

Collider phenomenology of the unparticle physics

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Plan

- Introduction
- Processes with real unparticle emissions
- Processes with virtual unparticle exchange
- Summary
- H. Georgi, PRL98, 221601 (2007); PLB650, 275 (2007), K. Cheung et al., PRL99, 051803 (2007); PRD76,055003 (2007), D. Choudhury, PLB658, 148 (2008); arXiv:0707.2074[hep-ph], accepted for publication in IJMPA, K. Huitu et al. PRD77,035015 (2008).

Scale Invariant Sector

- Scale invariance forbids mass terms in the theory.
- A free massless particle is a simple example of scale invariant theory.
- There could be an exactly scale invariant sector at a high scale above TeV that could be probed at the LHC or ILC.
- Such a sector might be strongly interacting and highly non-trivial, but can weakly interact with the rest of the SM.
- One expects that such a sector decouples effectively from the low energy and can use the technique of effective field theory to describe its low energy effects.

Scale Invariant Sector

- Imagine that at a very high energy scale there exists an exact scale-invariant sector with non-trivial IR fixed point. Let us call the fields of the scale-invariant sector as BZ fields.
- The interactions between the SM sector and the BZ sector is introduced just in an effective field theory way, by exchanging heavy messenger particles (of mass scale M_U) leading to non-renormalizable interaction below the scale M_U

$$\mathcal{L}_{eff} = \frac{1}{M_U^{d_{BZ} + d_{SM} - 4}} \mathcal{O}_{BZ} \mathcal{O}_{SM}$$

- As in massless non-Abelian gauge theories, renormalization effects the scale invariant BZ sector induce dimensional transmutation at an energy scale Λ_U .

Scale Invariant Sector

- In the effective theory below the scale Λ_U , the BZ operators must match onto new (unparticle) operators which have the following form of the interaction:

$$\mathcal{L}_{eff} = c_U \frac{\Lambda_U^{d_{BZ} - d_U}}{M_U^{d_{BZ} + d_{SM} - 4}} \mathcal{O}_{SM} \mathcal{O}_U$$

- d_U is the scale dimension of the unparticle operator \mathcal{O}_U and c_U is a coefficient fixed by the matching.
- Three unparticle operators with different Lorentz structure were addressed by Georgi : $\mathcal{O}_U, \mathcal{O}_U^\mu, \mathcal{O}_U^{\mu\nu}$ which correspond to scalar, vector and tensor operators respectively.
- We assume that these operators are SM singlet.

Scale Invariant Sector

- If one assumes the scale invariant BZ theory at Λ_U to be strongly coupled, the BZ particles will be confined into a composite states.
- Therefor a natural interpretation of the unparticles is that they are composite states made of confined BZ particles.
- The strongly interacting sector is scale invariant \implies no distinct mass or length scale associated with these states.
- The unparticles will correspond normal particles with states of continuous mass.
- This continuous mass states leads to a very unconventional phenomenology.

Phase space for real emission of unparticle

- Consider a two-point function for a scalar unparticle operator \mathcal{O}_U :

$$\begin{aligned}\langle 0|\mathcal{O}_U(x)\mathcal{O}_U^\dagger(0)|0\rangle &= \langle 0|e^{i\hat{P}\cdot x}\mathcal{O}_U(0)e^{-i\hat{P}\cdot x}\mathcal{O}_U^\dagger(0)|0\rangle \\ &= \int d\beta \int d\beta' \langle 0|\mathcal{O}_U(0)|\beta'\rangle \langle \beta'|e^{-i\hat{P}\cdot x}|\beta\rangle \langle \beta|\mathcal{O}_U^\dagger(0)|0\rangle \\ &= \int \frac{d^4P}{(2\pi)^4} e^{-iP\cdot x} \rho_U(P^2)\end{aligned}$$

- $\rho_U(P^2)$ is the spectral density and is formally given by :

$$\rho_U(P^2) = (2\pi)^4 \int d\beta \delta^4(P - p_\beta) |\langle 0|\mathcal{O}_U(0)|\lambda\rangle|^2.$$

- The function $\rho_U(P^2)$ is real valued, positive definite, and Lorentz invariant. It depends on the scalar variable P^2 .

Phase space for real emission of unparticle

- Scale invariance \implies :

$$\rho_{\mathcal{U}}(P^2) = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2) (P^2)^{d_{\mathcal{U}}-2}$$

- This is similar to the phase space for n massless particles:

$$(2\pi)^4 \delta^4 \left(P - \sum_{j=1}^n p_j \right) \prod_{j=1}^n \delta(p_j^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^3} = A_n \theta(P^0) \theta(P^2) (P^2)^{n-2}$$

- where $A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}} \frac{\Gamma(n+1/2)}{\Gamma(n-1)\Gamma(2n)}$
- Identify $n \Rightarrow d_{\mathcal{U}}$. Unparticles of scale dimension $d_{\mathcal{U}}$ behaves like a collection of $d_{\mathcal{U}}$ of massless particles.
- $d_{\mathcal{U}} = 1 \Rightarrow$ unparticle = one massless particle which couples weakly to the SM particles.

Scalar unparticle propagator

- Let us first calculate the scalar unparticle propagator :
- From Wightman 2-point correlation function we have after imposing scale invariance:

$$\int d^4x e^{iPx} \langle 0 | \mathcal{T} [\mathcal{O}_{\mathcal{U}}(x) \mathcal{O}_{\mathcal{U}}^\dagger(0)] | 0 \rangle = \frac{A_{d_{\mathcal{U}}}}{(2\pi)} \int_0^\infty \frac{dM^2 (M^2)^{d_{\mathcal{U}}-2} i}{(P^2 - M^2 + i\epsilon)}$$

- Integration of the R.H.S. gives

$$\frac{A_{d_{\mathcal{U}}}}{2} \frac{i(-P^2 - i\epsilon)^{d_{\mathcal{U}}-2}}{\sin(\pi d_{\mathcal{U}})}, \quad A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}} \frac{\Gamma(n + 1/2)}{\Gamma(n - 1) \Gamma(2n)}$$

- $(-P^2 - i\epsilon)^{d_{\mathcal{U}}-2} = |P^2|^{d_{\mathcal{U}}-2} \exp(-id_{\mathcal{U}}\pi)$ for $P^2 > 0 \implies$ Complex phase
- $(-P^2 - i\epsilon)^{d_{\mathcal{U}}-2} = |P^2|^{d_{\mathcal{U}}-2}$ for $P^2 < 0 \implies$ No Complex phase

Vector unparticle propagator

- spin-1 transverse ($\partial_\mu \mathcal{O}_U^\mu = 0$) unparticle propagator :
- In this case the 2-point correlation function is :

$$\begin{aligned}
 \Pi^{\mu\nu}(q) &= \int d^4x e^{iqx} \langle 0 | \mathcal{O}_U^\mu(x) \mathcal{O}_U^{\nu\dagger}(0) | 0 \rangle \\
 &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) \frac{A_{d_U}}{(2\pi)} \int_0^\infty \frac{dM^2 (M^2)^{d_U-2} i}{(q^2 - M^2 + i\epsilon)} \\
 &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) i\Delta_U
 \end{aligned}$$

