Can we get the Standard Model from String Theory?

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Plan of the talk

1. String Theory and Experiments
2. Intersecting and magnetized D branes
3. A simple phenomenological model
4. Conclusions

String theory and Experiments

- The strongest motivation for string theory is the fact that it provides a consistent quantum theory of gravity unified with the gauge interactions.
- This is because string theory has a parameter α' of the dimension of a $(length)^2$ that acts as an ultraviolet cutoff $\Lambda = \frac{1}{\sqrt{\alpha'}}$.
- Because of it all loop integrals are finite in the UV.
- The string tension T is equal to $T = \frac{1}{2\pi\alpha'}$.
- String theory is an extension of field theory !

$$\text{Quantum Mechanics} \quad \underset{h \to 0}{\Longrightarrow} \quad \text{Classical Mechanics}$$

$$\text{Special Relativity} \quad \underset{c \to \infty}{\Longrightarrow} \quad \text{Galilean Mechanics}$$

$$\text{String Theory} \quad \underset{\alpha' \to 0}{\Longrightarrow} \quad \text{Field Theory}$$

- In the limit $\alpha' \to 0$ one recovers all UV divergences of quantum gravity unified with gauge theories.
- They are due to the point-like structure of the elementary constituents.
- The possibility of seeing stringy effects in experiments depends then on the energy E available.
- If $\alpha' E^2 << 1$, then one will see only the limiting field theory.
- α' is a parameter that tells us how much a string theory differs from field theory based on point-like objects.
- The simplest string theory is the bosonic string that is, however, not consistent because it contains tachyons in the spectrum.
- Around 1985 it was clear that we have 5 ten-dimensional consistent string theories: IIA, IIB, I, Het. $E_8 \times E_8$ and Het. $SO(32)$.
- They are inequivalent in string perturbation theory ($g_s < 1$), supersymmetric and unify in a consistent quantum theory gauge theories with gravity.

- Unlike α' the string coupling constant g_s is not a parameter to be fixed from experiments.
- It corresponds to the vacuum expectation value of a string excitation, called the dilaton, $g_s = e^{\langle \phi \rangle}$, that should be fixed by the minima of the dilaton potential.
- But the potential for the dilaton is flat in any order of string perturbation theory.
- For each value of $\langle \phi \rangle$ we have an inequivalent theory.
- This is unsatisfactory for a theory, as string theory, that pretends to explain everything.......
- But this is not the only problem....
- If string theory is the fundamental theory unifying all interactions, why do we have 5 theories instead of just one?

- The key to solve this problem came from the discovery of new p-dim. states, called D(irichlet)p branes.
- The spectrum of massless states of the II theories is given in the table

$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ	NS-NS sector
Metric	Kalb-Ramond	Dilaton	
C_0, C_2	C_4, C_6	C_8	RR sector IIB
C_1, C_3	C_5	C_7	RR sector IIA

- the RR C_i stands for an antisymmetric tensor $C_{\mu_1\mu_2\dots\mu_i}$
- They are generalizations of the electromagnetic potential A_μ

$$\int A_\mu dx^\mu \Longrightarrow \int A_{\mu_1\mu_2\dots\mu_{p+1}} d\sigma^{\mu_1\mu_2\dots\mu_{p+1}}$$

As the electromagnetic field is coupled to point-like particles so they are coupled to p-dimensional objects.

- There exist classical solutions of the low-energy string effective action that are coupled to the metric, the dilaton and are charged with respect a RR field. For them we get

$$C_{01\ldots p} \sim \frac{1}{r^{d-3-p}} \Longleftrightarrow C_0 \sim \frac{1}{r} \ \text{ if } d=4, p=0$$

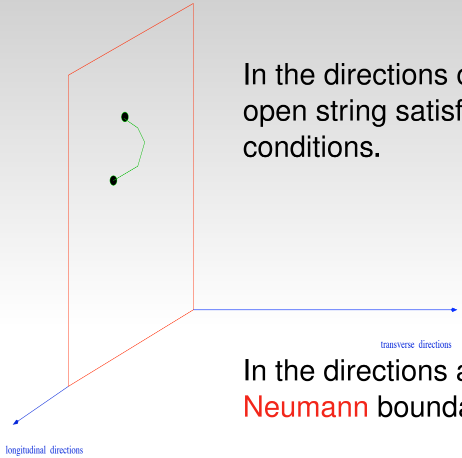
  They are additional non-perturbative states of string theory with tension and RR charge given by:

$$\tau_p = \frac{Mass}{p-volume} = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} \quad ; \quad \mu_p = \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p}$$

