Defects and D-branes as integrable boundary states

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Based on:

- M. de Leeuw, T. Gombor, C.K., G. Linardopoulos & B. Pozsgay ArXiv:1912.09338[hep-th], JHEP 01, (2020) 176
- C.K., D. Müller & K. Zarembo, arXiv:2106.08116[hep-th],JHEP 09 (2021) 004
- Earlier work
- Work in progress

Strings, Fields and Branes

NORDITA, Stockholm

November 23rd, 2021

AdS/CFT

$\mathcal{N} = 4$ SYM in 4D $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

$\mathcal{N} = 4$ SYM in 4D with 3D domain wall	$\longleftrightarrow$	IIB strings on $AdS_5 \times S^5$ with probe brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

Motivation

- Insights on the interplay between conformal symmetry, supersymmetry and integrability
- Novel examples of integrable boundary states, novel characterization at the discrete level
- Exact results for novel types of observables such as one-point functions and three-point functions -- via overlaps
- Novel microscopic duality relations for correlation functions
- Interesting connections to statistical physics: matrix product states and quantum quenches – via overlaps
- Positive tests of AdS/CFT dictionary for set-ups with supersymmetry partially or completely broken
- Possible cross-fertilization with the boundary conformal bootstrap program.

Plan of the talk

I. Quantum Quenches and overlaps

II. Overlaps and AdS/dCFT

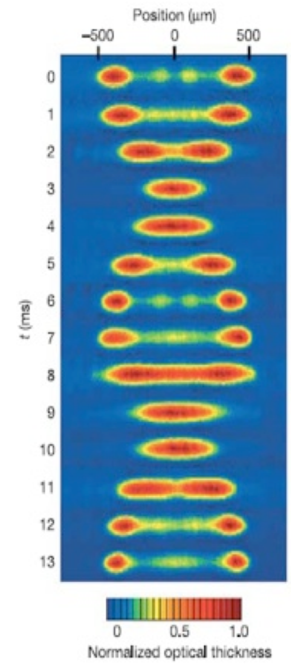
III. Integrable boundary states in AdS/dCFT

IV. Exact results for overlaps

V. Duality relations for overlaps

IV. Future directions

Overlaps and Quantum Quenches



Set out quantum system in initial state $|\Psi_0\rangle$
which is not an eigenstate of its Hamiltonian $\mathcal{H}_0$

Study time development of local observable

$$\begin{aligned}\langle \mathcal{O}(t)\rangle &= \langle \Psi_0|e^{i\mathcal{H}_0 t}\mathcal{O}e^{-i\mathcal{H}_0 t}|\Psi_0\rangle \\ &= \sum_{\mathbf{u},\mathbf{v}} \langle \Psi_0|\mathbf{u}\rangle\langle \mathbf{u}|\mathcal{O}|\mathbf{v}\rangle\langle \mathbf{v}|\Psi_0\rangle e^{-i(E_{\mathbf{v}}-E_{\mathbf{u}})t},\end{aligned}$$

$$\mathcal{H}_0|\mathbf{u}\rangle = E_{\mathbf{u}}|\mathbf{u}\rangle$$

Assume $\mathcal{H}_0$ Hamiltonian of an integrable system

When and how can $\langle \Psi_0|\mathbf{u}\rangle$ be calculated in closed form?

Of relevance for

- Time development after quantum quench
- Correlation functions in AdS/dCFT

AdS/CFT

$\mathcal{N} = 4$ SYM in 4D $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

Conformal operators $\longleftrightarrow$ String states

$\updownarrow$ $\updownarrow$

Eigenstates of integrable super spin chain: $|\mathbf{u}\rangle$

Minahan. Zarembo '02

AdS/dCFT

$\mathcal{N} = 4$ SYM in 4D with co-dimension one defect $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$ Karch-Randall probe brane

$|\Psi_0\rangle$ (integrable) boundary state describing defect / probe brane

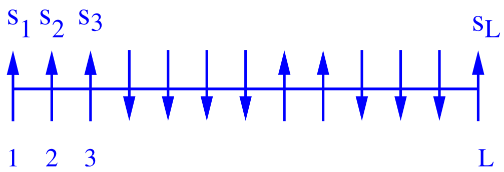
$\langle\Psi_0|\mathbf{u}\rangle$ is a one-point function

De Leeuw, C.K. Zarembo '15

Similar idea: $|\Psi_0\rangle \sim$ determinant operators/giant graviton

Jiang, Komatsu Vescovi '19

Integrable boundary states



$s_{L+m} = s_m$ $\quad |\Psi\rangle = |s_1 s_2 s_3 \ldots s_L\rangle$

Eigenstates: $H_0|\mathbf{u}\rangle = E_0|\mathbf{u}\rangle$

Integrable boundary state: $\langle\Psi_0|\mathbf{u}\rangle$ computable in closed form

Identified types of relevance for AdS/dCFT:

Matrix product states

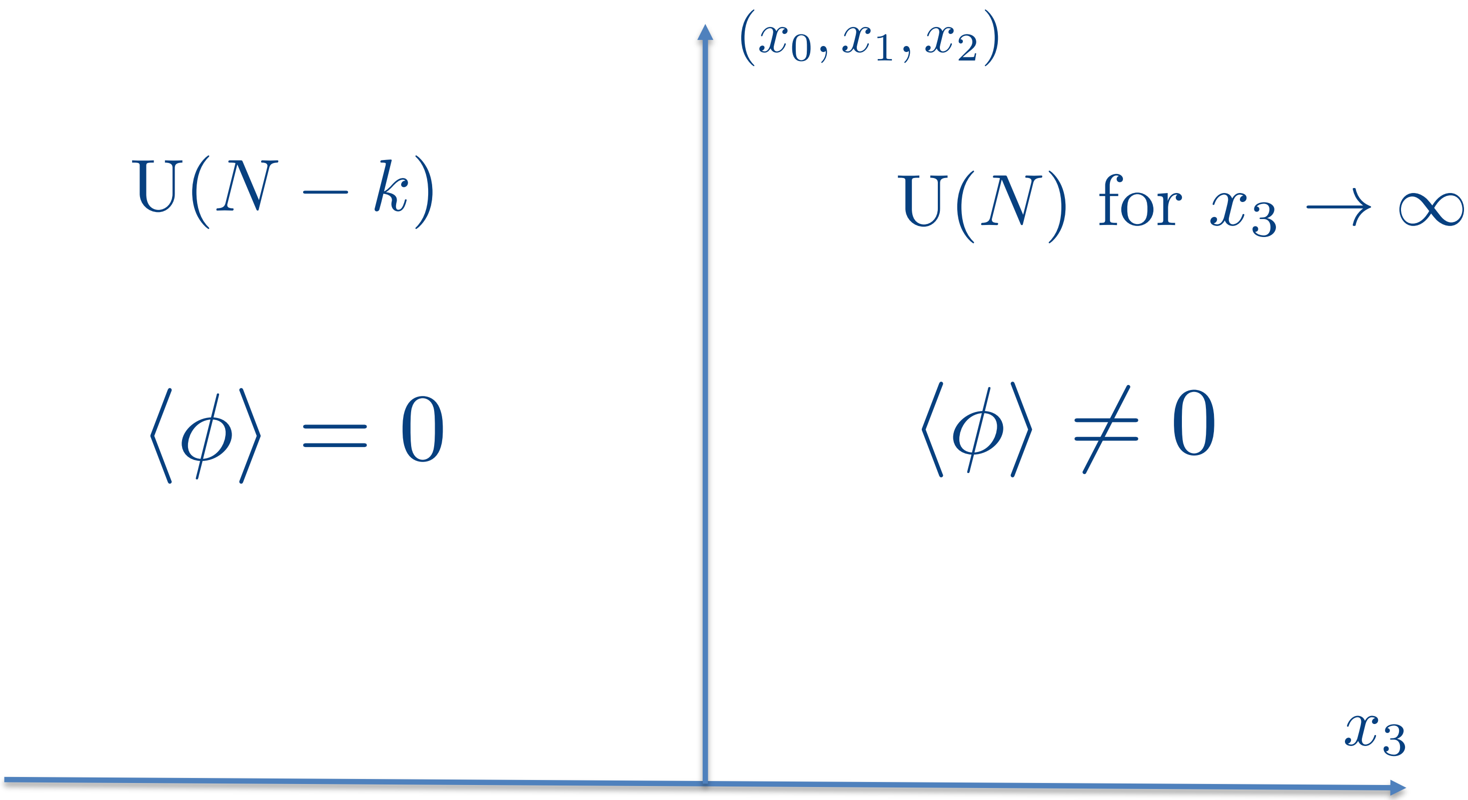
$$|B\rangle = |\text{MPS}\rangle = \sum_{\{s_i\}} \text{Tr}(t_{s_1} \ldots t_{s_L})|s_1 \ldots s_L\rangle$$

Valence Bond States

$$|\text{VBS}\rangle = |K\rangle^{\otimes \frac{L}{2}}, \qquad K = \sum_{s_1, s_2} K_{s_1, s_2}|s_1 s_2\rangle$$

The defect set-up of $|\mathrm{MPS}\rangle$

$$\mathcal{N}=4 \quad \mathrm{SYM}$$



Classical Fields (simplest case)

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\text{cl}} = 0, \qquad \Psi_A^{\text{cl}} = 0$

Classical e.o.m.:

(x$_3$ is distance to defect)

$$\frac{d^2\phi_i^{\text{cl}}}{dx_3^2} = \left[\phi_j^{\text{cl}}, \left[\phi_j^{\text{cl}}, \phi_i^{\text{cl}}\right]\right].$$

Solution:

$$\phi_i^{\text{cl}} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1,2,3$$

Constable, Myers & Tafjord '99

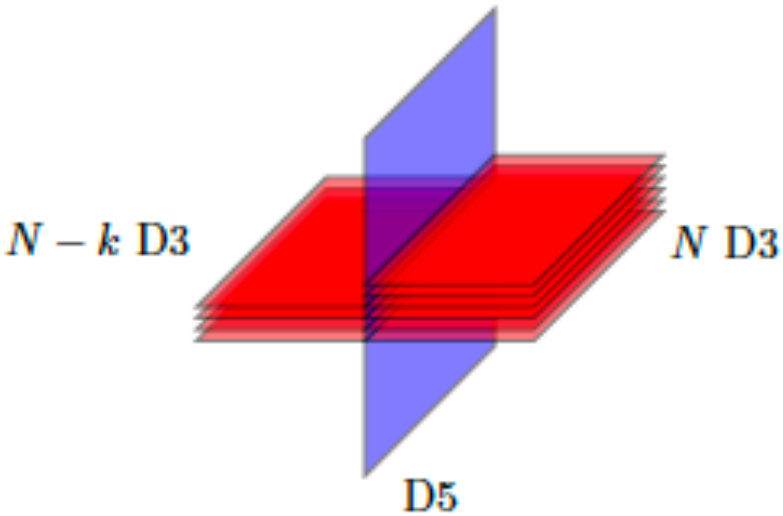
$$\phi_4^{\text{cl}} = \phi_5^{\text{cl}} = \phi_6^{\text{cl}} = 0$$

where t_i, i=1,2,3, constitute a k-dimensional irreducible repr. of SU(2). (Nahm eqns. also fulfilled.)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

AdS/dCFT --- The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	$\times$	$\times$	$\times$	$\times$						
D5	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$			

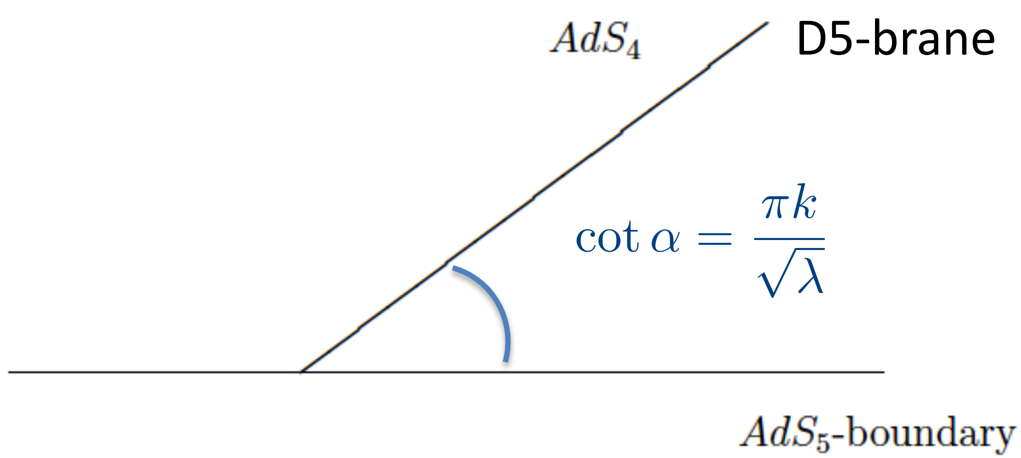


Karch & Randall '01,

Geometry of D5 brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

String embedding



D3-D5 probe brane system suggests a new double scaling limit Nagasaki & Yamaguchi '12,

$$\lambda \to \infty,\ k \to \infty,\ \frac{\lambda}{k^2}\ \text{finite} \qquad (N \to \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