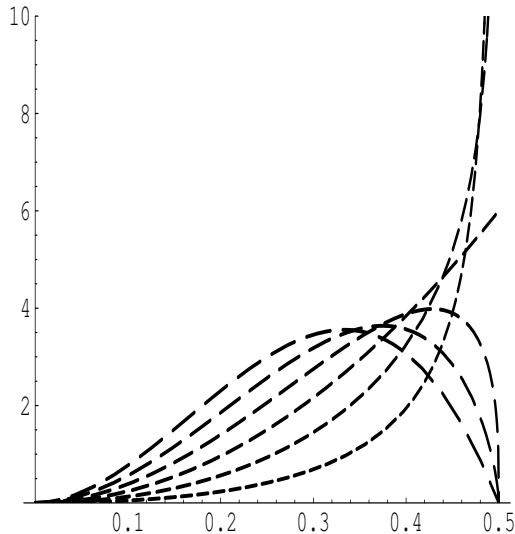
- Where, $\Delta_U = \frac{A_{d_U}}{2 \sin(\pi d_U)} (-q^2 - i\epsilon)^{d_U-2}$.
- Propagator has a singularity at integral values of $d_U = 2, 3, \dots$, these integer values of d_U describes multi particle cuts.

- Compare with Z propagator : $\pi_{\mu\nu}^Z(q) = \frac{i \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right)}{q^2 - M_Z^2 + iM_Z \Gamma_Z}$

Real Unparticle emission

$t \rightarrow u + \mathcal{U}$ [Georgi, PRL 98, 221601 (2007)]

- Unparticle stuff with scale dimension $d_{\mathcal{U}}$ looks like a non-integral number $d_{\mathcal{U}}$ of invisible particles.
- Consider $t \rightarrow u + \mathcal{U}$ through scalar Unparticle operator: $i \frac{\lambda}{\Lambda_{\mathcal{U}}^d} \bar{u} \gamma_{\mu} (1 - \gamma_5) t \partial^{\mu} \mathcal{O}_{\mathcal{U}} + h.c.$



- The energy spectrum of the u is given by :

$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_u} = 4d_{\mathcal{U}}(d_{\mathcal{U}}^2 - 1)(1 - 2E_u/m_t)^{d_{\mathcal{U}}-2} E_u^2/m_t^2$$

- Figure shows $d \ln \Gamma / d E_u$ versus E_u in units of m_t with $d_{\mathcal{U}} = j/3$ for $j = 4 - 9$. The dashes get longer as j increases.
- As $d_{\mathcal{U}} \rightarrow 1^+$, $d \ln(\Gamma) / d E_u$ becomes more peaked at $E_u = m_t/2$ matching smoothly unto the kinematics of a 2-particle decay in the limit.

Single γ production at e^+e^- Machine [K. Cheung etal. PRD76,055003 (2007)]

- The differential cross-section for $e^+(p)e^-(p') \rightarrow \gamma(k)\mathcal{U}(P_{\mathcal{U}})$ (Vector Unparticle) is :

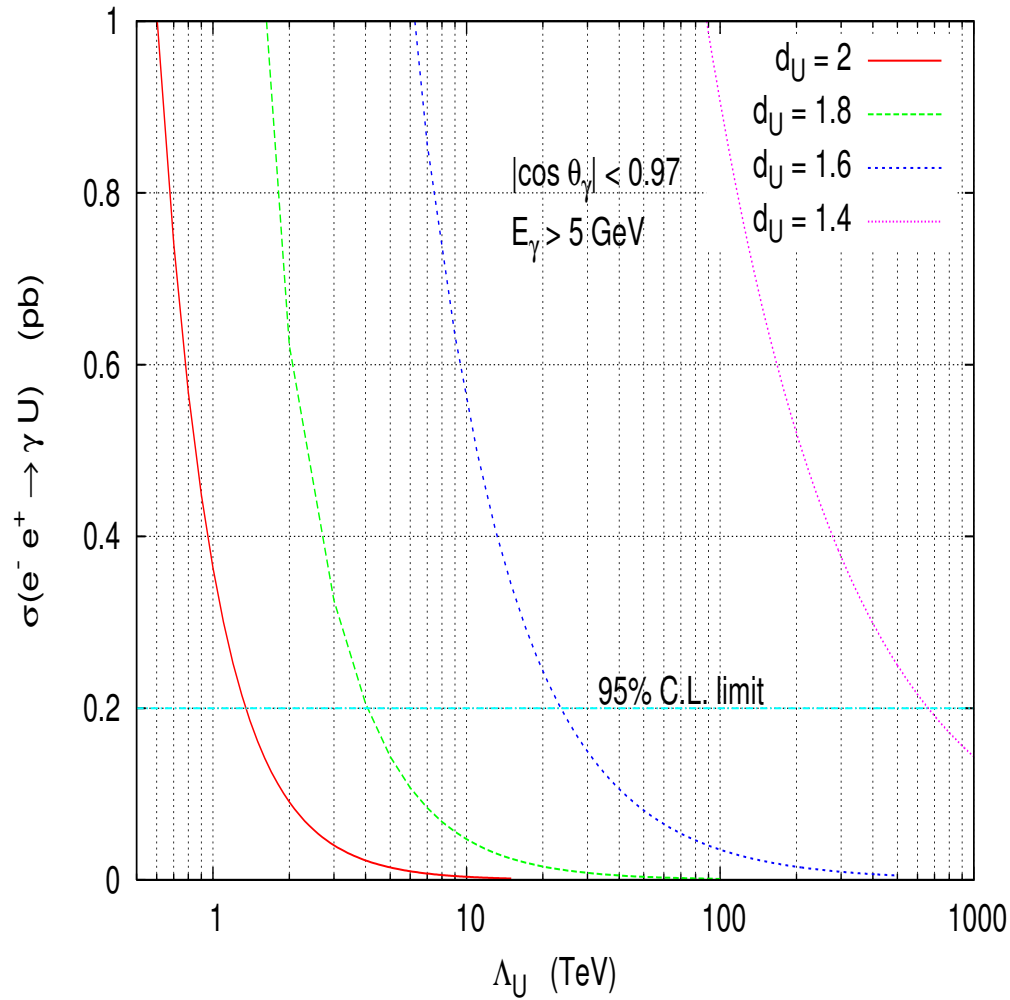
$$d\sigma = \frac{1}{2s} |\overline{\mathcal{M}}|^2 \frac{E_\gamma^2 A_{d_{\mathcal{U}}}}{16\pi^3 \Lambda_{\mathcal{U}}^2} \left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-2} dE_\gamma d\Omega$$

$$P_{\mathcal{U}}^2 = s - 2\sqrt{s}E_\gamma$$

$$|\overline{\mathcal{M}}|^2 = 2\lambda_1^2 e^2 \frac{u^2 + t^2 + 2sP_{\mathcal{U}}^2}{ut}$$

- LEP Collaborations had measured single photon plus missing energy in the context of different BSM scenarios.
- Strongest bound comes from L3: 95% C.L. upper limit on $\sigma(e^+e^- \rightarrow \gamma + X) \simeq 0.2$ pb, under the cuts $E_\gamma > 5$ GeV and $|\cos\theta_\gamma| < 0.97$ at $\sqrt{s} = 207$ GeV.
- Limits on $\Lambda_{\mathcal{U}} > 600(1.35)$ (TeV) for $d_{\mathcal{U}} = 1.4(2.0)$ for fixed $\lambda_1 = 1$.