- They are called D(irichlet)p branes because they have open strings attached to their (p+1)-dim. world-volume:

$$\partial_\sigma X^\mu(\sigma=0,\pi;\tau) = 0 \ \ \mu = 0\ldots p \qquad \textit{Neumann b.c.}$$
$$\partial_\tau X^i(\sigma=0,\pi;\tau) = 0 \ \ i = p+1\ldots 10 \qquad \textit{Dirichlet b.c.}$$

- Remember that a string is described by the string coordinate $X^\mu(\sigma,\tau)$ and $\sigma = 0, \pi$ correspond to the two end-points of an open string.

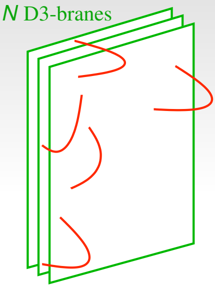


In the directions orthogonal to the brane the open string satisfies Dirichlet boundary conditions.

In the directions along the brane they satisfy Neumann boundary conditions.

- The open strings (gauge theory) live in the (p+1)-dim. volume of a Dp brane, while closed strings (gravity) live in the entire ten dimensional space.

- If we have a stack of N parallel D branes, then we have N^2 open strings having their endpoints on the D branes:



An open string attached to the same stack of D branes transforms according to the adjoint representation of $U(N)$

- The massless strings correspond to the gauge fields of $U(N)$.

- A stack of N D branes has a $U(N) = SU(N) \times U(1)$ gauge theory living on their worldvolume.

- The discovery of Dp branes opened the way in 1995 to the discovery of the string dualities.
- and this led to understand that the 5 string theories were actually part of a unique 11-dimensional theory: M theory.
- However, in the experiments we observe only 4 and not 10 or 11 non-compact directions.
- Therefore 6 of the 10 dimensions must be compactified and small: $R^{1,9} \rightarrow R^{1,3} \times M_6$ where M_6 is a compact manifold.
- In order to preserve at least $N = 1$ supersymmetry M_6 must be a Calabi-Yau manifold.
- But this means that the low-energy physics will depend not only on α' and g_s , but also on the size and shape of the manifold M_6.

- Originally the most promising string theory for phenomenology was considered the Heterotic $E_8 \times E_8$ that was studied intensively.
- But in this theory both the fundamental string length $\sqrt{\alpha'}$ and the size of the extra dimensions were supposed to be of the order of the Planck length $\left(\frac{1}{\sqrt{\alpha'}} \equiv M_s = \frac{M_{Pl.}\sqrt{\alpha_{GUT}}}{2} \sim \frac{M_{Pl.}}{10}\right.$ and $\frac{R}{\sqrt{\alpha'}} \sim 1$ if $g_s < 1$).
- Too small to be observed in present and even future experiments!
- One needs a very good control of the theory to be able to extrapolate to low energy.
- Later on in 1998 it became clear that in type I and in a brane world one could allow for much larger values for the string length $\sqrt{\alpha'}$ and for the size of the extra dimensions without being in contradiction with the experimental data.

- When we compactify 6 of the 10 dimensions, in addition to the dilaton, we generate a bunch of scalar fields (moduli) corresponding to the components of the metric and of the other closed string fields in the extra dimensions.
- Their vacuum expectation values, corresponding to the parameters of the compact manifold, are not fixed in any order of perturbation theory because their potential is flat.
- We get a continuum of string vacua for any value of the moduli ! No good for phenomenology !
- The problem of Moduli stabilization.
- In the last few years one has been able to stabilize the moduli by the introduction of non-zero fluxes for some of the NS-NS and R-R fields.

- But we still have a discrete (and huge) quantity of string vacua: "Landscape Problem".
- How do we fix the vacuum we live in?
- Anthropic principle or better understanding needed?
- Bottom-up approach: construct string extensions of the SM and of the MSSM.
- If we want to construct them in an explicit way we must limit ourselves to toroidal compactifications with orbifolds and orientifolds.
- and, most important, we need to have massless open strings corresponding to chiral fermions in four dimensions for describing quarks and leptons.
- The simplest models are those based on several stacks of intersecting branes and/or of their T-dual magnetized branes on $R^{3,1} \times \mathcal{T}^2 \times \mathcal{T}^2 \times \mathcal{T}^2$.

