In Search of Missing Stellar Physics

(using RANS analysis of 3D Fully-Compressible Hydrodynamic Simulations – no rotation, no magnetic fields)

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Challenging Observations

- Properties of supernova explosions studied by HST or Keck can not be linked to their progenitors conclusively [Smartt, 2009]. Such progenitors are known to have a structure interleaved by turbulent convection shells [Hirschi et al., 2004].

Motivation

- Limitations of the current 1D modelling of turbulence in stars
- Closures for approximated or neglected physics
  - what do we actually neglect?

- adiabatic convection in HSE

$$\partial_r M = + 4\pi r^2 \rho$$
$$\partial_r \overline{P} = -\overline{\rho}\, \overline{g}_r$$
$$\partial_r \widetilde{L} = -4\pi r^2 \widetilde{u}_r \overline{\rho}\, \overline{g}_r / \Gamma_1 + \widetilde{\epsilon}_t \partial_r 4\pi r^2 \overline{\rho} \widetilde{u}_r$$
$$\partial_r \overline{T} = -(\Gamma_3 - 1)\, \overline{\rho}\, \overline{T}\, \overline{g}_r / \Gamma_1 \overline{P}$$
$$\partial_t \widetilde{X}_i = \dot{\widetilde{X}}_i^{nuc} - (1/\overline{\rho}) \nabla_r f_i - \widetilde{u}_r \partial_r \widetilde{X}_i$$

Explanation of observed element abundances in AGB stars requires physically motivated but still inconclusive tuning for mixing between turbulent envelope convection and underlying hydrogen-free core [Herwig, 2005].

RANS analysis

- Theory: Reynolds and Favrian decomposition

$$A(r,\theta,\phi) = \overline{A}(r) + A'(r,\theta,\phi) \qquad \overline{A(r)} = \frac{1}{\Delta T \Delta\Omega} \int_{\Delta T} \int_{\Delta\Omega} A(r,\theta,\phi)\, dt\, d\Omega$$

$$F(r,\theta,\phi) = \widetilde{F}(r) + F''(r,\theta,\phi) \qquad \widetilde{F} = \overline{\rho F} / \overline{\rho}$$

$$\underbrace{\overline{u}_r}_{\text{mean velocity}} = \underbrace{\widetilde{u}_r}_{\text{expansion velocity } \partial_t M / 4\pi r^2 \overline{\rho}} - \underbrace{\overline{u''}_r}_{\text{turbulent mass flux } -\overline{\rho' u'_r}/\overline{\rho}}$$

- https://github.com/mmicromegas/ransX/tree/master/DOCS

The VLT is observing massive stars with unexplained chemical peculiarities, where rotational mixing was considered to be enough to explain observations [Evans et al., 2008].

- The Kepler spacecraft has found unexplained pulsations of δ Scuti and γ Doradus stars [Uytterhoeven, 2011], which depend heavily on properties of sub-surface stellar convection [Guzik et al., 2000].

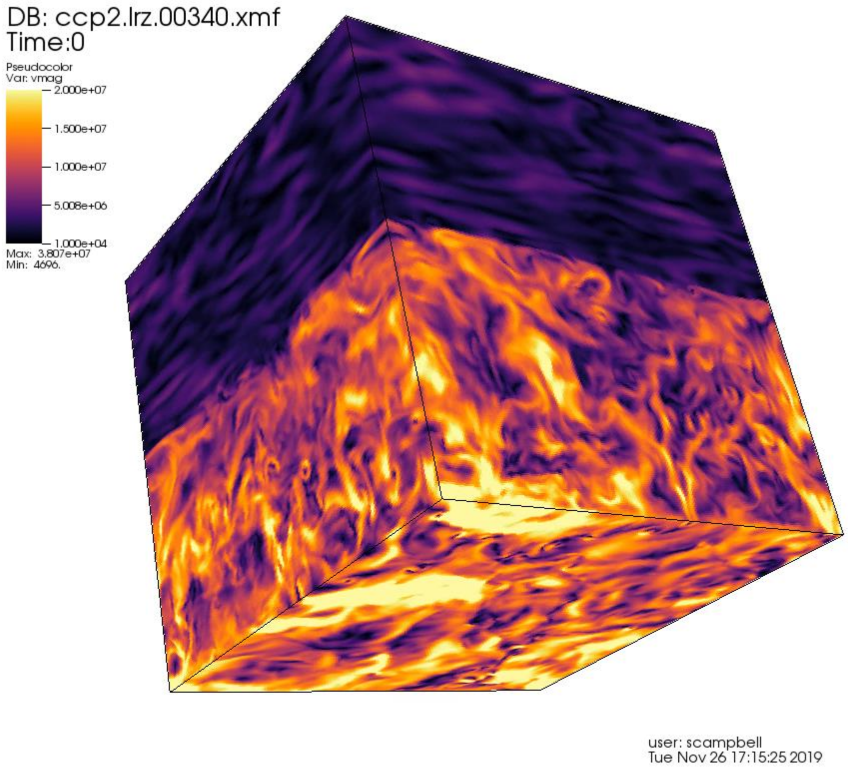
ransX framework (no rotation, no magnetic f.)