Similar idea works for the two D3-D7 set-ups C.K., Semenoff & Young '12,

One-point functions and MPS

$$\langle \mathcal{O}_{\Delta}^{\text{bulk}}(x) \rangle = \frac{C}{|x_3|^{\Delta}}$$

Cardy ´84

McAvity & Osborn '95

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_{\Delta}(x) \rangle = \left(\mathrm{Tr}(\phi_{i_1} \dots \phi_{i_\Delta}) + \dots\right)\Big|_{\phi_i \to \phi_i^{\text{cl}} = \frac{t_i}{x_3}}$$

$\mathcal{O}_{\Delta}(x) \sim$ eigenstate of integrable $SO(6)$ spin chain

Minahan & Zarembo '02

$$\mathrm{Tr}(\phi_{i_1}\phi_{i_2} \dots \phi_{i_L}) \sim |s_{i_1} s_{i_2} \dots s_{i_L}\rangle$$

Matrix Product State associated with the defect:

deLeeuw, C.K. & Zarembo '15,

$$|\mathrm{MPS_k}\rangle = \sum_{\vec{i}} \mathrm{tr}[t_{i_i} \dots t_{i_L}] |\phi_{i_1} \dots \phi_{i_L}\rangle,$$

Object to calculate:

Bethe eigenstate

$$C_k(\mathbf{u}) = \frac{\langle \mathrm{MPS}_k | \mathbf{u} \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{\frac{1}{2}}}$$

One-point functions and VBS

For $k = 1$: No vevs

Gaiotto & Witten, '08

Quantum fields
$$A_\mu, \Phi_i, \Psi_\alpha = \begin{array}{c} \begin{array}{cc} 1 & N-1 \end{array} \\ \left[\begin{array}{c|ccc} x & y & y & y \\ \hline y & z & z & z \\ y & z & z & z \\ y & z & z & z \end{array}\right] \end{array}$$

Boundary conditions (supersymmetric)

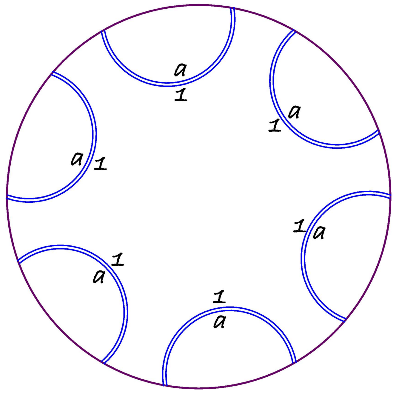
	$\Phi_{4,5,6}$	$\Phi_{1,2,3}$
x, y	Dirichlet	Neumann
z	no BCs	no BCs

Propagators
$$D_\kappa(x,y) = \frac{1}{4\pi^2}\left(\frac{1}{|x-y|^2} + \frac{\kappa}{|\bar{x}-y|^2}\right), \qquad \kappa = \begin{cases} 1 & \text{Neumann} \\ -1 & \text{Dirichlet} \\ 0 & \text{no BCs.} \end{cases}$$
$$\bar{x} = (x_0, x_1, x_2, -x_3)$$

For complex scalars ($X = \Phi_1 + i\Phi_4$, etc.)

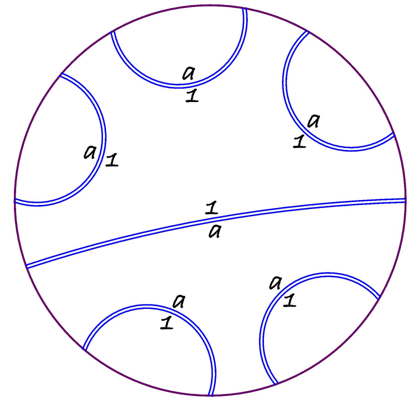
$$\langle X^{1a}(x) X^{b1}(y)\rangle = \frac{g_{\rm YM}^2 \delta^{ab}}{2}\Big(D_1(x,y) - D_{-1}(x,y)\Big) = \frac{g_{\rm YM}^2 \delta^{ab}}{4\pi^2|\bar{x}-y|^2}$$

Feynman diagrams



Leading for large-N

$\sim (g_{\mathrm{YM}}^2 N)^6$



Sub-leading for large-N

$\sim \dfrac{1}{N^2}(g_{\mathrm{YM}}^2 N)^6$

C.K., Müller, Zarembo '20

Object to calculate $\quad C_{k=1} = \dfrac{\langle \mathrm{VBS} | \mathbf{u} \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{1/2}}$

$\langle \mathrm{VBS} | = (\langle XX | + \langle YY |)^{\otimes L/2}, \qquad SU(2)$ sector

$\langle \mathrm{VBS} | = (\langle XX | + \langle YY | + \langle ZZ | + \langle \Psi_1 \Psi_2 | - \langle \Psi_2 \Psi_1 |)^{\otimes L/2}, \qquad SU(2|3)$ sector

$|\mathrm{VBS}\rangle$'s also of importance as initial steps in proofs of overlap formulas for $|\mathrm{MPS}\rangle$'s

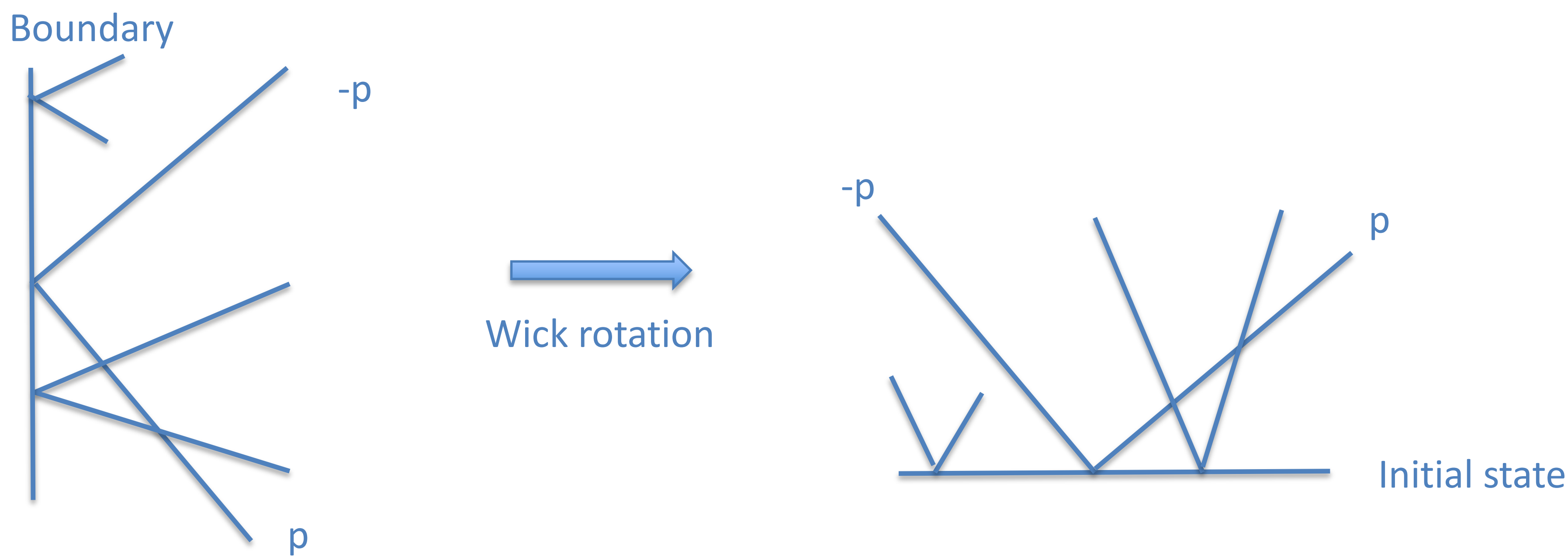
de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