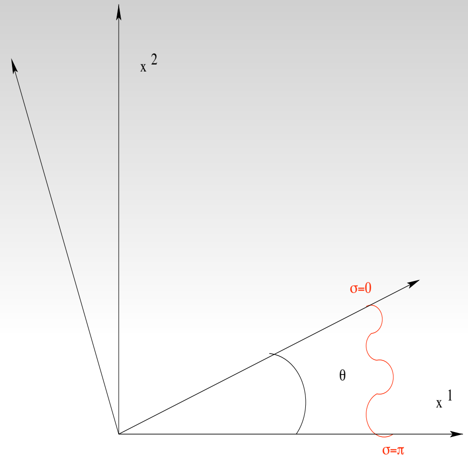
Intersecting and magnetized D branes

Intersecting branes

- Consider a rectangular torus $\mathcal{T}^2$ with radii R_1 and R_2.
- Assume that the two stacks of branes are parallel and lying along the axis x^1.
- An open string $(X^{1,2}(\sigma, \tau))$, having one end-point attached to one stack and the other end-point attached to the other stack, satisfies the following eq. of motion and boundary conditions:

$$\begin{aligned} \left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^i &= 0 \\ \partial_\sigma X^1|_{\sigma=\pi} = \partial_\tau X^2|_{\sigma=\pi} &= 0 \\ \partial_\sigma X^1|_{\sigma=0} = \partial_\tau X^2|_{\sigma=0} &= 0 \end{aligned} \tag{1}$$

- We keep now the first stack along the axis x^1, while we put the second stack at an angle θ with respect to the axis x^1.



First stack of branes along x^1.

Second stack at an angle θ with x^1

b.c. for an open string attached at $\sigma = \pi$ to the first stack and at $\sigma = 0$ to the second stack:

$$\partial_\sigma X^1|_{\sigma=\pi} = \partial_\tau X^2|_{\sigma=\pi} = 0$$

$$\partial_\sigma \left[\cos\theta X^1 - \sin\theta X^2\right]_{\sigma=0} = \partial_\tau \left[\sin\theta X^1 + \cos\theta X^2\right]_{\sigma=0} = 0$$

- If the brane at θ is wrapped $n(m)$ times along the cycle 1(2) of the torus, then the angle between the two stacks of branes is given by:

$$\tan\theta = \frac{mR_2}{nR_1}$$

- Performing a T-duality along x^2, that amounts to $\partial_\sigma X^2 \leftrightarrow \partial_\tau X^2$ and $R_2 \rightarrow \frac{\alpha'}{R_2}$, we get the following b.c.:

$$\partial_\sigma X^1|_{\sigma=\pi} = \partial_\sigma X^2|_{\sigma=\pi} = 0 \quad ; \quad \tan\theta = \frac{m\alpha'}{nR_1 R_2}$$
$$\left[\partial_\sigma X^1 - \tan\theta\partial_\tau X^2\right]_{\sigma=0} = \left[\partial_\sigma X^2 + \tan\theta\partial_\tau X^1\right]_{\sigma=0} = 0$$

- These are the b.c. for an open string with the end-point at $\sigma = 0$ attached to a magnetized brane.

Magnetized branes

- Assume that on the first (second) stack of branes there is a constant magnetic $F^{(\pi)}(F^{(0)})$.
- The action describing the interaction of an open string with its end-points attached to these two stacks of branes is given by:

$$S = S_{bulk} + S_{boundary}$$

$$S_{bulk} = -\frac{1}{4\pi\alpha'}\int d\tau \int_0^\pi d\sigma \left[G_{ab}\partial_\alpha X^a \partial_\beta X^b \eta^{\alpha\beta} - B_{ab}\epsilon^{\alpha\beta}\partial_\alpha X^a \partial_\beta X^b\right]$$

$$\begin{aligned} S_{boundary} &= -q_0 \int d\tau A_i^{(0)} \partial_\tau X^i|_{\sigma=0} + q_\pi \int d\tau A_i^{(\pi)} \partial_\tau X^i|_{\sigma=\pi} \\ &= \frac{q_0}{2}\int d\tau F_{ij}^{(0)} X^j \dot{X}^i|_{\sigma=0} - \frac{q_\pi}{2}\int d\tau F_{ij}^{(\pi)} X^j \dot{X}^i|_{\sigma=\pi} \end{aligned}$$