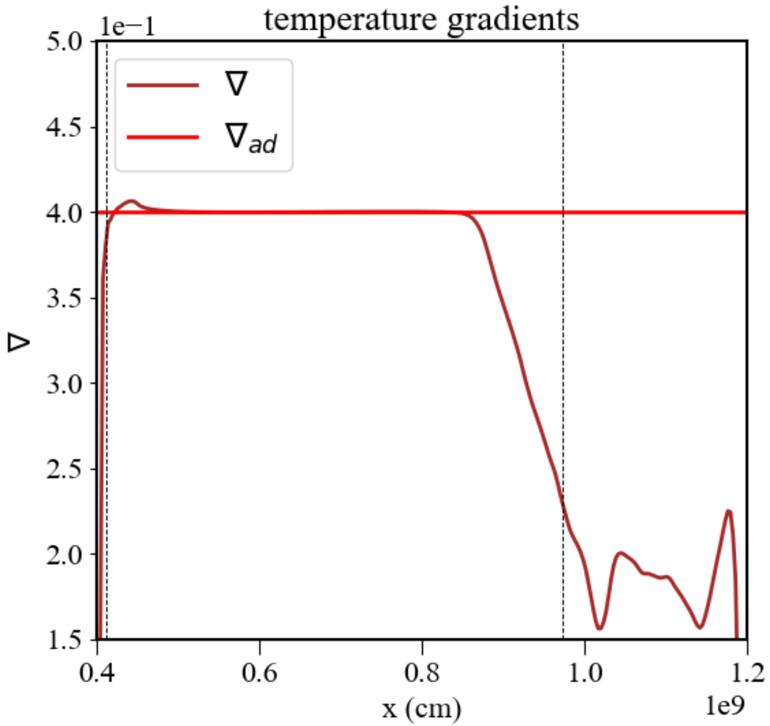
- https://github.com/mmicromegas/ransX
- Transport/Flux/Variance equations for evolution of density, momenta, kinetic/internal/total energy, temperature, enthalpy, pressure
- Transport/Flux/Variance equations for evolution of chemical composition
- Hydrodynamic stellar structure equations (3 versions)

  - general
  - simplified (based on flux evolution equations)
  - simplified (for adiabatic flow in HSE)

  * all of them well validated with our oxygen-neon burning simulation
  * for more details see https://github.com/mmicromegas/ransX/tree/master/DOCS/RANDOM

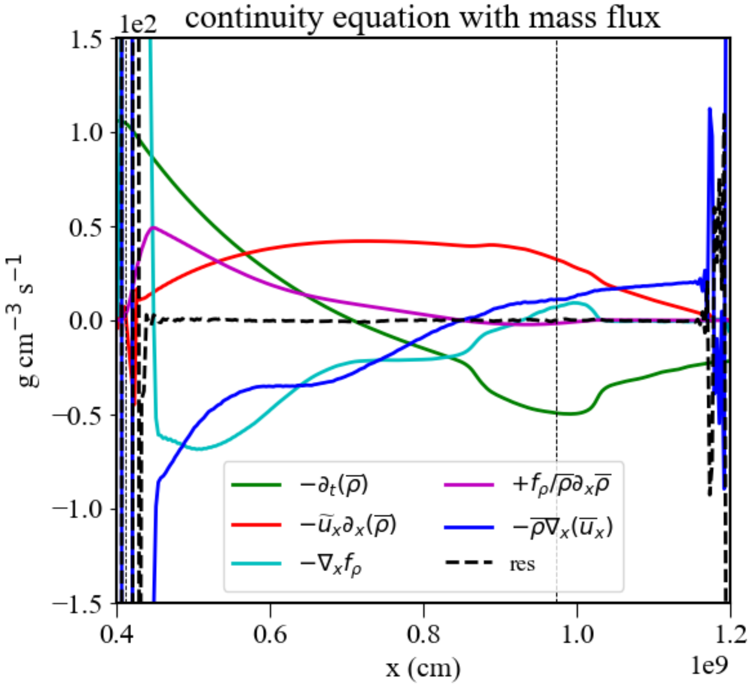
Code comparison project (stable-unstable)