Gombor & Bajnok '20

Integrable boundaries in integrable QFTs

Ghoshal, Zamolodchikov '93

- No particle production or annihilation
- Pure reflection, possibly change of internal quantum numbers
- Yang-Baxter relations fulfilled (order of reflection does not matter)



Pure reflection
+BYB for reflection matrix

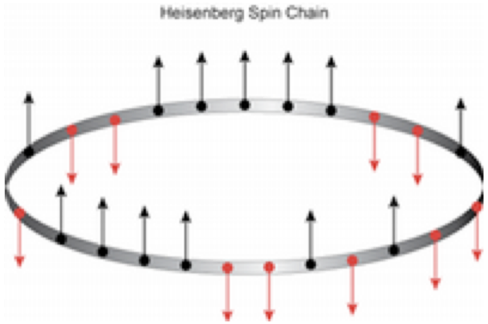
Entangled (p,-p) pairs
+BYB for initial state

Integrable spin chain boundary states

Piroli, Pozsgay Vernier '17

Heisenberg spin chain encodes conformal single trace operators built from two complex fields X (vacuum, $\uparrow$) and Y (excitation, $\downarrow$)

$$H = \sum_{n=1}^{L} (1 - P_{n,n+1})$$



Eigenstates: $|\{u_i\}_{i=1}^{K}\rangle \equiv |\mathbf{u}\rangle$, K excitations

$$u_i = \frac{1}{2}\cot\left(\frac{p_i}{2}\right)$$

L conserved charges, $\hat{Q}_n$, with eigenvalues Q_n

$$Q_n(\{p_i\}) = (-1)^n Q_n(\{-p_i\})$$

Integrable initial state

$$Q_{2m+1}|\Psi_0\rangle = 0$$

(BYB observed to be fulfilled for all cases considered)

Integrability of $|\mathrm{MPS}\rangle$

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$\mathrm{AdS}_4 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^2 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^4$
Symmetry of vev's	$SU(2)$	$SU(2) \times SU(2)$	$SO(5)$
Dim. of rep./ Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
$\|\mathrm{MPS}\rangle$	Integrable	Non-integrable	Integrable
Overlaps	Exact formula derived	—	Exact formula derived

Overlap Formulas

Selection rule

$\langle \Psi_0 | \mathbf{u} \rangle \neq \mathbf{0} \iff \{\mathbf{u_j}\} = \{-\mathbf{u_i}, \mathbf{u_i}\}$ Parity invariance

Ingredients: de Leeuw, C.K. & Zarembo '15 de Leeuw, C.K. & Mori '16 de Leeuw, C.K. & Linardopoulos '18 de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

For $|\mathrm{MPS}_k\rangle$:

- Superdeterminant of Gaudin matrix: $\frac{\det(G_+)}{\det(G_-)}$
- Ratios of Baxter polynomiums (reduced): $Q(u) = \prod_i (u^2 - u_i^2)$
- "Transfer matrices": Sums of ratios of Baxter polynomials: $\sum_{a=-\frac{k}{2}}^{a=\frac{k}{2}}$

For $|\mathrm{VBS}\rangle$:

- No sums involved Poszgay '18 Gombor '21

Gaudin matrix

Eigenstates: $|\mathbf{u}\rangle = |\{u_i\}_{i=1}^K\rangle$

$$1 = \left(\frac{u_k - \frac{i}{2}}{u_k + \frac{i}{2}}\right)^L \prod_{j\neq k}^{K} \frac{u_k - u_j + \frac{i}{2}}{u_k - u_j - \frac{i}{2}} = e^{i\chi_k}, \qquad k = 1, \ldots, K$$

$$\langle \mathbf{u}|\mathbf{u}\rangle \propto \det G, \qquad G_{kj} = \frac{\partial \chi_k}{\partial u_j}$$

For parity invariant states: block structure

$$\det G = \begin{vmatrix} A & B \\ B & A \end{vmatrix} = \begin{vmatrix} A+B & B \\ B+A & A \end{vmatrix} = \begin{vmatrix} A+B & B \\ 0 & A-B \end{vmatrix} = \det(A+B)\cdot\det(A-B)$$

$$= \det G_+ \cdot \det G_-$$

Quantity entering overlap formulas

$$\mathrm{SDet}\, G = \frac{\det G_+}{\det G_-} \equiv \mathbb{D}$$

One-point functions of D3-D5 set-up

Operators built from two scalar fields, X and Y

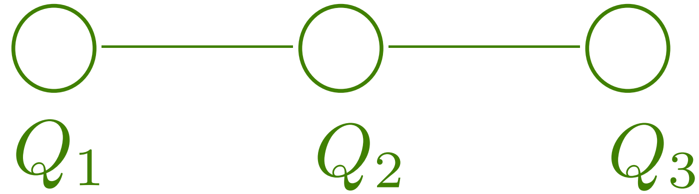
$$C_k(\mathbf{u}) = \frac{\langle \mathrm{MPS}_k | \mathbf{u} \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{\frac{1}{2}}} = \mathbb{T}_k \cdot Q\left(\frac{ik}{2}\right) \sqrt{Q(0)\, Q\left(\frac{i}{2}\right) \frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_k = \sum_{a=-\frac{k-1}{2}}^{\frac{k-1}{2}} \frac{a^L}{Q\left(\frac{2a+1}{2} i\right) Q\left(\frac{2a-1}{2} i\right)}, \quad k \geq 2$$