- The two gauge field strengths are constant:

$$A_i^{(0,\pi)} = -\frac{1}{2}F_{ij}^{(0,\pi)} x^j \ .$$

- The data of the torus $\mathcal{T}^2$, called moduli, are included in the constant G_{ij} and B_{ij}.
- They are the complex and Kähler structures of the torus:

$$U \equiv U_1 + iU_2 = \frac{G_{12}}{G_{11}} + i\frac{\sqrt{G}}{G_{11}} \quad ; \quad T \equiv T_1 + iT_2 = B_{12} + i\sqrt{G}$$

  by

$$G_{ij} = \frac{T_2}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix} \quad \text{and} \quad B_{ij} = \begin{pmatrix} 0 & -T_1 \\ T_1 & 0 \end{pmatrix}$$

  They are the closed string moduli.
- F is constrained by the fact that its flux is an integer:

$$\int Tr\left(\frac{qF}{2\pi}\right) = m \Longrightarrow 2\pi\alpha' qF_{12} = \frac{m}{n}$$

  They are the open string moduli.
- The D brane is wrapped n times on the torus and the flux of F, on a compact space as T^2, must be an integer m (magnetic charge).

- The most general motion of an open string in this constant background can be determined and the theory can be explicitly quantized.
- One gets a string extension of the motion of an electron in a constant magnetic field on a torus (Landau levels).
- The ground state is degenerate and the degeneracy is given by the number of Landau levels.
- When $\alpha' \to 0$ one goes back to the problem of an electron in a constant magnetic field.
- The mass spectrum of the string states can be exactly determined:

$$\alpha' M^2 = N_4^X + N_4^\psi + N_{comp.}^X + N_{comp}^\psi + \frac{x}{2}\sum_{i=1}^{3}\nu_i - \frac{x}{2}$$

$x = 0$ for fermions (R sector) and $x = 1$ for bosons (NS sector)

$$N_4^X = \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n \ ; \ N_4^\psi = \sum_{n=\frac{x}{2}}^{\infty} n b_n^\dagger \cdot b_n$$

$$N^X_{comp} = \sum_{i=1}^{3} \left[\sum_{n=0}^{\infty} (n+\nu_i) A^{\dagger i}_{n+\nu_i} A^i_{n+\nu_i} + \sum_{n=1}^{\infty} (n-\nu_i) A^{\dagger i}_{n-\nu_i} A^i_{n-\nu_i} \right]$$

$$N^\psi_{comp} = \sum_{i=1}^{3} \left[\sum_{n=\frac{x}{2}}^{\infty} (n+\nu_i) B^{\dagger i}_{n+\nu_i} B^i_{n+\nu_i} + \sum_{n=1-\frac{x}{2}}^{\infty} (n-\nu_i) B^{\dagger i}_{n-\nu_i} B^i_{n-\nu_i} \right]$$

- where

$$\nu_i = \nu_i^0 - \nu_i^\pi \quad ; \quad \tan \pi \nu_i^{0,\pi} = \frac{m_i^{(0,\pi)}}{n_i^{(0,\pi)} T_2^{(i)}}$$

$T_2^{(i)}$ is the volume of one of the three tori.

- In the fermionic sector the lowest state is the vacuum state.
- It is a 4-dimensional massless chiral spinor!!

- For generic values of ν_1, ν_2, ν_3 there is no massless state in the bosonic sector.
- In general the original 10-dim supersymmetry is broken.
- The lowest bosonic states are

$$B^{\dagger i}_{\frac{1}{2}-\nu}|0> \quad ; \quad \alpha' M^2 = \frac{1}{2}\sum_{j=1}^{3} \nu_j - \nu_i \;\; ; \;\; i = 1, 2, 3$$

$$B^{\dagger 1}_{\frac{1}{2}-\nu_1} B^{\dagger 2}_{\frac{1}{2}-\nu_2} B^{\dagger 3}_{\frac{1}{2}-\nu_3}|0> \quad ; \quad \alpha' M^2 = \frac{2 - \nu_1 - \nu_2 - \nu_3}{2}$$