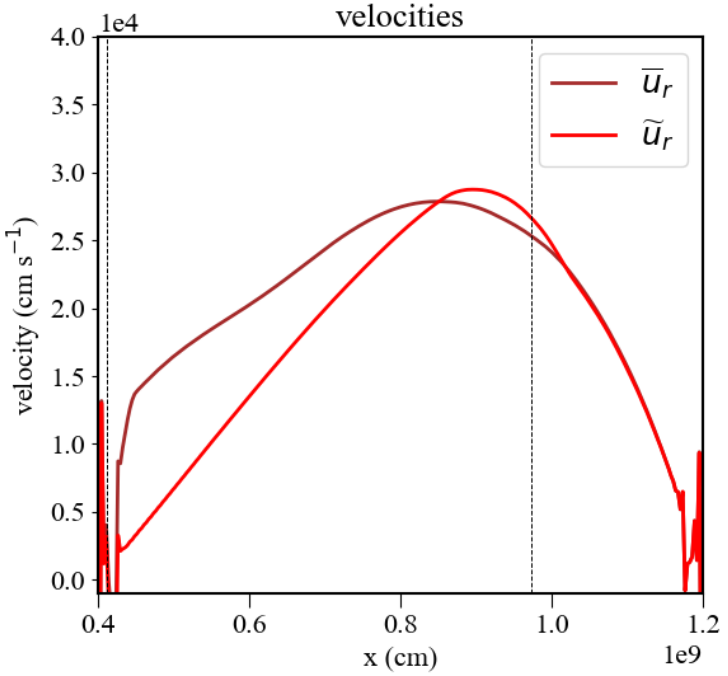


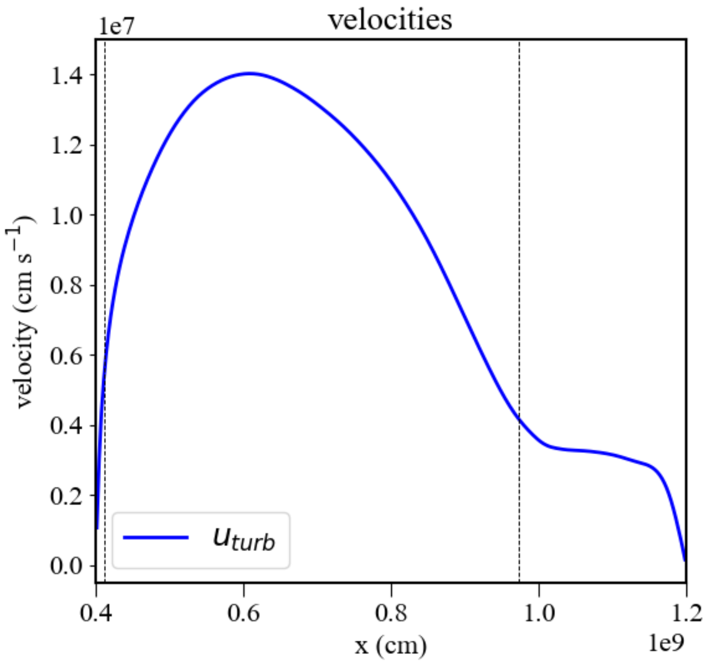


PROMPI code, fine-volume method, PPM, 256^3 run, Mach numbers around 0.01, Reynolds number 10^3, Rayleigh number 10^7