Buhl-Mortensen, de Leeuw, C.K. & Zarembo '16

The full scalar sector of the D3-D5 set-up

$SO(6)$ spin chain



$$C_k\left(\mathbf{u}\right)=\frac{\langle \mathrm{MPS}_k\,|\mathbf{u}\rangle}{\langle \mathbf{u}\,|\mathbf{u}\,\rangle^{\frac{1}{2}}}\;=\;\mathbb{T}_k\cdot Q_2\left(\frac{ik}{2}\right)\sqrt{\frac{Q_2(0)Q_2(\frac{i}{2})}{Q_1(0)Q_1(\frac{i}{2})Q_3(0)Q_3(\frac{i}{2})}}\cdot\sqrt{\frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_k=\sum_{a=-\frac{k-1}{2}}^{\frac{k-1}{2}} a^L\,\frac{Q_1(ia)Q_3(ia)}{Q_2(\frac{2a+1}{2}i)Q_2(\frac{2a-1}{2}i)},\quad k\geq 2$$

This is the complete answer at tree level

Higher loops & other sectors: Start with $|\mathrm{VBS}\rangle$

Poszgay '18

$$SU(2): \quad |\mathrm{VBS}\rangle = (|XX\rangle + |YY\rangle)^{\otimes L/2}, \quad C = \frac{Q(0)}{Q(\frac{i}{2})} S \det G$$

$$SO(6): \quad |\mathrm{VBS}\rangle = (|XX\rangle + |YY\rangle + |ZZ\rangle + |\bar{X}\bar{X}\rangle + |\bar{Y}\bar{Y}\rangle + |\bar{Z}\bar{Z}\rangle)^{\otimes L/2} \; ,$$

$$C = \frac{Q_1(0) Q_2(0) Q_3(0)}{Q_1\left(\frac{i}{2}\right) Q_2\left(\frac{i}{2}\right) Q_3\left(\frac{i}{2}\right)} S \det G$$

de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

Gombor '21

$$SU(2|1): \quad |\mathrm{VBS}\rangle = (|XX\rangle + |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)^{\otimes L/2}$$

$$C = \frac{Q_1(0)}{Q_1\left(\frac{i}{2}\right) Q_2(0)} S \det G$$

C.K., Müller, Zarembo '20

All loops & all sectors by bootstrap

Making use of:

- Symmetries: $PSU(2,2|4)$
- Knowledge of the S-matrix from the spectral problem
- Consistency requirements (BYB, unitarity, crossing)

From the string theory perspective

Komatsu & Wang, '20

Proof at the classical level: Linardopoulos, Zarembo '21

- Assume integrability (factorization into two-particle overlaps)
- Input from one-loop perturbative calculation
- Obtain the result in the $SU(2)$ sub-sector

Buhl-Mortensen, de Leeuw, Ipsen, Wilhelm '17

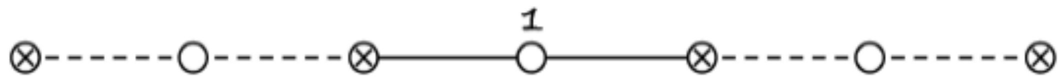
From the gauge theory (spin chain) perspective

Gombor & Bajnok '20 I & II

- Assume factorized from of overlaps for $|\mathrm{VBS}\rangle$
- Obtain the result for the entire theory

The leading order result for $|\text{VBS}\rangle$

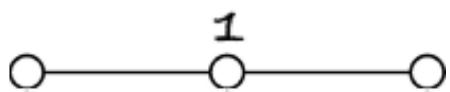
$PSU(2,2|4)$:



Bajnok & Gombor ‘20

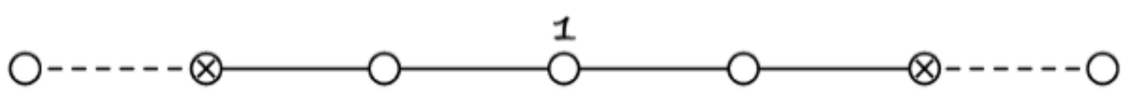
$$C = \frac{Q_1(0)Q_3(0)Q_4(0)Q_5(0)Q_7(0)}{Q_2(0)Q_2(\frac{i}{2})Q_4(\frac{i}{2})Q_6(0)Q_6(\frac{i}{2})}\, S \det G$$

- How to compare to previous results ?

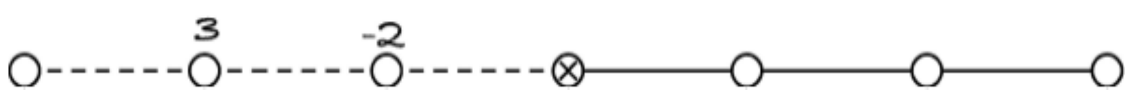


- What does the expression look like for other gradings ?
  - The Beauty

Beisert & Staudacher ‘04



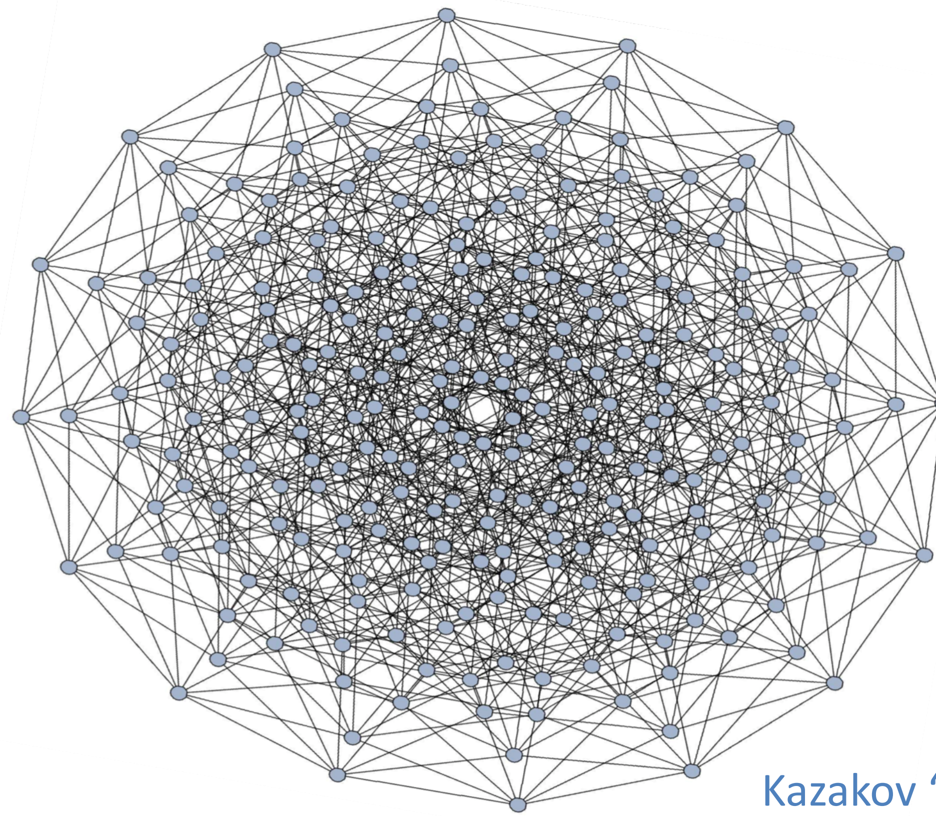
  - The Beast



QQ-system

Many equivalent ways of writing the Bethe equations

For $\mathcal{N} = 4$ SYM, # different choices of Q-functions $= 2^8$



Kazakov '18

Connected via dualities

- Fermionic (Change of Dynkin diagram)
- Bosonic

$|\mathrm{VBS}\rangle$ of relevance for AdS/dCFT singled out by transforming covariantly under fermionic duality

