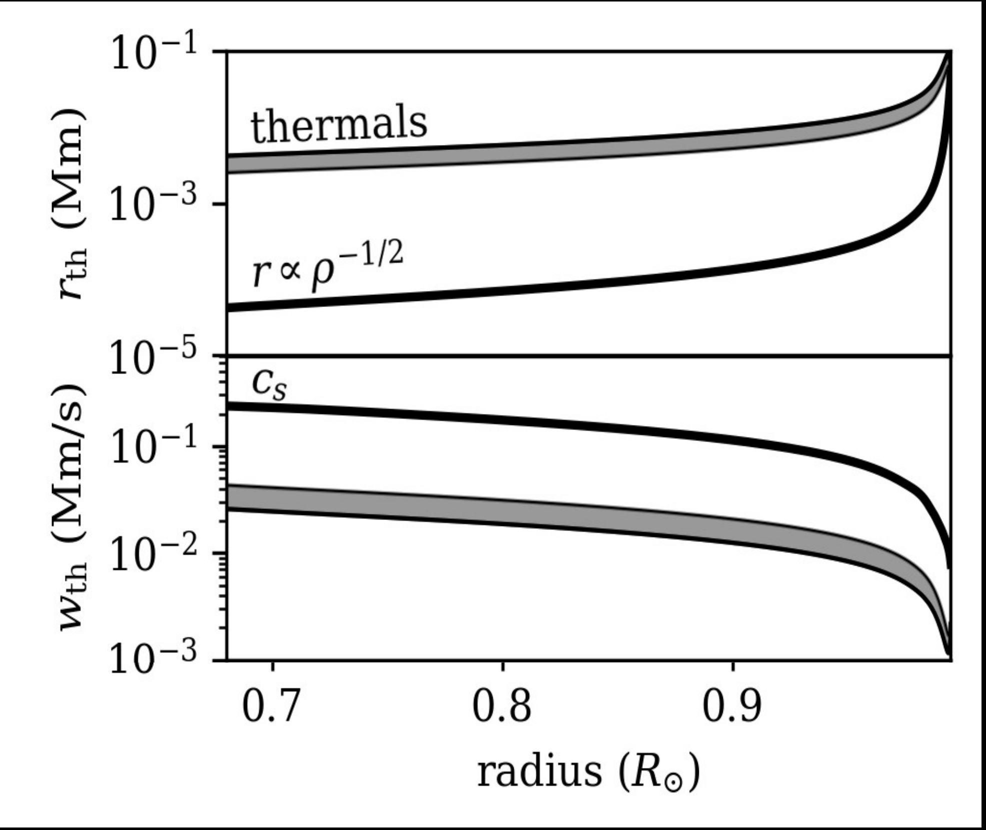
Behavior of range of estimated thermals

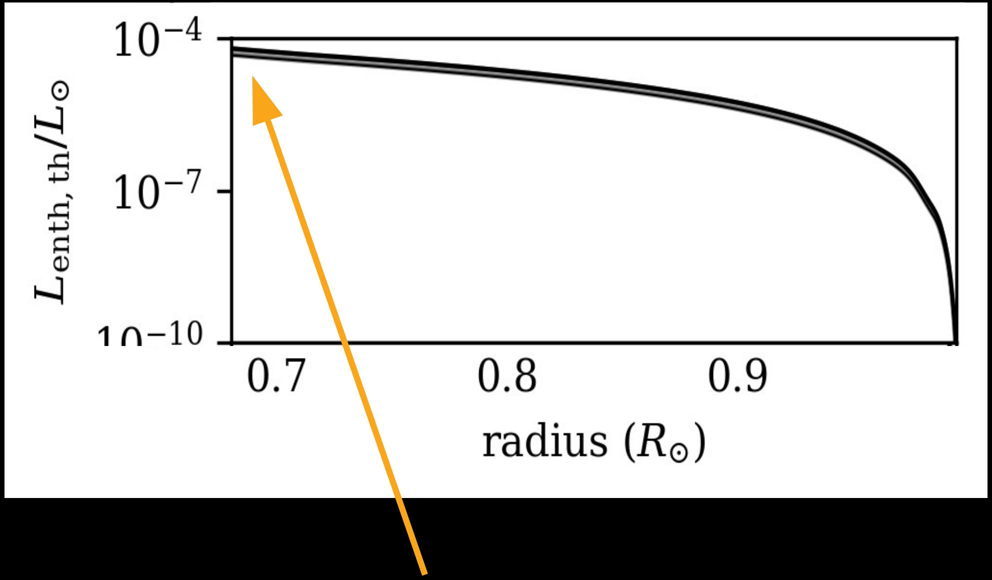
Thermals travel this way (from surface to RZ)

