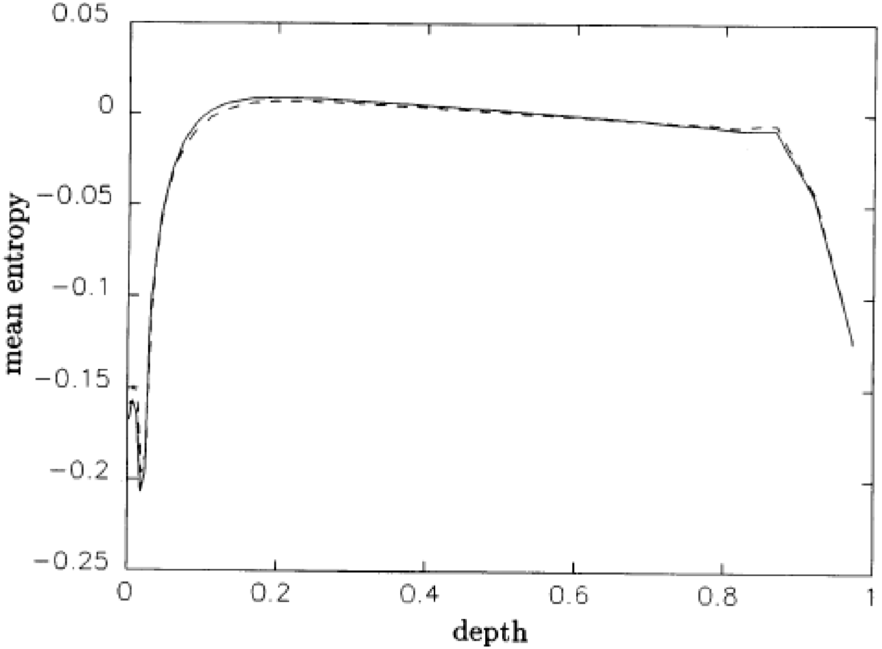
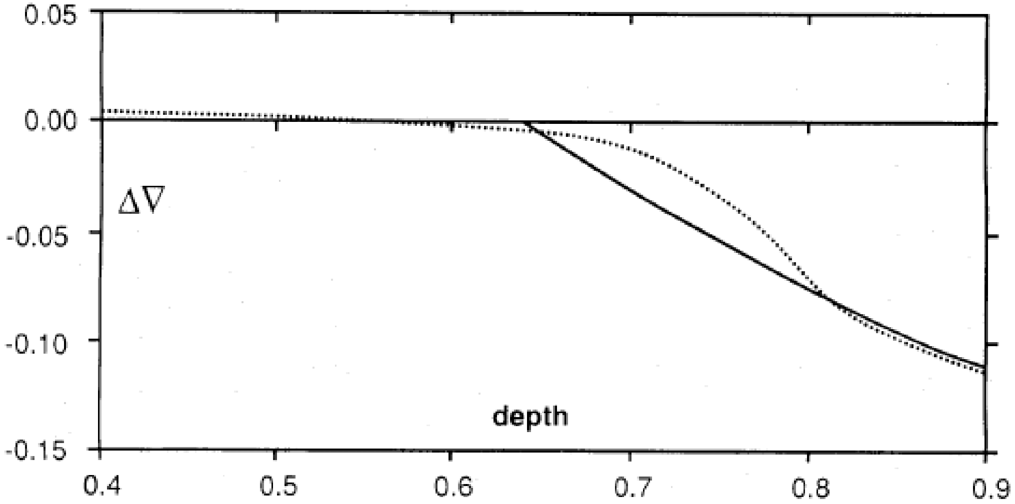
Subadiabatic convection



Chan & Gigas (1992), *ApJL*, **389**, L87

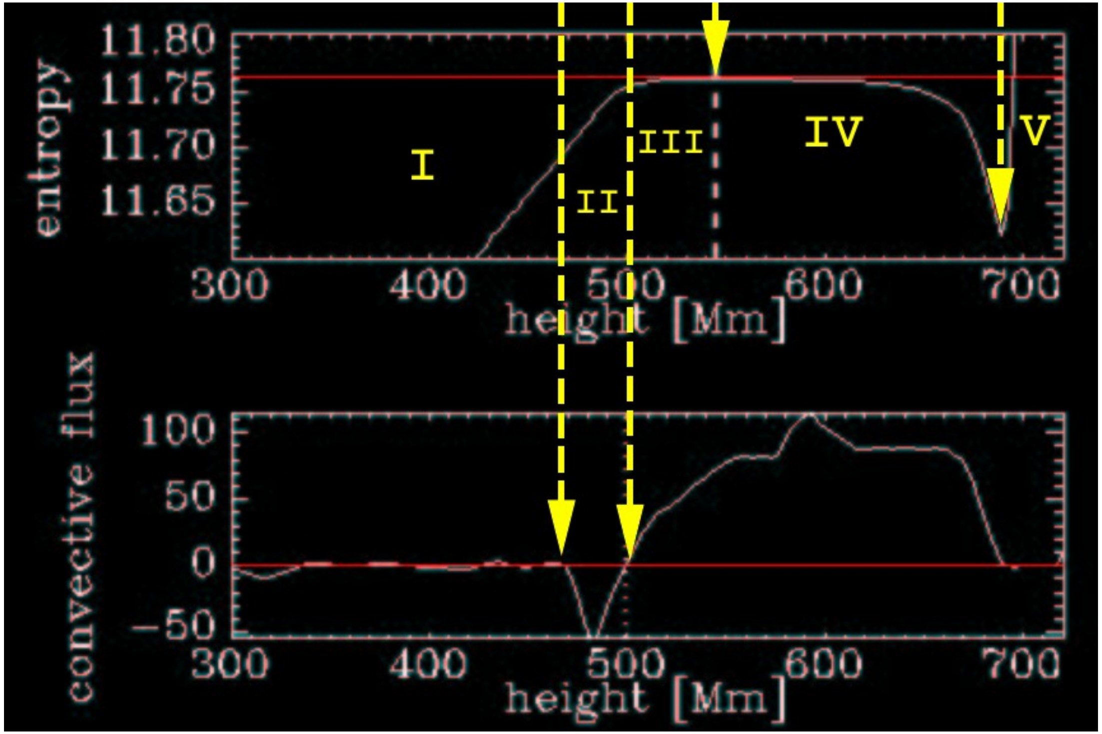


Roxburgh & Simmons (1993), *A&A*, **277**, 93

$$K(T) = K_0 \left[\left(\frac{T}{T_0} \right)^3 + \frac{3}{5} \left(\frac{T_0}{T} \right)^5 \right]$$

Tremblay et al. (2015), *ApJ*, **799**, 142; Hotta (2017), *ApJ*, **843**, 52.

Subadiabatic convection



$$K(\rho, T) = K_0 \rho^{-2} T^{6.5}$$

Brandenburg, Nordlund & Stein (2000), in *Astrophysical Convection and Dynamos*, 85

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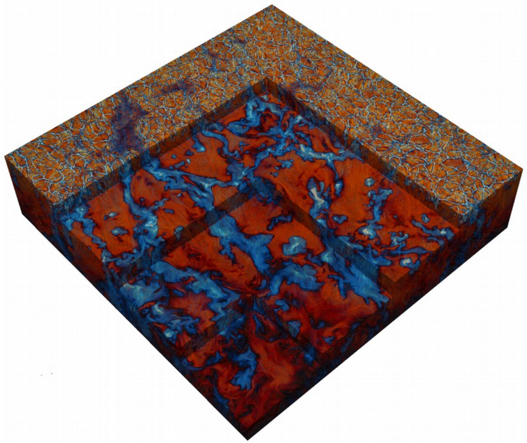
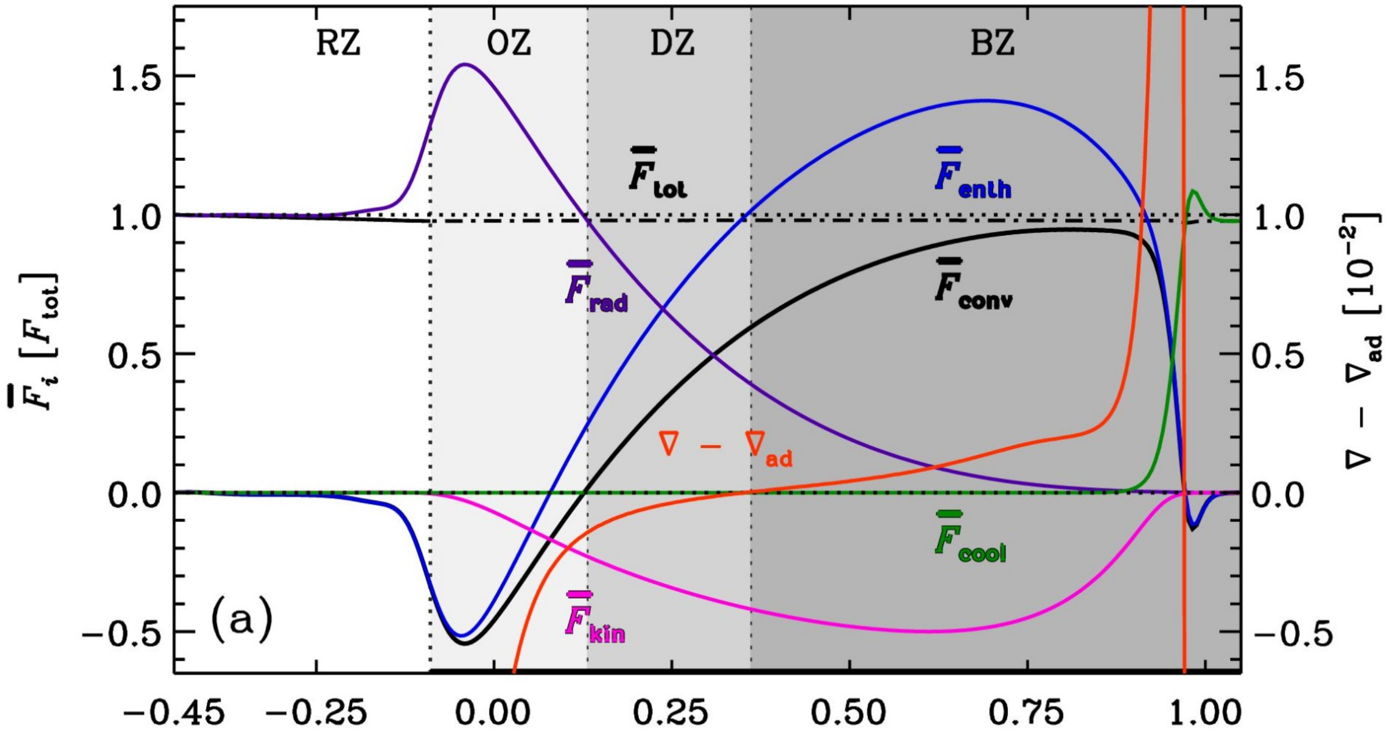
Subadiabatic convection