Single γ production at e^+e^- Machine [K. Cheung et al. PRD76,055003 (2007)]



$\mu \rightarrow e + \mathcal{U}$ (Scalar unparticle operator)

- In the SM $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ is allowed.
- This mode can be faked by $\mu^- \rightarrow e^- + \mathcal{U}$ decay in the Unparticle model.
- The simplest term in the effective Lagrangian which can lead to the above decay involves a scalar unparticle:

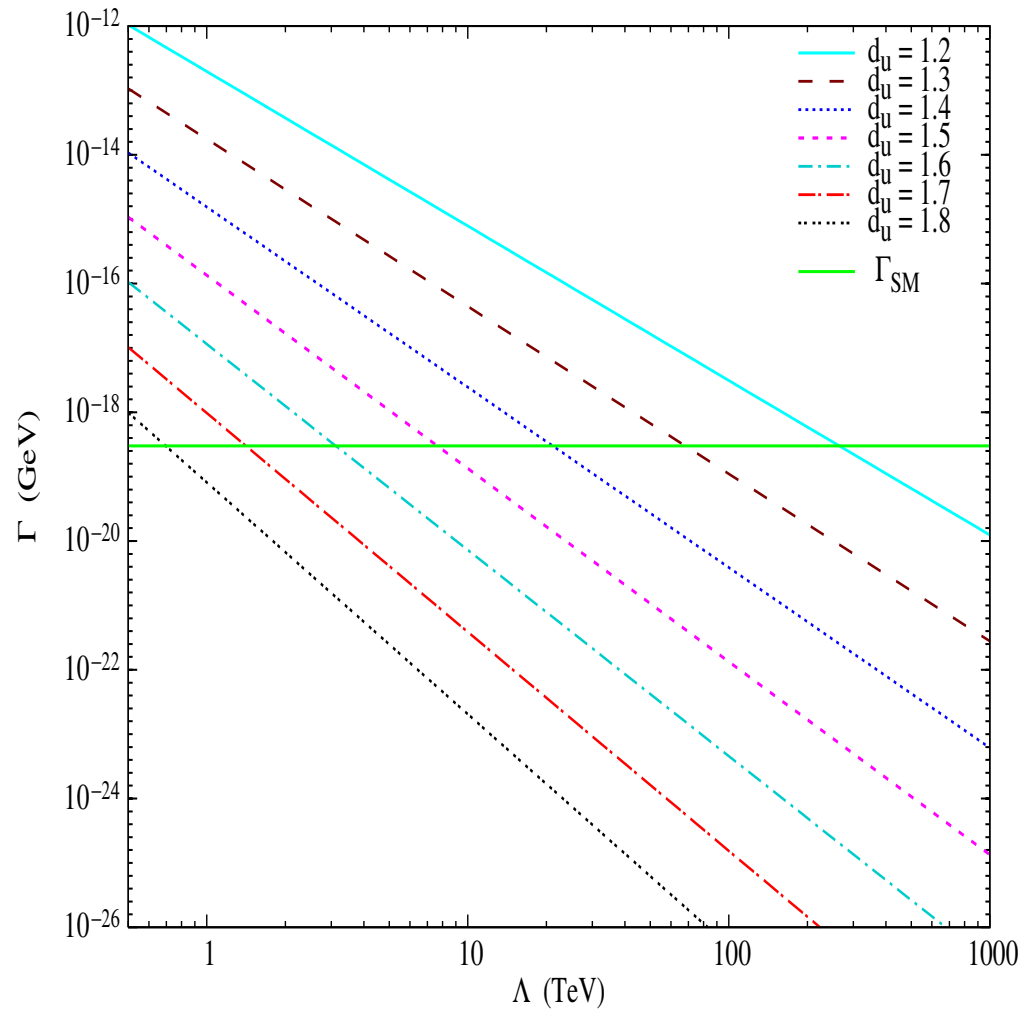
$$\mathcal{L}_1 = \Lambda^{-d_u} \bar{e} \gamma_\eta (c_1 + c_2 \gamma_5) \mu \partial^\eta \mathcal{O}_\mathcal{U}$$

- The decay profile is given by :

$$\begin{aligned} \frac{d\Gamma_S}{dE_e}(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{4 \pi^2} (c_1^2 + c_2^2) m_\mu^2 E_e^2 \left(m_\mu^2 - 2 m_\mu E_e \right)^{d_u-2} \Lambda^{-2 d_u} \Theta(m_\mu - 2 E_e) \\ \Gamma_S(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{16 \pi^2} \frac{c_1^2 + c_2^2}{d_u^3 - d_u} m_\mu \left(\frac{m_\mu}{\Lambda} \right)^{2 d_u} \end{aligned}$$

where the mass of the electron has been neglected and the second equality follows only for $d_u > 1$.

$\mu \rightarrow e + \mathcal{U}$ (Scalar unparticle operator)



$\mu \rightarrow e + \mathcal{U}$ (Vector Unparticle operator)

- Now consider a different possible couplings of the unparticles to the muon-electron current, namely a vector one:

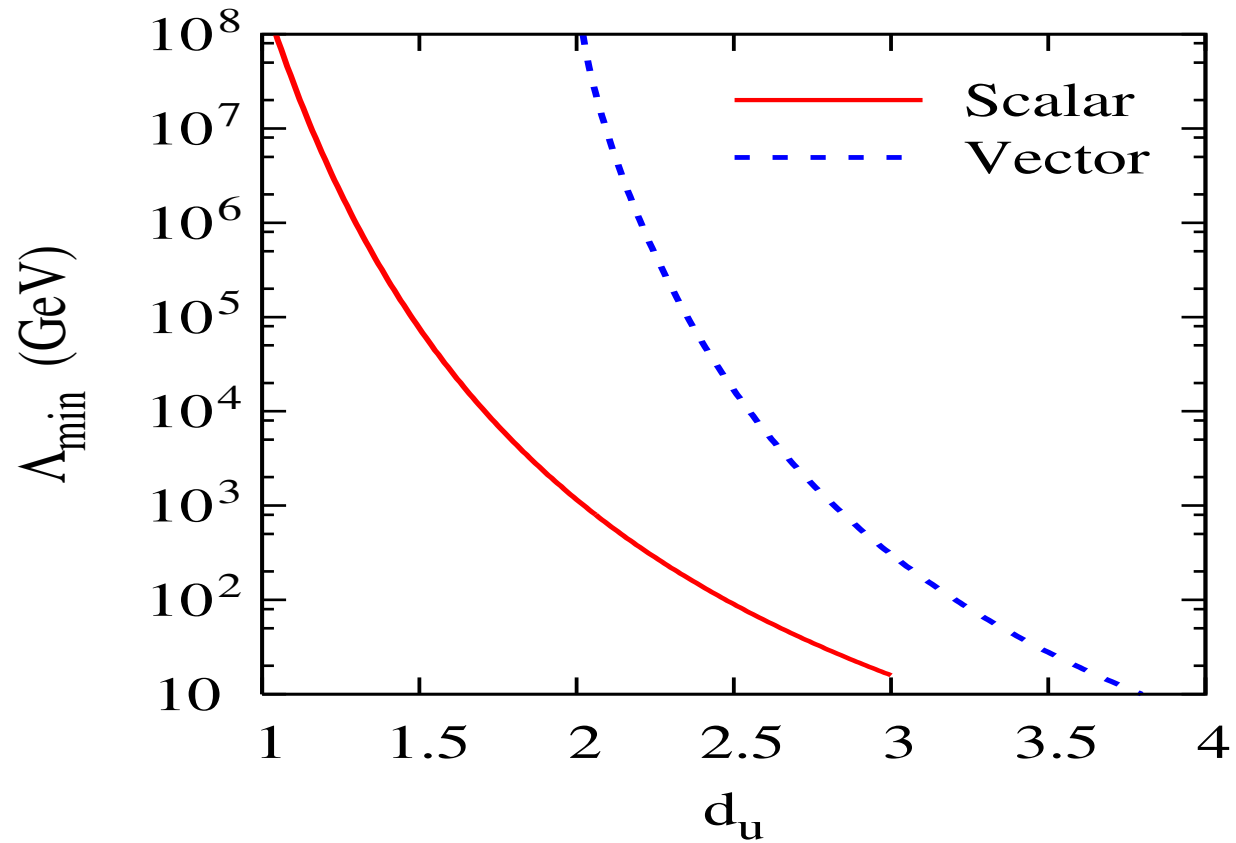
$$\mathcal{L}_2 = \Lambda^{1-d_u} \bar{e} \gamma_\eta (c_3 + c_4 \gamma_5) \mu \mathcal{O}_U^\eta$$

- The decay profile is given by :

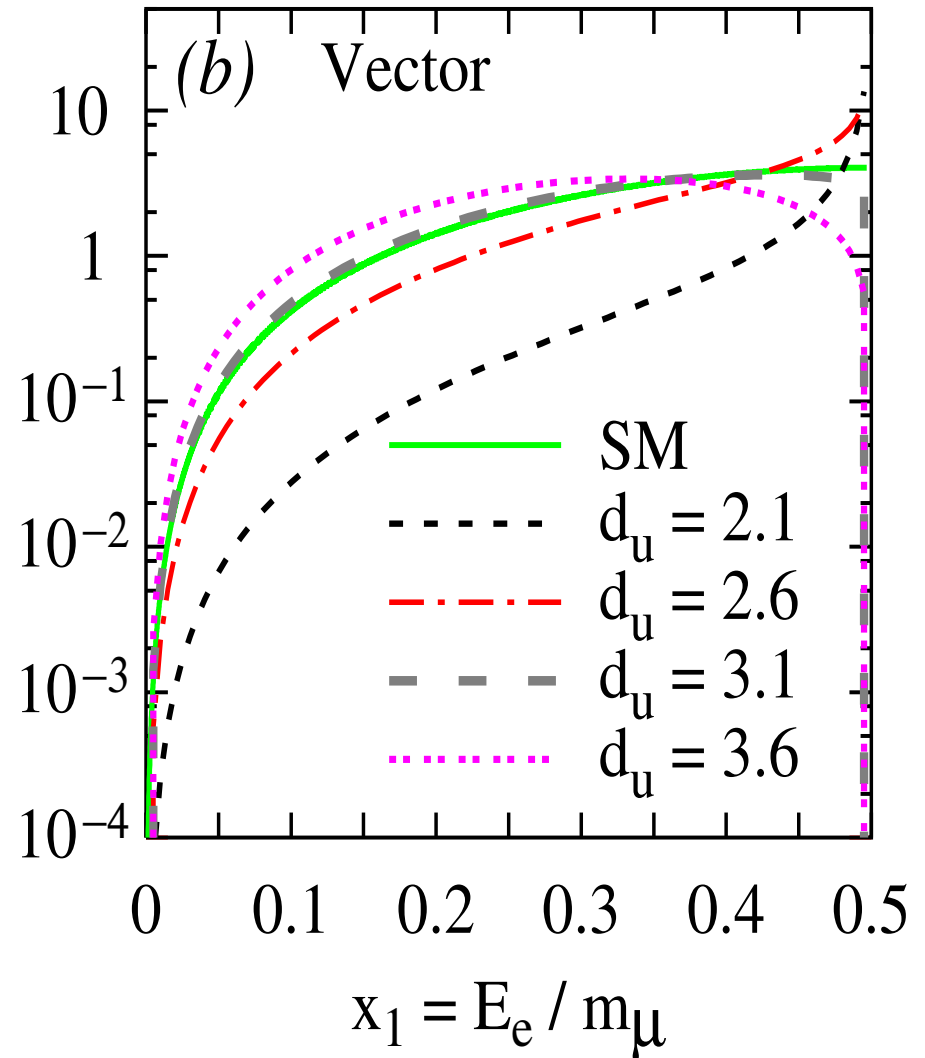
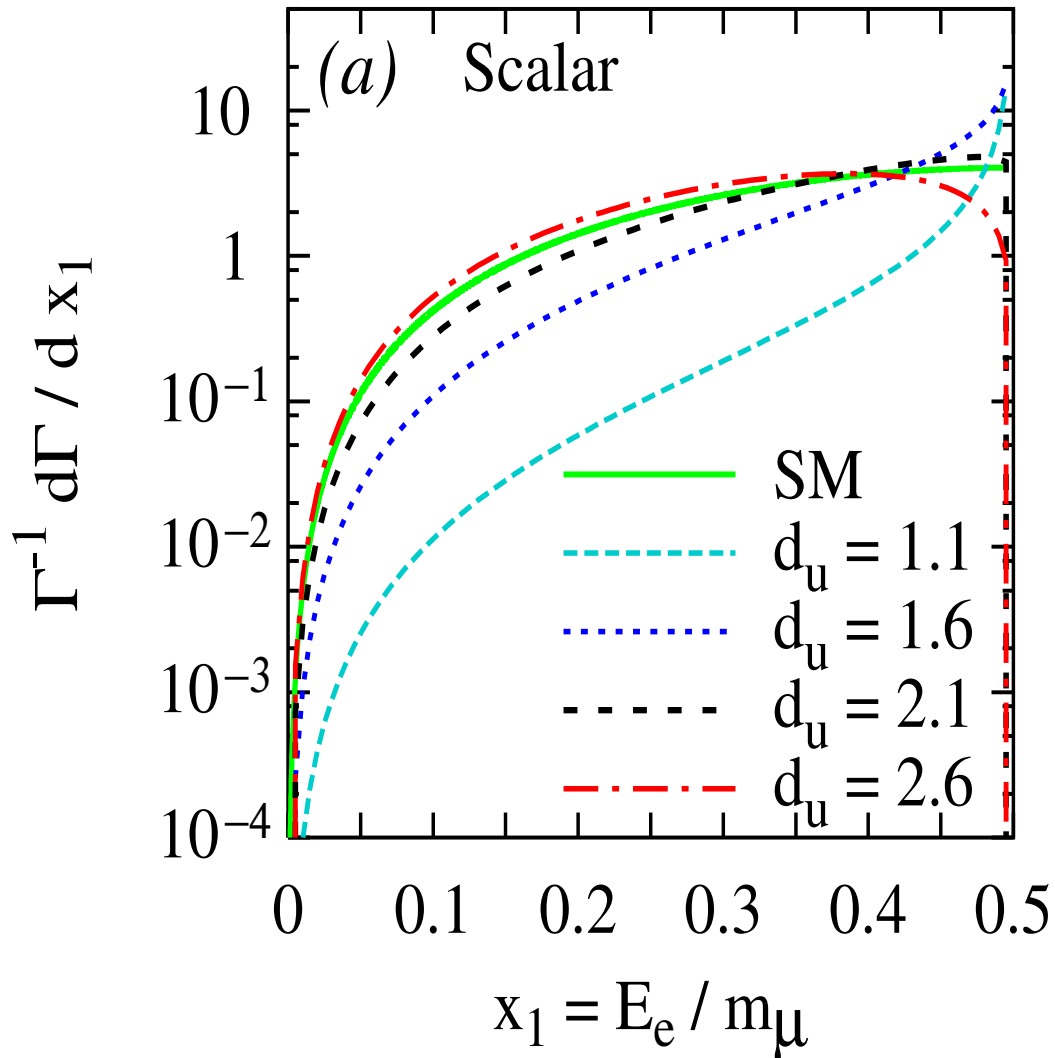
$$\begin{aligned} \frac{d\Gamma_V}{dE_e}(\mu \rightarrow e + \mathcal{U}) &= \frac{A_{d_u}}{4\pi^2} (c_3^2 + c_4^2) m_\mu E_e^2 (m_\mu^2 - 2m_\mu E_e)^{d_u-3} \Lambda^{2-2d_u} \\ &\quad (3m_\mu - 4E_e) \Theta(m_\mu - 2E_e) \\ \Gamma_V(\mu \rightarrow e + \mathcal{U}) &= \frac{3A_{d_u}}{16\pi^2} \frac{c_3^2 + c_4^2}{d_u^3 - d_u^2 - 2d_u} m_\mu \left(\frac{m_\mu}{\Lambda}\right)^{2d_u-2} \end{aligned}$$