Extrapolation to solar parameters



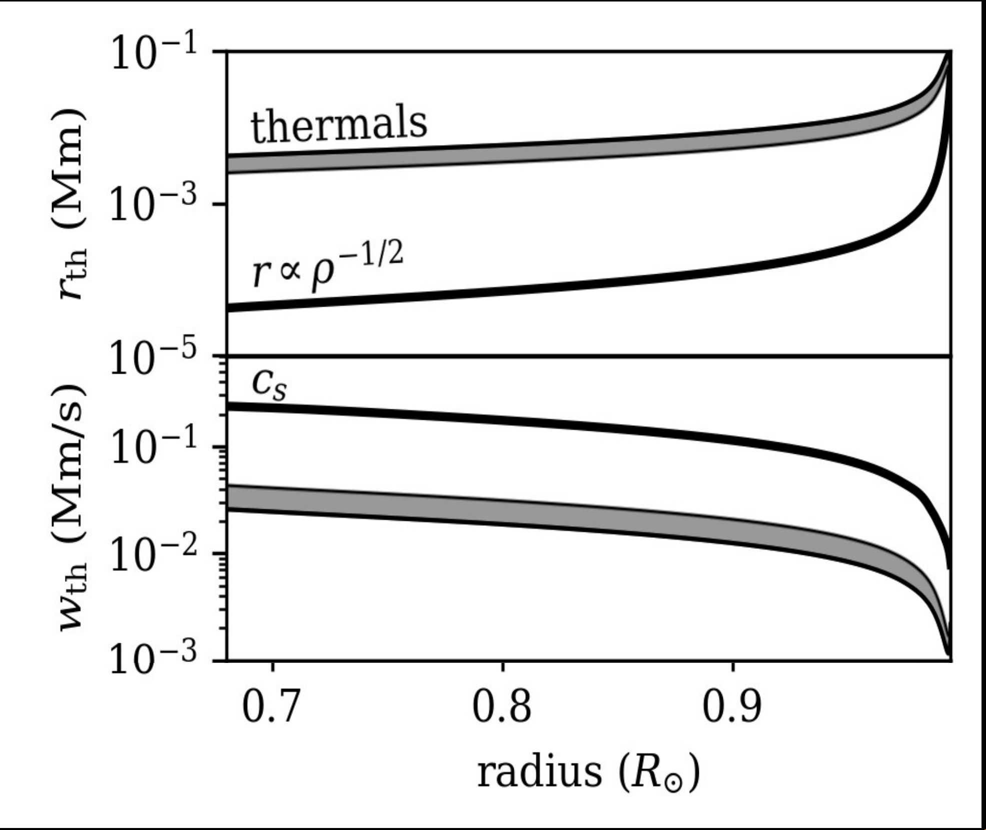
Extrapolation to solar parameters

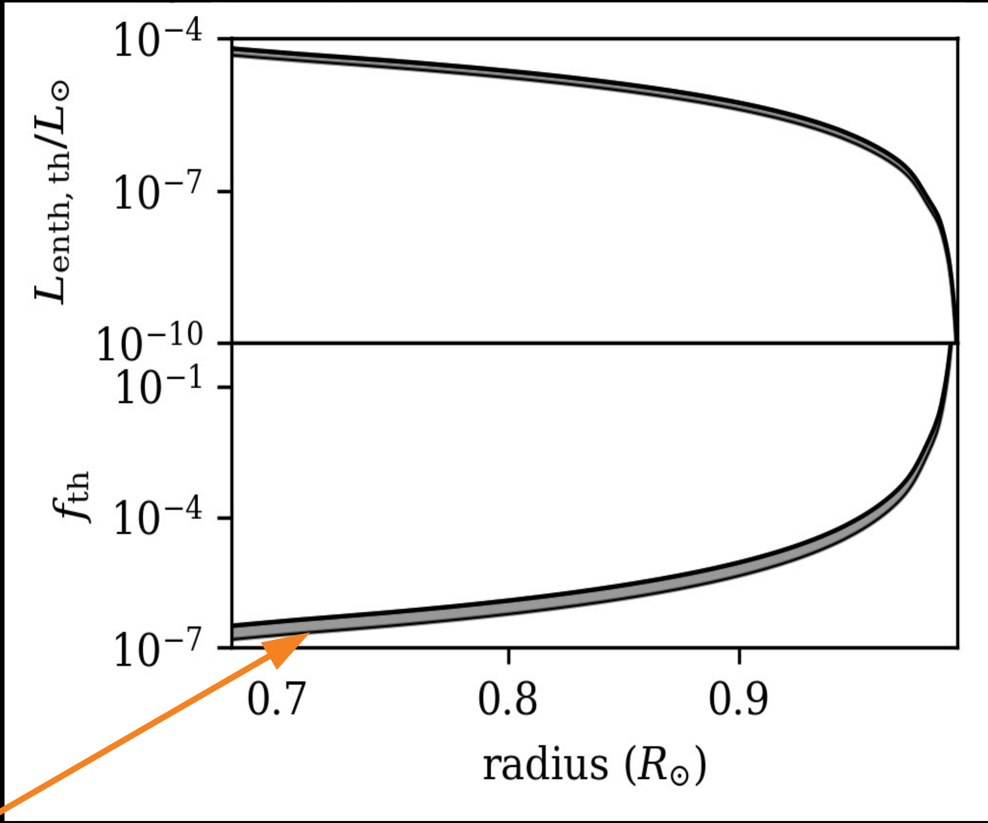




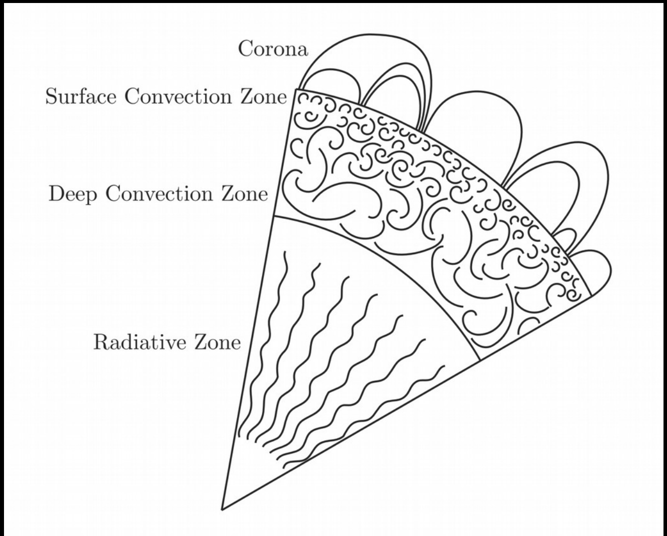
A single thermal carries 0.01% of the solar luminosity at base of CZ.