$$\overline{F}_{\rm rad} = -\overline{K}\frac{\partial \overline{T}}{\partial z},$$
$$\overline{F}_{\rm enth} = c_{\rm P}\overline{(\rho u_z)'T'},$$
$$\overline{F}_{\rm kin} = \tfrac{1}{2}\overline{\rho \boldsymbol{u}^2 u_z'},$$
$$\overline{F}_{\rm visc} = -2\nu\overline{\rho u_i \mathsf{S}_{iz}}$$
$$\overline{F}_{\rm cool} = \int_{z_{\rm bot}}^{z_{\rm top}} \Gamma_{\rm cool} {\rm d}z.$$

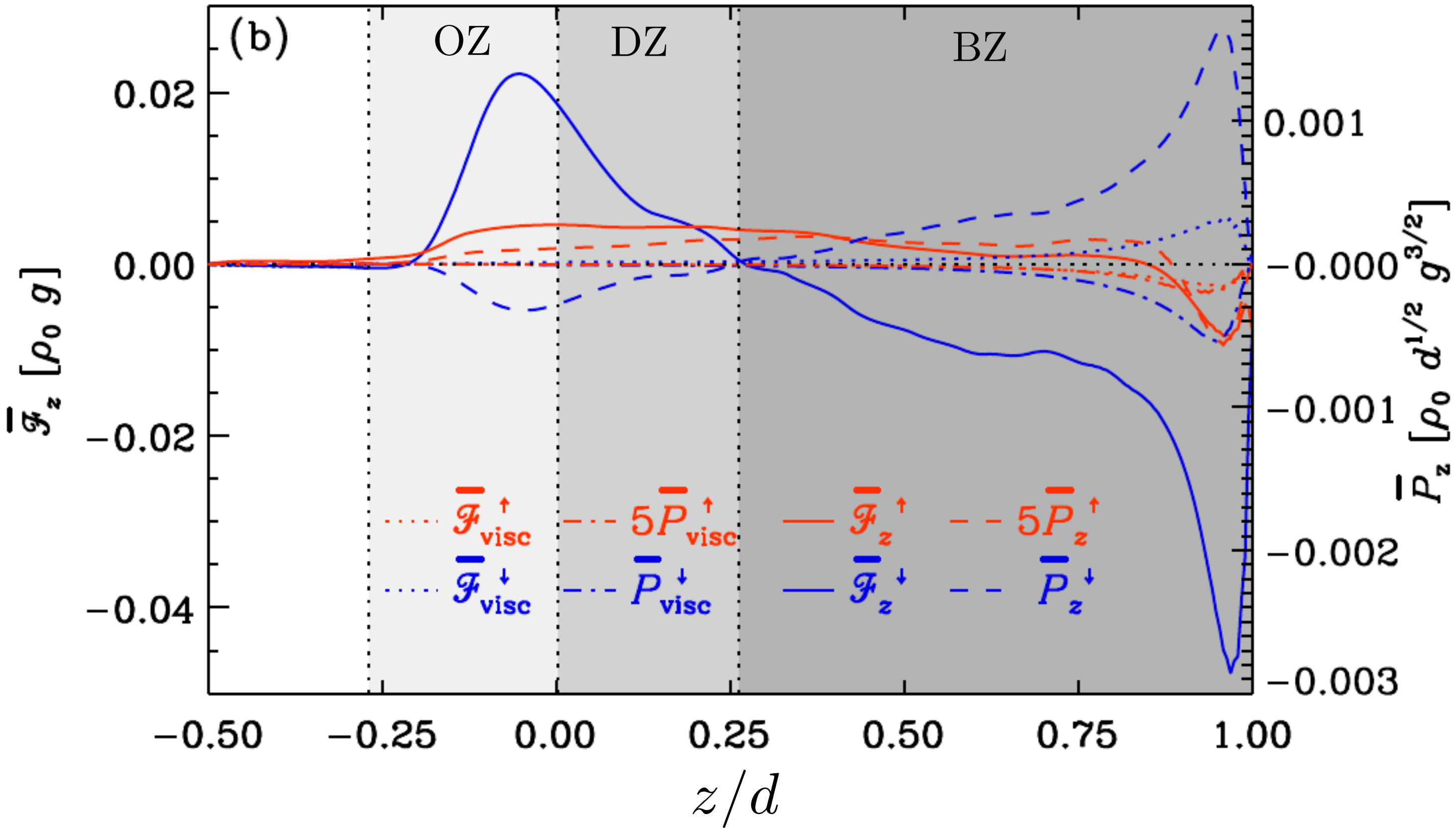
$$\overline{F}_{\rm conv} = \overline{F}_{\rm enth} + \overline{F}_{\rm kin}$$



Käpylä et al. (2017), *ApJL*, **845**, L23

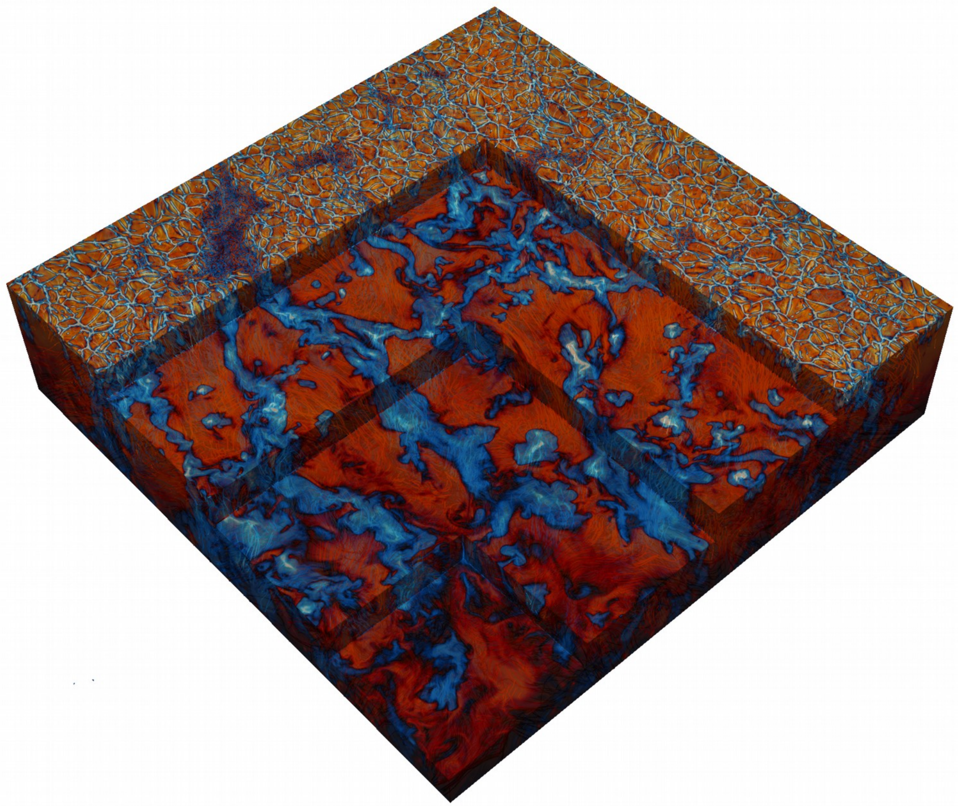
Käpylä (2019), *A&A*, **631**, 122

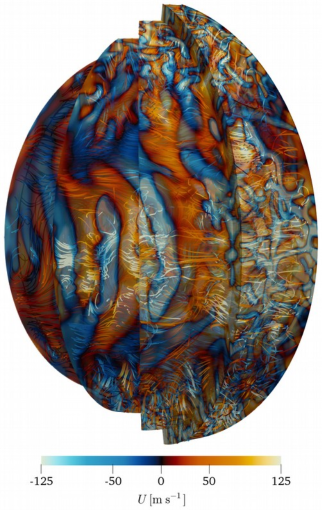
Force balance

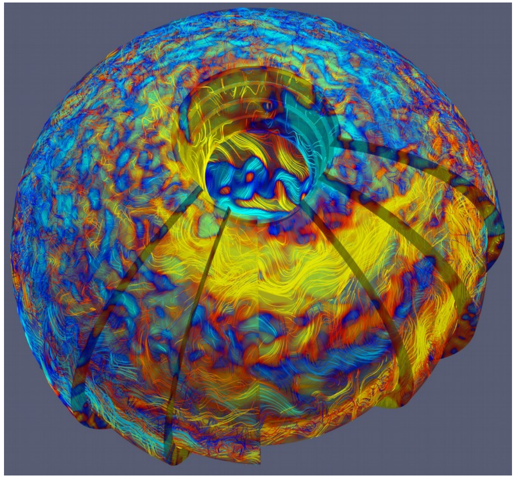


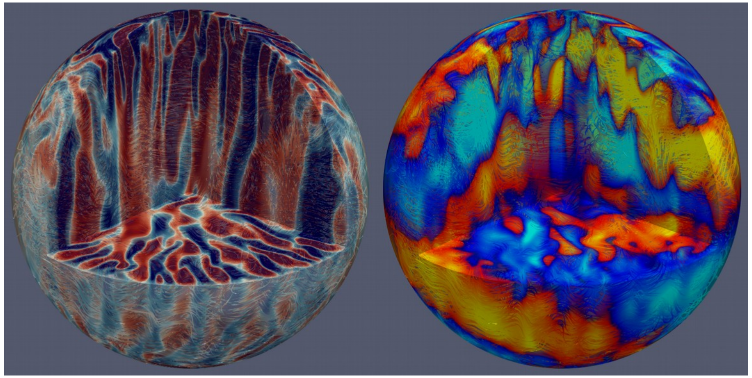
Käpylä et al. (2017), *ApJL*, **845**, L23

Pencil Code









https://github.com/pencil-code

Nordita 6th March 2020

Convective conundrum...