Continuity Equation





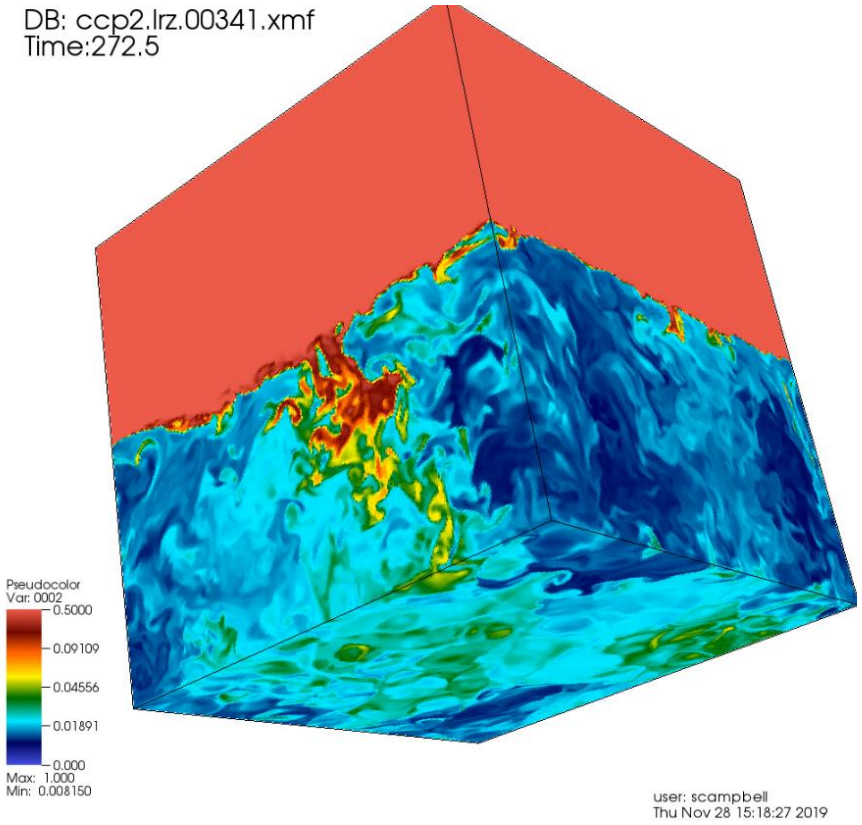


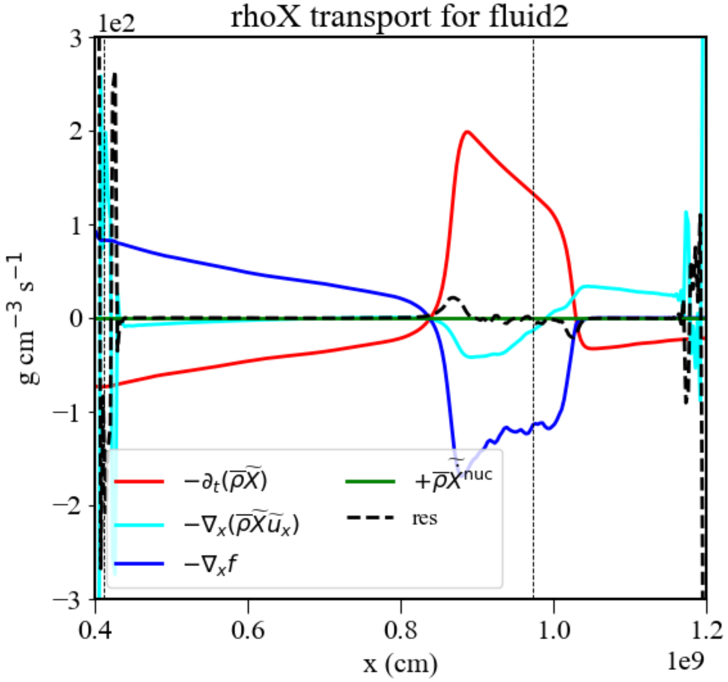
$\widetilde{D}_t\overline{\rho} = -\nabla_r f_\rho + (f_\rho/\overline{\rho})\partial_r\overline{\rho} - \overline{\rho}\overline{d}$ $\quad$ $\widetilde{D}_t(.) = \partial_t(.) + \widetilde{u}_n\partial_n(.)$ $\quad$ $f_\rho = \overline{\rho' u_r'}$ (turbulent mass flux) $\quad$ $d = \nabla \cdot \mathbf{u}$ dilatation

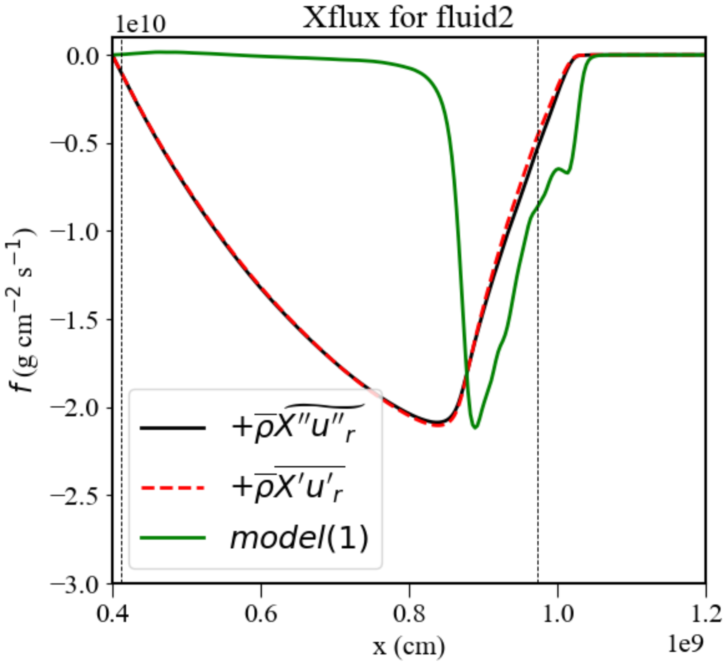
- time-dependent background state, compressibility effects (stars are gas, gas is compressible, solids are incompressible), non-local transport of density turbulent field are important parts of underlying physics of stellar turbulence.

Transport Equation for Chemical Elements

$$\overline{\rho}\widetilde{D}_t\widetilde{X}_\alpha = -\nabla_r f_\alpha + \overline{\rho}\widetilde{\dot{X}}_\alpha^{\rm nuc} + \mathcal{N}_\alpha$$







$$\overline{X'_\alpha u'_r} = -D_{rr}\,\partial_r\overline{X}_\alpha = -\left[(\tau/C_D)\,\overline{u'_r u'_r} - S_{r\theta}\,\tau\,(\tau/C_D^2)\,\overline{u'_r u'_\theta}\right]\partial_r\overline{X}_\alpha \qquad \text{model (1)}$$

Rogers et al. [1989]