where the mass of the electron has been neglected and the second equality holds for $d_u > 2$.

$\mu \rightarrow e + \mathcal{U}$ (Bound on Λ)



$$\mu^- \rightarrow e^- + \mathcal{U}$$



$$\mu^- \rightarrow e^- + \mathcal{U}$$

- Can MEG distinguish between the possible unparticle operators if there is a discrepancy in $\mu^- \rightarrow e^- + \text{nothing}$ in the forthcoming experiment.
- For small $d_{\mathcal{U}}$, the distributions are naturally peaked at $E_e = m_{\mu}/2$ as is expected for a decay into two massless particles.
- For vector case, the peaking persists to much larger values of $d_{\mathcal{U}}$ is the reflection of the differing powers of P^2 in the two cases ($d_{\mathcal{U}} - 3$ for vector vs. $d_{\mathcal{U}} - 2$ for scalar).
- $d_{\mathcal{U}} \rightarrow 1^+$ for scalar correspond to the two body decay in the case of the vector, it is the limit $d_{\mathcal{U}} \rightarrow 2^+$ that corresponds to the same.
- The limit $d_{\mathcal{U}} \rightarrow 2^+$ in the scalar case corresponds to a 3 body decay (close to SM curve in Fig a).
- For vector case, this feature is exhibited in the limit $d_{\mathcal{U}} \rightarrow 3$.

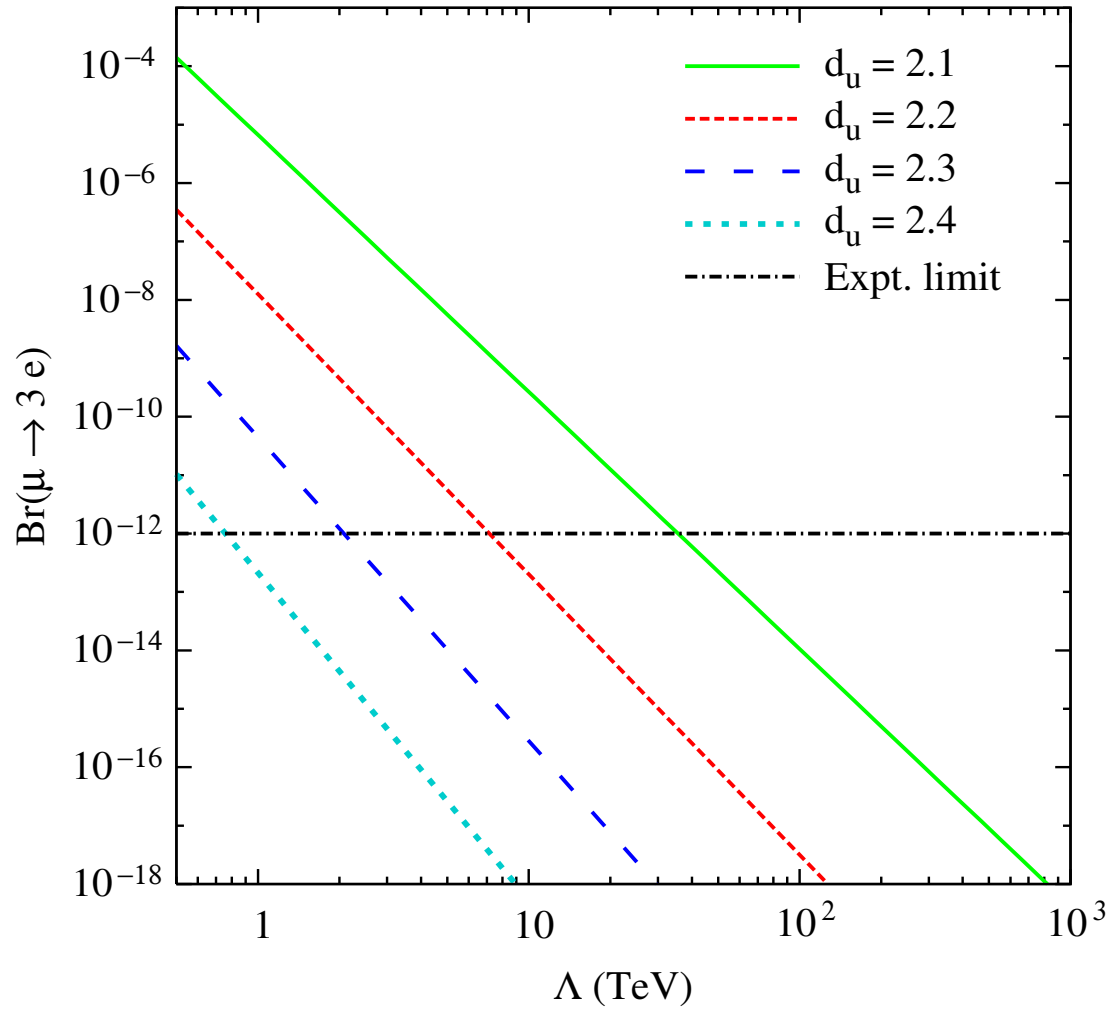
Virtual Unparticle exchange

$\mu^- \rightarrow 3e$

- Unparticle sector can also couple to lepton flavour conserving currents
- Restricting ourselves to the vector operator, an additional term relevant for μ decay :

$$\mathcal{L}_3 = \Lambda^{1-d_U} \bar{e} \gamma_\alpha (c_5 + c_6 \gamma_5) e \mathcal{O}_U^\alpha \quad (1)$$