...or how I stopped worrying and learned to love the Prandtl number

Nordita 6th March 2020

Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\chi},$$

Prandtl number in the Sun is $\mathrm{Pr} \lesssim 10^{-6}$, in sims $\mathrm{Pr} \approx 1$.

Why? $\mathrm{Pe} = \dfrac{u\ell}{\chi} = \mathrm{Pr}\mathrm{Re}$, with $\mathrm{Pe}, \mathrm{Re} \gg 1$.

Pr-dependence is ~~bullshit~~ ~~wrong~~ completely irrelevant.

Why? Because for fully developed turbulence $\mathrm{Pe}, \mathrm{Re} \gg 1$ and the influence of the small scales is negligible.

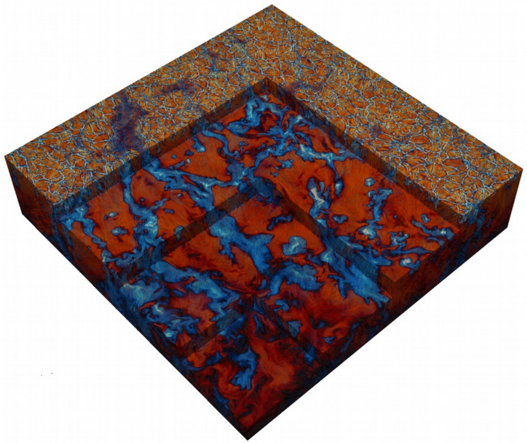
But how can this be known a priori?

Results (Cartesian)

$$\begin{aligned}
\frac{D\ln\rho}{Dt} &= -\boldsymbol{\nabla}\cdot\boldsymbol{u}, \\
\frac{D\boldsymbol{u}}{Dt} &= \boldsymbol{g} - \frac{1}{\rho}(\boldsymbol{\nabla} p - \boldsymbol{\nabla}\cdot 2\nu\rho\mathbf{S}), \\
T\frac{Ds}{Dt} &= -\frac{1}{\rho}\left[\boldsymbol{\nabla}\cdot(\boldsymbol{F}_{\rm rad} + \boldsymbol{F}_{\rm SGS})\right] + 2\nu\mathbf{S}^2.
\end{aligned}$$

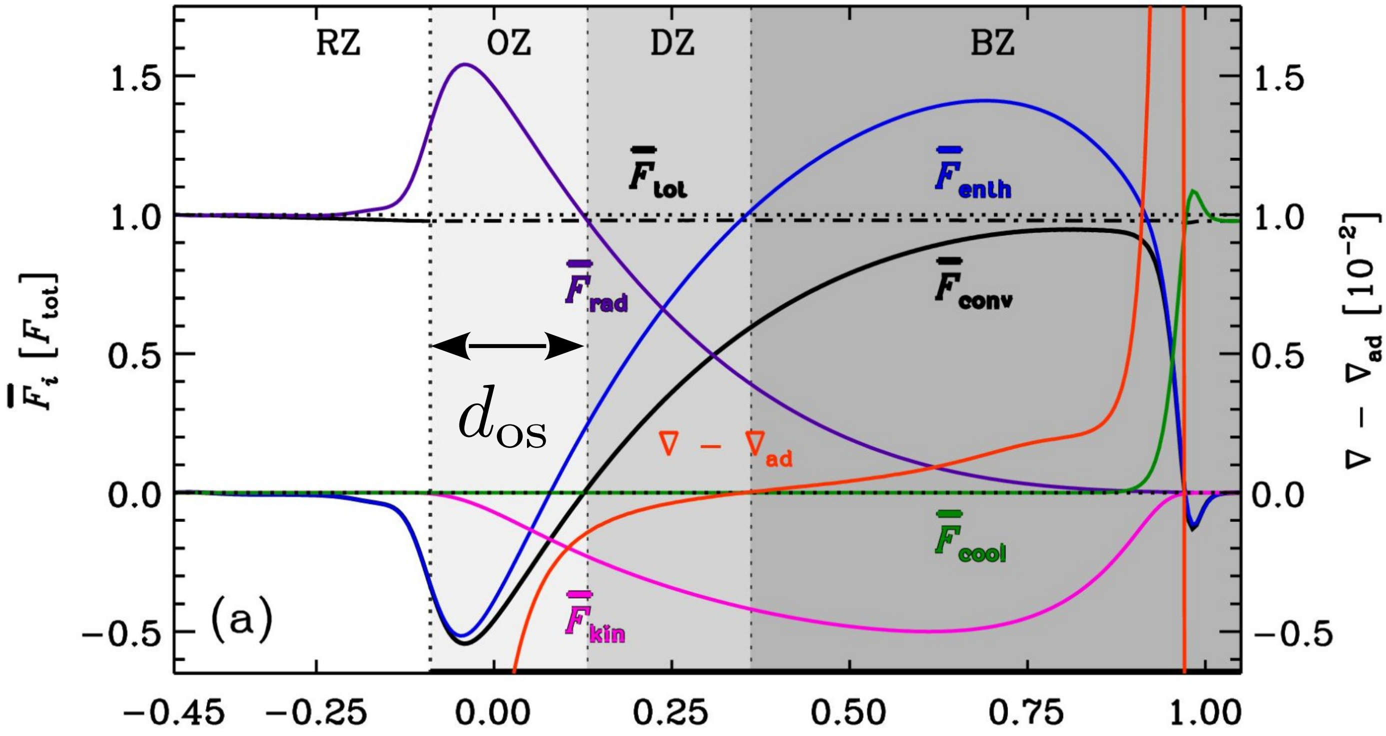
$$\boldsymbol{F}_{\rm rad} = -K\boldsymbol{\nabla} T, \quad \boldsymbol{F}_{\rm SGS} = -\chi_{\rm SGS}\rho T\boldsymbol{\nabla} s'.$$

Results (Cartesian)



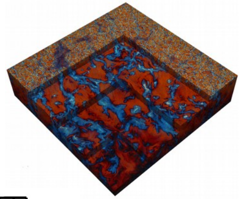
$$\overline{F}_{\rm rad} = -\overline{K}\frac{\partial\overline{T}}{\partial z},$$
$$\overline{F}_{\rm enth} = c_{\rm P}\overline{(\rho u_z)'T'},$$
$$\overline{F}_{\rm kin} = \tfrac{1}{2}\overline{\rho \boldsymbol{u}^2 u_z'},$$
$$\overline{F}_{\rm visc} = -2\nu\overline{\rho u_i \mathsf{S}_{iz}}$$
$$\overline{F}_{\rm cool} = \int_{z_{\rm bot}}^{z_{\rm top}} \Gamma_{\rm cool} {\rm d}z.$$

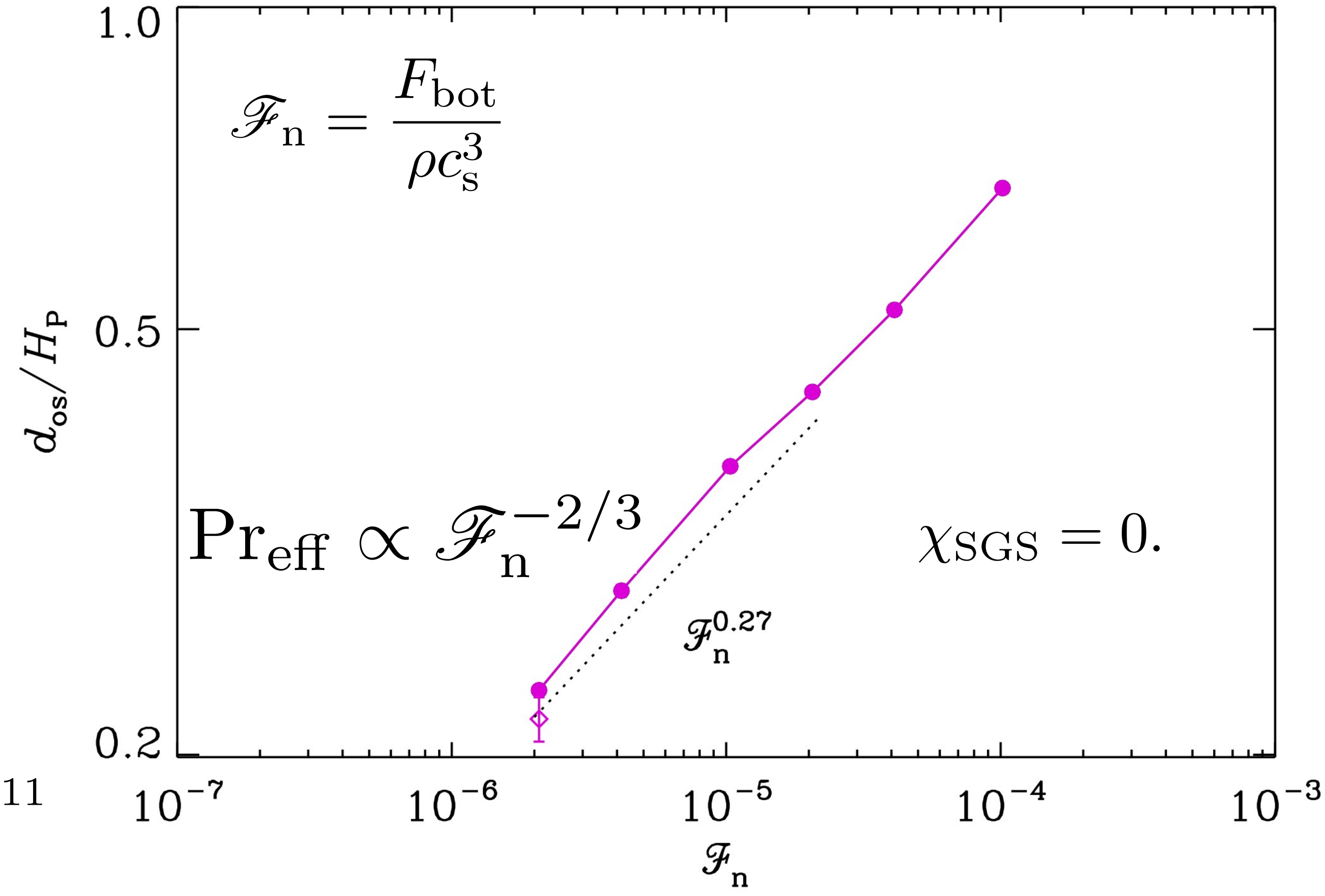
$$\overline{F}_{\rm conv} = \overline{F}_{\rm enth} + \overline{F}_{\rm kin}$$