Integrable Super Spin Chains (of type $SU(M|N)$)

Cartan matrix M_{ab}, Dynkin labels q_a

$a, b = 1, \ldots, M + N - 1 = n =$ # nodes in Dynkin diagram

Q-functions: $Q_a(u) = \prod_{j=1}^{K_a} (u - u_{a,j})$

Bethe equations:

$$(-1)^{M_{aa}+1} = \left(\frac{u_{a,j} - \frac{iq_a}{2}}{u_{a,j} + \frac{iq_a}{2}} \right)^L \prod_{b,k} \frac{u_{a,j} - u_{b,k} + \frac{iM_{ab}}{2}}{u_{a,j} - u_{b,k} - \frac{iM_{ab}}{2}} \equiv \mathrm{e}^{\,i\chi_{a,j}}$$

Can also be expressed in terms of Q-functions

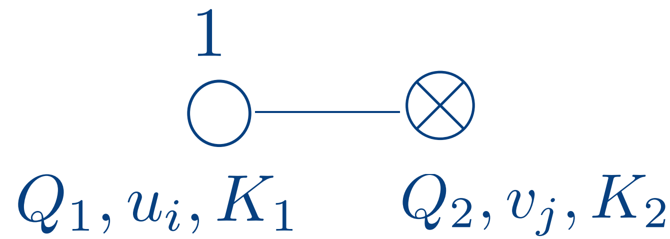
Gaudin matrix: $G_{aj,bk} = \frac{\partial \chi_{aj}}{\partial u_{bk}}$, of size $\sum_a K_a \times \sum_a K_a$

Example: $SU(2|1)$ super spin chain

Encodes conformal single trace operators built from fields
X (bosonic), Ψ_1, Ψ_2 (fermionic) in $\mathcal{N} = 4$ SYM

Cartan matrix   Dynkin label

$$M = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, \qquad q = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$H = \sum_{n=1}^{L} (1 - \Pi_{n,n+1})$$

graded
permutation

Baxter polynomials

$$Q_1(u) = \prod_{i=1}^{K_1} (u - u_i), \qquad Q_2(u) = \prod_{j=1}^{K_2} (v - v_j)$$ (plus two trivial ones)

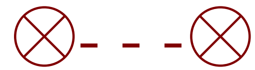
Vacuum: $|\Psi_1 \Psi_1 \ldots \rangle$,   Excitations at level 1 and 2: Ψ_2, X

Fermionic Duality: Ex: $SU(2|1)$

Beisert, Kazakov, ,
Sakai, Zarembo ‘05

Q_1, u_i, K_1 Q_2, v_j, K_2

$$M = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, \; q = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



Q_1, u_i, K_1 $\widetilde{Q}_2, \tilde{v}_j, \widetilde{K}_2$

$$\widetilde{M} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \; \tilde{q} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Change of variables (from v_j to $\tilde{v}_j$)

$$\begin{array}{c} K_2 \text{ roots } v_j \\ \downarrow \end{array}$$

$$Q_1^-(v) - Q_1^+(v) = Q_2(v) \cdot \widetilde{Q}_2(v)$$

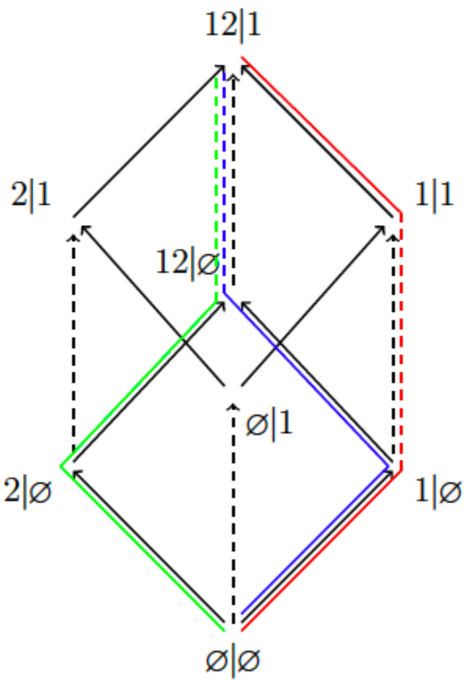
$$\widetilde{K}_2 = K_1 - K_2 - 1 \text{ roots } \tilde{v}_j$$

$$1 = \frac{Q_1^-(v_k)}{Q_1^+(v_k)} \longrightarrow \frac{Q_1^+(\tilde{v}_k)}{Q_1^-(\tilde{v}_k)} = 1$$

$$-1 = \frac{Q_1^{++}(u_k)}{Q_1^{--}(u_k)} \cdot \frac{Q_2^-(u_k)}{Q_2^+(u_k)} \left(\frac{Q_\theta^-(u_k)}{Q_\theta^+(u_k)} \right)^L \longrightarrow \frac{\widetilde{Q}_2^+(u_k)}{\widetilde{Q}_2^-(u_k)} \left(\frac{Q_\theta^-(u_k)}{Q_\theta^+(u_k)} \right)^L = 1$$

A Web of Dualities: Ex: $SU(1|2)$

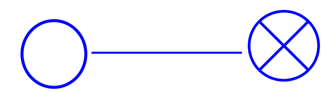
Tsuboi '98



2^3 Q-functions, 2 fixed

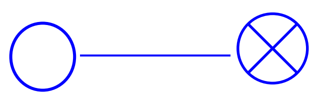
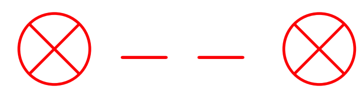
$$Q_{\emptyset|\emptyset} = u^L, \;\; Q_{12|1} = 1$$

$6 = 3 \times 2$ versions of the BE's ($\sim$ paths)

Standard choice: Blue path 

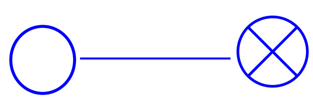
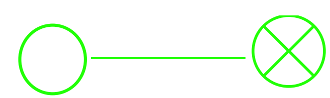
Fermionic Duality

(Change of variables
in the Bethe equations)