$f_\alpha = \overline{\rho}\widetilde{X''_\alpha u''_r}$ X_α flux

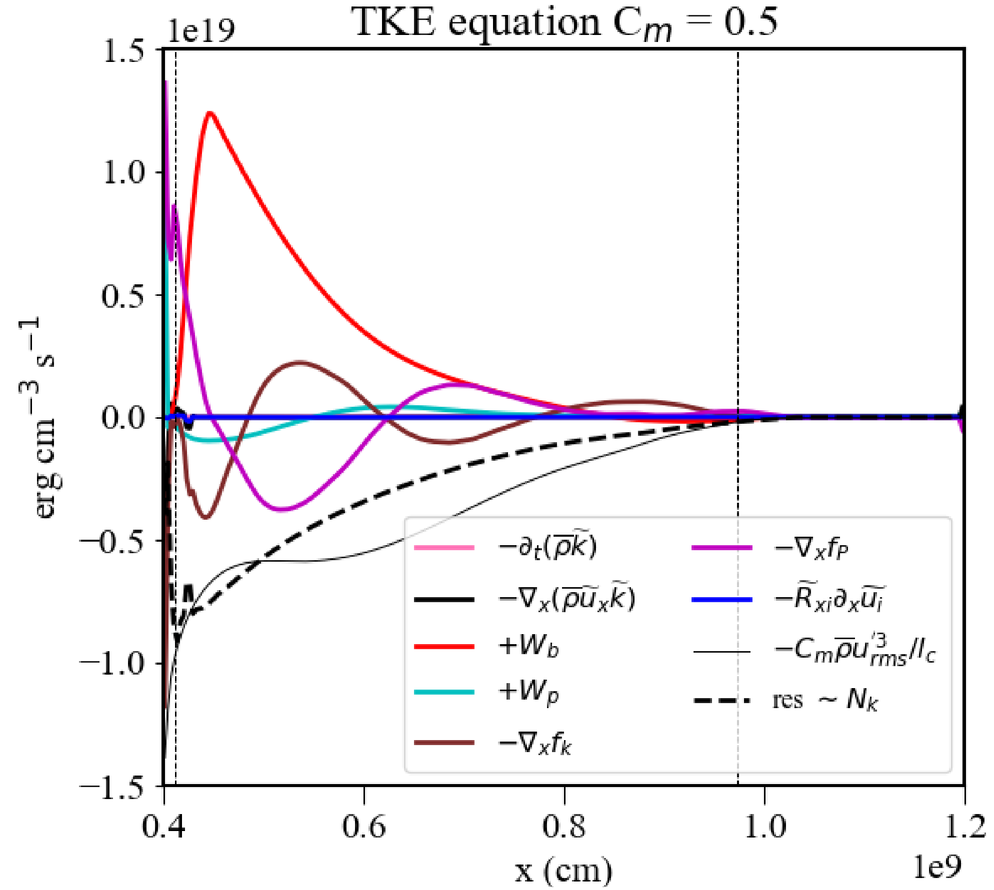
X_α mass fraction of isotope α

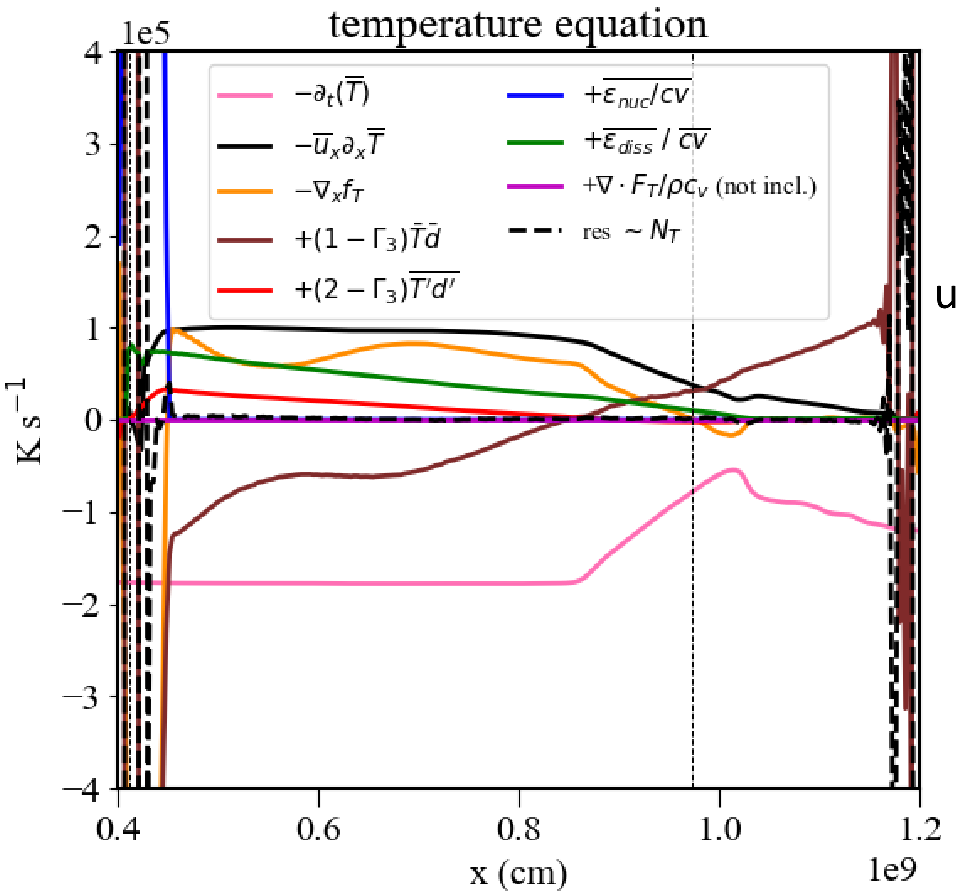
$\dot{X}_\alpha^{\rm nuc}$ rate of change of X_α