Käpylä (2019), *A&A*, **631**, 122

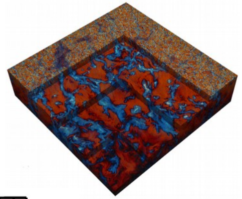
Results (Cartesian)

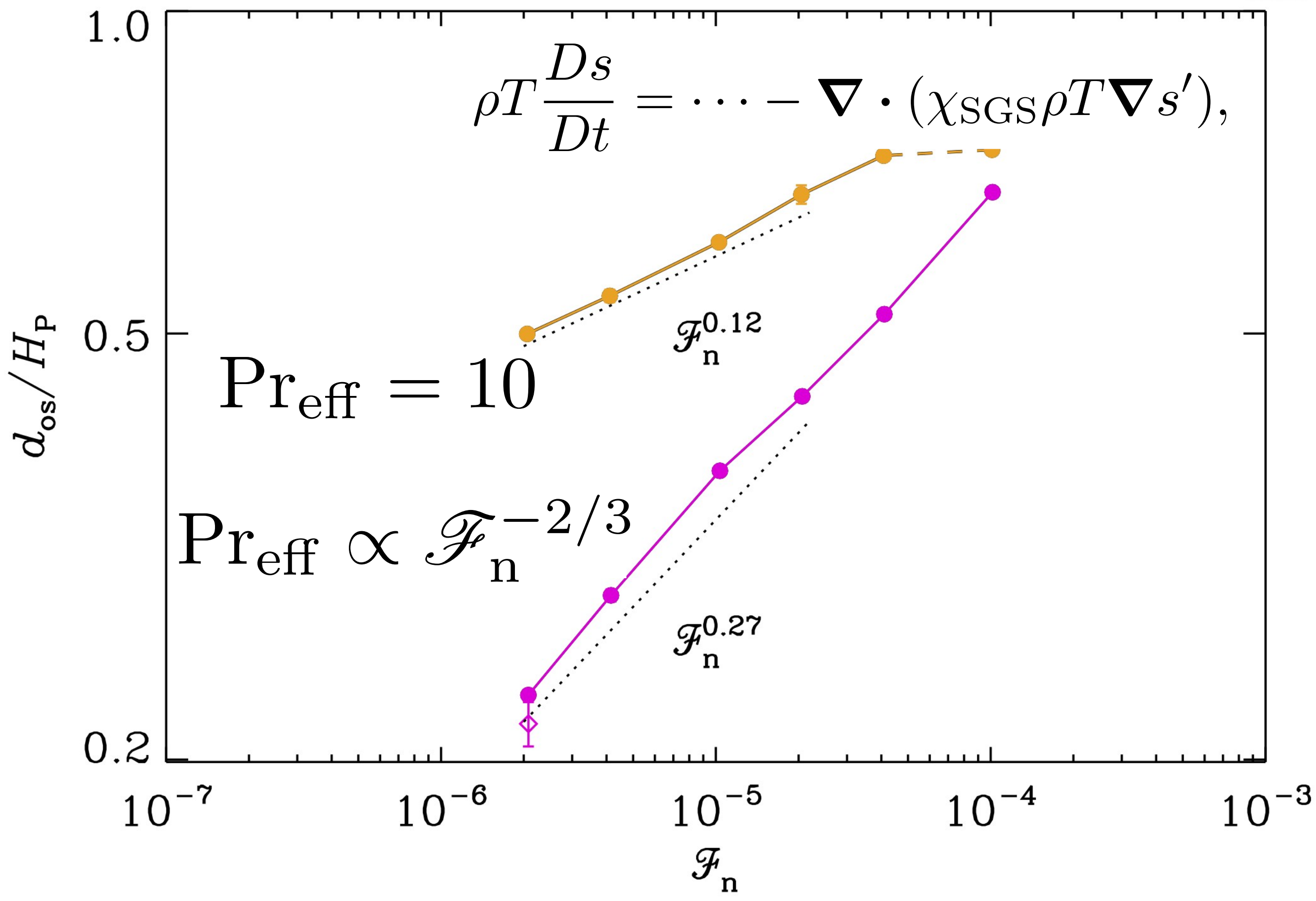




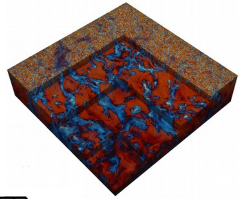
$\mathscr{F}_\mathrm{n}^{\odot} \approx 4 \cdot 10^{-11}$

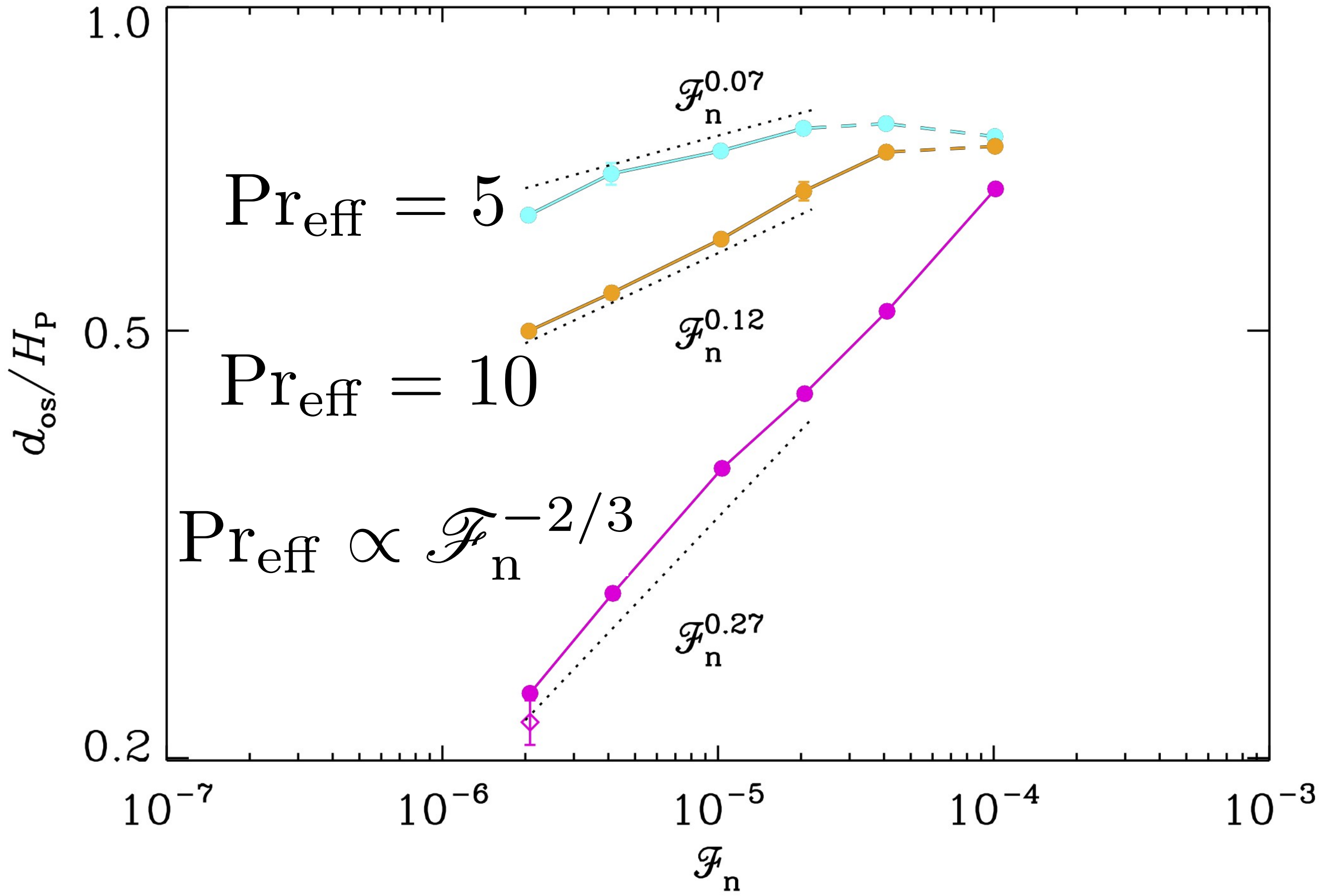
Results (Cartesian)



