In Search of Missing Stellar Physics

(using RANS analysis of 3D Fully-Compressible Hydrodynamic Simulations – no rotation, no magnetic fields)

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Motivation

- Limitations of the current 1D modelling of turbulence in stars
- Closures for approximated or neglected physics
  - what do we actually neglect?

RANS analysis

- Theory: Reynolds and Favrian decomposition

$$\begin{align*}
A(r, \theta, \phi) &= \overline{A}(r) + A'(r, \theta, \phi) \\
\overline{A}(r) &= \frac{1}{\Delta T \Delta \Omega} \int_{\Delta T} \int_{\Delta \Omega} A(r, \theta, \phi) \, dt \, d\Omega \\
F(r, \theta, \phi) &= \overline{F}(r) + F''(r, \theta, \phi) \\
\overline{u}_r &= \overline{u}_r \\
\overline{u}_r &= \overline{u}_r \\
\overline{w}_r &= \overline{u}_r \\
\text{mean velocity} &= \text{expansion velocity} \\
\partial_t M / 4\pi r^2 \rho &= \text{turbulent mass flux} - \rho' u_r' / \rho
\end{align*}$$

- https://github.com/mmicromegas/ransX/tree/master/DOCS

Challenging Observations

- Properties of supernova explosions studied by HST or Keck cannot be linked to their progenitors conclusively [Smartt, 2009]. Such progenitors are known to have a structure interleaved by turbulent convection shells [Hirschi et al., 2004].

- adiabatic convection in HSE

$$\begin{align*}
\partial_t M &= + 4\pi r^2 \rho \\
\partial_t \overline{p} &= - \overline{p} \overline{g}_r \\
\partial_t \overline{L} &= - 4\pi r^2 \overline{u}_r \overline{g}_r / \Gamma_1 + \overline{c}_t \partial_r 4\pi r^2 \overline{p} \overline{u}_r \\
\partial_t \overline{T} &= - (\Gamma_3 - 1) \overline{p} \overline{T} \overline{g}_r / \Gamma_1 \overline{P} \\
\partial_t \overline{\chi}_i &= \overline{\chi}_i^{nuc} - (1/\overline{p}) \nabla_r f_i - \overline{u}_r \partial_r \overline{\chi}_i
\end{align*}$$

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The VLT is observing massive stars with unexplained chemical peculiarities, where rotational mixing was considered to be enough to explain observations [Evans et al., 2008].

- The Kepler spacecraft has found unexplained pulsations of δ Scuti and γ Doradus stars [Uytterhoeven, 2011], which depend heavily on properties of sub-surface stellar convection [Guzik et al., 2006].
ransX framework (no rotation, no magnetic f.)

- [https://github.com/mmicromegas/ransX](https://github.com/mmicromegas/ransX)
- Transport/Flux/Variance equations for evolution of density, momenta, kinetic/internal/total energy, temperature, enthalpy, pressure
- Transport/Flux/Variance equations for evolution of chemical composition
- Hydrodynamic stellar structure equations (3 versions)
  - general
  - simplified (based on flux evolution equations)
  - simplified (for adiabatic flow in HSE)
* all of them well validated with our oxygen-neon burning simulation
* for more details see [https://github.com/mmicromegas/ransX/tree/master/DOCS/RANDOM](https://github.com/mmicromegas/ransX/tree/master/DOCS/RANDOM)
Code comparison project (stable-unstable)

PROMPI code, fine-volume method, PPM, 256^3 run, Mach numbers around 0.01, Reynolds number 10^3, Rayleigh number 10^7
Continuity Equation

\[ \dot{\bar{\rho}} = - \nabla \bar{f}_\rho + (f_\rho / \bar{\rho}) \partial_t \bar{\rho} - \bar{\rho} \bar{d} \]

\[ \dot{D}_t \bar{\rho} = \bar{\rho} \bar{u}_t \] (turbulent mass flux)

\[ d = \nabla \cdot u \] dilatation

- time-dependent background state, compressibility effects (stars are gas, gas is compressible, solids are incompressible), non-local transport of density turbulent field are important parts of underlying physics of stellar turbulence.
Transport Equation for Chemical Elements

\[ \rho \frac{D}{Dt} \tilde{X}_\alpha = -\nabla f_\alpha + \tilde{\rho} \dot{X}_\alpha^{\text{exc}} + \mathcal{N}_\alpha \]