- Simultaneous presence of both sets of operators \mathcal{L}_2 and \mathcal{L}_3 would lead to unparticle mediated $\mu \rightarrow 3e$ decay.
- We concentrate on $d_U > 2$.
- The experimental limit on $Br(\mu \rightarrow 3e) < 10^{-12}$ **PDG (2006)**.
- We set $c_3 = c_4 = c_5 = c_6 = \frac{1}{\sqrt{2}}$
- The branching ratio is very sensitive to the scaling dimension d_U .

$\mu \rightarrow 3e$ 

Drell-Yan process [K. Cheung et al. PRL99,051803 (2007)]

- The virtual exchange of vector unparticle between two fermionic current can be expressed as 4-fermion interactions (assuming massless fermions):

$$\mathcal{A} = \lambda^2 \frac{A_{d_U}}{2 \sin(d_U \pi) \Lambda_U^2} \left(-\frac{Q_U^2}{\Lambda_U^2} \right)^{d_U-2} (\bar{f}_2 \gamma_\alpha P_\mu f_1) (\bar{f}_4 \gamma^\alpha P_\nu f_3)$$

- $Q_U^2 > 0 \implies$ phase factor $\exp(-i\pi d_U)$.
- No such phase factor for $Q_U^2 < 0$

Drell-Yan process [K. Cheung et al. PRL99,051803 (2007)]

- The differential cross-section for DY process (through Vector Unparticle) is :

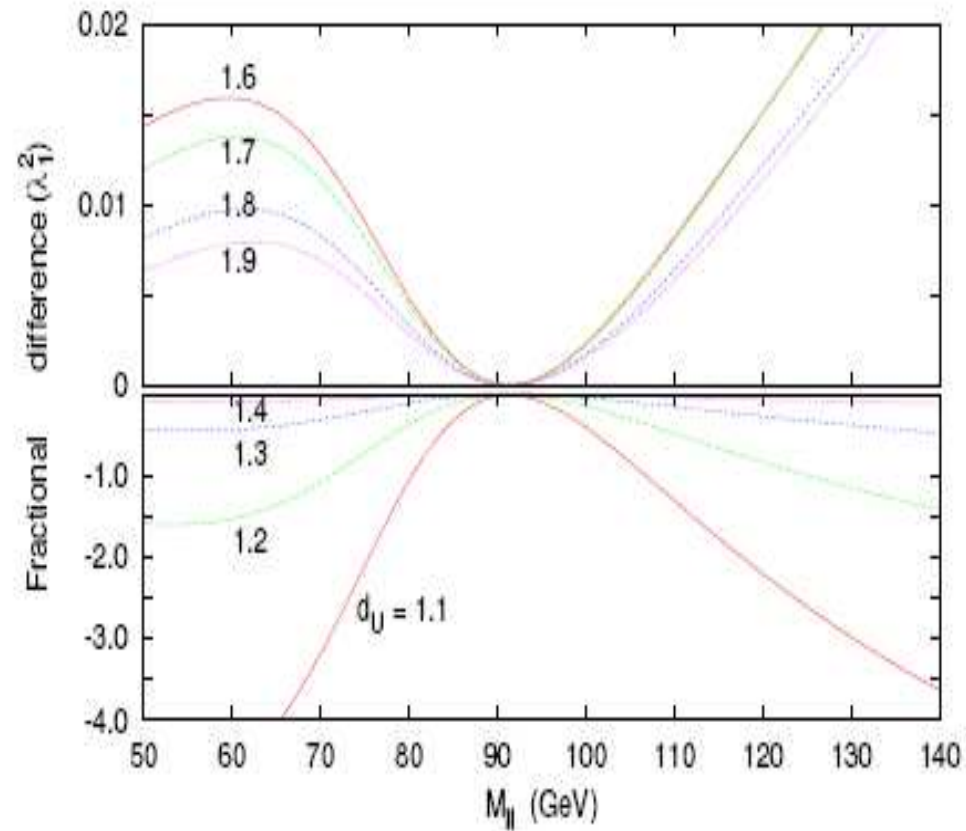
$$\frac{d^2\sigma}{dM_{\ell\ell} dy} = K \frac{M_{\ell\ell}^3}{72\pi s} \sum_q f_q(x_1) f_{\bar{q}}(x_2) \times (|M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2)$$

where,

$$M_{\alpha\beta} = \frac{\lambda^2 A_{d\mathcal{U}}}{2 \sin(d\mathcal{U}\pi) \Lambda_{\mathcal{U}}^2} \left(-\frac{\hat{s}}{\Lambda_{\mathcal{U}}^2} \right)^{d\mathcal{U}-2} + \frac{e^2 Q_\ell Q_q}{\hat{s}} + \frac{e^2 g_\alpha^\ell g_\beta^q}{\sin^2 \theta_w \cos^2 \theta_w} \frac{1}{\hat{s} - M_Z^2 + iM_Z \Gamma_Z}$$

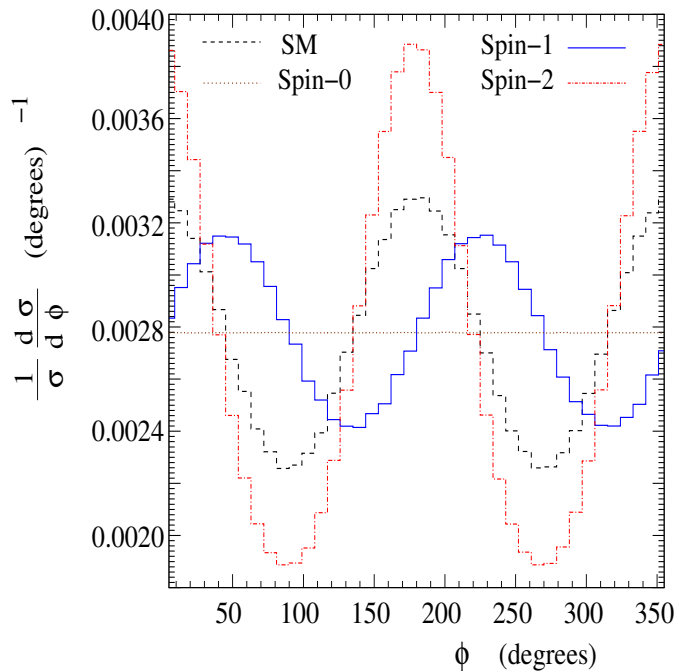
- where $\hat{s} = M_{\ell\ell}^2$ and $K = 1 + \frac{4\alpha_s}{6\pi} \left(1 + \frac{4\pi^2}{3} \right)$.
- $\hat{s} > 0$, the phase of the Unparticle propagator $\exp(-i\pi d\mathcal{U})$ will interfere with both the real γ propagator and also the real and imaginary parts of the Z propagator in a non-trivial way.