- model(1) is downgradient approximation for passive scalar flux of incompressible and homogeneous flow

Heating Due to Kinetic Energy Dissipation

$$\overline{\rho}\widetilde{D}_t\widetilde{k} = -\nabla_r(f_k + f_P) - \widetilde{R}_{ir}\partial_r\widetilde{u}_i + W_b + W_P + \mathcal{N}_k \qquad \overline{D}_t\overline{T} = -\nabla_r f_T + (1-\Gamma_3)\overline{T}\,\overline{d} + (2-\Gamma_3)\overline{T'd'} + \overline{(\nabla\cdot F_T)/\rho c_v} + \overline{(\tau_{ij}\partial_i u_j)/\rho c_v} + \overline{\epsilon_{\rm nuc}/c_v} + \mathcal{N}_T$$





unknown physics?

$(2-\Gamma_3)\overline{T'd'}$

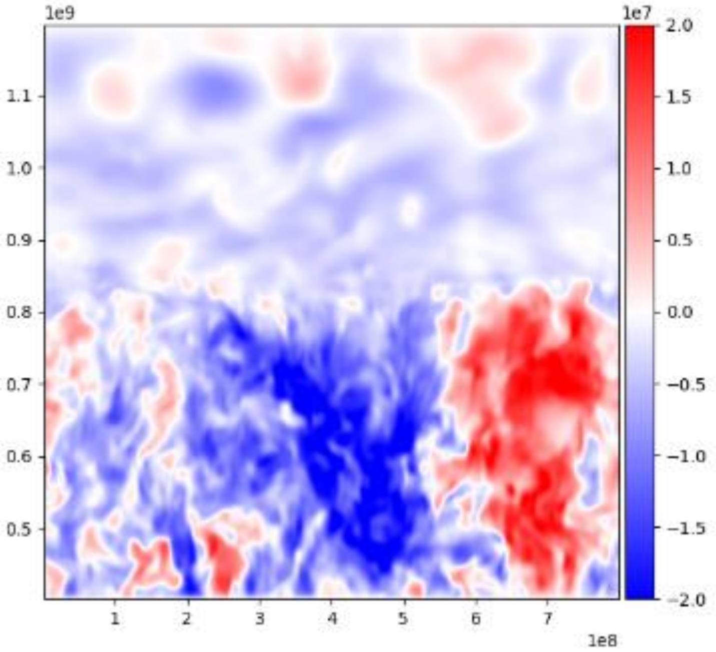
$(1-\Gamma_3)\overline{T}\,\overline{d}$

$k = (1/2)\widetilde{u''_i u''_i}$ turbulent kinetic energy $\quad \widetilde{R}_{ij} = \overline{\rho}\widetilde{u''_i u''_j}$ Reynolds stress tensor $\quad f_T = -\overline{T'u'_r}$ heat flux

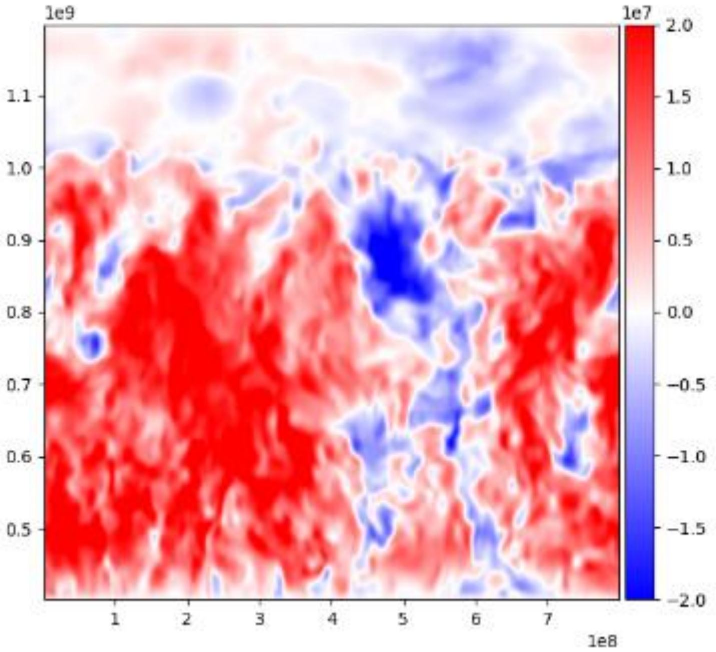
$f_k = (1/2)\overline{\rho}\widetilde{u''_i u''_i u''_r}$ turbulent kinetic energy flux $\quad W_b = \overline{\rho}\widetilde{u''_r}\widetilde{g}_r$ buoyancy $\quad W_p = \overline{P'd''}$ turbulent pressure dilatation