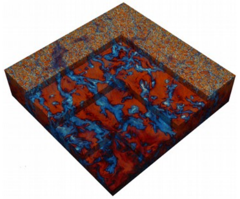


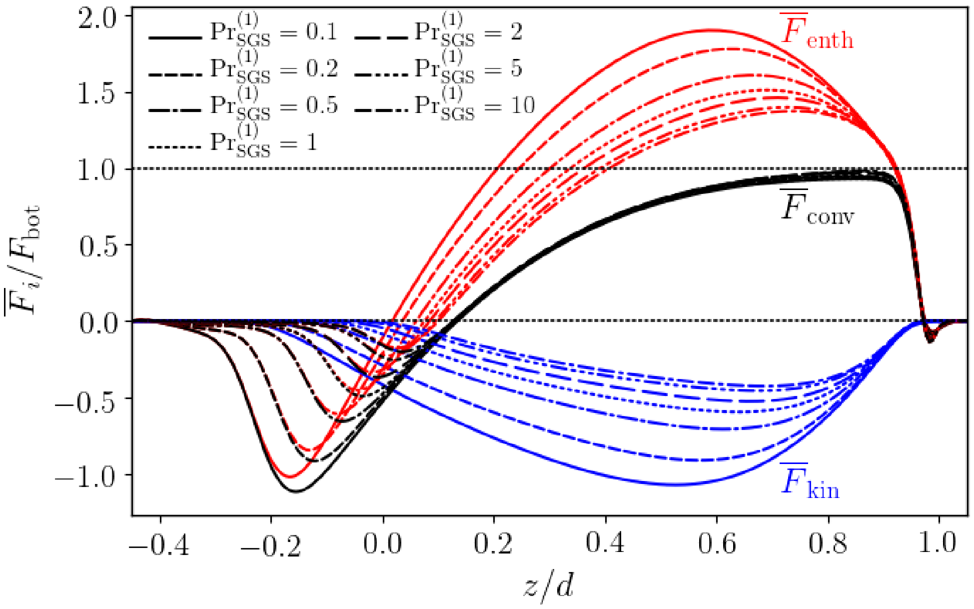
Results (Cartesian)

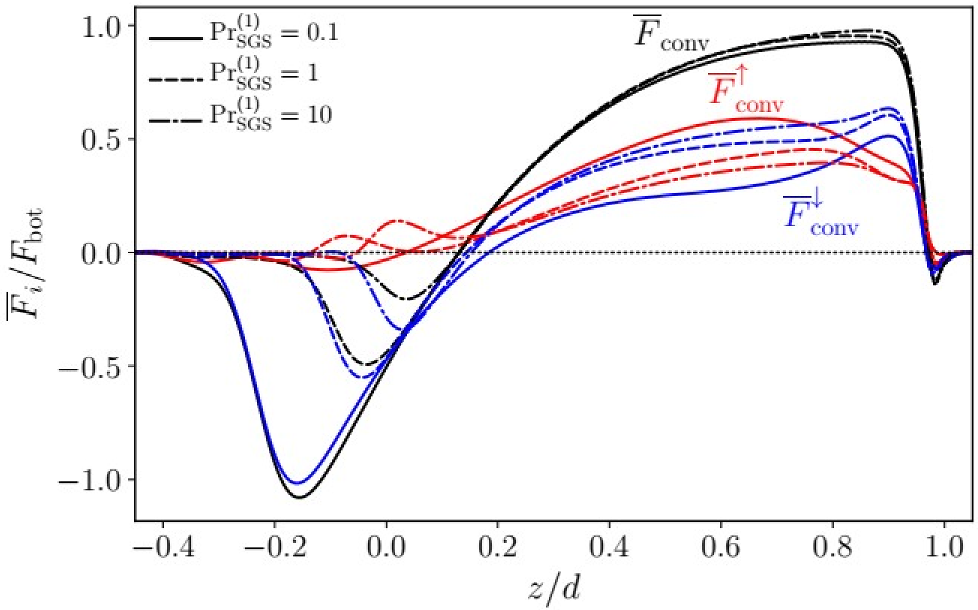




Results (Cartesian)



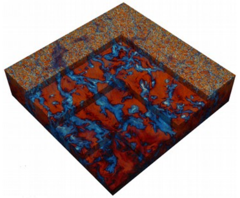




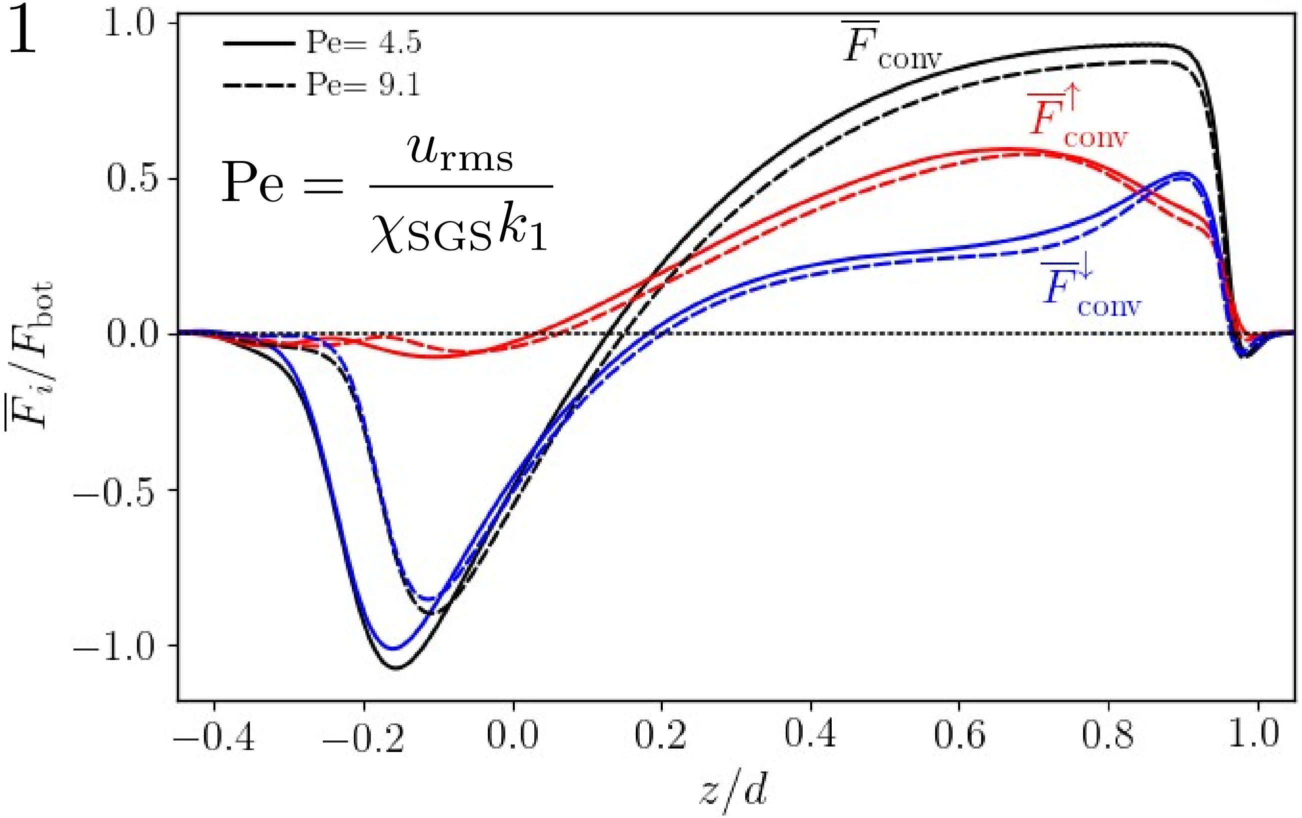
$$\overline{F}_{\mathrm{conv}} = \overline{F}_{\mathrm{enth}} + \overline{F}_{\mathrm{kin}} \quad \overline{F}_{\mathrm{enth}} = c_{\mathrm{P}}\overline{(\rho u_z)'T'} \quad \overline{F}_{\mathrm{kin}} = \tfrac{1}{2}\overline{\rho \boldsymbol{u}^2 u_z}$$

(Work in progress)

Results (Cartesian)

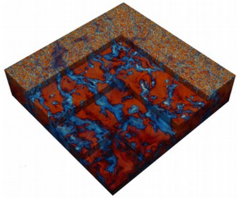


$\mathrm{Pr}_{\mathrm{SGS}} = 0.1$



(Work in progress)

Results (Cartesian)