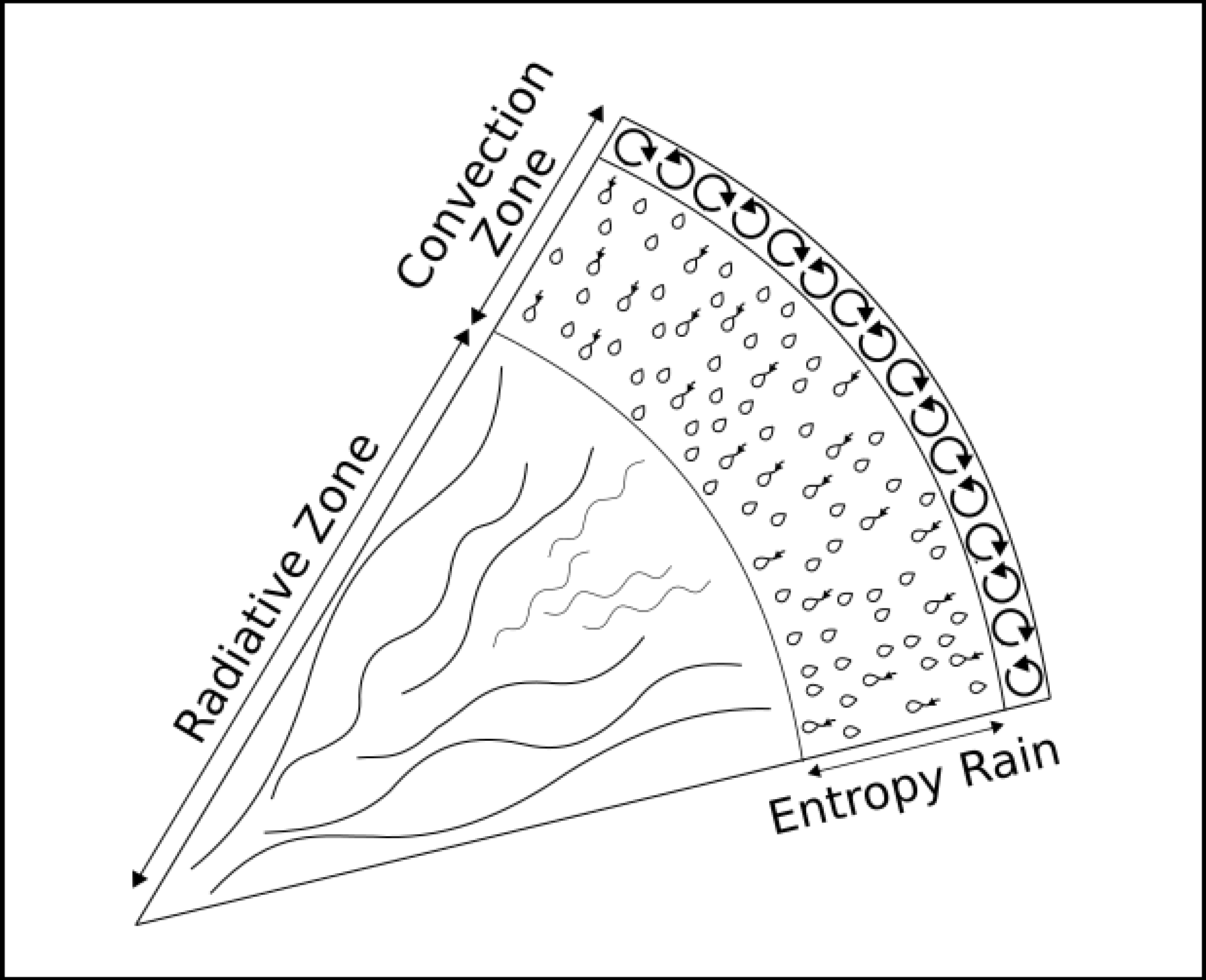
Extrapolation to solar parameters





Only a small fraction of the area of the CZ is required to carry the solar luminosity using thermals


Corona
Surface Convection Zone
Deep Convection Zone
Radiative Zone


Convection Zone
Radiative Zone
Entropy Rain

Takeaways

1) There is an absence of large-scale convective power in observations of the Sun.
2) Entropy rain (the nonlocal transport of heat by downflows) may make giant cells unnecessary
3) Buoyancy makes vortex rings expand & entrain, while stratification compresses them. Entropy rain feels both of these effects.
4) We develop a theory that describes the size & depth of entropy rain vs. time and verify that theory with simulations. There are two regimes: “stalling” and “falling”
5) **The enthalpy fluxes carried by solar downflows is sufficient to carry the solar luminosity in the deep CZ**

For more specifics, see the paper:
[Anders, Lecoanet & Brown 2019; ApJ 884; arXiv: 1906.02342]

Preliminary Turbulent Results

- Entrainment behavior ~the same
- Acceleration & velocity behavior is quite different than the laminar cases
- Hunch: baroclinic torques

$$\frac{d\Gamma}{dt} = \oint_{\mathcal{C}} g \frac{S_1}{c_P} \hat{z} \cdot d\boldsymbol{x},$$

+ viscous heating

$$\rho \frac{\partial S_1}{\partial t} \sim \rho \frac{\nu}{T} \nabla^2 |\boldsymbol{u}|^2; \mu = \rho\nu$$