DY process [K. Cheung et al. PRL99,051803 (2007)]



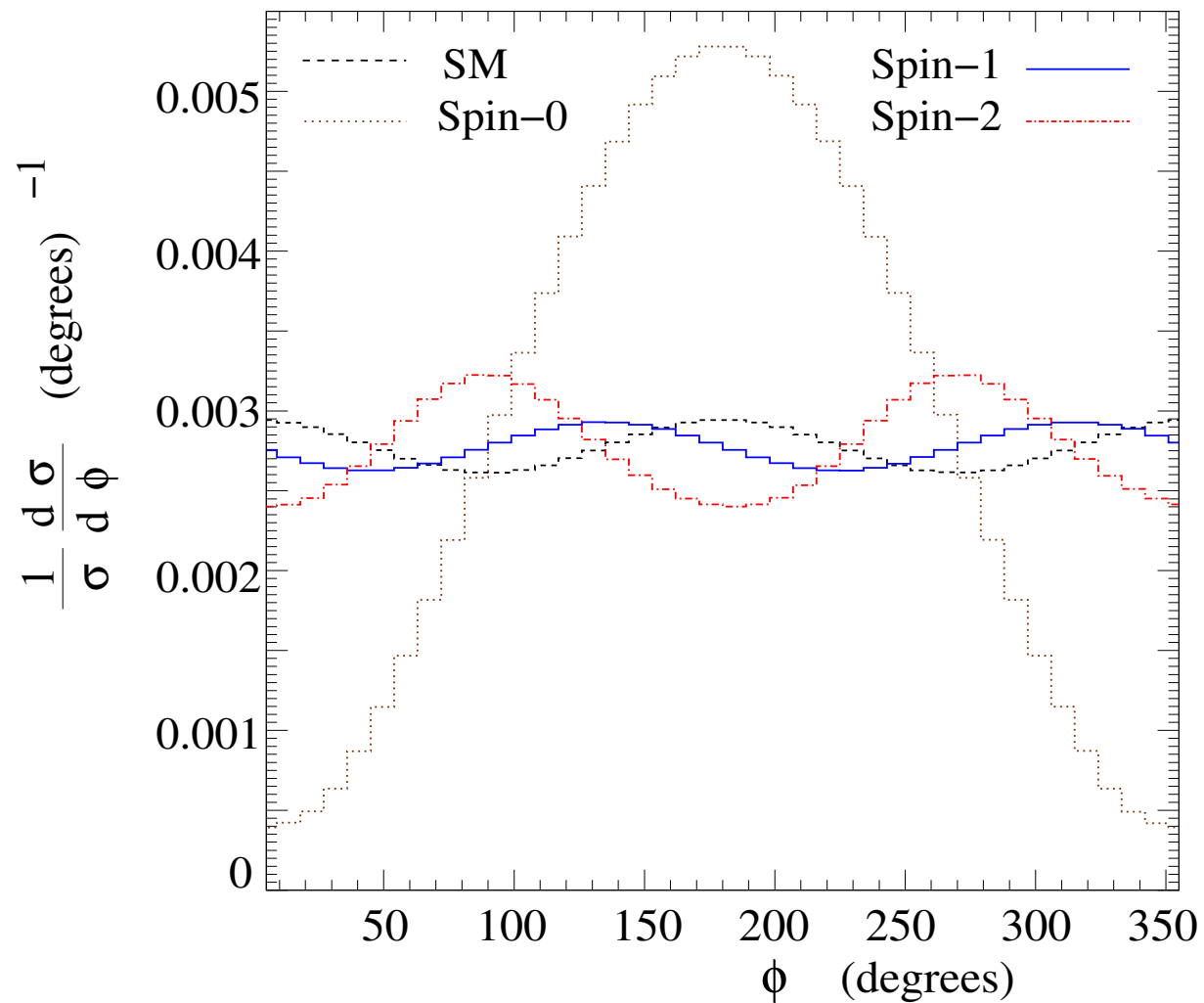
Interference of U with γ , Z propagators

Disentangling the unparticles with polarized beams at e^+e^- Machine [K. Huitu and S. Rai, PRD77, 035015 (2008)]



- The effect of complex phase in the virtual Unparticle propagator can be probed by studying the azimuthal distribution of the final state particles using the transverse beam polarizations.
- The azimuthal angle is defined by the directions of the e^\pm transverse polarization and the plane of the momenta of the outgoing fermions in the $e^+e^- \rightarrow f\bar{f}$ process.
- Here the azimuthal angle distribution for the μ final state is shown.
- $\Lambda_{\mathcal{U}} = 10$ TeV, $d_{\mathcal{U}} = 1.5$ (spin-0 and spin-1).
- $\Lambda_{\mathcal{U}} = 1$ TeV, $d_{\mathcal{U}} = 1.3$ (spin-2).

Disentangling the unparticles with polarized beams....



Relevant Unparticle operators for $t\bar{t}$ process

- The relevant operators in the effective Lagrangian are given by :

$$\mathcal{L}_{scalar} = \Lambda^{-d_U} \left[\bar{t} \gamma_\mu (a + b \gamma_5) t \partial^\mu \mathcal{O}_S + G_{\mu\nu} \left(\tilde{c}_1 G^{\mu\nu} + \tilde{c}_2 \tilde{G}^{\mu\nu} \right) \mathcal{O}_S \right] \quad (2)$$

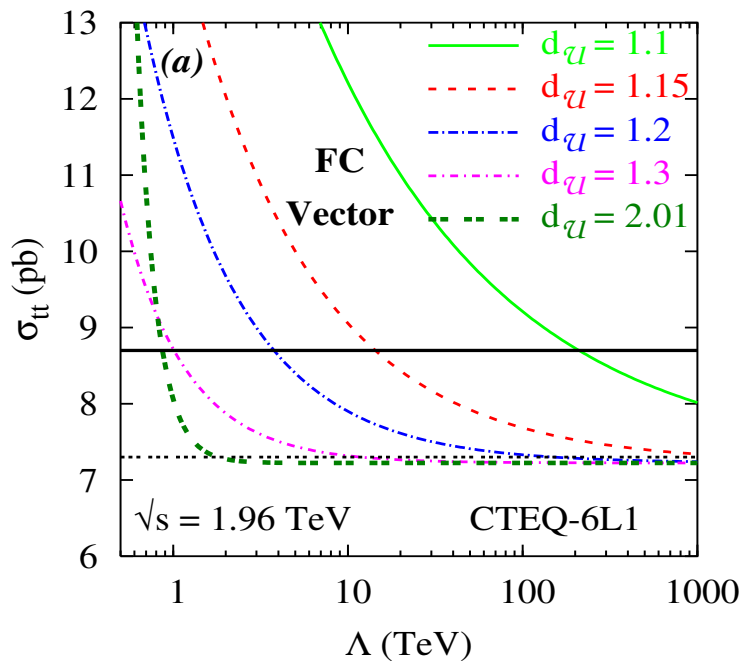
$$\mathcal{L}_{vector} = \Lambda^{1-d_U} \sum_q \bar{q} \gamma_\mu (\tilde{v}^q + \tilde{a}^q \gamma_5) q \mathcal{O}_V^\mu \quad (3)$$

$$\mathcal{L}_{tensor}^1 = \Lambda^{-d_U} \mathcal{O}_T^{\mu\nu} \left[\sum_q \bar{q} \left(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu \right) (a_q^T + b_q^T \gamma_5) q \right] \quad (4)$$

$$\mathcal{L}_{tensor}^2 = \Lambda^{-d_U} \mathcal{O}_T^{\mu\nu} \left[G_\nu^\alpha \left(a_g G_{\mu\alpha} + b_g \tilde{G}_{\mu\alpha} \right) \right] \quad (5)$$