 $\longrightarrow$ 

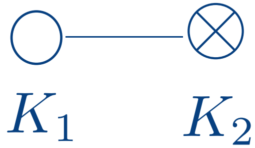
$$Q_{12|\emptyset} Q_{1|1} = Q^-_{1|\emptyset} - Q^+_{1|\emptyset}$$

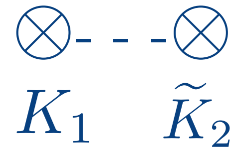
Bosonic duality

 $\longrightarrow$ 

$$Q^+_{1|\emptyset} Q^-_{2|\emptyset} - Q^-_{1|\emptyset} Q^+_{2|\emptyset} = Q_{\emptyset|\emptyset} Q_{12|\emptyset}$$

Transformation formula: Ex: $SU(2|1)$





$K_1,\ K_2$ even $\implies \widetilde{K_2} = K_1 - K_2 - 1$ odd, i.e. $\tilde{v}$'s contain a single zero

Det $\widetilde{G}$ still factorizes

$$Q_1^+(u) - Q_1^-(u) = iK_1\, u\, Q_2(u)\, \widetilde{Q}_2(u),$$ with reduced Baxter polynomials

$$\boxed{\widetilde{\mathbb{D}} = K_1 \frac{\widetilde{Q}_2(0) Q_2(0)}{Q_1(\frac{i}{2})}\, \mathbb{D}}$$

Found numerically

C.K., Müller, Zarembo '20

Analytical proof in progress

Notice:

- Holds semi-on-shell (the $\{u_i, -u_i\}$'s can be chosen at random)
- Covariance of overlap formula which involves $Q_2(0)\mathbb{D}$
- Factor K_1 signals that a hws is mapped to a descendent

de Leeuw, Ipsen, C.K., Vardinghus, Wilhelm 17

Fermionic dualities in general

- Allow one to move between any two Dynkin diagrams of a super Lie algebra (of type $GL(N|M)$)

- Involve a fermionic node and its neighbours only

- Changes the nature of neighbouring nodes $\otimes \longleftrightarrow \bigcirc$ and the connections $\overline{} \longleftrightarrow$ - - -

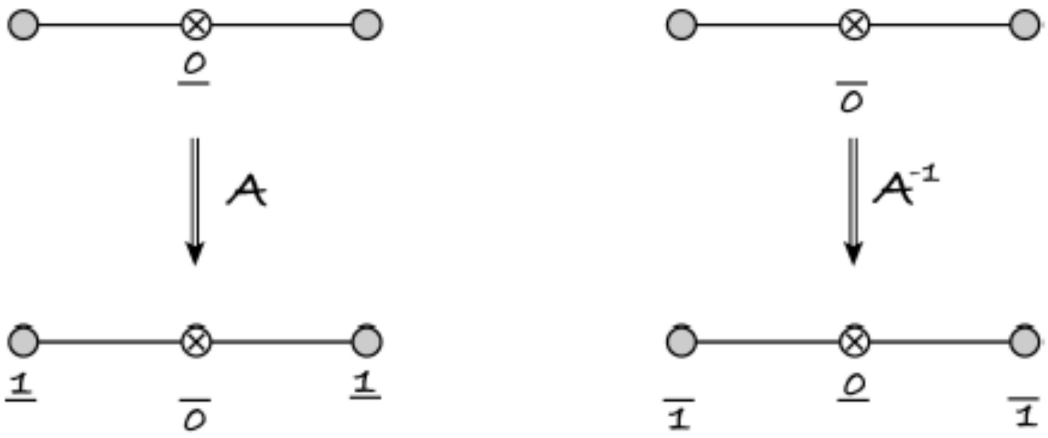
- Dualized node non-momentum carrying $\implies$ Dynkin labels unchanged

- Dualized node momentum carrying $\implies$ Dynkin labels change

$$\begin{bmatrix} 0 \\ V \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} V \pm 1 \\ -V \\ V \mp 1 \end{bmatrix} \quad \text{for}$$

Most general case

$$\mathbb{D} \propto \frac{Q_{a-1}\left(\frac{i}{2}\right) Q_{a+1}\left(\frac{i}{2}\right)}{\widetilde{Q}_a(0) Q_a(0)} \widetilde{\mathbb{D}}$$

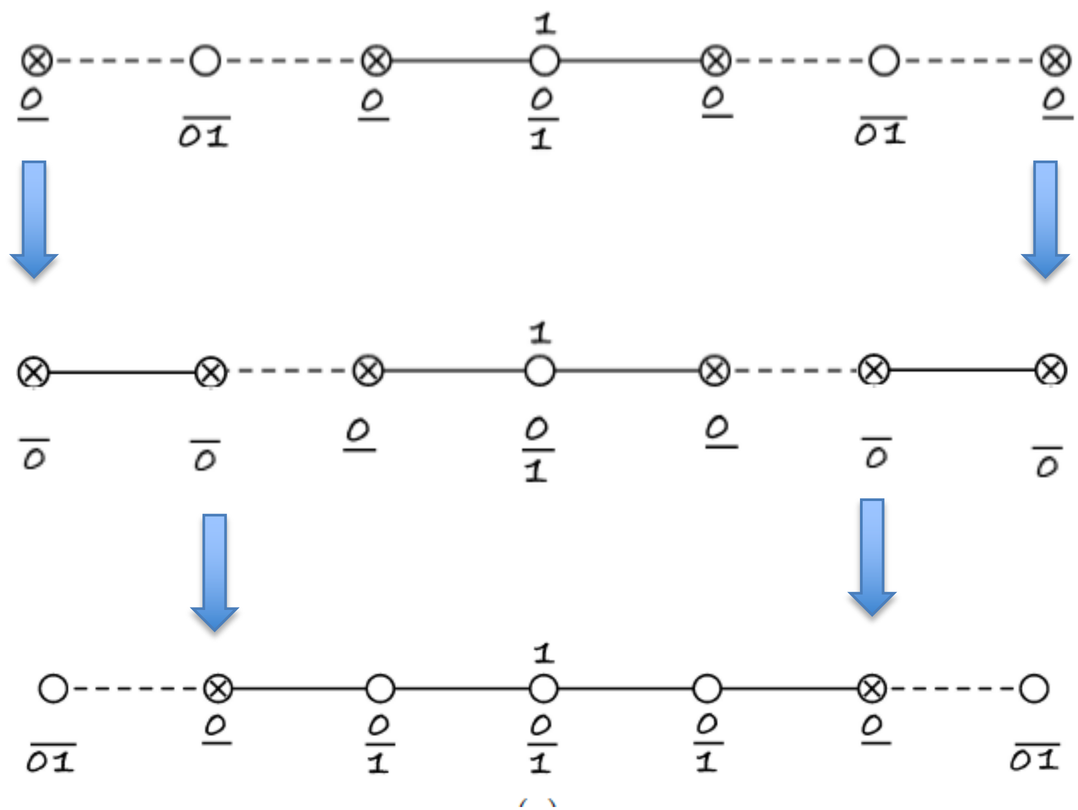


Covariance of overlap formulas very constraining

Dualizing the overlap formula

$PSU(2,2|4)$ overlap formula, alternating grading

Gombor &
Bajnok '20



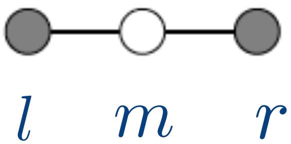
Agrees with field theory result in $SO(6)$ sector