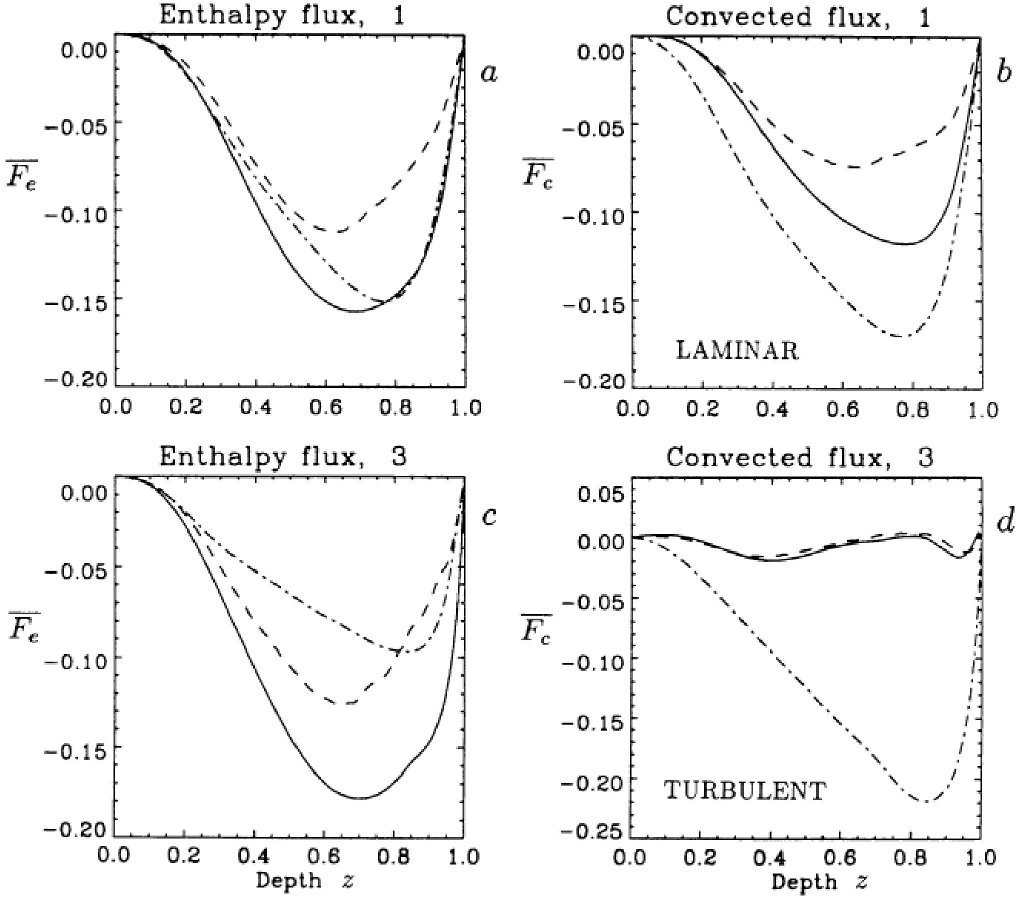


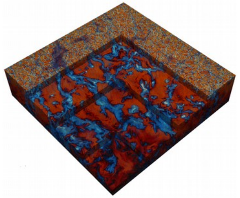
TABLE 1

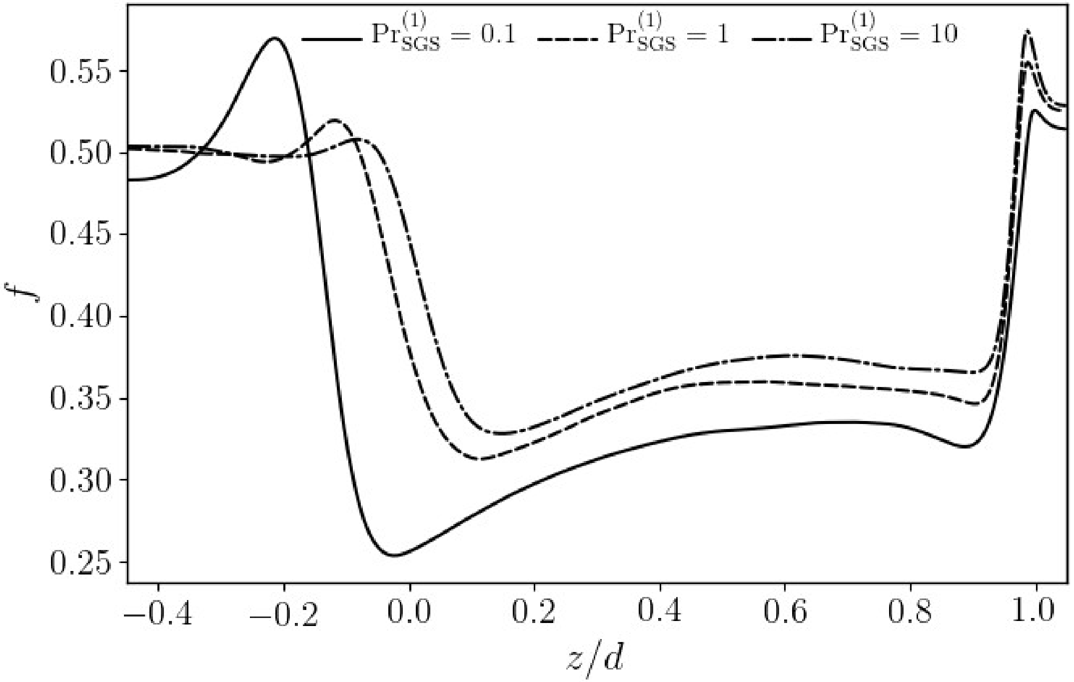
PARAMETERS IN THE FOUR SIMULATIONS

Case	σ	γ	Ra	X_{mx}, Y_{mx}	$NX:NY:NZ$
1………	1.0	5/3	4.9×10^4	6	64:64:62
2………	0.3	5/3	1.6×10^5	6	64:64:62
3………	0.1	5/3	4.9×10^5	6	96:96:96
4………	$0.013\bar{\rho}$	1.4	1.3×10^6	4	96:96:56

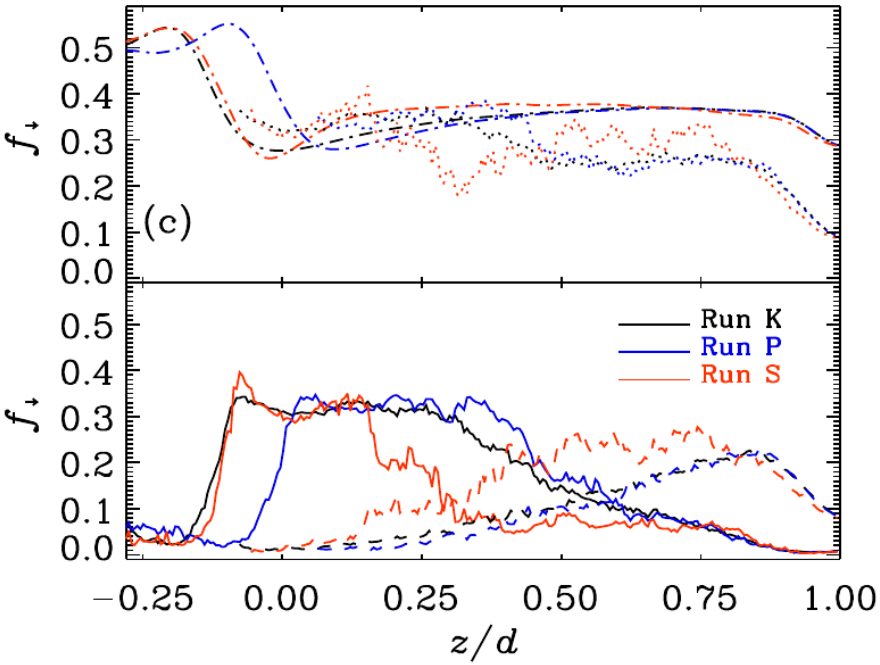
Cattaneo et al. (1991), *ApJ*, **370**, 282

Results (Cartesian)





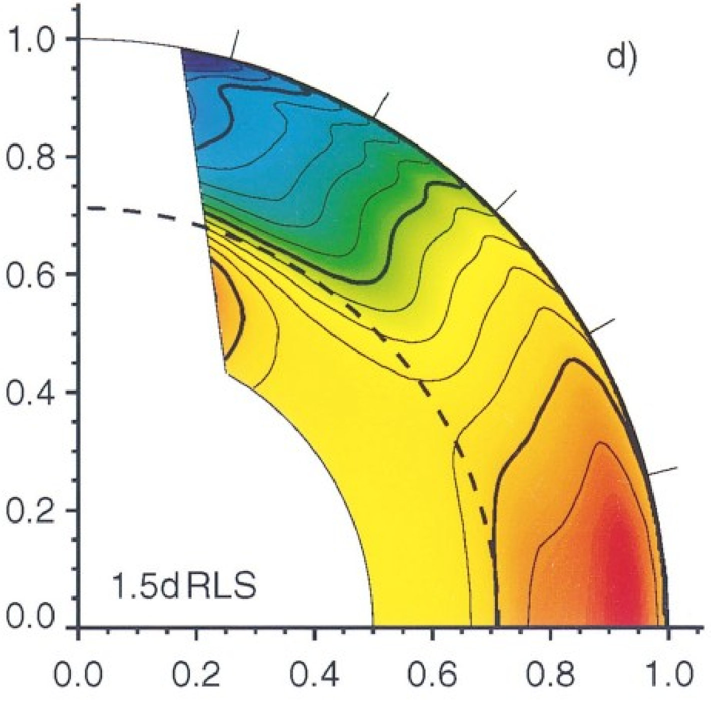
(Work in progress)



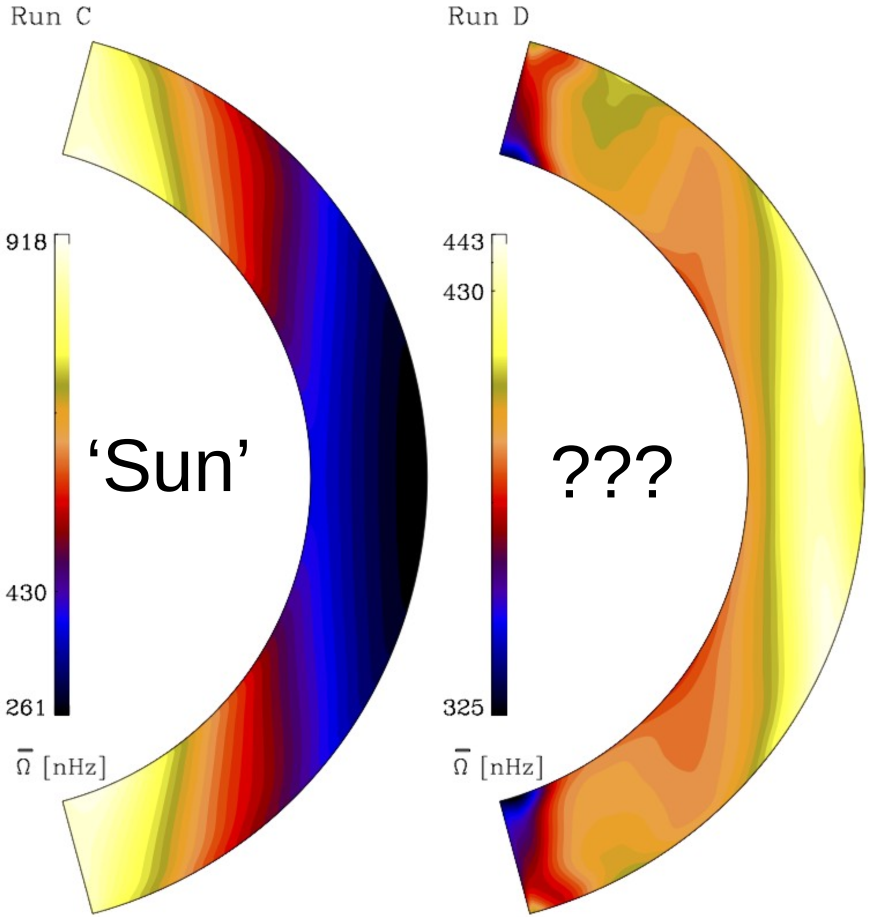
Käpylä et al. (2017), *ApJL*, **845**, L23

Convective conundrum

Why is the differential rotation anti-solar in simulations?