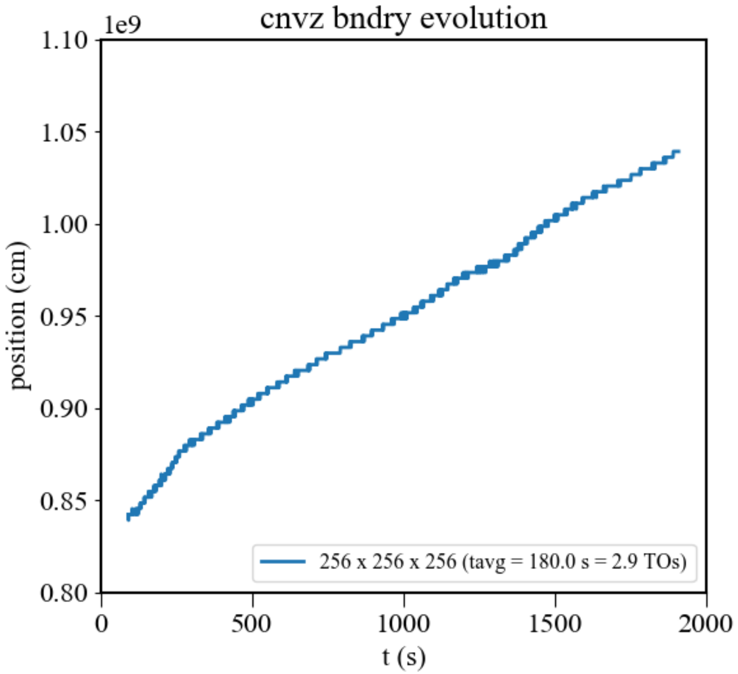
Width of the mixing layer (convection zone)

t ~ 200 s



t ~ 1500 s





Stellar evolution calculations are 1D and will remain so in near future. So, how can we get the width of convection zones in 1D stars right?

- overshooting? $d_{\mathrm{ov}} = \alpha_{\mathrm{ov}} H_p,$ - turbulent entrainment? $E = A\mathrm{Ri}_B^{-n}.$ - something else, can RANS help?

(Meakin et al., 2006)

It is essential, because stars do not retain its initial composition in atmosphere, but are often enriched by mixing from below

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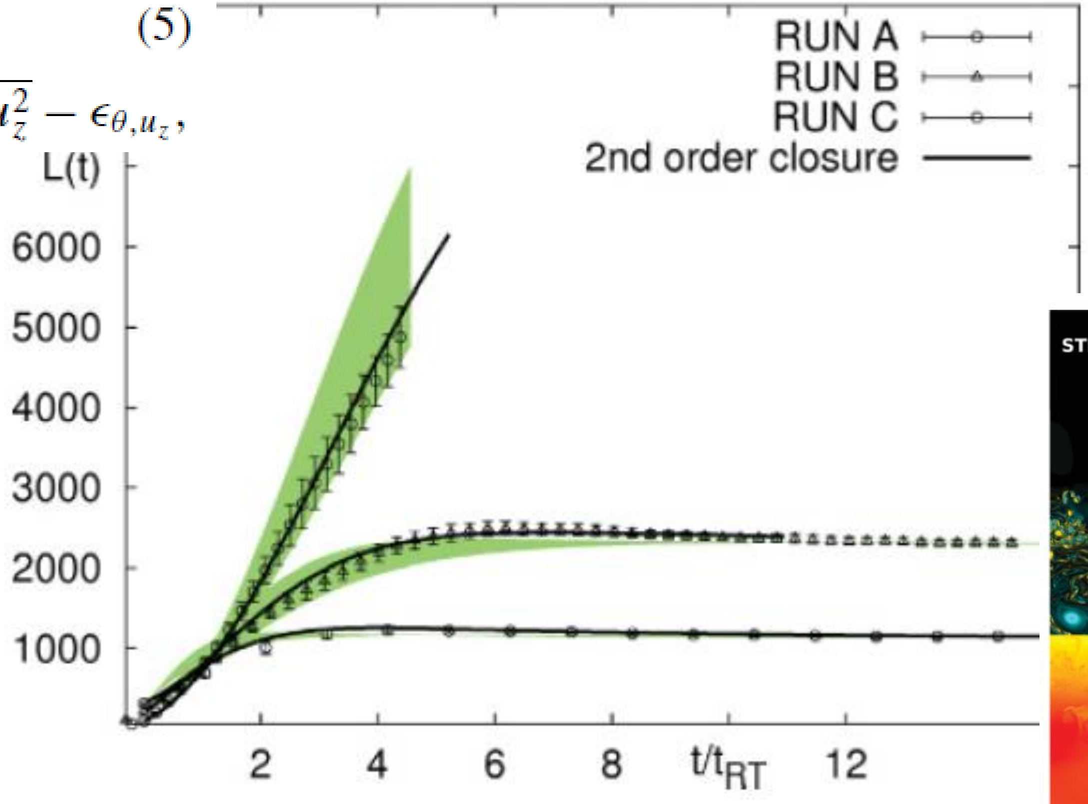
Second-order closure in stratified turbulence: Simulations and modeling of bulk and entrainment regions