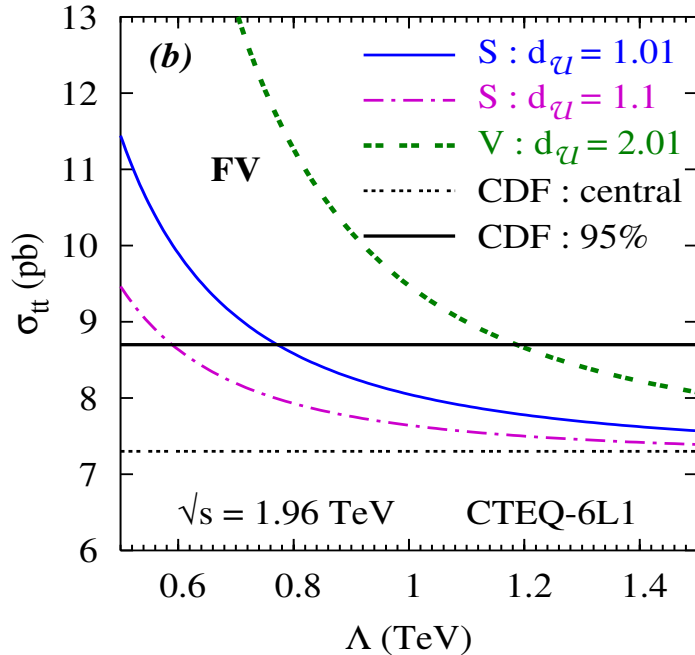
- We shall consider the coefficients to be order unity.
- The coupling of \mathcal{O}_S to light fermions are negligible.

$t\bar{t}$ pair cross-section at Tevatron Run II



- Variation of the total $\sigma_{t\bar{t}}$ at the Tevatron Run II with Λ in the presence of a vector unparticle operator.
- New physics contribution : through s -channel, No interference with the (dominant) QCD amplitude.
- Thus unparticle effect can become appreciable only when the amplitude becomes comparable to the QCD one.
- EW contribution is small \longrightarrow the phase factor $\exp(-i\pi d_U)$ in the unparticle propagator is of little consequence.
- $\sigma(t\bar{t}) = 7.3 \pm 0.5(stat) \pm 0.6(syst) + \pm 0.4(lum)$ pb, (CDF Run II results averaged over all channels)

$t\bar{t}$ pair cross-section at Tevatron Run II ...

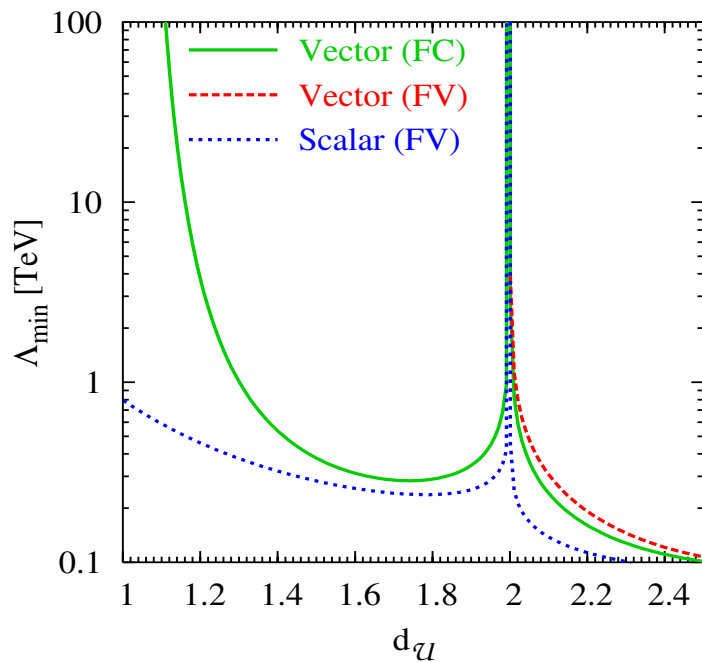


- Variation of the total $\sigma_{t\bar{t}}$ at the Tevatron Run II with Λ in the presence of either scalar or vector flavour violating unparticle operator.
- This being only an effective theory, flavour-violating (FV) unparticle couplings are also possible. For example, the vector coupling can be generalized to :

$$\mathcal{L}_{FV} \supset \Lambda^{1-d_{\mathcal{U}}} \bar{q} \gamma_{\eta} (\tilde{v}^{qq'} + \tilde{a}^{qq'} \gamma_5) q' \mathcal{O}_V^{\eta}$$

- Similarly for the scalar and tensor operators.
- These operators result in a t -channel diagram for $u\bar{u} \rightarrow t\bar{t}$, which interferes with the QCD amplitude.

Bound on Λ from $\sigma_{t\bar{t}}$ at Tevatron Run II



- Using the $\sigma(t\bar{t})^{\text{exp}}$ (with errors added in quadrature), one may impose bounds on the unparticle parameter space for a given choice of operators.
- Here we show the 95% C.L. bounds assuming that only one of the operators, whether FC or FV contributes.
- The constraint is strongly dependent on the structure of the operator involved as well as on the scale dimension d_U .
- The sharp rise of the limit at $d_U = 2$ is but a manifestation of the presence of physical poles in the unparticle propagators at all integral values of $d_U > 1$.

summary

- Unparticle is a hidden scale invariant sector which couples to the SM particles only through higher dimensional operators.
- An EFT can be used to explore the unparticle physics.
- It can lead to a very interesting phenomenological consequences due to non-integral value of the scale dimension.
- Production of real unparticle at the colliders give rise to missing energy.
- The unparticle propagator in the time like region contains an additional phase.
- In our study of unparticle physics in the LFV $\mu \rightarrow e + \mathcal{U}$, we obtained strong limit on the unparticle parameter space.
- We have also shown that the LFV decay width $\mu^- \rightarrow e^+ e^- e^-$ is highly sensitive to the virtual unparticle effects.

summary

- The current Tevatron measurements on top pair production cross-section can be used to impose significant constraints on the unparticle physics
- The bounds are strongly dependent on the Lorentz structure of the relevant unparticle operator as well as its mass dimension $d_{\mathcal{U}}$.