- One of these states becomes massless if one of the following identities is satisfied:

$$\nu_1 = \nu_2 + \nu_3 \;\; ; \;\; \nu_2 = \nu_1 + \nu_3 \;\; ; \;\; \nu_3 = \nu_1 + \nu_2 \;\; ; \;\; \nu_1 + \nu_2 + \nu_3 = 2$$

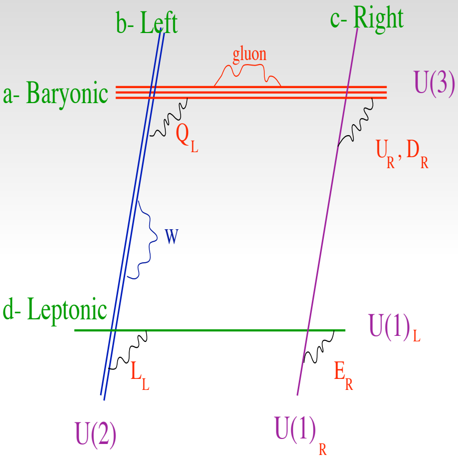
- In each of these cases four-dimensional $\mathcal{N} = 1$ supersymmetry is restaured!

- In general the ground state for the open strings, having their end-points respectively on stacks a and b, is degenerate.
- Its degeneracy is given by the number of Landau levels as in the case of a point-like particle:

$$I_{ab} = \prod_{i=1}^{3} \left\{ n_i^{(a)} n_i^{(b)} \int \left[\frac{q_a F_i^{(a)} - q_b F_i^{(b)}}{2\pi} \right] \right\} = \prod_{i=1}^{3} \left[m_i^{(a)} n_i^{(b)} - m_i^{(b)} n_i^{(a)} \right]$$

  that gives the number of families in the phenomenological applications.
- It corresponds to the number of intersections in the case of intersecting branes.

A simple phenomenological model



Marchesano, thesis, 2003

Four stacks of magnetized branes: a, b, c, d.

$SU(3)_a \times SU(2)_b \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$

- Having a chiral theory we must be careful to cancel all anomalies.
- Need to introduce an orientifold projection.
- For each stack of D branes we must introduce its image.
- Choose intersecting numbers or number of Landau levels as follows:

$$\begin{aligned} I_{ab} &= 1 \quad ; \quad I_{ab^*} = 2 \\ I_{ac} &= -3 \quad ; \quad I_{ac^*} = -3 \\ I_{bd} &= -3 \quad ; \quad I_{bd^*} = 0 \\ I_{cd} &= 3 \quad ; \quad I_{cd^*} = -3 \end{aligned} \qquad (2)$$

  with all others being zero.
- The previous numbers insure that there is no non-abelian anomaly $\Longrightarrow$ Tadpole cancellation conditions.
- The anomaly cancellation requires that the number of generations be equal to the number of colors!!

- But there are mixed and $U(1)$ anomalies that, however, are eliminated by a stringy "Green-Schwarz" mechanism.
- In addition to the non-abelian gauge symmetries $SU(3) \times SU(2)$ we have four additional $U(1)$ gauge symmetries instead of only one.
- It turns out that the gauge boson, corresponding to a combination of the $U(1)$'s,

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$$

  is massless $\Longrightarrow$ hypercharge $U(1)$.
- On the other hand the gauge bosons corresponding to the other $U(1)$'s get a mass by a generalized Stückelberg mechanism
- The gauge symmetry corresponding to the $U(1)$'s with a massive gauge bosons becomes a global symmetry.
- They correspond to

$$Q_a = 3B \quad ; \quad Q_d = -L \quad ; \quad Q_b \to PQ \;\; symm.$$

- These $U(1)$'s are exact global symmetries at each order of string perturbation theory.
- The baryon and lepton numbers are exactly preserved.
- Majorana neutrino masses are also not allowed at each order of perturbation theory.
- However, they can be broken by instantons.
- They may be pure stringy effects that disappear in the field theory limit ($\alpha' \to 0$).

Conclusions

- I have presented the problems that one encounters in connecting string theory to experiments.
- I have discussed intersecting and magnetized D branes and used them for constructing string extensions of the Standard Model.
- A lot more work should be done to clarify their properties.