L. Biferale,[1] F. Mantovani,[2] M. Sbragaglia,[3] A. Scagliarini,[3] F. Toschi,[4] and R. Tripiccione[5]

Width of the mixing layer

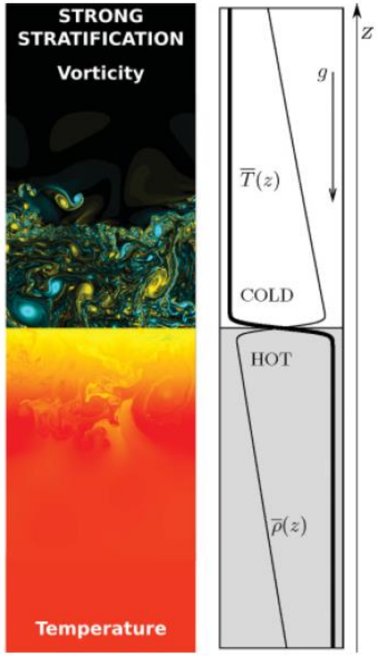
$$\tfrac{1}{2}\partial_t\overline{\theta^2} + \tfrac{1}{2}\partial_z\overline{\theta^2 u_z} + \overline{u_z\theta}(\partial_z\bar{T} - \gamma) = \kappa\overline{\theta\partial_{jj}\theta},$$
$$\tfrac{1}{2}\partial_t\overline{u^2} + \partial_z[\overline{u^2 u_z} + \overline{u_z p}] = -g\overline{\theta u_z} - \epsilon_\nu, \qquad (5)$$
$$\partial_t\overline{\theta u_z} + \partial_z[\overline{\theta u_z^2} + \overline{\theta p}] = -g\overline{\theta^2} + \beta(z)\overline{u_z^2} - \epsilon_{\theta,u_z},$$

$$\theta = T - \overline{T}(z), \qquad \partial_t\overline{T} + \partial_z\overline{u_z\theta} = \kappa\partial_{zz}\overline{T},$$

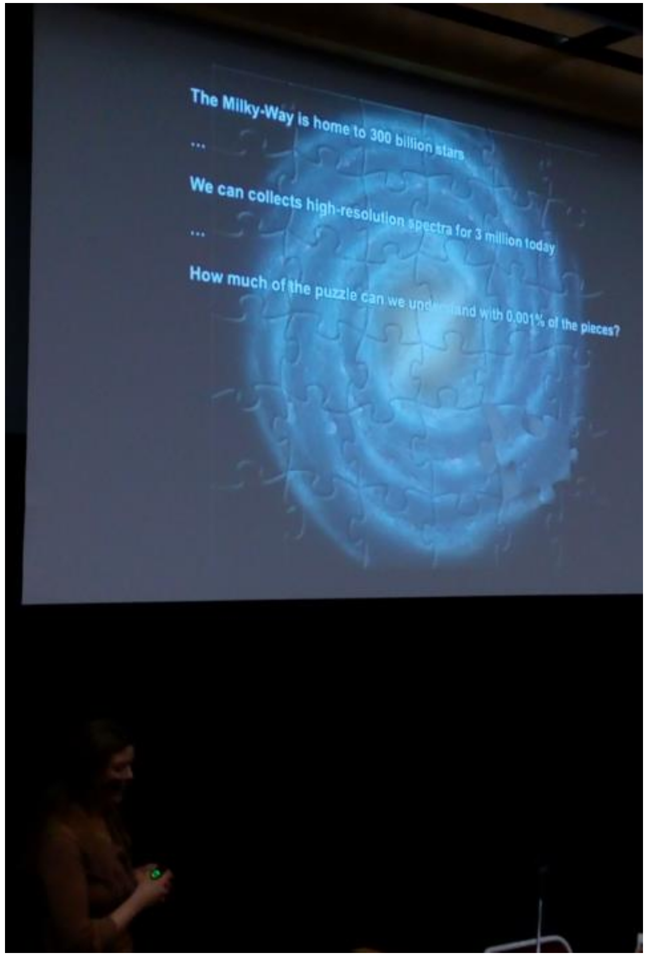


Second-order moment closures (diffusion-like)

$$\overline{\theta^2 u_z} = a_\theta L|\dot{L}|\partial_{zz}\overline{\theta^2},$$
$$\overline{(u^2+p)u_z} = a_{u_z} L|\dot{L}|\partial_{zz}\overline{u_z^2},$$
$$\overline{\theta(u_z^2+p)} = a_{\theta u_z} L|\dot{L}|\partial_{zz}\overline{\theta u_z},$$
$$\epsilon_\theta = b_\theta\overline{\theta^2}/\tau(z,t),$$
$$\epsilon_\nu = b_{u_z}\overline{u_z^2}/\tau(z,t),$$
$$\epsilon_{\theta,u_z} = b_{\theta u_z}\overline{\theta u_z}/\tau(z,t),$$



In the future, it can get worse.



Karin Lindt, yesterday

- we have good spectra for only 0.001% of all stars in galaxy

>> there is a good chance, that in time the number of observable peculiarities will increase significantly

>> more models for unknown physics

>> and, of coarse, stars rotate and have magnetic fields