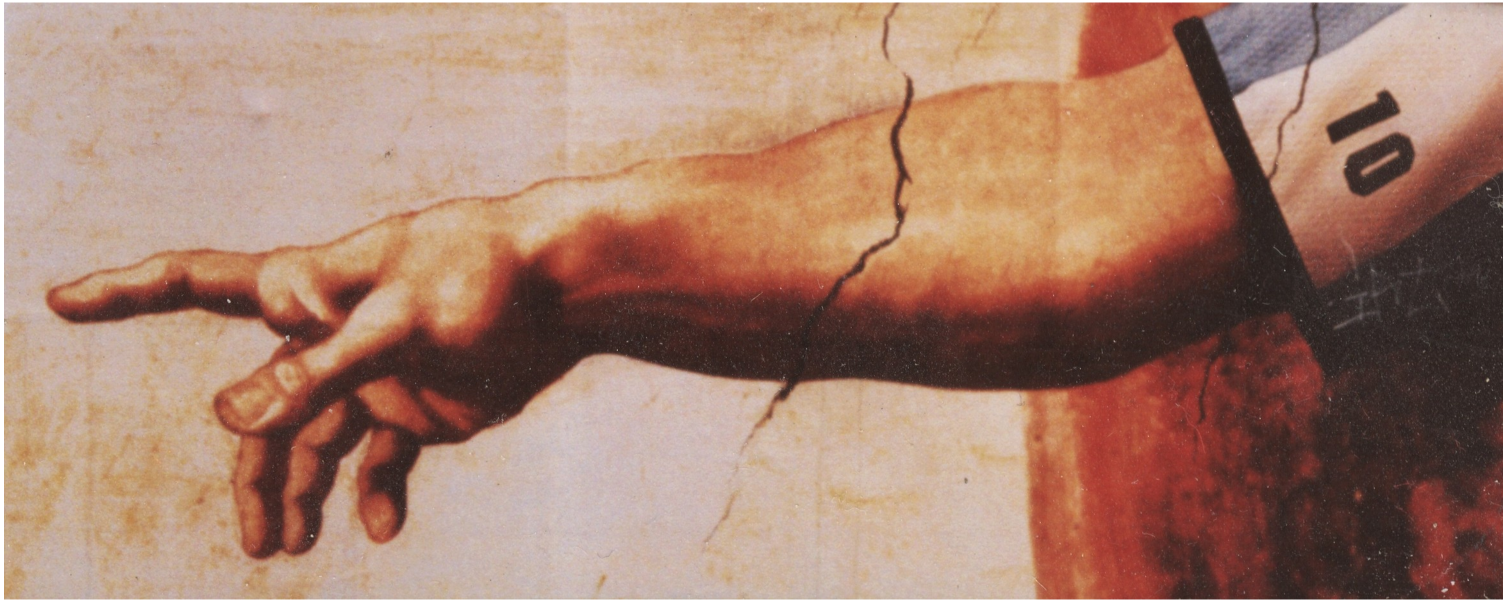


Schou et al. (1998), *ApJ*, **505**, 390



Käpylä et al. (2014), *A&A*, **570**, 43

The Maradona effect



A little with the head of Maradona, a little with the hand of god.

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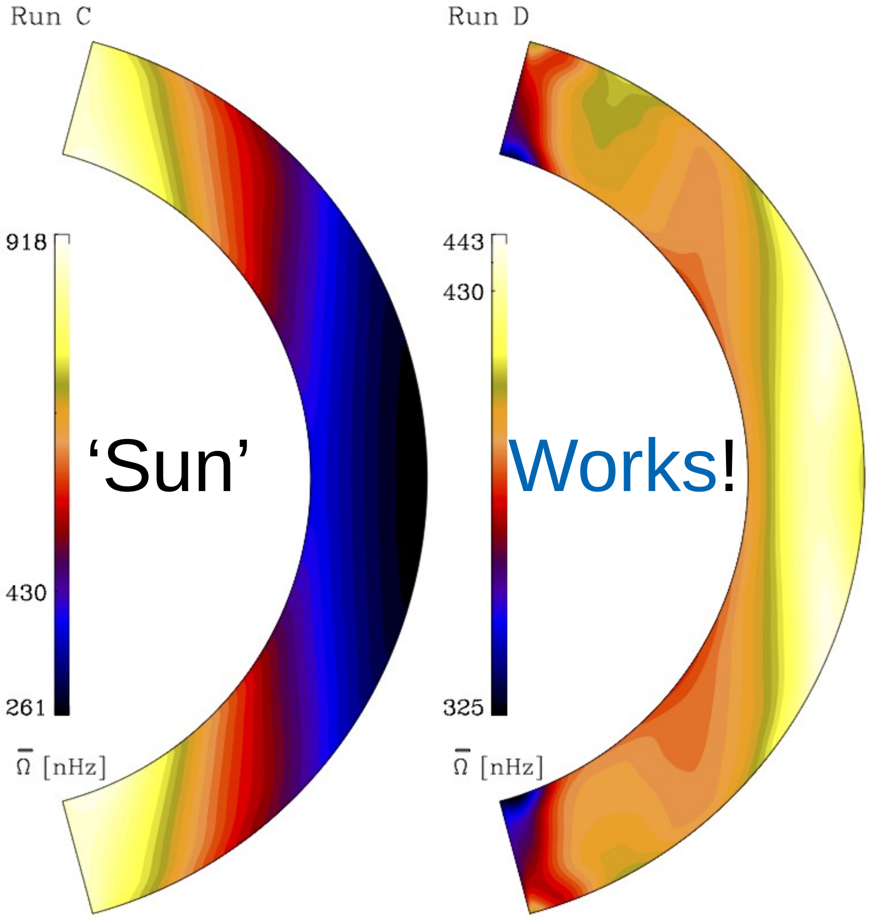
Convective conundrum

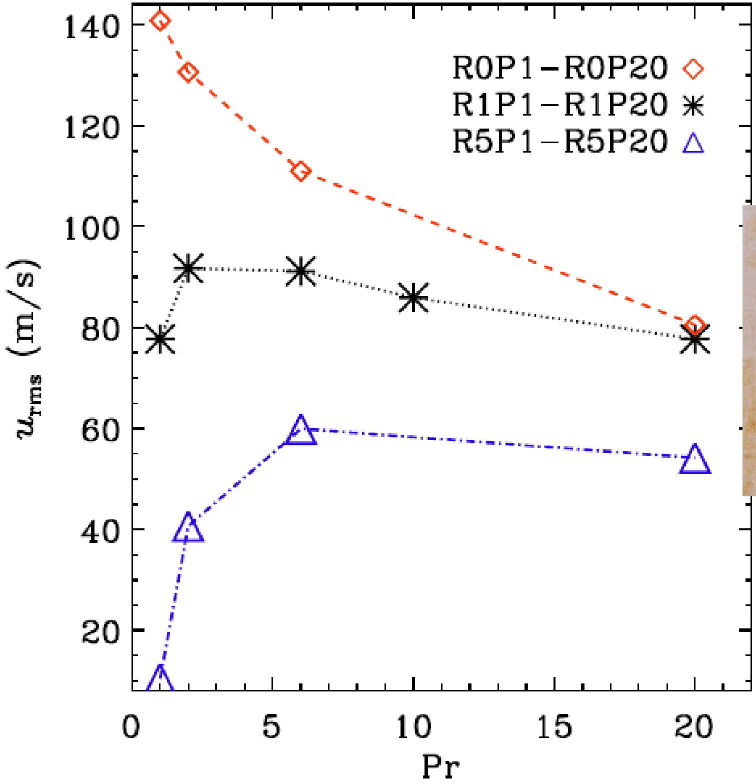
Convective velocities too high: enhance radiation to reduce (Maradona effect!).

Works, but no physical reason whatsoever to do this.

More usual suspects: too low resolution, missing surface layers, unrealistic Prandtl number.