- model(1) is downgradient approximation for passive scalar flux of incompressible and homogeneous flow

\[ f_\alpha = \bar{\rho} \dot{X}_\alpha^{\text{exc}} \]  
\[ X_\alpha = \text{mass fraction of isotope } \alpha \]  
\[ \dot{X}_\alpha^{\text{exc}} = \text{rate of change of } X_\alpha \]

Rogers et al. [1989]
Heating Due to Kinetic Energy Dissipation

\[ \bar{p} \tilde{D}_k \tilde{k} = -\nabla_r (f_k + f_P) - \tilde{R}_{ij} \partial_r \tilde{u}_i + W_b + W_P + N_k \]

\[ \tilde{D}_k T = -\nabla_r f_T + \left( 1 - \Gamma_3 \right) T \tilde{d} + \left( 2 - \Gamma_3 \right) T' \tilde{d}' + \frac{(\nabla \cdot F_T)}{\rho c_v} + \frac{(\tau_{ij} \partial_i u_j)}{\rho c_v} + \frac{\epsilon_{\text{mix}}}{c_v} + N_T \]

**TKE equation** \( C_m = 0.5 \)

**temperature equation**

\[ \begin{align*}
\frac{-\partial T}{\partial t} &+ \frac{c_v}{\rho} \frac{\partial T}{\partial t} + \frac{\rho}{\rho} \frac{\partial T}{\partial t} = \left( 1 - \Gamma_3 \right) T' \tilde{d}' \\
&+ \left( 2 - \Gamma_3 \right) T' \tilde{d}' \\
&+ \rho T' \tilde{d}' \\
&+ \left( 1 - \Gamma_3 \right) T \tilde{d} \\
&+ \left( 2 - \Gamma_3 \right) T \tilde{d}'
\end{align*} \]

**unknown physics?**

\[ (2 - \Gamma_3) T' \tilde{d}' \]

\[ (1 - \Gamma_3) T \tilde{d} \]

**turbulent kinetic energy** \( k = (1/2) u'_i u'_i \)

**Reynolds stress tensor** \( \tilde{R}_{ij} = \bar{p} u'_i u'_j \)

**heat flux** \( f_T = -T' u'_i \)

**buoyancy** \( W_b = \bar{p} u'_i \tilde{g}_r \)

**turbulent pressure dilatation** \( W_P = \bar{P}' \tilde{d}' \)
Width of the mixing layer (convection zone)

Stellar evolution calculations are 1D and will remain so in near future. So, how can we get the width of convection zones in 1D stars right?

- overshooting? \( d_{ov} = \alpha_{ov} H_P \)
- turbulent entrainment? \( E = A R_i^n \)
- something else, can RANS help?

(Meakin et al., 2006)

It is essential, because stars do not retain its initial composition in atmosphere, but are often enriched by mixing from below.
Width of the mixing layer

\[
\frac{1}{2} \partial_t \overline{\theta^2} + \frac{1}{2} \partial_z \overline{\theta^2 u_z} + \overline{u_z \theta (\partial_z T - \gamma)} = \kappa \theta \partial_{jj} \theta, \\
\frac{1}{2} \partial_t \overline{u^2} + \partial_z [u^2 u_z + \overline{u_z p}] = -g \overline{\theta u_z} - \epsilon_v, \\
\partial_t \overline{\theta u_z} + \partial_z [\overline{\theta u_z^2} + \overline{\theta p}] = -g \overline{\theta^2} + \beta(z) \overline{u_z^2} - \epsilon_{\theta, u_z},
\]

Second-order moment closures (diffusion-like)

\[
\overline{\theta^2 u_z} = a_\theta L |\dot{L}| \partial_{zz} \overline{\theta^2}, \\
(u^2 + p) u_z = a_{u_z} L |\dot{L}| \partial_{zz} \overline{u_z^2}, \\
\overline{\theta (u_z^2 + p)} = a_{\theta u_z} L |\dot{L}| \partial_{zz} \overline{\theta u_z}, \\
\epsilon_\theta = b_\theta \overline{\theta^2} / \tau(z, t), \\
\epsilon_v = b_{u_z} \overline{u_z^2} / \tau(z, t), \\
\epsilon_{\theta, u_z} = b_{\theta u_z} \overline{\theta u_z} / \tau(z, t),
\]
In the future, it can get worse.

- we have good spectra for only 0.001% of all stars in galaxy

  >> there is a good chance, that in time the number of observable peculiarities will increase significantly

  >> more models for unknown physics

  >> and, of coarse, stars rotate and have magnetic fields

Karin Lindt, yesterday