C.K., Müller,
Zarembo '20

Covariance requirement fixes the overlap formula from $SO(6)$ result

Bosonic dualities in general

- Involve a bosonic node and its neighbours only



- Do not change the Dynkin diagram or the Dynkin labels

- Only involves Q-functions of the dualized node

$$\widetilde{\mathbb{D}} \sim \frac{Q_m(0)\widetilde{Q}_m(i/2)}{Q_m\left(\frac{i}{2}\right)\widetilde{Q}_m(0)}\,\mathbb{D}$$

- Overlaps in the scalar $SO(6)$ sector invariant (up to pre-factor)

Integrability of $|\mathrm{MPS}\rangle$

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$\mathrm{AdS}_4 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^2 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^4$
Symmetry of vev's	$SU(2)$	$SU(2) \times SU(2)$	$SO(5)$
Dim. of rep./ Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
$\|\mathrm{MPS}\rangle$	Integrable asymptotically	Non-integrable	Integrable at tree level
Overlaps	Exact formula derived	—	Exact formula derived

Solution SO(5) symmetric D3-D7 brane case. Tree level

$$\frac{\langle \mathbf{u} | \mathrm{MPS}_n \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{1/2}} = \Lambda_n \cdot \sqrt{\frac{Q_0(0)\, Q_0\left(\frac{1}{2}\right)}{\bar{Q}_+(0)\, \bar{Q}_+\left(\frac{1}{2}\right) \bar{Q}_-(0)\, \bar{Q}_-\left(\frac{1}{2}\right)}} \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\Lambda_n = 2^L \sum_{q=-\frac{n}{2}}^{\frac{n}{2}} q^L \left[\sum_{p=-\frac{n}{2}}^{q} \frac{Q_0(p-\frac{1}{2})}{Q_0(q-\frac{1}{2})} \frac{Q_-(q) Q_-(\frac{n}{2}+1)}{Q_-(p) Q_-(p-1)} \right] \left[\sum_{r=q}^{\frac{n}{2}} \frac{Q_0(r+\frac{1}{2})}{Q_0(q+\frac{1}{2})} \frac{Q_+(q) Q_+(\frac{n}{2}+1)}{Q_+(r) Q_+(r+1)} \right].$$

de Leeuw, C.K & Linardopoulos,'18. de Leeuw, Gombor C.K & Linardopoulos, Pozsgay '19.

Argument against higher loop integrability in Gombor & Bajnok, '20.

Perturbative program set up in: Gimenez-Grau, C.K, Volk & Wilhelm '19

Match to next to leading order in d.s.l. for *chiral primary* of length L

D3-D7 set-up with SO(5) symmetry (non-supersymmetric and integrable)

$$\frac{\langle \mathrm{Tr}\, Z^L \rangle}{\langle \mathrm{Tr}\, Z^L \rangle|_{\text{tree}}} = 1 + \frac{\lambda}{4\pi^2 n^2} \frac{L(L+3)}{(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 n^2} \right)^2 \right)$$

Gimenez-Grau, C.K, Volk & Wilhelm, '19

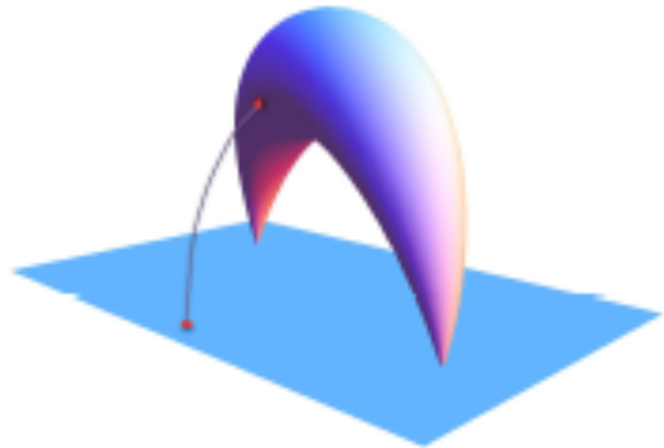
D3-D7 set-up with SO(3)× SO(3) symmetry (non-susy and non-integrable)

$$\begin{aligned}\frac{\langle \mathrm{Tr}\, Z^L \rangle}{\langle \mathrm{Tr}\, Z^L \rangle|_{\text{tree}}} = 1 + \frac{\lambda}{4\pi^2 (k_1^2 + k_2^2)} & \frac{1}{(L-1)\sin(L+2)\phi\, [k_1^2 + k_2^2]^3} \Big[\\ & + 4L k_1 k_2 \left[(k_1)^4 + (k_2)^4 + (k_1 k_2)^2 (L+1) \right] \cos L\phi \\ & + \left[(k_2)^2 - (k_1)^2 \right] \left[4(k_1 k_2)^2 (L^2 + L - 1) + ((k_1)^4 + (k_2)^4)(L^2 + 3L - 2) \right] \sin L\phi \Big] \\ & + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 (k_1^2 + k_2^2)} \right)^2 \right), \qquad \phi = \arctan\left(\frac{k_1}{k_2} \right)\end{aligned}$$

Gimenez-Grau, C.K, Volk & Wilhelm, '18

Other integrable boundary states in AdS/CFT

Giant gravitons



Overlaps: $\sim$ 3-point fcts

Bissi, C.K,
Young Zoubos '11

Two determinant operators and one single trace

Jiang, Komatsu
Vescovi, '19

$\mathcal{N} = 4$ SYM: Full non-perturbative expression by TBA (in principle)

ABJM: Leading order result

Yang, Jiang,
Komatsu, Wu '21

Hirano, C.K,
Young '12

Future directions

- Other integrable defect set-ups (ABJM, Coulomb branch, co-dimension-2 defects….)
- Constraining overlap formulas by fermionic duality
- Classification of integrable boundary states in N=4 SYM (VBS, MPS,…)
- Proof of duality transformation formulas
- Proof of factorized overlap formulas for super spin chains
- Higher loop integrability for D3-D7?
- Derive the TBA for overlaps (Finite size effects).

Thank you