Results (spherical)



Karak et al. (2018), *Phys. Fluids*, **30**, 046602

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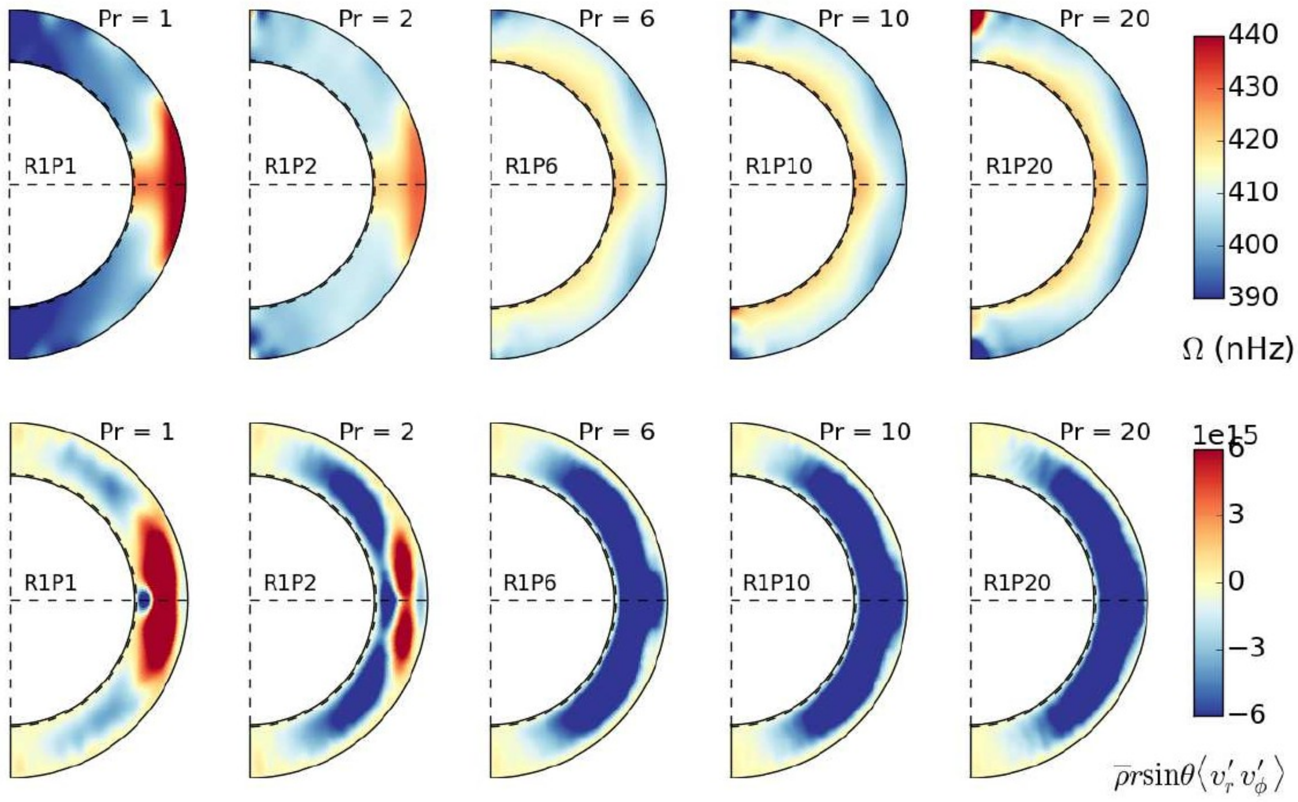
Results (spherical)



Karak et al. (2018), *Phys. Fluids*, **30**, 046602

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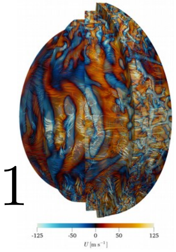
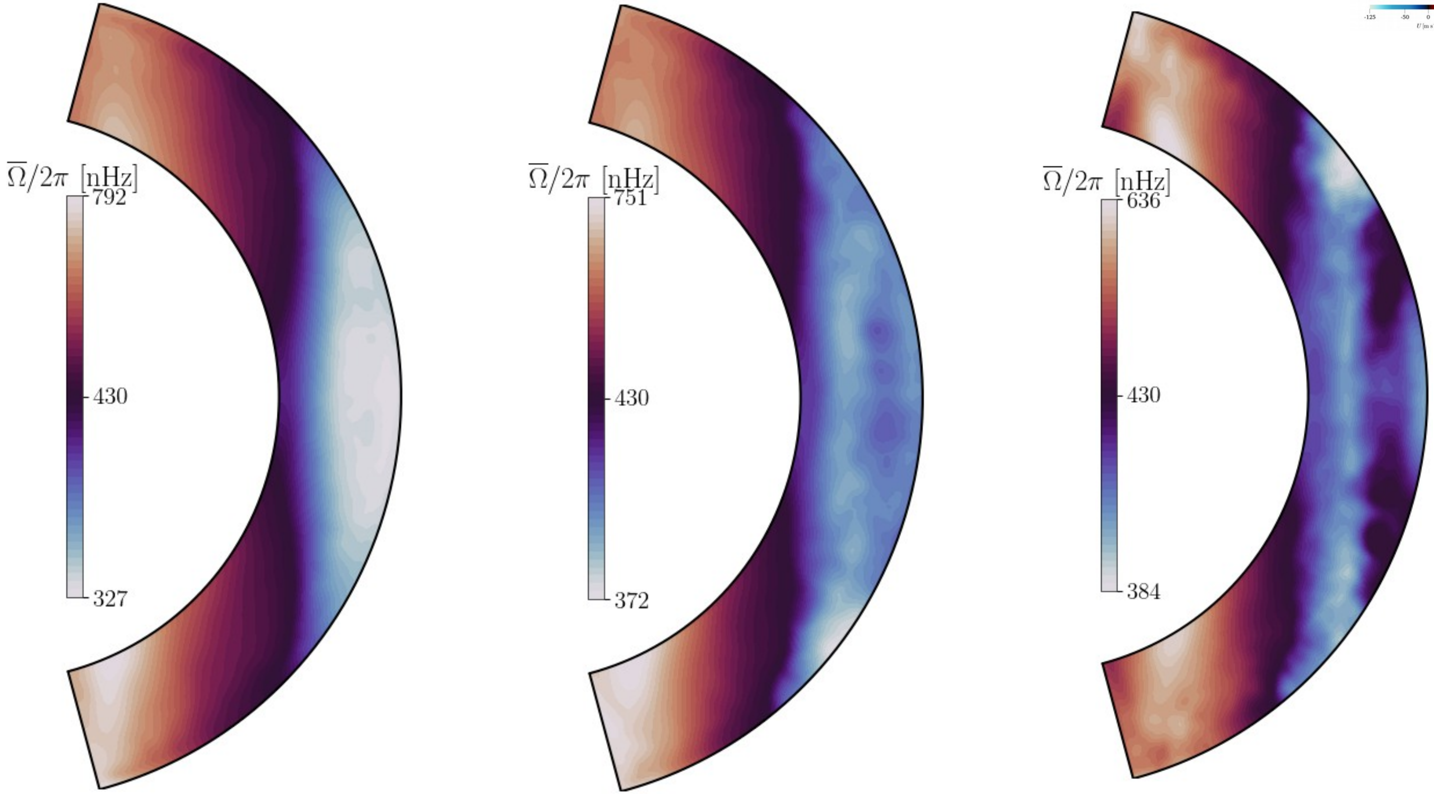
Results (spherical)



$\mathrm{Pr}_{\mathrm{SGS}} = 0.5$ $\qquad$ $\mathrm{Pr}_{\mathrm{SGS}} = 0.2$ $\qquad$ $\mathrm{Pr}_{\mathrm{SGS}} = 0.1$



(Work very much in progress)

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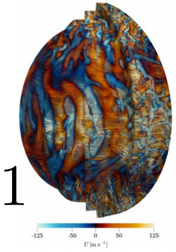
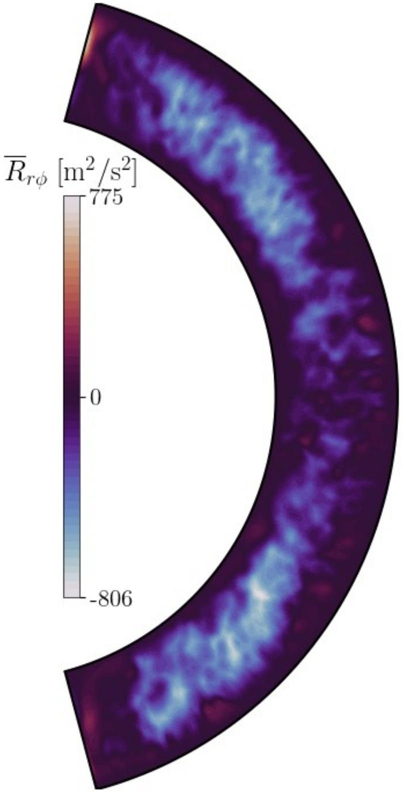
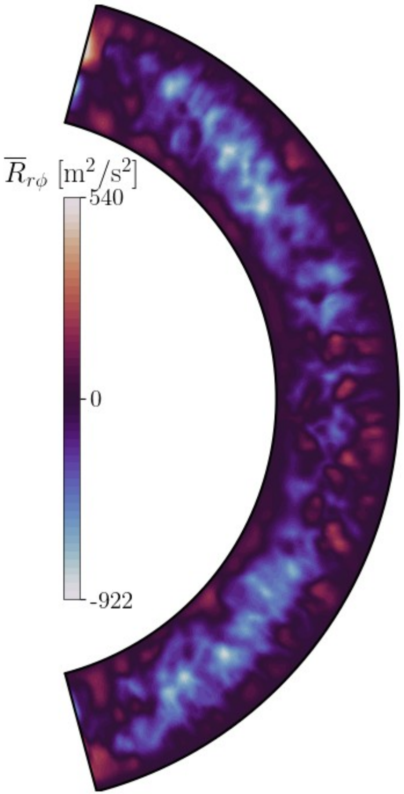
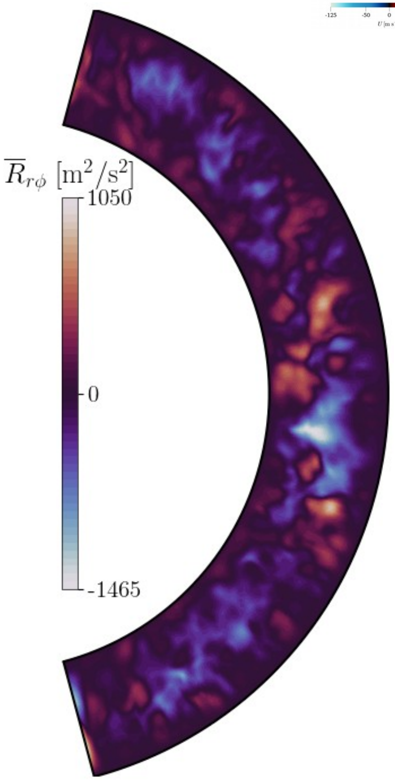
Results (spherical)



$\mathrm{Pr}_{\mathrm{SGS}} = 0.5$	$\mathrm{Pr}_{\mathrm{SGS}} = 0.2$	$\mathrm{Pr}_{\mathrm{SGS}} = 0.1$







(Work very much in progress)

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